Georgia Tech Research Institute
Problem. Solved.

Radar Frequencies and Waveforms

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Based on material created by
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Waveforms Extract "Target" Information

- A radar system probes its environment with specially designed waveforms to identify and characterize targets of interest.
- Detection
  - For a given range, angle, and/or Doppler, decide if a target is or is not present.
  - Example: Moving target indication (MTI) radar
- Estimation
  - For a given range, angle, and/or Doppler, estimate
  - Example: Synthetic aperture radar (SAR) imaging

Overview

- Radar frequencies
- Radar waveform taxonomy
- CW: Measuring Doppler
- Single Pulse: Measuring range
- Ambiguity function
- Pulse compression waveforms (FM and PM)
- Coherent pulse trains

Radar Frequencies

Radar Bands

Large ↔ Small

Long-Range ↔ Short-Range

Poor Angular Resolution ↔ Good Angular Resolution

Radar Band	Frequency
HF	3 – 30 MHz
VHF	30 – 300 MHz
UHF	300 – 1000 MHz
L	1 – 2 GHz
S	2 -4 GHz
C	4 – 8 GHz
X	8 – 12 GHz
Ku	12 – 18 GHz
Ka	27 – 40 GHz
mm (V & W)	40 – 300 GHz

Long-Range Air Surveillance

FOPEN

SAR / GMTI

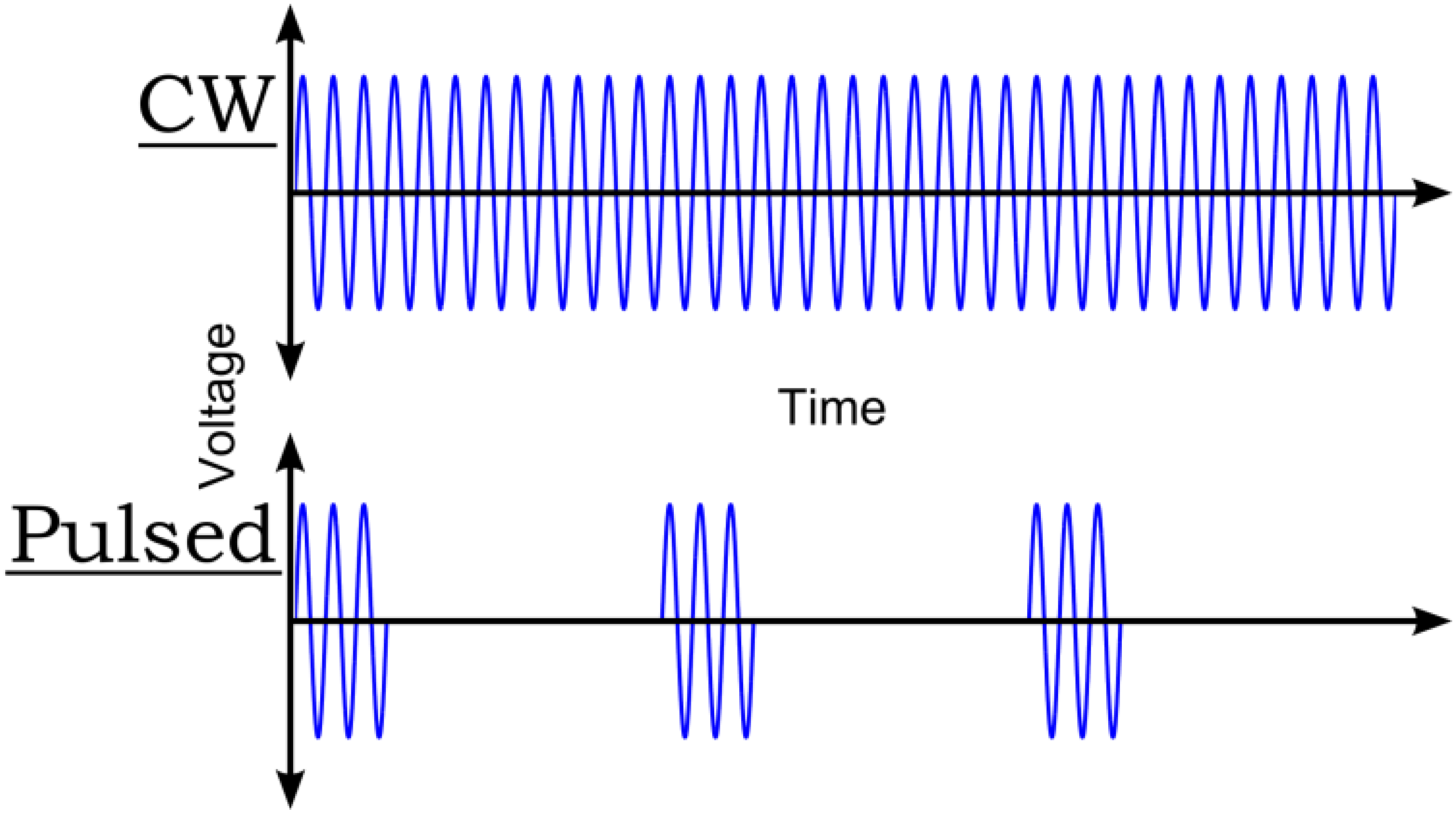
Air-to-Air

Fire Control / Munitions

Radar Waveform Taxonomy

Continuous Wave (CW) vs. Pulsed

- CW: Simultaneously transmit and receive
- Pulsed: Interleave transmit and receive periods

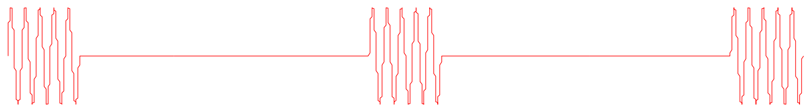


Continuous Wave (CW) vs. Pulsed

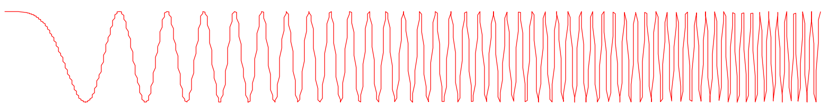
Continuous Wave	Pulsed
Requires separate transmit and receive antennas.	Same antenna is used for transmit and receive.
Isolation requirements limit transmit power.	Time-multiplexing relaxes isolation requirements to allow high power.
Radar has no blind ranges.	Radar has blind ranges due to “eclipsing” during transmit events.

Modulated vs. Unmodulated

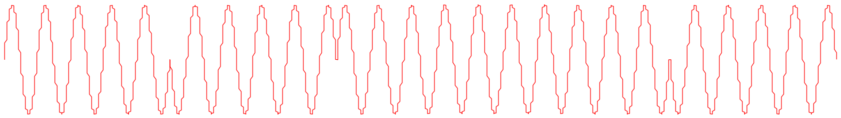
- Modulation may be applied to each pulse (<u>intrapulse modulation</u>) or from pulse-to-pulse (<u>interpulse modulation</u>)
- Classes of Modulation
  - Amplitude
  - Phase
  - Frequency
  - Polarization



“ON-OFF” Amplitude Modulation



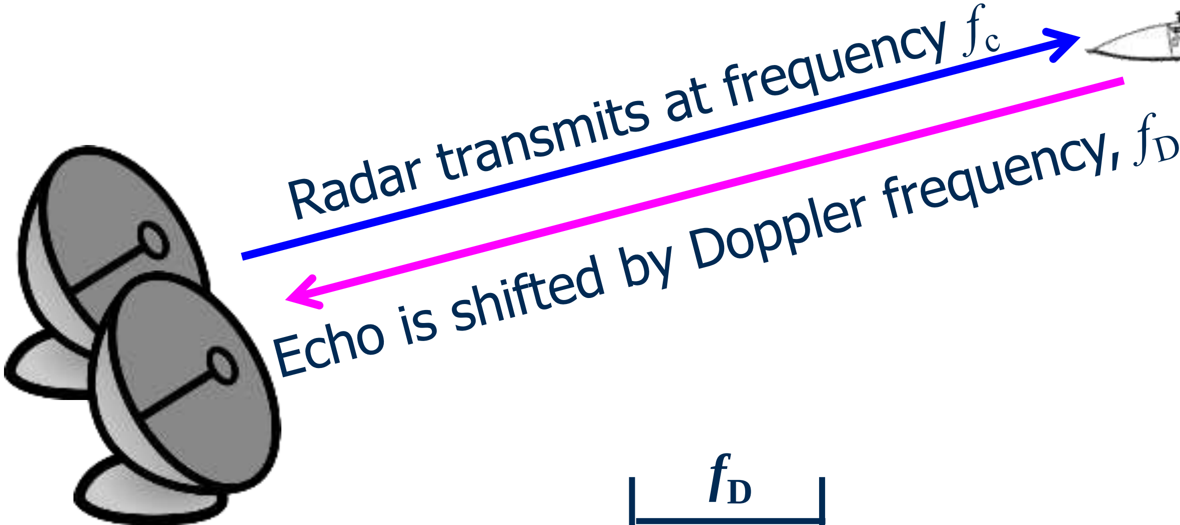
Frequency Modulation

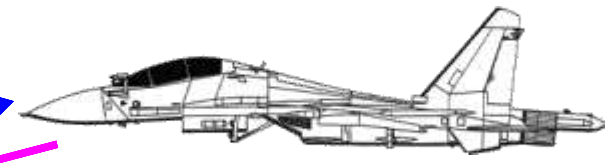


Phase Modulation

Measuring Doppler with CW Waveform

Measuring Doppler with a CW Radar



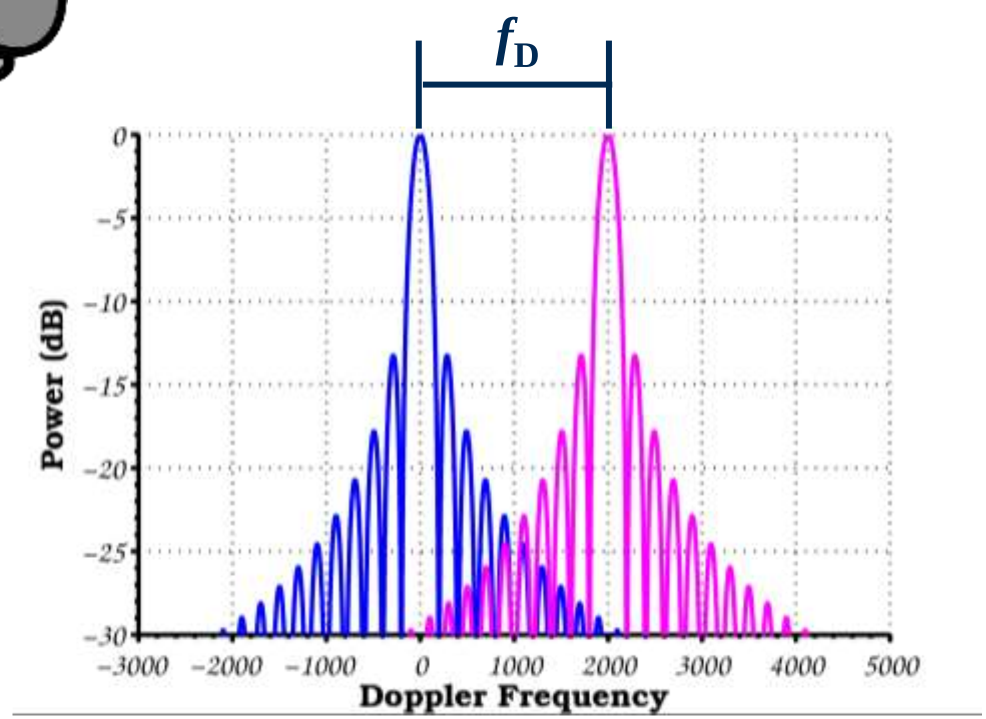


Doppler Shift

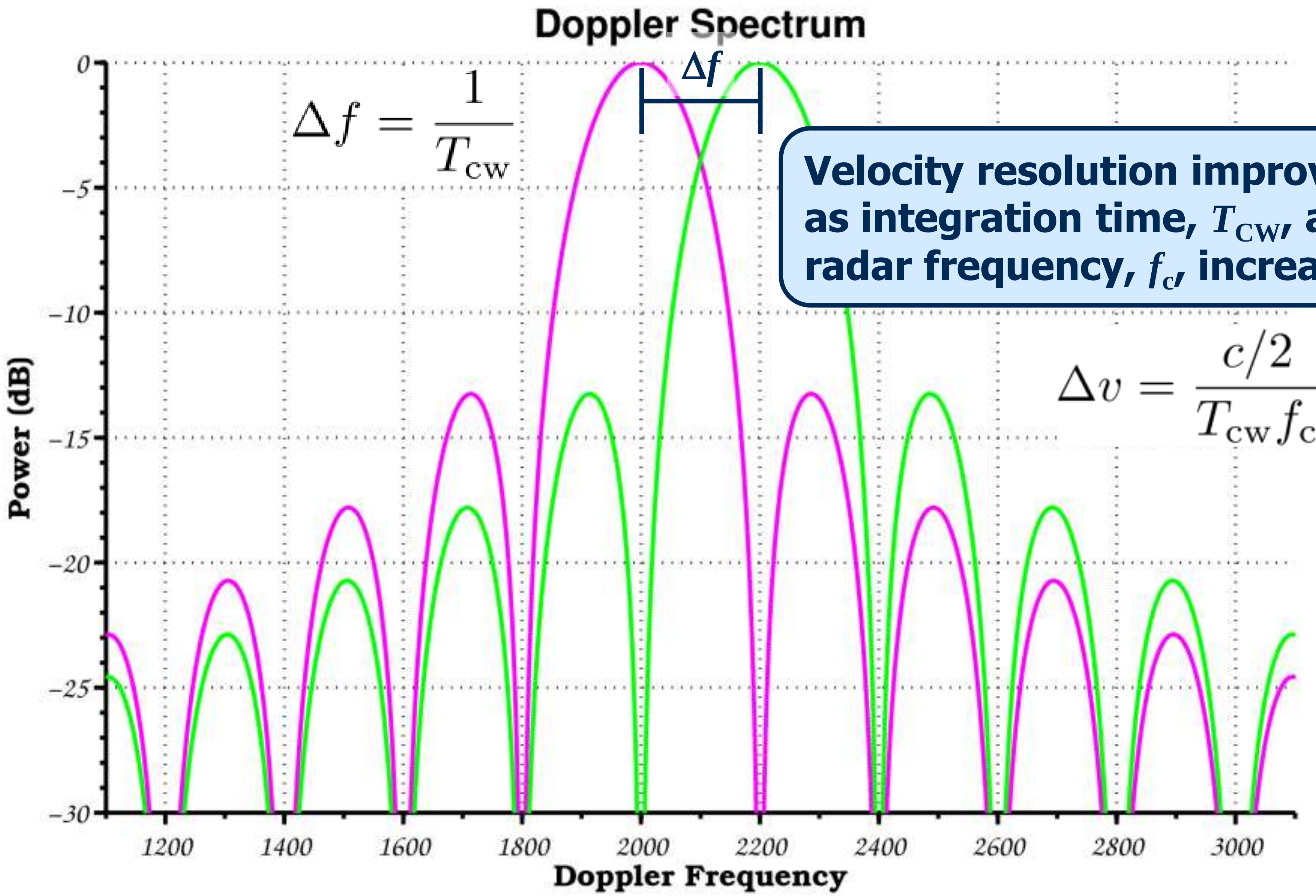
$$f_D \triangleq \frac{v}{c/2} f_c$$

v Target radial velocity

f_c Radar frequency



CW Doppler Resolution

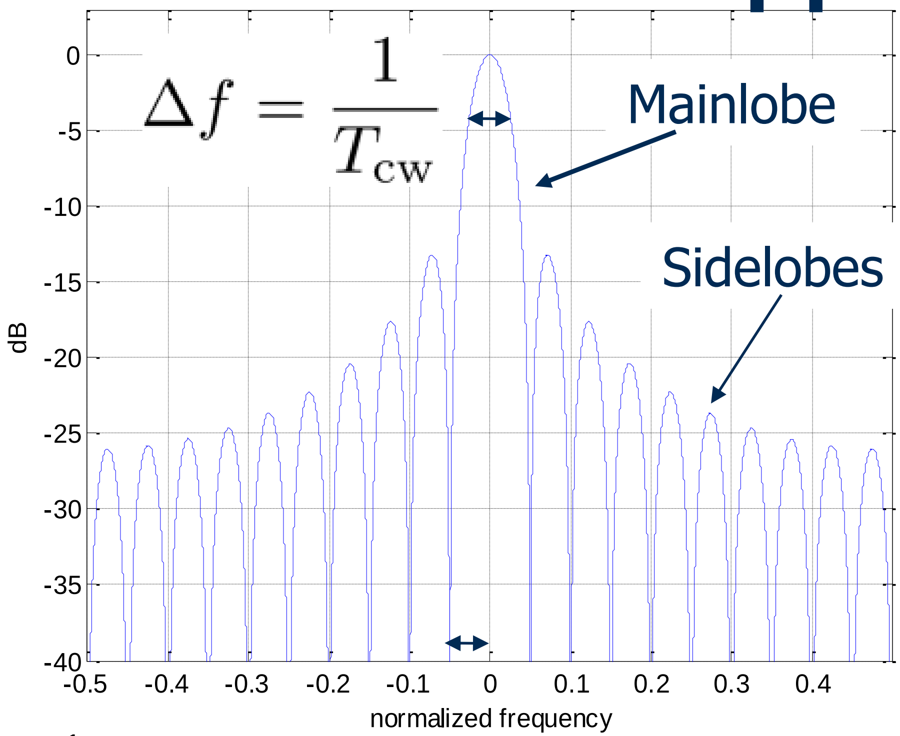


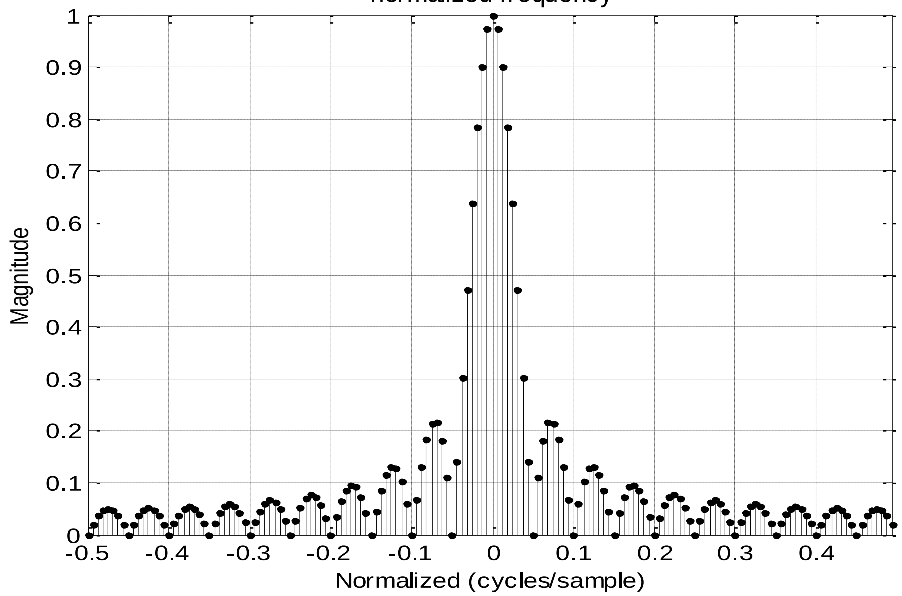
$$\Delta f = \frac{1}{T_{\mathrm{cw}}}$$

Velocity resolution improves as integration time, T_{CW}, and radar frequency, f_c, increases.

$$\Delta v = \frac{c/2}{T_{\mathrm{cw}} f_{\mathrm{c}}}$$

CW Doppler Processing





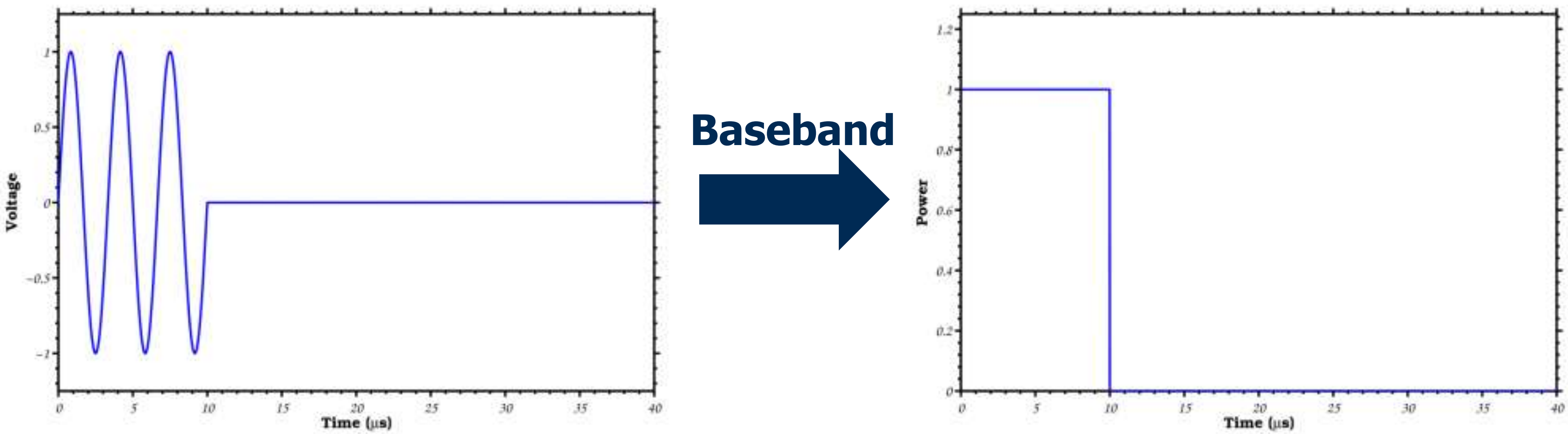
- **DFT processing**
  - **Sample CW returns discretely in time**
  - **Generate spectrum via Fourier analysis (e.g., FFT)**
  - **Results in sinc shaped response**
- **Weighting can be applied to reduce Doppler sidelobes**
  - **SNR loss**
  - **Resolution degradation**
- **Sampling of DFT response a function of**
  - **Bin spacing**
  - **Frequency**
- **Zero padding reduces bin spacing; does not improve resolution**

Measuring Range with a Single (Unmodulated) Pulse

Unmodulated Pulse

$$s(t) \triangleq \begin{cases} \sqrt{P_{\mathrm{TX}}} \sin(2\pi f_c t), & \text{for } t \in [0, T_{\mathrm{p}}) \\ 0, & \text{for } t \notin [0, T_{\mathrm{p}}) \end{cases}$$

P_{TX}	**Peak transmit power**
f_c	**Center frequency**
T_p	**Pulse width**



The Matched Filter

- **Observe a known signal, *s*(*t*), in noise**

$$y(t) = As(t) + \eta(t)$$

- **Apply matched filter to maximize signal-to-noise ratio (SNR)**

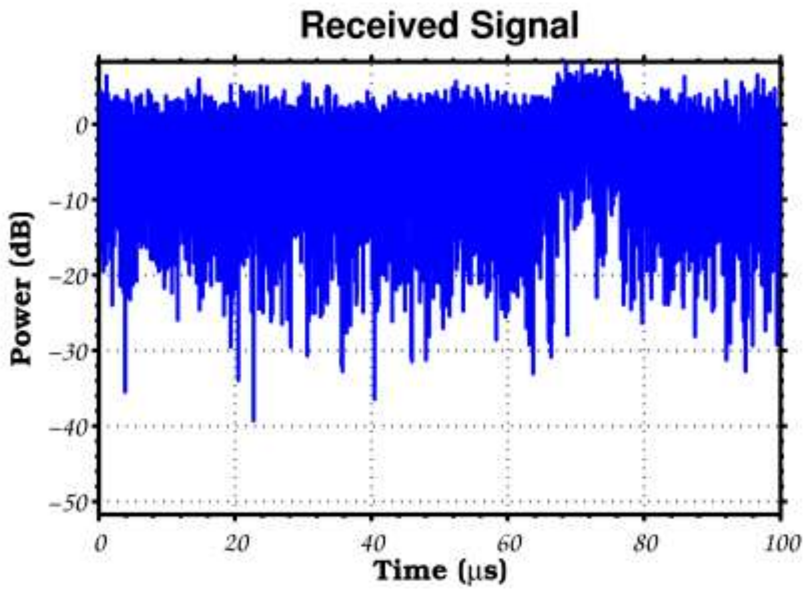
$$z \triangleq \int_{-\infty}^{\infty} y(t)\, s^*(t)\; dt$$
$$= A + \int_{-\infty}^{\infty} \eta(t)\, s^*(t)\; dt$$

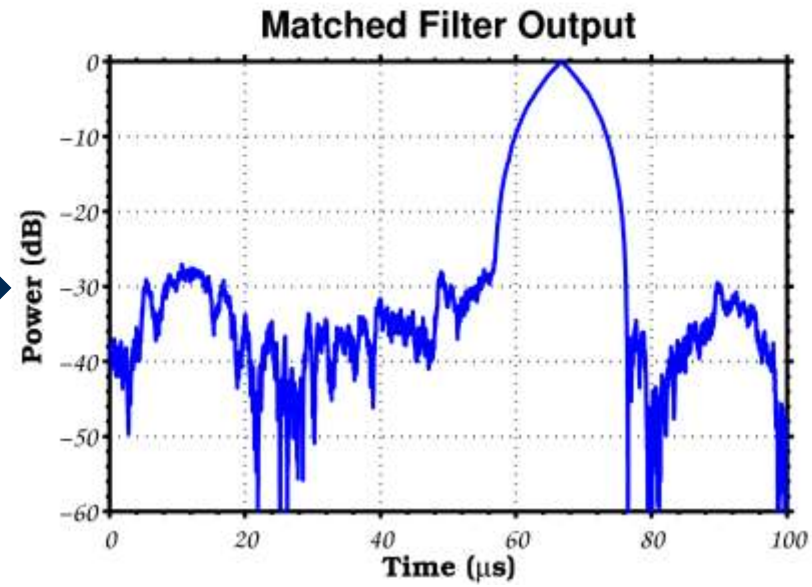
$$\text{SNR} = \frac{A^2}{\sigma_\eta^2}$$

assuming that signal has unit power, i.e.,

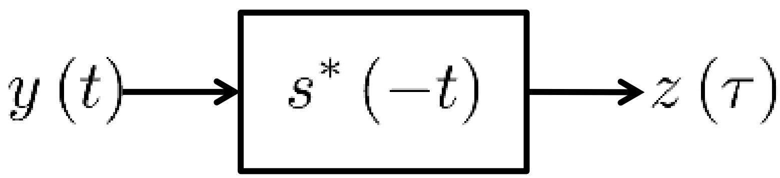
$$\int\limits_{-\infty}^{\infty} |s(t)|^2\; dt = 1$$

The Matched Filter

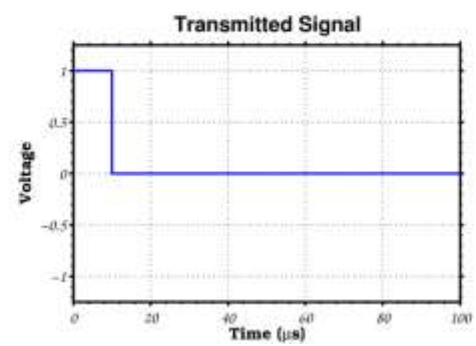




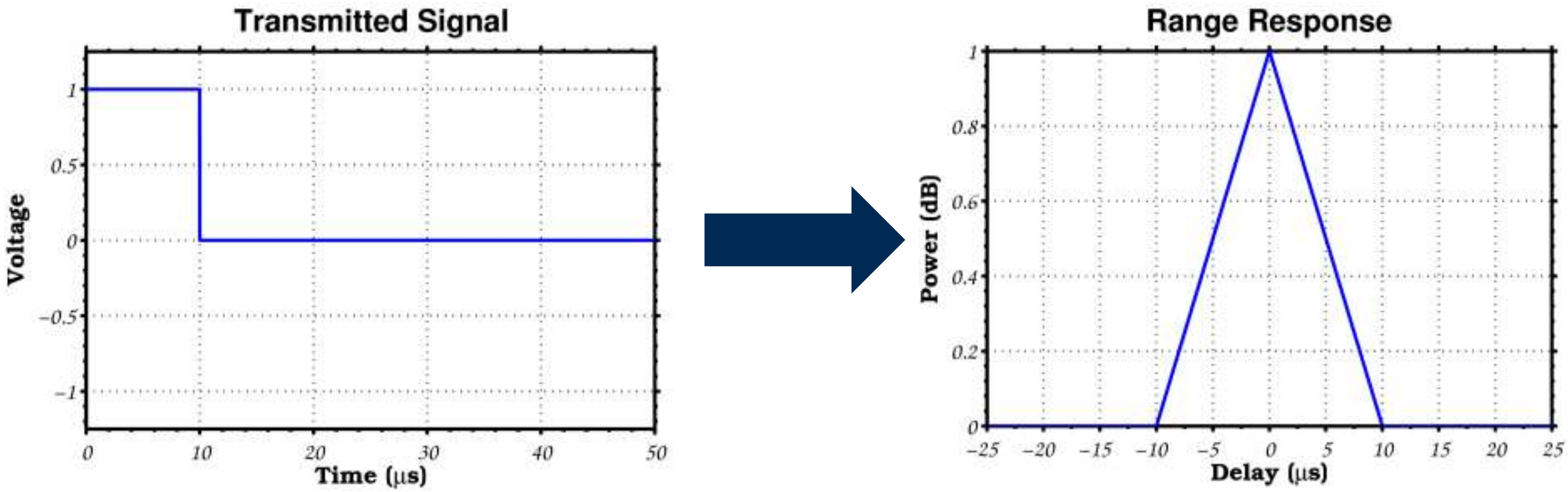
Matched Filter



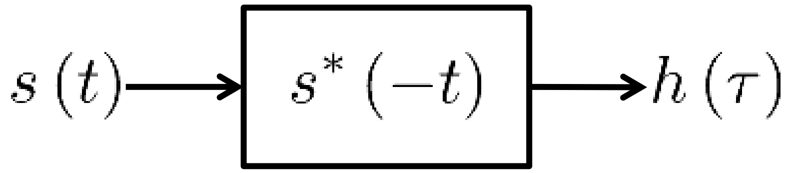
$$z(\tau) = \int_{-\infty}^{\infty} y(t)\, s^*(t-\tau)\, dt$$



Waveform Range Response



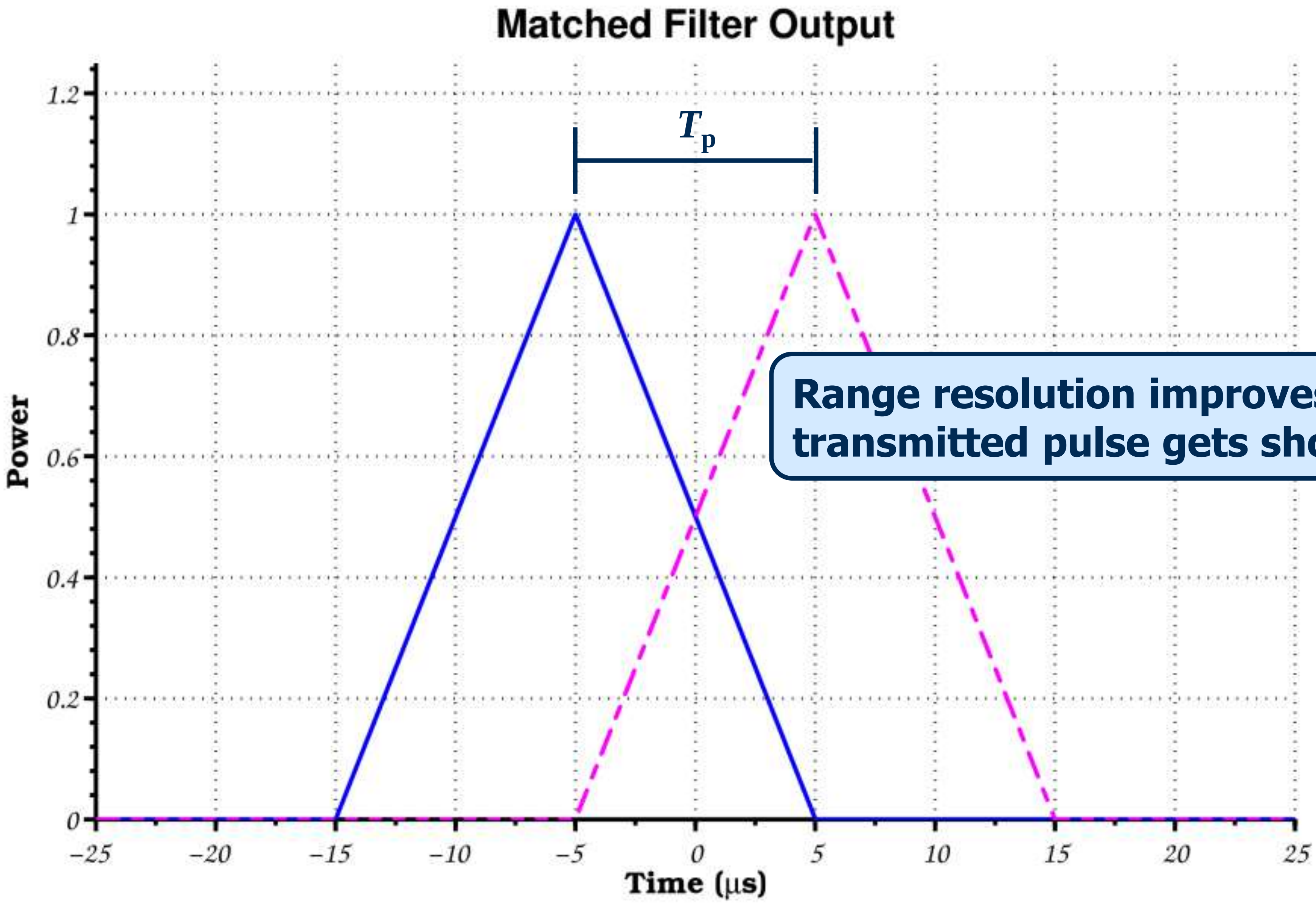
Matched Filter



The <u>range response</u>, $h(\tau)$, of a waveform is the auto-correlation function of the transmitted signal.

$$h(\tau) \triangleq \int_{-\infty}^{\infty} s(t-\tau)\, s^*(t)\; dt$$

Range Resolution: Unmodulated Pulse



Range resolution improves as transmitted pulse gets shorter.

Ambiguity Function

Ambiguity Function

Range Response (No Uncompensated Doppler)

$$h(\tau) \triangleq \int_{-\infty}^{\infty} s(t-\tau)\, s^*(t)\; dt$$

Ambiguity Function

$$\chi(\tau, f_{\mathrm{D}}) \triangleq \int_{-\infty}^{\infty} s(t-\tau)\, e^{i2\pi f_{\mathrm{D}} t} s^*(t)\; dt$$

↑ Doppler shift

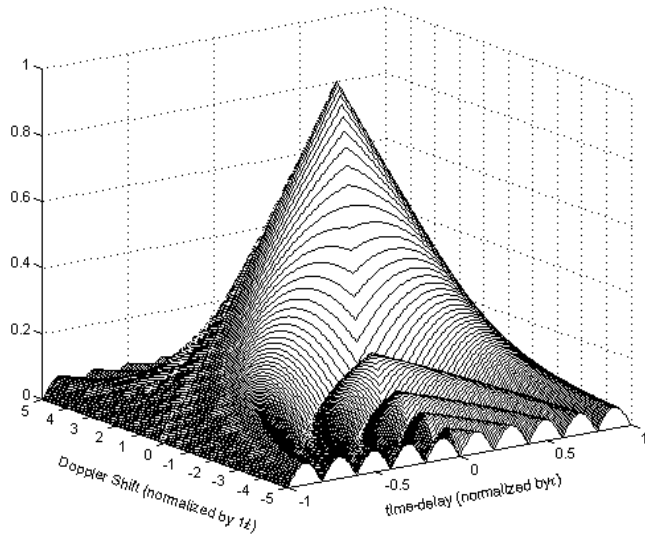
The ambiguity function characterizes the filtered response when the received signal contains an *uncompensated* Doppler shift

Ambiguity Function for a Simple Pulse

$$x(t) = \frac{1}{\sqrt{\tau}} \quad 0 \le t \le \tau$$ **Simple Pulse**

$$A(t, f_d) = \left| \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left(\pi f_d \tau \left(1 - \frac{|t|}{\tau}\right)\right)}{\pi f_d \tau \left(1 - \frac{|t|}{\tau}\right)} \right| \quad |t| \le \tau$$

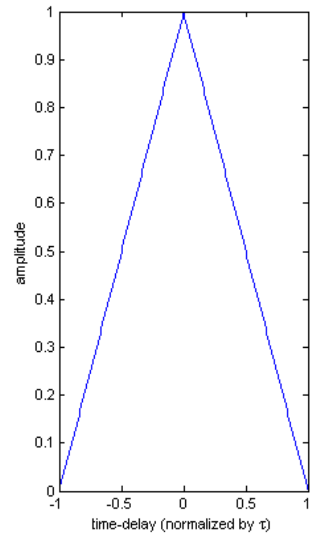
Simple Pulse Ambiguity Function



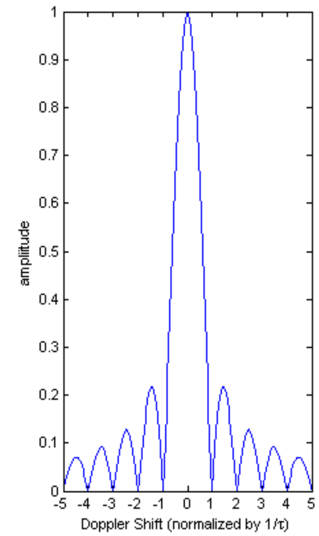
Zero Doppler Cut $$A(t, 0) = \left| 1 - \frac{|t|}{\tau} \right| \quad |t| \le \tau$$

Zero Time-Delay Cut $$A(0, f_d) = \left| \frac{\sin(\pi f_d \tau)}{\pi f_d \tau} \right| \quad |t| \le \tau$$

Zero Doppler Cut



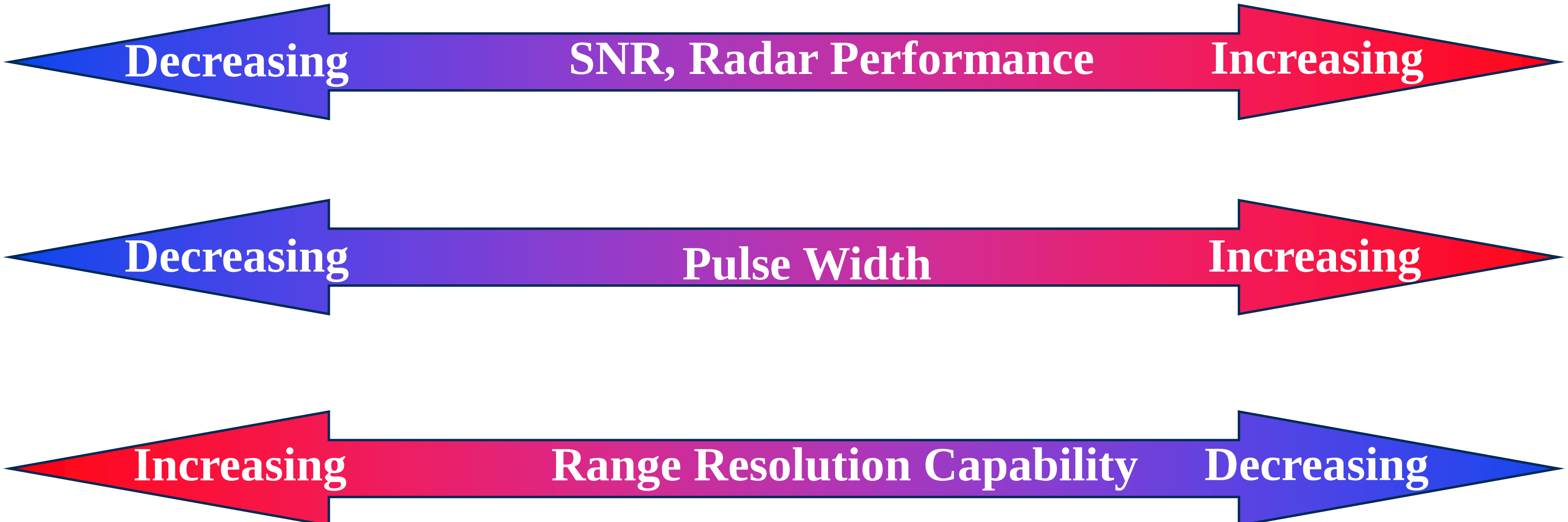
Zero Time-Delay Cut



Improving Range Resolution with Pulse Compression

Limitations of the Unmodulated Pulse

$$\mathrm{SNR} = \frac{P_{\mathrm{TX}} T_{\mathrm{p}} G^2 \lambda^2 \sigma}{k T_0 \left(4\pi\right)^3 R^4}$$

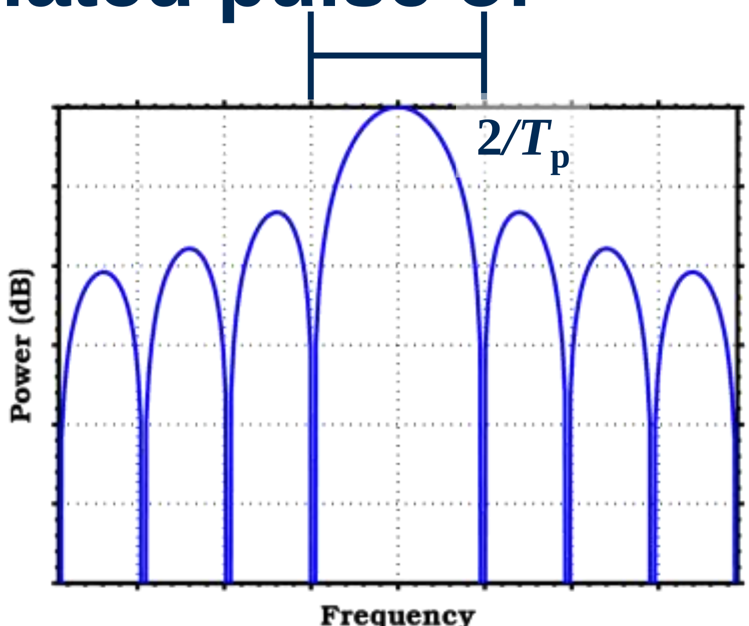


$$\Delta_{\mathrm{R}} = \frac{c}{2} T_{\mathrm{p}}$$

For an unmodulated pulse there exists a coupling between range resolution and waveform energy

Pulse Compression

- Range response is the auto-correlation of the transmitted signal.
- To have "narrow" in range (time) domain, the waveform must have "wide" bandwidth in frequency domain
- The bandwidth of an unmodulated pulse of duration T_p is $1/T_p$
- Pulse Compression
  Use modulated pulses to get better range resolution.



Pulse Compression Waveforms

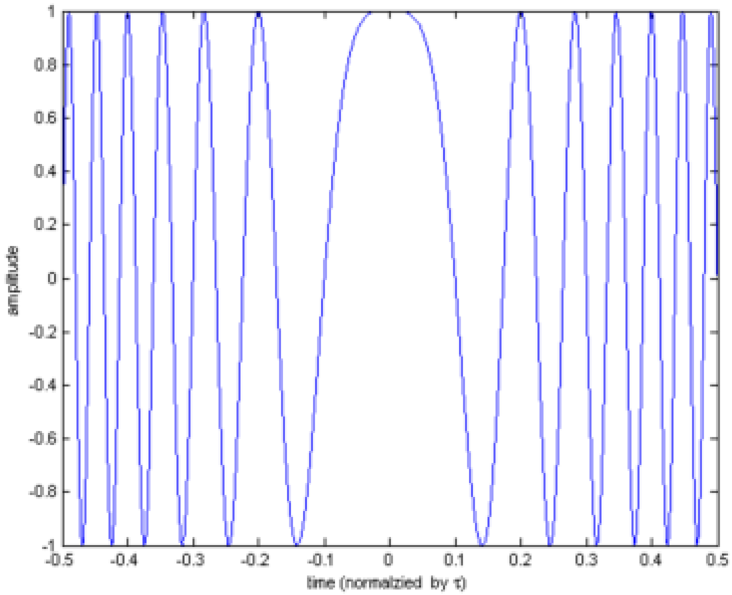
- Permit a de-coupling between range resolution and waveform energy.
- Apply modulation to increase bandwidth.
- Range resolution, Δ_R, improves as bandwidth, W, increases.

$$\Delta_{\mathrm{R}} = \frac{c}{2W}$$

- SNR is unchanged if pulse width remains the same.

Linear Frequency Modulated (LFM) Waveforms

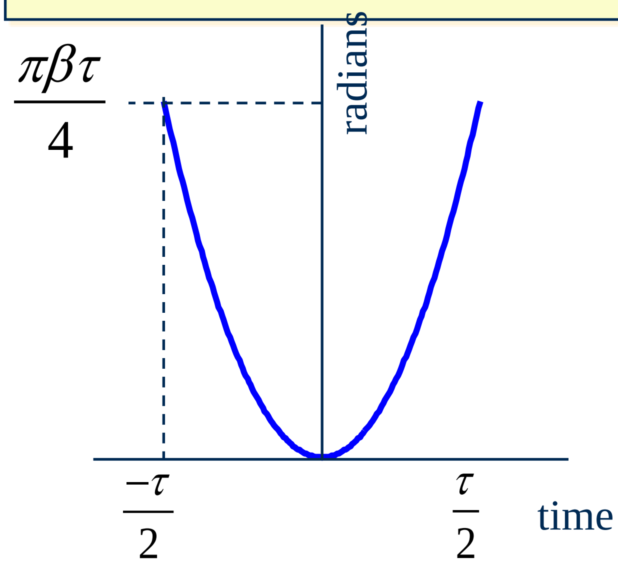
LFM Phase and Frequency Characteristics



$$x(t) = \cos\left(\pi \frac{\beta}{\tau} t^2\right) \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

Linear Frequency Modulated Waveforms

- **LFM phase is quadratic**
- **Instantaneous frequency is defined as the time derivate of the phase**
- **The instantaneous frequency is linear**

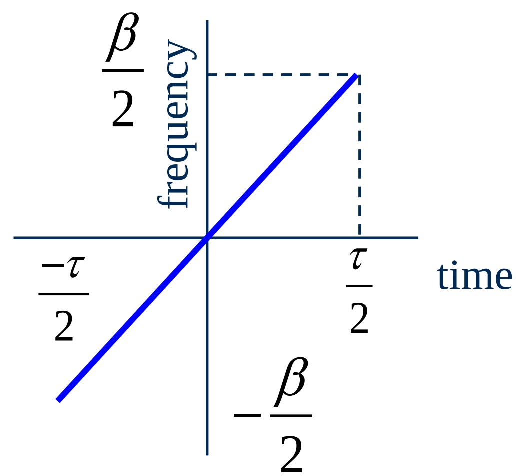


Quadratic Term

Linear Term

$$\frac{d}{dt}\left(\pi \frac{\beta}{\tau} t^2\right) = 2\pi \frac{\beta}{\tau} t$$

$$f \equiv \frac{1}{2\pi} \frac{d\phi}{dt} = \frac{\beta}{\tau} t$$

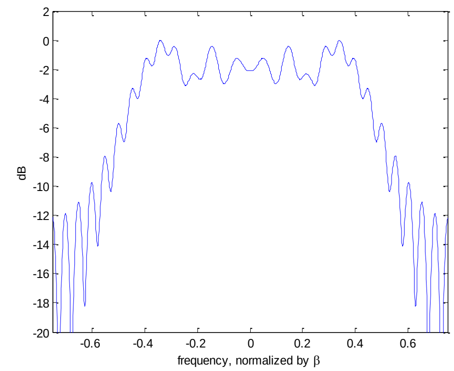


Components of LFM Spectrum

$$X(\omega) = |X(\omega)| \exp\big(j\theta(\omega)\big) \exp\big(j\phi(\omega)\big)$$

3 Key Terms

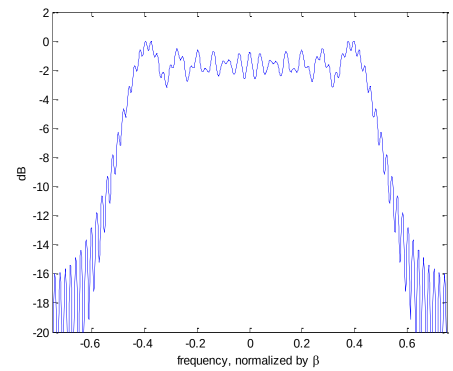
Magnitude Response **Quadratic Phase** **Residual Phase**



$\tau\beta = 20$

$$|X(\omega)| \approx 1 \quad -\pi\beta \le \omega \le \pi\beta$$

For large time-bandwidth products



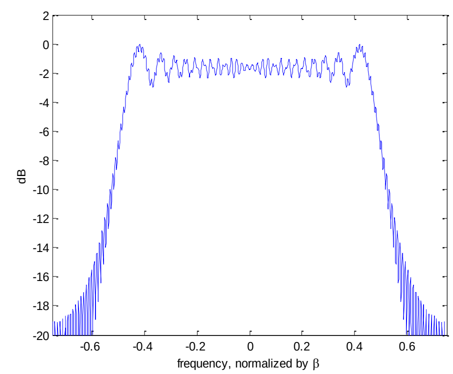
$\tau\beta = 50$

$$\theta(\omega) = -\frac{1}{4\pi}\frac{\tau}{\beta}\omega^2$$

Quadratic phase term

$$\phi(\omega) \approx \frac{\pi}{4}$$

Residual phase term



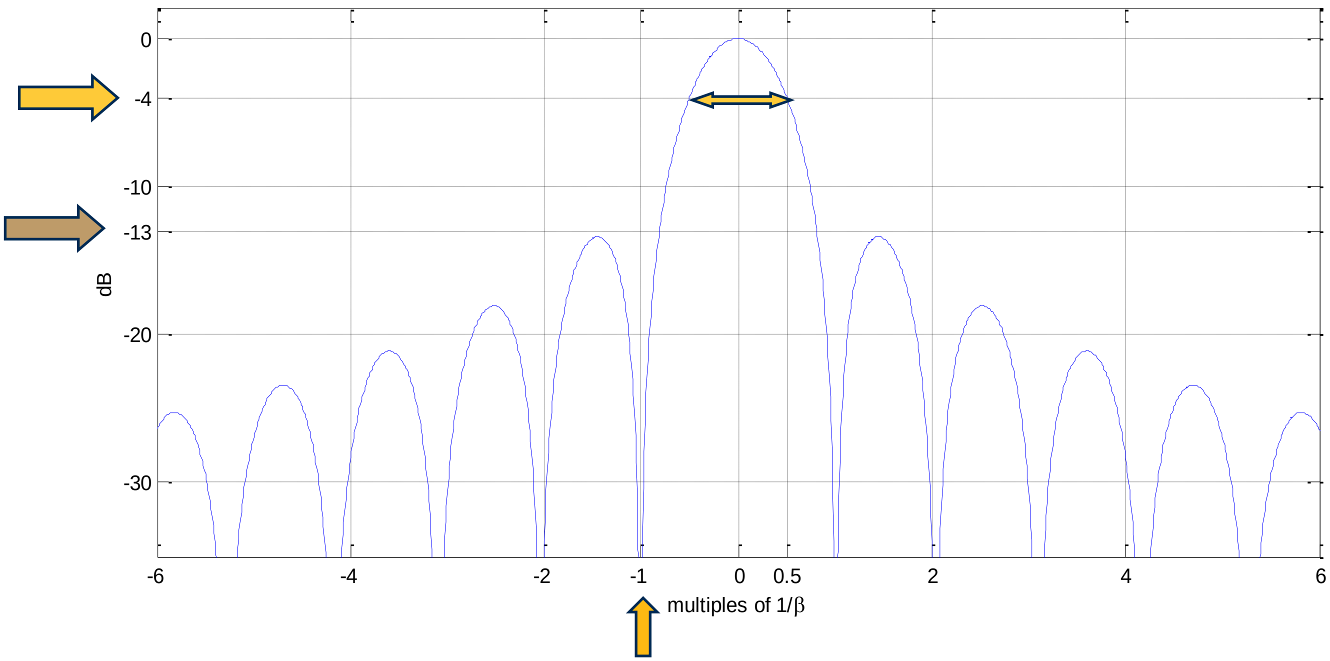
$\tau\beta = 100$

$$X(\omega) \approx \exp\left(-j\frac{1}{4\pi}\frac{\tau}{\beta}\omega^2\right) \quad -\pi\beta \le \omega \le \pi\beta$$

Reference: Cook, Bernfeld, "Radar Signals, An Introduction to Theory and Application", Artech House, 1993, p. 49

LFM Match Filtered Response

Bandwidth (MHz)	Range Resolution (m)
0.1	1500
1	150
2	75
5	30
10	15
20	7.5
50	3
100	1.5
200	0.75
500	0.3
1000	0.15

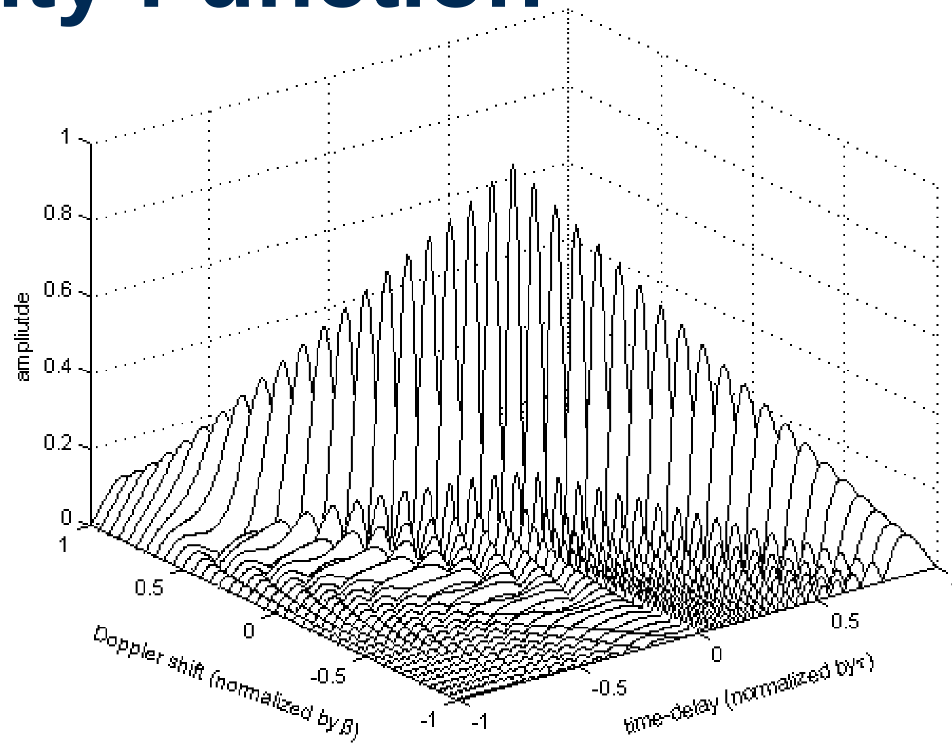


$$y(t) = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\pi \frac{\beta}{\tau} t\left(\tau - |t|\right)\right]}{\pi \frac{\beta}{\tau} t\left(\tau - |t|\right)}$$

- **For βτ ≥ 20, match filtered response approximates a sinc**
  - **~ -13 dB peak sidelobes** $\delta t = \frac{1}{\beta}$ resolution in time $\delta r = \frac{c}{2\beta}$ range resolution
  - **Rayleigh resolution:**
  - **Rayleigh resolution equivalent to 4 dB width**

LFM Ambiguity Function

$$y(t, f_d) = \left| \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left(\pi\tau\left(1 - \frac{|t|}{\tau}\right)\left(f_d + \frac{\beta}{\tau} t\right)\right)}{\pi\tau\left(1 - \frac{|t|}{\tau}\right)\left(f_d + \frac{\beta}{\tau} t\right)} \right| \quad |t| \le \tau$$



$$y(t, f_d) = \left| \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left(\left(1 - \frac{|t|}{\tau}\right)\pi\beta\left(t + \frac{\tau}{\beta} f_d\right)\right)}{\left(1 - \frac{|t|}{\tau}\right)\pi\beta\left(t + \frac{\tau}{\beta} f_d\right)} \right| \quad |t| \le \tau$$

Sinc functions are time shifted versions of one another

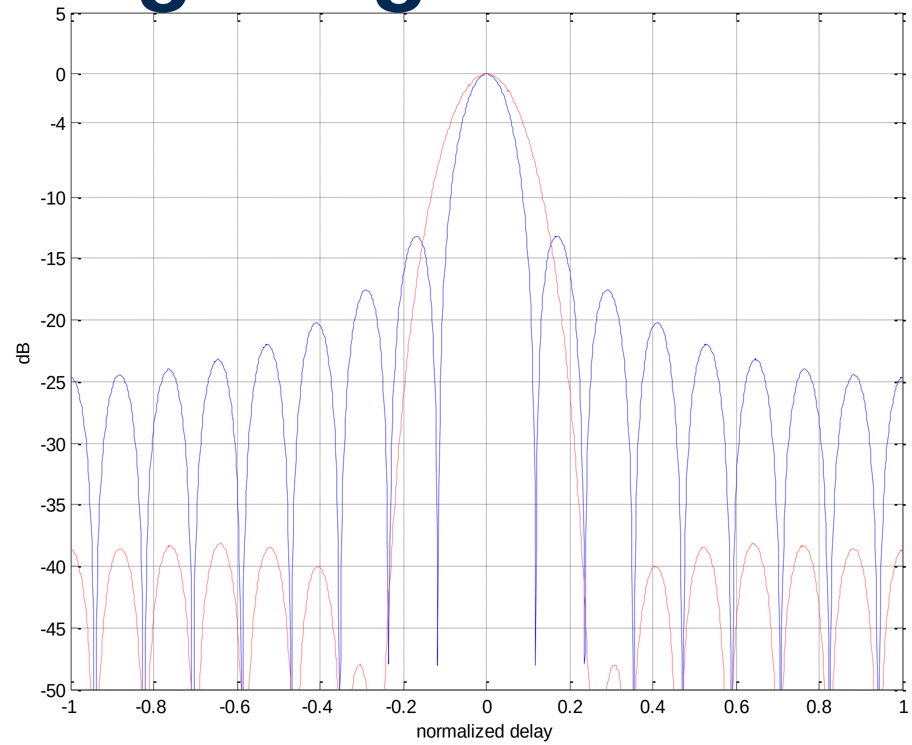
$$y(t, 0) = \left| \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left(\left(1 - \frac{|t|}{\tau}\right)\pi\beta(t)\right)}{\left(1 - \frac{|t|}{\tau}\right)\pi\beta(t)} \right| \quad |t| \le \tau$$

$$t_d = -\frac{f_d}{\beta}\tau$$

The triangle shaped response does not shift with the sinc response

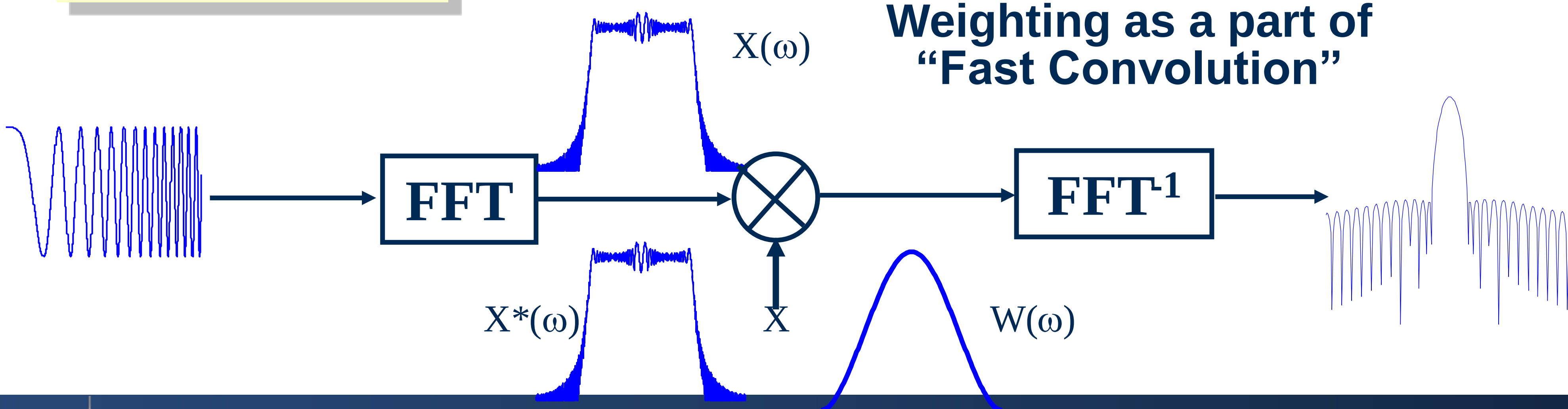
Amplitude Weighting

- Amplitude weighting
  - reduces peak sidelobe levels
  - reduces straddle loss
- Price paid
  - increased mainlobe width (degraded resolution)
  - loss in SNR (loss computable from weighting coefficients)



$$SNR_{Loss} = \frac{\left|\sum_{n=0}^{N-1} w(n)\right|^2}{N\sum_{n=0}^{N-1}\left|w(n)\right|^2}$$

Weighting as a part of "Fast Convolution"



Taylor Weighting Function

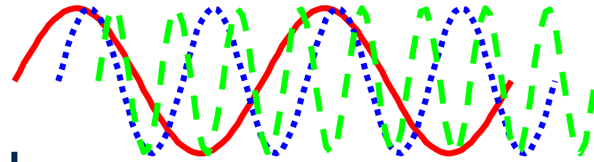
	Peak Sidelobe Level (dB)								
	-20	-25	-30	-35	-40	-45	-50	-55	-60
nbar	**SNR Loss (dB)**								
2	-0.21	-0.38	-0.51						
3	-0.21	-0.45	-0.67	-0.85					
4	-0.18	-0.43	-0.69	-0.91	-1.11	-1.27			
5	-0.16	-0.41	-0.68	-0.93	-1.14	-1.33	-1.49		
6	-0.15	-0.39	-0.66	-0.92	-1.15	-1.35	-1.53	-1.68	
7	-0.15	-0.37	-0.65	-0.91	-1.15	-1.36	-1.54	-1.71	-1.85
8	-0.16	-0.36	-0.63	-0.90	-1.14	-1.36	-1.55	-1.72	-1.87
9	-0.16	-0.36	-0.63	-0.90	-1.14	-1.36	-1.55	-1.72	-1.87

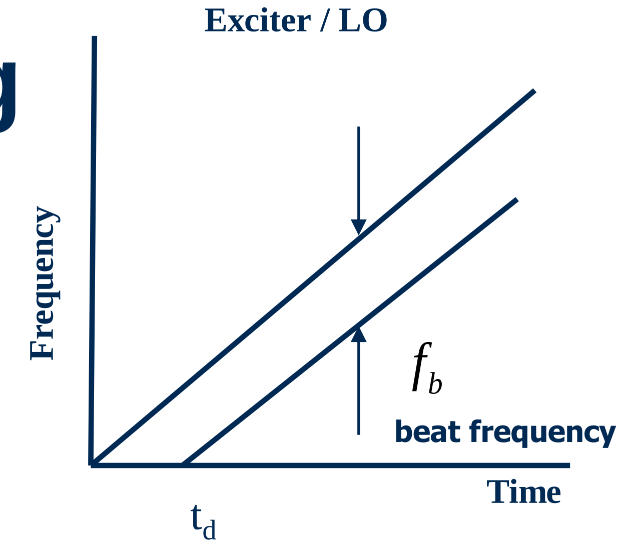
	Peak Sidelobe Level (dB)								
	-20	-25	-30	-35	-40	-45	-50	-55	-60
nbar	**4 dB Resolution Normalized by c/2**								
2	1.15	1.19	1.21						
3	1.14	1.22	1.28	1.33					
4	1.12	1.22	1.29	1.36	1.42	1.46			
5	1.11	1.20	1.29	1.36	1.43	1.49	1.54		
6	1.10	1.19	1.28	1.36	1.43	1.50	1.56	1.61	
7	1.09	1.19	1.28	1.36	1.43	1.50	1.56	1.62	1.67
8	1.08	1.18	1.27	1.35	1.43	1.50	1.57	1.63	1.68

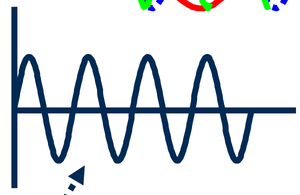
Stretch Processing

Stretch Processing:

a technique for converting time-delay into frequency

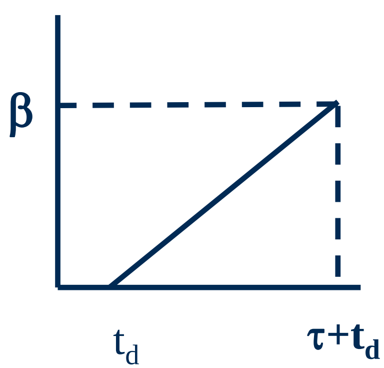


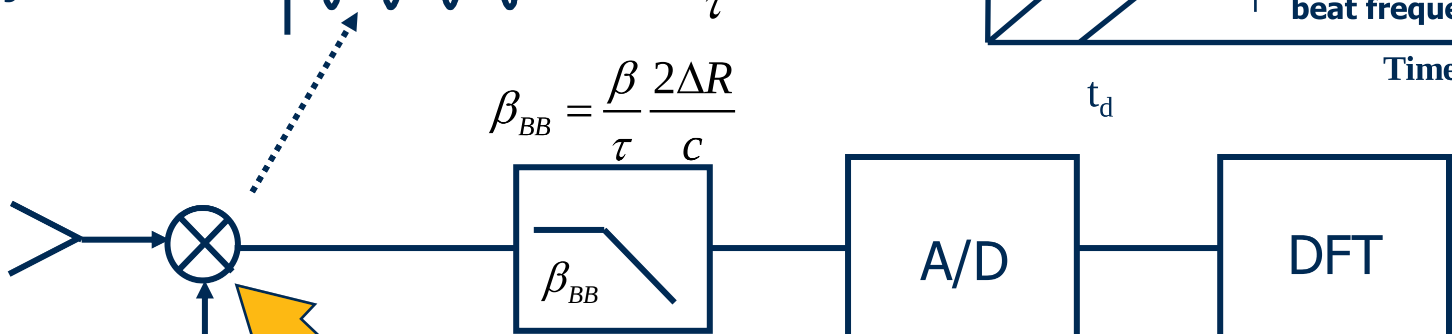




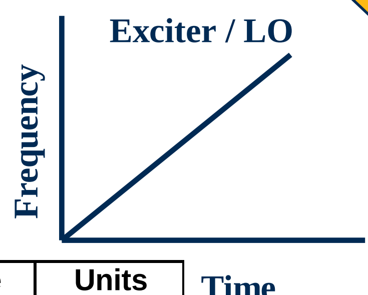
$$f_b = \frac{\beta}{\tau} t_d$$

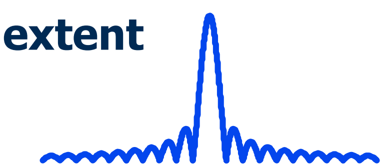
$$\beta_{BB} = \frac{\beta}{\tau}\frac{2\Delta R}{c}$$





Filter limits range of frequencies
Filter defines A/D requirements
Filter defines range window extent





Commonly referred to as a de-ramp operation

Example Calculation

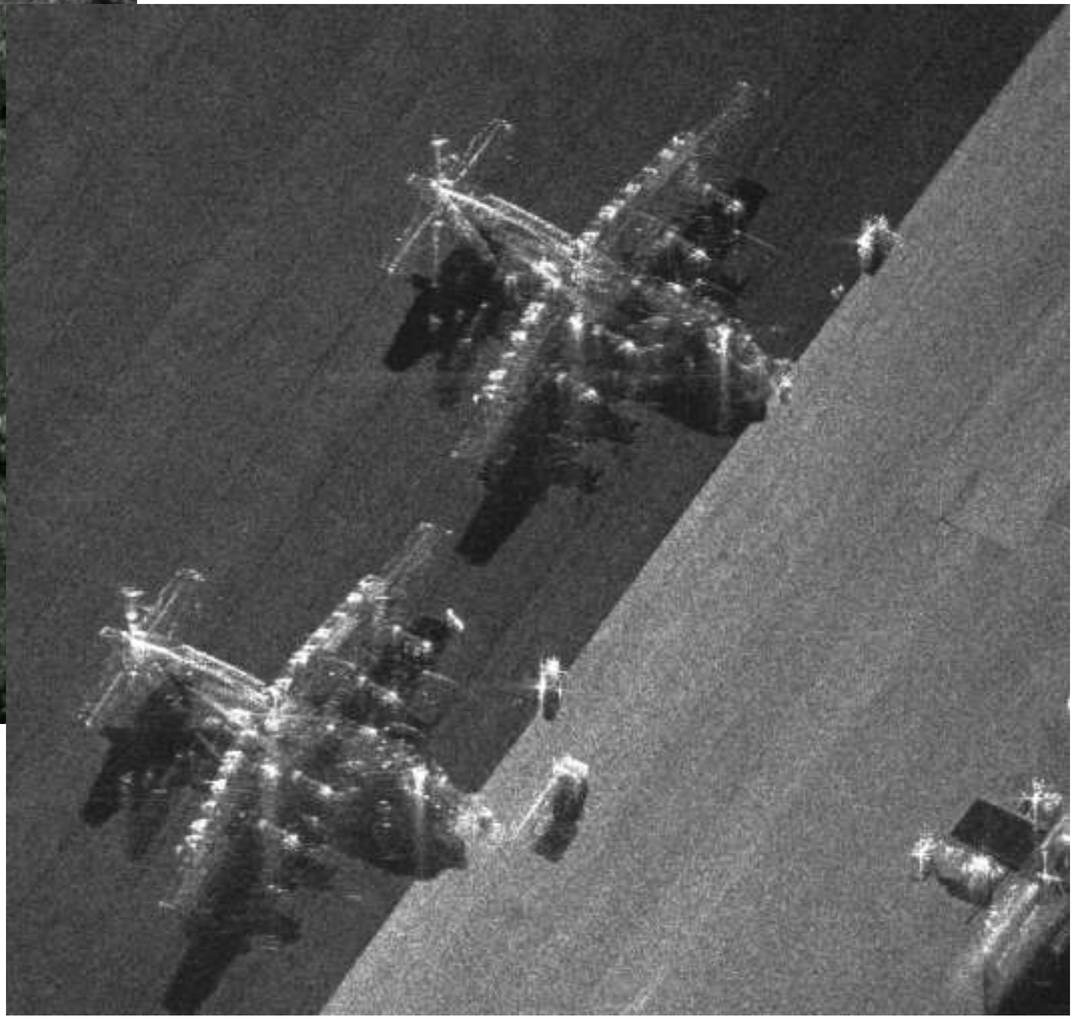
Parameter	Value	Units
Pulse Length	50	usec
Waveform Bandwidth	500	MHz
Filter Bandwidth	40	MHz
A/D Sampling Rate	40	MHz
Range Window Extent	600	m
Nominal Range Resolution	0.3	m

Caputi, "Stretch: A Time-Transformation Technique", March 1971

$$\Delta R = \frac{c\tau}{2}\frac{\beta_{BB}}{\beta}$$ **Range Window Extent**

Range Resolution and SAR Imagery

1 m resolution (> 150 MHz bandwidth)



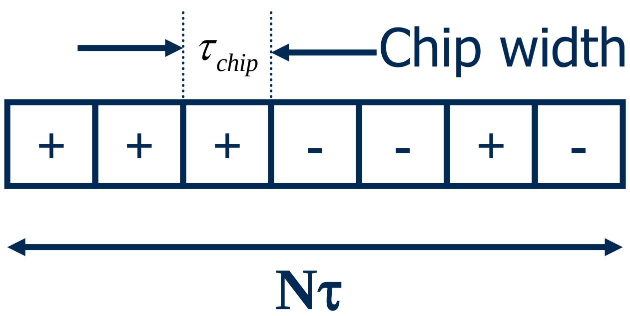
K_u band

10 cm resolution (> 1.5 GHz bandwidth)

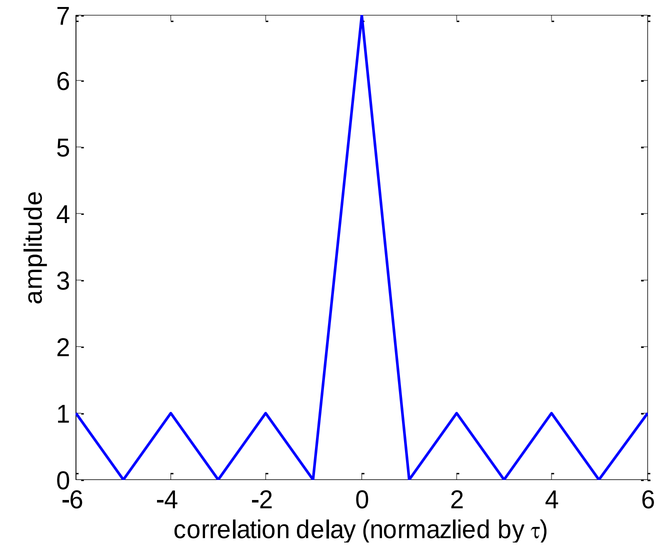
Phase Coded Waveforms

Phase Code Waveforms

- Composed of concatenated sub-pulses (or chips)
- Chip-to-chip phase modulation applied to achieve desired compressed response (e.g., mainlobe, sidelobes, & Doppler tolerance)
- Phase modulation
  - Bi-phase codes (only 2 phase states)
  - Poly-phase codes (exhibit more than 2 phase states)



$$\delta r = \frac{c\tau_{chip}}{2}$$

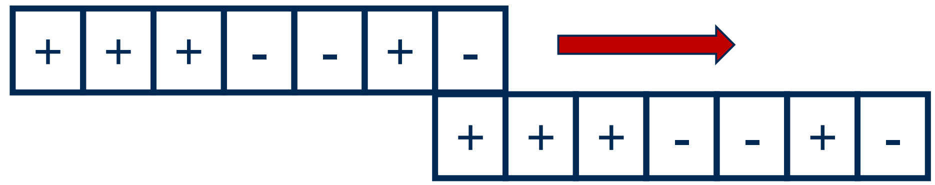


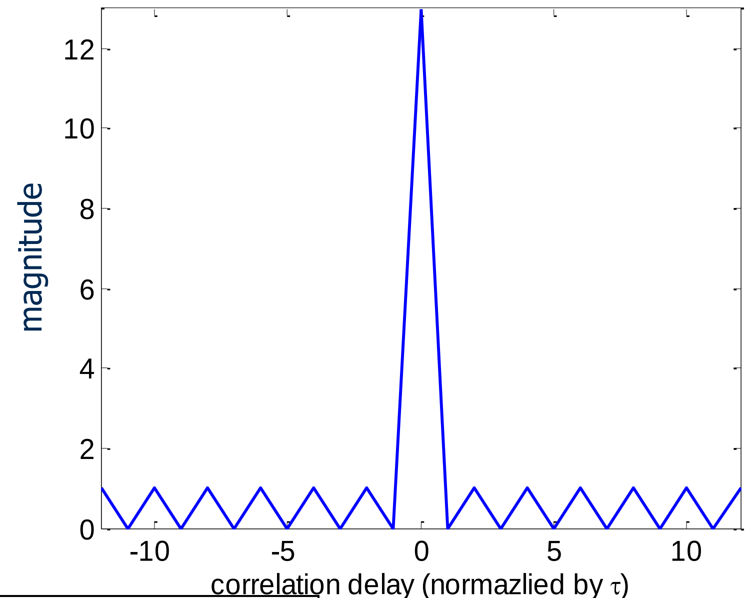
- Consists of N chips each with duration, τ_{chip}
- For appropriately chosen codes, the Rayleigh range resolution is equal to the chip width
- Energy in the waveform is proportional to the number of chips
- In general, sidelobe levels are inversely proportional to the number of chips

Barker Codes

- Perfect bi-phase aperiodic codes
- Belief that no Barker code exists above length 13
  - Has been proven for odd length sequences
- Barker codes are applied in radar applications
- Desire for longer codes however has driven the community to consider longer sub-optimum codes

Correlation





Code Length	Code Sequence	Peak Sidelobe Level, dB	Integrated Sidelobe Levels, dB
2	+−, ++	-6.0	-3.0
3	++−	-9.5	-6.5
4	++−+,+++−	-12.0	-6.0
5	+++−+	-14.0	-8.0
7	+++−−+−	-16.9	-9.1
11	+++−−−+−−+−	-20.8	-10.8
13	+++++−−++−+−+	-22.3	-11.5

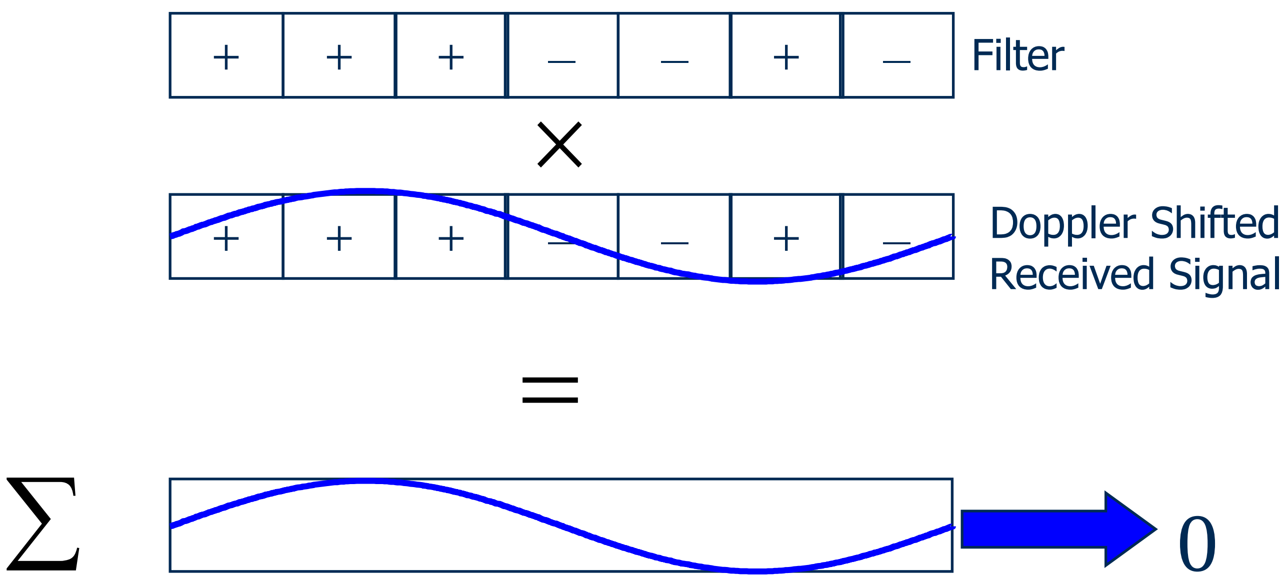
Longer code = Lower PSL

Minimum Peak Sidelobe Codes

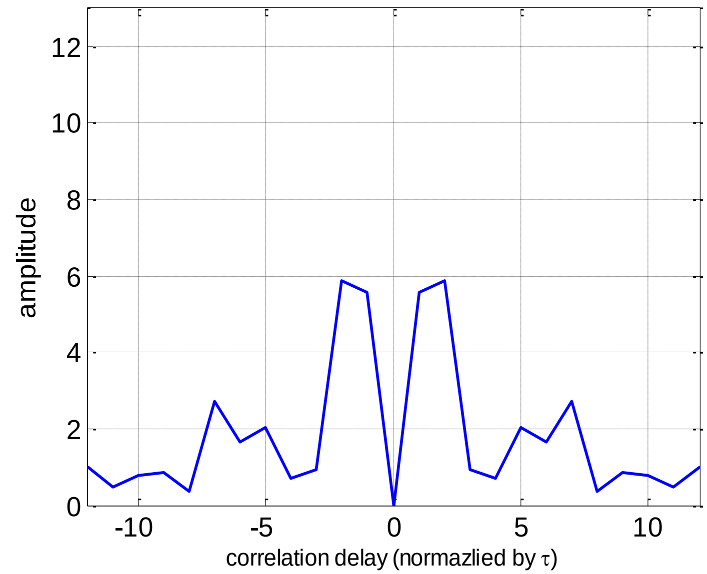
- Binary codes yielding minimum peak sidelobes for a given sequence length
  - Identified through exhaustive searches
  - MPS codes identified through length 69
  - Peak sidelobe levels
    - = 1 for the Barker length sequences N = 2,3,4,5,7,11, &13
    - = 2 for N <= 28 (excluding Barker codes & N = 22,23,24,26,27)
    - = 3 for N = (22,23,24,26,27) & 29 <= N <= 48, and N = 51
    - = 4 for N = 50, and 52 <= N <= 70
  - Does not ensure optimum integrated sidelobe level
- Nunn and Coxson (IEEE AES 2008) found codes with peak sidelobe levels
  - = 4 for N = 71 through 82
  - = 5 for N = 83 through 105
- Longer codes with low peak sidelobes have been identified (not necessarily optimum)

Doppler Intolerance of Bi-Phase Codes

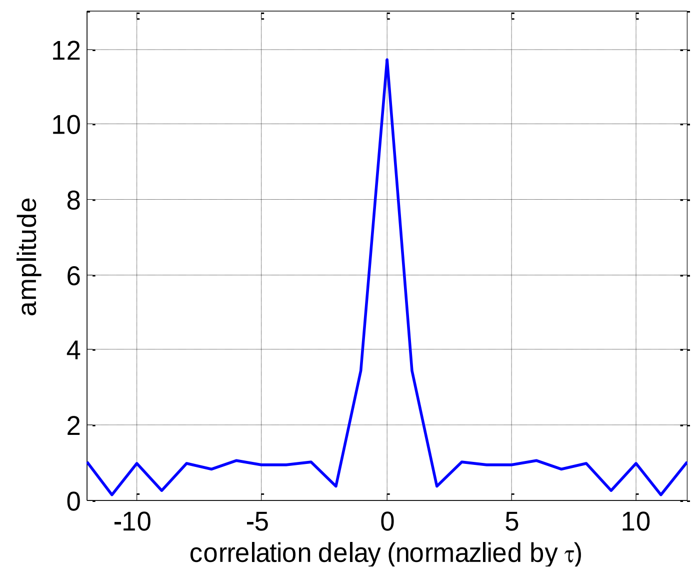
- Bi-phase codes are Doppler intolerant
  - Mainlobe is not preserved
  - Sidelobes increase
- Waveform designed to limit maximum Doppler shift to ¼ cycle
  - Corresponds to 1 dB loss in peak amplitude
- Poly-phase, quadratic phase response required to achieve Doppler tolerance



1 Cycle of Doppler

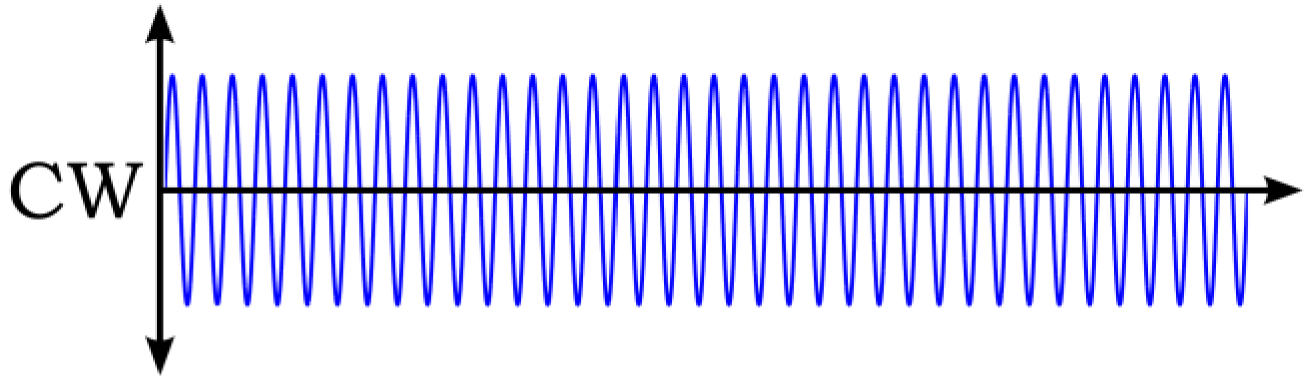


¼ Cycle of Doppler



Measuring Range and Doppler with Coherent Pulse Train

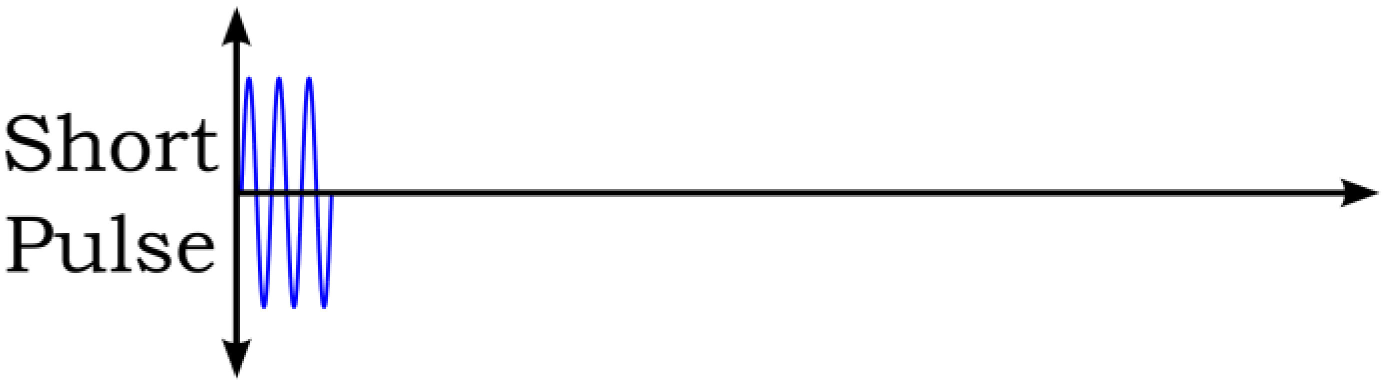
Coherent Pulse Train



Good Doppler resolution

No range resolution

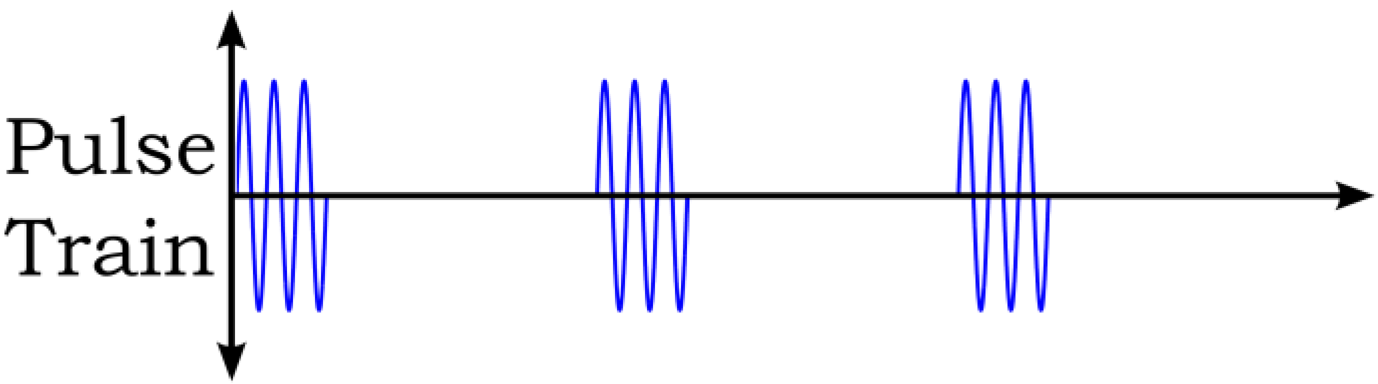
$$\Delta f = \frac{1}{T_{\mathrm{cw}}}$$



Good range resolution

Poor Doppler resolution

$$\Delta R = \frac{c}{2} T_{\mathrm{p}}$$



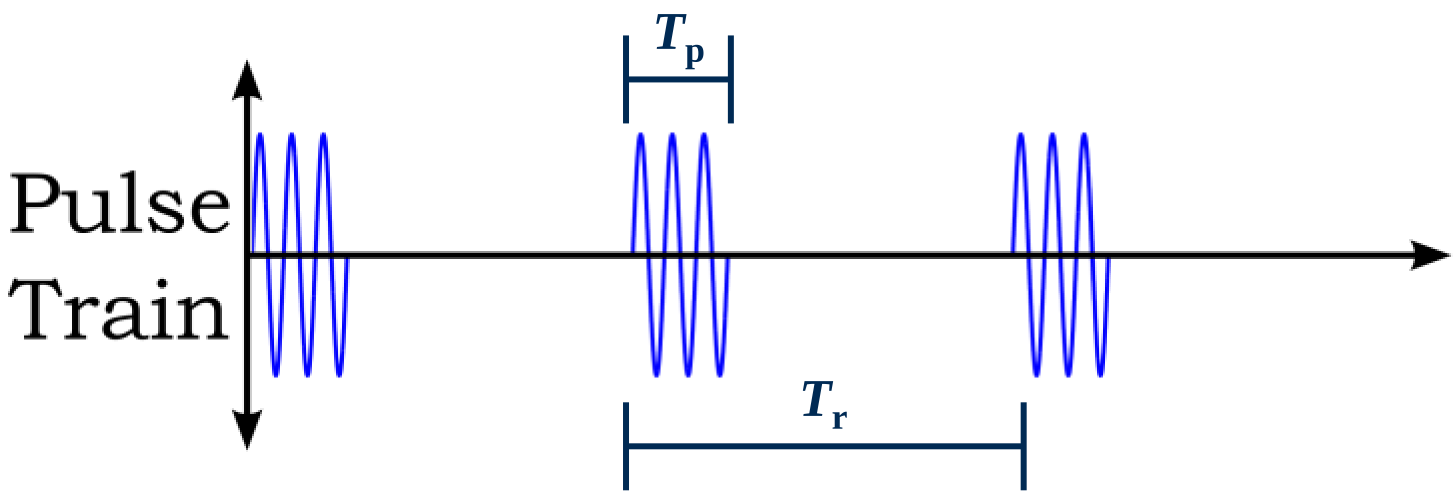
Good range resolution

Good Doppler resolution

$$\Delta f = \frac{1}{N_{\mathrm{p}} T_{\mathrm{p}}}, \Delta_R = \frac{c}{2} T_{\mathrm{p}}$$

Coherent Pulse Train

Parameter	Symbol
Pulse Width	T_p
Pulse Repetition Interval (PRI)	T_r
Number of Pulses	N_p

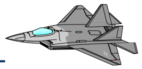


Duty Cycle: $d \triangleq \dfrac{T_\mathrm{p}}{T_\mathrm{r}}$

Pulse Repetition Frequency (PRF): $f_\mathrm{r} \triangleq \dfrac{1}{T_\mathrm{r}}$

Measuring Phase & Radial Velocity

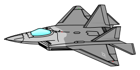
1



R_0

$$\exp\left(j2\pi f_c \frac{2R_0}{c}\right)$$

2

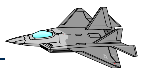


$R_0 - vT$

$$\exp\left(j2\pi f_c \frac{2\left(R_0 - vT\right)}{c}\right)$$

A pulsed Doppler waveform measures the phase change between pulses

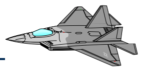
3



$R_0 - v2T$

$$\exp\left(j2\pi f_c \frac{2\left(R_0 - v2T\right)}{c}\right)$$

n



$R_0 - vnT$

$$\exp\left(j2\pi f_c \frac{2\left(R_0 - vnT\right)}{c}\right)$$

$$n = 0, \ldots, N-1$$

$$\exp\left(j2\pi f_c \frac{2R_0}{c}\right)\exp\left(-j2\pi f_c \frac{2vnT}{c}\right)$$

$$\exp\left(-j2\pi f_c \frac{2vnT}{c}\right) = \exp\left(-j2\pi \frac{2v}{\lambda} nT\right)$$

$$\left|f_d\right| = \left|\frac{2v}{\lambda}\right|$$

Processing Doppler

- The Discrete Fourier Transform represents a bank of matched filters
- The filters are only applied at the zero time-delay lag

$$h(n) = \exp(j2\pi f_k nT)$$

$$f_k = \frac{k}{N'} F_S$$

Note: $F_s = PRF$

$$y(k) = \sum_{n=0}^{N-1} x(n) \exp\left(-j2\pi \frac{kF_S}{N'} nT_S\right) \quad k = 0, \ldots, (N'-1), \quad N' \geq N$$

Measured signal from N pulses

N' filters

$$F_S T_S = 1$$

$$y(k) = \sum_{n=0}^{N-1} x(n) \exp\left(-j2\pi \frac{k}{N'} n\right) \quad k = 0, \ldots, (N'-1), \quad N' \geq N$$

Measured signal from N pulses

N' filters

$$f_k = \frac{k}{N'} F_S$$

Discrete Fourier Transform

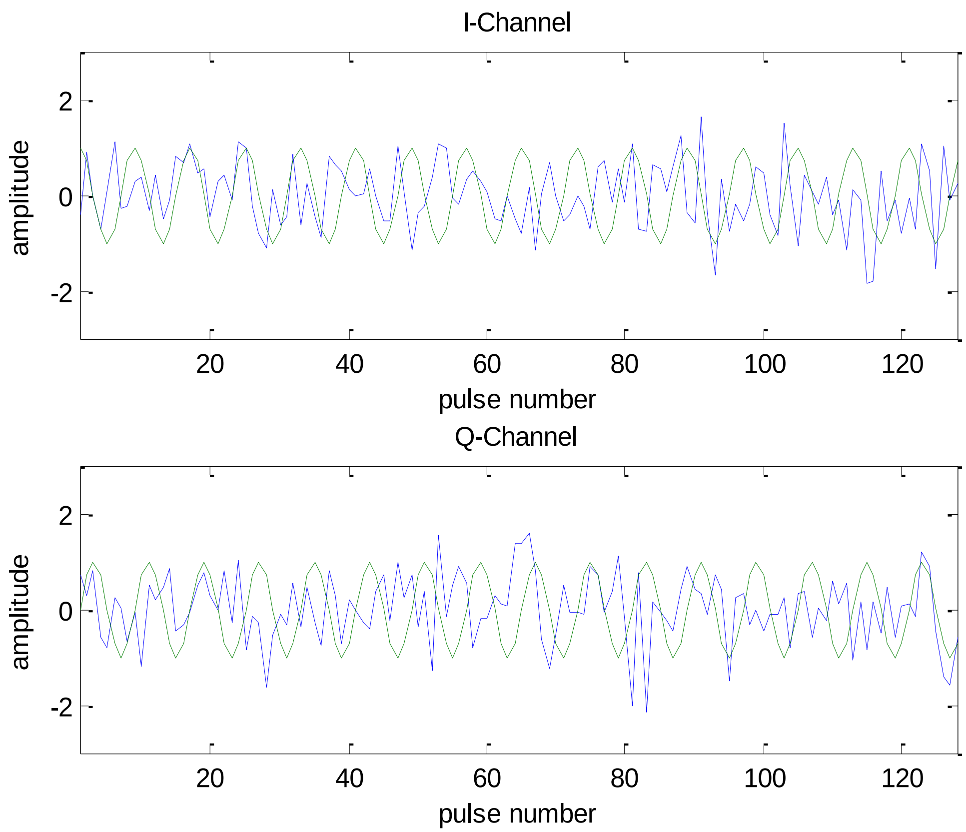
Signal-to-Noise Ratio

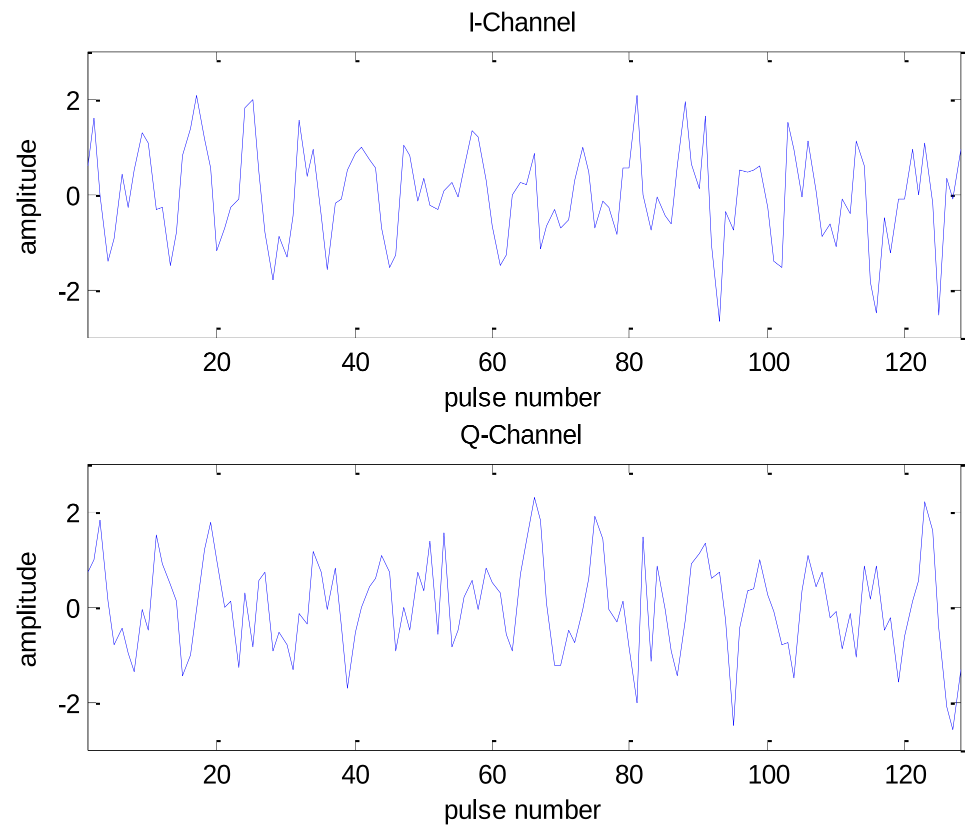
Prior to applying the Doppler matched filter, the SNR may be less than zero

$$x(n) = \exp\left(j2\pi(0.125PRF)(nT)\right) \quad n = 0,\ldots,(N-1)$$

$$SNR = \frac{A^2}{\sigma^2}$$

$$SNR = 0 \text{ dB}$$





Blue – noise
Green – Doppler shifted signal

Blue – noise +signal

SNR Gain Associated with Doppler Processing

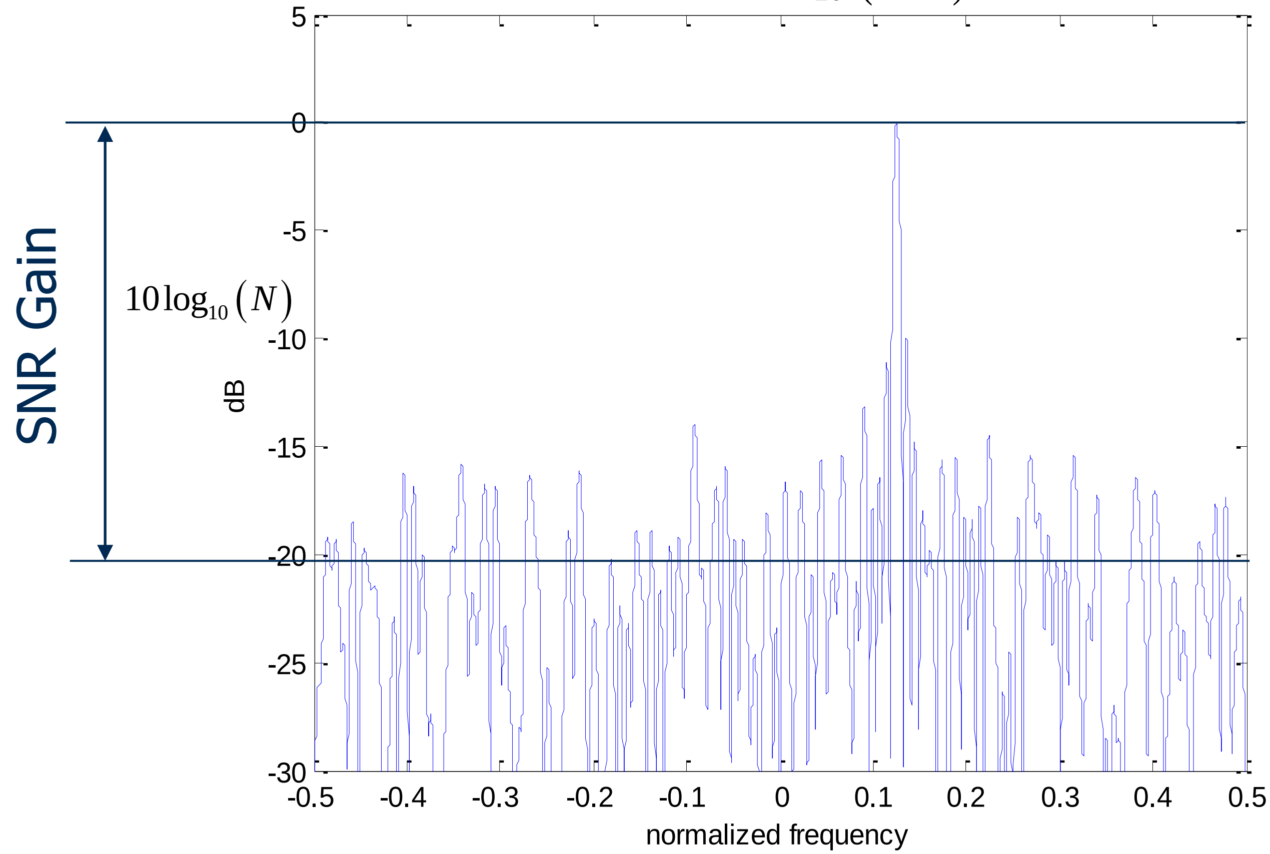
SNR Gain due to Doppler processing
Often referred to as coherent processing gain

$$SNR \text{ GAIN} = 10\log_{10}(128) \approx 21 \text{ dB}$$

$$SNR = \frac{A^2}{\sigma^2}$$

$$SNR = \frac{N^2 A^2}{N\sigma^2} = \frac{NA^2}{\sigma^2}$$

$$SNR \text{ GAIN} = 10\log_{10}(N)$$



- **Example of radar modes benefiting from coherent integration**
  - **SAR:** 100s to 1000s of pulses (20 to 30 dB of SNR gain or more)
  - **GMTI:** 10s to 100s of pulses (10 to 20 dB of SNR gain or more)

Pulse-Doppler Design Considerations

- Ambiguities
  - Range
  - Doppler
- Blind Zones
  - Range eclipsing occurs since radar cannot receive while transmitting.
  - Doppler blind zones occur when target is observed with same Doppler as clutter.

Pulsed Doppler Waveform Modes

- Low PRF
  - Range unambiguous
  - Doppler ambiguous
- High PRF
  - Range ambiguous
  - Doppler unambiguous
- Medium PRF
  - Range ambiguous
  - Doppler ambiguous
- Process multiple PRFs to
  - Resolve range and Doppler ambiguities
  - Move range and Doppler blind zones

Summary

- Radar frequencies
- Radar waveform taxonomy
- CW: Measuring Doppler
- Single Pulse: Measuring range
- Ambiguity function
- Pulse compression waveforms (FM and PM)
- Coherent pulse trains