Radar Fundamentals

A Tutorial Presented to the 12th Annual International Symposium on Advanced Radio Technologies (ISART)

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Rules of Engagement

- This tutorial is "radar according to Greg"
  - Based on my experiences and understanding
  - Tailored to ISART-11
- This tutorial is not exhaustive
  - There are always exceptions
  - Shooting for the "90th-percentile"

Outline

- Background and Range Measurement
- SNR and the Matched Filter
- Radar Range Equation
- Detection in Noise
- Pulse Compression
- Multiple Pulses
- Antenna Effects

Radar Applications and Functions

Airborne interceptor

Air Defense

- **Point Targets**
  - **Detection**
  - **Estimation**
  - **Tracking**
  - **<u>Targets</u>: Cars (police radar), aircraft (air traffic control), reentry vehicles (ballistic missile early warning)**
- **Continuum Targets**
  - **Real-Beam Imaging**
  - **Synthetic Aperture Imaging**
  - **<u>“Targets”</u>: 1-D target (range profile), 2-D clutter (terrain) and 3-D clutter (weather)"**

Air-to-Ground Surveillance

Anti-Air Warfare

Electromagnetic (EM) Spectrum

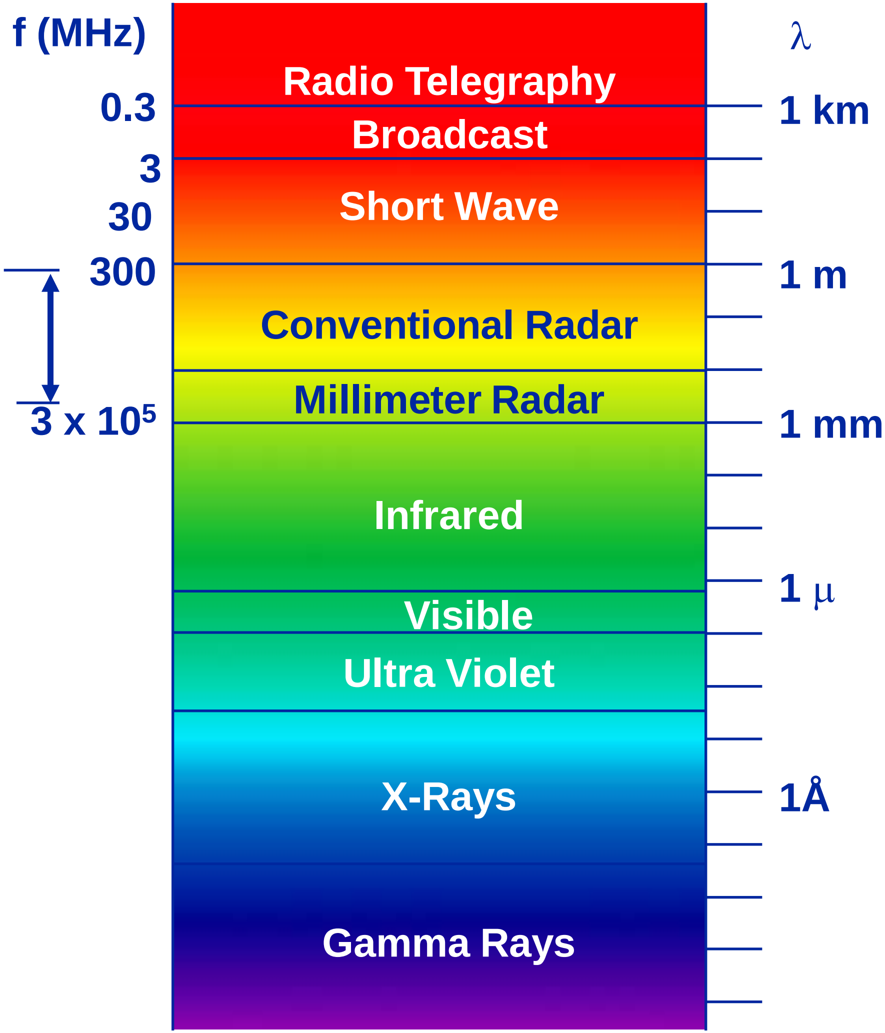
Radar Applications / Bands

Long Range / Search:
- HF = 3 - 30 MHz
- VHF = 30 - 300 MHz
- UHF = 300 - 1000 MHz

Short Range:
- L-Band = 1 - 2 GHz
- S-Band = 2 - 4 GHz
- C-Band = 4 - 8 GHz

Fire Control (Trackers):
- X-Band = 8 - 12 GHz
- Ku-Band = 12 - 18 GHz

Seekers:
- Ka-Band = 27 - 40 GHz
- W-Band = 75 - 110 GHz



$f\lambda = c$

f=frequency (cycles per second)

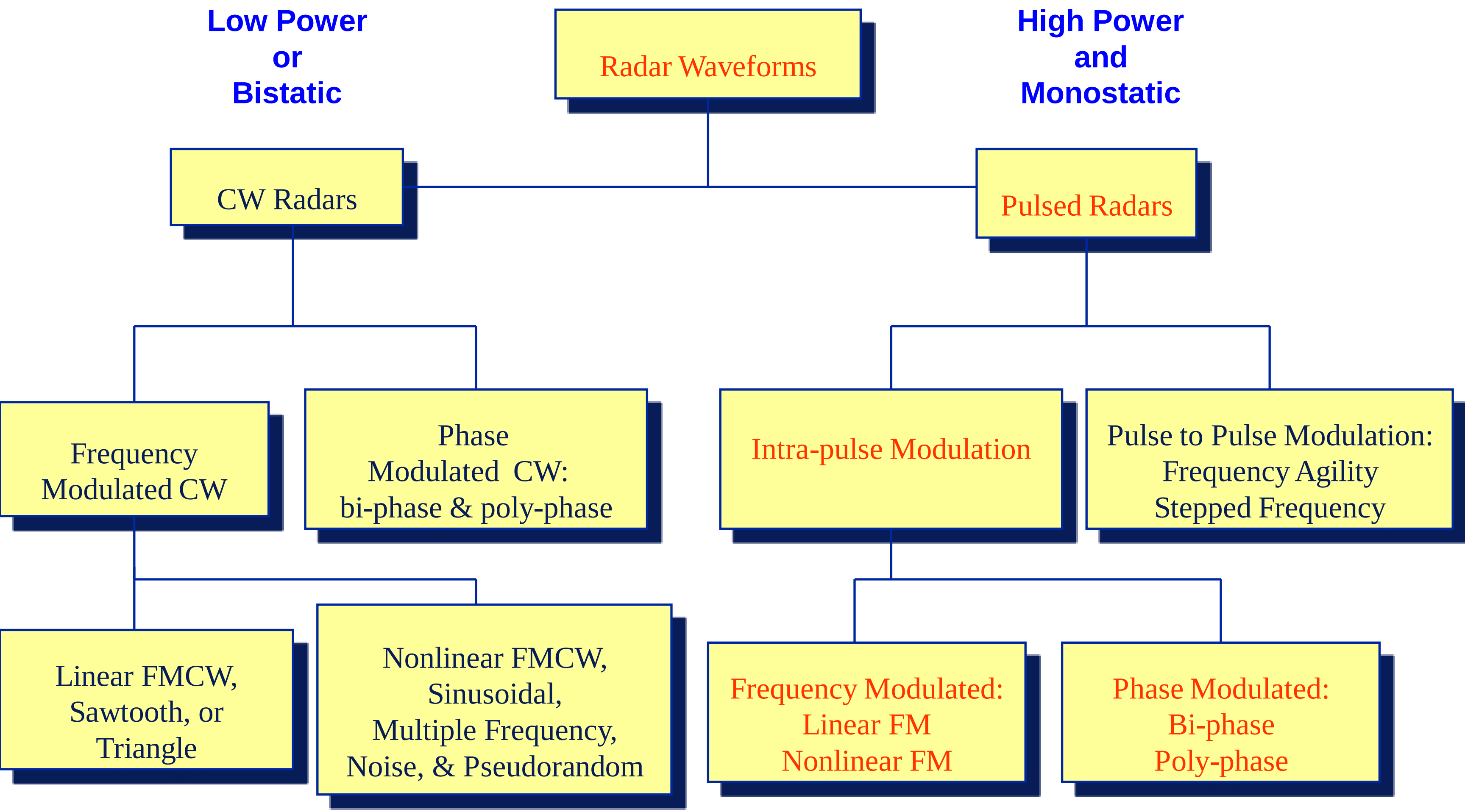
λ=wavelength (m)

c= speed of light (3×10^8 m/s)

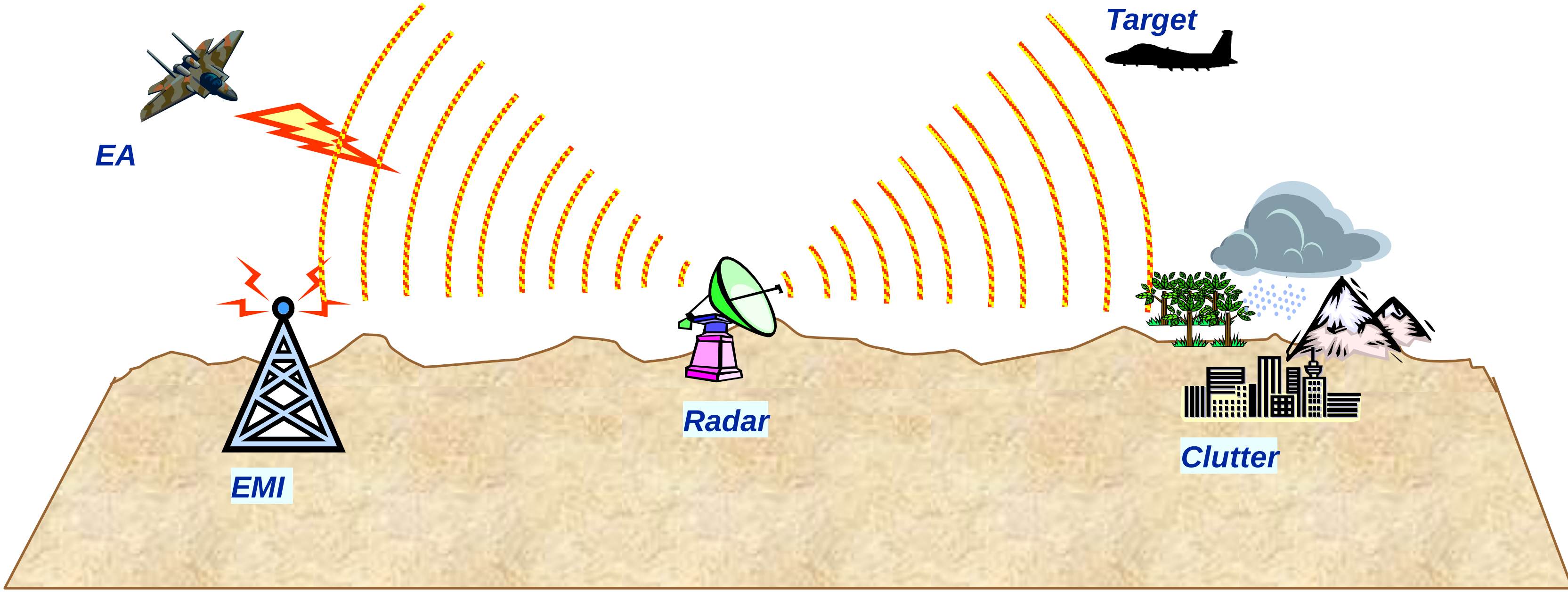
Continuous Wave (CW) vs. Pulsed

- **Continuous Wave (CW) operation properties**
  - Simultaneous TX and RX
  - Simple TX and RX hardware
  - Good Doppler measurements
  - BUT, operation is limited to
    - Low power ⇒ short ranges
    - Or bistatic ⇒ complicated implementation
- **Pulsed operation properties**
  - Time multiplexing of high-power TX and sensitive RX
  - Sharing of TX and RX apertures
  - Good range measurements

Waveform Hierarchy



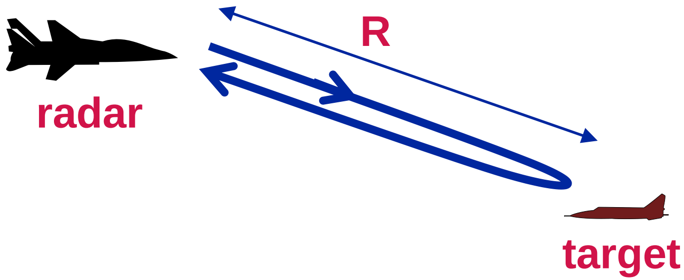
Radar Environment



- Transmits: Electromagnetic wave
- Receives:
    - Echoes from targets, clutter
    - Electromagnetic interference (EMI)
    - Electronic attack (EA)
    - External environmental noise
    - Internal system noise

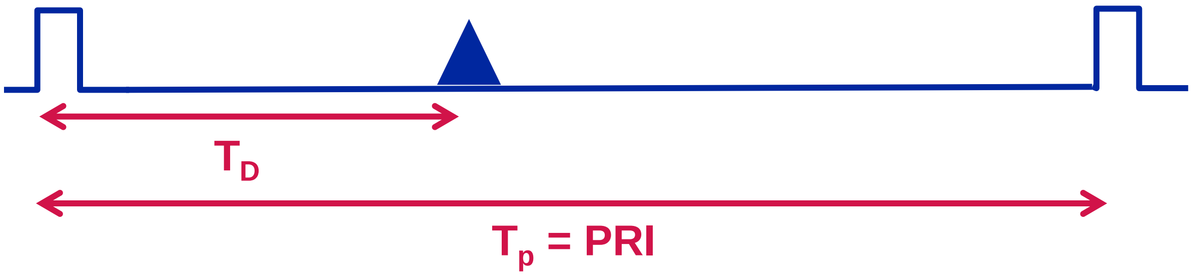
- Measures (varies with radar):
    - Azimuth, Elevation
    - Range
    - Doppler shift
    - Amplitude
    - Polarization

Radio Detection and Ranging



Round-trip path length: $c\,T_D = 2\,R$

Range: $R = c\,T_D / 2$



T_D=Time delay between transmission and reception of echo

$T_D = 1\ \mu s \Leftrightarrow R = 150\ m$

$T_D = 10\ \mu s \Leftrightarrow R = 1.5\ km$

$T_D = 100\ \mu s \Leftrightarrow R = 15\ km$

$T_D = 1\ ms \Leftrightarrow R = 150\ km$

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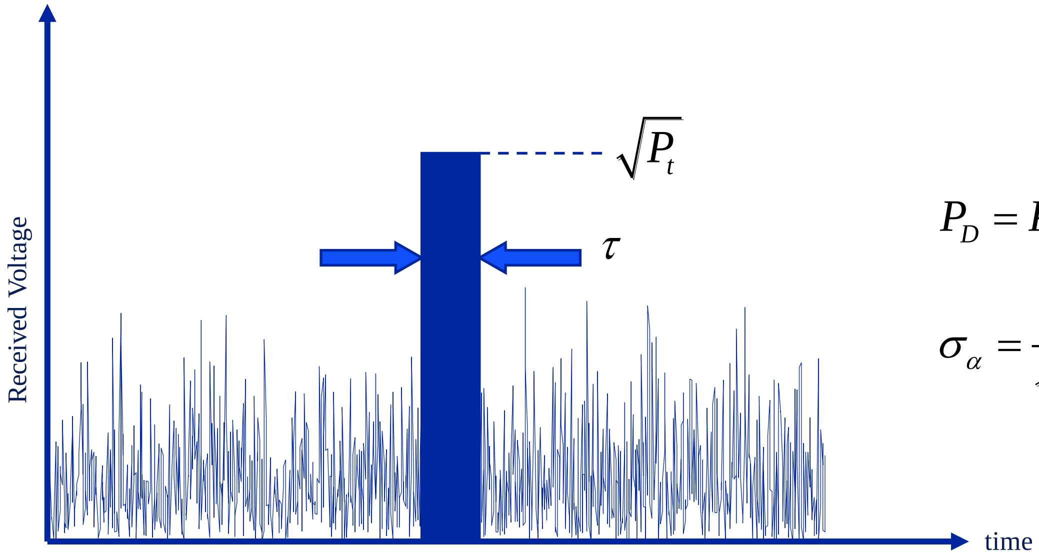
Signal-To-Noise Ratio

- **Receiver thermal noise is always present and limits radar performance**
- **Target detection and measurement accuracy are a functions of target signal-to-noise ratio (*SNR*)**
- **Desire is to maximize *SNR***



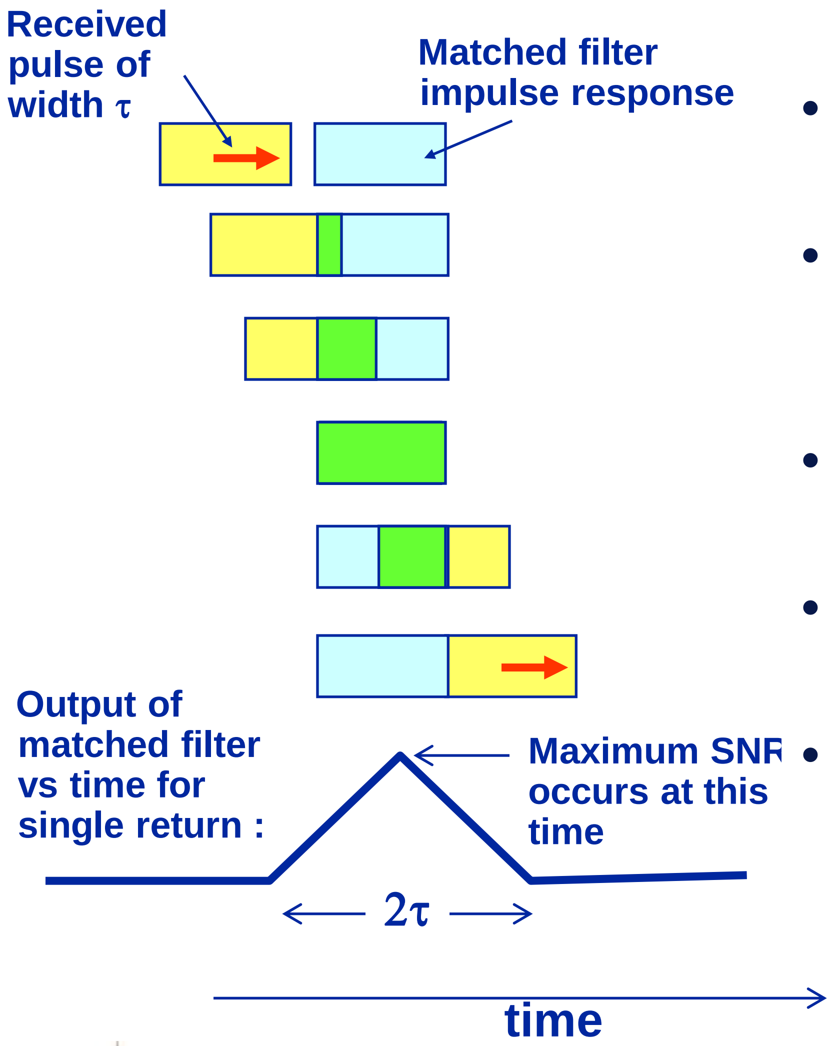
$$P_D = P_{FA}^{\frac{1}{1+SNR}}$$

Probability of detection of a target given a desired probability of false alarm

$$\sigma_\alpha = \frac{\alpha}{\sqrt{SNR}}$$

General measurement standard deviation

Matched Filter Response



- Filter received returns with the "matched filter"
- Matched Filter = the time-reversed conjugate of the transmitted waveform
- Filtering is performed as a convolution in time
- Therefore, matched filtering = a conjugate correlation
- The matched filter maximizes SNR when filter is aligned with a point target return

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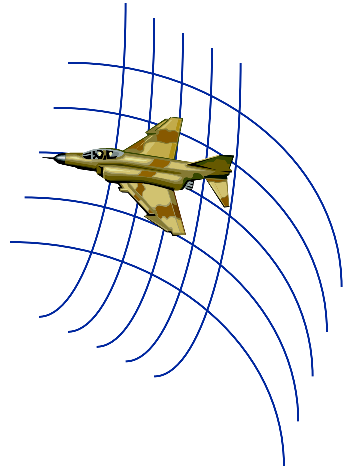
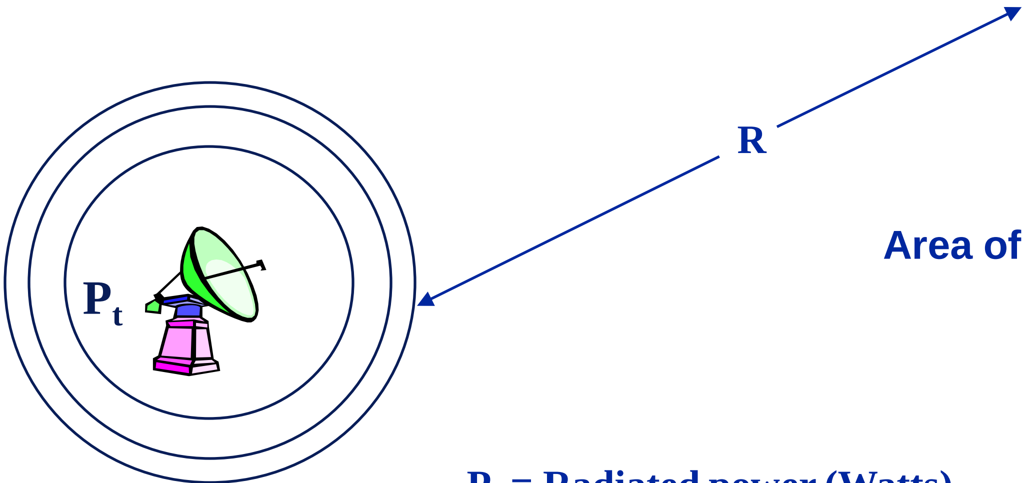
RADAR RANGE EQUATION (RRE):
Power Density at Range R for Isotropic Radiation

$$D_{isotropic} = P_t \left(\frac{1}{4\pi R^2} \right)$$

(Watts/m^2)





Area of sphere = $4\pi R^2$

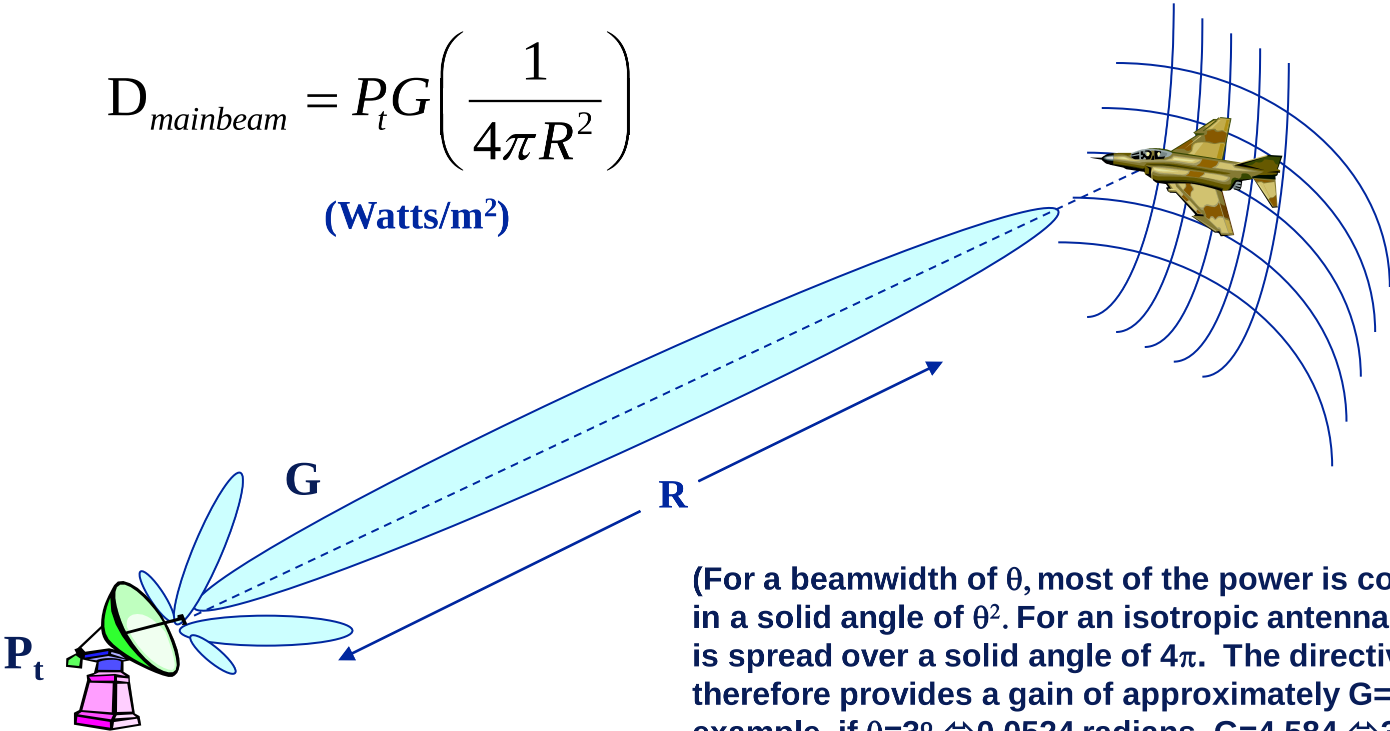
P_t = Radiated power (Watts)

RADAR RANGE EQUATION :

Power Density at Range R for Main Beam of Directive Antenna

$$D_{mainbeam} = P_t G \left(\frac{1}{4\pi R^2} \right)$$

(Watts/m²)



(For a beamwidth of θ, most of the power is concentrated in a solid angle of θ^2. For an isotropic antenna, the power is spread over a solid angle of 4π. The directive antenna therefore provides a gain of approximately $G=4\pi/\theta^2$. For example, if θ=3º ⇔0.0524 radians, G=4,584 ⇔36.6 dBi.)

P_t G = Effective radiated power (ERP) (Watts)

RADAR RANGE EQUATION :
Power Captured by Target

(Watts)

$$\sigma = \lim_{R\to\infty}\left(4\pi R^2 \frac{|E_r|^2}{|E_t|^2}\right)$$

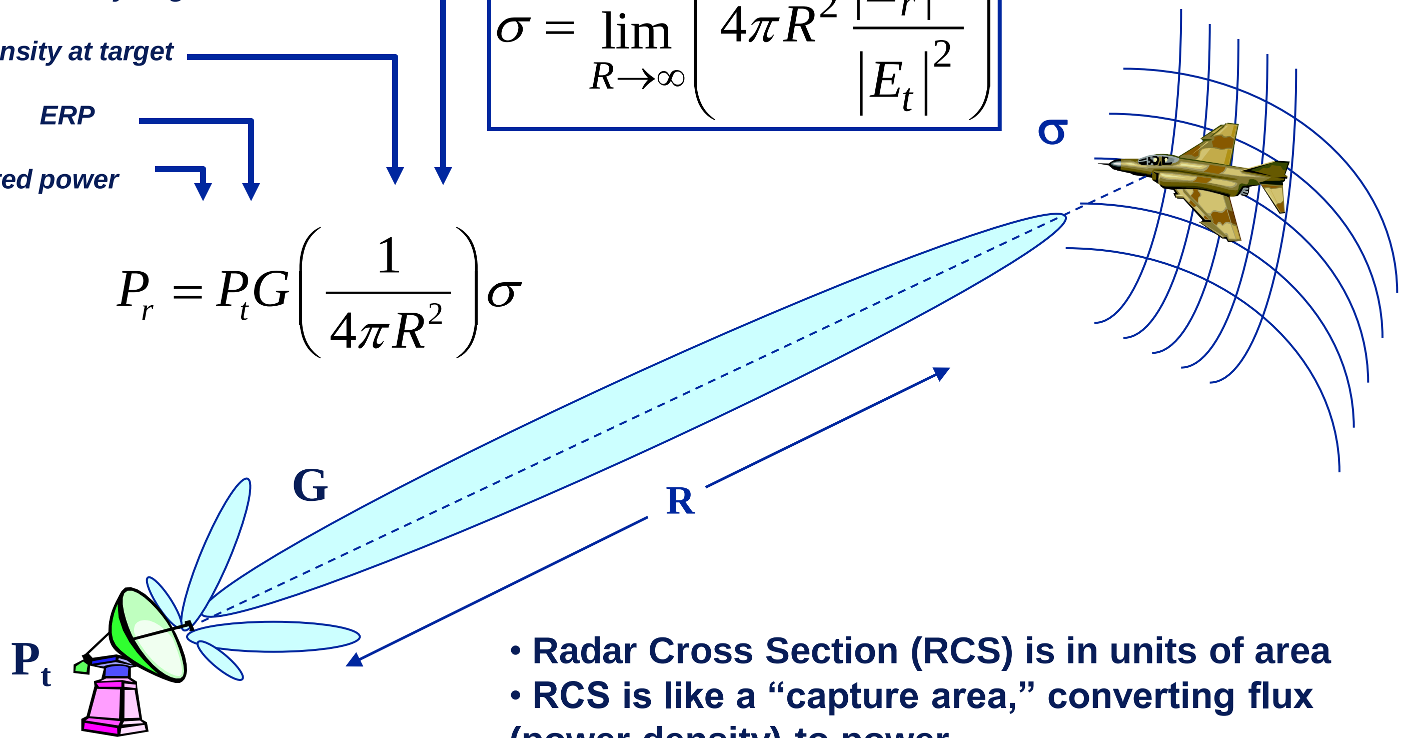
Power reflected by target

Power density at target

ERP

Radiated power

$$P_r = P_t G \left(\frac{1}{4\pi R^2}\right)\sigma$$



- Radar Cross Section (RCS) is in units of area
- RCS is like a "capture area," converting flux (power density) to power
- This captured power is assumed to be uniformly scattered over 4π steradians

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Representative RCS Values

Mean Radar Cross Section of Typical Targets at 1.3-10 GHz

• **Aircraft (Nose and Tail Aspect)**	
Small General Aviation	0.6 - 3 m^2
Small Fighters	1.5 - 4 m^2
Medium Fighters (F-4, *et cetera*)	4.0 - 10 m^2
Small Commercial (DC-9)	10 - 20 m^2
Medium Commercial (707, DC-8)	20 - 40 m^2
Large Commercial (DC-10, 747)	40 - 100 m^2
• **Ships (5-10 GHz; Approximate Frequency Dependence = $f^{1/2}$)**	
Sailboats -- Small	0.5 - 5 plus mast
Military Power Boats	20 - 500 m^2
Frigates (1-2 ktons)	0.5 - 1.0 x 10^4 m^2
Destroyers (3-5 ktons)	3.0 - 6.0 x 10^4 m^2
Cruisers (7-20 ktons)	10 - 40 x 10^4 m^2
Carriers (20-40 ktons)	30 - 100 x 10^4 m^2
• **Tanks**	20 - 200 m^2
• **Personnel**	0.3 - 1.2 m^2
• **Birds**	
Sparrow and Starling	0.0003 - 0.001 m^2
Pigeon (25-40 knots)	0.001 - 0.01 m^2
Mallard (25-40 knots)	0.01 m^2
• **Insects**	
Moths	0.0001 to 0.000001 m^2 (-40 to -70 dBsm)

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RADAR RANGE EQUATION :

Received Power Density at Radar Antenna

(Watts)

Power density at radar

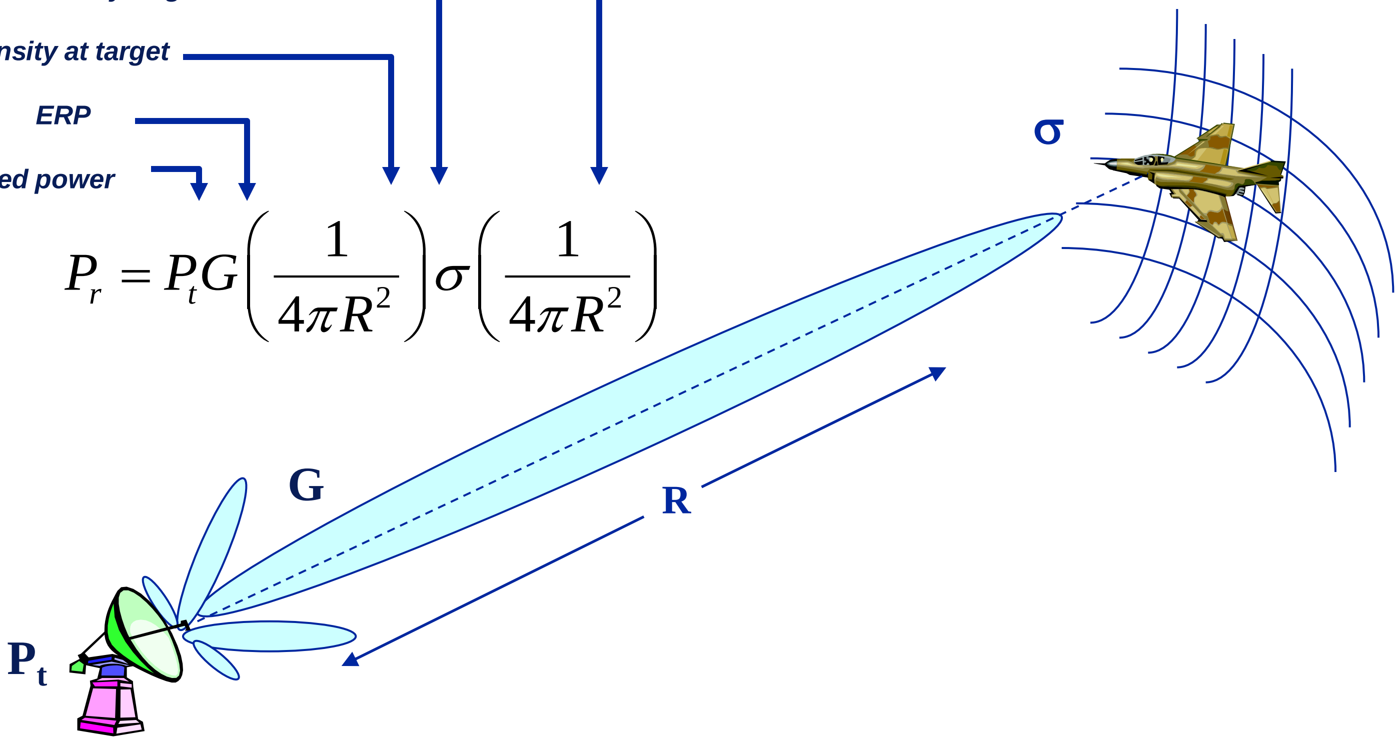
Power reflected by target

Power density at target

ERP

Radiated power

$$P_r = P_t G \left(\frac{1}{4\pi R^2} \right) \sigma \left(\frac{1}{4\pi R^2} \right)$$



RADAR RANGE EQUATION :

Peak Received Power through Radar Antenna

Power captured by radar antenna

Power density at radar

Power reflected by target

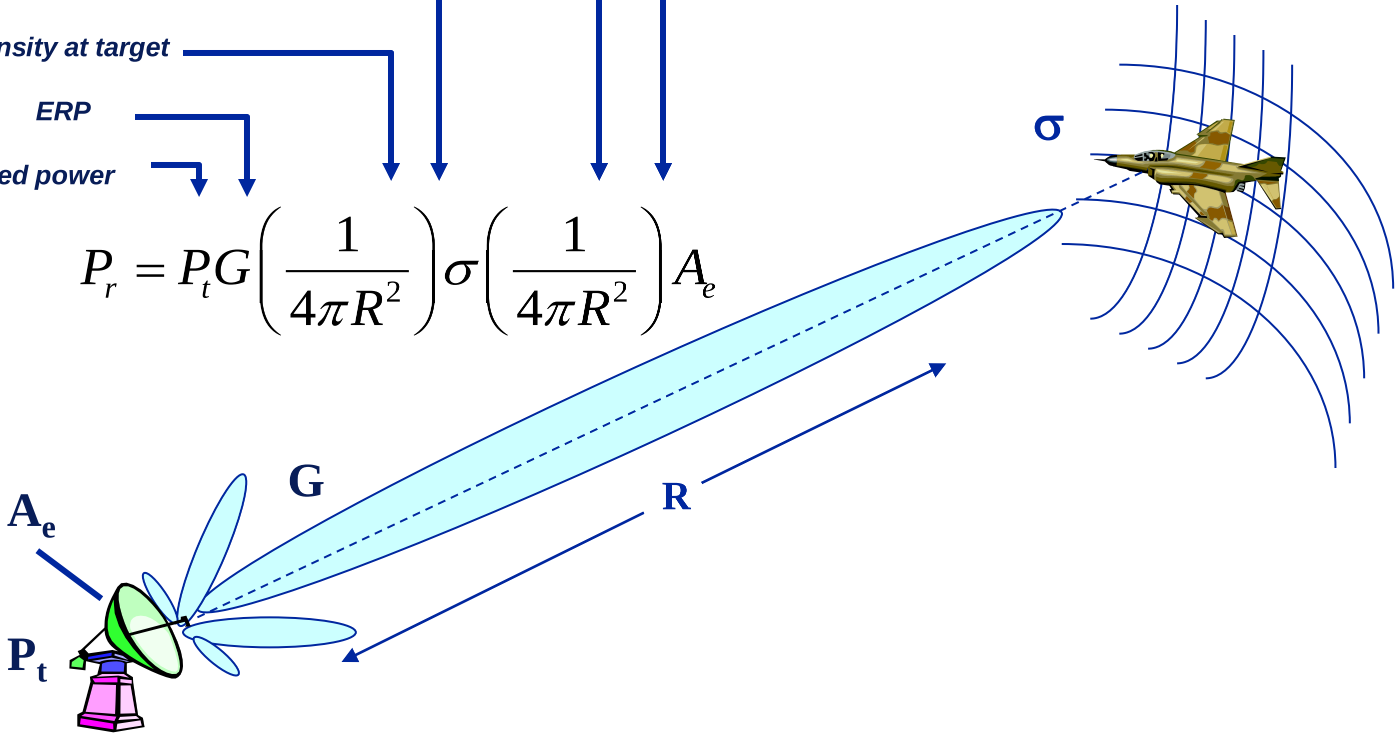
Power density at target

ERP

Radiated power

(Watts)

$$P_r = P_t G \left(\frac{1}{4\pi R^2} \right) \sigma \left(\frac{1}{4\pi R^2} \right) A_e$$



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Power density at radar

Power reflected by target

Power density at target

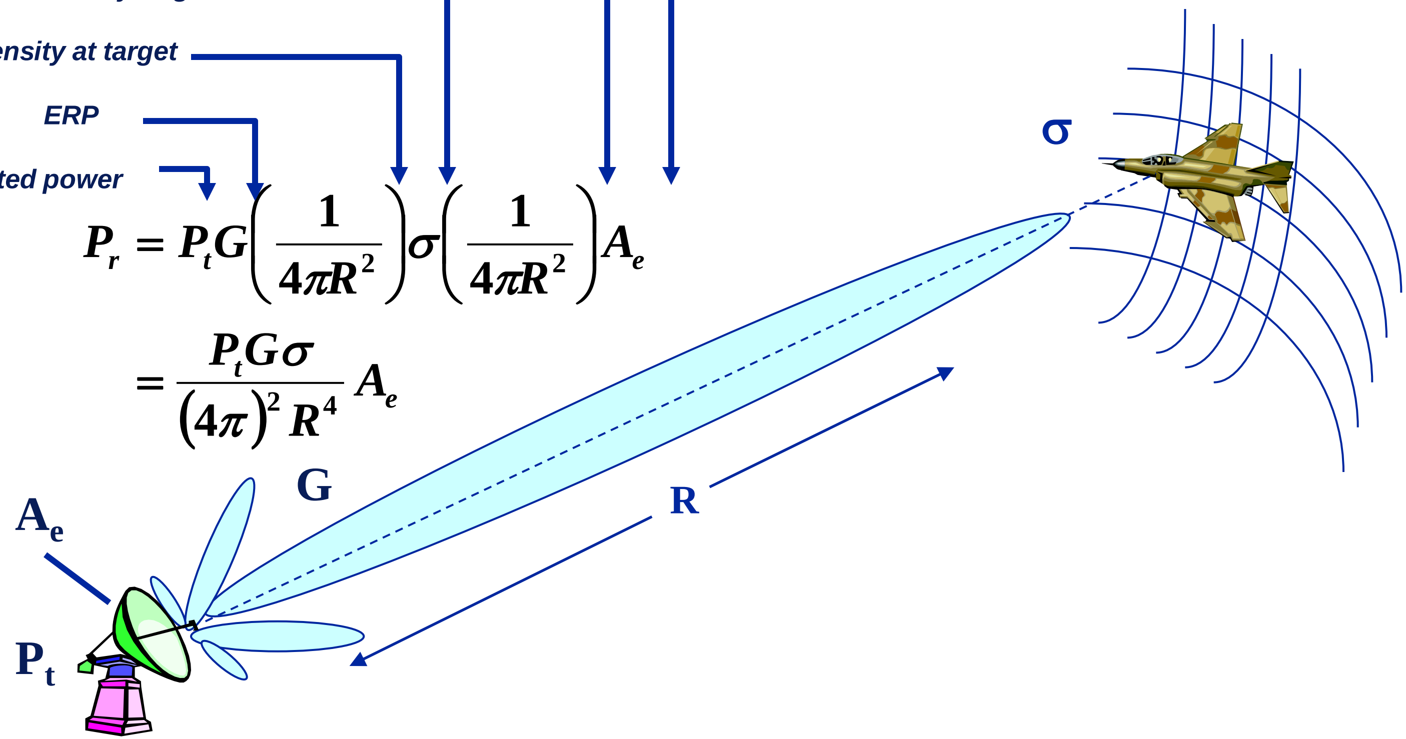
ERP

Radiated power

(Watts)

$$P_r = P_t G \left(\frac{1}{4\pi R^2} \right) \sigma \left(\frac{1}{4\pi R^2} \right) A_e$$

$$= \frac{P_t G \sigma}{(4\pi)^2 R^4} A_e$$



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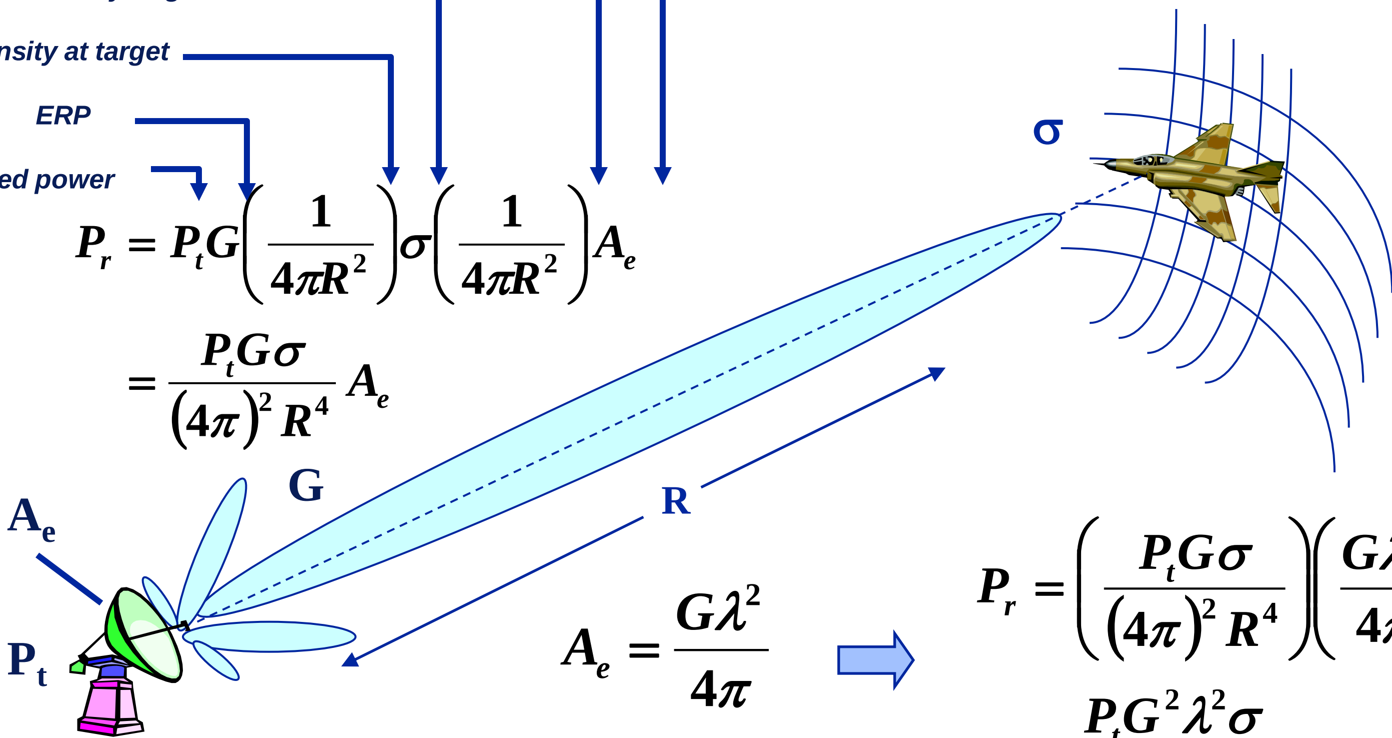


$$A_e = \frac{G\lambda^2}{4\pi}$$

$$P_r = \left(\frac{P_t G \sigma}{(4\pi)^2 R^4} \right) \left(\frac{G\lambda^2}{4\pi} \right)$$

$$= \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Receiver Noise

The target signal competes with receiver thermal noise:

Thermal noise power = kT_0BF

$k = 1.38 \times 10^{-23}$ J/°K (Boltzman's constant)
T_0 = 290 °K
F = receiver noise factor (noise figure)
T_S = system noise temperature = T_0F
B = receiver bandwidth (Hz)

Bandwidth	kTB (dBm)
1 Hz	-174
10 Hz	-164
100 Hz	-154
1 kHz	-144
10 kHz	-134
100 kHz	-124
1 MHz	-114
10 MHz	-104
100 MHz	-94
1 GHz	-84

- This is an average power level
- Noise power randomly fluctuates above and below this level according to its statistical properties

RADAR RANGE EQUATION:

Single Pulse Signal-to-Noise Power Ratio

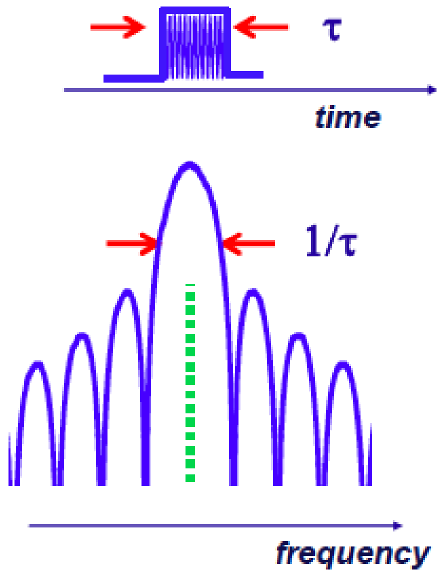
$$\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_s k T_0 B F}$$

System losses

- **Note the dependence on "R-to-the-4th"**
  - **Doubling range decreases SNR 12 dB**
  - **100 km to 10 km is a 40 dB increase in SNR**

Bandwidth and Pulsewidth

The bandwidth of an unmodulated narrow pulse is approximately B=1/τ



RADAR RANGE EQUATION:

Single Pulse Energy-to-Noise-Density Ratio

$$\frac{S}{N} = \frac{E}{N_0} = \frac{P_t \tau G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_s k T_0 F}$$

- SNR power looks more like energy
  - Form is reminiscent of Eb/No in comms

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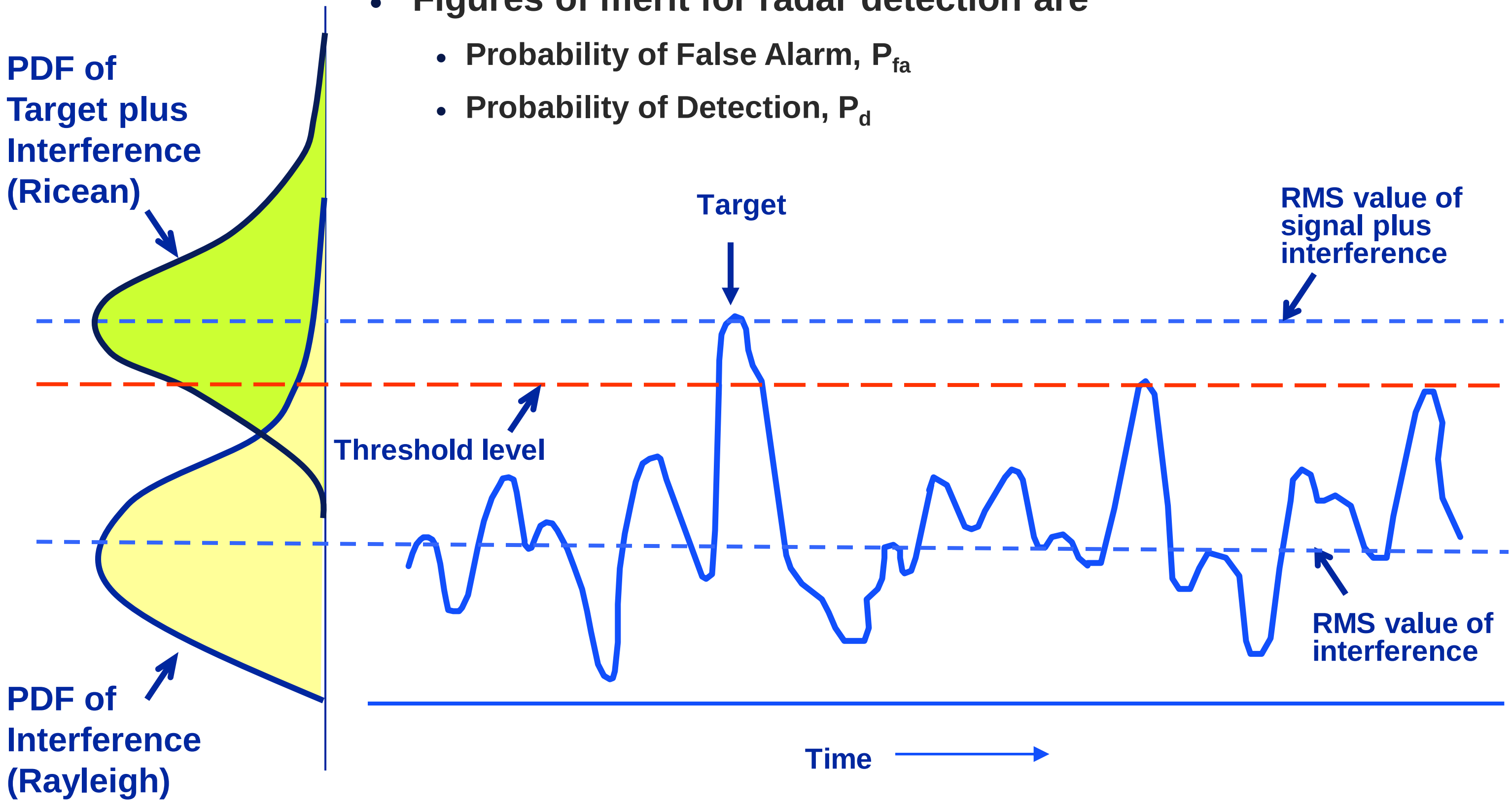
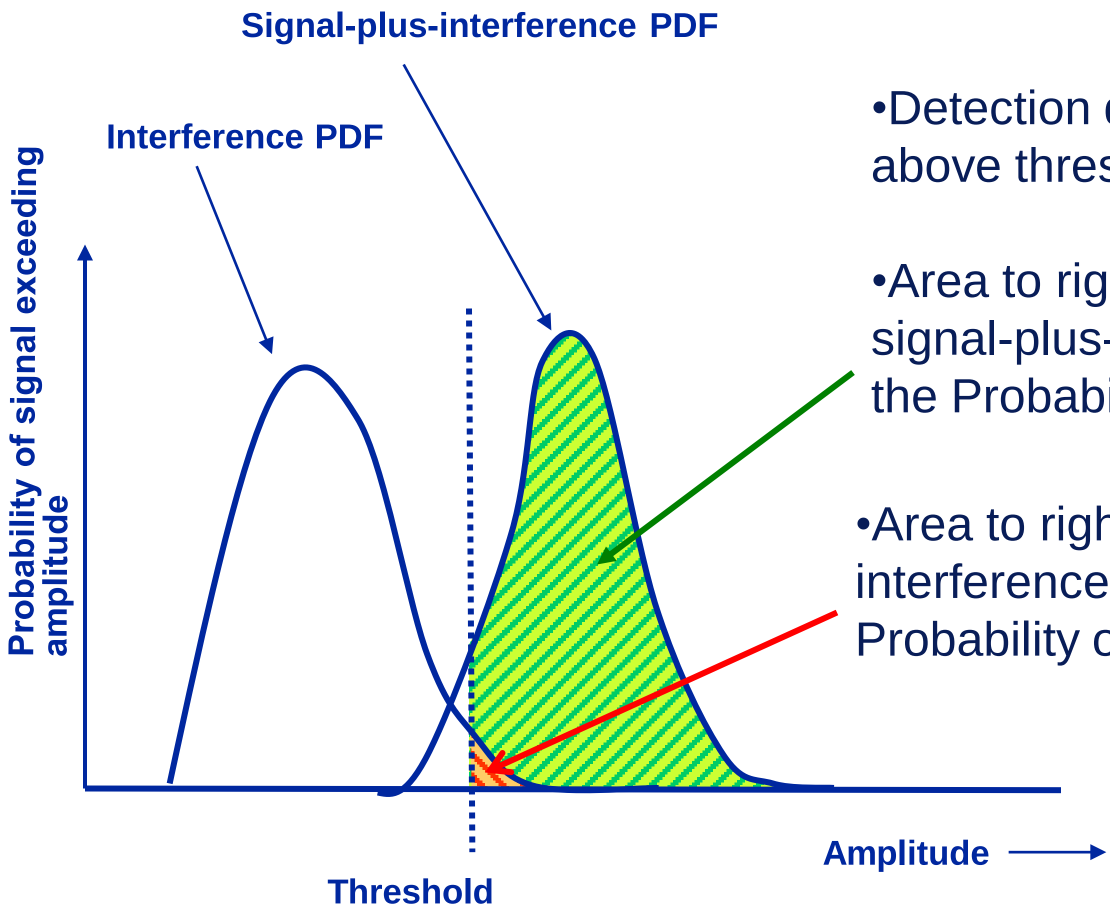
Detection

- Radar signals and noise are random variables
- Radar range equation represents **mean** signal-to-noise ratio
- Figures of merit for radar detection are
  - Probability of False Alarm, P_{fa}
  - Probability of Detection, P_d



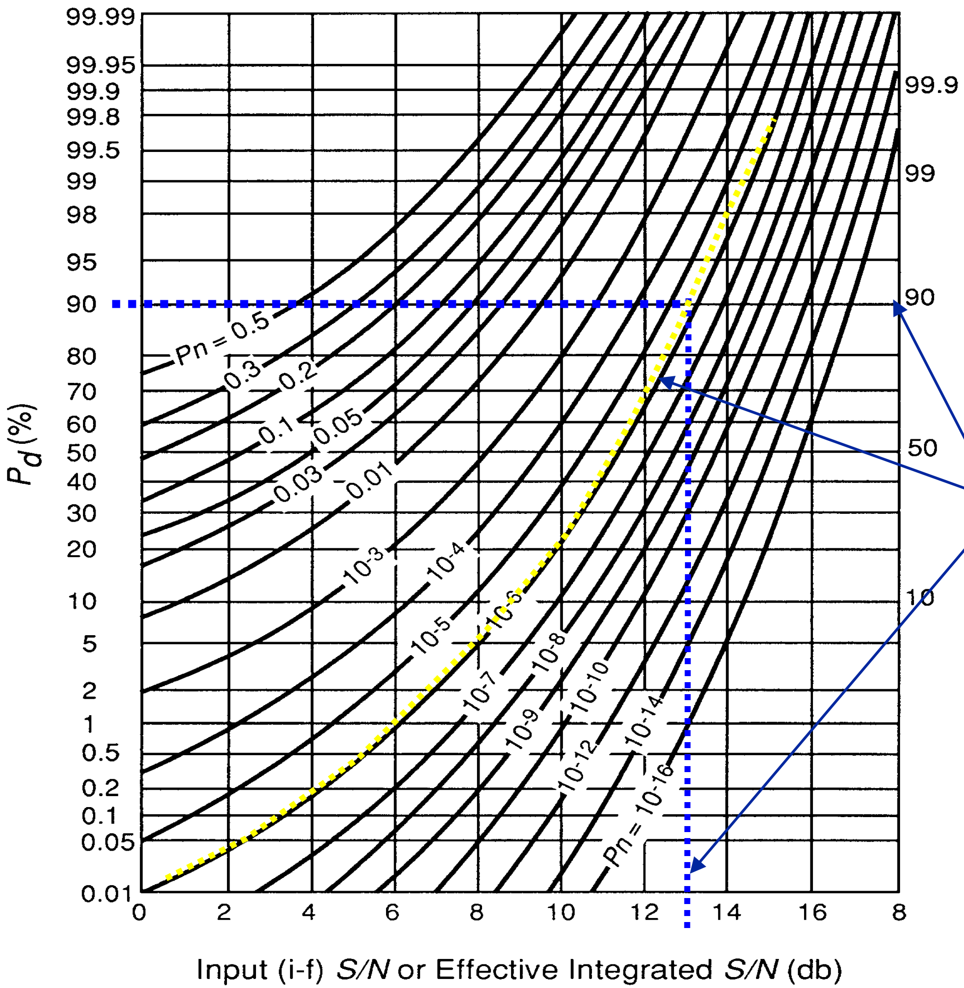
(PDF = probability density function)

Detection Processing
Probability Density Functions (PDFs)



- Detection declared if signal is above threshold
- Area to right of threshold under signal-plus-interference PDF is the Probability of Detection (Pd)
- Area to right of threshold under interference PDF is the Probability of False Alarm (Pfa)

Radio Detection and Ranging

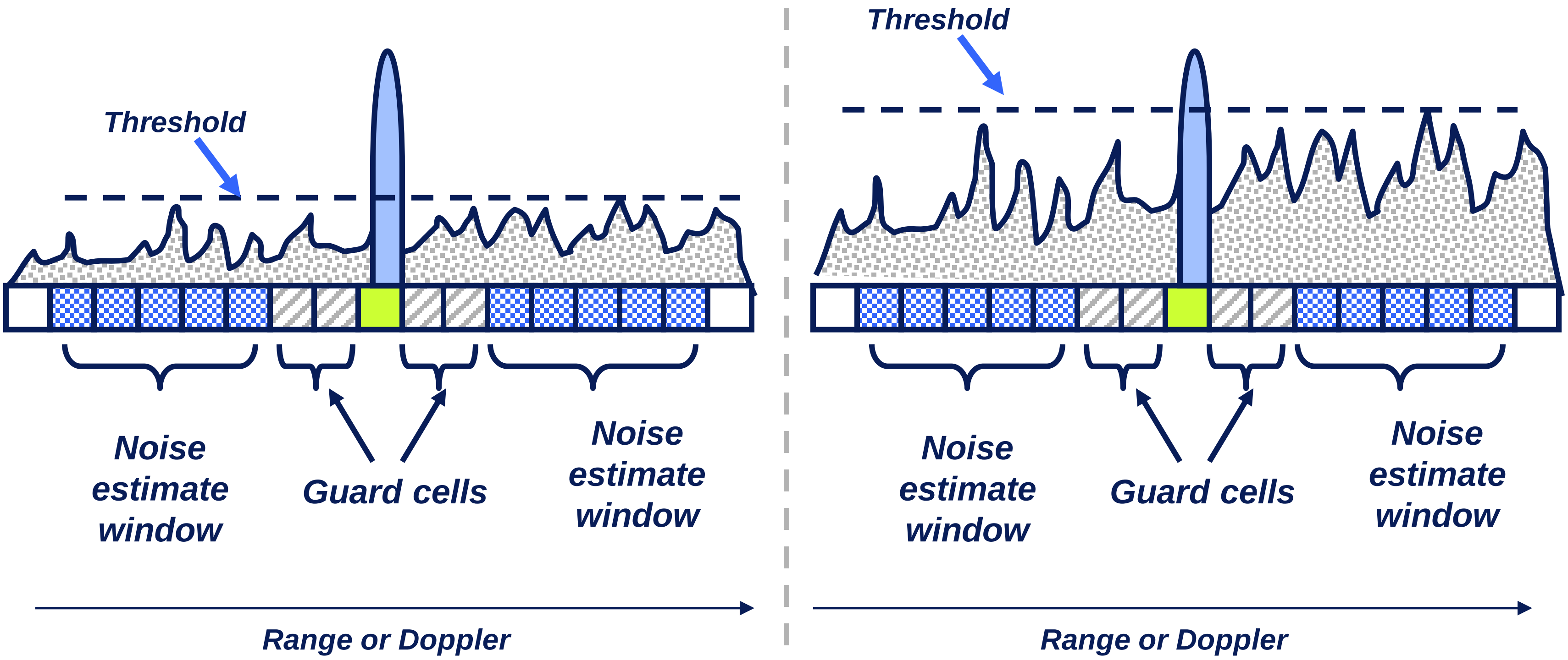


Non-fluctuating target
Thermal (Gaussian) noise

13.2 dB SNR provides:
90% P_d with 10^{-6} Pfa.

Higher SNR is required for fluctuating target and clutter interference.

Constant False Alarm Rate (CFAR) Receiver: Threshold Adjusts to Estimated Noise Level



- **Radar detection theory and application is 2-step**
  1. **Set threshold to meet false alarm requirements**
  2. **Predict/realize detections – it is what it is!**

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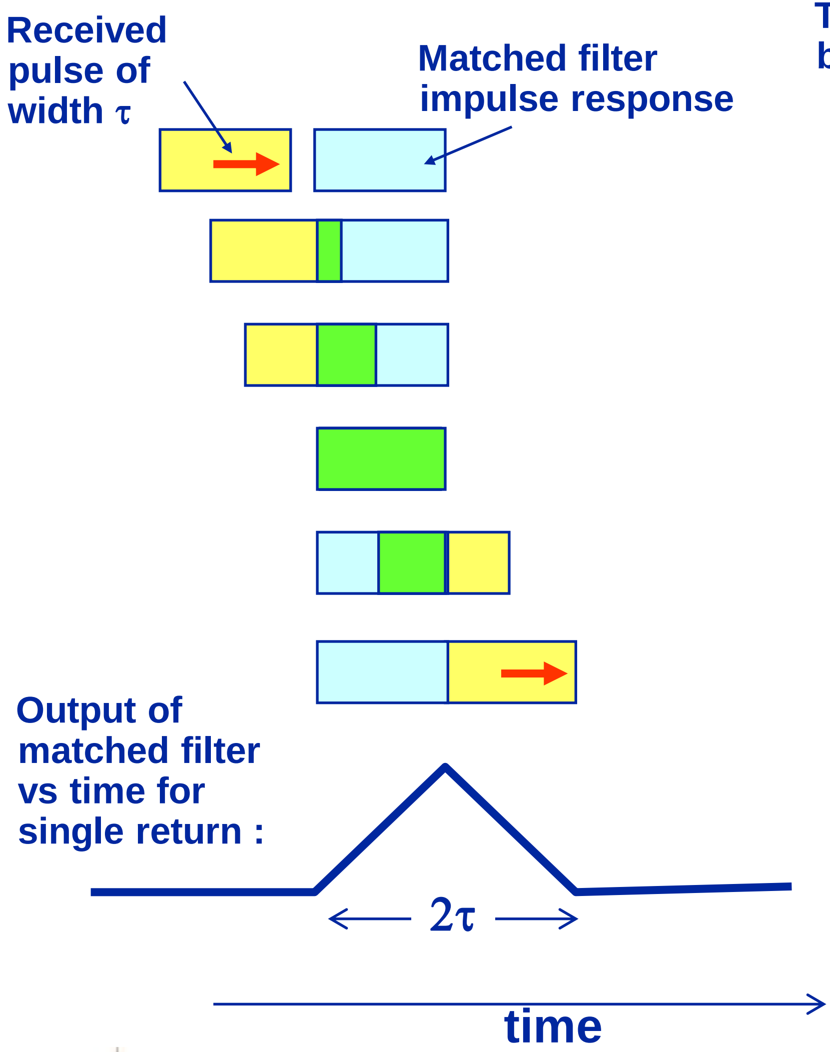
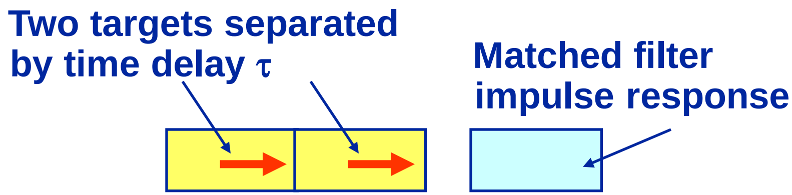
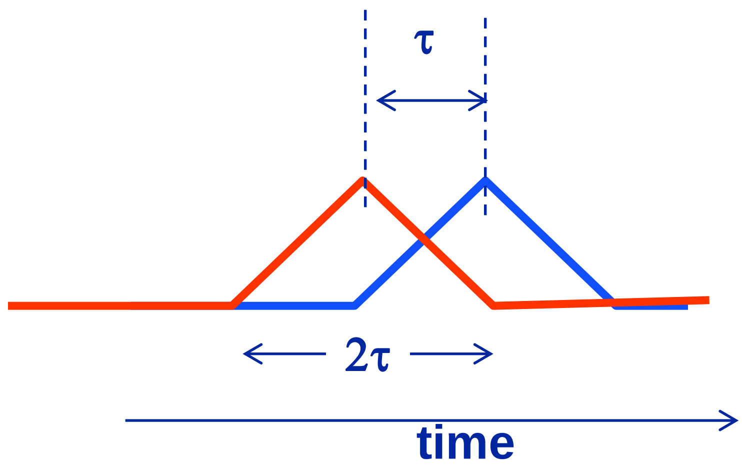
Matched Filter Response



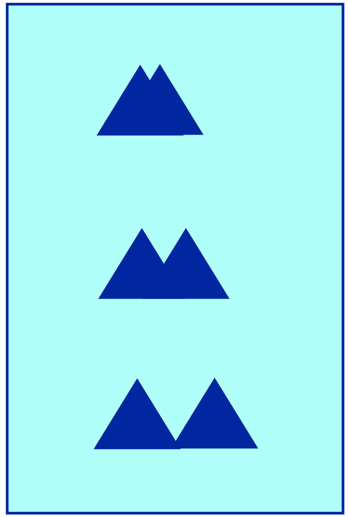


Output of matched filter vs time for two returns separated in time by τ. Peak of one roughly coincides to null of other (“Rayleigh criterion”).



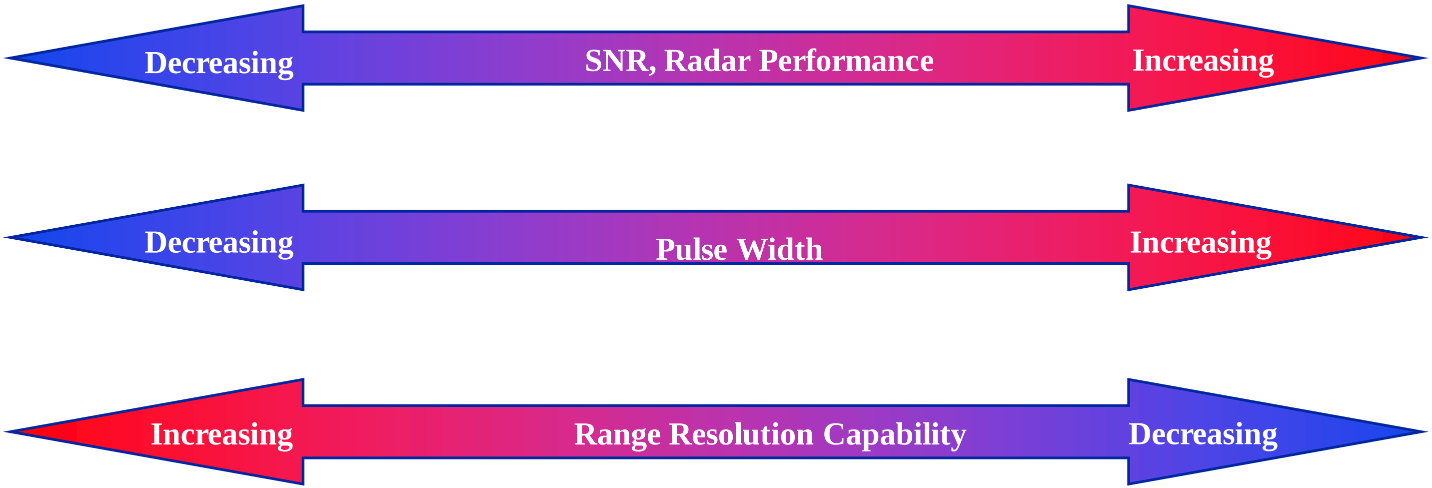
RANGE RESOLUTION OF UNMODULATED PULSE

- Range resolution is the minimum range separation at which two point scatterers of equal size are distinguishable as separate scatters
- Increased range resolution is desirable:
  - Range measurement accuracy
  - Multiple target detection/tracking
  - Clutter reduction
  - Target ID
  - EP vs certain types of jammers
- For an unmodulated pulse of width τ, the resolution is $\delta_R = c\tau/2$
- Example:
  - τ = 1 μs ⇔ δ_R=150 m
  - τ = 0.1 μs ⇔ δ_R=15 m



Competing Relationships

$$SNR = \frac{P_t \tau G^2 \lambda^2 \sigma}{k T_s (4\pi)^3 R^4}$$



$$\delta R = \frac{c\tau}{2}$$

For an unmodulated pulse there exists a coupling between range resolution andwaveform energy

Pulse Compression Waveforms

- **Permit a de-coupling between range resolution and waveform energy**

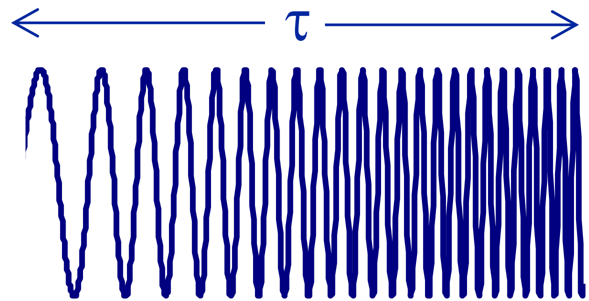
$$\delta r = k\,\frac{c}{2B}$$

$$SNR \propto \tau$$

  - **Resolution really depends on waveform bandwidth**
  - **SNR still depends on pulsewidth**
- **This de-coupling is achieved via some form of modulation**
    - **Amplitude**
    - **Phase**
    - **Frequency**
- **For pulse compression waveforms:** $B >> \frac{1}{\tau}$

Pulse Compression Techniques

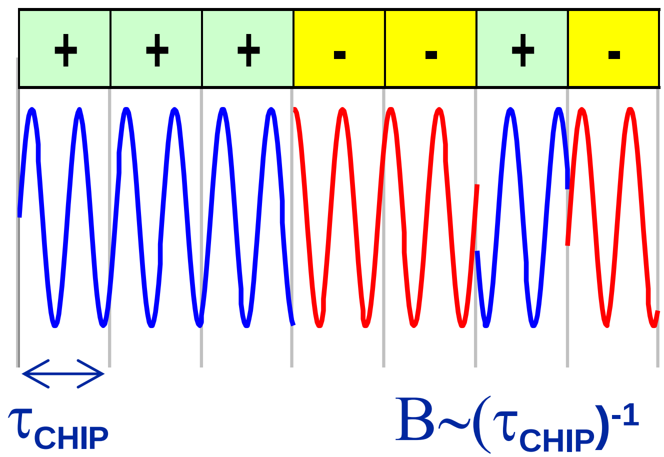
- Frequency Modulation
  - Linear FM
  - Non-linear FM



$$\phi(t) = 2\pi(f_0 t + \tfrac{1}{2}\nu t^2),\ [0 \le t \le \tau]$$

$$f(t) = f_0 + \nu t \qquad B = \nu\tau$$

- Phase Code
  - Bi-phase (0º/180º)
  - Pseudo-random "Noise" (PRN)
  - Quad-phase (0º/90º/180º/270º)



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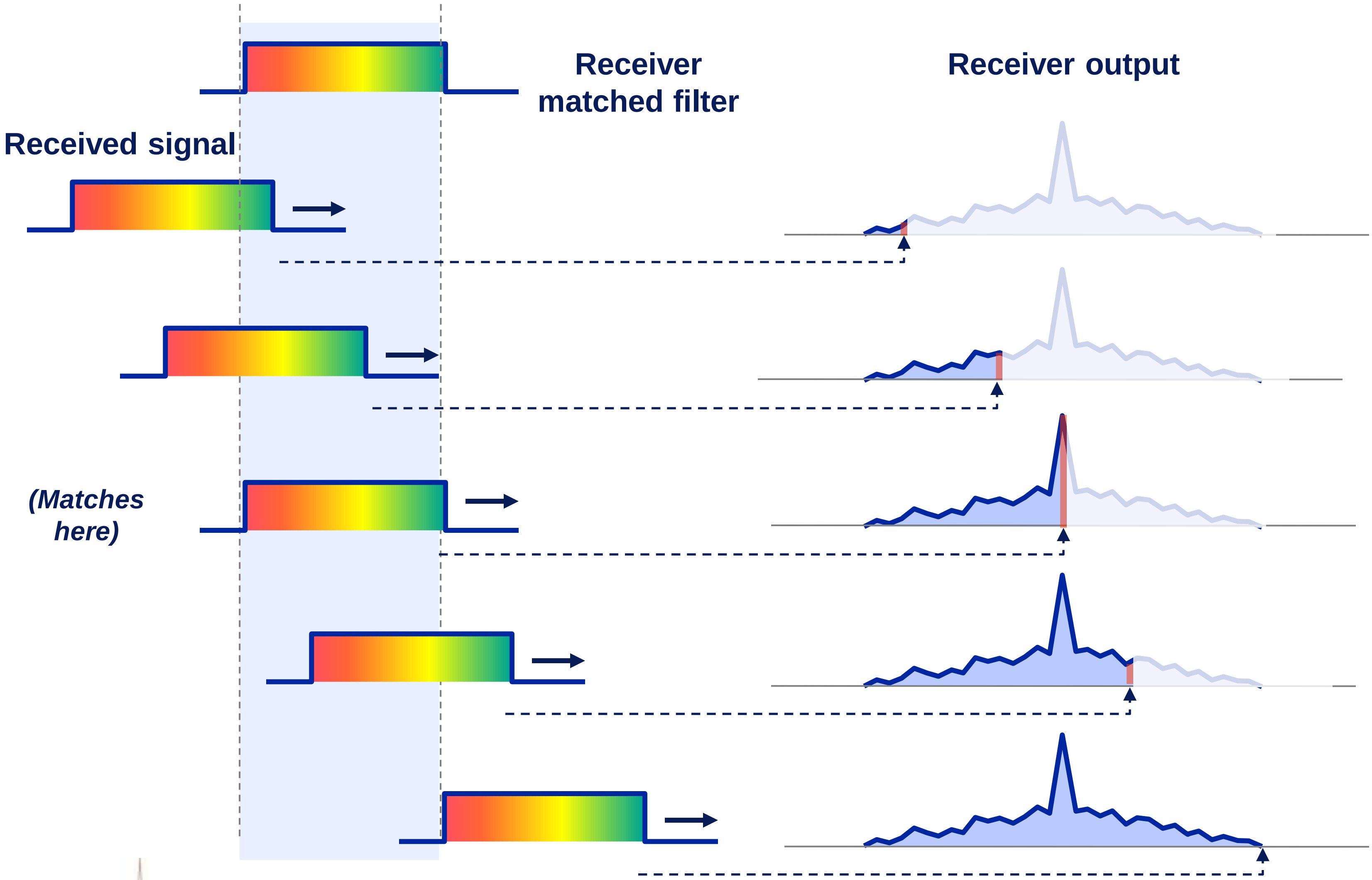
Pulse Compression Time Response of Single-Point Return



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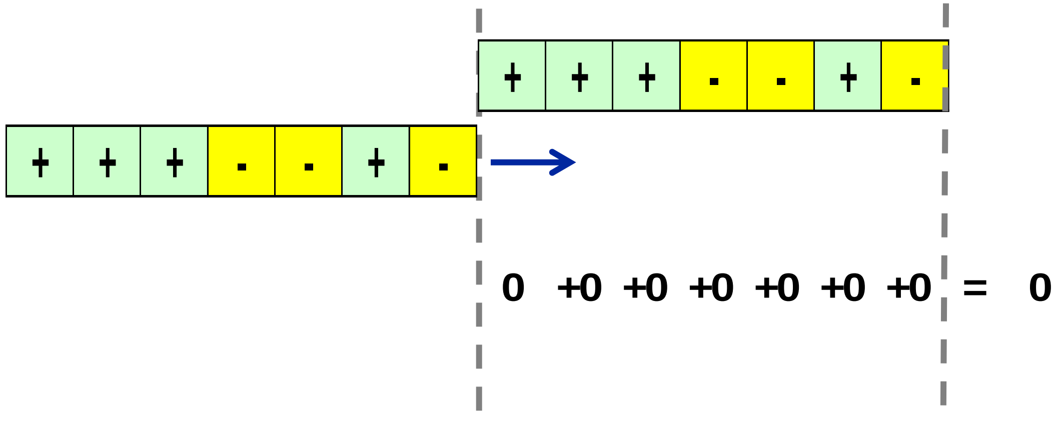
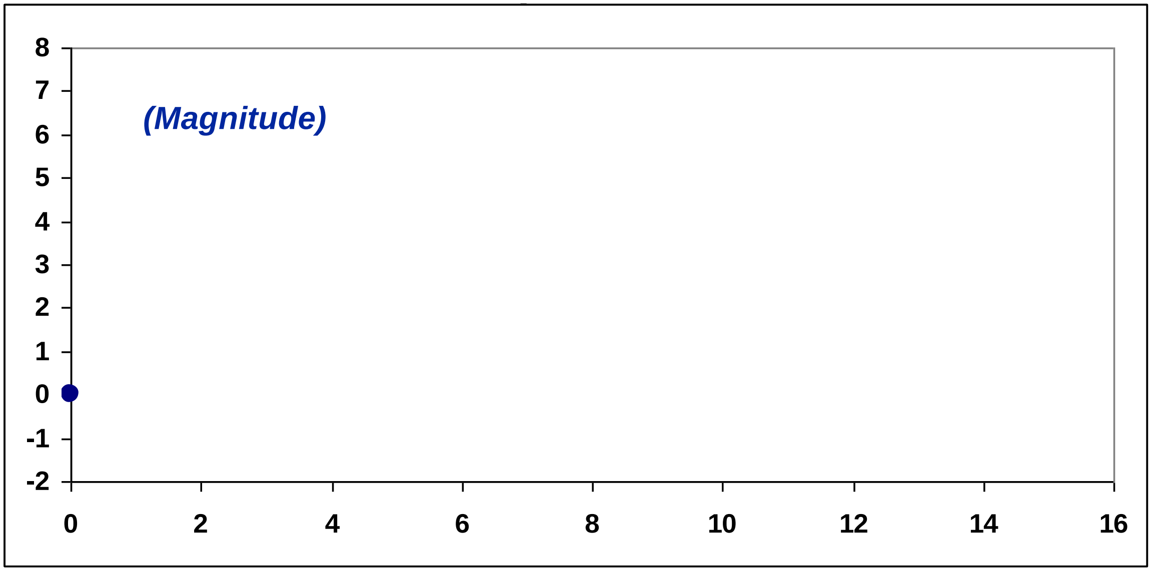
7-BIT BARKER CODE PULSE COMPRESSION





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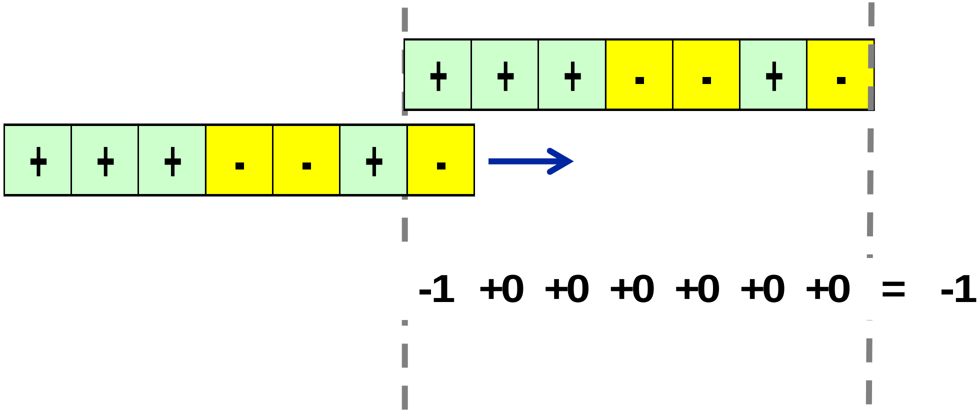

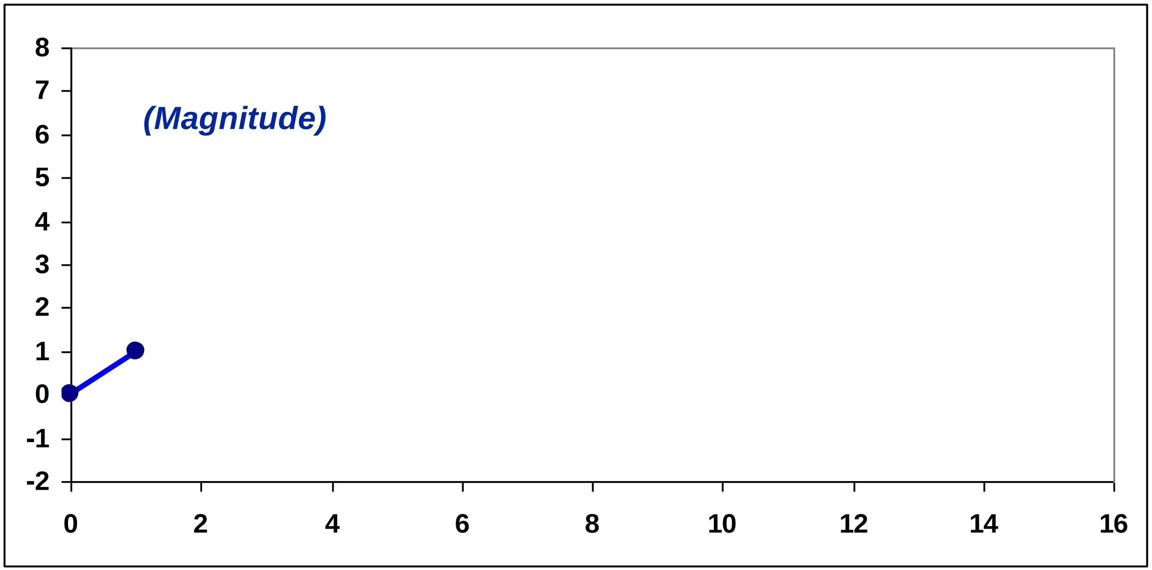

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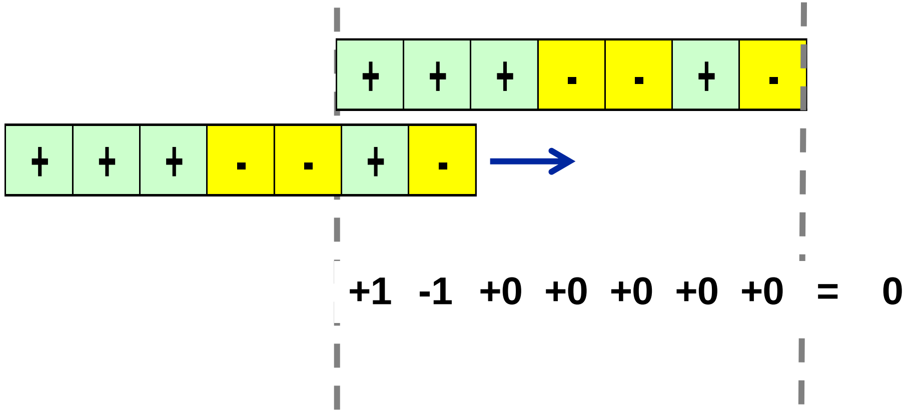

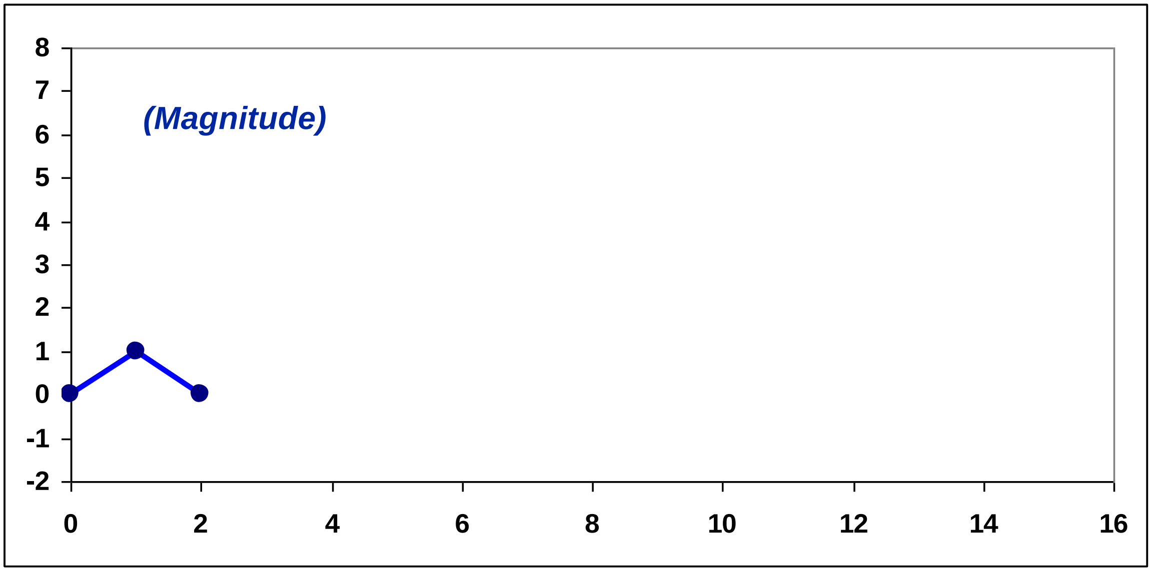

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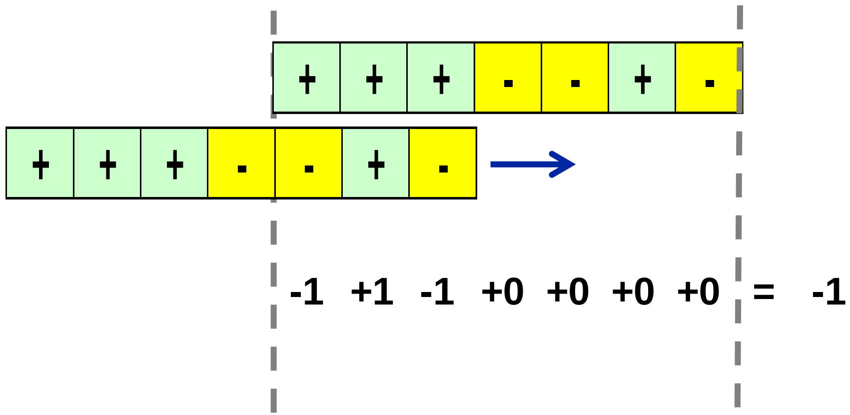

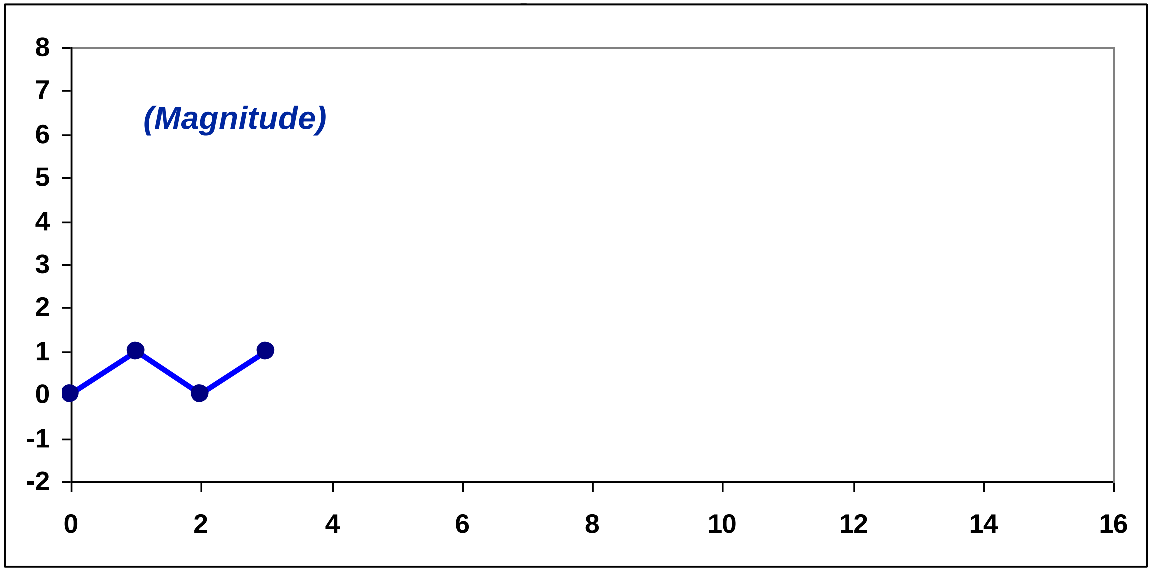

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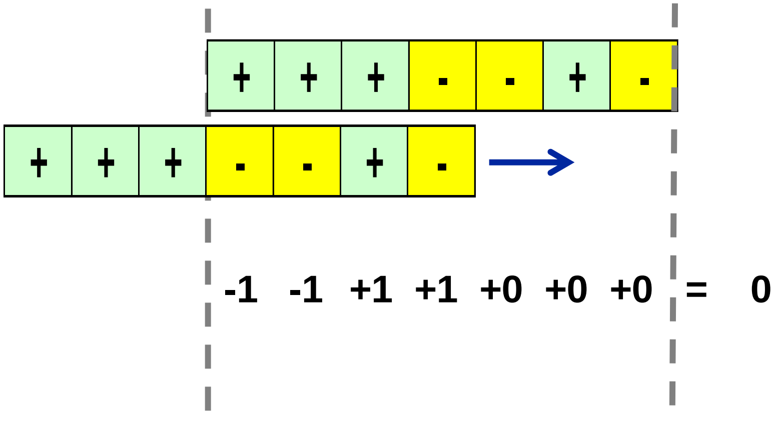
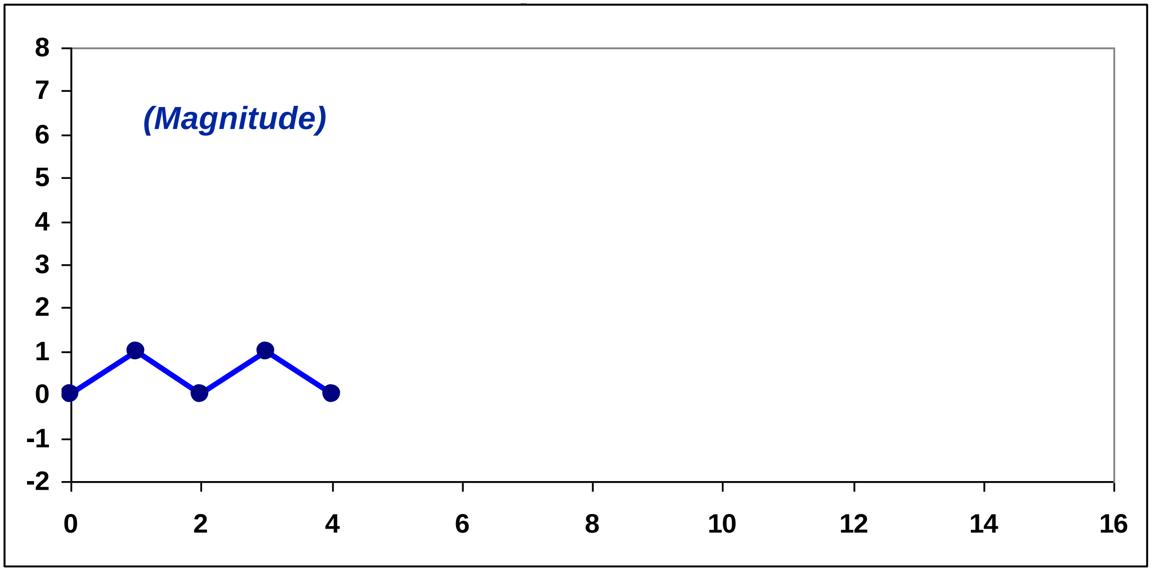
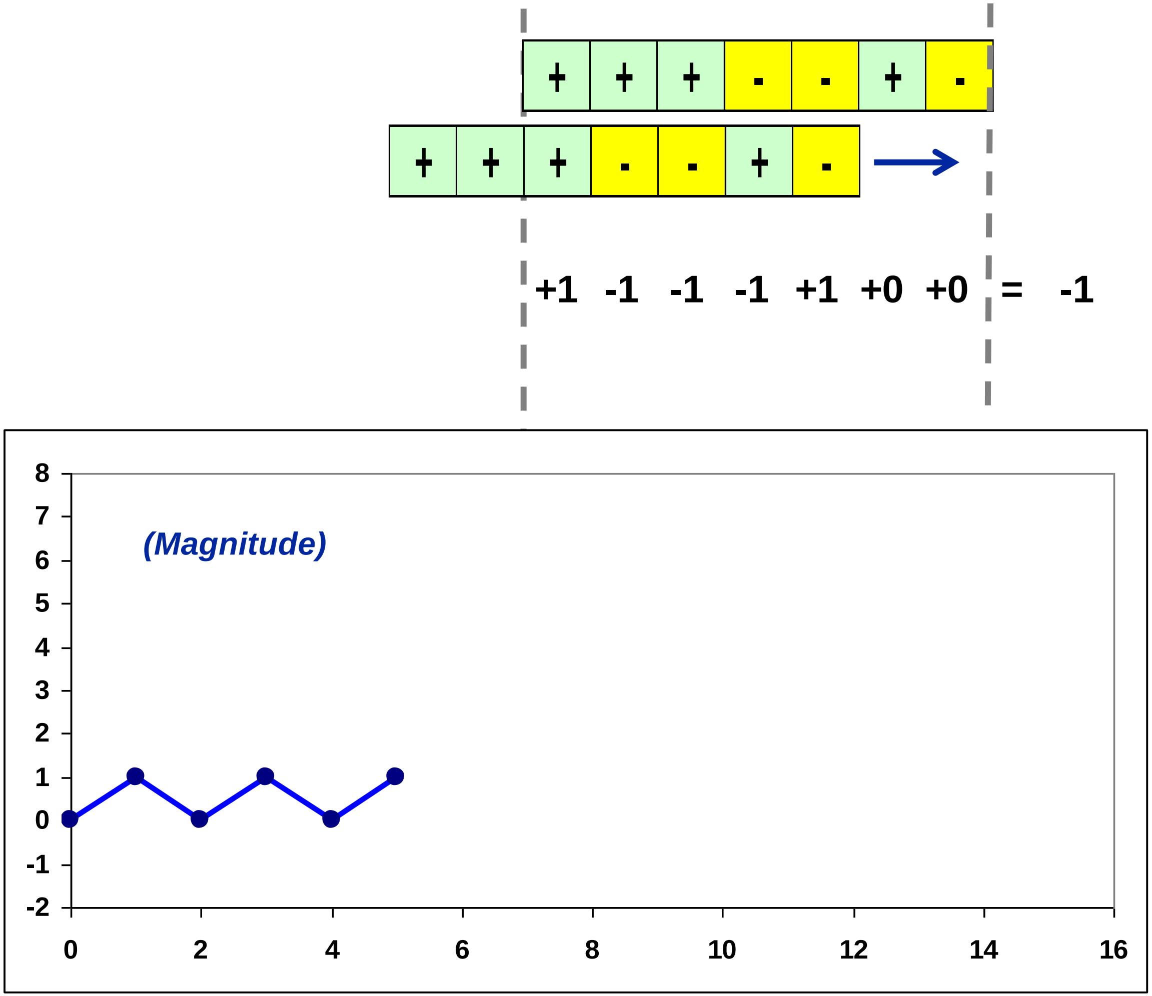




7-BIT BARKER CODE PULSE COMPRESSION



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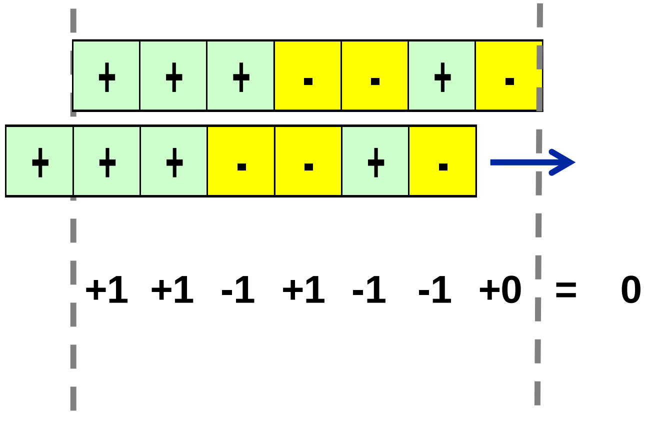

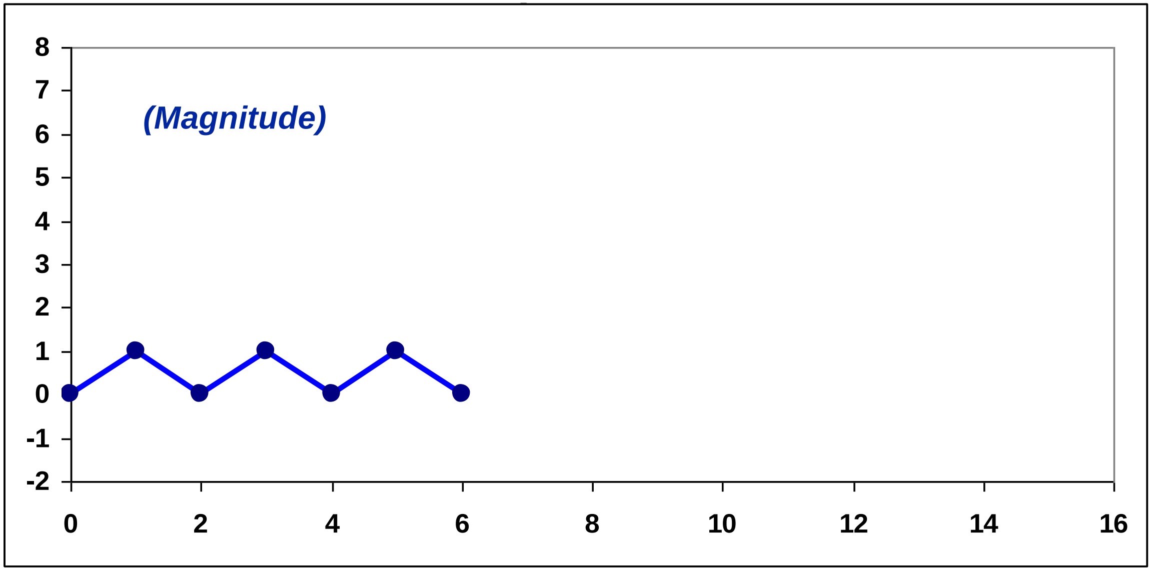

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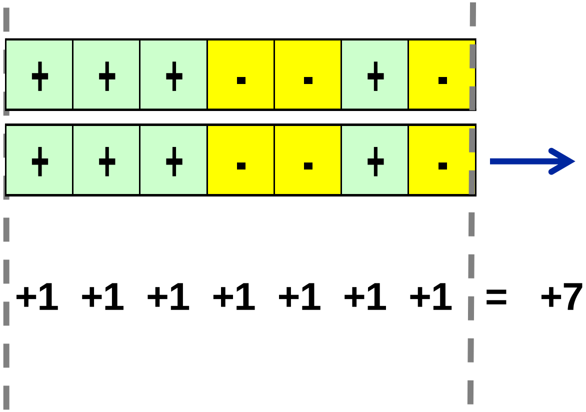

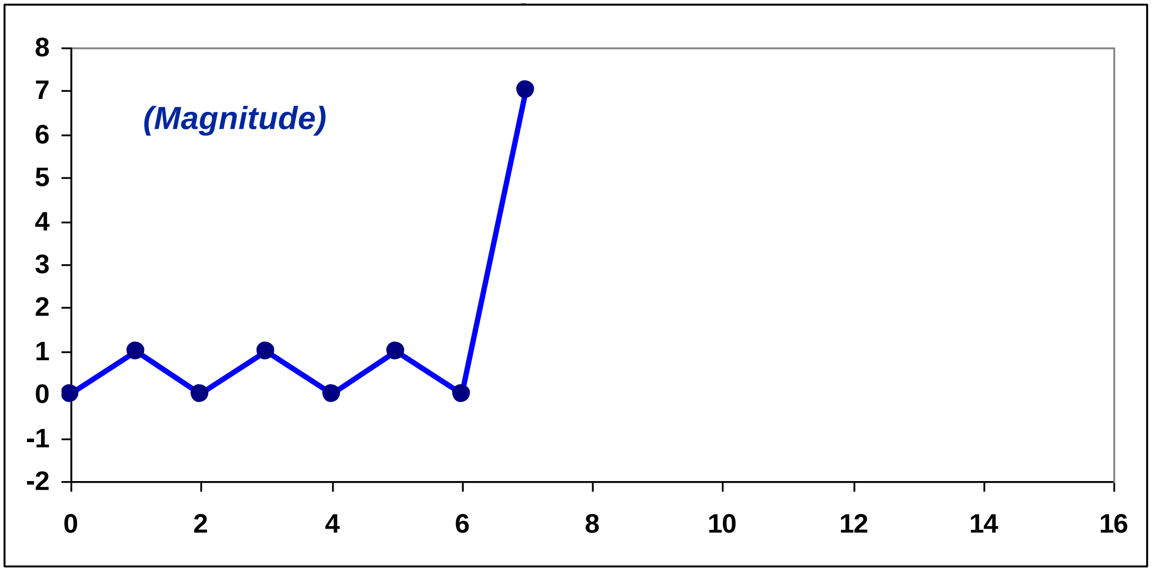

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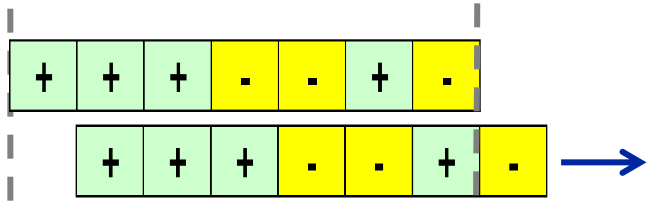

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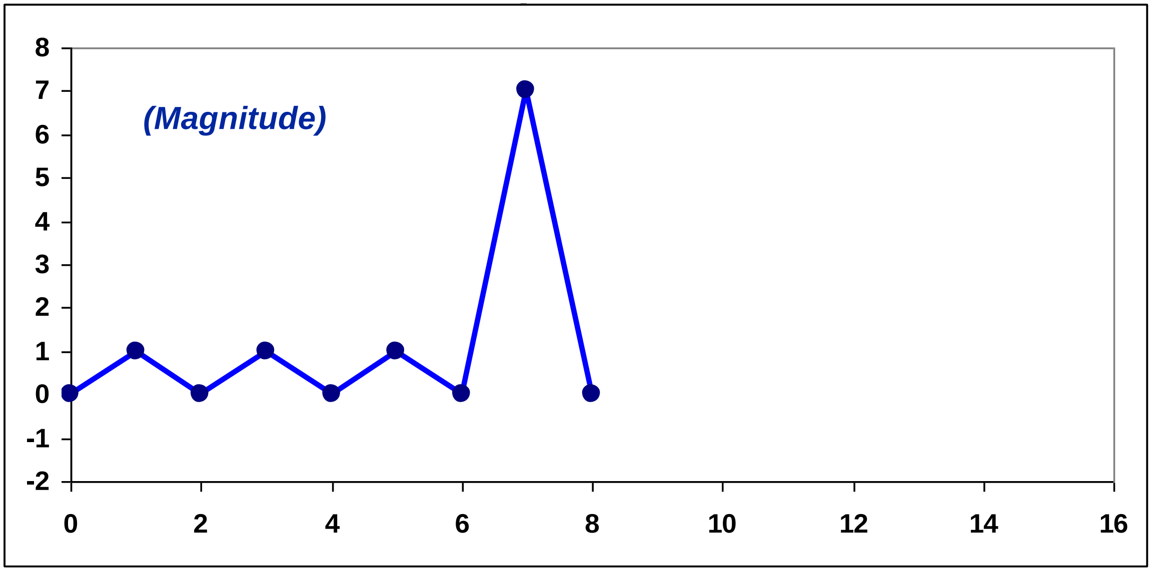

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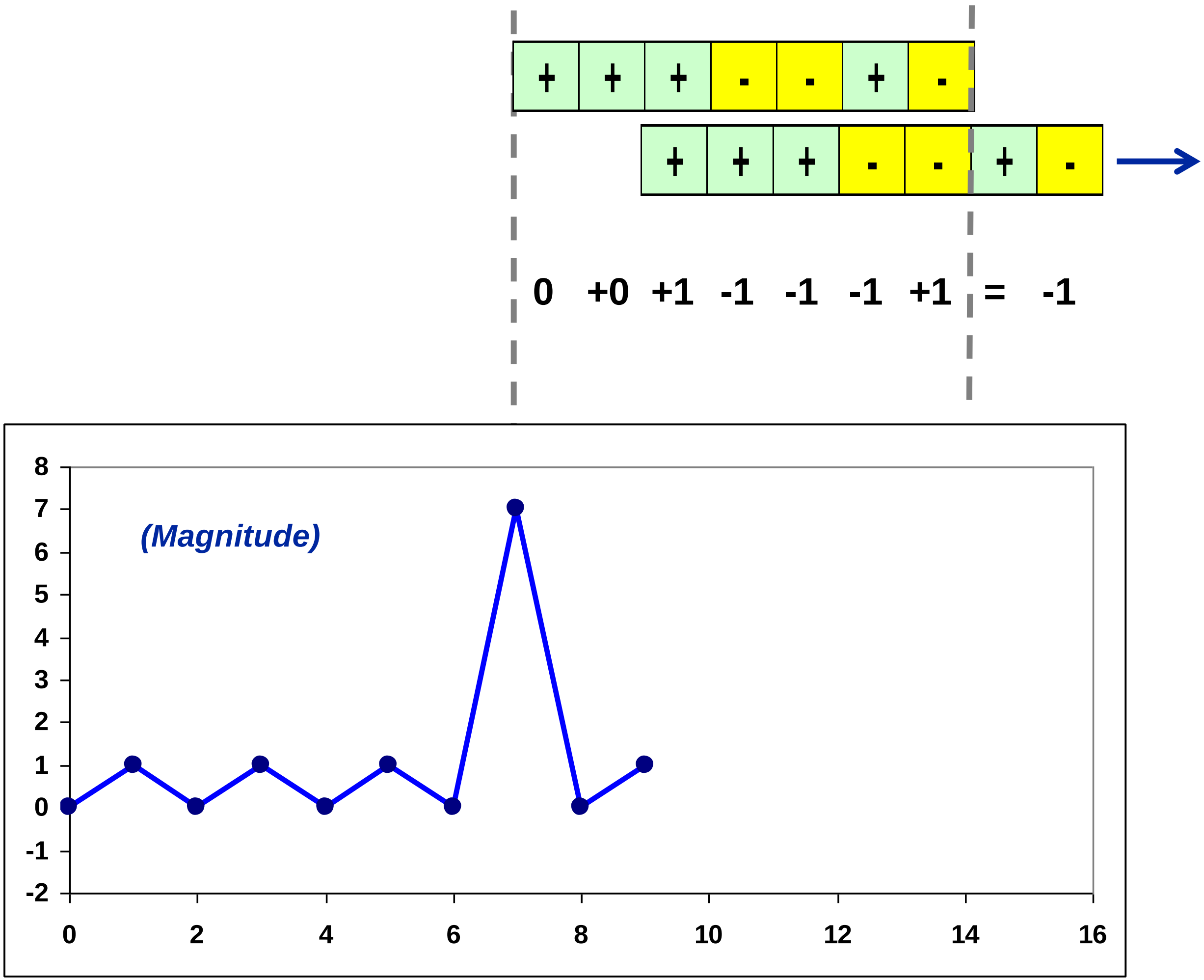

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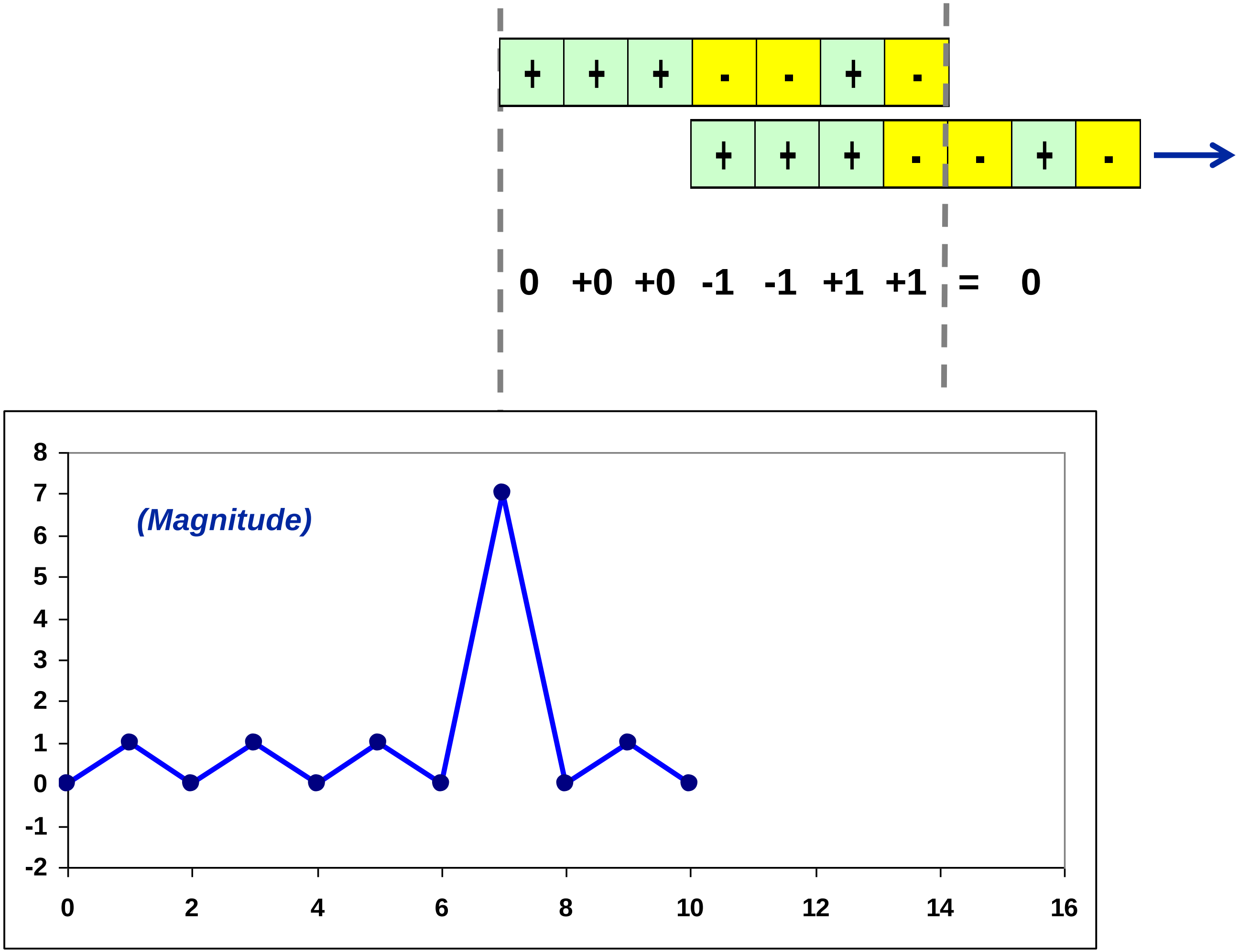

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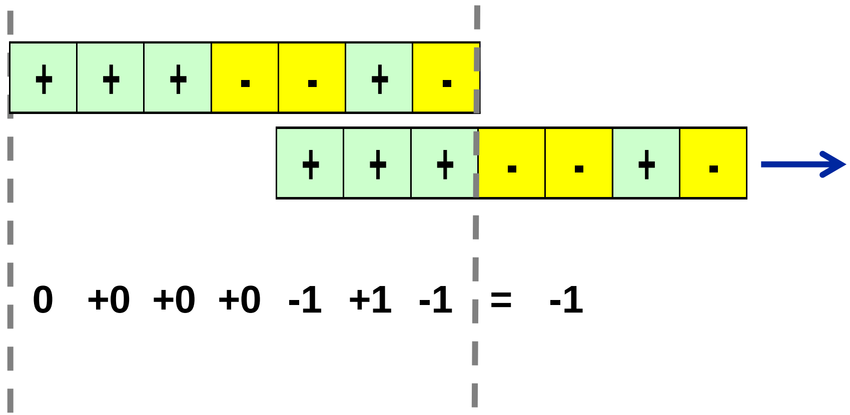

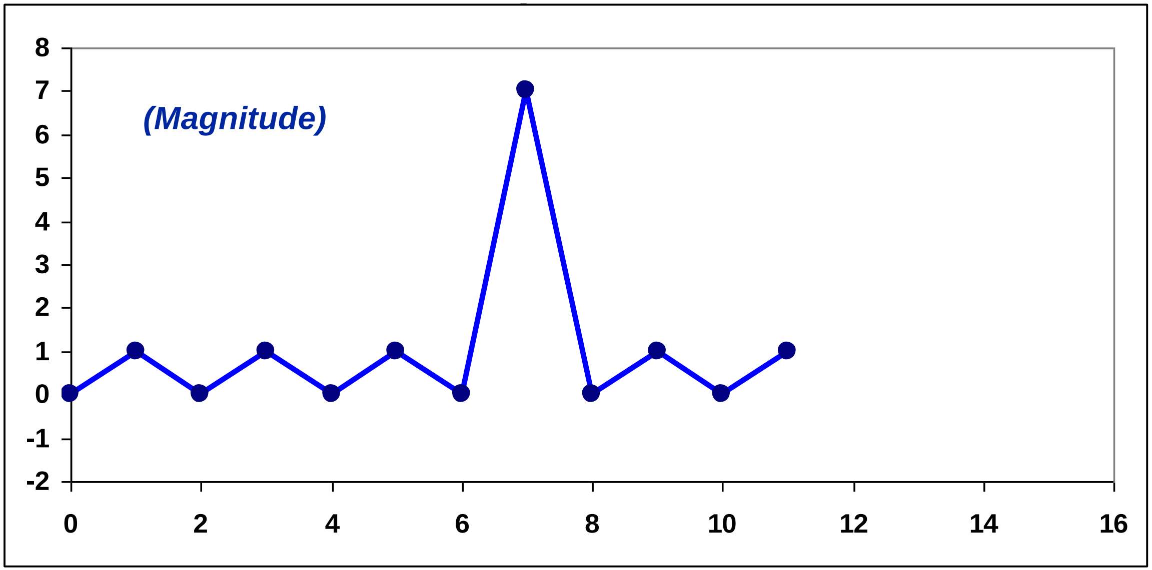

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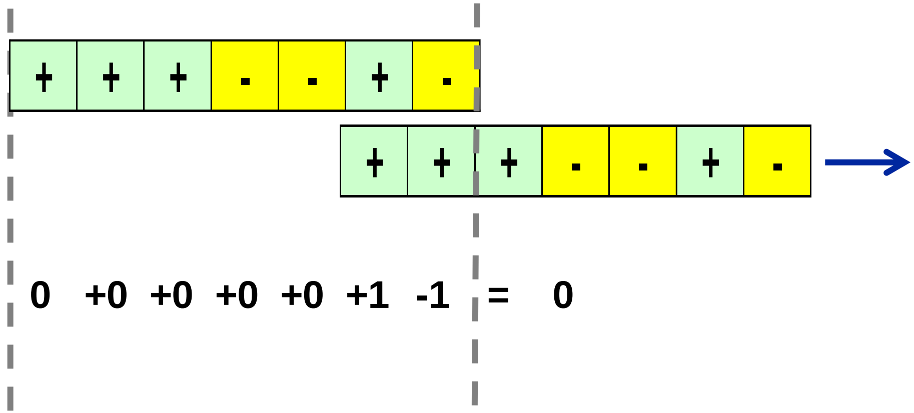

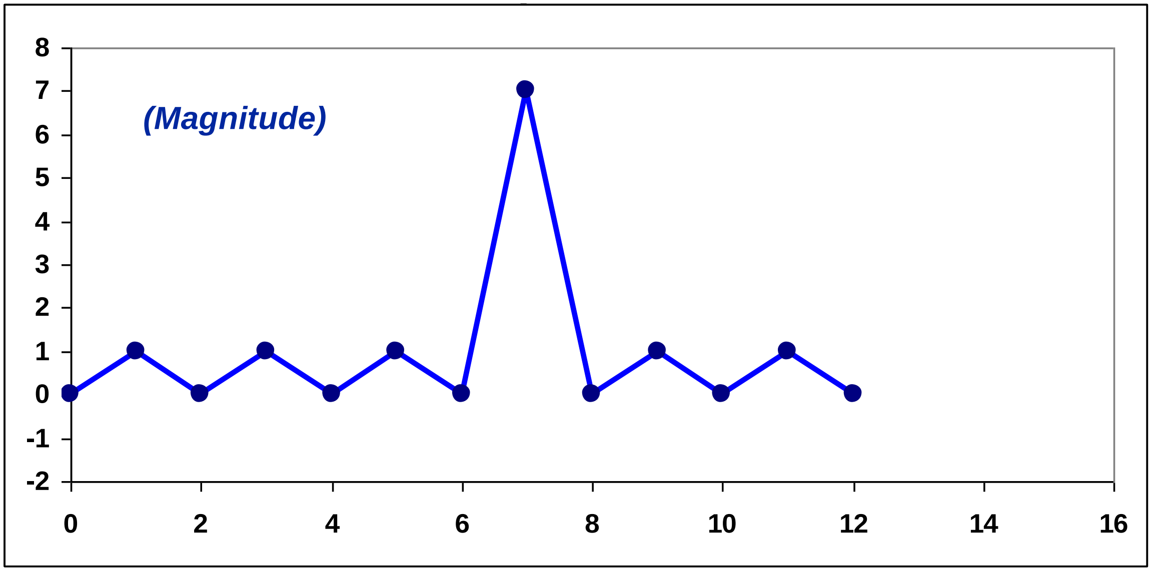

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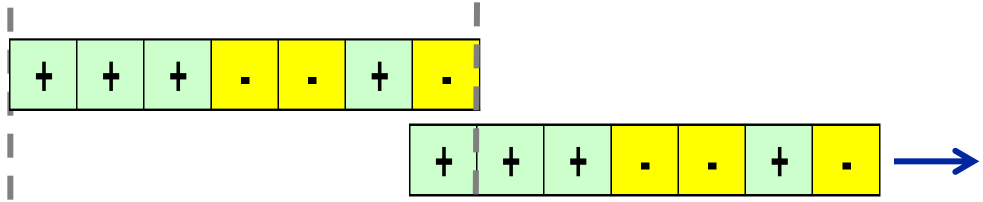
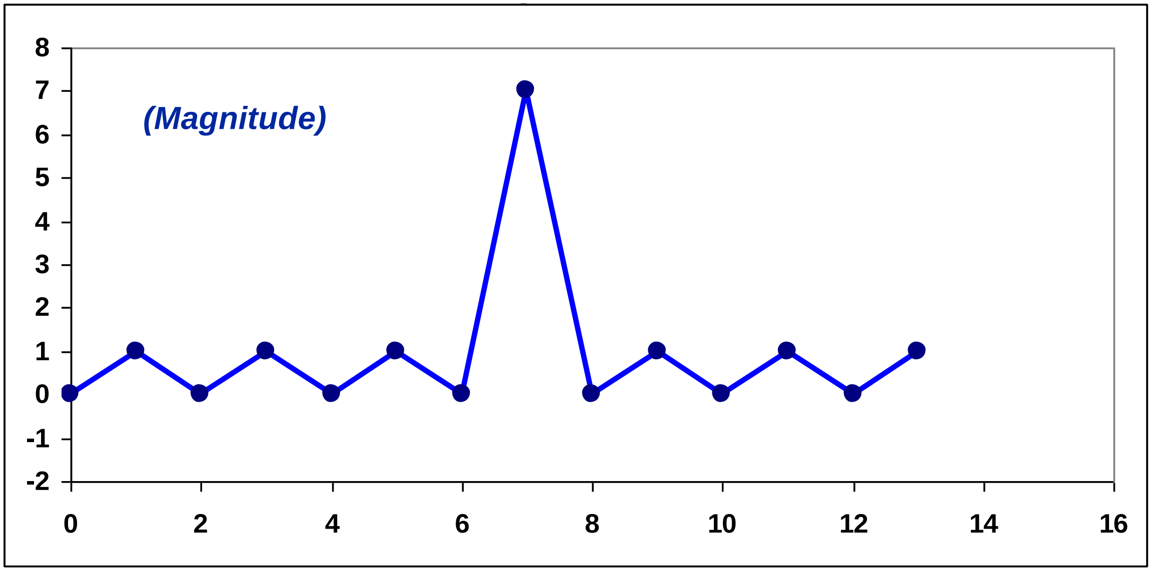


0 +0 +0 +0 +0 +0 -1 = -1



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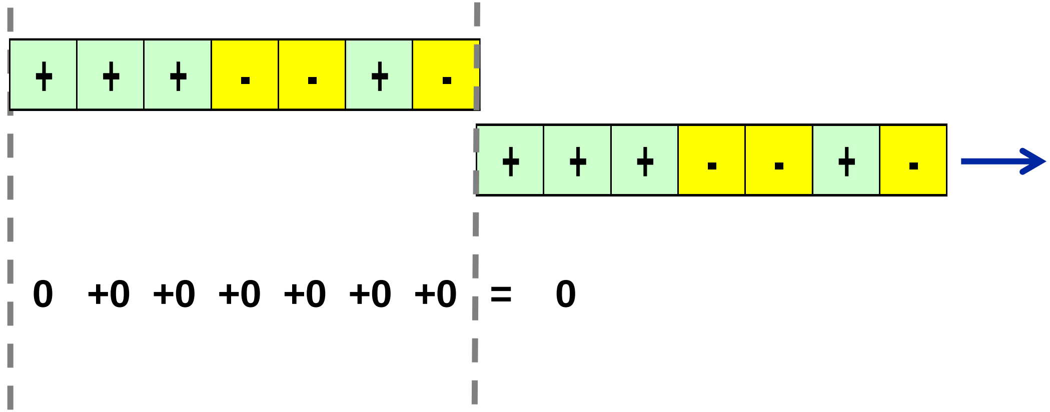
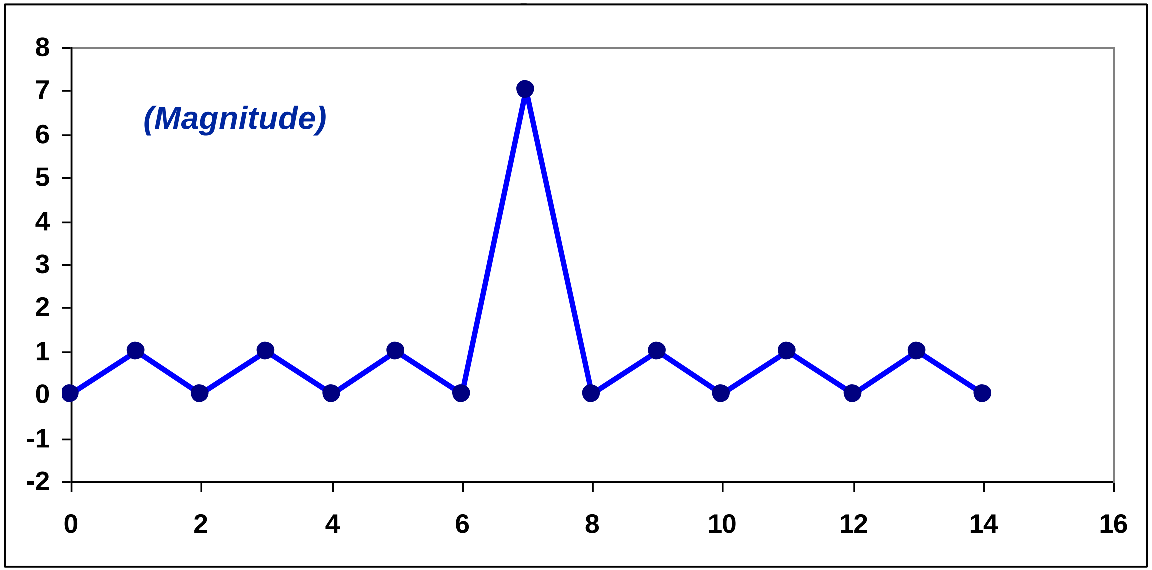
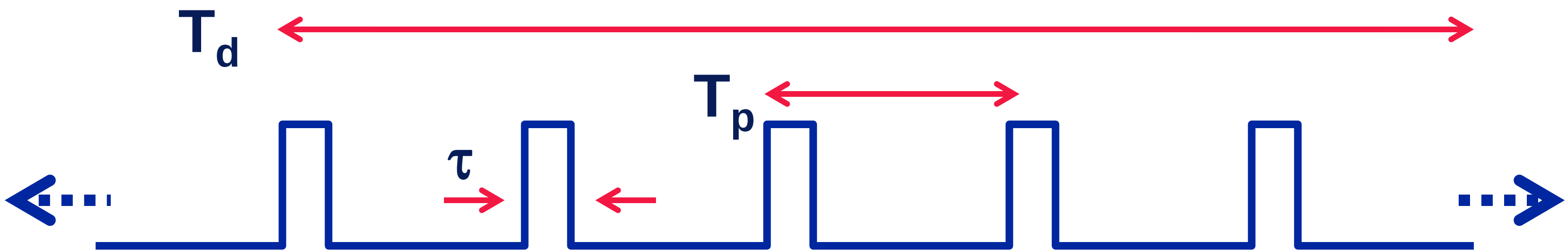


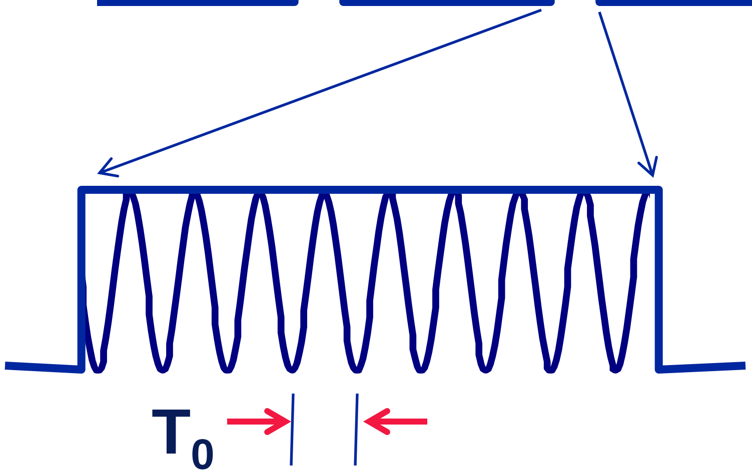


Outline

- Background and Range Measurement
- SNR and the Matched Filter
- Radar Range Equation
- Detection in Noise
- Pulse Compression
- ➡ Multiple Pulses
- Antenna Effects

Pulsed Waveform Parameters





τ = Pulse Width

T_p = Inter-Pulse Period
= Pulse Repetition Interval (PRI)

f_p = Pulse Repetition Frequency (PRF)
= $1/T_p$

T_d = Coherent dwell time

T_o = RF Carrier Period

f_0 = Carrier frequency = $1/T_o$

λ= Wavelength = cT_o = c/f_o

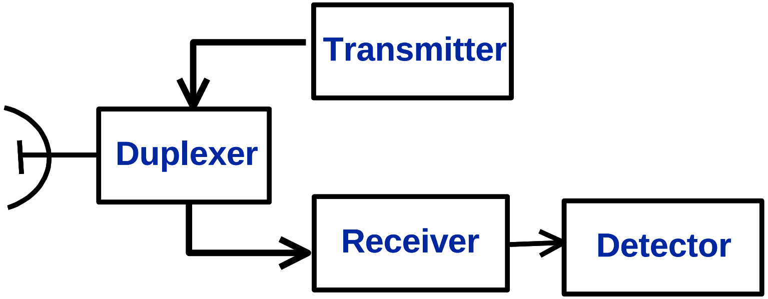
c = Speed of light in vacuum = $3\text{x}10^8$ m/s

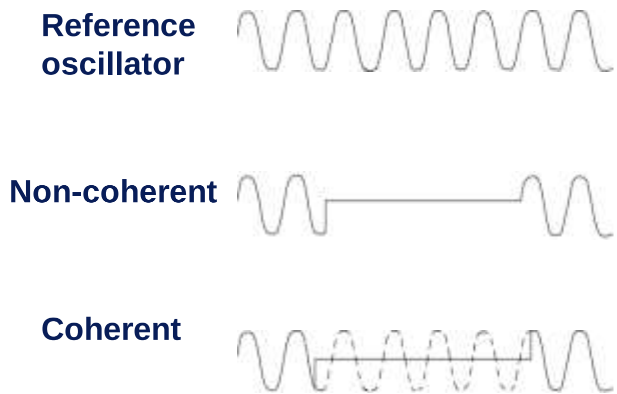
f_0 = 1 GHz ⇔ λ= .3 m

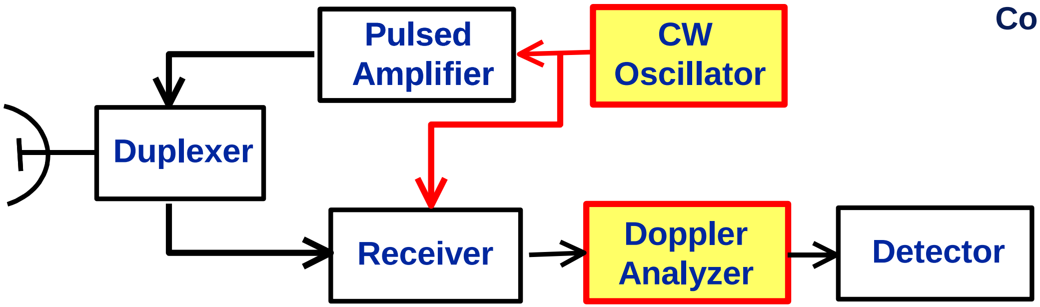
f_0 = 10 GHz ⇔ λ= .03 m

Noncoherent vs Coherent Radar

Noncoherent radar uses amplitude only to process returns



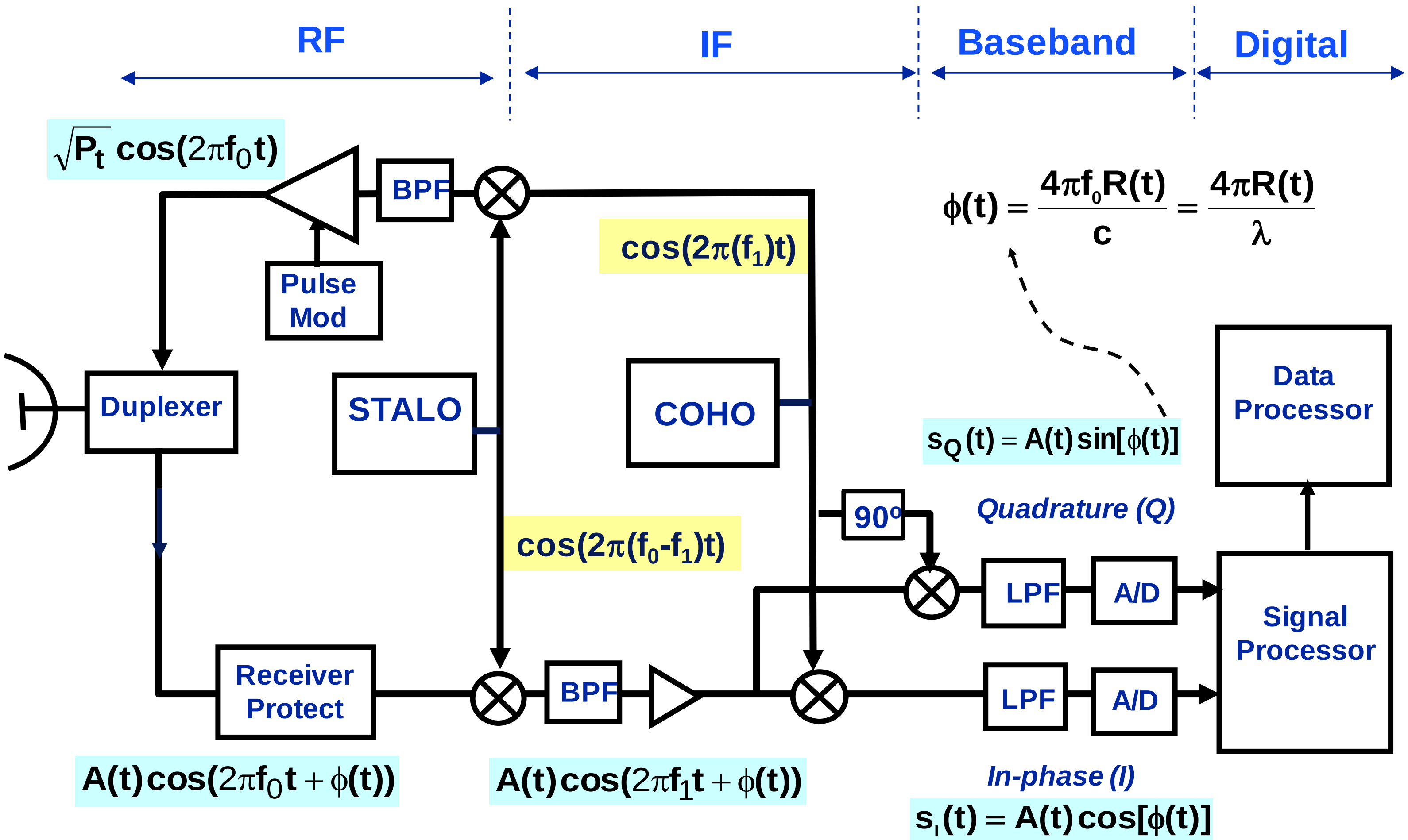




Coherent radar uses both amplitude and phase to process returns.

Specifically, radar uses time variation of phase to infer target motion.

Notional Pulse-Doppler Radar Block Diagram



•STALO and COHO remove carrier frequency, f_c.

•Baseband is DC except for phase/amplitude variation of target (plus noise).

RADAR RANGE EQUATION:
Multiple-Pulse Energy-to-Noise-Density Ratio

$$\frac{E}{N_0} = \frac{P_t \left(dT_d \right) G^2 \lambda^2 \sigma}{\left(4\pi \right)^3 R^4 L_s k T_0 F}$$

- **Presumes coherent integration**
- **Extend pulswidth τ to**
  - **CPI integration time T_d**
  - **But reduce by the TX duty factor $d = \tau / T_p$**

RADAR RANGE EQUATION:

Multiple-Pulse Energy-to-Noise-Density Ratio

$$\frac{E}{N_0} = \frac{P_t d G^2 \lambda^2 \sigma T_d}{(4\pi)^3 R^4 L_s k T_0 F} = \frac{P_{avg} G^2 \lambda^2 \sigma T_d}{(4\pi)^3 R^4 L_s k T_0 F}$$

$$= \frac{P_t \tau G^2 \lambda^2 \sigma n}{(4\pi)^3 R^4 L_s k T_0 F}$$

- **Variations**
  - P_{avg} is average TX power
  - n = number of pulses coherently integrated

Motivations for Coherent Integration (Doppler Processing)

- Efficient increase in target SNR
- Ability to measure range rate

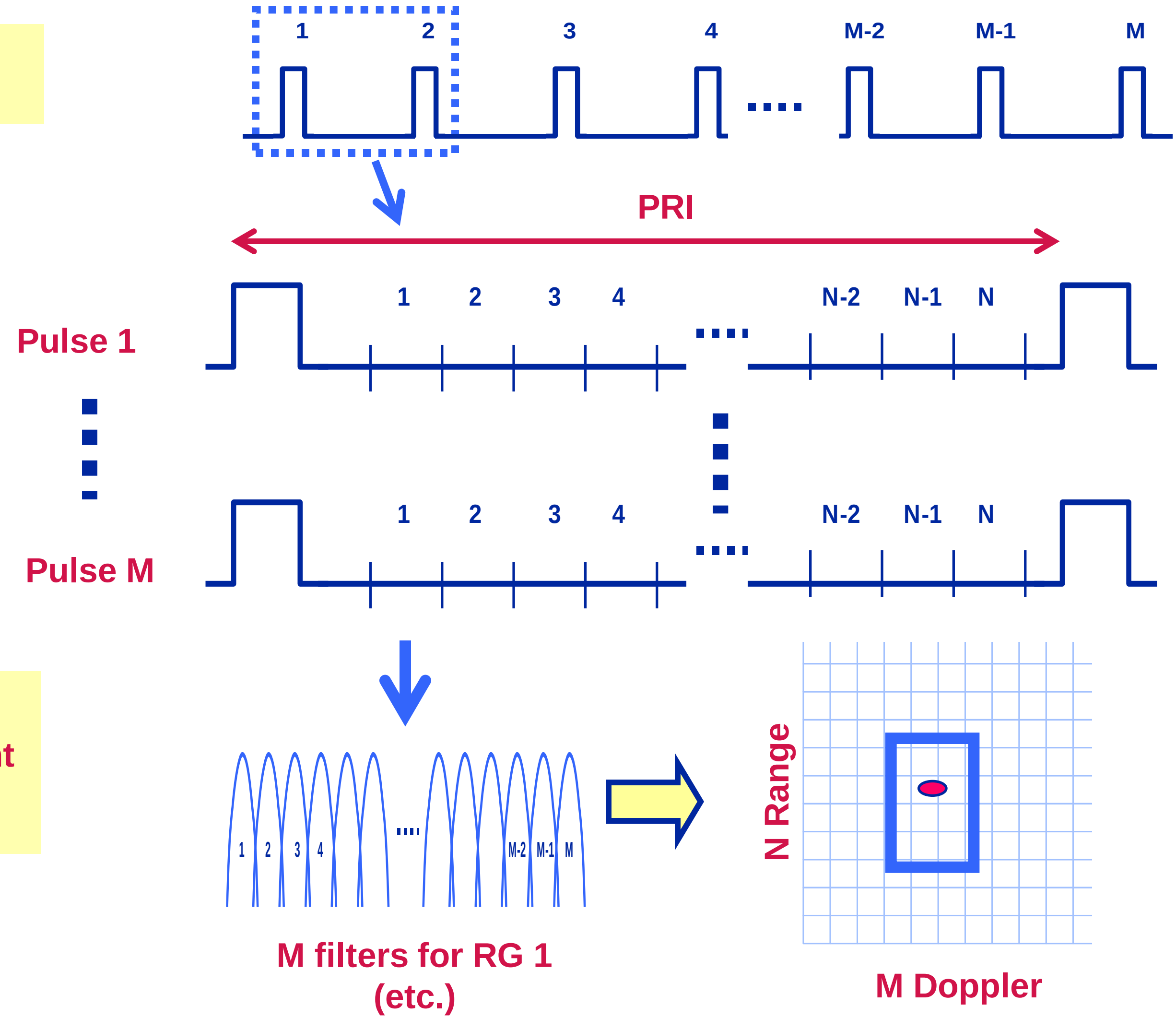
- Separation (by Doppler frequency) of ground clutter and moving targets
- Fine-resolution imaging via SAR and ISAR

Pulse-Doppler Waveform Concept

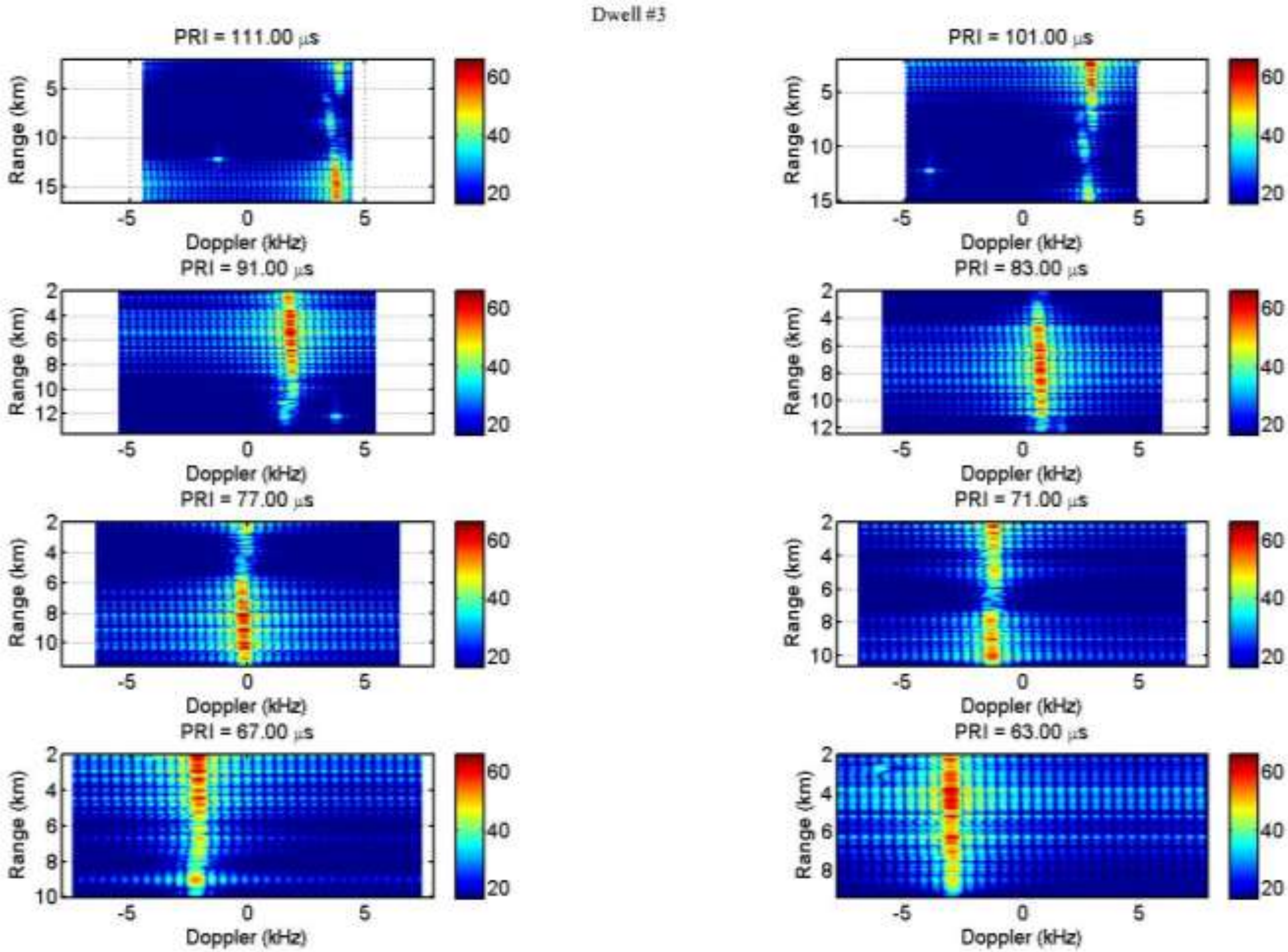
Coherent pulse train of M pulses.

Collect digital I/Q samples from each of N range gates for M pulses.

For each of N range gates, perform M-point FFT to obtain Doppler spectrum.

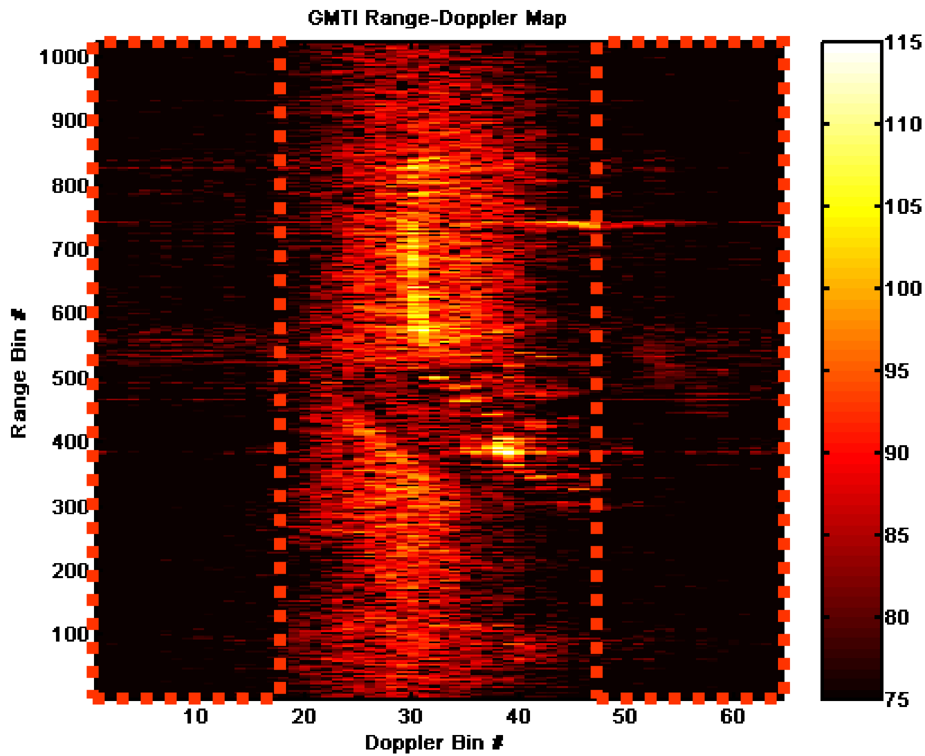


Air-to-Air Range-Doppler Maps



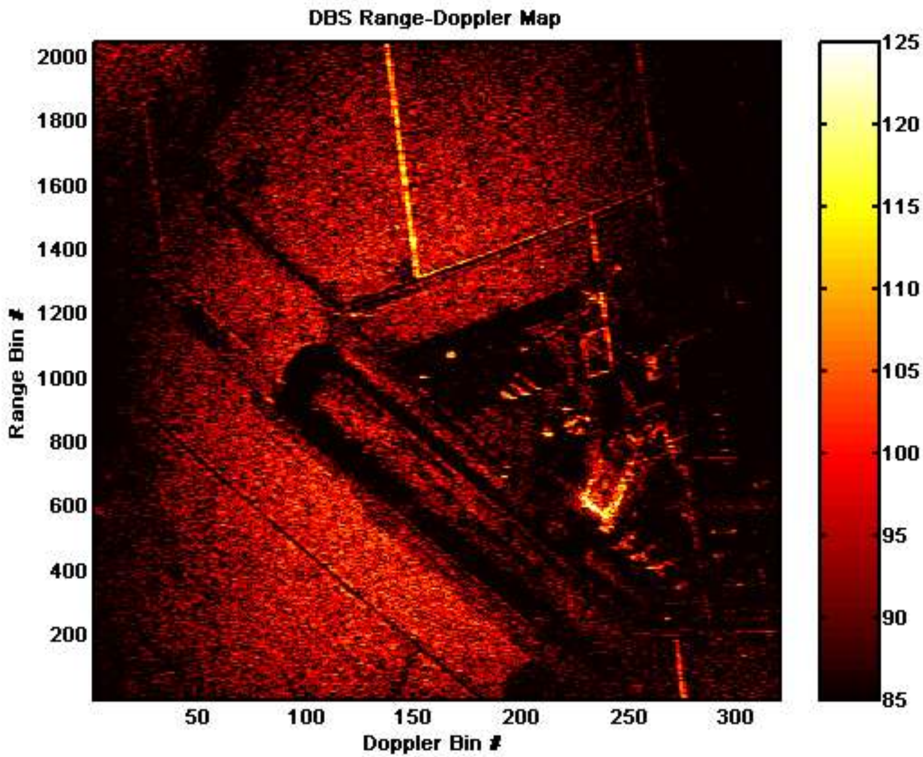
GMTI Range-Doppler Map

- Example of a 64-pulse GMTI range-Doppler map
- Note substantial exo-clutter extent (boxes) for moving target detection
- (Courtesy Raythoen Corporation)



Range-Doppler Map

- Example of a DBS range-Doppler map
- DBS (Doppler Beam Sharpening) is a crude form of SAR
- Note small exo-clutter extent
- (Courtesy Raythoen Corporation)



Fine-Resolution SAR Image

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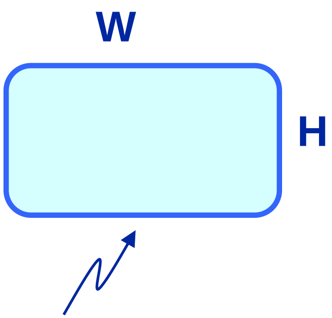
Antenna Gain and Area



Antenna

Area A=W H

Azimuth beamwidth $\theta_{AZ} \cong \dfrac{\lambda}{W}$

Elevation beamwidth $\theta_{EL} \cong \dfrac{\lambda}{H}$

Antenna gain $G \cong \dfrac{4\pi}{(\lambda / W)(\lambda / H)} = \dfrac{4\pi A}{\lambda^2}$

- Large antennas provide
  - High gain on TX
  - Large collection area on RX
  - Low energy on TX to other users outside the mainbeam
  - Reduced power on RX from sidelobe returns
  - Improved angle *resolution* against multiple targets
  - Increased angle measurement *accuracy* on one target

RADAR RANGE EQUATION:
Search Version

$$\frac{E}{N_0} = \frac{P_t d A_e \sigma}{(4\pi) R^4 L_s k T_0 F} \left(\frac{T_{frame}}{\Omega_{search}} \right)$$

- **Search parameters**
  - **T_{frame} = "frame time," time between revisits**
  - **Ω_{search} = search volume (steradians)**
- **Search is all about "power-aperture"**
  - **High average power**
  - **Large antenna area**

Georgia Tech Research Institute

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