Design of a New Multiuser Line Code

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Objectives

- Find a code that allows multiple users to simultaneous access the same data channel
- Maximize efficiency, trading off
  - number of users
  - number of symbols per user
  - number of bits per symbol
- Compare to other schemes

Results

- May use as a single user line code by partitioning data stream into “users’” symbols.
- Produces higher rates than QAM and TDM because higher dimensional signal space is used
- Up to 4.4 bits/baud for 22 users, 2 symbols/user, and 5 bits/symbol

Methodology

- Similar to collaborative coding multiple access (CCMA) techniques
- Permit simultaneous transmission by multiple users over a common binary adder channel or over a shared radio channel
- Combined users' symbol codes must be uniquely decodeable

Example coding process

U = 3 users' source symbols,

M = 2 symbols/user, assigned to

2-bit words, m (only for cc)

1 → 0 1

0 → 0 1

1 → 1 1

Lookup table combines m_i

B = 2-time-slot data word V transmitted

1 3

Problems

- What is the maximum number of users U each choosing from M symbols, that can use B bit transmission words?

- Alternately, given U users, what is the minimum number B of bits that is required.

- How do we decode the M user symbols associated with each V?

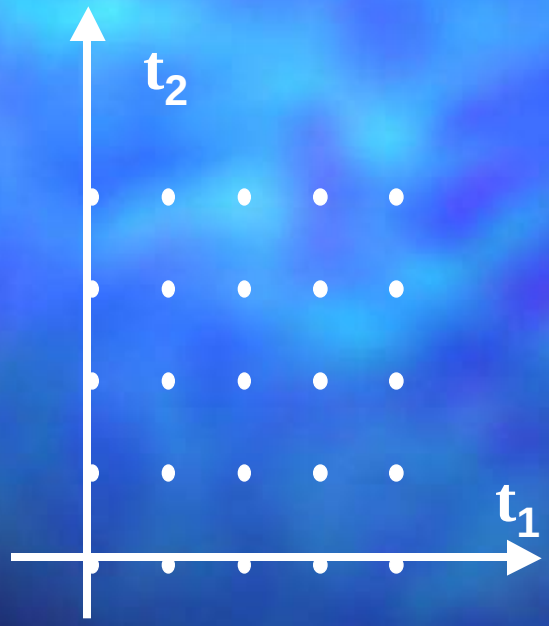
EXAMPLE

- Consider the U = 4-users, M = 2 symbols/users, B = 2-bits/symbol code.

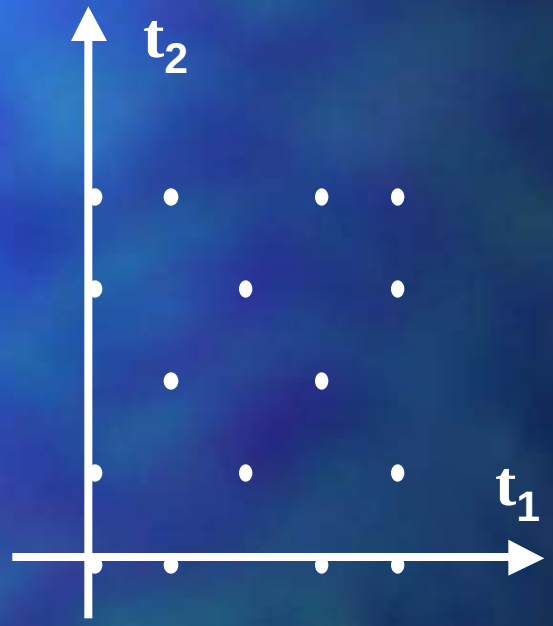
- Needs only $M^U = 2^4 = 16$ points from the $N_v = (U+1)^B = 25$ possible combinations $\underline{V}$ in the 2-dimensional space, i.e. from the grid of size $(U+1)^B = 5^2 \Rightarrow 25$ points ...

Example showing size and use of signal space;

U = 4, M = 2, B = 2

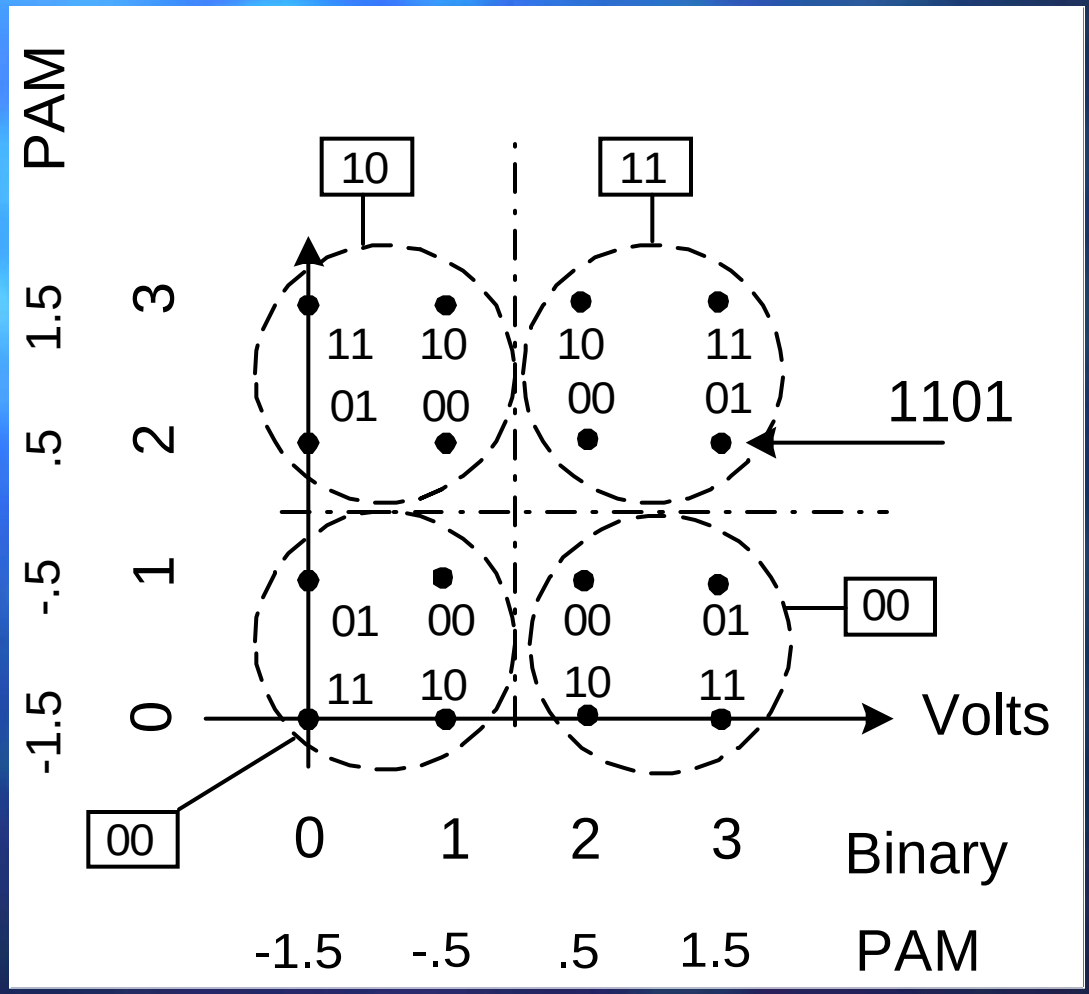


U=4, All $(U+1)^B$ = 25 possible points



U=4, M^U = 16 points selected for transmission

We may use "QAM-like" constellations, as shown for the U = 4 users B = 2 time slots



Alternate constellation diagram for U=4 , B=2, M = 2 symbols/user illustrating both direct transmit level coding and PAM transmission.

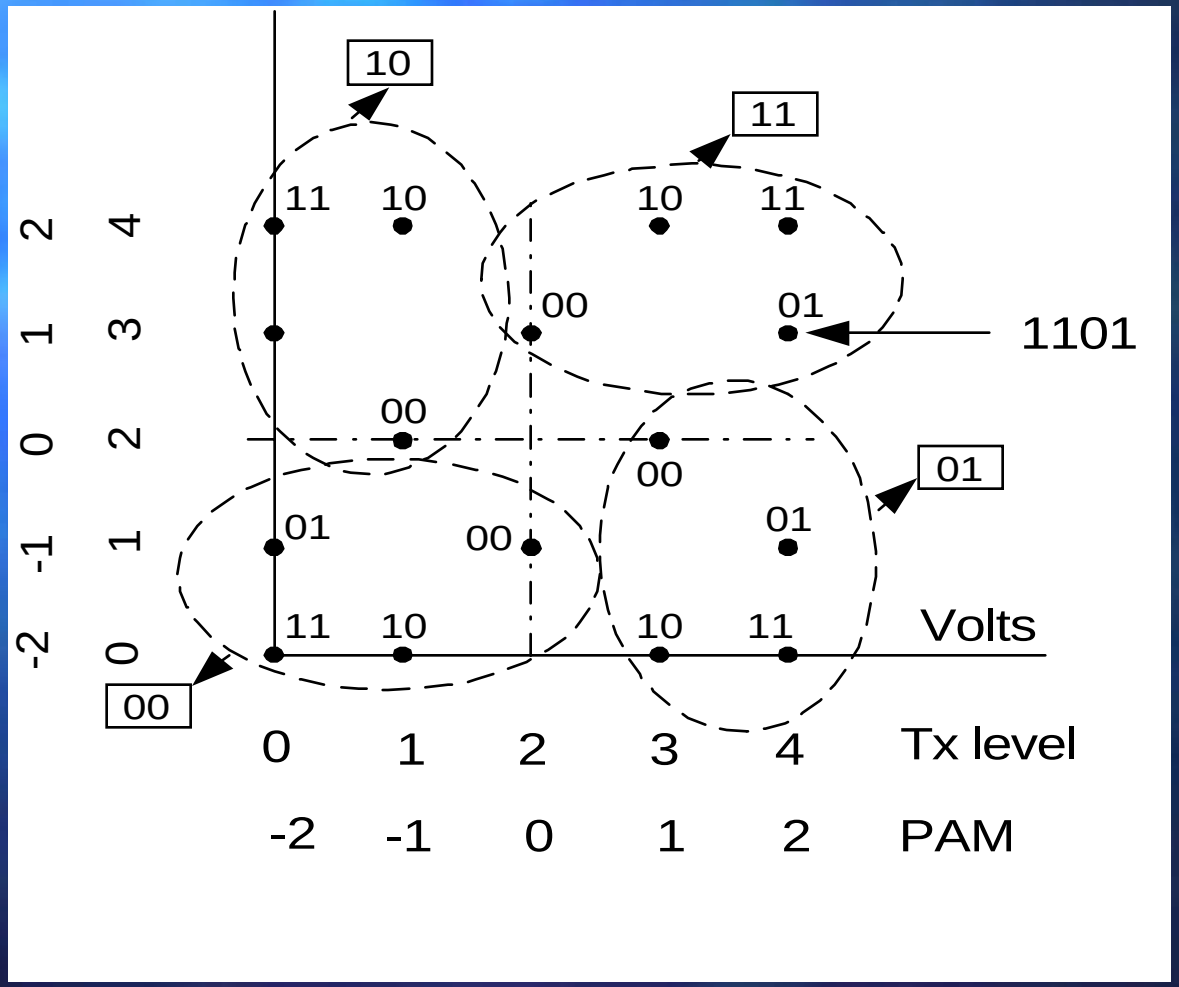


Table 1: Constellations for 4-user code

Constellation assignment	user symbols u_1 u_2 u_3 u_4	Direct Tx levels	Symmetric PAM
1	0000	2 1	0 –1
2	0001	0 1	-2 –1
3	0010	1 0	-1 –2
4	0011	0 0	-2 –2
5	0100	3 2	1 0
6	0101	4 1	2 –1
7	0110	3 0	1 –2
8	0111	4 0	2 –2
9	1000	1 2	-1 0
10	1001	0 3	-2 1
11	1010	1 4	-1 2
12	1011	0 4	-2 2
13	1100	2 3	0 1
14	1101	4 3	2 1
15	1110	3 4	1 2
16	1111	4 4	2 2

Implications

- The number of dimensions in the signal space is B.

- The number of possible unique values (levels) of the sum vectors $\underline{V}$ is $N_{Vmax} = (U+1)^B$.

- The maximum number of unique symbols available to all users is $M_{max} = 2^{B.}$

- The maximum number N_w of symbol combinations is $M_{max}{}^U$.

- Because the actual number N_w words used must be less than or equal to N_{Vmax} , we have the constraint:

$$N_w = M^U \leq N_{Vmax} = (U+1)^B$$

A most important point :

We can use the constraint two ways:

either

1) For fixed U and B we can select an M small enough that $M^U \leq (U+1)^B$

or

2) For a fixed M and U we may increase B so that $M^U \leq (U+1)^B$.

- **Information Rate is defined as**

$$R = \log_2 M * (U/B)$$

user information bits / time-slot

- **Code Efficiency is defined as the symbol combinations used by the U users divided by the number of symbols available given B-bit symbols,**

$$\eta = M^U/(U+1)^B \leq 1$$

Table 2. Code rate and efficiency for various U, M = 2, B =2.

U	$M^U = 2^U$	$(U+1)^B$	Rate (R) = $\log_2 M * U/B$	Efficiency (η) = $M^U/(U+1)^B$	$\eta * R$
2	4	9	1	.444	.444
3	8	16	1.5	.5	.75
4	16	25	2	.64	1.28
5	32	36	2.5	.889	2.222
6	64	49		Bound not satisfied	

Table 4. Maximum M for varying U with B =2.

U	M calculated $(U+1)^{B/U}$	M Selected	M^U	$(U+1)^B$	Rate (R) = $\log_2 M^* U/B$	Efficiency (η) = $M^U/(U+1)^B$	η R
2	3	3	9	9	1	1	1
2	3	2	4	9	1	.444	.444
3	2.52	2	8	16	1.5	.5	.75
4	2.24	2	16	25	2	.64	1.28
5	2.048	2	32	36	2.5	.889	2.222
6	1.91	-	64	49	-	M * < 2	

* Each user must have at least 2 symbols for transmission

Table 5. Maximum M for varying U with B =3.

U	$(U+1)^{B/U}$	M Selected	M^U	$(U+1)^B$	Rate (R) = $\log_2 M$ (U/B)	Efficiency (η) = $M^U/(U+1)^B$	η R
2	5.196	4	16	27	2 * (2/3)	.593	.79
3	4	4	64	64	2 * (1)	1	2
4	3.343	2	16	125	4/3	.128	.171
5	2.93	2	32	216	5/3	.148	.247
6	2.645	2	64	343	2	.187	.373
7	2.438	2	128	512	7/3	.25	.583
8	2.28	2	256	729	8/3	.351	.936
9	2.154	2	512	1000	3	.512	1.536
10	2.053	2	1024	1331	10/3	.769	2.564
11	1.96	-	-	-	M < 2		

Table 6. Maximum U for M = 4, B =2.

U	$M^U = 4^U$	$(U+1)^B$	Rate (R) = $\log_2 M$ *(U/B)	Efficiency (η) = $M^U/(U+1)^B$	η * R
2	16	9	Bound not satisfied		

Table 7. Maximum U for M = 4, B =3.

U	$M^U = 4^U$	$(U+1)^B$	Rate (R) = $\log_2 M$ *(U/B)	Efficiency (η) = $M^U/(U+1)^B$	η * R
2	16	27	2*(2/3)	.593	0.395
3	64	64	2*1	1	1
4	256	125	Bound not satisfied		

Table 8. Upper rate limits for M=2 symbols/user

B	U	Rate $\log_2 M * (U/B)$	$\eta = M^U/(U+1)^B$
2	5	2.5	.889
3	10	3.33	.769
4	16	4	.785
5	22	4.4	.6517

Table 9. Upper rate limits for M=4 symbols /user

B	U	Rate	η
3	3	2	1
4	5	2.5	.79
5	7	2.8	.50

Table 10. Upper rate limits for M=8 symbols /user

B	U	Rate	η
4	2	1.5	.79
5	3	1.8	.5
6	4	2	.26
6	5	2.5	.7

Conclusions

- The code may be used as a single user line code by partitioning data stream into blocks of U "users'" symbols.

- U users' simultaneous source symbols are encoded by lookup table according to the design, and decoded similarly.

- It produces higher bits/baud or bits/dimension than QAM and TDM because a higher dimensional signal space is used (B bits / symbol).

- Maximum M , symbols/user and maximum number of users U, have been tabulated for most realistic cases when other parameters are fixed.
- Up to 4.4 bits/baud (bits/time slot) results with U = 22 users, M = 2 symbols/user, and B = 5 bits/symbol.
- Design objectives have been achieved