A Methodology for Approximating BPSK Demodulator Performance in the Presence of Various Undesired Signals

NTIA/ITS Research in support of

The President's Spectrum Policy Initiative

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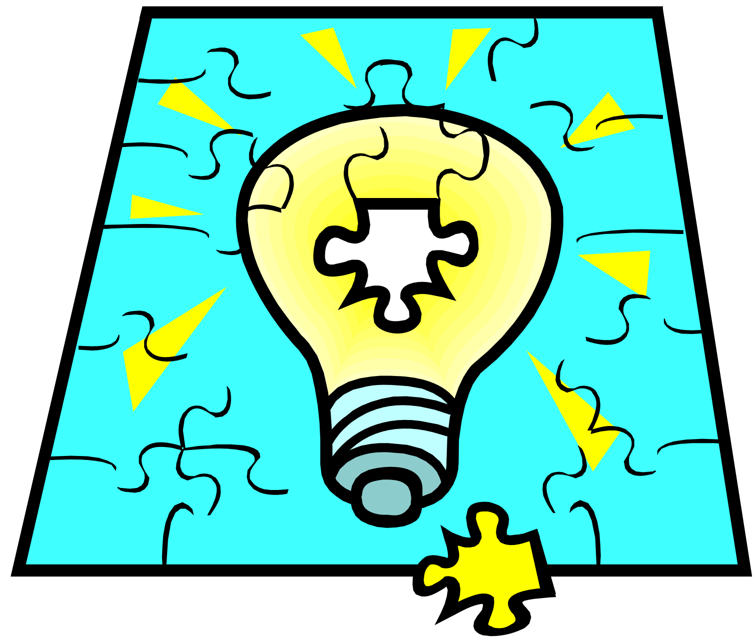
Organization

- Background and Motivation
- Scope
- Methodology
- Application and Results
- Summary and Continuing Work

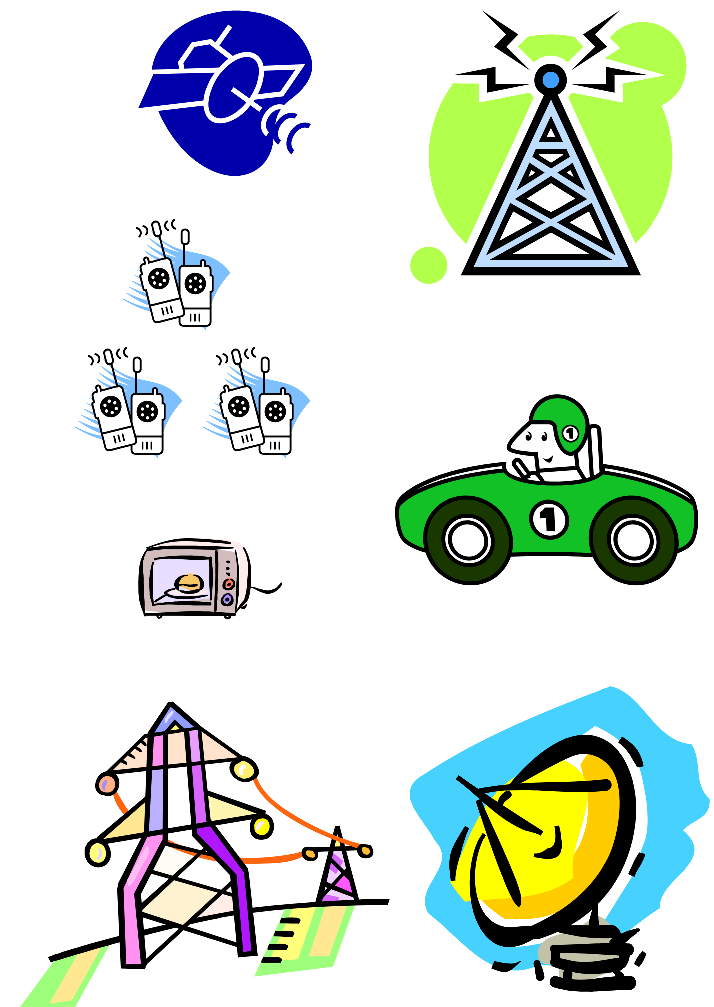


Background

- NTIA and FCC work on behalf of Federal agencies and the private sector, respectively, to provide spectrum planning, allocation, and assignment .
- The goal is to develop policy and regulations that foster a healthy competitive telecom industry by
  - Promoting new innovative technologies,
  - Preventing interference to legacy systems.
- A key element to this process is to identify the appropriate interference protection criteria (IPC) that protects critical infrastructure communications systems operating in new and proposed telecom scenarios.

Today’s Spectrum Challenges

- Increased interference risks due to higher densities radiating devices.
- New technologies and spectrum sharing paradigms, e.g., ultrawideband and dynamic spectrum access, change how undesired signals affect legacy systems.



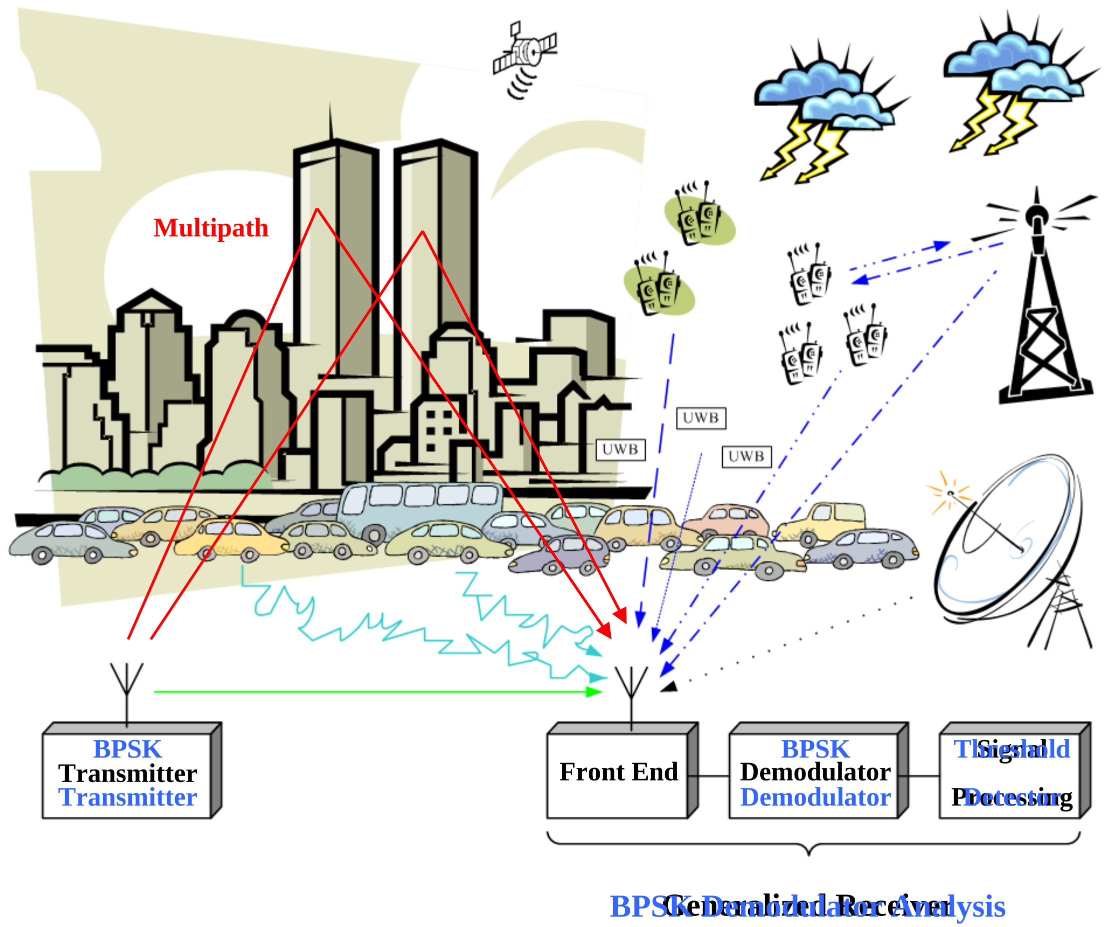
Receiver Susceptibility Testing

- NTIA/ITS provides receiver susceptibility data to support the development of IPC.
- Significant time is required to acquire this data, .e.g., it took several years to obtain sufficient measurement and analytical information on the potential interference from UWB devices in order to define and apply the appropriate IPC.
- Restricted access to internal functions of receivers limits our ability to generalize the results.

Satellite DTV Susceptibility to undesired UWB signals.

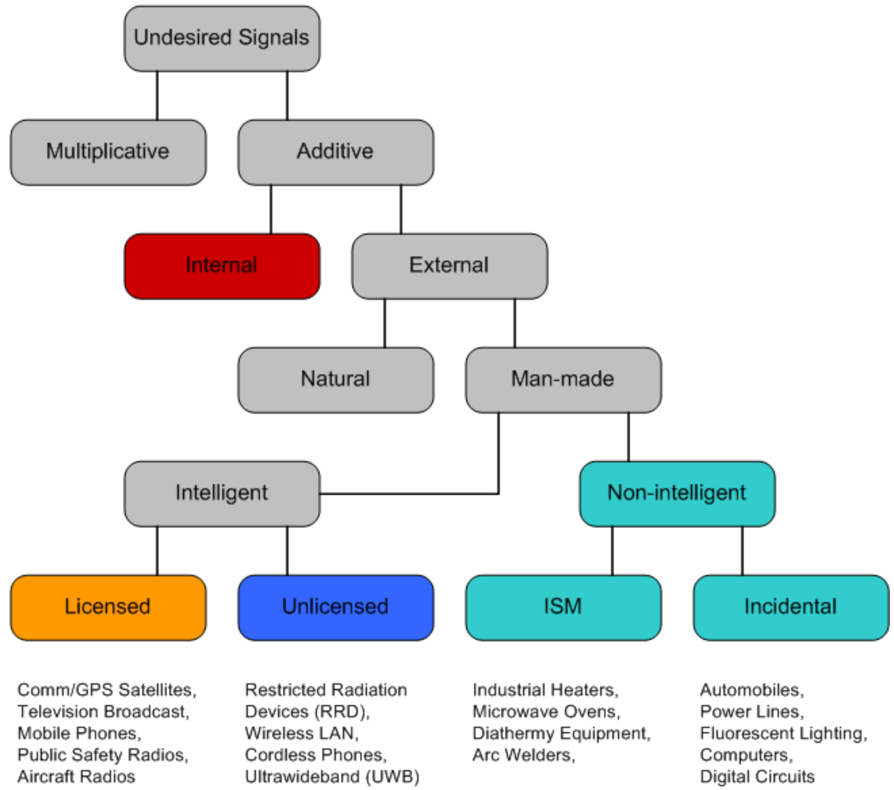
Tools to Aid IPC Development

- As part of *The President's Spectrum Policy Initiative*, analytic and simulation tools are being developed to facilitate IPC development.
- In parallel, NTIA/ITS Generalized Receiver Research is analyzing how undesired signals affect certain receiver components.



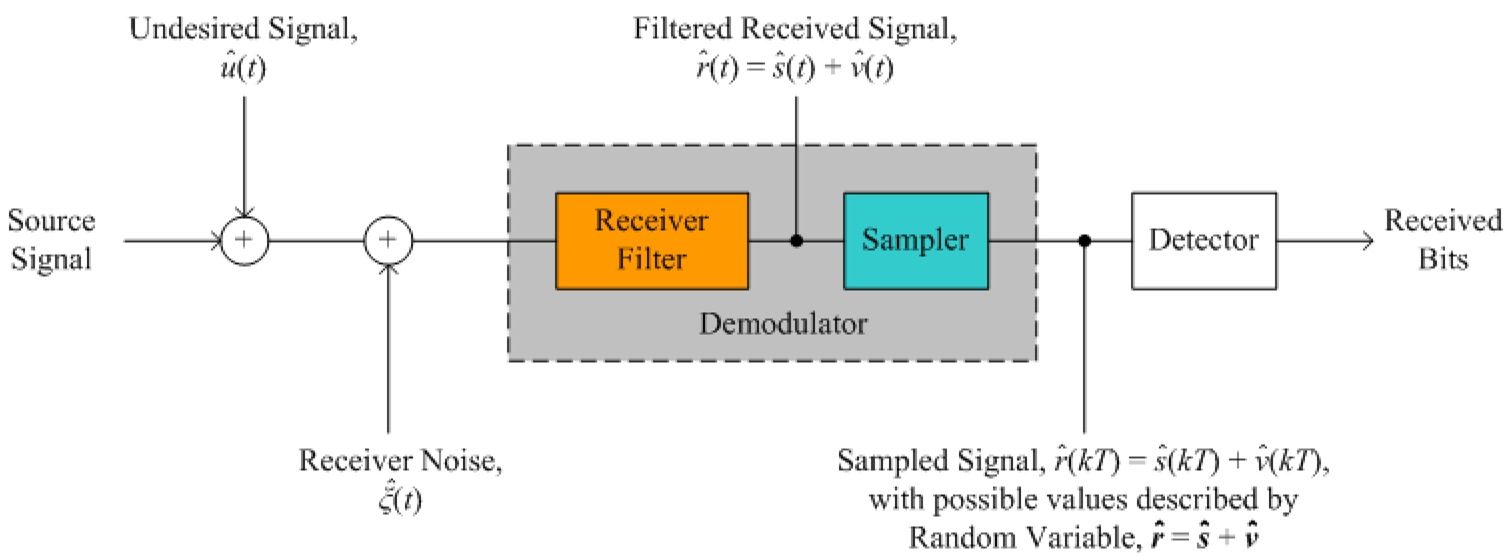
Undesired Signals

- Receiver CGN
- Undesired CGN
  + Receiver CGN
- Undesired Constant
  + Receiver CGN
- Undesired MPSK
  + Receiver CGN
- Undesired Impulsive Noise
  + Receiver CGN
- Undesired Gated CGN
  + Receiver CGN



BPSK Demodulator

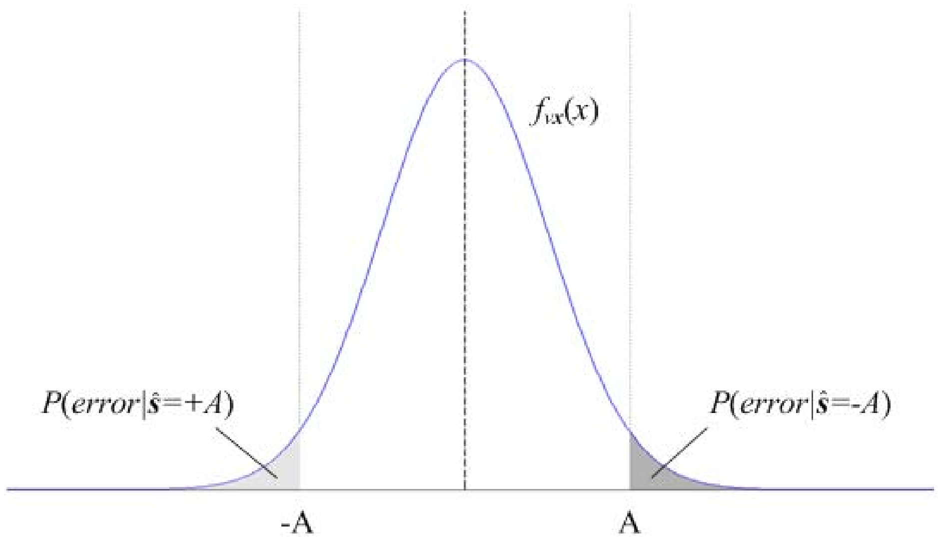
- Receiver filter is matched RRC filter with $a = 0.35$ and $b = 1$ MHz.
- Undesired-plus-noise signal is $\hat{v}(t) = \hat{u}(t) * \hat{h}_R(t) + \hat{n}(t)$.
- Optimal sampling produces received source signal $\hat{s}(kT) = \pm A$, when a binary 1 and 0 were sent.
- Sampled signal is a discrete random process with possible values at time *kT* described by a random variable (denoted by bold font).



Error Probability ($\mathcal{P}_e$)

- From the theorem of total probability, $\mathcal{P}_e$ can be expressed in integral form as

$$\mathcal{P}_e = \frac{1}{2}\int_{A}^{\infty} f_{\mathbf{v_x}}(x)dx + \frac{1}{2}\int_{-\infty}^{-A} f_{\mathbf{v_x}}(x)dx$$



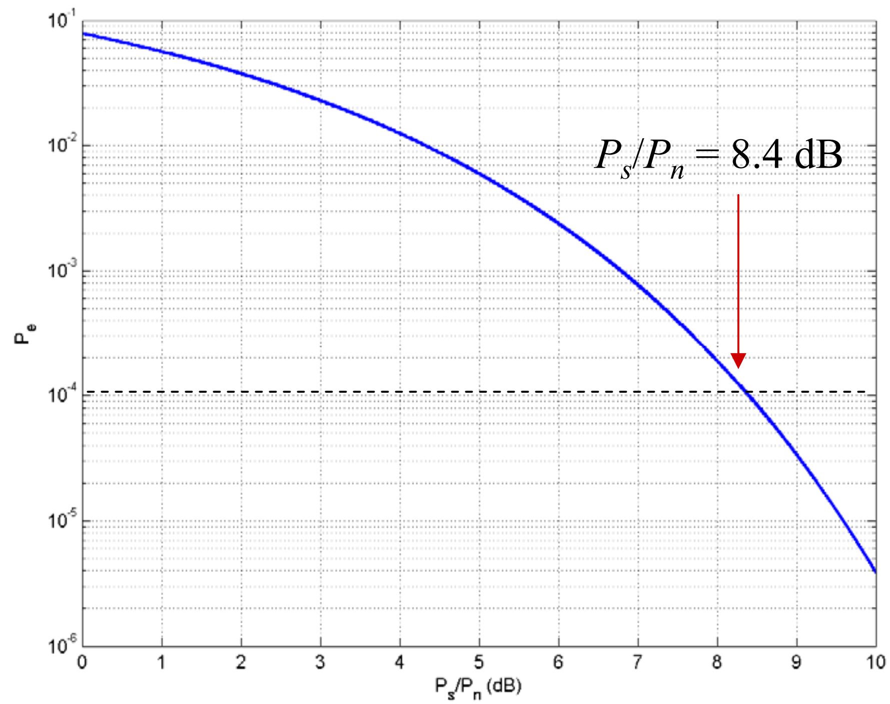
Receiver Noise

- W/o the influence of an undesired signal, $\hat{\mathbf{v}}$ is CGN and $\mathbf{v_x} = \mathbf{N}[0, \sigma_n^2]$.
- Substitution yields

$$\mathcal{P}_e = Q\left(\sqrt{2\frac{P_s}{P_n}}\right)$$

$$Q(\xi) = \int_\xi^\infty e^{-\lambda^2/2} d\lambda = \frac{1}{2}\text{erfc}\left(\frac{\xi}{\sqrt{2}}\right)$$

- Operational scenario is set to $P_s/P_n = 8.4$ dB, which corresponds to $\mathcal{P}_e \approx 10^{-4}$.

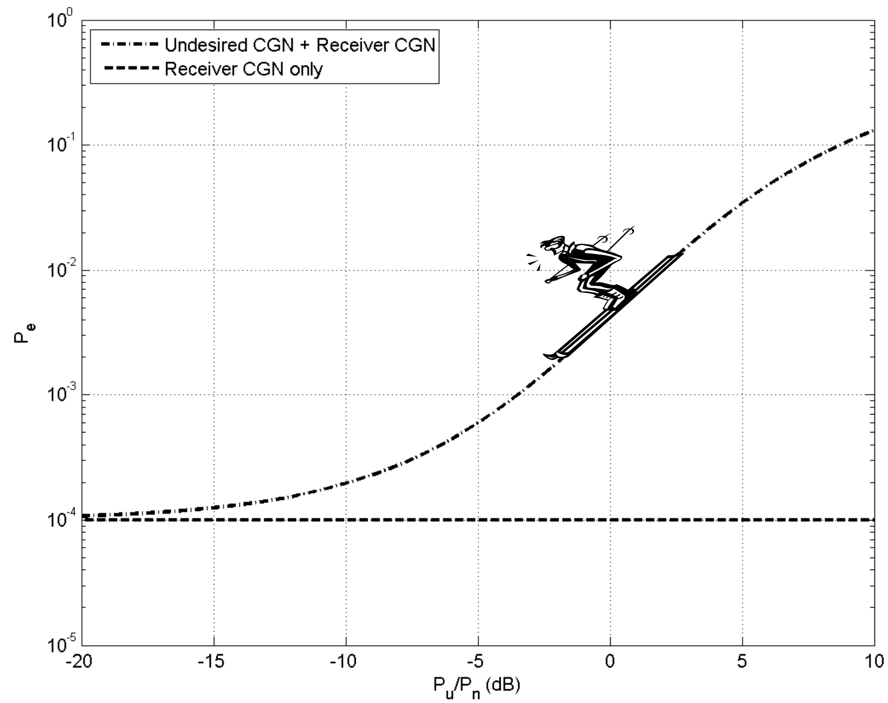


Undesired CGN

- Undesired CGN emulates the noisy nature of many modern UWB devices.
- Summing 2 CGN RVs gives $\mathbf{v}_{\mathbf{x}} = \mathbf{N}[0, \sigma_u^{\,2} + \sigma_n^{\,2}]$.
- Substitution yields

$$\mathcal{P}_e = Q\left(\frac{A}{\sqrt{\sigma_n^2+\sigma_u^2}}\right) = Q\left(\sqrt{\frac{2\frac{P_s}{P_n}}{1+\frac{P_u}{P_n}}}\right)$$

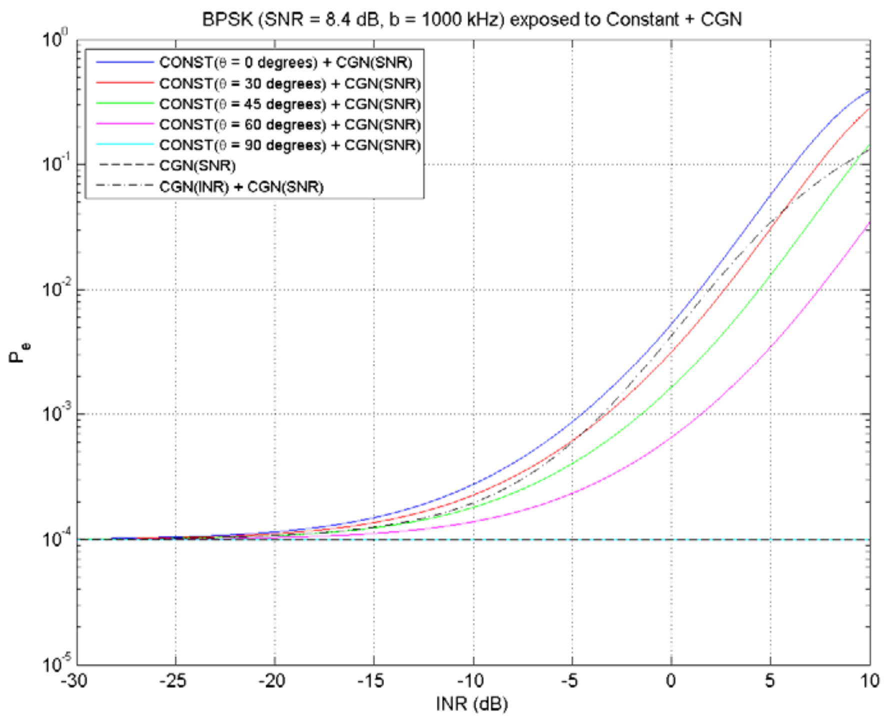
- Undesired CGN is reference curve used to compare other undesired signals.



Undesired Constant

- Undesired constant emulates an interfering spectral line (centered in the band) due to a CW signal or a periodic signal.
- Adding a constant to CGN gives $\mathbf{v_x} = \mathbf{N}[v_c\cos\theta_c, \sigma_n^2]$.
- Substitution yields

$$\mathcal{P}_e = \frac{1}{2}\sum_{k=0}^{1} Q\left(\sqrt{2\frac{P_s}{P_n}} - (-1)^k\sqrt{2\frac{P_u}{P_n}\cos^2\theta_c}\right)$$



Sample Function Analysis

- When $f_{\mathbf{v_x}}$ is too complicated to integrate analytically, a solution is available by generating a sample function of the undesired signal plus receiver noise, i.e.,

$$\hat{v}_k = \hat{u}_k * \hat{h}_{R,k} + \hat{n}_k$$

- Error probability is calculated with

$$\mathcal{P}_e = \frac{1}{2}\mathcal{P}\{\mathbf{v_x} > A\} + \frac{1}{2}\mathcal{P}\{\mathbf{v_x} \leq -A\}$$

where the probabilities are approximated with

$$\mathcal{P}\{\mathbf{v_x} > A\} \approx \frac{\ell_+}{L} \quad \text{and} \quad \mathcal{P}\{\mathbf{v_x} \leq -A\} \approx \frac{\ell_-}{L}$$

ℓ_+ is the number of samples where Re[v_k] is greater than A, ℓ_- is the number of samples where Re[v_k] is less than or equal to -A, and L is the total number of samples.

- A sample function of receiver CGN can be generated with $\hat{n}_k = z_k e^{j\theta_k}$ where z_k are Rayleigh-distributed amplitudes and θ_k are uniformly-distributed phase.

- The remaining slides provide statistical models for generating $\hat{u}_k$ for other signal types.

Undesired MPSK

- Undesired MPSK is representative of co-channel, interference-limited scenarios.

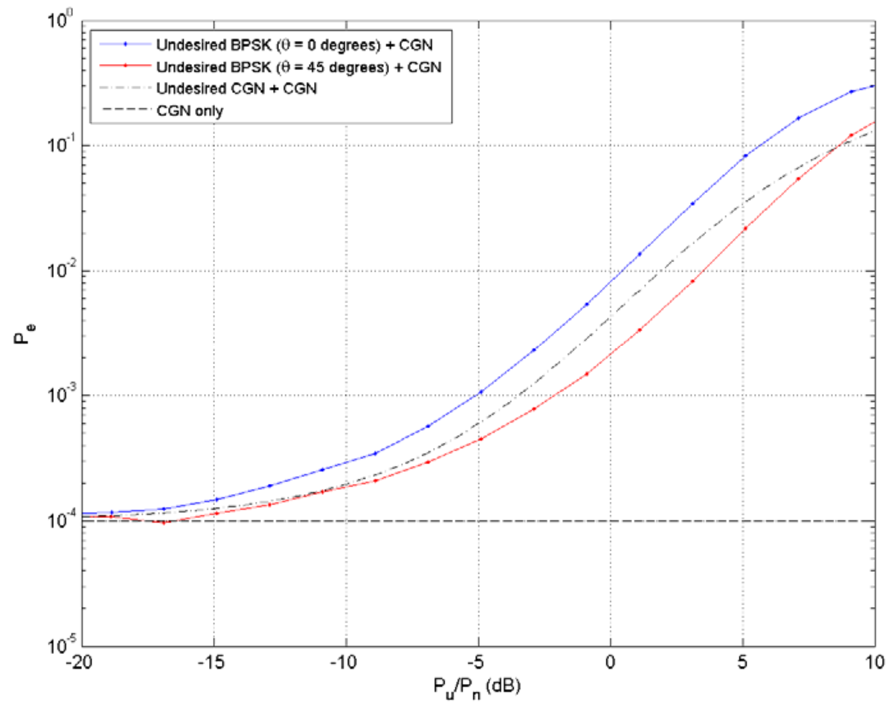
- An MPSK sample function can be generated with

$$\hat{u}_k = \hat{h}_{T,k} * A_u \exp\left[j\left(\frac{2\pi m_k}{M} + \theta_u\right)\right]$$

M = number of symbols,

m_k = uniformly-distributed integers between 0 and M-1

θ_u = offset angle



Undesired Impulsive Noise

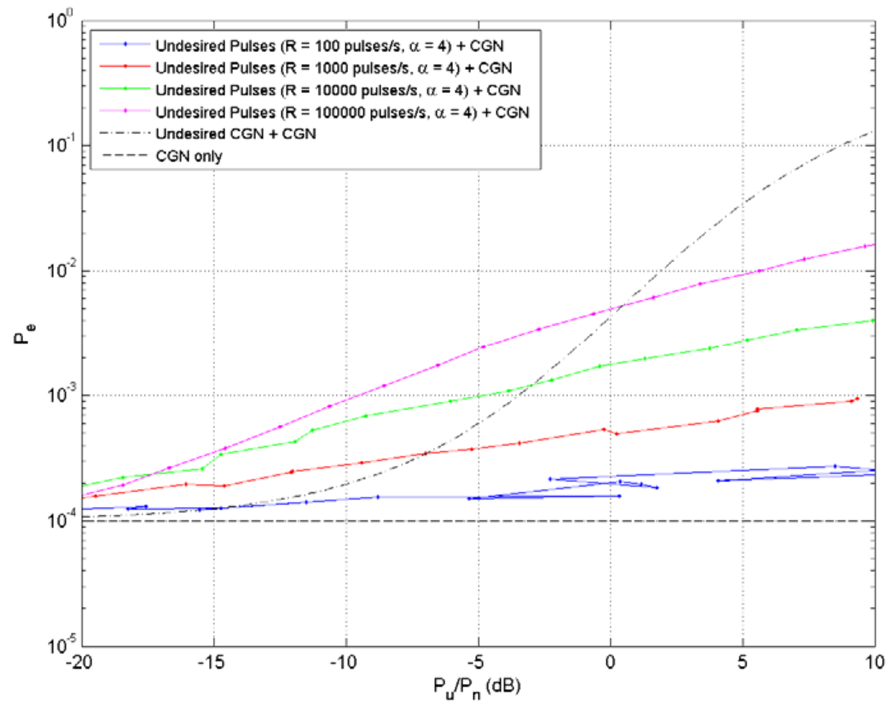
- Undesired impulsive noise is representative of man-made noise limited scenarios.
- A sample function of impulsive noise can be generated with

$$\hat{u}_k = z_k^{(\alpha)} \chi_k e^{j\theta_k}$$

$z_k^{(\alpha)}$ = Weibull-distributed amps

χ_k = binary sample function that determines presence of a pulse

θ_k = uniform phase



* R.J. Achatz, et. al., "Man-made noise in the 136 to 138-MHz VHF meteorological satellite band," NTIA Report 98-355, Sep. 1998.

Undesired Gated Noise

- Undesired gated noise is representative of UWB signals that periodically turn on/off or hop in and out of band.

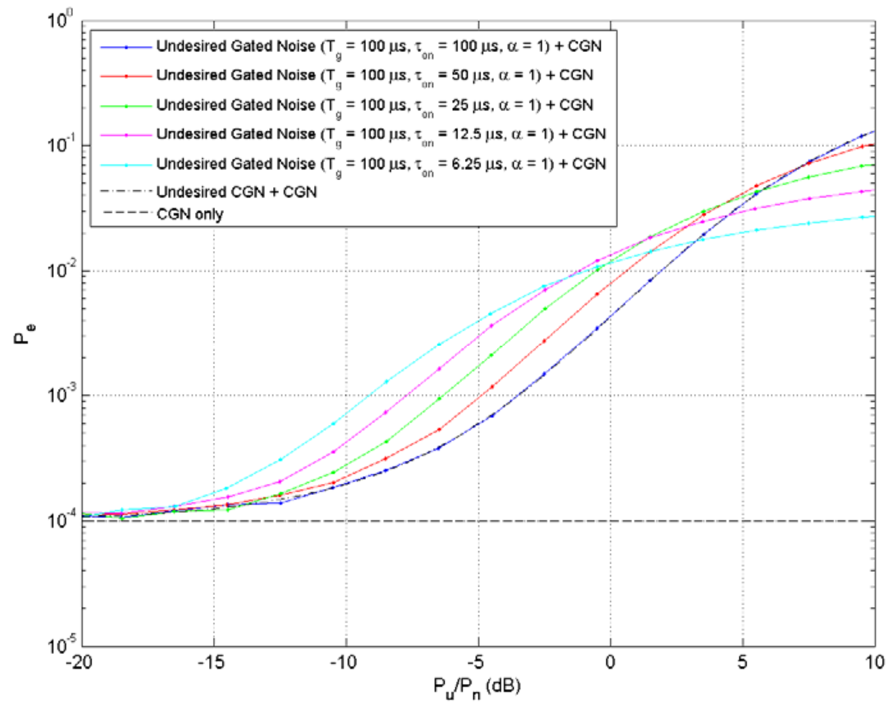
- A sample function of gated noise can be generated with

$$\hat{u}_k = z_k^{(\alpha)} g_k(T_g, \tau_{on}) e^{j\theta_k}$$

$z_k^{(\alpha)}$ = Weibull-distributed amps

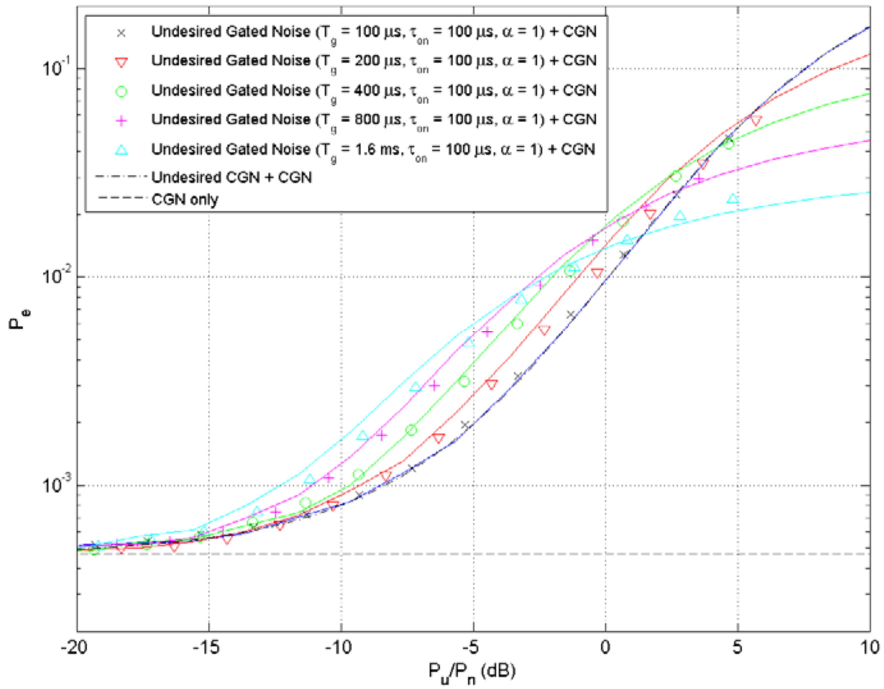
$g_k(T_g, \tau_{on})$ = gating function

θ_k = uniform phase



Applications

- Assess and possibly categorize the interference effects of new spectrum sharing technologies.
- Provide theoretical validation of receiver susceptibility tests.
- Translate statistical descriptions of certain additive undesired signals, e.g., man-man noise, to tangible digital receiver performance metrics.



Summary

- A methodology for assessing BPSK demodulator performance in the presence of various types of undesired signals was presented.
- The goal is to establish a best practice for spectrum engineers to help identify the appropriate interference protection criteria for a given telecom scenario in order to develop fair spectrum policy.
- Future work will include
  - Uncertainty analyses for the sample function approach,
  - Analyses of other undesired signals and other victim receivers, and
  - Parametric study to find correlations between bit error probability and statistical metrics of the undesired signal plus receiver noise.