The ITS Irregular Terrain Model, version 1.2.2
The Algorithm

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In this report I have attempted to write down the algorithm of the ITM (the Irregular Terrain Model) as it is currently implemented. There are probably, however, some features of the model that are not documented here. I hope they are not many, and that one need not refer too often to the original FORTRAN source code.

The purpose of the model is to estimate some of the characteristics of a received signal level for a radio link. This usually means cumulative distributions for what really appears to be a random phenomenon.

The original model was developed in the late 1960's when land mobile radio and television broadcasting were important systems that required better engineering. Perhaps that explains the emphasis on low and hidden antennas and on the long-distance fields that might cause interference.

Historical Bibliography.

The reports listed here are probably the more important mileposts in the development of the ITM. They are not necessarily relevant to the present algorithm, but are included here for their historical interest.

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Longley, A. G., and P. L. Rice (1968), Prediction of tropospheric radio transmission loss over irregular terrain—a computer method 1968, ESSA Tech. Report ERL65-ITS67. Available from NTIS, Access. No. AD-676-874.

Longley, A. G., and R. K. Reasoner (1970), Comparison of propagation measurements with predicted values in the 20 to 10,000 MHz range, ESSA Tech. Report ERL148-ITS97. Available from NTIS, Access. No. ??

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Longley, A. G., and G. A. Hufford (1979), Memorandum to users of ARPROP, of its subroutine ACR, or of the Longley-Rice model in its latest version (version 1.2); subject: Modifications. Unpublished.

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Hufford, G. A. (1985), Memorandum to users of the ITS Irregular Terrain Model; subject: Modifications. Unpublished.

1. Input.

We picture a radio link located in some region on the earth. Then the input needed by the model is a proper description of that link. The two modes of the model—the *area prediction mode* and the *point-to-point mode*—are distinguished mostly by the amount of input data required. The point-to-point mode must provide details of the terrain profile of the link that the area prediction mode will estimate using empirical medians. Since in other respects the two modes follow very similar paths we shall try here to treat both in parallel.

The two terminals of the link we denote as terminals 1 and 2, leaving it to the user to identify which is transmitter and which receiver.

We try always to use consistent units and we think the user can modify our statements to fit any desired basic units. For ourselves, we prefer SI units and express lengths (and distances) in meters. Exceptions to the rule of consistent units might include our measure of atmospheric refractivity (N-units or parts per million) and our measure of losses, attenuations, gains, etc. (in decibels).

1.1. General input for both modes of usage.

d	Distance between the two terminals.
h_{g1}, h_{g2}	Antenna structural heights.
k	Wave number, measured in units of reciprocal lengths; see Note 1.
Δh	Terrain irregularity parameter.
N_s	Minimum monthly mean surface refractivity, measured in N-units; see Note 2.
γ_e	The earth's effective curvature, measured in units of reciprocal length; see Note 3.
Z_g	Surface transfer impedance of the ground—a complex, dimensionless number; see Note 4.
radio climate	Expressed qualitatively as one of a number of discrete climate types.

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Note 1. The wave number is that of the carrier or central frequency. It is defined to be

$$k = 2\pi/\lambda = f/f_0 \qquad \text{with } f_0 = 47.70 \text{ MHz} \cdot \text{m} \tag{1.1}$$

where λ is the wave length, f the frequency. (Here and elsewhere we have assumed the speed of light in air is 299.7 m/μs.)

Note 2. To simplify its representation, the surface refractivity is sometimes given in terms of N_0, the surface refractivity "reduced to sea level." When this is the situation, one must know the general elevation z_s of the region involved, and then

$$N_s = N_0 e^{-z_s/z_1} \qquad \text{with } z_1 = 9.46 \text{ km.} \tag{1.2}$$

Note 3. The earth's effective curvature is the reciprocal of the earth's effective radius and may be expressed as

$$\gamma_e = \gamma_a/K$$

where γ_a is the earth's actual curvature and K is the "effective earth radius factor." The value is normally determined from the surface refractivity using the empirical formula

$$\gamma_e = \gamma_a(1 - 0.04665 \, e^{N_s/N_1}) \tag{1.3}$$

where

$$N_1 = 179.3 \text{ N-units, and} \quad \gamma_a = 157 \cdot 10^{-9} \text{ m}^{-1} = 157 \text{ N-units/km.}$$

Note 4. The "surface transfer impedance" is normally defined in terms of the relative permittivity ϵ_r and conductivity σ of the ground, and the polarization of the radio waves involved. In these terms, we have

$$Z_g = \begin{cases} \sqrt{\epsilon'_r - 1} & \text{horizontal polarization} \\ \sqrt{\epsilon'_r - 1}/\epsilon'_r & \text{vertical polarization} \end{cases} \tag{1.4}$$

where ϵ'_r is the "complex relative permittivity" defined by

$$\epsilon'_r = \epsilon_r + iZ_0\sigma/k, \qquad Z_0 = 376.62 \text{ ohm.} \tag{1.5}$$

The conductivity σ is normally expressed in siemens (reciprocal ohms) per meter.

1.2. Additional input for the area prediction mode.

siting criteria   Criteria describing the care taken at each terminal to assure good radio propagation conditions. This is expressed qualitatively in three steps: at random, with care, and with great care.

1.3. Additional input for the point-to-point mode.

$h_{e1}, \; h_{e2}$ Antenna effective heights.

$d_{L1}, \; d_{L2}$ Distances from each terminal to its radio horizon.

$\theta_{e1}, \; \theta_{e2}$ Elevation angles of the horizons from each terminal at the height of the antennas. These are measured in radians.

These quantities, together with Δh, are all geometric and should be determined from the terrain profile that lies between the two terminals. We shall not go into detail here.

The "effective height" of an antenna is its height above an "effective reflecting plane" or above the "intermediate foreground" between the antenna and its horizon. A difficulty with the model is that there is no explicit definition of this quantity, and the accuracy of the model sometimes depends on the skill of the user in estimating values for these effective heights.

In the case of a line-of-sight path there are no horizons, but the model still requires values for $d_{Lj}, \; \theta_{ej}, \; j = 1, 2$. They should be determined from the formulas used in the area prediction mode and listed in Section 3 below. Now it may happen that after these computations one discovers $d > d_L = d_{L1} + d_{L2}$, implying that the path is a beyond-horizon one. Noting that d_L is a monotone increasing function of the h_{ej} we can assume these latter have been underestimated and that they should be increased by a common factor until $d_L = d$.

2. Output.

The output from the model may take on one of several forms at the user's option. Simplest of these forms is just the *reference attenuation* A_{ref}. This is the *median* attenuation relative to a free space signal that should be observed on the set of all similar paths during times when the atmospheric conditions correspond to a standard, well-mixed, atmosphere.

The second form of output provides the two- or three-dimensional cumulative distribution of attenuation in which time, location, and situation variability are all accounted for. This is done by giving the *quantile* $A(q_T, q_L, q_S)$, the attenuation that will not be exceeded as a function of the fractions of time, locations, and situations. One says *In q_S of the situations there will be at least q_L of the locations where the attenuation does not exceed $A(q_T, q_L, q_S)$ for at least q_T of the time.*

When the point-to-point mode is used on particular, well-defined paths with definitely fixed terminals, there is no location variability, and one must use a two-dimensional description of cumulative distributions. One can now say *With probability (or confidence) q_S the attenuation will not exceed $A(q_T, q_S)$ for at least q_T of the time.* The same effect can be achieved by setting $q_L = 0.5$ in the three-dimensional formulation.

On some occassions it will be desireable to go beyond the three-dimensional quantiles and to treat directly the underlying model of variability. For example, consider the case of a

communications link that is to be used once and once only. For such a "one-shot" system one is interested only in what probability or confidence an adequate signal is received that once. The three-dimensional distributions used above must now be combined into one.

3. Preparatory Calculations.

We start with some preliminary calculations of a geometric nature.

3.1. Preparatory calculations for the area prediction mode.

The parameters h_{ej}, d_{Lj}, θ_{ej}, $j = 1, 2$, which are part of the input in the point-to-point mode are, in the area prediction mode, estimated using empirical formulas in which Δh plays an important role.

First, consider the effective heights. This is where the siting criteria are used. We have

$$h_{ej} = h_{gj} \qquad \text{if terminal } j \text{ is sited at random.} \tag{3.1}$$

Otherwise, let

$$B_j = \begin{cases} 5 \text{ m} & \text{if terminal } j \text{ is sited with care} \\ 10 \text{ m} & \text{if terminal } j \text{ is sited with great care.} \end{cases}$$

Then

$$B'_j = (B_j - H_1)\sin\left(\frac{\pi}{2}\min(h_{g1}/H_2, 1)\right) + H_1 \qquad \text{with } H_1 = 1 \text{ m}, \ H_2 = 5 \text{ m},$$

and

$$h_{ej} = h_{gj} + B'_j e^{-2 h_{gj}/\Delta h}. \tag{3.2}$$

The remaining parameters are quickly determined.

$$d_{Lsj} = \sqrt{2h_{ej}/\gamma_e}$$
$$d_{Lj} = d_{Lsj}\exp\left[-0.07\sqrt{\Delta h/\max(h_{ej}, H_3)}\right] \qquad \text{with } H_3 = 5 \text{ m}, \tag{3.3}$$

and finally,

$$\theta_{ej} = [0.65\,\Delta h(d_{Lsj}/d_{Lj} - 1) - 2h_{ej}]/d_{Lsj}. \tag{3.4}$$

3.2. Preparatory calculations for both modes.

$$d_{Lsj} = \sqrt{2h_{ej}/\gamma_e}, \qquad j = 1, 2 \tag{3.5}$$
$$d_{Ls} = d_{Ls1} + d_{Ls2} \tag{3.6}$$
$$d_L = d_{L1} + d_{L2} \tag{3.7}$$
$$\theta_e = \max(\theta_{e1} + \theta_{e2}, -d_L\gamma_e). \tag{3.8}$$

We also note here the definitions of two functions of a distance s:

$$\Delta h(s) = (1 - 0.8\,e^{-s/D})\Delta h \qquad \text{with } D = 50 \text{ km}, \tag{3.9}$$

and

$$\sigma_h(s) = 0.78\,\Delta h(s)\exp\left[-(\Delta h(s)/H)^{1/4}\right] \qquad \text{with } H = 16 \text{ m}. \tag{3.10}$$

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4. The Reference Attenuation.

The reference attenuation is determined as a function of the distance d from the piecewise formula

$$A_{\text{ref}} = \begin{cases} \max\bigl(0, A_{el} + K_1 d + K_2 \ln(d/d_{Ls})\bigr) & d \leq d_{Ls} \\ A_{ed} + m_d d & d_{Ls} \leq d \leq d_x \\ A_{es} + m_s d & d_x \leq d \end{cases} \tag{4.1}$$

where the coefficients A_{el}, K_1, K_2, A_{ed}, m_d, A_{es}, m_s, and the distance d_x are calculated using the algorithms below. The three intervals defined here are called the line-of-sight, diffraction, and scatter regions, respectively. The function in (4.1) is continuous so that at the two endpoints where $d = d_{Ls}$ or d_x the two formulas give the same results. It follows that instead of seven independent coefficients there are really only five.

4.1. Coefficients for the diffraction range.

Set

$$X_{ae} = (k\gamma_e^2)^{-1/3} \tag{4.2}$$

$$d_3 = \max(d_{Ls}, d_L + 1.3787\, X_{ae}) \tag{4.3}$$

$$d_4 = d_3 + 2.7574\, X_{ae} \tag{4.4}$$

$$A_3 = A_{\text{diff}}(d_3) \tag{4.5}$$

$$A_4 = A_{\text{diff}}(d_4) \tag{4.6}$$

where A_{diff} is the function defined below. The formula for A_{ref} in the diffraction range is then just the linear function having the values A_3 and A_4 at the distances d_3 and d_4, respectively. Thus

$$m_d = (A_4 - A_3)/(d_4 - d_3) \tag{4.7}$$

$$A_{ed} = A_3 - m_d d_3. \tag{4.8}$$

4.1.1. The function $A_{\text{diff}}(s)$.

We first define the weighting factor

$$w = \frac{1}{1 + 0.1\,\sqrt{Q}} \tag{4.9}$$

with

$$Q = \min\bigl(\frac{k}{2\pi}\Delta h(s), 1000\bigr)\left(\frac{h_{e1}h_{e2} + C}{h_{g1}h_{g2} + C}\right)^{1/2} + \frac{d_L + \theta_e/\gamma_e}{s}$$

and

$$C = \begin{cases} 0 & \text{in the area prediction mode} \\ 10 \text{ m}^2 & \text{in the point-to-point mode} \end{cases}$$

and where $\Delta h(s)$ is the function defined in (3.9) above. Next we define a "clutter factor"

$$A_{fo} = \min\bigl[15, 5\log\bigl(1 + \alpha k h_{g1} h_{g2} \sigma_h(d_{Ls})\bigr)\bigr] \qquad \text{with } \alpha = 4.77 \cdot 10^{-4} \text{ m}^{-2} \tag{4.10}$$

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and with $\sigma_h(s)$ defined in (3.10) above.

Then
$$A_{\mathrm{diff}}(s) = (1 - w)A_k + wA_r + A_{fo} \tag{4.11}$$
where the "double knife edge attenuation" A_k and the "rounded earth attenuation" A_r are yet to be defined. Set
$$\theta = \theta_e + s\gamma_e \tag{4.12}$$
$$v_j = \frac{\theta}{2} \left(\frac{k}{\pi} \frac{d_{Lj}(s - d_L)}{s - d_L + d_{Lj}} \right)^{1/2}, \qquad j = 1, 2 \tag{4.13}$$
and then
$$A_k = \mathrm{Fn}(v_1) + \mathrm{Fn}(v_2) \tag{4.14}$$
where $\mathrm{Fn}(v)$ is the Fresnel integral defined below.

For the rounded earth attenuation we use a "three radii" method applied to Vogler's formulation of the solution to the smooth, spherical earth problem. We set
$$\gamma_0 = \theta/(s - d_L) \qquad \gamma_j = 2h_{ej}/d_{Lj}^2, \quad j = 1, 2 \tag{4.15}$$

$$\alpha_j = (k/\gamma_j)^{1/3}, \qquad j = 0, 1, 2 \tag{4.16}$$
$$K_j = \frac{1}{i\alpha_j Z_g}, \qquad j = 0, 1, 2. \tag{4.17}$$
Note that the K_j are complex numbers. To continue, we set
$$x_j = AB(K_j)\alpha_j\gamma_j d_{Lj}, \qquad j = 1.2 \tag{4.18}$$
$$x_0 = AB(K_0)\alpha_0\theta + x_1 + x_2 \tag{4.19}$$
and then
$$A_r = G(x_0) - F(x_1, K_1) - F(x_2, K_2) - C_1(K_0) \tag{4.20}$$
where $A = 151.03$ is a dimensionless constant and the functions $B(K)$, $G(x)$, $F(x, K)$, and $C_1(K)$ are those defined by Vogler.

In (4.14) and (4.20) we have finished the definition of A_{diff}. We should like, however, to complete the subject by defining more precisely the more or less standard functions mentioned above. The Fresnel integral, for example, may be written as
$$\mathrm{Fn}(v) = 20 \log \left| \frac{1}{\sqrt{2i}} \int_v^\infty e^{i\pi u^2/2} \, du \right|. \tag{4.21}$$

For Vogler's formulation to the solution to the spherical earth problem, we first introduce the special Airy function
$$\mathrm{Wi}(z) = \mathrm{Ai}(z) + i\mathrm{Bi}(z)$$
$$= 2\mathrm{Ai}(e^{2\pi i/3}z)$$

where $\mathrm{Ai}(z)$ and $\mathrm{Bi}(z)$ are the two standard Airy functions defined in many texts. They are analytic in the entire complex plane and are particular solutions to the differential equation

$$w''(z) - zw(z) = 0.$$

First, to define the function $B(K)$ we find the smallest solution to the modal equation

$$\mathrm{Wi}(t_0) = 2^{1/3} K \, \mathrm{Wi}'(t_0)$$

and then

$$B = 2^{-1/3} \mathrm{Im}\{t_0\}. \tag{4.22}$$

Finally, we also have

$$G(x) = 20 \, \log(x^{-1/2} e^{x/A}) \tag{4.23}$$

$$F(x,K) = 20 \, \log\left|(\pi/(2^{1/3}AB))^{1/2} \, \mathrm{Wi}(t_0 - (x/(2^{1/3}AB))^2)\right| \tag{4.24}$$

$$C_1(K) = 20 \, \log\left|\frac{1}{2}(\pi/(2^{1/3}AB))^{1/2}(2^{2/3}K^2 t_0 - 1)\mathrm{Wi}'(t_0)^2\right| \tag{4.25}$$

where A is again the constant defined above.

It is of interest to note that for large x we find $F(x,K) \sim G(x)$, and that for those values of K in which we are interested it is a good approximation to say $C_1(K) = 20$ dB.

4.2. Coefficients for the line-of-sight range.

We begin by setting

$$d_2 = d_{Ls} \tag{4.26}$$

$$A_2 = A_{ed} + m_d d_2. \tag{4.27}$$

Then there are two general cases. First, if $A_{ed} \geq 0$

$$d_0 = \min\left(\frac{1}{2}d_L, 1.908 \, k h_{e1} h_{e2}\right) \tag{4.28}$$

$$d_1 = \frac{3}{4}d_0 + \frac{1}{4}d_L \tag{4.29}$$

$$A_0 = A_{\mathrm{los}}(d_0) \tag{4.30}$$

$$A_1 = A_{\mathrm{los}}(d_1) \tag{4.31}$$

where the function $A_{\mathrm{los}}(s)$ is defined below. The idea, now, is to devise a curve of the form

$$A_{el} + K_1 d + K_2 \ln(d/d_{Ls})$$

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that passes through the three values A_0, A_1, A_2 at d_0, d_1, d_2, respectively,. In doing this, however, we require K_1, $K_2 \geq 0$, and sometimes this forces us to abandon one or both of the values A_0, A_1. We first define

$$K_2' = \max \left[0, \frac{(d_2 - d_0)(A_1 - A_0) - (d_1 - d_0)(A_2 - A_0)}{(d_2 - d_0)\ln(d_1/d_0) - (d_1 - d_0)\ln(d_2/d_0)} \right] \tag{4.32}$$

$$K_1' = (A_2 - A_0 - K_2'\ln(d_2/d_0))/(d_2 - d_0) \tag{4.33}$$

which, except for the possibility that the first calculation for K_2' results in a negative value, is simply the straightforward solution for the two corresponding coefficients. If $K_1' \geq 0$ we then have

$$K_1 = K_1', \qquad K_2 = K_2'. \tag{4.34}$$

If, however, $K_1' < 0$, we define

$$K_2'' = (A_2 - A_0)/\ln(d_2/d_0), \tag{4.35}$$

and if now $K_2'' \geq 0$ then

$$K_1 = 0, \qquad K_2 = K_2''. \tag{4.36}$$

Otherwise, we abandon both A_0 and A_1 and set

$$K_1 = m_d, \qquad K_2 = 0. \tag{4.37}$$

In the second general case we have $A_{ed} < 0$. We then set

$$d_0 = 1.908 \, k h_{e1} h_{e2} \tag{4.38}$$

$$d_1 = \max(-A_{ed}/m_d, d_L/4). \tag{4.39}$$

If $d_0 < d_1$ we again evaluate A_0, A_1, and K_2' as before. If $K_2' > 0$ we also evaluate K_1' and proceed exactly as before. If, however, we have either $d_0 \geq d_1$ or $K_2' = 0$, we evaluate A_1 and define

$$K_1'' = (A_2 - A_1)/(d_2 - d_1). \tag{4.40}$$

If now $K_1'' > 0$ we set

$$K_1 = K_1'', \qquad K_2 = 0; \tag{4.41}$$

and otherwise we use (4.37).

At this point we will have defined the coefficients K_1 and K_2. We finally set

$$A_{el} = A_2 - K_1 d_2. \tag{4.42}$$

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4.2.1. The function $A_{\mathrm{los}}(s)$

First we define the weighting factor

$$w = 1/\big(1 + D_1 k \Delta h / \max(D_2, d_{Ls})\big) \qquad \text{with } D_1 = 47.7 \text{ m}, \ D_2 = 10 \text{ km}. \tag{4.43}$$

Then

$$A_{\mathrm{los}} = (1 - w)A_d + wA_t \tag{4.44}$$

where the "extended diffraction attenuation" A_d and the "two-ray attenuation" A_t are yet to be defined.

First, the extended diffraction attenuation is given very simply by

$$A_d = A_{ed} + m_d s. \tag{4.45}$$

For the two-ray attenuation, we set

$$\sin \psi = \frac{h_{e1} + h_{e2}}{\sqrt{s^2 + (h_{e1} + h_{e2})^2}} \tag{4.46}$$

and

$$R'_e = \frac{\sin \psi - Z_g}{\sin \psi + Z_g} \exp[-k\sigma_h(s) \sin \psi] \tag{4.47}$$

where $\sigma_h(s)$ is the function defined in (3.10) above. Note that R'_e is complex since it uses the complex surface transfer impedance Z_g. Then

$$R_e = \begin{cases} R'_e & \text{if } |R'_e| \geq \max(1/2, \sqrt{\sin \psi}) \\ (R'_e / |R'_e|) \sqrt{\sin \psi} & \text{otherwise} \end{cases} \tag{4.48}$$

We also set

$$\delta' = 2k h_{e1} h_{e2} / s \tag{4.49}$$

and

$$\delta = \begin{cases} \delta' & \text{if } \delta' \leq \pi/2 \\ \pi - (\pi/2)^2 / \delta' & \text{otherwise} \end{cases} \tag{4.50}$$

Then finally

$$A_t = -20 \log |1 + R_e e^{i\delta}|. \tag{4.51}$$

4.3. Coefficients for the scatter range.

Set

$$d_5 = d_L + D_s \tag{4.52}$$

$$d_6 = d_5 + D_s \qquad \text{with } D_s = 200 \text{ km.} \tag{4.53}$$

Then define

$$A_5 = A_{\text{scat}}(d_5) \tag{4.54}$$

$$A_6 = A_{\text{scat}}(d_6), \tag{4.55}$$

where $A_{\text{scat}}(s)$ is defined below. There are, however, some sets of parameters for which A_{scat} is not defined, and it may happen that either or both A_5, A_6 is undefined. If this is so, one merely sets

$$d_x = +\infty \tag{4.56}$$

and one can let A_{es}, m_s remain undefined. In the more normal situation one has

$$m_s = (A_6 - A_5)/D_s \tag{4.57}$$

$$d_x = \max\left[d_{Ls}, d_L + X_{ae}\log(kH_s), (A_5 - A_{ed} - m_s d_5)/(m_d - m_s)\right] \tag{4.58}$$

$$A_{es} = A_{ed} + (m_d - m_s)d_x \tag{4.59}$$

where D_s is the distance given above, where X_{ae} has been defined in (4.2), and where $H_s = 47.7$ m.

4.3.1. The function A_{scat}.

Computation of this function uses an abbreviated version of the methods described in Section 9 and Annex III.5 of NBS TN101. First, set

$$\theta = \theta_e + \gamma_e s \tag{4.60}$$

$$\theta' = \theta_{e1} + \theta_{e2} + \gamma_e s \tag{4.61}$$

$$r_j = 2 k \theta' h_{ej}, \qquad j = 1, 2. \tag{4.62}$$

If both r_1 and r_2 are less than 0.2 the function A_{scat} is not defined (or is infinite). Otherwise we put

$$A_{\text{scat}}(s) = 10 \log(kH\theta^4) + F(\theta s, N_s) + H_0 \tag{4.63}$$

where $F(\theta s, N_s)$ is the function shown in Figure 9.1 of TN101, H_0 is the "frequency gain function", and $H = 47.7$ m.

The frequency gain function H_0 is a function of r_1, r_2, the scatter efficiency factor η_s, and the "asymmetry factor" which we shall here call s_s. A difficulty with the present model is that there is not sufficient geometric data in the input variables to determine where the crossover point is. This is resolved by assuming it to be midway between the

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two horizons. The asymmetry factor, for example, is found by first defining the distance between horizons

$$d_s = s - d_{L1} - d_{L2} \tag{4.64}$$

whereupon

$$s_s = \frac{d_{L2} + d_s/2}{d_{L1} + d_s/2}. \tag{4.65}$$

There then follows that the height of the crossover point is

$$z_0 = \frac{s_s d\theta'}{(1 + s_s)^2} \tag{4.66}$$

and then

$$\eta_s = \frac{z_0}{Z_0} \left[1 + (0.031 - N_s 2.32 \cdot 10^{-3} + N_s^2 5.67 \cdot 10^{-6}) e^{-(z_0/Z_1)^6} \right] \tag{4.67}$$

where

$$Z_0 = 1.756 \text{ km} \qquad Z_1 = 8.0 \text{ km}$$

The computation of H_0 then proceeds according to the rules in Section 9.3 and Figure 9.3 of TN101.

The model requires these results at the two distances $s = d_5$, d_6, described above. One further precaution is taken to prevent anomalous results. If, at d_5, calculations show that H_0 will exceed 15 dB, they are replaced by the value it has at d_6. This helps keep the scatter-mode slope within reasonable bounds.

5. Variability—the quantiles of attenuation.

We want now to compute the quantiles $A(q_T, q_L, q_S)$ where q_T, q_L, q_S, are the desired fractions of time, locations, and situations, respectively. In the point-to-point mode, we would want a two-fold quantile $A(q_T, q_S)$, but in the present model this is done simply by computing the three-fold quantile with q_L equal to 0.5.

Because the distributions involved are all normal, or nearly normal, it simplifies the calculations to rescale the desired fractions and to express them in terms of "standard normal deviates." We use the complementary normal distribution

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-t^2/2} dt$$

and then the deviate is simply the inverse function

$$z(q) = Q^{-1}(q).$$

Thus if the random variable x is normally distributed with mean X_0 and standard deviation σ, its quantiles are given by

$$X(q) = X_0 + \sigma z(q).$$

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Setting
$$z_T = z(q_T), \quad z_L = z(q_L), \quad z_S = z(q_S),$$
we now ask for the quantiles $A(z_T, z_L, z_S)$. In these rescaled variables, it is as though all probabilities are to be plotted on normal probability paper. In the case of the point-to-point mode we will simply suppose $z_L = 0$.

First we define
$$A' = A_{\text{ref}} - V_{\text{med}} - Y_T - Y_L - Y_S \tag{5.1}$$
where A_{ref} is the reference attenuation defined in Section 4, and where the adjustment V_{med} and the deviations Y_T, Y_L, Y_S, are defined below. The values of Y_T and Y_L depend on the single variables z_T and z_L, respectively. The value of Y_S, on the other hand, depends on all three standard normal deviates.

The final quantile is a modification of A' given by

$$A(z_T, z_L, z_S) = \begin{cases} A' & \text{if } A' \geq 0 \\ A' \dfrac{29 - A'}{29 - 10A'} & \text{otherwise.} \end{cases} \tag{5.2}$$

An important quantity used below is the "effective distance." We set

$$d_{ex} = \sqrt{2a_1 h_{e1}} + \sqrt{2a_1 h_{e2}} + a_1 (kD_1)^{-1/3} \tag{5.3}$$

with
$$a_1 = 9000 \text{ km}, \qquad D_1 = 1266 \text{ km}.$$
Then the effective distance is given by

$$d_e = \begin{cases} D_0 d/d_{ex} & \text{for } d \leq d_{ex} \\ D_0 + d - d_{ex} & \text{for } d \geq d_{ex} \end{cases} \tag{5.4}$$

with $D_0 = 130$ km.

5.1. Time variability.

Quantiles of time variability are computed using a variation of the methods described in Section 10 and Annex III.7 of NBS TN101, and also in CCIR Report 238-3. Those methods speak of eight or nine discrete radio climates, of which seven have been documented with corresponding empirical curves. It is these empirical curves to which we refer below. They are all curves of quantiles of deviations versus the effective distance d_e.

The adjustment from the reference attenuation to the all-year median is

$$V_{\text{med}} = V_{\text{med}}(d_e, \text{clim}) \tag{5.5}$$

where the function is described in Figure 10.13 of TN101.

The deviation Y_T is piecewise linear in z_T, and may be written in the form

$$Y_T = \begin{cases} \sigma_{T-} z_T & z_T \leq 0 \\ \sigma_{T+} z_T & 0 \leq z_T \leq z_D \\ \sigma_{T+} z_D + \sigma_{TD}(z_T - z_D) & z_D \leq z_T \end{cases} \tag{5.6}$$

The slopes (or "pseudo-standard deviations")

$$\begin{aligned} \sigma_{T-} &= \sigma_{T-}(d_e, \text{clim}) \\ \sigma_{T+} &= \sigma_{T+}(d_e, \text{clim}) \end{aligned} \tag{5.7}$$

are obtained from TN101 in the following way. For σ_{T-} we use the .90 quantile and divide the corresponding ordinates by $z(.90) = -1.282$. For σ_{T+} we use the .10 quantile and divide by $z(.10) = 1.282$.

The remaining constants in (5.6) pertain to the "ducting," or low probability, case. We write

$$z_D = z_D(\text{clim}), \qquad \sigma_{TD} = C_D(\text{clim})\sigma_{T+} \tag{5.8}$$

where values of z_D and C_D are given in Table 5.1. In that table we have also listed values of $q_D = Q(z_D)$.

Table 5.1. Ducting (low probability) constants

Climate	q_D	z_D	C_D
Equatorial	.10	1.282	1.224
Continental Subtropical	$\approx$.015	2.161	.801
Maritime Subtropical	.10	1.282	1.380
Desert	0	∞	—
Continental Temperate	.10	1.282	1.224
Maritime Temperate Overland	.10	1.282	1.518
Maritime Temperate Oversea	.10	1.282	1.518

5.2. Location variability.

We set

$$Y_L = \sigma_L z_L \tag{5.9}$$

where

$$\sigma_L = 10k\Delta h(d)/(k\Delta h(d) + 13)$$

and $\Delta h(s)$ is defined in (3.9) above.

5.3. Situation variability.

Set

$$\sigma_S = 5 + 3e^{-d_e/D} \tag{5.10}$$

where $D = 100$ km. Then

$$Y_S = \left(\sigma_S^2 + \frac{Y_T^2}{7.8 + z_S^2} + \frac{Y_L^2}{24 + z_S^2} \right)^{1/2} z_S \tag{5.11}$$

This latter is intended to reveal how the uncertainties become greater in the wings of the distributions.

6. Addenda—numerical approximations.

Part of the algorithm for the ITM consists in approximations for the standard functions that have been used. In these approximations, computational simplicity has often taken greater priority than accuracy.

The Fresnel integral is used in §4.1.1 and is defined in (4.21). We have (for $v > 0$)

$$\text{Fn}(v) \approx \begin{cases} 6.02 + 9.11v - 1.27v^2 & \text{if } v \leq 2.40 \\ 12.953 + 20 \log v & \text{otherwise} \end{cases} \tag{6.1}$$

The functions $B(K)$, $G(x)$, $F(x, K)$, $C_1(K)$, which are used in diffraction around a smooth earth, are also used in §4.1.1 and are defined in (4.22)–(4.25). We have

$$B(K) \approx 1.607 - |K| \tag{6.2}$$

$$G(x) = .05751x - 10 \log x \tag{6.3}$$

$$F(x, K) \approx \begin{cases} F_2(x, K) & \text{if } 0 < x \leq 200 \\ G(x) + 0.0134xe^{-x/200}(F_1(x) - G(x)) & \text{if } 200 < x < 2000 \\ G(x) & \text{if } 2000 \leq x \end{cases} \tag{6.4}$$

where

$$F_1(x) = 40 \log(\max(x, 1)) - 117 \tag{6.5}$$

$$F_2(x, K) = \begin{cases} F_1(x) & \text{if } |K| < 10^{-5} \text{ or } x(-\log |K|)^3 > 450 \\ 2.5 \cdot 10^{-5}x^2/|K| + 20 \log |K| - 15 & \text{otherwise} \end{cases} \tag{6.6}$$

The final approximation here is

$$C_1(K) \approx 20 \tag{6.7}$$

To complete this section we have the two functions, $F(\theta d)$ and H_0, used for tropospheric scatter. First,

$$F(D, N_s) = F_0(D) - 0.1(N_s - 301)e^{-D/D_0} \tag{6.8}$$

where

$$D_0 = 40 \text{ km}$$

and (when D is given in meters)

$$F_0(D) = \begin{cases} 133.4 + 0.332 \cdot 10^{-3}D - 10 \ \log D & \text{for } 0 < D \leq 10 \text{ km} \\ 104.6 + 0.212 \cdot 10^{-3}D - 2.5 \ \log D & \text{for } 10 < D \leq 70 \text{ km} \\ 71.8 + 0.157 \cdot 10^{-3}D + 5 \ \log D & \text{otherwise} \end{cases} \qquad (6.9)$$

The frequency gain function may be written as

$$H_0 = H_{00}(r_1, r_2, \eta_s) + \Delta H_0 \qquad (6.10)$$

where

$$\Delta H_0 = 6(0.6 - \log \eta_s) \log s_s \ \log r_2/s_s r_1 \qquad (6.11)$$

and where H_{00} is obtained by linear interpolation between its values when η_s is an integer. For $\eta_s = 1, \ldots, 5$ we set

$$H_{00}(r_1, r_2, j) = \frac{1}{2}[H_{01}(r_1, j) + H_{01}(r_2, j)] \qquad (6.12)$$

with

$$H_{01}(r, j) = \begin{cases} 10 \ \log(1 + 24r^{-2} + 25r^{-4}) & j = 1 \\ 10 \ \log(1 + 45r^{-2} + 80r^{-4}) & j = 2 \\ 10 \ \log(1 + 68r^{-2} + 177r^{-4}) & j = 3 \\ 10 \ \log(1 + 80r^{-2} + 395r^{-4}) & j = 4 \\ 10 \ \log(1 + 105r^{-2} + 705r^{-4}) & j = 5 \end{cases} \qquad (6.13)$$

For $\eta_s > 5$ we use the value for $\eta_s = 5$, and for $\eta_s = 0$ we suppose

$$H_{00}(r_1, r_2, 0) = 10 \ \log \left[\left(1 + \frac{\sqrt{2}}{r_1}\right)^2 \left(1 + \frac{\sqrt{2}}{r_2}\right)^2 \frac{r_1 + r_2}{r_1 + r_2 + 2\sqrt{2}} \right] \qquad (6.14)$$

In all of this, we truncate the values of s_s and $q = r_2/s_s r_1$ at 0.1 and 10.

end

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