First-Principles Modeling and Statistical Analyses Wave Propagation in Complex Environments

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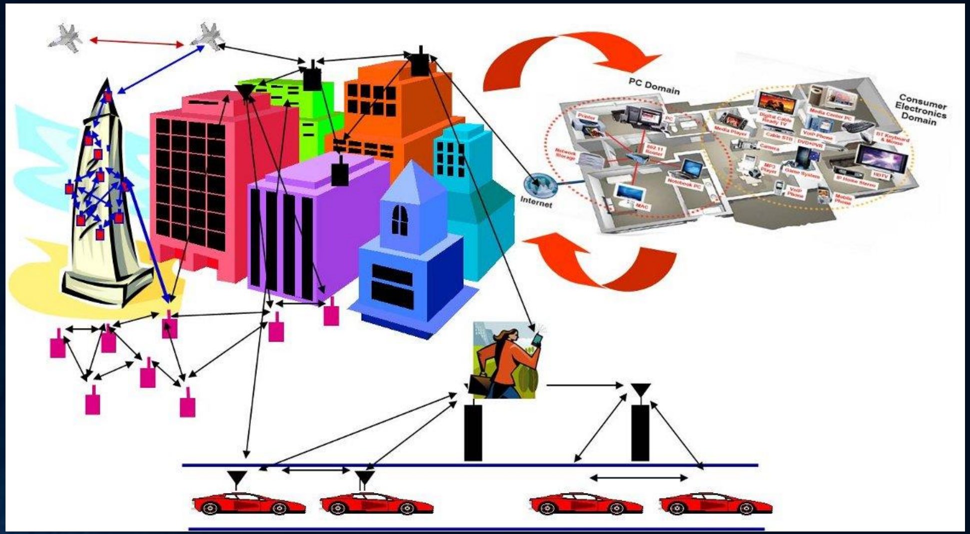
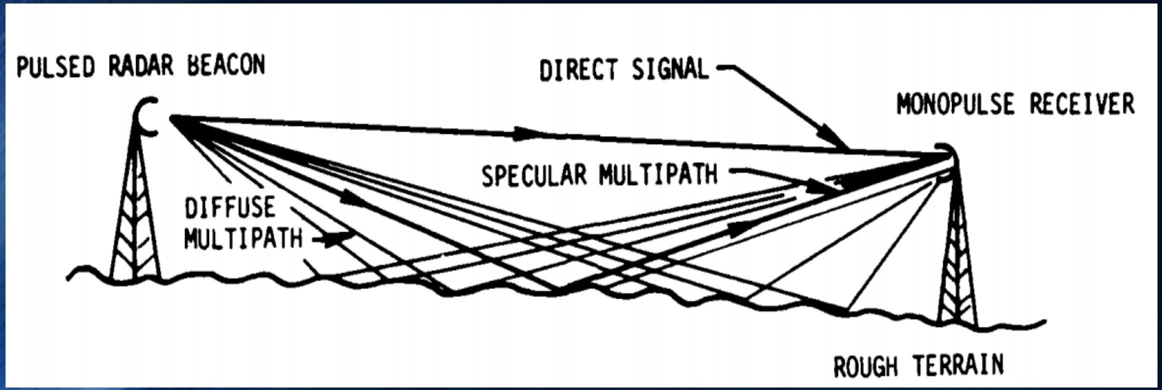
Communication through Diffuse Multipath Environments

Open environments:

Wireless channels in urban environments

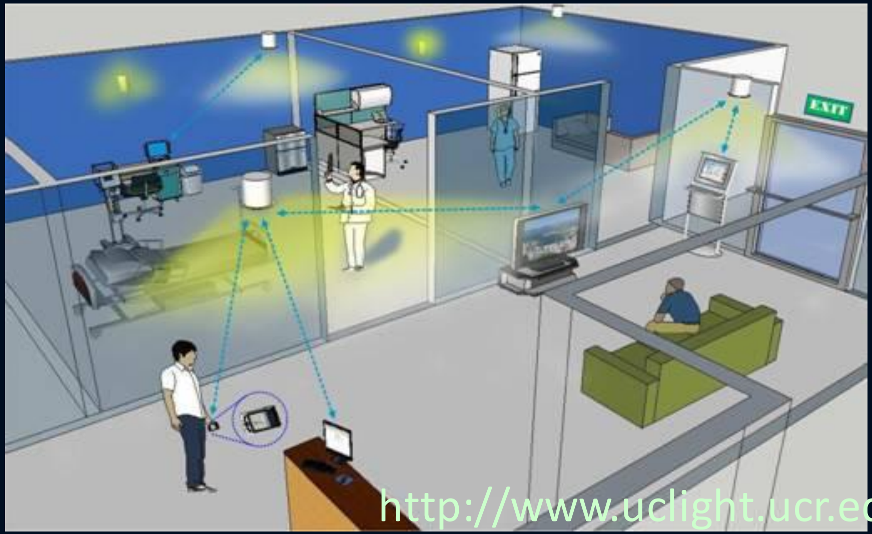


Radar systems and SAR imaging



Confined systems:

Indoor communication and



http://www.uclight.ucr.edu

Naval ship compartments

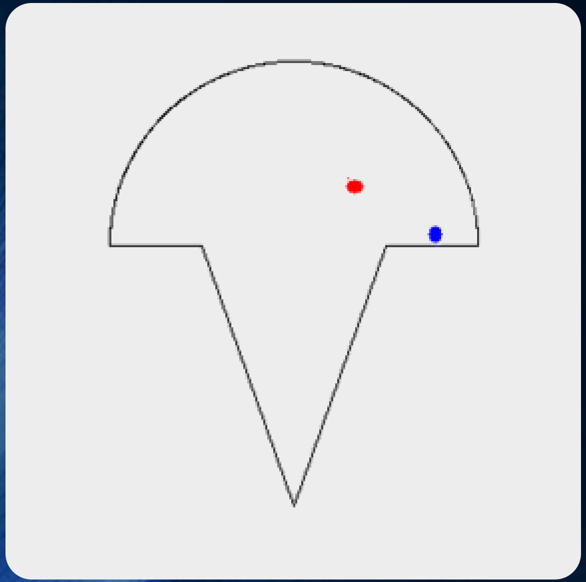
Communication through Diffuse Multipath Environments

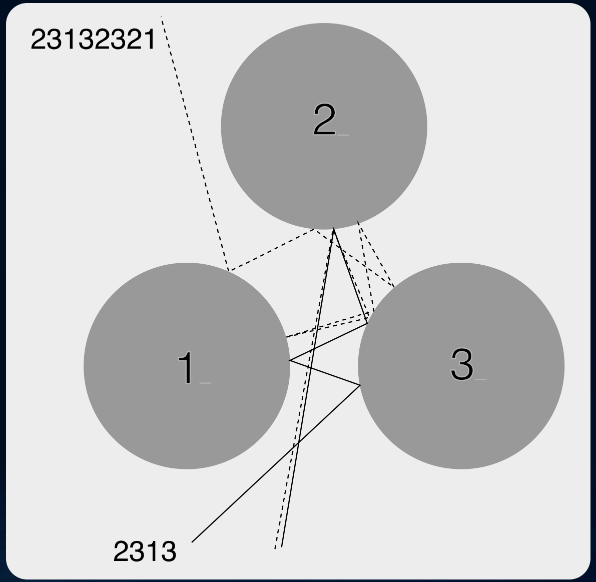
Chaotic ray dynamics:

- Extreme multiple path
- Exponential sensitivity
- Ergodicity
- Broadband spectra of dynamics in phase space

Advanced applications:

- MIMO communications
- Time-reversal systems
- Wavefront shaping and focusing
- Sensing and targeting





http://wamoresearch.org *Chaos: Classical and Quantum*

Overview of Proposed Work

Objective: first-principles mathematical model which statistically replicates the multipath, ray-chaotic interactions between transmitters and receivers

Stochastic Green's Function	Stochastic Integral Equation	Hybrid Formulation
The fundamental solution of wave equations in wave-chaotic media	The statistical characterization of complex multipath environments	Incorporation of the component-specific information

Methodology and contributions:

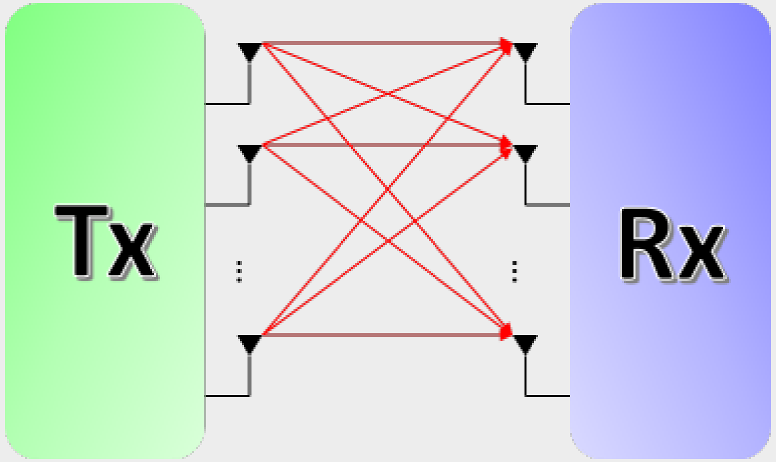
- Physics-oriented statistical representations of complex multipath environments
- Quantitative statistical analysis of EM systems exhibiting chaotic dynamics
- Encoding the governing physics into the mathematical information theory

Motivation for Stochastic Green's Function

Multi-antenna information theory:

$$I = \log_2 det\left(I + G^{\dagger} G/N\right)$$

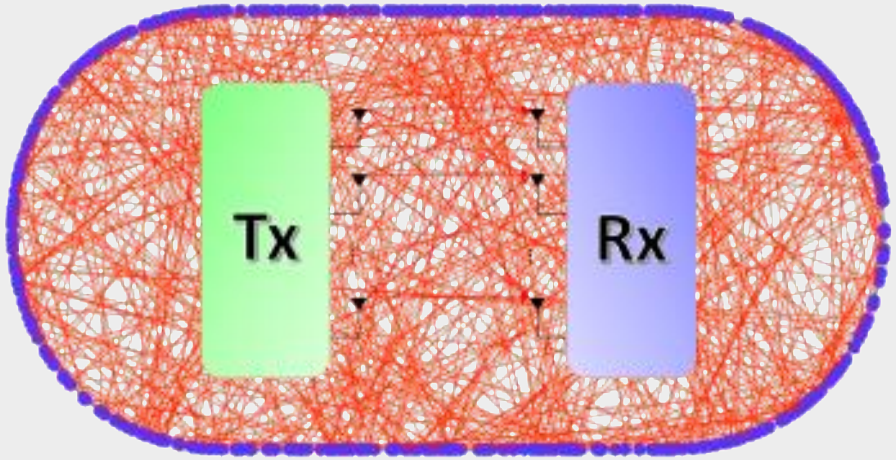
Free-space environment



Free-space Green's Function

$$G_0 = \frac{e^{-jk|r' - r|}}{4\pi|r' - r|}$$

Ray-chaotic environment



Chaotic Green's Function

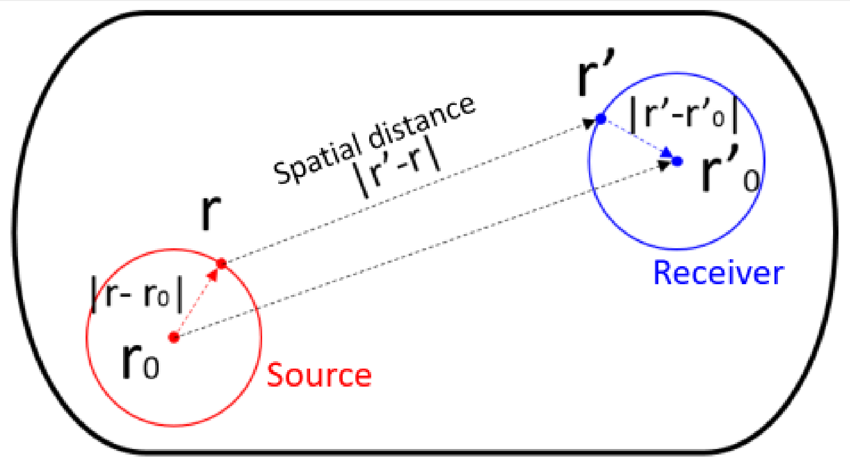
$$G_S ?$$

Stochastic Green's Function for Wave Chaotic Media

Two theoretical tools:

Random Plane Wave Hypothesis and Random Matrix Theory (RMT)

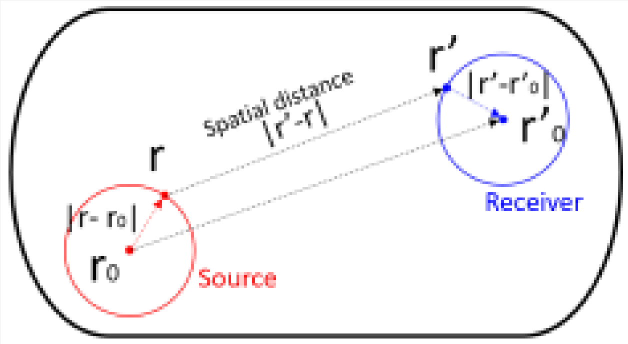
$$G_S(r, r') = \underbrace{\frac{1}{\pi}\sum_i \frac{\omega_i^2}{k^2 - k_i^2}\,\frac{k\Delta}{k_i}\,\frac{\sin k_i|\vec{r}' - \vec{r}|}{4\pi|\vec{r}' - \vec{r}|}}_{\text{correlated } G_{co}} + \underbrace{\sum_i \frac{\omega_i \omega_i'}{k^2 - k_i^2}\,\frac{k\Delta}{4\pi^2}\,\frac{\sin k_i\left|\vec{r}' - \vec{r}_0'\right|}{k_i\left|\vec{r}' - \vec{r}_0'\right|}\,\frac{\sin k_i|\vec{r} - \vec{r}_0|}{k_i|\vec{r} - \vec{r}_0|}}_{\text{uncorrelated } G_{uc}}$$



- ω_i and ω_i': two independent Gaussian random variables
- k_i : eigenvalues distributed based on random matrix theory (RMT)
- Δ : mean-spacing between adjacent eigenvalues (Weyl Formula)

Stochastic Green's Function for Wave Chaotic Media

$$G(r, r') = \underbrace{\frac{1}{\pi}\sum_i \frac{\omega_i^2}{k^2 - k_i^2}\,\frac{k\Delta}{k_i}\,\frac{\sin k_i\,|r' - r|}{4\pi\,|r' - r|}}_{\text{correlated } G_{co}} + \underbrace{\sum_i \frac{\omega_i\,\omega_i'}{k^2 - k_i^2}\,\frac{k\Delta}{4\pi^2}\,\frac{\sin k_i\,|r' - r_0'|}{k_i\,|r' - r_0'|}\,\frac{\sin k_i\,|r - r_0|}{k_i\,|r - r_0|}}_{\text{uncorrelated } G_{uc}}$$



Correlated G_{co}:

- spatial correlation function
- decay as the distance $|r' - r|$ increases
- coherent propagation

Uncorrelated G_{uc}:

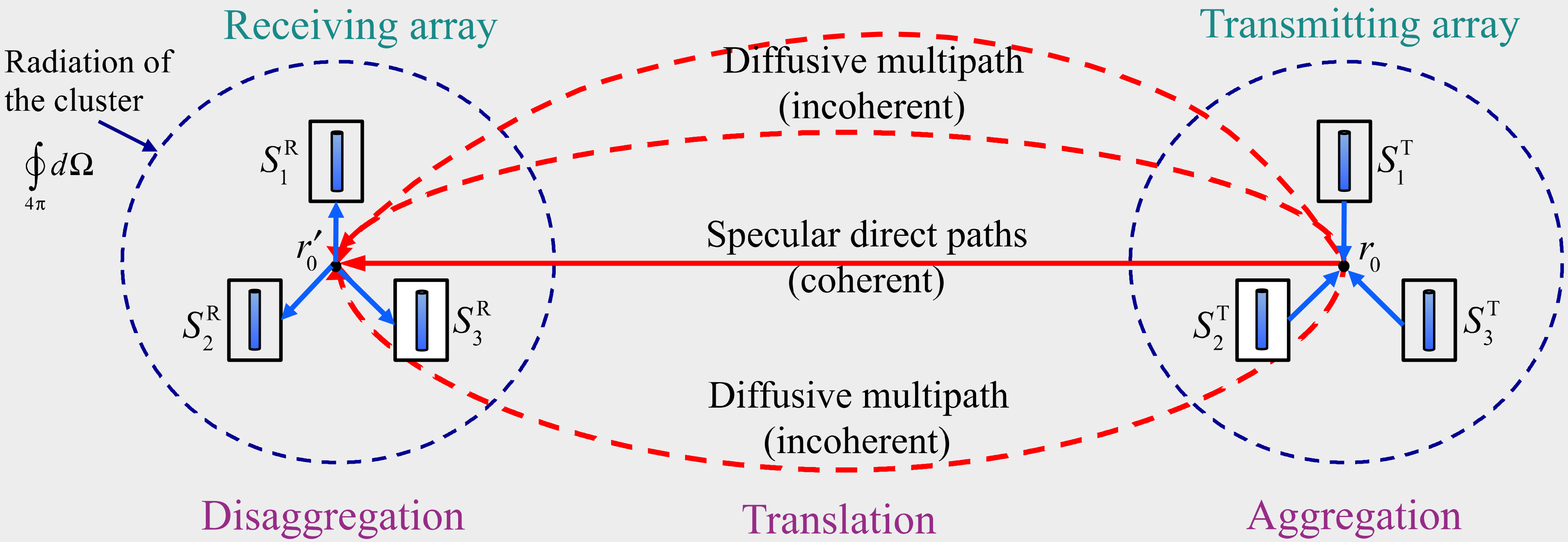
- independent of the spatial distance $|r' - r|$
- ergodicity
- Gaussian diffuse energy propagator

$|r' - r| \nearrow \Rightarrow$ correlated: $\frac{\sin k_i\,|r' - r|}{|r' - r|} \searrow \Rightarrow$ uncorrelated

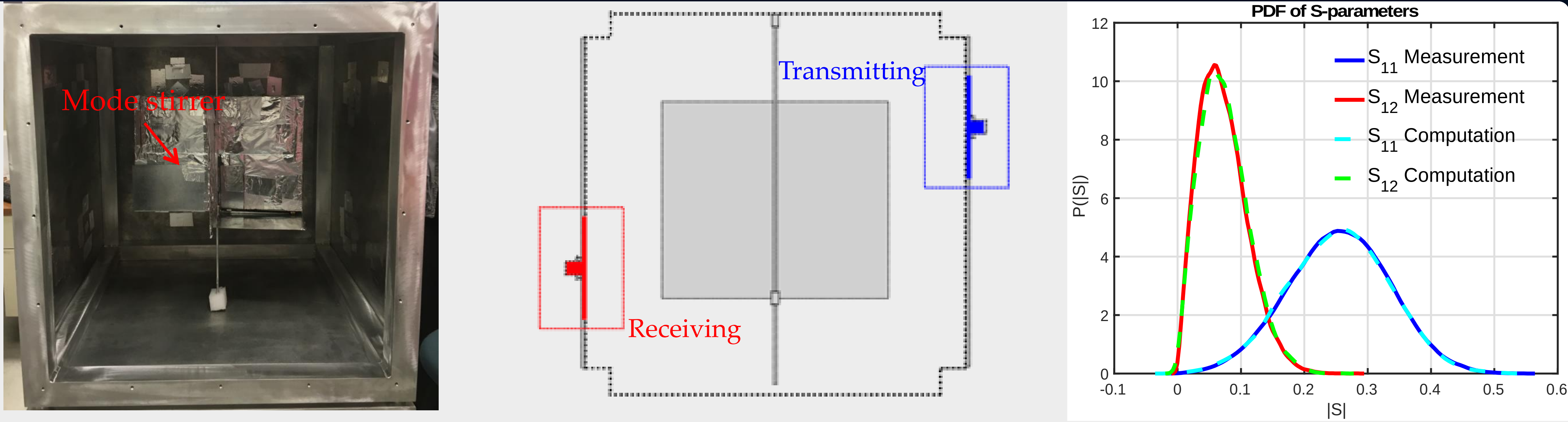
Stochastic Integral Equation

$$\mathbf{Z}^{RT} = \Re\left[\mathbf{Z}_0^{RT}\right] + j\mathbf{D}_0^{R} \cdot \underbrace{\left[\mathbf{T}_0^{RT}\mathbf{g}_{co} + \mathbf{g}_{uc}\right]}_{\text{Translation}} \cdot \mathbf{A}_0^{T}$$

- $\mathbf{Z}_0^{**}, \mathbf{D}_0^{*}, \mathbf{A}_0^{*}$: Impedance matrix, Disaggregation matrix, Aggregation matrix
- $\mathbf{T}_0^{**}$: Translation matrix (direct trajectory contributions)
- $\mathbf{g}_{**}$: Multipath chaotic rays with random phases (RMT)



Experimental Validation



(a) Internal view of the 3D box (b) Computational setup (c) Probability density function

(a) WG & aperture are modeled in first-principle using finite element method

(b) Plate is big enough to capture the near-field effects (deterministic)

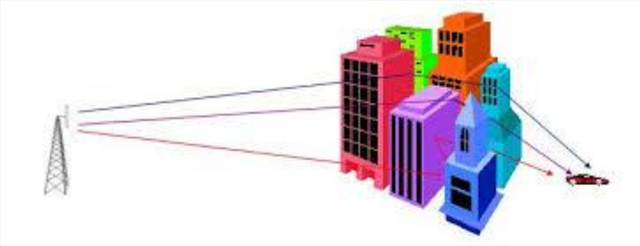
(c) Incorporate the universal statistical chaotic effects in $\mathbf{A}_{BI}$ at aperture

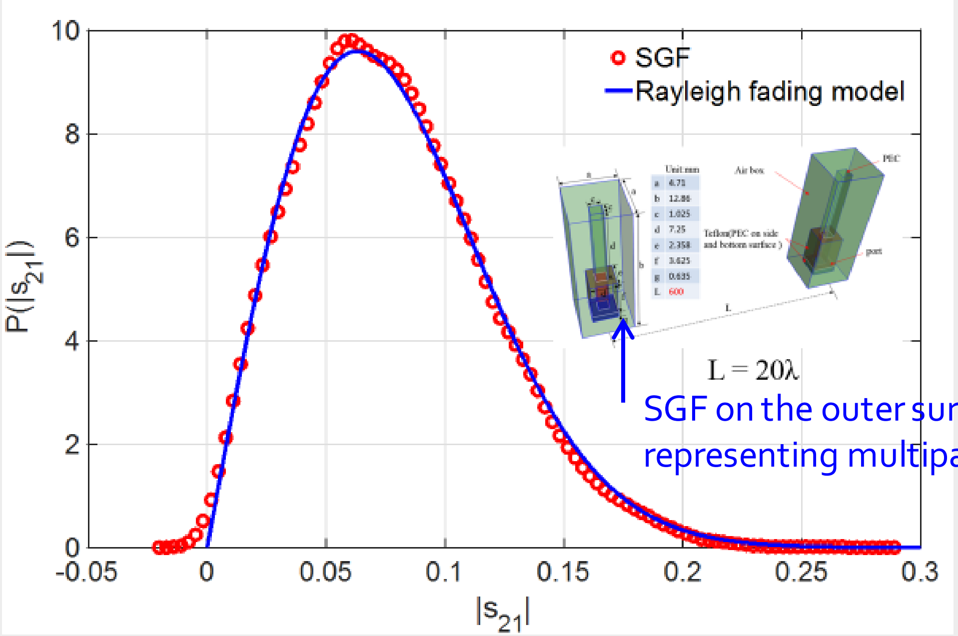
(d) Solve the stochastic matrix equation with random parameter g

Experimental Validation

two decoupled monopoles
Rayleigh fading
(without line-of-sight signal)

$$P(R;\sigma) = \frac{R}{\sigma^2} e^{-R^2/2\sigma^2}$$

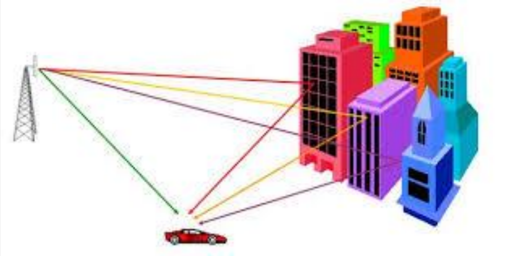


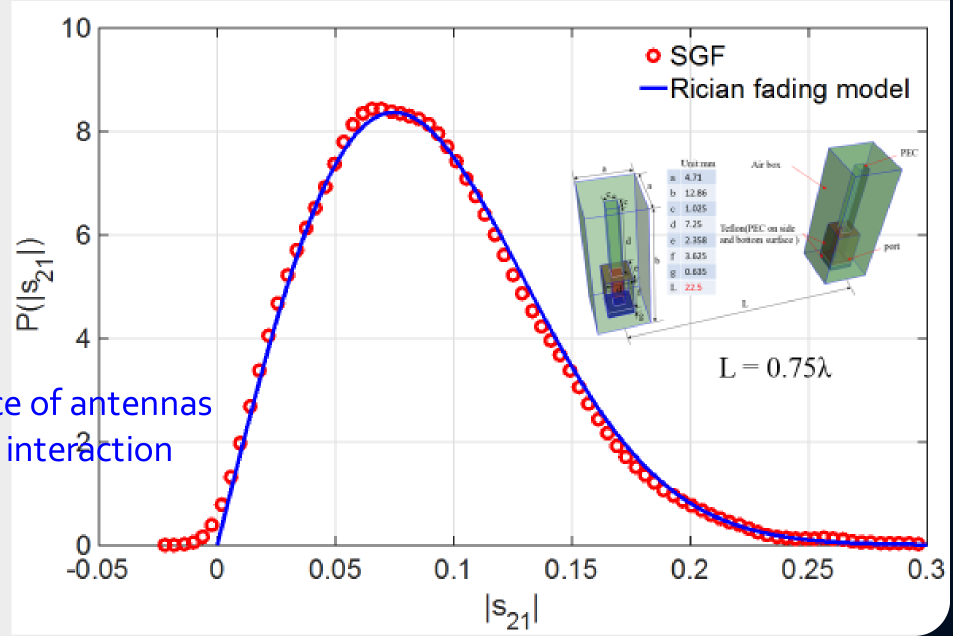


SGF on the outer surface of antennas representing multipath interaction

two coupled monopoles
Rician fading
(with strong line-of-sight signal)

$$P(R;\sigma,\nu) = \frac{R}{\sigma^2} e^{-(R^2+\nu^2)/2\sigma^2} I_0\left(\frac{R\nu}{\sigma^2}\right)$$





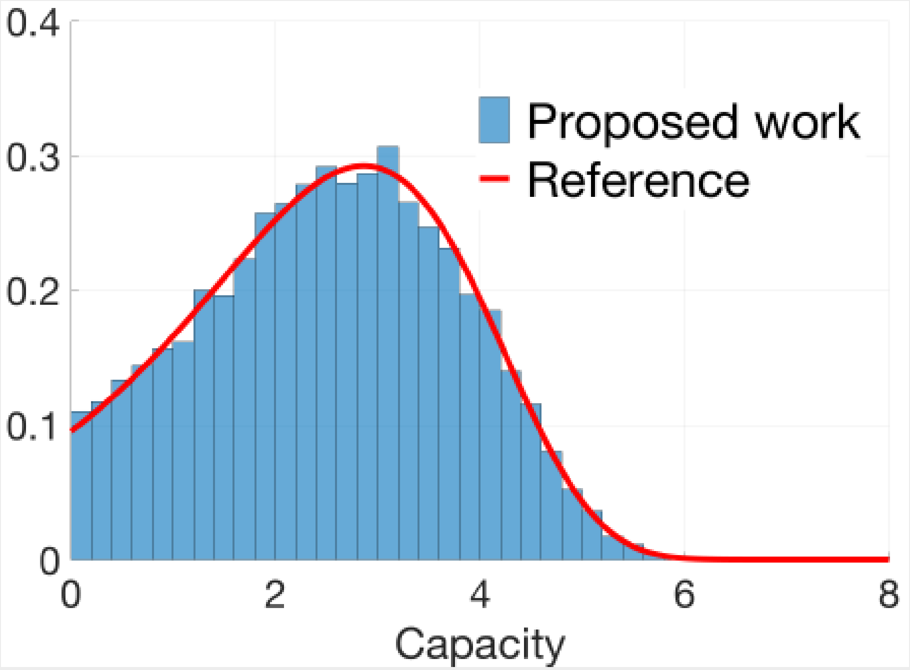
Experimental Verification

The information capacity for M^T transmitting (Tx) and M^R receiving (Rx) antennas:

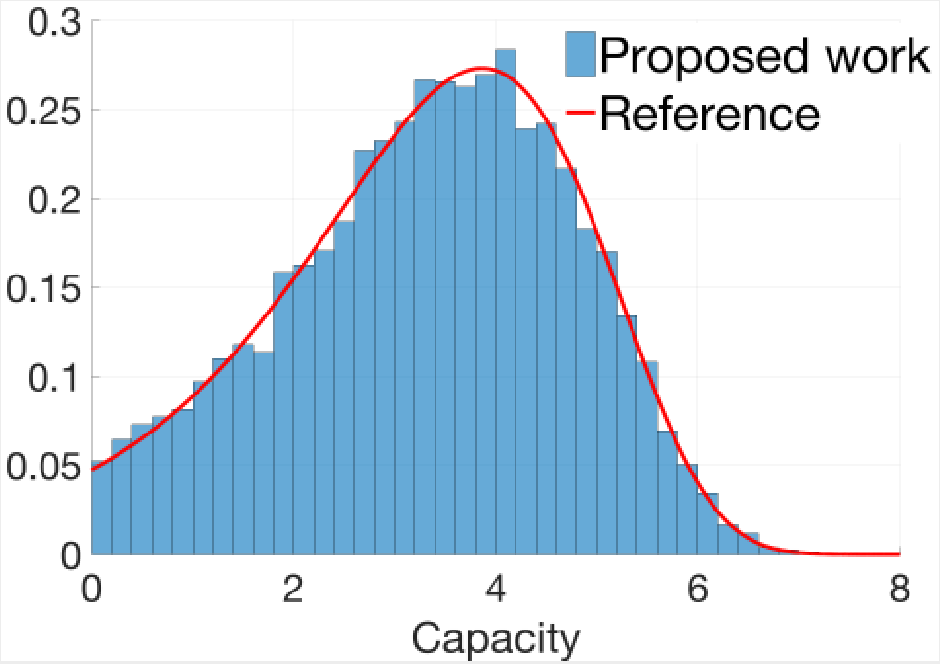
$$C = \log_2 \det\left(\vec{I}_{M^R} + \frac{\rho}{M^T}\vec{H}^+\vec{H}\right)$$

Single-input-single-output (SISO)[1]: $M^T = 1$

SNR: $\rho = 2$



SNR: $\rho = 4$



[1] A. F. Molisch, Wireless Communications, 2nd Edition. Wiley-IEEE Press, New York, 2011

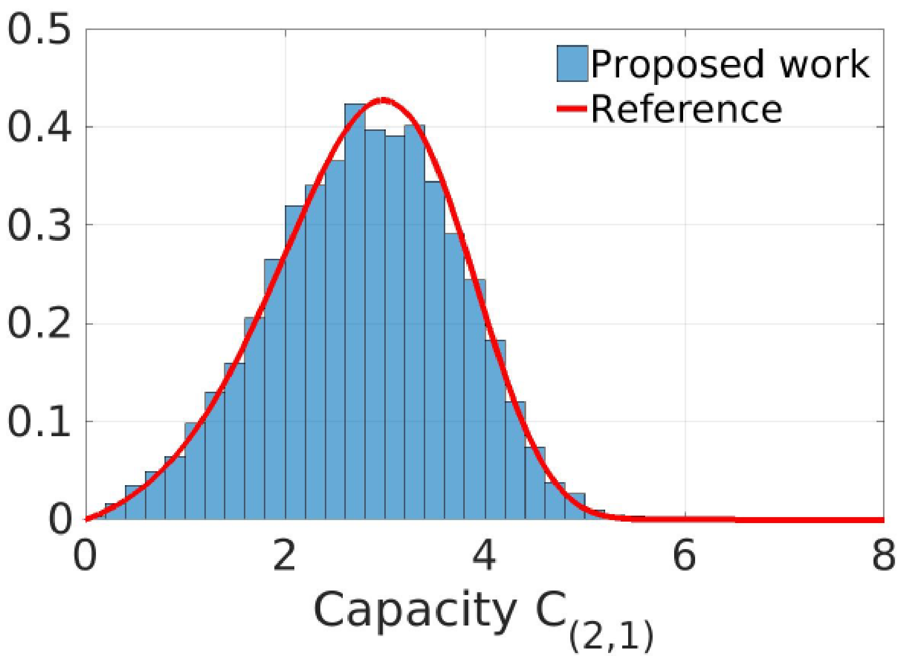
Experimental Verification

The information capacity for M^T transmitting (Tx) and M^R receiving (Rx) antennas:

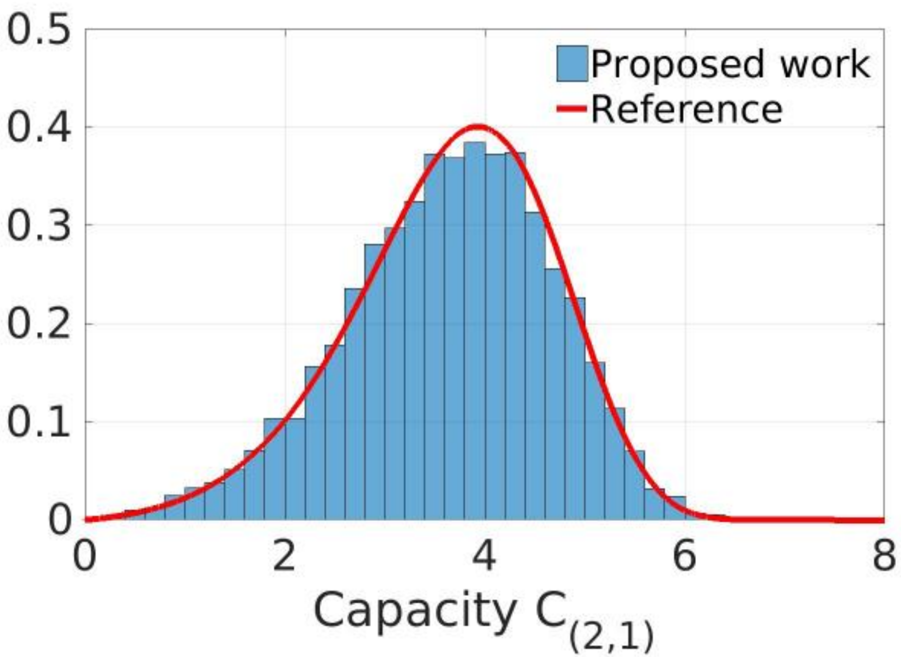
$$C = \log_2 det\left(\vec{I}_{M^R} + \frac{\rho}{M^T}\vec{H}^+\vec{H}\right)$$

Multi-input-single-output (MISO)[2]: $M^T = 2$

SNR: $\rho = 2$



SNR: $\rho = 4$



[2] Reference: courtesy of Dr. Gabriele Gradoni, the University of Nottingham

Remarks and Conclusion

- A physics-oriented statistical wave model for predicting the performance of wireless antennas in the multipath environment.
- SGF statistically replicates the propagation phenomena including both direct orbits and uniform,isotropic multipath fading
- The computational costs are comparable to the free-space environment, since the SIE-SGF are placed only at the exterior surface of antennas
- It doesn't make the ansatz for special cases (isotropic fading, far-field separations, etc.), and there are no semi-empirical fitting parameters required.

The advancements will result in a reliable, reconfigurable, and repeatable testbed for emerging wireless devices and systems in complex environments, beyond the confines of the laboratory and measurements