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ABBREVIATIONS/ACRONYMS

FM	frequency modulation
FMCW	frequency modulated carrier wave
ITS	Institute for Telecommunication Sciences (NTIA)
LFM	linear frequency modulation (linear chirp)
NLFM	non-linear frequency modulation (non-linear chirp)
NTIA	National Telecommunications and Information Administration
P0N	amplitude-only pulse modulation (pronounced "p-on")
pri	pulse repetition interval
RBW	resolution bandwidth
RF	radio frequency
SA	spectrum analyzer
VBW	video bandwidth
VSA	vector signal analyzer
VSG	vector signal generator

RELATIONSHIPS BETWEEN MEASURED POWER AND MEASUREMENT BANDWIDTH FOR FREQUENCY-MODULATED (CHIRPED) PULSES

Frank H. Sanders and Roger A. Dalke[1]

Measured power levels for radio frequency (RF) pulses that are frequency modulated (chirped) vary as a function of the bandwidth in which the measurement is performed; if chirped pulses cause RF interference, the power levels of the pulses in victim receivers will likewise vary as a function of receiver bandwidth. This report provides both heuristic and rigorous derivations of the relationships among chirped pulse parameters and the measured peak and average power levels of chirped pulses as a function of measurement bandwidth. These relationships may be best understood via a single graph (Figure 3) presented in this report. This report supplements NTIA Technical Reports TR-05-420, TR-10-465 and TR-10-466, in which the formula for minimum bandwidth needed for measurement of full peak power in chirped pulses is presented but not derived.

Key words: chirped pulses; electromagnetic compatibility; frequency-modulated carrier wave (FMCW chirped) pulses; pulse power measurements; radar power measurement; radar spectrum measurement; radio frequency (RF) interference; RF measurement

1 HEURISTIC DERIVATION OF FREQUENCY-MODULATED (CHIRPED) PULSE EMISSION BANDWIDTH

1.1 Introduction

Most radar signals and some radio jamming signals consist of radio frequency (RF) carriers that are formed into repetitive pulse sequences. Other than being switched on and off (amplitude modulated) to form trapezoidal time-domain envelopes, pulse carriers are not necessarily additionally modulated; such are said to have P0N (pronounced "P-on") modulation [1]. Some types of radar and jamming pulses are, however, additionally modulated in either frequency or phase. Frequency modulated carrier wave (FMCW) pulses, called chirped because of the sound they make when they are demodulated at audio frequencies, show variation in their measured power levels as a function of the bandwidth used for the power measurement.

Meaningful power measurements of chirped pulses depend on accurate knowledge of the variation of measured power as a function of measurement bandwidth. This knowledge depends in turn on understanding the emission bandwidths of chirped pulses as a function of the pertinent parameters (frequency range and time interval) of the chirp. In this report we derive the relationships between chirped-pulse parameters and measured power levels as a function of measurement bandwidth. We provide the derivations twice, in heuristic and rigorous mathematical forms, and compare the results to measurements of actual chirped pulses. This

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report supplements NTIA Technical Reports [2], [3] and [4] in which a formula for the minimum bandwidth for measurement of full peak power in chirped pulses is presented but not derived.[2]

1.2 Heuristic Approach to Understanding Power-Bandwidth Relationships in Chirped Pulses

While a rigorous mathematical derivation is required to definitively demonstrate the relationships between power and bandwidth for chirped pulses, we provide this heuristic presentation first because it may be more accessible by a wider audience than the mathematical derivation that follows in Section 2 of this report.

Consider the frequency-in-time modulation of chirped pulses of two operational radars, as measured with a vector signal analyzer (VSA) and shown in Figure 1. The frequency may be ramped upward or downward in time (up-chirped or down-chirped) during each pulse, and the frequency modulation may be either linear or non-linear (LFM or NLFM, respectively) in time.

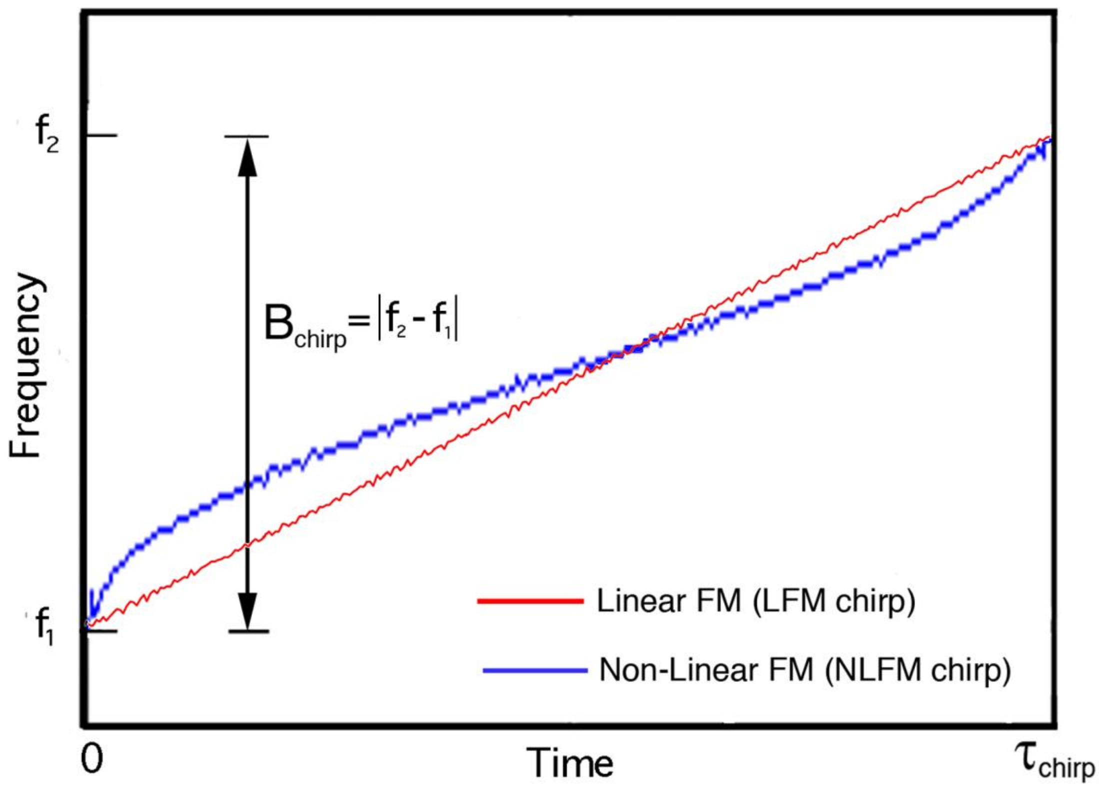

Figure 1. Measured frequency modulation versus time behavior for linear and non-linear chirped radar pulses. Many modern radars use non-linear chirped pulses.

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[2] Results will vary slightly depending on the exact shapes of measurement or receiver bandwidths. We use a rectangular, brick-wall shape for theoretical analysis as it is more mathematically tractable. Corroborating measurement data in this report, however, were taken with Gaussian filter shapes in a spectrum analyzer. As shown in Section 3, the brick-wall theoretical assumption agrees closely with the Gaussian implementation results. This indicates that the results are insensitive to details of filter shape.

In this analysis we assume that pulses of RF energy are transmitted in a continuous sequence with a pulse repetition interval (pri) of T_{pri}. The pulse envelopes in the time domain are assumed to be trapezoidal with a center-pulse width of τ_{chirp}; their rise and fall times are short compared to τ_{chirp}. The bandwidth of the chirped-frequency modulation is B_{chirp},

$$B_{chirp} = |f_2 - f_1|, \tag{1}$$

where f_2 and f_1 are the extreme frequencies of the chirp modulation.

As for any periodic waveform, the pulse sequence produces emission lines in the frequency domain that are spaced ($1/T_{pri}$) apart. Just as for simple P0N pulses, the amplitude (voltage) envelope of these lines follows a *sin(x)/x* or *sinc* curve, and the frequency domain envelope of the lines' intensity (power) spectrum is a $sinc^2$ ("sinc squared") function. (If the pulse repetition interval is irregular (staggered) instead of constant, then the average pulse interval is substituted for T_{pri} and the $sinc^2$ envelope still holds.)

The measurement bandwidth B_{meas} (or victim receiver bandwidth in interference studies) is centered on the central lobe of the emission spectrum. This bandwidth may be varied; as it expands or contracts, the number of convolved emission spectrum lines varies proportionately. The minimum value of B_{meas} is ($1/T_{pri}$), so that it always contains at least one emission line. If the measurement bandwidth is less than the half-power points of the spectrum central lobe, the amplitude and power levels of the convolved emission spectrum lines are approximately constant within the measurement bandwidth.

1.3 Measurement Bandwidth Required to Obtain Full Peak Power of Chirped Pulses

From general Fourier transform relationships between the length of a pulse in the time domain and the spread of the same pulse in the frequency domain,

$$\Delta f \approx \frac{1}{\tau}, \tag{2}$$

where τ is the pulse width and Δf is the approximate spectral width of the majority of the pulsed energy. Therefore to measure the total power of simple P0N pulsed energy, the measurement bandwidth $B_{meas\_total\_power}$ must be at least as wide as the inverse of the pulse widths:

$$B_{meas\_total\_power} \geq \frac{1}{\tau}. \tag{3}$$

This is the condition for measurement of total peak power in a pulse or equivalently for the convolution of the full power of a pulse to be coupled into a radio receiver's circuitry. (The condition for measurement of the total average power in the pulse sequence is addressed below.) To obtain the minimum bandwidth required to observe the full peak power of a chirped pulse, consider Figure 2, in which B_{meas} is drawn as an arbitrary bandwidth relative to the chirp bandwidth, B_{chirp}, of a pulse.

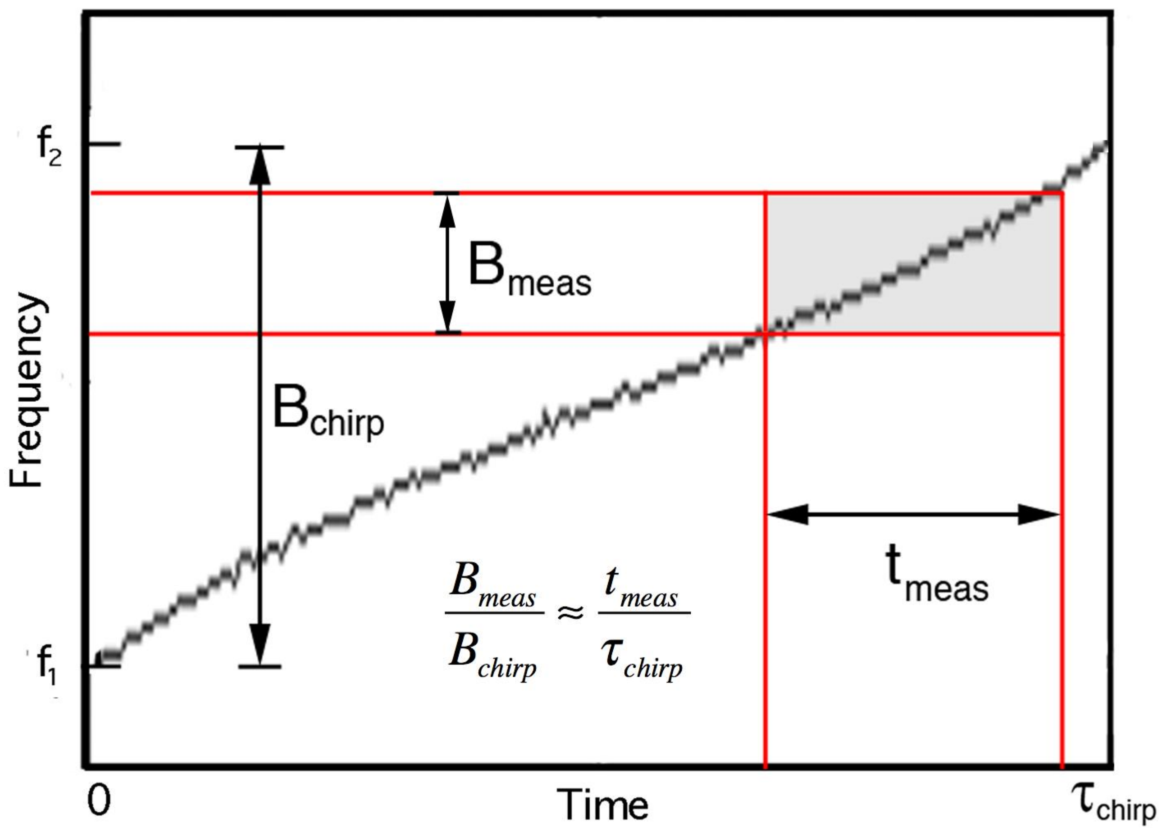

Figure 2. Diagram of proportional geometric relationships among B_{meas}, B_{chirp}, t_{meas} and τ_{chirp}. The relationship is exact for LFM pulses and approximate for NLFM pulses.

In this figure, the pulse width τ_{chirp} is the time interval of the chirping from the initial frequency f_1 to the final frequency f_2: $\tau = \tau_{chirp}$. Since $B_{chirp} = |f_2 - f_1|$ and the chirp bandwidth is swept linearly (or nearly so, even for NLFM pulses) during the time interval τ_{chirp}, the following geometric proportionality[3] holds:

$$\frac{t_{meas}}{B_{meas}} = \frac{\tau_{chirp}}{B_{chirp}}, \tag{4}$$

where t_{meas} is the interval of time during which the chirped pulse is observed in the measurement bandwidth B_{meas}. If $B_{chirp} \geq B_{meas}$ then of course $\tau_{chirp} \geq t_{meas}$. Rearranging (4), the proportionality relationship among these parameters is

$$t_{meas} = \frac{B_{meas}}{B_{chirp}} \cdot \tau_{chirp}. \tag{5a}$$

From the fundamental frequency-time relationship of (2) the minimum bandwidth in which a full-power measurement may be obtained must be

[3] This proportionality is only approximately true for NLFM pulses.

$$B_{meas} \geq \frac{1}{t_{meas}}. \tag{5b}$$

Combining (5a) and (5b):

$$\left(B_{meas} \cdot \frac{\tau_{chirp}}{B_{chirp}} \right) = \left(t_{meas} \right) \geq \left(\frac{1}{B_{meas}} \right) \tag{5c}$$

and therefore the full power can only be measured if

$$B_{meas}^2 \geq \frac{B_{chirp}}{\tau_{chirp}}. \tag{6a}$$

Introducing a shorthand variable, α, for the ratio (B_{chirp}/τ_{chirp}), the bandwidth condition for measurement of total chirped-pulse peak power is:

$$B_{meas} \geq \sqrt{\alpha}\,. \tag{6b}$$

Equation (6b) is used in NTIA Reports [2], [3] and [4], but we have not published its derivation until now. The equation is only approximately true for NLFM pulses because the proportionality relationship of (4) is only approximately true under that condition.[4]

When using (6b) it is often convenient to render bandwidths in units of megahertz and chirp intervals in units of microseconds; likewise units of kilohertz and milliseconds may be used together. Any units that are reciprocals of each other may legitimately be used together in the equation. The relationship of (6b) is also true for the total power convolved in the circuitry of radio receivers that might experience interference from chirped pulses: only victim receivers that have an effective bandwidth equaling or exceeding the value given in (6b) will couple the full power of the chirped pulses into their circuitry.

1.4 Power Per Spectrum Emission Line Relative to Total Power

1.4.1 Power Per Spectrum Emission Line Relative to Total Peak Power

The number of emission lines contributing to the peak power within $\sqrt{\alpha}$ (treating this as a brick-wall bandwidth) is the ratio of that bandwidth to the line spacing of $(1/T_{pri})$:

[4] As a practical matter NLFM pulses in modern radar systems are close to being linear through most of the chirp cycle. This near-linear characteristic is seen in the NLFM curves of Figures 1 and 2, which were both measured from operational radars.

$$N_{peak\_lines} = \left(\frac{\sqrt{\alpha}}{1/T_{pri}} \right) = \frac{T_{pri}}{\tau_{chirp}} \sqrt{B_{chirp}\tau_{chirp}} = \frac{\sqrt{B_{chirp}\tau_{chirp}}}{dc}, \tag{7}$$

where *dc*, the ratio of the pulse width to the pulse repetition interval, is the *duty cycle* of the pulse sequence.

Power goes as the square of voltage. For an emission spectrum line with voltage V_{line} the power measured in the line, p_{line}, goes as V_{line}^2. (Peak and average power values are identical for single emission lines.) The peak-detected power, $p_{peak\_detected}$, measured across *all* of the spectrum lines, n_{lines}, occurring within the measurement bandwidth (or receiver bandwidth) goes as the square of the number of lines within the measurement bandwidth:

$$p_{peak\_detected} \propto \left(n_{lines} \cdot V_{line}\right)^2 = n_{lines}^2 \cdot p_{line}. \tag{8}$$

Since n_{lines} is proportional to B_{meas},

$$p_{peak\_detected} \propto B_{meas}^2 \cdot p_{line} \tag{9a}$$

or, in decibel terms,

$$P_{peak\_detected} \propto P_{line} + 20\log B_{meas} \tag{9b}$$

where $P_{peak\_detected} = 10\log(p_{peak\_detected})$ and $P_{line} = 10\log(p_{line})$.

Combining (7) and (8), the relationship between the power in a spectrum line and total peak-detected power is:

$$p_{total\_peak} = \left(\frac{T_{pri}}{\tau_{chirp}} \sqrt{B_{chirp}\tau_{chirp}} \right)^2 \cdot p_{line} = \frac{B_{chirp}T_{pri}^2}{\tau_{chirp}} \cdot p_{line} = \alpha T_{pri}^2 \cdot p_{line} \tag{10a}$$

or, in decibel terms,

$$P_{total\_peak} = 10\log\left(B_{chirp}\right) - 10\log\left(\tau_{chirp}\right) + 20\log\left(T_{pri}\right) + P_{line}. \tag{10b}$$

1.4.2 Power Per Spectrum Emission Line Relative to Total Average Power

As already noted, the power measured in a line is identical for peak and average detection. But the average power within B_{meas} goes in direct proportion to B_{meas}. Since n_{lines} is proportional to B_{meas},

$$p_{average} \propto B_{meas} \cdot p_{line} \tag{11a}$$

or, in decibel terms,

$$P_{average} \propto P_{line} + 10\log B_{meas}. \tag{11b}$$

The total average power is distributed across B_{chirp}. The number of lines within this bandwidth is

$$n_{average\_lines} = \left(\frac{B_{chirp}}{1/T_{pri}} \right) = B_{chirp} T_{pri}. \tag{12}$$

The total average power is the total peak power times the duty cycle:

$$p_{total\_average} = dc \cdot p_{peak\_total} = \frac{\tau_{chirp}}{T_{pri}} \cdot p_{total\_peak}, \tag{13}$$

so the total average power is

$$p_{total\_average} = B_{chirp} T_{pri} \cdot p_{line} \tag{14a}$$

or, in decibel terms,

$$P_{total\_average} = P_{line} + 10\log\left(B_{chirp} T_{pri}\right). \tag{14b}$$

1.5 Summary of Relationships Among Peak Power, Average Power and B_{meas}

Total peak power occurs in bandwidths of $B_{meas} \geq \sqrt{\alpha}$, while total average power occurs in bandwidths of $B_{meas} \geq B_{chirp}$. If the total peak power is normalized to unity (zero decibels), then the power (peak-detected equaling average) per line in the central lobe of the spectrum is, from (10a):

$$p_{line\_normalized\_to\_total\_peak} = \frac{\tau_{chirp}}{B_{chirp} T_{pri}^2}. \tag{15a}$$

In decibel terms,

$$P_{line\_normalized\_to\_total\_peak} = 10\log\left(\tau_{chirp}\right) - 10\log\left(B_{chirp}\right) - 20\log\left(T_{pri}\right). \tag{15b}$$

The peak-detected power measured in bandwidths $\left(1/T_{pri}\right) < B_{meas} \leq \sqrt{\alpha}$ varies at a rate of 20log(B_{meas}).

The power per line normalized to the average power is the peak power times the duty cycle, or

$$p_{line\_normalized\_to\_average} = \frac{1}{B_{chirp} T_{pri}} . \tag{16a}$$

In decibel terms,

$$P_{line\_normalized\_to\_average} = -10\log\left(B_{chirp}\right) - 10\log\left(T_{pri}\right). \tag{16b}$$

The average power measured in bandwidths $\left(1/T_{pri}\right) < B_{meas} \le \sqrt{\alpha}$ varies at a rate of *10log(*B_{meas}*)*. Peak and average power are related to each other by the duty cycle:

$$p_{average} = dc \cdot p_{peak} \tag{17a}$$

or, in decibel terms,

$$P_{average} = P_{peak} + DC = P_{peak} + 10\log\left(\tau_{chirp}\right) - 10\log\left(T_{pri}\right). \tag{17b}$$

The relationships among chirp pulse parameters, B_{meas}, and the measured peak and average power levels of chirped pulses are illustrated graphically in the log-log plot of Figure 3.

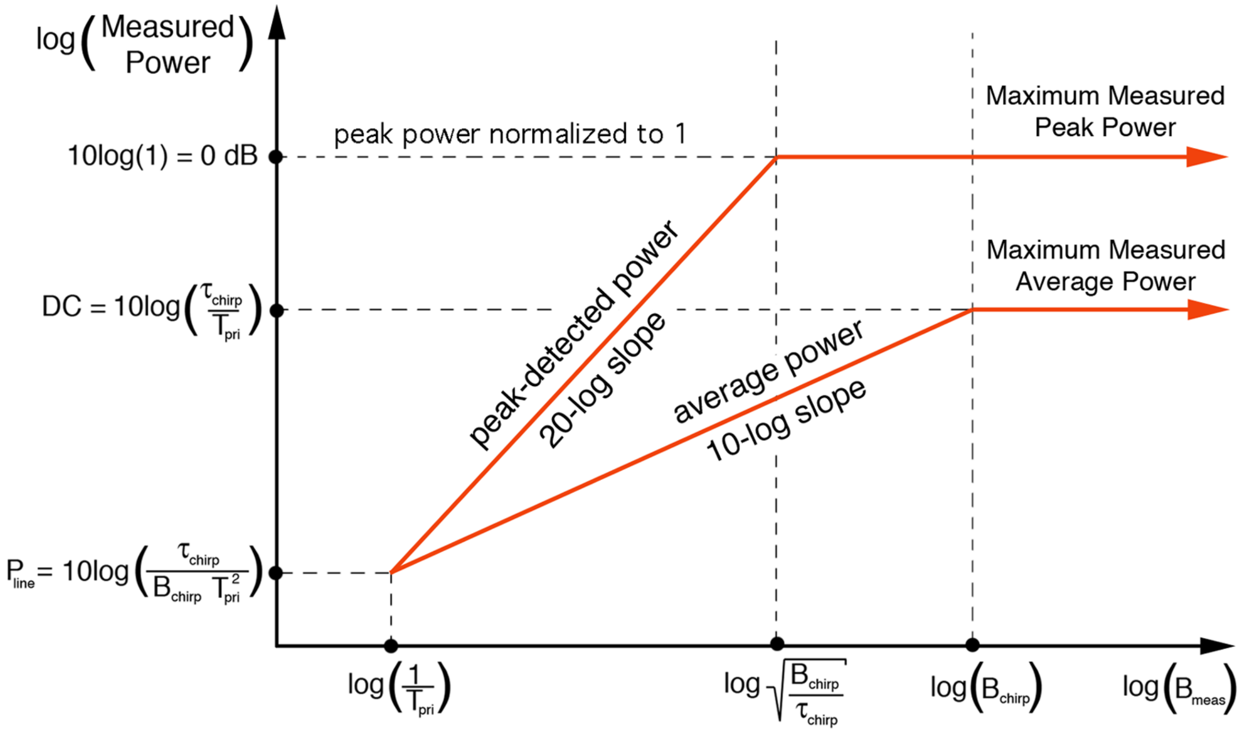

Figure 3. Relationships among B_{meas}, chirp pulse parameters, and measured peak and average power.

2 RIGOROUS DERIVATION OF CHIRPED PULSE EMISSION BANDWIDTH

2.1 Introduction

In this section we rigorously derive (6b) for the minimum measurement bandwidth required to obtain the peak envelope voltage of a periodic chirped pulsed radar signal. First we define the signal. Then we obtain a mathematical expression for its Fourier transform. The Fourier transforms of the chirped pulse and the measurement filter are used to derive an expression for the peak measured voltage as a function of bandwidth and hence (6b). The results given here assume an ideal brick-wall filter and use the large argument approximation for Fresnel integrals. Hence, (6b) applies to situations that are reasonably characterized by these assumptions.

2.2 Chirped Pulsed FMCW Radar Signal and Fourier Transform

The chirped pulsed FMCW radar signal is defined as

$$s(t) = \sum_{n=-\infty}^{\infty} p\left(t - nT_{pri}\right)\cos\left(2\pi f_c t + \pi\alpha\left(t - nT_{pri}\right)^2\right) \tag{18}$$

where $p(t)$ defines the envelope of the chirp, f_c is the center frequency and T_{pri} is the repetition period. The parameter α is the ratio of the range of the frequency sweep $(f_c - \Delta f) \leq f \leq (f_c + \Delta f)$ to the duration of the sweep $2T$ or $\alpha = (\Delta f / T) = (B_{chirp} / \tau_{chirp})$ and we assume $T_{pri} \geq 2T$.

We can derive (18) by starting with the baseband expression for a single chirp,

$$c(t) = p(t)e^{i\pi\alpha t^2}. \tag{19}$$

To obtain (18), the chirp is convolved with an infinite series of Dirac delta functions regularly spaced at time intervals T_{pri} and up-converted to f_c. The FMCW signal of (18) is just the real part. Since convolution with an infinite series of delta functions in the time domain is equivalent to sampling in the frequency domain, the baseband FMCW spectrum is obtained from the spectrum for a single chirp by sampling at the repetition frequency $1/T_{pri}$.

We consider the case of a rectangular pulse, $p(t)$,

$$p(t) = 1 \text{ where } (-T \leq t \leq T), \text{ and } p(t) = 0 \text{ everywhere else} \tag{20}$$

and a linear frequency sweep (α is a constant). The baseband spectrum of a single chirp is given by

$$C(f) = \int_{-T}^{T} e^{i\pi\alpha t^2 + i2\pi f t}\, dt. \tag{21}$$

Equation (21) reminds us of Fresnel integrals that are usually defined as follows [5]:

$$F(z) = \int_0^z e^{i\pi t^2/2} dt. \tag{22}$$

Note the symmetry property $F(-z) = -F(z)$ which will be used in the sequel. Completing the square of the integrand in (21) and a change in variables yields

$$C(f) = \frac{e^{i\pi f^2/\alpha}}{\sqrt{\alpha}} \int_{\nu_-}^{\nu_+} e^{i\pi t^2} dt \tag{23}$$

where $\nu_- = (f - \Delta f)/\sqrt{\alpha}$ and $\nu_+ = (f + \Delta f)/\sqrt{\alpha}$. Hence, the Fourier transform can be written in terms of Fresnel integrals as follows:

$$C(f) = \frac{e^{i\pi f^2/\alpha}}{\sqrt{2\alpha}} \left[F\left(\sqrt{2}\nu_+\right) - F\left(\sqrt{2}\nu_-\right) \right]. \tag{24}$$

We will use both the small and large argument approximations for the Fresnel integrals [5]. For small arguments

$$\left|F(z)\right| = |z| + O\left(|z|^7\right) \tag{25}$$

and for large arguments where $|\arg z| < \pi/2$

$$F(z) \approx \frac{1+i}{2} - \frac{e^{i\pi z^2/2}}{\pi z} \left(i + \frac{1}{\pi z^2} + \sum_{m=0}^{\infty} (-1)^m \frac{1 \cdot 3 \cdots (4m-1)}{\left(\pi z^2\right)^{2m}} \left(i + \frac{4m(4m+1)}{\pi z^2} \right) \right). \tag{26}$$

In the derivation that follows, we will use only the first term of this series. If we keep only the first term and $z \geq 5$, the maximum error is about 12 per cent and if $z \geq 10$, the maximum error is about 6 per cent.

2.3 Minimum Bandwidth Required to Obtain Peak Voltage of the Chirp

We assume that the measurement system simply filters the periodic chirped pulsed FMCW signal and the measured voltage is given by

$$\upsilon(t) = \frac{1}{T_{pri}} \sum_{n=-\infty}^{\infty} C\left(\frac{n}{T_{pri}}\right) H\left(\frac{n}{T_{pri}}\right) e^{i2\pi nt/T_{pri}} \tag{27}$$

where $H(f)$ is the measurement system transfer function. To obtain the minimum measurement bandwidth required to obtain the peak envelope voltage of the chirp, we assume an ideal rectangular brick-wall filter, i.e.,

$$H(f) = 1 \text{ where } (-B/2 \le f \le B/2), \text{ and } H(f) = 0 \text{ everywhere else} \tag{28}$$

and B is the bandwidth of the filter. For this case, the measured voltage can be written as

$$\upsilon(t) = \frac{1}{T_{pri}} \sum_{n \in I} C\left(\frac{n}{T_{pri}}\right) e^{i2\pi nt / T_{pri}} \tag{29}$$

where $I = [-T_{pri} \cdot B/2, T_{pri} \cdot B/2]$.

Returning to (24), the arguments of the Fresnel integrals can be written as

$$2\nu_{\pm} = \sqrt{2}\left(\frac{f}{\sqrt{\alpha}}\right) \pm \sqrt{\Delta f T}\,. \tag{30}$$

If we restrict ourselves to $|f| < \sqrt{\alpha}$, and chirp parameters so that $\sqrt{\Delta f T}$ is sufficiently large, we can use the first term of (26) as an approximation and obtain the following result for the Fourier transform of a chirp:

$$C(f) \approx \frac{(1+i)e^{-i\pi f^2/\alpha}}{\sqrt{2\alpha}}. \tag{31}$$

Furthermore, when $\sqrt{\Delta f T_0}$ is large, we can reasonably approximate the sum in (29) by an integral

$$|\upsilon(t)| = \left|\frac{1}{\sqrt{\alpha}} \int_{-B/2}^{B/2} e^{i\pi f^2/\alpha + i2\pi ft} df\right| \tag{32}$$

which can also be written in terms of Fresnel integrals. For convenience we define

$$B' = \frac{B}{\sqrt{2\alpha}}, \qquad t' = \sqrt{2\alpha}\,t\,, \tag{33}$$

which can also be written in terms of Fresnel integrals. For convenience we define

$$|\upsilon(t')| \approx \frac{1}{\sqrt{2}}\left|F(B'+t') + F(B'-t')\right|. \tag{34}$$

Now if we define B'_m so that $|F(B'_m)| = max\ |F(B')|$, then $|F(B'_m + t') + F(B'_m - t')| \le 2|F(B'_m)|$ and equality is achieved only when $t' = 0$. Furthermore, it can be shown (e.g., using numerical methods) that $max\ |\upsilon(t')| = |\upsilon(0)|$ for $0 \le B' \le B'_m$ and $B'_m \approx 1.2$. Hence, the peak measured voltage is given by the following approximation:

$$|\upsilon(0)| \approx \sqrt{2}\,|F(B')| \quad for \quad 0 \le B' \le B'_m \quad and \quad \sqrt{\Delta f T} >> 1 \tag{35}$$

and for $B' \le 1/\sqrt{2}$, we can use the small argument approximation

$$|F(B') \approx B'|. \tag{36}$$

We now have the desired rule for the minimum bandwidth required to measure the peak voltage of the chirp. For measurement bandwidths in the range $0 \le B \le \sqrt{\alpha}$

$$|\upsilon(0)| \approx \frac{B}{\sqrt{\alpha}}, \tag{37}$$

i.e., the peak measured voltage is proportional to the measurement bandwidth and when $B \ge \sqrt{\alpha}$, the peak measured voltage is approximately equal to the peak envelope voltage of the chirp (i.e. *max* |*c(t)*| given in (19)).

Since $\alpha = (\Delta f/T) = (B_{chirp}/\tau_{chirp})$, the condition $B \ge \sqrt{\alpha}$ (i.e, $B \ge \sqrt{B_{chirp}/\tau_{chirp}}$) for the peak measured voltage to equal the peak envelope voltage of the chirp in the rigorously derived (37) is identical to the requirement for the measurement bandwidth to capture the full power of a chirped pulse, as derived heuristically as (6b). In the identity of (6b) and (37) we have met our goal of independently deriving this relationship both ways.

2.4 Miscellaneous Relations for Peak and Average Power When $0 \le B \le \sqrt{\alpha}$

Having derived (37), we now briefly extend that work to provide some miscellaneous additional equations that parallel several results in the heuristic development of (7) through (17).

In Sections 2.2 and 2.3, the amplitude of the chirped pulse was taken to be unity. If we let the amplitude be denoted as *A*, the previous results scale accordingly, e.g., (31) and (37) are scaled as follows:

$$|\upsilon(0)| \approx A\frac{B}{\sqrt{\alpha}} \quad and \quad C(f) \approx A\frac{(1+i)e^{-i\pi f/\alpha}}{\sqrt{2\alpha}} \quad . \tag{38}$$

As noted previously, all results are approximate. Recognizing this, we use equal signs in what follows. Using these results, the peak power in the measurement bandwidth is

$$P_{peak} = |\upsilon(0)|^2 = \frac{A^2B^2}{\alpha} \to A^2 \quad as \quad B \to \sqrt{\alpha} \quad . \tag{39}$$

We need to examine periodic signals in terms of their spectral lines and average power. The number of lines in the full measurement bandwidth B_{meas} is $n_{lines} = \lfloor B_{meas} T_{pri} \rfloor$ (the floor function, or integer value of $B_{meas} T_{pri}$) or $B_{meas} \approx n_{lines} / T_{pri}$ for $B_{meas} T_{pri} \geq 1$. Since the power per line is

$$p_{line} = \frac{\left|C(0)\right|^2}{T_{pri}^2} = \frac{A^2}{\alpha T_{pri}^2} \tag{40}$$

the average power is

$$p_{avg} = n_{lines} \cdot p_{line} = \frac{A^2 B}{\alpha T_{pri}} \tag{41}$$

and

$$p_{peak} = B^2 T_{pri}^2 p_{line} = n_{lines}^2 p_{line}. \tag{42}$$

The ratio of peak to average power, p_r, is

$$p_r = \frac{p_{peak}}{p_{avg}} = n_{lines} \tag{43}$$

and

$$\frac{p_{line}}{p_{peak}} = \frac{1}{n_{lines}^2}. \tag{44}$$

3 POWER-MEASUREMENT DATA SUPPORTING THEORETICAL RESULTS

3.1 Introduction

To illustrate the relationships among peak power, average power and B_{meas} with a physical chirped pulsed FMCW signal, we set up RF-generation hardware to produce chirped pulses in a repeating sequence. We measured the RF emissions with peak and average detection in bandwidths ranging from ($1/T_{pri}$) to somewhat wider than B_{chirp}; we provide the results of those measurements in this section.

3.2 Measurement Setup

We configured a vector signal generator (VSG) to generate linearly chirped pulses. We adjusted the chirp bandwidth to1 MHz, set the pulse width at 100 μs, and set the pulse repetition rate at 2500/sec = 2.5 kHz. We calculated the quantity $\alpha = \sqrt{B_{chirp}/\tau_{chirp}}$ to be 100 kHz. We connected a spectrum analyzer to the VSG output and measured the spectrum in bandwidths of 1 kHz, 3 kHz, 10 kHz, 30 kHz, 100 kHz, 300 kHz, 1 MHz and 3 MHz with average detection, and again with peak detection in all of the same bandwidths except 3 MHz. We used an Agilent E4440A spectrum analyzer with maximum-hold trace mode, 50 msec sweep time and 10 dB input attenuation. The measured pulse repetition sequence is documented in the time domain (from the E4440A, operated in zero-hertz span) in Figure 4. The measured linear chirp is documented in the modulation domain in Figure 5; we acquired the Figure 5 data with an Agilent E9600 Series VSA.

3.3 Measurement Results

Measurement results are shown in Figures 6–9. Figure 6 shows a detailed portion of the emission line spectrum, with the lines spaced 2.5 kHz apart at a constant amplitude from one to the next. Figures 7 and 8 show emission spectra measured with average and peak detection, respectively, in a variety of bandwidths B_{meas}. Using the data in these two graphs, we graphed the power levels observed in each bandwidth and detection mode as a function of B_{meas} in Figure 9.

3.4 Comparison of Theoretical and Measured Results

The theoretical predictions graphed in Figure 3 can be compared directly to the measurement values shown in Figure 9. Both figures show identical power-measurement results at bandwidths that resolve individual emission lines. The line power relative to the maximum peak power is

$$10log\left(100\ \mu s/(1\ MHz \cdot (400\ \mu s)^2)\right) = \text{-32 dB},$$

as predicted by (15), and relative to the maximum average power the line power is

$$10log\left(1/(1\ MHz \cdot 400\ \mu s)\right) = \text{-26 dB},$$

as predicted by (16). The peak power maximizes at a normalized level of 0 dB when

$$B_{meas} = \sqrt{1\ MHz/100\ \mu s} = 0.1\ \text{MHz} = 100\ \text{kHz},$$

as predicted by (6b) and (37), and average power maximizes when $B_{meas} = B_{chirp} = 1$ MHz, also as predicted. The measured difference between maximum peak and average power is -6 dB, as predicted by (17) from the time domain duty cycle of

(100 μs/400 μs) = 1/4.

As bandwidths increase from the line spacing, the average power levels go as $10\log B_{meas}$ and peak power levels go as $20\log B_{meas}$, as predicted by (9) and (11) and the parallel (41) and (42).

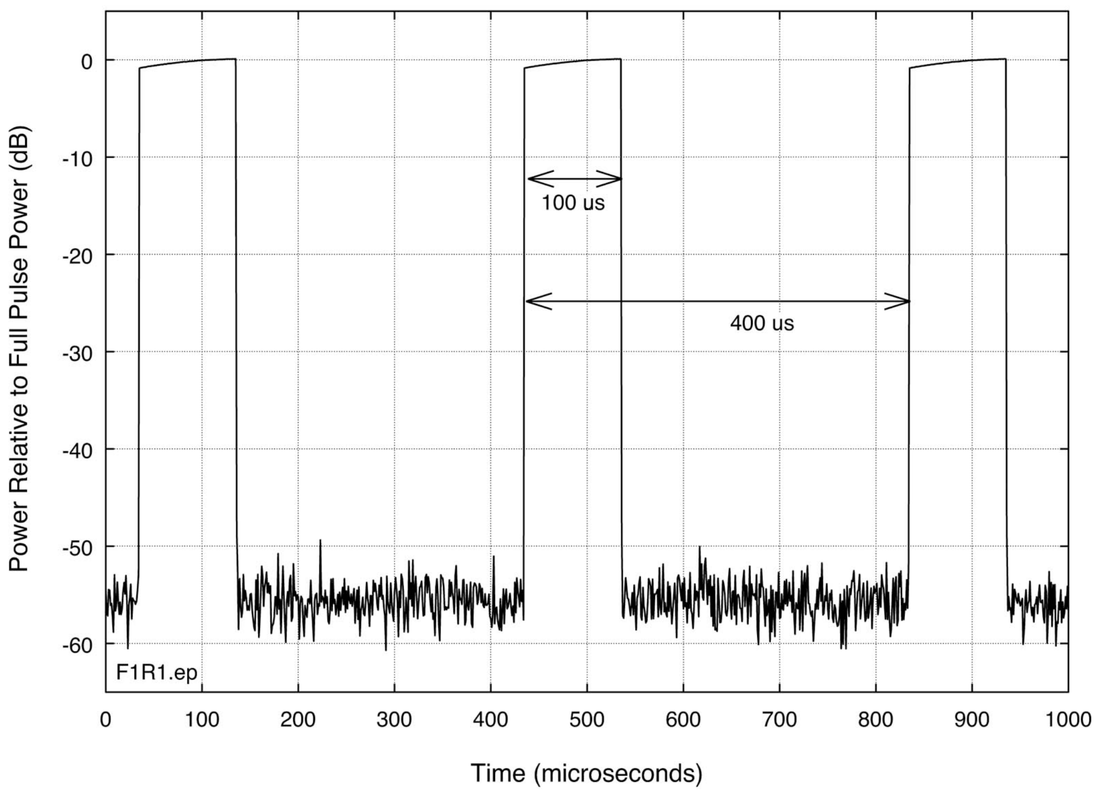

Figure 4. Measured time-domain envelopes of a series of chirped pulses. The full pulse width is observed in this graph because the measurement bandwidth of 8 MHz exceeded the chirp bandwidth of 1 MHz.

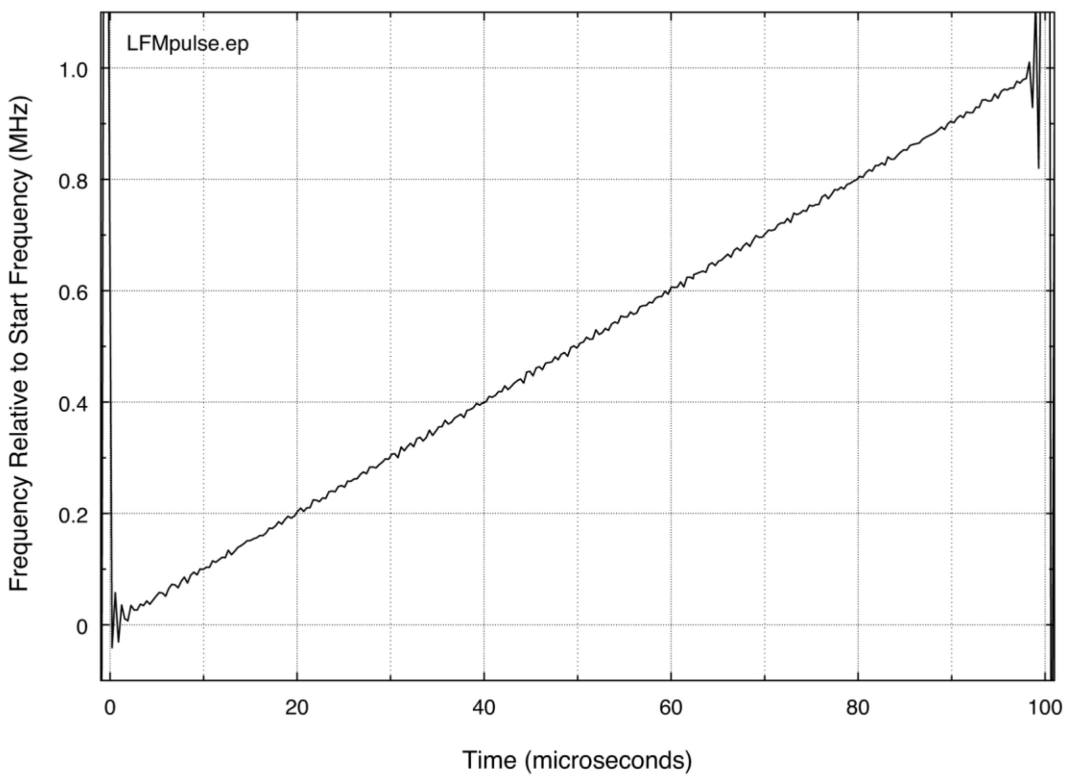

Figure 5. Measured linear chirp (1 MHz in 100 μs) of one pulse from the sequence of Figure 4.

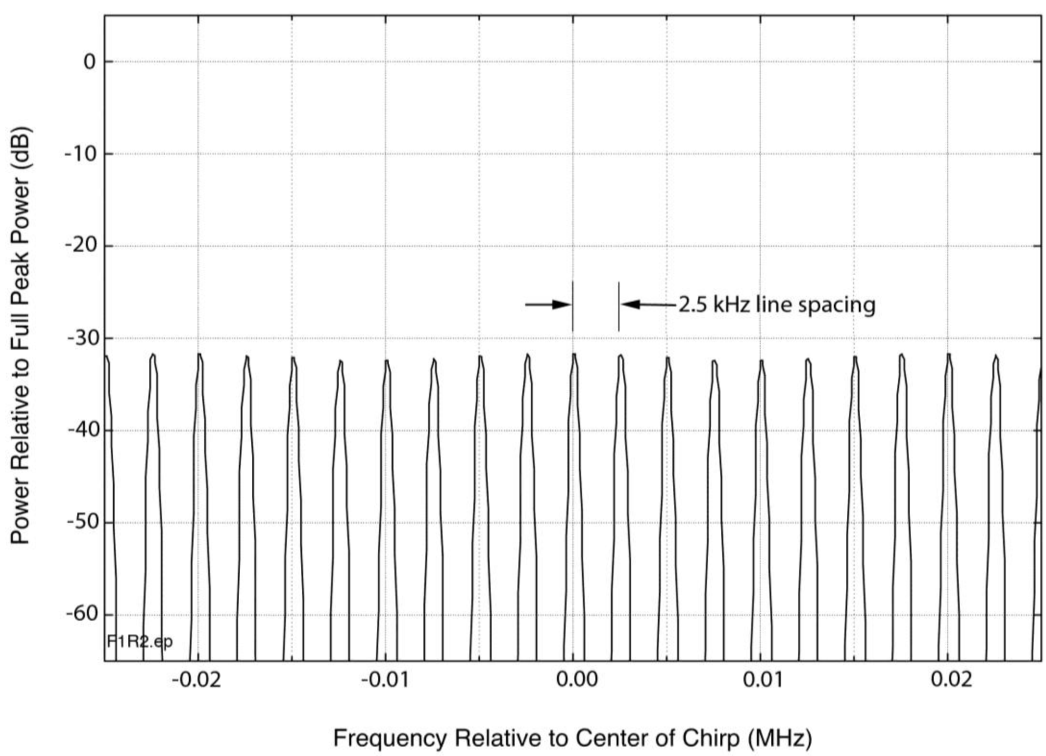

Figure 6. Measured line spectrum detail for the chirped-pulse sequence of Figures 4 and 5.

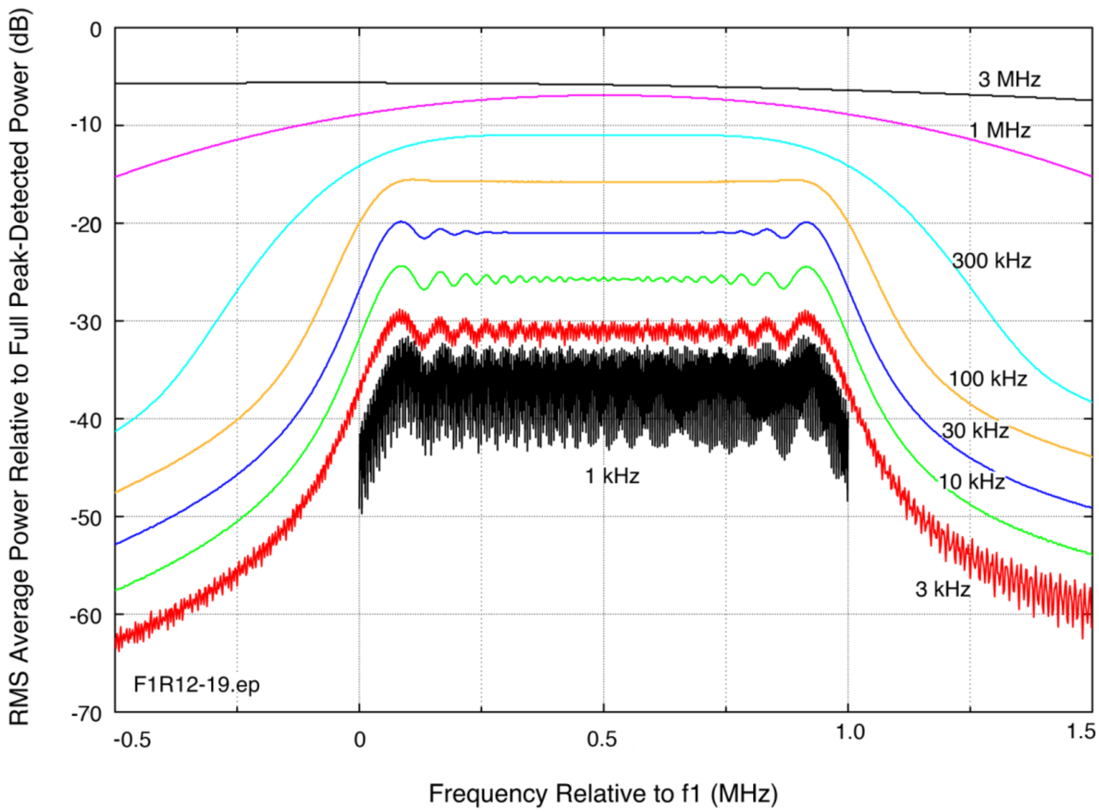

Figure 7. Measured average-power chirped-pulse spectra as a function of B_{meas}.

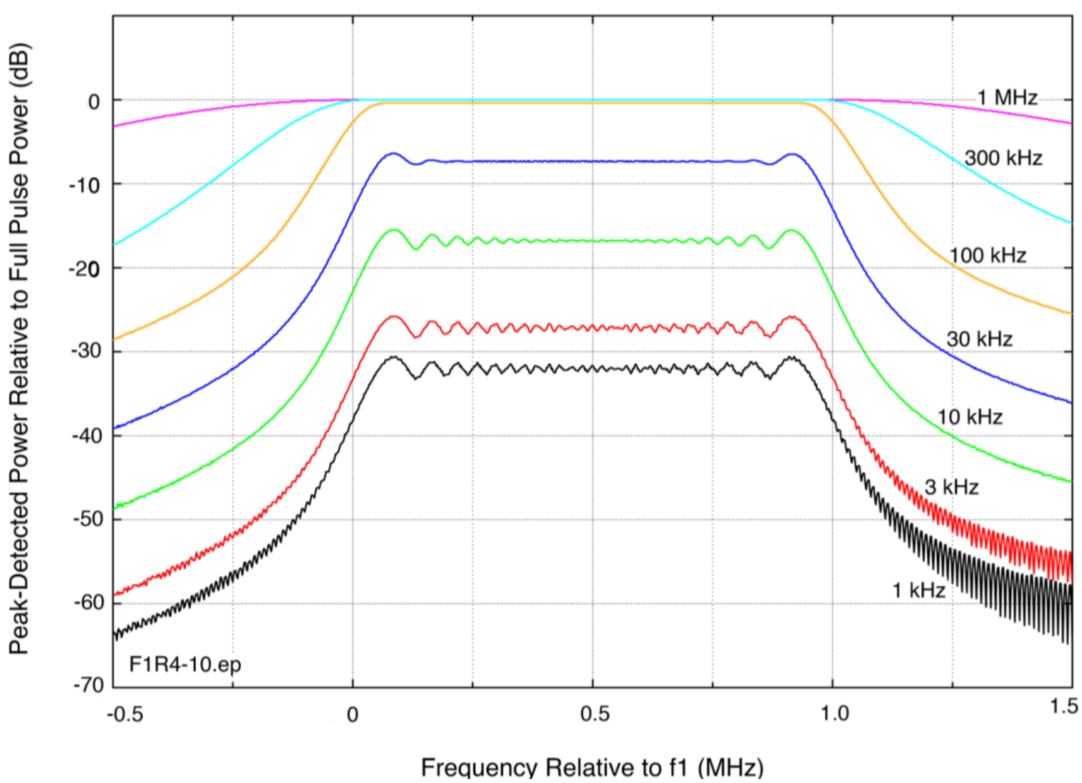

Figure 8. Measured peak-power chirped-pulse spectra as a function of B_{meas}.

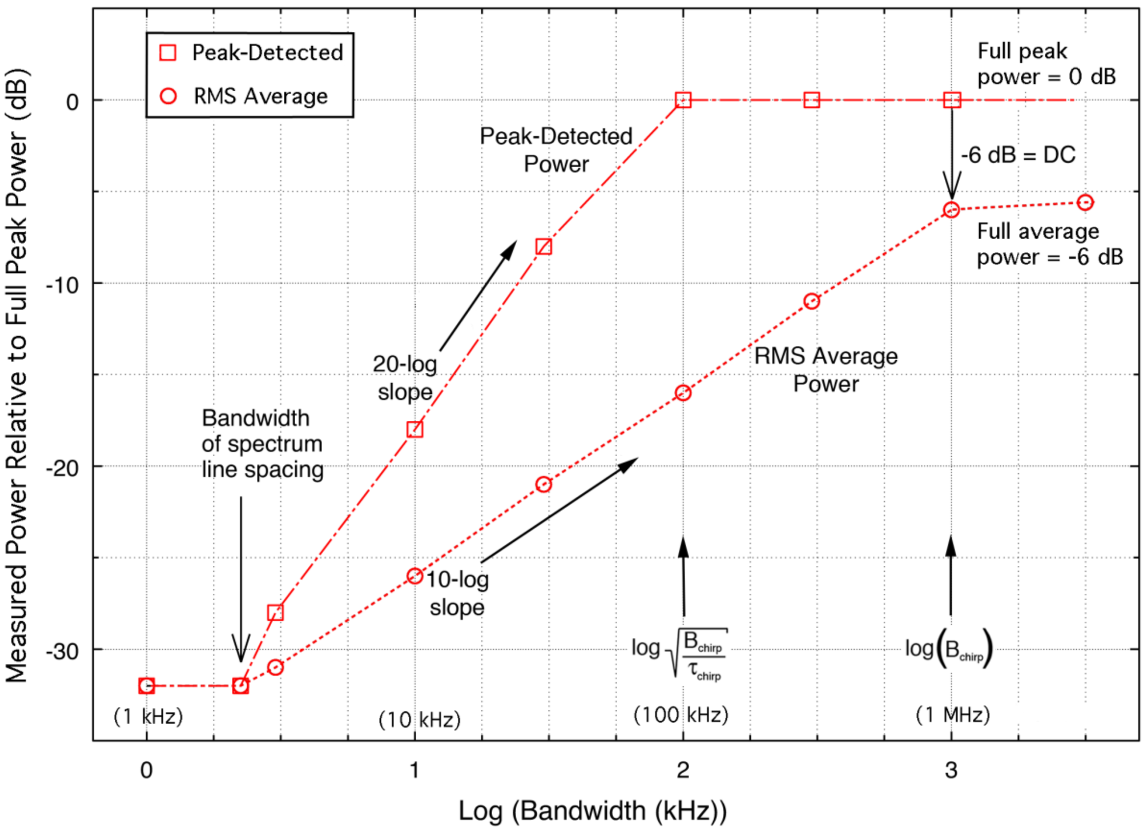

Figure 9. Measured peak and average power for example chirped pulses as a function of measurement bandwidth. Measured power levels are from Figures 7 and 8. Comparison of this figure to Figure 3 shows strong agreement between theoretical predictions and measured results.

3.5 Summary

We have derived the relationships among peak power, average power and measurement bandwidth for chirped FMCW pulses both heuristically and rigorously in this report. These relationships may be best understood and utilized via the graph of Figure 3. We see strong agreement between theoretical predictions and measurements of actual chirped pulsed emissions in comparison of our prediction graph (Figure 3) with measurement results (Figure 9).

Unlike the case for unmodulated P0N pulses, the bandwidths at which measured peak and average power are maximized for chirped pulses are *not* necessarily identical, although peak and average power measurements always yield identical power levels for individual emission spectrum lines. As is the case for P0N pulses, peak power measurements of chirped pulses vary as *20log* of the measurement bandwidth, while average power measurements of these pulses vary as *10log* of the measurement bandwidth.

The relationships derived in this report may be used for electromagnetic compatibility studies of interference from chirped-pulsed emitters to victim radio receivers. In such studies, victim receiver bandwidth should be substituted for the variable B_{meas} in the equations in this report.

4 ACKNOWLEDGMENTS

Discussions with Dr. Michael Spencer of the Jet Propulsion Laboratory (JPL) in Pasadena, California prompted our creation of this report; Dr. Spencer identified the need to have this material published in a referenceable document. We thank Geoff Sanders and John Carroll of the NTIA/ITS laboratory for setting up the VSG that created the chirped RF pulses for the measurements we have presented in this report.

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14. SUPPLEMENTARY NOTES

15. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

Measured power levels for radio frequency (RF) pulses that are frequency modulated (chirped) vary as a function of the bandwidth in which the measurement is performed; if chirped pulses cause RF interference, the power levels of the pulses in victim receivers will likewise vary as a function of receiver bandwidth. This report provides both heuristic and rigorous derivations of the relationships among chirped pulse parameters and the measured peak and average power levels of chirped pulses as a function of measurement bandwidth. These relationships may be best understood via a single graph (Figure 3) presented in this report. This report supplements NTIA Technical Reports TR-05-420, TR-10-465 and TR-10-466, in which the formula for minimum bandwidth needed for measurement of full peak power in chirped pulses is presented but not derived.

16. Key Words (Alphabetical order, separated by semicolons)
chirped pulses; electromagnetic compatibility; frequency-modulated carrier wave (FMCW chirped) pulses; pulse power measurements; radar power measurement; radar spectrum measurement; radio frequency (RF) interference; RF measurement

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