Correspondence

A General Model for Signal Level Variability

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Abstract—**A general model of received signal variability is outlined in which the signal level is represented as a simple algebraic combination of random functions which, in turn, vary with time, location, and the "situation." A study of signal variability should then be a study of the statistics of these separate random functions. Among other things, the model inspires extended definitions of the terms "reliability" and "confidence."**

Introduction

In this correspondence we want to outline a model that helps describe how received signal levels vary randomly with time, location, frequency, and miscellaneous other quantities. It is a general model that might unify the presently disparate approaches used in the different radio services. We hope we are continuing a tradition [1]–[4], but whereas previous discussions were usually concerned with long, point-to-point communication links, here we shall emphasize the broadcast and land-mobile services in the VHF and UHF bands.

The aim of the model will be to represent the received signal level w as a random variable and as a random (or stochastic) function of such variables as time and location. For definiteness we suppose that w is the power available at the terminals of the receiving antenna and that it is measured in decibels relative to 1 mW. In the notation we shall use, and with which we have already begun, random variables are represented in lowercase italic letters while statistics of random variables are denoted by uppercase italic letters.

Small-Scale Variability

In developing the model, we take the customary first step of separating the signal level process into two superimposed processes representing multipath fading and power fading. Normally the two are easy to separate, for multipath fading is rapid and corresponds to short-term or small-scale variability while power fading is slow and corresponds to long-term or large-scale variability. Multipath fading is also frequency selective and the source of multiplicative noise. How a given radio system reacts to it depends very much on the system. Some systems, for example, will use spread-spectrum modulation or some diversity scheme to actually take advantage of the variations. Other systems will find that multiplicative noise is ruinous and so will avoid regions or operating modes where it is present. For example, a television receiver will try to avoid ghosts by putting its antenna on a rooftop and by using directivity to null out extraneous signal paths.

To represent the superposition of the two processes we write

$$w = w_0 + r \tag{1}$$

where w_0 is a smoothed version of w and the residual r then represents the small-scale variations. As indicated by the lowercase italic letters, both components here are to be considered random variables; w_0 is measured in dBm and r in simple decibels. Furthermore, we may suppose that r always has a median value of 0 dB.

The variable r can be thought of as a "random function," $r = r(t, x, f)$, of time t, position x, and frequency f. Its statistical properties as a function of f will provide many of the "channel characteristics" that are important. The behavior with respect to any two of the variables often displays a similarity relation. This is probably especially true for the frequency and position variables. When it comes to the time variable, the common assumption for troposcatter links is that time and position variations are related by similarity; but on short paths such as those found in the land-mobile services, there is usually no short-term (or even long-term) variability except for that induced by motion of the mobile unit.

The form of (1) implies a multiplicative relation for the corresponding amplitudes or powers. This is by design. As a function of frequency, the received signal level provides a frequency transfer function, and it seems quite proper that it should equal a general overall level multiplied by a normalized random function.

When the multipath has many components all of about the same size, then, of course, we have the case of *Rayleigh fading*. The first-order statistics (those that describe how large the variations are but not how fast they change) satisfy the *Rayleigh Law*. For the quantity r, the (complementary) cumulative distribution is given by the simple formula

$$q = \Pr[r > R] = 2^{-10^{R/10}}. \tag{2}$$

We might call this the *Rayleigh decibel distribution* since the term "Rayleigh distribution" is usually reserved for the distribution of the underlying voltage amplitudes. Solving (2) for R we obtain the *quantile*

$$R(q) = 10 \log \frac{\ln 1/q}{\ln 2} \tag{3}$$

so that $R(q)$ is the value that r exceeds for the fraction q of the trials. Note that, particularly since we have taken care of the median by the term w_0 in (1), the distribution has no parameters. If multipath is severe enough to give rise to Rayleigh fading, the first-order statistics are fixed. The distribution has a mean of 0.92 dB, a mode of 1.59 dB, a standard deviation of 5.57 dB, and an interdecile range $\Delta r = R(0.1) - R(0.9) = 13.40$ dB. Because we have not seen it elsewhere, we have plotted in Fig. 1 the corresponding density function—the negative derivative of (2).

A somewhat more general condition arises when there are still many multipath components but one of them (probably the direct wave) is much stronger than the others. This condition leads to first-order statistics that satisfy what is called the *Nakagami–Rice distribution* (and also the "Rician" or the "constant-plus-a-Rayleigh" distribution). If the median is again fixed at 0 db, this distribution has but a single parameter that corresponds to the constant-to-scattered power ratio.

An interesting fact appears when one plots the Nakagami–Rice cumulative distributions on Rayleigh paper. This latter is the kind of graph paper that has a nonlinear probability scale devised so that the Rayleigh decibel distribution plots out as a straight line with slope -1. When it comes to the Nakagami–Rice distributions, their graphs [4], [5] turn out to look much like straight lines with slopes in the 0 to -1 range. They look so much like straight lines that it seems a good approximation to simply replace them by straight lines. It would certainly simplify many calculations.

As it happens, there is a distribution used in the theory of reliability known as the *Weibull distribution* [6], which may be most

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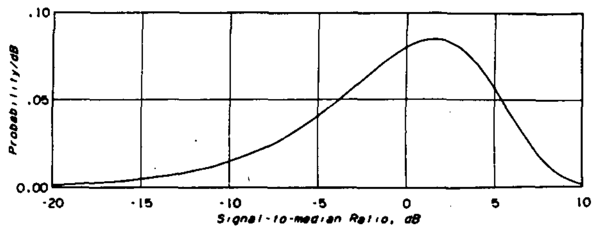

Fig. 1. The density function of the Rayleigh decibel distribution.

simply defined as one whose cumulative distribution plotted on Rayleigh paper is a straight line of arbitrary slope. It appears appropriate, then, to use this distribution as an approximation to the Nakagami-Rice distributions. Note that statistics such as the quantiles are easily determined for the Weibull (decibel) distribution. If the slope of the cumulative distribution is $-\alpha$ then the quantiles are given again by (3) after the right-hand side is multiplied by α. These remarks also suggest a new representation for the small-scale variability of (1) in which we set

$$r = \alpha_0 r_0 \tag{4}$$

where r_0 is a rapidly varying random function that satisfies the Rayleigh law and α_0 is locally a constant, and hence, like w_0, a slowly varying random function. This *multiplier* should in principle lie between 0 and 1 although Shepherd [7] reports measurements that exceed 1. In many cases, it is probably correct to assume that α_0 equals either 0 or 1; in other words, that multipath is either severe or absent.

Large-Scale Variability

For the first component of (1) we write

$$w_0 = W_0 + y_S(s) + \alpha_{TS}(s)\, y_T(t) \tag{5}$$

where W_0 is the overall median received signal (hence, the uppercase letter); y_S and y_T are two random variables called *deviations*; and the multiplier α_{TS} is a third random variable. The deviations are measured in decibels and their median values are 0 dB. The multiplier is dimensionless, always positive, with median value equal to unity.

The deviation y_T is a random function of time. It changes from hour to hour mostly because of diurnal changes in the atmosphere. The other two variables in (5) are introduced in order to account for all other randomness. In one sense they seem to be introduced mostly to allow for errors; the value of W_0 and the statistics of y_T are probably derived from an inexact computation and these variables can be thought of as the adjustments needed to fit reality. In this sense, then, it should be the aim of research in radio propagation to improve the calculations so that the variances of these errors become ever smaller. It seems to us, however, that there will always be some not inconsequential residual variation that is due to unavoidable gaps in our knowledge of the propagation path or of the deployment of the measuring equipment.

Whatever the cause or source of this extra randomness, and whether or not it will someday be possible to remove it entirely, at the present time it must be retained. It is then most convenient to treat these variables as straightforward random functions, and to do that we need to attach a label to the associated argument. For lack of a better term, we shall here call that argument the *situation*, and by that we mean to imply there is indeed a particular situation in which one makes an observation of received signal levels. It may consist of the geographic region containing the propagation path, the antennas used, or the exact measurement process. Thus, y_S, for example, becomes a random function of the situation s. In (5) we have resolved the large-scale variability into a "time variability" and a "situation variability."

The values of W_0 and of the statistics of the three random functions depend on the parameters of the problem —the system parameters including radio frequency and antenna heights; the environmental parameters including the terrain, radio climate, and ground constants; and the deployment parameters such as how the antennas are mounted. One particular set of these parameters consists of a detailed description of the topography lying between the terminals and of the vegetation and buildings that are present. For many problems, however, this detail is unavailable or too expensive to obtain; for the conceptual design of a system it might even be meaningless. For such problems, one wants to abstract away from this one set of parameters, and the best way to do this is to randomize over the set just as it exists in nature. This introduces further variability, and to account for it we would propose to extend (5) so that it takes the form

$$\begin{aligned} w_0 = W_0 &+ y_S(s) + \alpha_{LS}(s)\, y_L(x) \\ &+ \alpha_{TS}(s)\, \alpha_{TL}(x)\, y_T(t) \end{aligned} \tag{6}$$

where x is to be interpreted as a "location" and the random functions with the subscript L provide us with a "location variability." The overall median W_0 and the random functions here with the same names as some of those in (5) are not meant to be the same as those in (5). The randomization process will have changed them.

Examples

The representations in (5) and (6) allow for two and three ways in which the received signal can vary. Sometimes this seems inappropriate. In the land-mobile services, for example, one might want to use (6) because one needs to study base station service throughout an area and obtaining (and utilizing) the required terrain information for an entire area might be too expensive. But the time variability in (6) is, first, small in size and, second, indistinguishable from location variability since motion of the mobile unit transforms the one variability into the other. Thus, one wants either to ignore the last term in (6) or to combine the last two terms obtaining a formula with only location and situation variability but in which location variability has a somewhat larger variance.

On the other hand, the full assortment of terms is used in the case of the design of a broadcast service, and this comes about in an interesting hierarchical series of steps. Consider first the individual user of the service. If the received signal w_0 (we assume no small-scale variability) exceeds some threshold W_a, then we can say he or she has adequate reception. But the individual is adequately served only if there is adequate reception for a large enough fraction q_T of the time. The user will be in a situation s and at a location x and so can measure the quantile

$$\begin{aligned} W_1(q_T, x, s) = W_0 &+ y_S(s) + \alpha_{LS}(s)\, y_L(x) \\ &+ \alpha_{TS}(s)\, \alpha_{TL}(x)\, Y_T(q_T) \end{aligned} \tag{7}$$

where $Y_T(q_T)$ is the q_T quantile of y_T. Then the individual is adequately served if W_1 exceeds W_a.

Next comes the broadcaster. That one will want to have a sufficiently large audience, and one tool to help determine whether that is possible is the fraction q_L of locations that receive adequate service. With the broadcaster and the broadcasting service area in a single situation s, one computes quantiles $W_2(q_T, q_L, s)$ from (7). This is a little more complicated than before since the last two terms in (7) both depend on x and so their statistics must be combined. (If the terms are statistically independent, we can use a convolution). However determined, if W_2 exceeds the threshold, the broadcaster will include another receiver location within the proposed service area.

Finally, the broadcasting industry will want to be assured that a sufficiently large fraction of the aspiring broadcasters can meet their objectives. If we assume that each broadcast station is in a separate situation, then this last fraction is the fraction q_S of situations that enters in evaluating the final threefold quantile $W_3(q_T, q_L, q_S)$. This quantile is thus the value of received signal level that will be exceeded for at least q_T of the time in at least q_L of the locations with probability q_S.

There are at least two radio propagation models in present use that try to estimate both W_0 and the first-order statistics of the random functions in (6). These are the ITS irregular terrian model (the ITM) [2], [3] and the TV and FM-radio field-intensity charts of the FCC [8].

The ITM uses language closely related to (6). The multiplier α_{TL} is assumed to have zero variance (and hence, is dropped from the formula) while α_{TS} and α_{LS} have standard deviations of about 0.35 and 0.20, respectively. The five random variables are assumed to be statistically independent and to follow a normal or nearly normal distribution. Depending on the parameters of the problem, y_S has a standard deviation of from 5 to 8 dB, y_L from 0 to 10 dB, and y_T from 0 to perhaps 12 dB.

The FCC curves do not provide explicit statistics, but background papers hint at them. Thus, from [8] we find that at UHF y_S has a standard deviation of 14 dB (9 dB if the roughness correction factor is used). From [9] we learn that y_L has a standard deviation of 12 dB. And by finding how the $F(50, 50)$ curves and the $F(50, 10)$ curves differ [8], we conclude that y_T has a standard deviation of from 0 to about 11 dB.

Reliability and Confidence

As used by many engineers, the term "reliability" quantifies how well a system performs, often by citing the fraction of time during which the signal is available. It seems to us that it would be useful to extend this notion somewhat. We should like to say that *reliability* is in general a quantitative description of what one means by "adequate service." This description will vary depending on the type of service contemplated.

The term "confidence" is used by statisticians to measure with what certainty one is to accept the truth of their hypotheses. Again, it seems that the notion can be usefully extended to the design of radio systems. In this spirit we say that *confidence* is the probability that a given reliability will be achieved.

The mechanics of estimating reliability and confidence will vary according to the service to be provided. For example, in the straightforward case of a point-to-point *communications link*, the required reliability is that a threshold signal level be available for some given large fraction of the time. Our model in (5) then provides exactly what is needed to estimate the confidence. In this case reliability is simply time availability while confidence is related to situation variability.

On the other hand, the *broadcaster* will say that a station provides reliable service if it provides an adquate signal for a sufficiently large fraction of the time at some large fraction of locations. The last qualification is a new one, but if we turn to the model in (6) the estimation of confidence proceeds very much as before with a study of situation variability. The threefold quantile W_3 plays an immediate role.

In still other circumstances we may need to use combinations of the variabilities. The *individual* who wants to receive a broadcast station will say again that reliability is an adquate signal for some fraction of the time. But if there is a lack of data concerning topography one will need to use (6); then reliability is time availability while confidence is measured by a combined location and situation variability. And, in the *mobile* service, we have already observed that time variability is indistinguishable from location variability. Thus, reliability will be related to a combined time and location variability while confidence will be measured by situation variability.

One approach to the design of a system might be to consider how confidence in an adequate signal varies with some parameter of interest. For example, in Fig. 2 we have treated a modest base station-to-mobile unit system and have plotted confidence as a function of range. The system is at 410 MHz, includes an allowance for small-scale Rayleigh fading, and requires a large-scale reliability of 85 percent. Calculations used the ITM.

Closely allied to the present notion of confidence is that of *service probability*. But whereas "confidence" as used here is concerned only with propagation losses, service probability provides the probability that a given service will perform adequately well and should account not only for variations in propagation loss but also for variations in equipment parameters. Hagn [10] has drawn figures very similar to our Fig. 2, and we may note that his "probability of successful communications" includes random variations in propagation loss, in equipment parameters, and also in ambient noise.

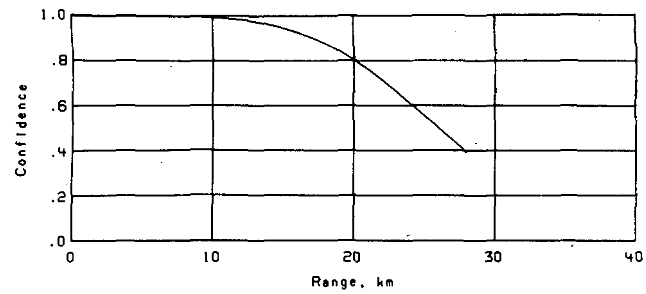

Fig. 2. Confidence versus range for a base station-to-mobile unit system in which 85 percent reliability is required.

Conclusions

We have outlined a general model for signal variability that can be used to resolve many radio engineering problems related to propagation. In particular, we have seen how the terms "reliability" and "confidence" can be extended to many kinds of services and how the model can be used in different ways to make estimates of these quantities.

It seems also that the model might be useful to sharpen our notions as to what kinds of experimental data are still needed. Most measurements to date have been concerned with time variability or small-scale location variability. There has been little notice of the role that "situation variability" might play, particularly in judging the accuracy and usefulness of a propagation model.

Finally, the model might also indicate what higher order statistics would be useful. The problem of correlations on two or more propagation paths is an important one in estimating results for both interference and network connectedness. To date, measurements have been skimpy and probably not well directed.

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