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PART I

FIRST-ORDER PROBABILITY MODELS OF THE INSTANTANEOUS AMPLITUDE

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PREFACE

The following report is the first of a series of studies whose general aims are threefold:

(1) To provide quantitative, statistical descriptions of primarily man-made electromagnetic noise or interference;

(2) To suggest and to guide experiments, which gain the needed data of the actual, physical environment; and

(3) To apply these quantitative and experimentally established results to the evolution of the performance of communication systems which operate, or will be expected to operate, in such noise environments (i.e., local and extended urban and regional areas.)

With the help of (1) and (2) one can then predict and determine interference characteristics of various selected regions of the radio spectrum. With the results of (3) one can establish rational performance criteria for successful, or unsuccessful, operation of communication links in various classes of interference. In combination, one has a quantitative procedure for spectral management.

Thus, in somewhat more detail, our overall aim is to achieve the capability of handling such typical problems as determining when a given communication link may be interfered with by other such links operating in both geographical and spectral proximity to it. Related questions concern the performance of such systems and how it may be affected by trade-offs in system parameters, such as signal level, source and receiver spacing, directionality, etc. Still other problems arise because of the EM interference produced by man-made devices, in particular, automobile ignition noise, which can exert adverse effects upon ground-to-air and air-collision avoidance systems, for example, especially with respect to planned broadband digital systems for such applications.

We distinguish two principal classes of interference generally:

<u>Type A Noise</u>: where the interference is spectrally comparable to or less than the desired signal; and

<u>Type B Noise</u>: where the interference is spectrally very broad vis-à-vis the desired signal.

Ignition noise, for example, belongs to Type B, as does the natural atmospheric noise (which is of concern only below about 30 MHz). On the other hand, the "intelligent" noise, represented by someone else's desired signal, belongs usually to Type A.

Although Type B noise has had a long history of investigation at various levels of detail, Type A interference has only recently been described by analytical models appropriate to the tasks required by (1)-(3) above. The present report is an initial step in this direction. The material following is primarily concerned with Type A interference (cf. p. 24) and first-order statistics of the instantaneous amplitude. In addition to providing noise models for this class of interference [cf. (1), (2)], these, in turn, are needed for the calculation of the performance of coherent systems, which is a task of major interest to the Institute for Telecommunication Science spectral management program. A second report in the series will deal with the corresponding envelope and phase statistics of both type A and B noise, while a third report will consider in detail the statistics of the instantaneous amplitude for Type B interference. Envelope statistics are required for (a), measurements, and (b) the analysis of systems which employ incoherent reception. A parallel program for experimental validation of the various interference models being developed here is being planned as part of the ITS/OT-OTP spectral occupancy investigation.

Finally, we remark that what makes these analyses a technically non-trivial exercise is the fact of the nonnormal or "impulsive" character of both classes of interference. The practical significance of these studies, of course, lies in the ability of the treatment to achieve the spectral management goals of (1)-(3) above.

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STATISTICAL-PHYSICAL MODELS OF MAN-MADE RADIO NOISE

PART I. FIRST-ORDER PROBABILITY MODELS OF THE INSTANTANEOUS AMPLITUDE

David Middleton*

Abstract

A general statistical-physical model of man-made radio noise processes appearing in the input stages of a typical receiver is described analytically. The first-order statistics of these random processes are developed in detail for narrow-band reception. These include, principally, the first-order probability densities and probability distributions for a) a purely impulsive (poisson) process, and b) an additive mixture of a gauss background noise and impulsive sources. Particular attention is given to the basic waveforms of the emissions, in the course of propagation, including such critical geometric and kinematic factors as the beam patterns of source and receiver, mutual location, doppler, far-field conditions, and the physical density of the sources, which are assumed independent and poisson distributed in space over a domain Λ.

Apart from specific analytic relations, the most important general results are that these first-order distributions are analytically tractable and canonical. They are not so complex as to be unusable in communication theory applications; they incorporate in an explicit way the controlling physical parameters and mechanisms which determine the actual radiated and received processes; and finally, they are formally invariant of the particular source location and density, waveform emission, propagation mode, etc., as long as the received disturbance is narrow-band, at least as it is passed by the initial stages of the typical receiver. The desired first-order distributions are represented by an asymptotic development, with additional terms dependent on the fourth and higher moments of the basic interference waveform, which in turn progressively affect the behavior at the larger amplitudes.

This first report constitutes an initial step in a program to provide workable analytical models of the general nongaussian channel ubiquitous in practical communications applications. Specifically treated here are the important classes of interference with bandwidths comparable to (or less than) the effective aperture-RF-IF bandwidth of the receiver, the common situation in the case of communication interference.

Key words: Man-made radio noise, Radio noise models, Statistical communications theory.

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STATISTICAL-PHYSICAL MODELS OF MAN-MADE RADIO NOISE

PART I. FIRST-ORDER PROBABILITY MODELS OF THE INSTANTANEOUS AMPLITUDE

David Middleton

1. INTRODUCTION

Man-made electromagnetic interference (or noise) has become a problem of great concern in the telecommunications community, particularly in the face of available bandwidth resources. Such noise is also, and will become more and more so, a major limiting factor for the successful functioning of communication systems, not only in urban environments but over larger regions as well. Effective analysis of system performance and design requirements demands tractable models of these noise mechanisms, so that the standard methods of Statistical Communications Theory (SCT) can then be employed for the desired system evaluations (Middleton, 1960; 1965; VanTrees, 1968). Our models are necessarily statistical, on the one hand, since the processes they describe are inherently random in time and space. On the other hand, since these processes are generated in the real world, for an adequate description we must also include the appropriate physics of the propagation and reception (Middleton, 1970).

Accordingly, we shall construct first-order probability distributions for some typical classes of man-made interference: (1) "unintelligent" noise, produced by the radio emissions from, say, mobile land vehicles (e.g., automobiles, trucks, buses, etc.), and (2) "intelligent" noise, which may appear in a communication link because of unwanted spectral overlap with, and physical proximity to, other communication links (Middleton, 1971). The general models are the same, but the specific characteristics of the interfering signals, e.g., their waveforms, frequencies, durations, source distributions and movement, geometries (location, beam patterns, etc.), are usually quite different. In this report our construction of explicit statistics will be for interference of type (2) only. Cases of type (1) are treated in a following study (Middleton, 1973c).

Technically, what has made a quantitative treatment very difficult in the past is the fundamentally impulsive, nongaussian character of these classes of noise. However, with new techniques (Middleton, 1970) and recently developed models (Middleton, 1973), this difficulty can be overcome, as the material below will indicate. There appears to be comparatively little earlier analytical work along these particular lines (Middleton, 1970, 1973a) regarding man-made noise. Important exceptions, however, devoted primarily to atmospheric models, are papers by Rice (1944), Middleton (1951),* Furutsu and Ishida (1961), the critical study by Hall (1966), and more recently, Disney and Spaulding (1970). Particularly to be noted, also, is the significant investigation of Giordano (1970), who establishes, among other results, the special conditions justifying the quasi-phenomonological distribution derived by Hall (1966).

The new results presented here are obtained by taking advantage of the above, and especially, the current studies of the author (Middleton, 1973a) on ocean reverberation models. It is found, generally, that as long as the received waveform (at or after the RF stages of the receiver) is narrow band, a canonical treatment is possible, which is analytically tractable. Thus, under practical operating conditions we show here that we can construct useable statistical physical models of man-made noise environments, which have the especially important

*See also, Sec. 11.2 of Middleton, 1960.

feature that they are founded in a physical model and are not an ad hoc statistical construction to be fittable only to particular, local, empirical results. An important consequence of this physical basis is that the statistical parameters of the model are specified in terms of the underlying physics. This first report indicates how one can derive a class of canonical, approximate first-order distributions for "intelligent" man-made noise of type (2) above, with this degree of generality and applicability.

The report is, accordingly, organized as follows:

A. Section 2 outlines the formulation of the basic statistical model (BSM), including gaussian background noise as well as impulsive effects. As before, the analytical starting point is an appropriately structured poisson process (Rice, 1944; Middleton, 1951; Furutsu and Ishida, 1961; Hall, 1966; Disney, 1970; and Giordano, 1970).

B. Section 3 is devoted to the specific calculation of the first-order characteristic function $F_1(i\xi, t)_X$ from the generalized poisson model, for both low (and high) impulsive densities, with and without an additive gauss process, for this class (2) noise type.

C. Section 4 gives explicit (exact) expressions for the lower order moments (up to and including the sixth), for this process.

D. Section 5 presents the desired first-order probability densities of the instantaneous amplitude of the receiver interference, and its distribution (in both normalized and unnormalized forms). Included is a calculation of the probability that a given threshold level will be exceeded. A number of characteristic curves illustrate the results.

E. Section 6 considers in a preliminary way the problem of determining the governing parameters of the distribution from empirical observations.

F. Section 7 concludes this first report with a brief resumé of what has been done, and a statement of various next steps to be taken in the development and validation of this class of canonical statistical-physical model.

2. FORMULATION

In this section we present the general forms of the first-order characteristic functions and probability distributions for impulsive noise and a mixture of impulsive noise and a gaussian (i.e., normal) background. For further development we need also some specific structure for the individual interference waveforms as they may be expected to appear following the RF stage of a typical receiver. Since the spatial distributions, as well as received waveforms, of these noise sources play a key rôle in determining the statistics of the resultant interference process, we shall need to examine its general form also, cf. Sec. (2.3). Finally, two parameters that appear explicitly in the desired probability distribution are the mean and mean intensity. These we shall determine in Sec. (2.4) below. Other important parameters of the distribution we considered later in Sections 3 and 4.

2.1 The Basic Statistical Model (BSM)

For most applications it is reasonable to postulate the familiar poisson mechanism (Middleton, 1951; 1970; 1972a; 1973b) for the initiation of the interfering signals that comprise the received waveform $X(t)$. As far as the receiver is concerned, each "event", representing an interfering signal U(t) is initiated independently here* in time and space, vis-à-vis all other such signals. For the moment we leave open the details of the individual waveforms U_j, except to remark that they have a deterministic structure and arbitrary durations. The received interfering process is

$$X(t) = \sum_j U_j(t, \underset{\sim}{\theta}), \tag{2.1}$$

where $\underset{\sim}{\theta}$ now represents a set of time-invariant random parameters descriptive of waveform scale and structure. (For simplicity, and without seriously restructuring the useful generality of our model, we shall assume that only one type of waveform, U, is generated. Variations in scale, duration, frequency, etc. may be subsumed under appropriate statistical treatment of the parameters $\underset{\sim}{\theta}$, cf. Sec. 2.4 below.

The first-order characteristic function of X(t) for these classes of space-time poisson process is known to be (Middleton, 1967; sec. 3; 1970, eq. (29); 1972, eq. 4.4; and 1973b)

$$F_1(i\xi; t_1)_X = \exp\left[\int_\Lambda \rho(\underset{\sim}{\lambda}) \left\langle e^{i\xi U(t_1;\underset{\sim}{\lambda},\underset{\sim}{\theta})} - 1 \right\rangle_{\underset{\sim}{\theta}} d\underset{\sim}{\lambda}\right] \tag{2.2}$$

where $\underset{\sim}{\lambda}$ $(=t, \theta, \phi)$ are coordinates of the source-receiver geometry; $d\underset{\sim}{\lambda} \equiv d\lambda\, d\theta\, d\phi$ for sources distributed in a volume, and $d\underset{\sim}{\lambda} = d\lambda\, d\phi$ for sources distributed on a surface (not necessarily flat); for the latter one has then $\theta = \theta(t, \phi)$; Λ is the physical domain in which the sources are located. The quantity $\int_\Lambda \rho(\underset{\sim}{\lambda})\, d\underset{\sim}{\lambda}$ is a "counting" functional,

---

*The usual poisson model may be readily extended to include (independent) sets of non-independent events, e.g., signals such as occur in atmospheric noise (Furutsu and Ishida, 1961; Hall, 1966; Giordano, 1970), or in various types of man-made interference where an initial disturbance produces a sequence of related transients, as for example in automobile ignition noise.

which adds up the contributions of the individual sources without regard to their magnitude (Middleton, 1967).

We shall refer to $\rho(\underset{\sim}{\lambda})$ as the <u>process density</u>, which is defined by (Middleton, 1967)

$$\rho_S(\underset{\sim}{\lambda}, \phi) = \sigma_S(\underset{\sim}{\lambda}) dS \big/ d\lambda \, d\phi ; \; \rho_V(\underset{\sim}{\lambda}, \theta, \phi) = \sigma_V(\underset{\sim}{\lambda}) \, dV \big/ d\lambda \, d\theta \, d\phi \qquad (2.3)$$

respectively for surface and volume distributions of sources; σ_S, σ_V are the source densities, per unit area or volume.

The desired probability density W_1 and distribution D_1 of the instantaneous, received process $X(t)$ are given formally by the indicated Fourier transforms:

$$W_1(X, t_1)_P = \underset{\approx}{\mathcal{F}}^{-1}_{\xi} \left\{ F_1(i\xi, t_1)_X \right\} =$$

$$\int_{-\infty}^{\infty} e^{-i\xi X} = \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} \exp\left[-\xi X + \int_{\Lambda} \rho \left\langle e^{i\xi U} - 1 \right\rangle d\lambda \right] \qquad (2.4)$$

and

$$D_1(X, t_1)_P = \int_{-\infty}^{X} W_1(X, t_1)_P dX = \int_{-\infty}^{X} \underset{\approx}{\mathcal{F}}^{-1}_{\xi} \left\{ F_1(i\xi, t_1)_X \right\} dX. \qquad (2.5)$$

The key technical problem, and the one to which we address ourselves primarily here, is now at once apparent: the <u>explicit evaluation of $W_1(X, t_1)_P$, cf. (2.4).</u> Because of the inherently <u>non</u>-normal nature of the poisson process, as reflected by the integrand of (2.4) (which is not limited to terms $O(\xi^2)$ in the exponent), this is a difficult task. However, as we shall see below (Sections 3 - 5), by taking advantage of waveform structure, source distribution, and the pertinent physics of the process generally, we can achieve tractable results.

Frequently, the source field can be regarded as consisting of two independent components: one, the poisson "impulsive" interference, containing only a few (0 (10) or less), discrete sources of relatively high level, and a second (zero-mean) normal background noise

(Middleton, 1972a), which stems either from receiver system noise directly (when there is negligible <u>external</u> background interference), or background interference itself, which is a high-density poisson process and thus (asymptotically) normal. This latter is the resultant of a large number of source emissions, similar to those producing the "impulsive", or poisson component, but none of which is sufficiently strong to exhibit the structural character of the former. Accordingly, since these two components are independent, we have directly for the sum process (gauss plus poisson):

$$W_1(X, t_1)_{P+G} = \int_{-\infty}^{\infty} F_1(i\xi, t_1)_P F_1(i\xi, t_1)_G e^{-i\xi X} \frac{d\xi}{2\pi} \tag{2.6a}$$

$$= \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} \exp\left[-i\xi X - \sigma_G^2 \xi^2/2 + \int_{\Lambda} \rho \left\langle e^{i\xi U} - 1 \right\rangle d\underset{\sim}{\lambda} \right] \tag{2.6b}$$

where $\sigma_G^2 \equiv \overline{X_G^2}$, $(\overline{X_G} = 0)$, and $X_G(t)$ is the gaussian component of $X(t) = X_G(t) + X_P(t)$ now. The distribution, $D_1(X, t_1)_{P+G}$, is defined by (2.5), with $W_1\big|_P$ replaced by $W_1\big|_{P+G}$, (2.6). The presence of the normal component does not remove the analytical difficulties attendant on the evaluation of $W_1(X, t_1)$, but, physically, it does eliminate the non-zero probability of $X = 0$ which is typical of impulse noise alone.

<u>2.2 Waveforms,</u> $U(t, \underset{\sim}{\theta})$:

Here we shall use results already obtained in a previous analysis (Middleton, 1972) to specify the received waveform U, which is a key element of the probability structure, cf. eqs. (2.1), (2.2), (2.4), (2.6). We shall need a certain degree of generality in this respect, to describe the major classes of mechanisms. We accordingly develop below a hierarchy of expressions for U, in decreasing order of complexity.

We make the following assumptions regarding the physical process of source emission:

(i) the various sources are independently radiating (as already noted above);

(ii) a far-field (Fraunhofer) condition (Middleton, 1970) applies, which in some respects permits us to treat both source and receiver as point elements;

(iii) there is a small doppler (sources and/or receiver moving in a fixed frame of reference);

(iv) the typical source has a beam pattern that is not necessarily omni-directional;

(v) the receiver generally has a directional beam pattern;

(vi) the typical source may have a time-variable mechanism, e. g., change in level, frequency, etc., with time;

(vii) the sources are distributed with density $\sigma(\underset{\sim}{\lambda})$, in some region, Λ.

For the moment no restriction is placed on the waveforms of the various sources or on their domain of distribution, Λ. Then, the waveform after entering and leaving the aperture-RF-IF portion of our receiver, arising from the $j^{\underline{th}}$ source ($j = 1, \ldots, J$), is given generally by [Middleton, 1972a, eq (3.28)]

$$
\text{(I).}\quad U_j(t) \doteq \frac{1}{4\pi R_j}\left\{\int_{-\infty i+d}^{\infty i+d} a_R(-\hat{\underset{\sim}{i}}_R s/2\pi i c, s/2\pi i)e^{st}\frac{ds}{2\pi i}\right.
$$

$$
\cdot\int_{V_I} d\underset{\sim}{\xi}\cdot\int_{-\infty i+d}^{\infty i+d} A_T(\underset{\sim}{\xi}, s'/2\pi c)\mathcal{Y}_I\left[(1+\beta)s'/2\pi i, \frac{s-(1+\beta)s'}{2\pi i}; \underset{\sim}{\xi}\,|\,\underset{\sim}{R}\right]
$$

$$
\left.\cdot\, e^{s'\underset{\sim}{i}_R\cdot\underset{\sim}{\xi}/c - s'T_o}\frac{ds'}{2\pi i}\right\}_j \qquad (2.7)
$$

where $\mathcal{Y}_I$ is the bi-frequency function (Middleton, 1967) of the (jth) time-variable source S_{Ij}, viz.

$$\mathcal{Y}_{I_j}(f, \nu; \underset{\sim}{\xi} | \underset{\sim}{R}) \equiv \underset{\approx t}{\mathcal{F}} \left\{ S_{Ij}(f, t; \underset{\sim}{\xi}, \underset{\sim}{R}) \right\} \equiv \underset{\approx t}{\mathcal{F}} \underset{\approx}{\mathcal{F}} \left\{ S_{Ij}(\tau, t; \underset{\sim}{\xi} | \underset{\sim}{R}) \right\}. \qquad (2.8)$$

Here, in addition, we have

(2.9)

$\mathcal{A}_R, \mathcal{A}_T$	$\equiv$	beam-patterns of receiver and interfering source; (These are the spatial Fourier transforms of the respective aperture weightings, A_R, A_T.)
A_T	$\equiv$	Aperture weighting of the (j^{th}) interference source;
$\hat{\underset{\sim}{1}}_R$	$\equiv$	$\underset{\sim}{R}/\lvert\underset{\sim}{R}\rvert$ = unit vector to the origin (0_R) of the receiver's coördinate system, from the origin (0_I) of the j^{th} source's coördinate system [cf. Fig. 2.1];
β	$\doteq$	$\beta_{RS} + \beta_{SR}$ = sum of dopplerized velocities, receiver to source and source to receiver, where $\beta_{SR} = \underset{\sim}{v}_S \cdot \hat{\underset{\sim}{1}}_R / c$; $\beta_{RS} = -\underset{\sim}{v}_R \cdot \hat{\underset{\sim}{1}}_R / c$, and $\underset{\sim}{v}_S$, $\underset{\sim}{v}_R$ are (linear), measured with respect to a stationary coördinate system.
c	=	speed of propagation (group velocity) of these (electromagnetic) emissions.
T_o	=	R/c = path delay, of emission wavefront to receiver.
R	=	distance between 0_I and 0_R, cf. Fig. (2.1).

There is a limitation on our model here: we treat the emissions as a scalar phenomenon, when in actuality they are a vector field. However, this is not a serious restriction vis-à-vis the receiver as long as we are not concerned with polarization effects: we consider the received field to have only one dominant component, E or H, which is whatever the receiving aperture is designed to couple to.

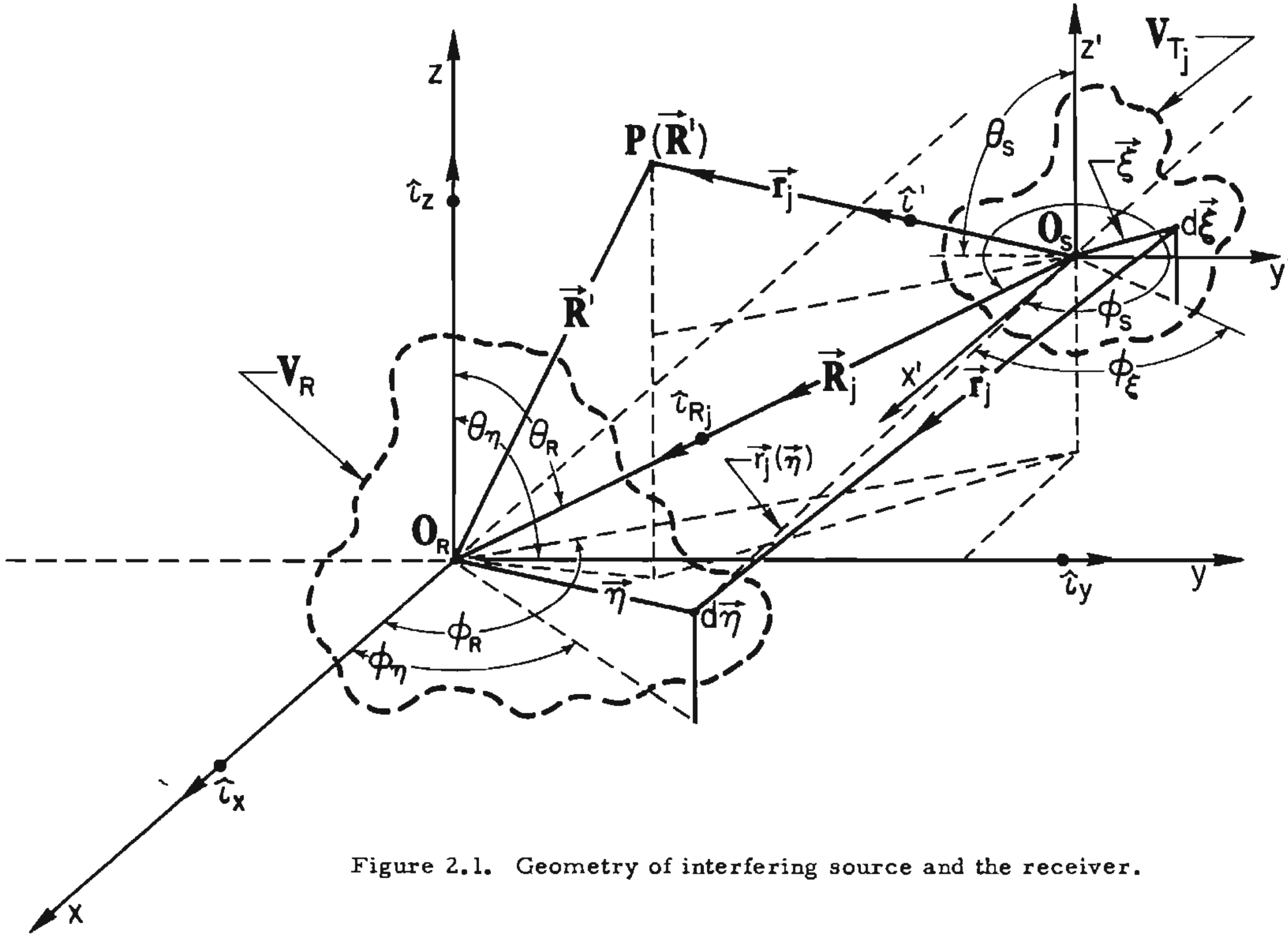

Figure 2.1. Geometry of interfering source and the receiver.

(Later, in considering possible means of interference reduction, we should remember the possibilities that may occur here if advantage can be taken of any systematic polarization in the interference field.)

Equation (2.7) is a result of considerable generality, including broadband* sources. However, except in some measurement situations where very broad-band receivers are employed, most man-made interference is comparatively narrow-band,* so that we can write, using now for convenience a complex representation

$$S_I(\tau, t|\underset{\sim}{\xi}|R) = \hat{S}_o(\tau, t;\underset{\sim}{\xi})e^{i\omega_o\tau}; \quad \hat{S}_o = A_o(\tau, t;\underset{\sim}{\xi})e^{i\phi_o(\tau, t;\underset{\sim}{\xi})} \qquad (2.10)$$

where $\hat{S}_o$ is a complex envelope; A_o and ϕ_o are a real envelope and phase. The bi-frequency function (2.8) accordingly reduces to

$$\left(\mathcal{Y}_I\right)_o = \int_{-\infty}^{\infty} S_o\left[(1+\beta)(s'-s_o)/2\pi i,\ t';\ \underset{\sim}{\xi}\right] e^{-[s-(1+\beta)s']t'}dt', \qquad (2.11)$$

$s_o = 2\pi i f_o$, as it appears in (2.7), with

$$S_o = \underset{\approx}{\mathcal{F}}_\tau\{\hat{S}_o\}. \qquad (2.11a)$$

Using (2.11) in (2.7) we get directly

$$\text{(II.)} \quad U_j(t)_{n.b.I.} \doteq \mathrm{Re}\left\{\frac{e^{-i\omega_o T_o}}{4\pi R}\int_{-\infty i+d}^{\infty i+d} \mathcal{A}_R(-\hat{\underset{\sim}{i}}_R s/2\pi i, s/2\pi i)e^{st}\frac{ds}{2\pi i}\right.$$

$$\cdot\int_{V_I} d\underset{\sim}{\xi}\, e^{i\hat{\underset{\sim}{i}}_R\cdot\underset{\sim}{\xi}\omega_o/c} \cdot \int_{-\infty i+d}^{\infty i+d} A_T\left(\underset{\sim}{\xi}, \frac{s_o+s''}{2\pi i}\right)e^{s''(\hat{\underset{\sim}{i}}_R\cdot\underset{\sim}{\xi}/c - T_o)}$$

$$\left.\mathcal{Y}_I\left[(1+\beta)s''/2\pi i, \frac{s-(1+\beta)(s''+s_o)}{2\pi i}; \xi|R_o\right]\frac{ds''}{2\pi i}\right\}_j. \qquad (2.12)$$

---

* "Broadband" here means a bandwidth Δf comparable to the center frequency, f_o, e.g., $f_o \approx \Delta f$. Conversely, "narrow-band" means that $f_o \gg \Delta f$; f_o "much greater than Δf" may be $f_o = O(5\Delta f)$ or more.

This relation is needed when the input signals, though narrow-band, are still spectrally comparable to or broader than the receiving (and transmitting) apertures, which are frequency selective, <u>and</u> the RF stage of the receiver, which is spectrally comparable to these apertures.

Frequently, however, we can simplify (3.12) very considerably. This happens under a variety of circumstances:

1) the input field, though broad or narrow, is still very much wider, spectrally, than the RF stage of the receiver.

2) the receiving apertures are spectrally insensitive over the domain of the input, which is narrow-band, as defined here. (The bandwidth of the RF is again the controlling factor.)

Then, we may regard $\mathcal{A}_R$ in (2.7) and (2.12) as effectively frequency invariant: in effect we are considering only the frequency range Δf, determined by the RF stage, about some receiver carrier frequency, f_{0R}, to be of interest, and that $\mathcal{A}_R$ is invariant over this interval. The magnitude of $\mathcal{A}_R$, of course, will depend on the particular f_{0R} chosen. Thus, for systems with specified RF bandwidths, we are mainly concerned with the interference process, <u>as it leaves the RF(-IF) stages</u> for subsequent processing in the receiver; e.g.,

$$X(t)_{RI} = \int_{-\infty}^{\infty} X(t-\tau) h_{RI}(\tau) d\tau = \sum_j \int_{-\infty}^{\infty} U_j(t-\tau; \underline{\theta}) h_{RI}(\tau) d\tau = \sum_j U_j * h_{RI} \equiv \sum_j (U_j)_{RI} . \quad (2.13)$$

Then, in (2.7) and (2.12) we replace $s/2\pi i$ by $f_{0R} = f_0$ in $\mathcal{A}_R$ and remove $\mathcal{A}_R$ from under the integral.

Next, we integrate over s to obtain (narrowband, RF and IF):

$$\text{(III).}\quad \left[U_j(t)_{nb}\right] \doteq \left\{ \frac{a_R(\hat{i}_R f_o/c, f_o)}{4\pi R} \int_{V_I} d\xi \int_{-\infty i+d}^{\infty i+d} A_T(\xi, \frac{s'}{2\pi i})\, e^{\mu s'} \right.$$

$$\left. \cdot\, Y_I(\mu s'/2\pi i; t + \frac{\hat{i}_R \cdot \xi}{c} - T_o | \xi, R)\, e^{\mu s'(t+\hat{i}_R \cdot \xi/c - T_o)} \frac{ds'}{2\pi i} \right\}_j \quad , \qquad (2.14)$$

$$\mu \equiv 1 + \beta \ .$$

for general sources and source apertures. Here Y_I is the time-varying frequency response (Middleton, 1967)

$$Y_I(f, t; \xi, R) = \mathcal{F}_\nu^{-1}\left\{\mathcal{Y}_I(f, \nu; \xi, R)\right\} = \mathcal{F}_\tau\left\{h_I(\tau, t | \xi, R)\right\} \text{, etc.} \qquad (2.14a)$$

The analogous version of (2.12) [i.e., when the sources are narrow band, cf. (2.12), and a_R is again frequency insensitive], is at once

$$\text{(IV).}\quad \left[U_j(t)_{n.b.}\right] \doteq \text{Re}\left\{ e^{i\omega_o(\mu t - T_o)} \frac{a_R(-\hat{i}_R f_o/c, f_o)}{4\pi R} \int_{V_I} d\xi\, e^{i\omega_o \hat{i}_R \cdot \xi/c} \right.$$

$$\left. \cdot \int_{-\infty i+d}^{\infty i+d} A_T(\xi, \frac{s_o + s''}{2\pi i})\, Y_I(\mu s''/2\pi i, t; \xi, R)_o\, e^{s''(\mu t + \hat{i}_R \cdot \xi/c - T_o)} \frac{ds''}{2\pi i} \right\}_j \ , \qquad (2.15)$$

with $(Y_I)_o = \mathcal{F}_\tau^{-1}\left\{(\mathcal{Y}_I)\right\}_o$, cf. (2.14a).

A further simplification of (2.14), (2.15) is possible if the obliquity factor $\hat{i}_R \cdot \xi/c$ is small compared to the path delay T_o, which is insured by the fact that our postulated far-field condition, and the interfering source, or its complex envelope $\hat{S}_o$ do not change noticeably in periods of time $O(L_{max}/c)$, where L_{max} is the largest dimension of the source's radiating aperture. Then we may replace the term $\hat{i}_R \cdot \xi/c - T_o$ by T_o alone in (2.14), (2.15). For example, suppose that L_{max} is 3 meters, then $L_{max}/c = (3/3 \cdot 10^{-8})$ seconds $=$ 10 nanoseconds, during which time we may expect ignorable change in S_I or $\hat{S}_o$ for most classes of interference.

Finally, when we can postulate the essential frequency insensitivity of the source aperture so that $A_T(\xi, s/2\pi i) \doteq A_T(\xi, s_o/2\pi i)$, or $A_T(\xi, f_o)$, we get for the narrow band receiver, (2.14)

$$\text{(V).}\quad \left[U_j(t)_{n.b.}\right] \doteq \frac{\mathcal{a}_R(-\hat{i}_R f_o/c, f_o)}{4\pi R\mu} \cdot$$

$$\cdot \int_{V_I} d\xi\, S_I\left(\mu\left[t + \frac{\hat{i}_R \xi}{c} - T_o\right], \left[t + \frac{\hat{i}_R \cdot \xi}{c} - T_o\right] \Big| \xi, R\right) A_T(\xi, f_o). \qquad (2.16)$$

and with narrow-band sources, in addition, (2.16) becomes

$$\text{(VI).}\quad \left[U_j(t)_{nb}\right] \doteq \mathrm{Re}\left\{ \frac{e^{i\omega_o(\mu t - T_o)} \mathcal{a}_R(-\hat{i}_R f_o/c, f_o)}{4\pi R\mu} \right.$$

$$\left. \cdot \int_{V_I} d\xi\, \hat{S}_{0I}(\mu t + \hat{i}_R \cdot \xi/c - T_o,\ t;\ \xi, R) A_T(\xi, f_o)\ e^{i\omega_o \hat{i}_{R_i} \cdot \xi/c} \right\}_j \qquad (2.17a)$$

$$\doteq \operatorname{Re}\left\{\frac{e^{i\omega_0(\mu t - T_0)}\mathcal{A}_R(-\hat{\underline{i}}_R f_0/c, f_0)}{4\pi R\mu}\right. \tag{2.17b}$$

$$\left.\cdot\int_{-\infty i+d}^{\infty i+d} S_{0I}(s/2\pi i,\ t;\ \underline{\xi},\underline{R})A_T(\underline{\xi}, f_0)e^{\hat{\underline{i}}_R\cdot\underline{\xi}(s_0+s)/c + s(\mu t - T_0)}\right\}_j \frac{ds}{2\pi i},$$

alternatively, where S_0 is the (amplitude) spectral density of the complex envelope $\hat{S}_0$, cf. (2.11a).

Frequently, all of the individual conditions illustrated above are obeyed in practise. In addition, it is not unreasonable to postulate that the driving source, S_I, of the typical interference mechanism is applied equally at each element $d\underline{\xi}$ of the source's aperture, i.e., S_I is independent of $\underline{\xi}$, cf. (2.8). For "intelligent" man-made interference, e.g., other radio communications, this is certainly an acceptable approximation in view of the far-field relation between the sources and the receiver, so that each may be regarded in this respect as point sources (with, of course, directional beam patterns still). [For the "non-intelligent" man-made noise, e.g., ignition noise, the full mechanism is not yet entirely clear, but again by the same sort of argument we may reasonably approximate S_I in the above way.]

With the usual small dopplers ($\mu \doteq 1$) and small apertures A_T [cf. remarks following eq. (2.15)], we have the still simpler relation

$$\text{(VII).}\quad \boxed{U_j(t) \doteq \operatorname{Re}\left\{\frac{e^{i\omega_0\mu(t-T_0)}}{4\pi R}\hat{S}_{0I}(t-T_0, t)\mathcal{A}_R(-\hat{\underline{i}}_R f_0/c, f_0)\mathcal{A}_I(\hat{\underline{i}}_R f_0/c, f_0)\right\}_j,} \tag{2.18}$$

which is the one we shall use in the remainder of this Report.* The conditions under which eq (2.18) applies are

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*See the comments under condition (iii), (2.18a) below, and remarks following (2.21).

(i) transmitting (i.e., source) and receiving apertures are frequency insensitive for the bandwidths of emissions passed by the RF stage of the receiver;

(ii) the source aperture is electrically small: $L_{max}/c \ll \left\{\overline{|\dot{S}_o|^2}\right\}^{1/2}$;

(iii) both the source emissions and the (aperture-RF-IF) stages of the receiver are of comparable bandwidth in this study (see remarks pp. 2, 3) and are narrow-band: e.g., $f_o \gg \Delta f_{RI}, \Delta f_I$;

(2.18a) [We do not need to restrict the emission from the source to be narrow-band, e.g., automobile ignition noise, atmospherics, etc. Then the receiving aperture, of course, acts like a filter, whose effects we can equivalently lump into those of the RF-IF stages of the receiver, provided the shape of the beam pattern, $\mathcal{A}_R$, is not strongly affected over the region of significant frequency response of the aperture, which is a reasonable assumption. Otherwise, of course, we must use the more general form (2.20), (2.21). In any event, we reserve to a subsequent report the detailed study of these cases (Middleton, 1973c).]

(iv) doppler is small, e.g., $\beta \doteq 0$ (or $\mu \doteq 1$);

(v) the source mechanism driving its aperture does not depend on aperture geometry, i.e., S_I is independent of ξ.

For measurement and study of typical sources, some form of (I), (II), for U_j may be needed, with $\mathcal{Y}_I$ independent of ξ, and the usually acceptable condition of time-invariance of S_I over properly chosen periods of observation. Then we have

$$\mathcal{Y}_I \rightarrow \mathcal{Y}_I(f, \nu; \underset{\sim}{R}) = S_I(f; \underset{\sim}{R})\, \delta(\nu - 0), \quad S_I(f) = \mathcal{F}_t\left\{S_I(t)\right\}, \tag{2.19}$$

and (2.7) reduces to

$$\text{(VIII).}\quad U_j(t) \doteq \left\{ \frac{1}{4\pi R} \int_{-\infty i+d}^{\infty i+d} \mathcal{A}_R\left(\frac{\hat{i}_R \mu s'}{-2\pi i c}, \frac{\mu s'}{2\pi i} \right) \mathcal{A}_T(\hat{i}_R s'/2\pi i c,\ s'/2\pi i) \cdot S_I(\mu s'/2\pi i;\ \underset{\sim}{R})\, e^{\mu s'(t - T_o/\mu)} \frac{ds'}{2\pi i} \right\}_j . \qquad (2.20)$$

With narrow-band invariant sources (and small doppler), (II), eq (2.12) reduces similarly to

$$\text{(IX).}\quad \left[U_j(t)_{nbI} \right] \doteq \mathrm{Re} \left\{ \frac{e^{\mu\omega_o(t-T_o)}}{4\pi R} \int_{-\infty i+d}^{\infty i+d} \mathcal{A}_R\left(\frac{-\hat{i}_R(s_o+s)}{2\pi i c}, \frac{s_o+s}{2\pi i} \right) \mathcal{A}_T\left(\frac{\hat{i}_R(s_o+s)}{2\pi i c}, \frac{s_o+s}{2\pi i} \right) \cdot S_I'(s/2\pi i\,|\underset{\sim}{R})_o\, e^{\mu s(t-T_o)} \frac{ds}{2\pi i} \right\}_j . \qquad (2.21)$$

Reductions to the primary form (2.18) follow directly when conditions (i) - (v) warrant.*

Note in all cases (I) - (IX) that these results are canonical in S_I, $\mathcal{A}_R$, $\mathcal{A}_T$: we need not specify for our purposes in this Report, precise source waveforms, beam patterns, etc. It is sufficient to delineate their general properties, e.g., bandwidths, beamwidths, directionalities, etc.

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*See the comments (iii), (2.18): in subsequent applications (Middleton, 1973c) we shall assume that if the emitted source is broadband, the filtering action of the receiver aperture can be included in the RF-IF stages, as a composite filter effect. This is permissible because (a), these stages of the receiver are "linear", and (b), the receiving beam pattern is not noticeably changed over the effective frequency range of the aperture response. The net effect of this is to permit the use of (2.18) quite generally, which we take advantage of in a subsequent study (Middleton, 1973c).

2.3 The Process Density, $\rho(\underset{\sim}{\lambda})$:

The process densities (2.3) depend on the geometry of the receiver vis-à-vis the various sources. In the case where these are distributed on a surface, we shall assume that the surface is flat; the receiver may be above or on the surface, in the manner of Fig. 2.2. Following the analysis of sec. 3.1, Middleton (1967), with L = 0 therein [cf. eq (3.9)], we find that

$$\rho_S(\underset{\sim}{\lambda}) = \sigma_S(\lambda, \theta_R, \varphi_R)\lambda c^2 \; ; \quad c\lambda \ge h_R ; \; \cos\theta'_R = h_R/R = h_R/c\lambda . \quad (2.22)$$

Similarly [cf. eq (3.16), Middleton (1967)] we get for sources distributed in a volume

$$\rho_V(\underset{\sim}{\lambda}) = \varsigma_V(\lambda, \theta_R, \varphi_R)\; \lambda^2 c^3 \sin\theta_R ; \quad \lambda \ge 0; \quad 0 \le \theta_R < \pi . \qquad (2.23)$$

Both ρ_S and ρ_V are non-negative functions, since the physical density of scatterers, σ_S, ς_V, are non-negative necessarily. These physical densities of radio noise sources, (i.e., number per unit spatial element) are quantities which must be determined for the various urban and larger regions (Λ) under consideration. Their magnitudes are time-dependent usually, on a scale much slower than the duration of particular message or other man-made interference waveforms. However, these secular variations over periods in the day, week, season, etc., need to be studied, as they can change the scale of ρ noticeably: as we shall see in Sec. 3 following, small ρ means highly "impulsive" noise, whereas sufficiently large ρ is indicative of an effectively gaussian process.

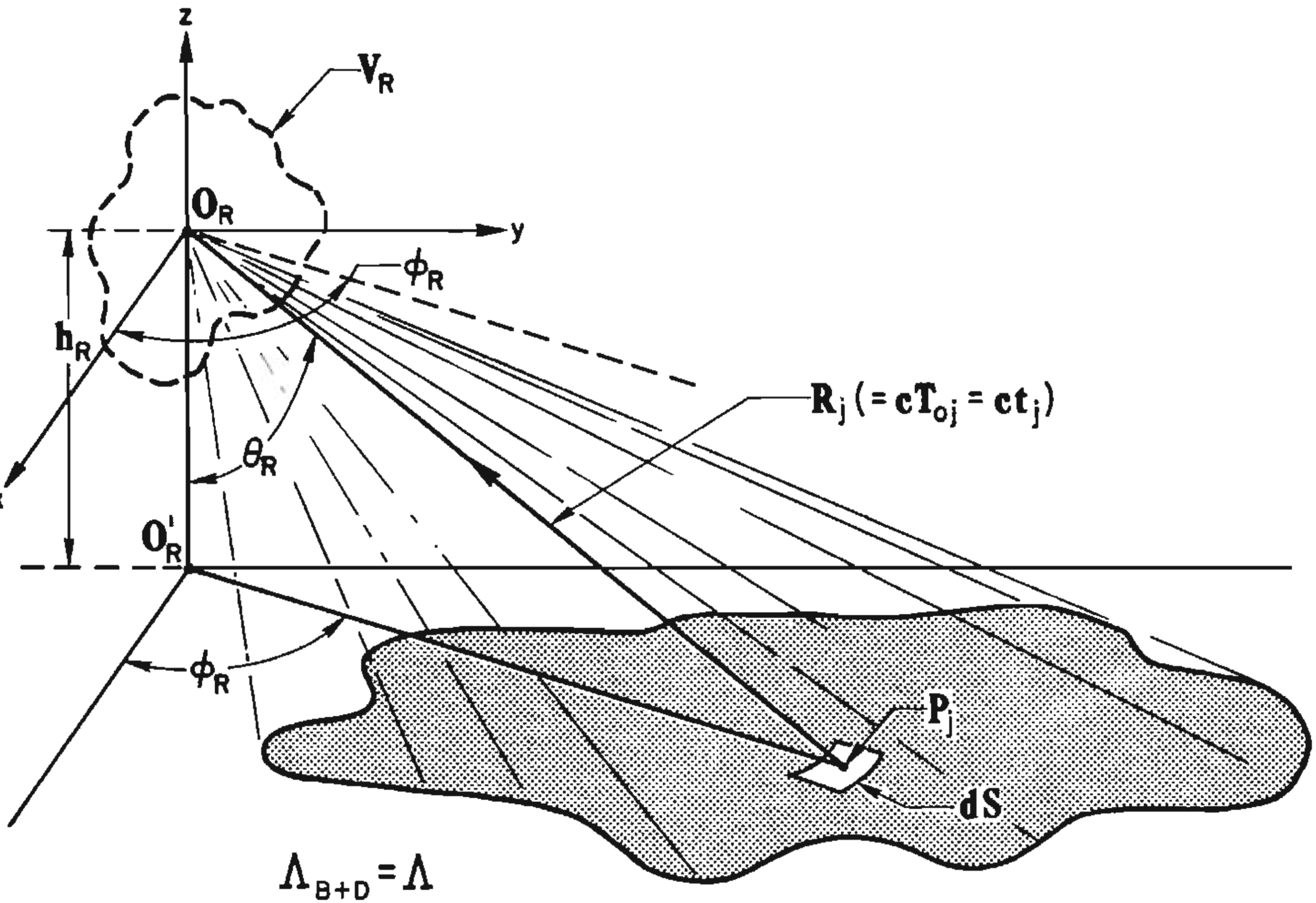

Figure 2.2. Geometry of an interfering source (at P_j) and a receiver at height ($h_R \geq 0$), for an urban region Λ containing independent radio noise sources. (Exaggerated in the vertical direction.)

2.4 The Mean Intensity $\overline{X^2}$, $\overline{X^2_{ARI}}$

A parameter of basic importance to both our models and to measurement is the mean intensity

$$\overline{X^2(t)} = -\frac{d^2}{d\xi^2} F_1(i\xi, t)\Big|_{\xi=0} = \int_\Lambda \rho(\lambda) \left\langle U(t;\lambda,\theta)^2 \right\rangle_\theta d\lambda + \overline{X(t)}^2, \quad (2.24)$$

obtained from (2.2), where the mean, if any (representing a specular component, e.g., resolvable multipath) is

$$\overline{X(t)} = -\frac{d^2}{d\xi} F_1(i\xi, t)\Big|_{\xi=0} = \int_\Lambda \rho(\lambda) \left\langle U(t;\lambda,\theta) \right\rangle_\theta d\lambda, \qquad (2.25)$$

in the usual way [Middleton (1951), (1960) Chapter 11, (1967) Sec. 2, etc.]. Formally, inserting eqs (2.7), (2.12), (2.14) - (2.18), (2.20), (2.21) respectively in (2.24), (2.25) yields the desired moments for these different broad- and narrow-band situations.

In our present study (Part I) we assume no specular component, e.g., $\overline{X} = 0$, and so $\overline{X^2} = \sigma^2$, the variance of this Poisson interfering process. In the usual case of aperture, RF, and IF filtering in the receiver [cf. (2.13) and remarks after (2.21)], we have [Middleton, (1960) eqs (3.101), (3.102), and (3.87)]

$$\overline{X^2(t)}_{ARI} = \iint_{-\infty}^{\infty} K_X(t-\tau, t-\tau') h_{ARI}(\tau) h_{ARI}(\tau') d\tau\, d\tau' \qquad (2.26a)$$

generally, where $K_X = \overline{X(t-\tau)X(t-\tau')}$ is the covariance frunction of the input process X(t) to the composite aperture-RF-IF filter (h_{ARI})

$$\overline{X^2(t)}_{ARI} \doteq \int_{-\infty}^{\infty} \rho_{ARI}(\tau) K_X(\tau) d\tau,$$

$$(2.26b)$$

$$\rho_{ARI}(\tau) \equiv \int_{-\infty} h_{ARI}(x) h_{ARI}(x+\tau) dx,$$

where X is "macrostationary" [cf. sec. 4, (8), of Middleton, 1972], and ρ_{ARI} is the autocorrelation function of the composite aperture-RF-IF filter. Note that in this formulation we need the covariance K_X, which requires the second-order, second-degree moment

$$K_X(t_1, t_2) = \int_\Lambda \rho(\underset{\sim}{\lambda}) \left\langle U(t_1; \underset{\sim}{\lambda}, \underset{\sim}{\theta}) \right\rangle_{\underset{\sim}{\theta}} d\underset{\sim}{\lambda}, \qquad (2.27)$$

[Middleton (1967), sec. 2]. However, we can easily avoid this formal introduction of a second-order theory by noting that $U_{ARI} = h_{ARI} * U$ [cf. (2.13), remarks after (2.4), (2.5)]; thus, we have

$$\overline{X(t)^2} = \int_\Lambda \rho(\underset{\sim}{\lambda}) \left\langle U_{RF}(t; \underset{\sim}{\lambda}, \underset{\sim}{\theta})^2 \right\rangle_{\underset{\sim}{\theta}} d\underset{\sim}{\lambda}, \text{ etc.} \qquad (2.28)$$

Accordingly, in our subsequent development of the statistics of these nongaussian processes in Sections 3-5 following, we consider X and U to be observed after the aperture, RF, and IF filters. We shall also limit ourselves, by way of illustration, to the usual cases of small apertures, uniform drives, and narrow-band sources, so that (2.18) represents the generic waveform, now with $U = U_{ARI}$, when we need to include the aperture, RF, and IF filter effects.

3. THE CHARACTERISTIC FUNCTION $F_1(i\xi, t)$

Starting with the narrow-band waveform (2.18), (and remembering the comments after (2.21)), let us write this waveform in envelope and phase form:

$$U(t) = B_o(t, \underset{\sim}{\lambda} | \underset{\sim}{\theta}) \cos \mu \Psi(t, \underset{\sim}{\lambda} | \underset{\sim}{\theta}), \tag{3.1}$$

where specifically it is now found that

$$\left.\begin{aligned} B_o(t, \lambda | \underset{\sim}{\theta}) &= \frac{|\mathcal{A}_R(\lambda, f_o) \mathcal{A}_I(\lambda, f_o)|}{(4 \pi c \lambda)} A_I(t - \lambda - \hat{\epsilon} | \underset{\sim}{\theta}_s); \\ &\qquad\qquad |t-\lambda| \in T_s: \text{ signal "on"}; \\ &= 0, \ |t - \lambda| \notin T_s: \text{ signal "off"} \\ \Psi(t, \underset{\sim}{\lambda} | \underset{\sim}{\theta}) &= \omega_o(t - \lambda - \hat{\epsilon}) - \Phi_I(t - \lambda - \hat{\epsilon} | \underset{\sim}{\theta}_s) - \varphi_I(\underset{\sim}{\lambda}) - \varphi_R(\underset{\sim}{\lambda}) \end{aligned}\right\} \tag{3.2}$$

and $\mu = 1 + \beta$, with β a sum of doppler velocities, cf. (2.9), and A_{0I}, cf. (2.10), the real envelope of the emitted signal. The beam patterns $\mathcal{A}_R$, $\mathcal{A}_I$ are generally complex, e.g., $\mathcal{A}_R = |\mathcal{A}_R| e^{-i\varphi_R}$, etc.; and, in more detail $\mathcal{A}_R(\underset{\sim}{\lambda}, f_o) = \mathcal{A}_R(-\hat{\underset{\sim}{i}}_R f_o / c, f_o)$, etc., cf. (2.18), and (2.9) for $\hat{\underset{\sim}{i}}_R$. The quantity $\hat{\epsilon}$ is an epoch, representing (vis-à-vis the receiver's time scale) the instant at which the (typical $j\underline{th}$) source emits: $\hat{\epsilon}$ is a random variable over the ensemble of possible source configurations, as may be the carrier frequency f_o, and any other parameters of these deterministic signal emissions, represented by $\underset{\sim}{\theta}$, in (3.1). The source emissions are individually "on" only a finite time (T_s), i.e., have only a finite duration, characteristic for example of automobile emissions, messages, etc., and generally for the "intelligent" noise considered here.

The characteristic function (2.2) can now be written compactly as

$$F_1(i\xi, t)_P = \exp \{ A H_1(i\xi, t) \}, \tag{3.3}$$

where

$$H_1(i\xi, t) = \int_\Lambda \frac{\rho(\lambda)}{A} \left\langle e^{i\xi B_o(t,\lambda|\theta)\cos\mu\Psi(t,\lambda|\theta)} - 1 \right\rangle_{\theta = \hat{\epsilon},\ \mu,\ \text{signal},} d\lambda \tag{3.4a}$$

$$\equiv \left\langle e^{i\xi B_o \cos\mu\Psi} - 1 \right\rangle_{\hat{\epsilon}, \mu, \theta_{sig} | \lambda} \tag{3.4b}$$

The averages $\langle \ \rangle_{\hat{\epsilon}, \mu, \theta_{sig} | \lambda}$ are

$$\langle \ \rangle \equiv \int \cdots \int w_1(\hat{\epsilon}) w_1(\mu) w_1(\theta_{sig}) w_1(\lambda) [\ \ ]\, d\hat{\epsilon} \cdots d\lambda, \tag{3.5}$$

where the geometric probability density $w_1(\lambda)$ is

$$w_1(\lambda) \equiv \rho(\lambda)/A; \tag{3.5a}$$

and

$$A \equiv \int_\Lambda \rho(\lambda)\, d\lambda \ \ (>0). \tag{3.5b}$$

The quantity A is called the Impulsive Index and is defined at the appropriate point in the receiver: here at the output of the combination aperture-RF-IF stages. Specifically, A can be shown to be equal to $\nu_T \bar{T}_s$ [cf. Middleton, 1973c], where ν_T is the average rate of "signal" generation, and $\bar{T}_s$ is the mean duration of a typical interfering "signal". The Impulsive Index measures the amount of temporal overlap among the waveforms of the interfering sources.

When the spectral width of the receiver is greater than that of the generic interfering signal, then the overlap is the same as that in the receiver. However, for spectrally narrow receivers the smoothing action of the aperture-RF-IF stages spreads the input signals in time and thus increases the amount of temporal overlap vis-à-vis that of the input. The amount of temporal overlap (as measured by A) is a critical parameter in determining the character of the p.d.f.'s and p.d.'s of X(t). Very large values of A imply a high density of overlapping waveforms at any given instant, and, as noted before (asymptotically) normal statistics for X. For small A ($< 0(10^0)$), on the other hand, there is comparatively little overlapping, so that the composite contributions of only a few sources are significant at the given instant, leading to a highly "impulsive" or discretized character for the resultant waveform, now dominated by the basic (filtered) waveform, U.

In this Report we shall confine our attention to those important cases where the bandwidths of the interfering signals are comparable to or less than that of the composite aperture-RF-IF stages of the receiver. This is the usual situation when the interference consists of other man-made communications of comparable spectral width in a multi-link environment. Then the maximum signal duration, T_s, is effectively finite, allowing us to write $A\langle J_o - 1\rangle_{\hat{\epsilon},\dots}$ as $A\langle J_o\rangle_{\hat{\epsilon},\dots} - A$: thus, in this case $\langle 1\rangle_{\hat{\epsilon}}$ is unity [cf. Middleton, 1973c]. On the other hand, with highly impulsive interference, such as automobile ignition, or atmospheric noise, where the receiver is now shock-excited and thus generates waveforms which are simply the (temporal) response of the receivers weighting functions, h_{ARI}, these (exponentially decaying) waveforms are essentially infinite in duration, e.g., $T_s \to \infty$ (with, of course, $\overline{T}_s < \infty$). Then it is easy to show that $\langle 1 \rangle_{\hat{\epsilon},\ldots}$ is infinite: there are, as expected no "gaps in time", although the impulsive index $A\ (= \nu_T \overline{T}_s)$ remains finite. The analytic development now requires that we consider $A\langle (J_o - 1)\rangle_{\hat{\epsilon},\ldots}$ as a whole, cf. Middleton (1973c). [We shall consider these cases in Middleton (1973c) and subsequent reports.]

The central technical problem in these nongaussian cases is to reduce F_1 to a form which can yield analytically manageable probability densities, e.g., W_1. We begin with the average with respect to emission epoch $\hat{\epsilon}$, and write (cf. remarks above)

$$\left\langle e^{i\xi B_o \cos\mu\psi} \right\rangle_{\hat{\epsilon}} = \sum_{m=0}^{\infty} \epsilon_m\, i^m \left\langle J_m(\xi B_o)\, \overline{\cos m\mu\, \Psi((\hat{\epsilon}))} \right\rangle_{\hat{\epsilon}} \tag{3.6a}$$

$$= \left\langle J_o(\xi B_o) \right\rangle_{\hat{\epsilon}}\ , \tag{3.6b}$$

since $\hat{\epsilon}$ is assumed uniformly distributed over a typical carrier cycle $(\sim f_o^{-1})$ and B_o is any slowly varying function of $\hat{\epsilon}$ vis-à-vis $\cos m\mu\omega_o(t-\lambda-\hat{\epsilon})$. Thus, (3.4a, b) become

$$H_1(i\xi;t) = \left\langle J_o(\xi B_o[t,\lambda\,|\,\theta]) - 1 \right\rangle_{\hat{\epsilon},\underset{\sim}{\theta}_{sig},|\underset{\sim}{\lambda},} \tag{3.7}$$

<u>which for these small dopplers</u> [$v \le O(1000$ mph); $\therefore\ \epsilon = \frac{2v}{c} \lesssim O(10^{-6})$, with $c = 3\cdot 10^8$ m/sec] <u>is essentially an exact result.</u>

Our next step requires further physical insight and some ingenuity.

We begin by observing that a canonical reduction of the characteristic function (c.f.) is effected by the observation that in accordance with the statistical nature of "impulsive" noise (small values of A), the "tails" (i.e., $|X| \to \infty$) of the p.d.f. $W_1(X)$ fall off <u>less</u> rapidly than is the case for gaussian noise. This means that for large amplitudes the behavior of the c.f. in the finite (non-zero) neighborhood of $\xi = 0$ is critical: a development of the c.f., F_1, with a <u>more rapid</u> fall-off near $\xi = 0$ is required.* Conversely, for small amplitudes ($|X| \to 0$), or equivalently, large $|\xi|$ ($\to \infty$), we see that $J_o \to 0$, and that consequently, $F_1 \to \exp(-A)$. The corresponding Fourier transform gives $W_1(X) \to e^{-A}\delta(X-0)$. This is the expected phenomenon of "gaps-in-time", typical of this class of interference: a finite, non-zero probability of zero amplitudes.

At this point we are tempted to use the direct expansion of $\langle J_o(B_o\xi)\rangle$:

$$\langle J_o(B_o\xi)\rangle = 1 - \langle B_o^2\rangle \frac{\xi^2}{4} + \xi^4\langle B_o^4\rangle / 2^4 2!\, 2! + \dots \quad (3.8)$$

and so obtain

$$\exp\left[-A+A\langle J_o(B_o\xi)\rangle\right] = e^{-A\xi^2\langle B_o^2\rangle/4}\left\{1 + \frac{A\xi^4\langle B_o^4\rangle}{2^4(2!)^2} + \dots\right\}. \quad (3.9)$$

This, however, is the well-known Edgeworth expansion, asymptotically appropriate as the Impulsive Index $A \to \infty$, yielding the expected normal statistics for (very) large A, and is certainly not valid for small A ($< 10^0$, say). Note, too, the now complete absence of the zero amplitude probability, e^{-A}: there are no "gaps-in'time" when A is large, since the number of overlapping waveforms is now great enough to insure that all gaps of finite (non-zero) duration are nonexistent, i.e., have zero probability.

*Equivalently, a more rapid fall-off to zero of the exponent $\langle J_o(B_o\xi)\rangle$ about $\xi=0$.

Accordingly, we follow our observation above and seek a c.f. with a more rapid fall-off to zero in the finite neighborhood of $\xi=0$ than the gauss c.f. (leading term of (3.9)). The desired form of the exponential term $J_o(B_o\xi)$ yielding correct behavior of the p.d.f. for <u>both</u> large and small values of the amplitude is found by approximating $\langle J_o \rangle$ with a steepest-descent term, $\langle J_o \rangle \doteq 1 - \langle B_o^2 \rangle \xi^2/4 \doteq \exp[-\langle B_o^2 \rangle \xi^2/4]$. Thus, we write <u>exactly</u>

$$\langle J_o(\xi B_o) \rangle = e^{-\xi^2 \langle B_o^2 \rangle /4} \left\langle J_o(\xi B_o) e^{+\xi^2 \langle B_o^2 \rangle /4} \right\rangle . \qquad (3.10)$$

With the help of (13.107B) of Middleton (1960), one can show that

$$\langle J_o(\xi B_o) \rangle e^{\xi^2 \langle B_o^2 \rangle /4} = 1 + \sum_{\ell=2}^{\infty} \frac{\langle B_o^2 \rangle^{\ell} \xi^{2\ell}}{\ell !\, 2^{2\ell}} \left\langle {}_1F_1(-\ell; 1;\ B_o^2/\langle B_o^2 \rangle) \right\rangle, \qquad (3.11)$$

where ${}_1F_1$ is a confluent hypergeometric function, terminating after $\ell+1$ terms. It is convenient now to introduce a set of coefficients

$$C_{2\ell} \equiv \ell !\, (-1)^{\ell} \left\langle {}_1F_1(-\ell; 1;\ B_o^2/\langle B_o^2 \rangle) \right\rangle \qquad (3.12)$$

which contain the 2ℓ, $2\ell-2$, . . ., moments of the filtered envelope B_o. Specifically, we have

$$C_4 = [\langle B_o^4 \rangle - 2\langle B_o^2 \rangle^2] / \langle B_o^2 \rangle^2 \qquad (3.12a)$$

$$C_6 = [\langle B_o^6 \rangle - 9\langle B_o^4 \rangle \langle B_o^2 \rangle + 12\langle B_o^2 \rangle^3] / \langle B_o^2 \rangle^3 \qquad (3.12b)$$

$$C_8 = [\langle B_o^8 \rangle - 16\langle B_o^6 \rangle \langle B_o^2 \rangle + 72\langle B_o^4 \rangle \langle B_o^2 \rangle^2 - 72\langle B_o^2 \rangle^4] / \langle B_o^2 \rangle^4, \text{ etc.}$$

$$(3.12c)$$

Thus, $H(i\xi, t)$, (3.7) becomes, still exactly,

$$H(i\xi, t) = e^{-\xi^2\langle B_o^2\rangle/4}\left[1 + \sum_{\ell=2}^{\infty} \frac{C_{2\ell}(-1)^{\ell}\xi^{2\ell}\langle B_o^2\rangle^{\ell}}{2^{2\ell}(\ell!)^2}\right] - 1. \quad (3.13)$$

The coefficients $C_{2\ell}$, involving the higher even moments of the envelope B, can be progressively critical in determining the shape of the distribution (of X) at the higher amplitudes, e.g., for those "rare" events characteristic of the excursions of these impulsive waveforms much away from zero amplitude.

The final step in the reduction of the c.f. (3.3) to the desired manageable form, particularly for small values of A, is now a direct expansion of (3.3) with the help of (3.13), viz:

$$F_1(i\xi)_P = \exp[-A + A\langle J_o(B_o\xi)\rangle]$$

$$\doteq e^{-A}\sum_{m=0}^{\infty}\frac{A^m}{m!} e^{-m\xi^2\langle B_o^2\rangle/4}\left[1 + \right.$$

$$\left. + \frac{A\langle B_o^2\rangle^2\xi^4 C_4 e^{-\xi^2\langle B_o^2\rangle/4}}{4^3} + \frac{A\langle B_o^2\rangle^3\xi^6 C_6 e^{-\xi^2\langle B_o^2\rangle/4}}{4^3\,(3!)^2} + \cdots\right]. \quad (3.14)$$

The resulting p.d.f., and p.d., may then be evaluated term by term, as we shall do in Section 5.

Finally, as remarked upon at the end of section 2.1, a more general model of the man-made noise environment includes an (additive) independent gaussian process (the limit of a high density poisson process representing the contribution of the non-resolveable background sources). For this we have

$$F_1(i\xi, t)_{P+G} = F_1(i\xi, t)_P \cdot F_1(i\xi, t)_G \, , \tag{3.15}$$

where

$$F_1(i\xi, t)_G = e^{-\xi^2 \sigma_G^2/2} \tag{3.15a}$$

cf. (2.6b), and $\sigma_G^2 = \overline{X_G^2}$, $(\overline{X_G} = 0)$, cf. (2.24) or (2.28), with an appropriate specification of $\rho \sim \sigma_S(\underset{\sim}{\lambda})$, cf.(2.3). Combining (3.15), (3.15a), (3.14), we get the desired extension,

$$F_1(i\xi)_{P+G} = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-c_m^2 \xi^2/2} \left[1 + \frac{A\langle B_o^2 \rangle^2 \xi^4 C_4}{4^3} e^{-\xi^2 \langle B_o^2 \rangle/4} + \frac{A\langle B_o^2 \rangle^3 \xi^6 C_6}{4^3 (3!)^2} e^{-\xi^2 \langle B_o^2 \rangle/4} + \ldots \right] , \tag{3.16}$$

where

$$c_m^2 = m\langle B_o^2 \rangle/2 + \sigma_G^2 \, , \tag{3.17}$$

remembering that the moments $\langle B_o^2 \rangle$, $\langle B_o^4 \rangle$ are usually functions of time (t), cf. comments following eq (3.5b). With (3.14) and (3.16) we can proceed to the calculation of the respective first-order densities, W_1, in Sec. 5 following.

4. MOMENTS

The lower order moments are of particular interest. These are most easily obtained by differentiating the characteristic functions (3.14), (3.16), after expressing the exponents therein as a power series in ξ^2. We start with the poisson case and the exact expression (3.13) in (3.7), expanding the c.f. directly in a power series and identifying the desired moments from the relation

$$F_1(i\xi, t)_P = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \xi^{2k} \overline{X(t)^{2k}} , \tag{4.1}$$

cf. eq (1.31), Middleton (1960), where now all odd moments are seen to vanish (since (3.13) is even in ξ). We have

$$F_1(i\xi, t)_P = \exp A\left[\langle J_o(B_o\xi)\rangle - 1\right]$$

$$= \exp\left\{ \frac{-A\xi^2\langle B_o^2\rangle}{4} + \frac{\xi^4 A\langle B_o^4\rangle}{4^3} - \frac{\xi^6 A\langle B_o^6\rangle}{2^6\cdot 3!^2} + \ldots \right\} \tag{4.2a}$$

$$= 1 - \frac{\xi^2}{2!}\left[\frac{A\langle B_o^2\rangle}{2}\right] + \frac{\xi^4}{4!}\left[4!\frac{A\langle B_o^4\rangle}{4^3} + 4!\,\frac{A^2\langle B_o^2\rangle^2}{2\cdot 4^2}\right]$$

$$- \frac{\xi^6}{6!}\left[\frac{A\langle B_o^6\rangle 6!}{2^6\cdot(3!)^2} + \frac{A\langle B_o^2\rangle\langle B_o^4\rangle 6!}{4^4} + \frac{A^3\langle B_o^2\rangle^3\, 6!}{4^3\cdot 3!}\right] + \ldots \tag{4.2b}$$

Using the definition

$$\Omega_{2k} \equiv \frac{A\langle B_o^{2k}\rangle}{2^k} , \quad k \geq 1 , \tag{4.3}$$

we obtain for (4. 2b)

$$(F_1)_P = 1 - \frac{\xi^2}{2!}\Omega_2 + \frac{\xi^4}{4!}\left(\frac{3\Omega_4}{2} + 3\Omega_2^2\right) - \frac{\xi^6}{6!}\left(\frac{5\Omega_6}{2} + \frac{45\Omega_2\,\Omega_4}{2} + 15\Omega_2^3\right) + \dots \quad (4.4)$$

from which and (4. 1) we can write at once

$$\overline{X^2} = \Omega_2; \quad \overline{X^4} = \frac{3\Omega_4}{2} + 3\Omega_2^2; \quad \overline{X^6} = \frac{5\Omega_6}{2} + \frac{45}{2}\Omega_2\Omega_4 + 15\Omega_2^3, \text{ etc.} \quad (4.5)$$

[Since $\overline{X} = 0$; (and $\overline{X^{2k+1}} = 0$), Ω_2 is that physically important quantity, the variance and mean intensity of the poisson process X(t).]

A similar calculation for the mixed gauss-poisson process, cf. (3. 15), yields

$$F_1(i\xi, t)_{P+G} = \exp\left[-\frac{\xi^2}{2}(\Omega_2 + \sigma_G^2) + \frac{\xi^4}{4^2}\Omega_4 - \frac{\xi^6\Omega_6}{2^3(3!)^2} + \dots\right], \quad (4.6)$$

cf. (4. 2a), so that writing

$$\Omega_2' \equiv \Omega_2 + \sigma_G^2, \text{ and } \Gamma' \equiv \sigma_G^2 / \Omega_2, \quad (4.7)$$

we see at once that (4. 4), (4. 5) apply here if we replace Ω_2 by Ω_2' therein. Observe with the aid of (3. 5a) in (4. 3) that as the poisson process becomes more "dense", i. e., less impulsive and more gaussian ($A \to \infty$), we have as expected

$$(\overline{X^2} \to \Omega_2); \quad \overline{X^4} \to 3\Omega_2^2; \quad \overline{X^6} \to 15\Omega_2^3, \quad (4.8)$$

cf. eq (7. 7) (Middleton, 1960), for the higher moments in terms of the variance. In fact, and for all A, we can write (4. 5) as

$$\overline{X^2}/\Omega_2 = 1; \quad \frac{\overline{X^4}}{(\overline{X^2})^2} = 3\left(1 + \frac{\langle B_o^4\rangle}{2A\langle B_o^2\rangle^2}\right);$$

$$\frac{\overline{X^6}}{(\overline{X^2})^3} = 15\left(1 + \frac{3\langle B_o^4\rangle}{2A\langle B_o^2\rangle} + \frac{1}{6}\frac{\langle B_o^6\rangle}{A^2\langle B_o^2\rangle^3}\right), \quad (4.9)$$

which shows the explicit dependence on the impulsive index A. For the mixed process, (4.9) is modified to

$$\overline{X^2}/\Omega_2' = 1; \qquad \frac{\overline{X^4}}{(\overline{X^2})^2} = 3\left(1 + \frac{\langle B_o^4\rangle}{2A[\langle B_o^2\rangle + 2\sigma_G^2/A]^2}\right)$$

$$\frac{\overline{X^6}}{(\overline{X^2})^3} = 15\left(1 + \frac{3\langle B_o^4\rangle}{2A[\langle B_o^2\rangle + 2\sigma_G^2/A]^2} + \frac{\langle B_o^6\rangle}{6A^2[\langle B_o^2\rangle + 2\sigma_G^2/A]^3}\right)$$

(4.10)

Finally, in the approximate situation where we retain only terms $O(\xi^2)$ in the coefficient of $\exp(-\xi^2\langle B_o^2\rangle/4)$ in (3.13), so that the approximate c.f. is

$$F_1(i\xi, t)_P \doteq \exp(Ae^{-\xi^2\langle B_o^2\rangle/4} - A), \quad = \exp\left\{A(\exp[-\xi^2\Omega_2/2A] - 1\right\},$$

$$= \exp\left[-\xi^2\frac{\Omega_2}{2} + \xi^4\Omega_2^2/2^3A - \xi^6\Omega_2^3/3!2^3A^7 + \cdots\right]. \quad (4.11)$$

we get in comparison with $\Omega_2 \to \Omega_2$, $\Omega_4 \to 2\Omega_2^2/A$, $\Omega_6 \to 6\Omega_2^3/A^2$, so that

$$\overline{X^2} = \Omega_2\,; \quad \overline{X^4} \doteq 3\Omega_2^2[1 + \frac{1}{A}]\,; \quad \overline{X^6} \doteq 15\Omega_2^3[1 + \frac{3}{A} + \frac{1}{A^2}]. \quad (4.12)$$

For the mixed process, we have $\Omega_2 \to \Omega_2' = \Omega_2 + \sigma_G^2$, eq (4.7), again, in (4.12), with (4.11) now further modified to

$$F_1(i\xi, t)_{P+G} \doteq \exp[-\xi^2\sigma_G^2/2 + Ae^{-\xi^2\Omega_2/2A} - A], \quad (4.13)$$

which is the general approximation we shall use in some of the following work, when it is reasonable to omit the "correction terms" in C_4, C_6, . . . etc., for amplitudes ($|X|$) not too large vis-à-vis $\sqrt{\Omega_2'}$. Finally, from (3.2) into (4.3), we can write explicitly

$$\Omega_{2k} = \frac{1}{2^k} \int_\Lambda \frac{\rho(\underset{\sim}{\lambda}) \, | a_R a_T |^{2k}}{(4 \pi c \lambda)^{2k}} \left\langle A_o^{2k} (t - \lambda - \epsilon \, | \, \underset{\sim}{\theta}_c) \right\rangle_{\hat{\epsilon}, \underset{\sim}{\theta}_s} d\underset{\sim}{\lambda} \qquad (4.14)$$

for the higher-order moments, cf. (4.4) et seq.

It should be particularly emphasized that because of the explicit physical foundations of our model, the parameters (A, Γ', Ω_2, C_4, C_6 . . .) themselves are explicitly and quantitatively described in terms of the substantative physical quantities, e.g., source density, beam-patterns, propagation conditions, emission waveforms, etc., which specify the noise phenomenon in question. [See eqs (3.5a, b) with (2.22), (2.23), for A; (4.5) for Ω_2; (3.12) for C_4, C_6, etc., with (4.3), (4.14).]

5. FIRST-ORDER PROBABILITY DENSITIES AND DISTRIBUTIONS

We are now ready to apply (2.4) to (3.16) to achieve tractable forms of the first-order probability density and distribution of the instantaneous (received) amplitude X(t) of the man-made interference, when a gaussian background, as well as the "impulsive" component, is included. Writing (cf. (3.17a))

$$c_m^2 \equiv m B_o^2/2 + \sigma_G^2 = m\Omega_2/A + \sigma_G^2 = \Omega_2(m/A + \Gamma');$$
$$\Gamma' \equiv \sigma_G^2/\Omega_2 = \overline{X_G^2}/\overline{X_P^2}, \tag{5.1}$$

cf. (4.7), we obtain at once on inversion of (3.16)

$$W_1(X,t)_{P+G} \simeq e^{-A}\sum_{m=0}^{\infty}\frac{A^m}{m!}\left\{\frac{\phi^{(0)}(X/c_m)}{c_m} + \frac{C_4\,\Omega_2^2\,\phi^{(4)}(X/c_{m+1})}{16A\,c_{m+1}^5} - \frac{C_6\,\Omega_2^3\,\phi^{(6)}(X/c_{m+1})}{288\,A^2 c_{m+1}^7} + \dots\right\}, \tag{5.2}$$

where $\phi^{(\ell)}(z) \equiv (2\pi)^{-1/2}\frac{d^\ell}{dz^\ell}\left(e^{-z^2/2}\right), \quad \ell \geq 0,$ (5.2a)

are the l^{th} derivatives of the standardized normal probability density. [See Appendix 1, Middleton (1960), A.1.1, and ref. 3 therein, for tabulations of $\phi^{(\ell)}(x)$]. The coefficients C_4, C_6, . . . , are given by (3.12a, b), etc. The leading term in (5.2) is explicitly

$$\frac{\phi^{(0)}(X/c_m)}{c_m} = \frac{e^{-x^2/2c_m^2}}{\sqrt{2\pi c_m^2}} \tag{5.3}$$

The distribution of X, (2.5), is now explicitly

$$D_1(X,t)_{P+G} \simeq \int_{-\infty}^{X} W_1(X,t)_{P+G}\,dX = e^{-A}\sum_{m=0}^{\infty}\frac{A^m}{m!}\left\{\frac{1+\text{Ⓗ}(X/c_m\sqrt{2})}{2}\right.$$

$$+\frac{C_4\Omega_2^2}{16Ac_{m+1}^4}\,\phi^{(3)}(X/c_{m+1})$$

$$\left.-\frac{C_6\Omega_2^3}{288\,A^2c_{m+1}^6}\,\phi^{(5)}(X/c_{m+1})+\ldots\right\}. \tag{5.4}$$

Here Ⓗ(z) is the familiar error integral,

$$\text{Ⓗ}(z) \equiv \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\,dt, \tag{5.4a}$$

(Middleton [1960], Appendix A.1.1).

Sometimes it is more convenient to work with the ("false-alarm") probability that X will exceed some threshold X_o, e.g.,

$$P(X \geq X_o)_{P+G} = 1 - D_1(X_o,t)_{P+G}, \tag{5.4b}$$

which is at once from (5.4)

$$P(X \geq X_o)_{P+G} = \int_{X_o}^{\infty} W_1(X,t)_{P+G}\, dX$$

$$\simeq e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left\{ \frac{[1 - \Theta(X_o/c_m\sqrt{2}\,]}{2} \right. \tag{5.5}$$

$$\left. - \frac{C_4 A}{16(m+1+A\Gamma')^2} \phi^{(3)}(X_o/c_m) + \frac{C_6 A}{288(m+1+A\Gamma')^3} \phi^{(5)}(X_o/c_m) \ldots \right\}.$$

For computational purposes, and for discussions generally, we consider the standardized variable

$$z = X/\sqrt{\Omega_2 + \sigma_G^2} = X/\sqrt{\Omega_2(1+\Gamma')}, \tag{5.6}$$

with the jacobian $|dX/dz| = \sqrt{\Omega_2(1+\Gamma')}$, and we now define

$$\sigma_m^2 \equiv c_m^2/\Omega_2(1+\Gamma') = \frac{m+A\Gamma'}{A(1+\Gamma')} \tag{5.7}$$

Applying these to (5.2), (5.4), and (5.5), we get directly

$$W_1(z,t)_{P+G} \simeq e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left\{ \frac{\phi^{(0)}(z/\sigma_m)}{\sigma_m} \right. \tag{5.8}$$

$$\left. + \frac{C_4 \phi^{(4)}(z/a_m)}{16 a_m^5 A(1+\Gamma')^2} - \frac{C_6 \phi^{(6)}(z/a_m)}{288 a_m^7 A^2 (1+\Gamma')^3} + \ldots \right\},$$

with

$$a_m^2 = \sigma_m^2 + 1/A(1+\Gamma') = \frac{m+1+A\Gamma'}{A(1+\Gamma')}, \tag{5.8a}$$

for the probability density.

For the distributions we have

$$D_1(z,t)_{P+G} \simeq e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left\{ \frac{1 + \textcircled{H}(z/\sigma_m\sqrt{2})}{2} \right. \tag{5.9}$$

$$\left. + \frac{C_4}{16} \cdot \frac{\phi^{(3)}(z/a_m)}{a_m^4 A(1+\Gamma')^2} - \frac{C_6}{288} \cdot \frac{\phi^{(5)}(z/a_m)}{a_m^6 A^2(1+\Gamma')^3} + \dots \right\} .$$

For $P(z \geq z_o)$, $z_o \equiv X_o / \sqrt{\Omega_2(1+\Gamma')}$, we find that (5.5) becomes

$$P(z \geq z_o)_{P+G} \simeq e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m!} \left\{ \frac{1 - \textcircled{H}(z_o/\sigma_m\sqrt{2})}{2} \right. \tag{5.10}$$

$$\left. - \frac{C_4}{16} \cdot \frac{\phi^{(3)}(z_o/a_m)}{a_m^4 A(1+\Gamma')^2} + \frac{C_6}{288} \cdot \frac{\phi^{(5)}(z/a_m)}{a_m^6 A^2(1+\Gamma')^3} - \dots \right\}$$

Observe that as $|z| \to \infty$, each term of $w_1(z,t)_{P+G}$ vanishes, as expected, and (with $z_o \to \infty$) the leading terms of D_1, P are respectively 1 and 0, as required. Furthermore, each of the "correction terms" in these asymptotic expansions vanishes individually.

An important special case arises when the gaussian background vanishes, e.g., $\Gamma' = 0$. Now, from (5.1) and (5.7) we have

$$\underline{(\Gamma' = 0)}: \qquad c_m^2 = \Omega_2\, m/A \; ; \quad \sigma_m^2 = m/A \tag{5.11}$$

The results (5.2), (5.4), (5.5), and the corresponding standardized forms (5.8) - (5.10) now become

$$W_1(X,t)_P \simeq e^{-A}\left[\delta(X-0) + \sum_{m=1}^{\infty} \frac{A^m}{m!}\left\{\sqrt{\frac{A}{m\Omega_2}}\ \phi^{(0)}(X\sqrt{A/m\Omega_2})\right.\right.$$

$$+ \frac{C_4 \quad A^{3/2}}{16(m+1)^{5/2}\sqrt{\Omega_2}}\ \phi^{(4)}(X\sqrt{A/(m+1)\Omega_2})$$

$$\left.\left. - \frac{C_6 \quad A^{3/2}}{288\ (m+1)^{7/2}\sqrt{\Omega_2}}\ \phi^{(6)}\ (X\sqrt{A/(m+1)\Omega_2}) + \ldots \right\}\right] \qquad (5.12)$$

and

$$D_1(X,t)_P \simeq e^{-A}\left[\int_{-\infty}^{X} \delta(X-0)dX + \sum_{m=1}^{\infty} \frac{A^m}{m!}\left\{\frac{1 + \text{Ⓗ}[X\sqrt{A/2m\Omega_2}]}{2}\right.\right.$$

$$+ \frac{C_4 A\phi^{(3)}(X\sqrt{A/(m+1)\Omega_2})}{16(m+1)^2}$$

$$\left.\left. - \frac{C_6 A}{288(m+1)^3}\ \phi^{(5)}(X\sqrt{A/(m+1)\Omega_2}) + \ldots \right\}\right]. \qquad (5.13)$$

The probability of exceeding the threshold X_o is now

$$P(X \geq X_o) \simeq e^{-A}\left[\int_{X_o}^{\infty} \delta(X-0)dX + \sum_{m=1}^{\infty} \frac{A^m}{m!}\left\{\frac{1 - \text{Ⓗ}[X_o\sqrt{A/2m\Omega_2}]}{2}\right.\right.$$

$$\left.\left. - \frac{C_4 A}{16(m+1)^2}\ \phi^{(3)}(X_o\sqrt{A/m\Omega_2}) + \frac{C_6 A}{288\ (m+1)^3}\ \phi^{(5)}(X_o\sqrt{A/m\Omega_2}) + \ldots \right\}\right]$$

(5.14)

cf. (5.5). Similarly, we get for the standardized cases

$$w_1(z,t)_P \simeq e^{-A}\left\{\delta(z-0) + \sum_{m=1}^{\infty}\frac{A^m}{m!}\left[\frac{e^{-z^2A/2m}}{\sqrt{2\pi m/A}} + \frac{A^{3/2}C_4\phi^{(4)}(z\sqrt{\frac{A}{m+1}})}{16\,(m+1)^{5/2}} - \frac{C_6A^{3/2}}{288(m+1)^{7/2}}\phi^{(6)}(z\sqrt{\frac{A}{m+1}}) + \ldots\right]\right\} \tag{5.15}$$

and

$$D_1(z,t)_P \simeq e^{-A}\left\{\int_{-\infty}^{z}\delta(z-0)dz + \sum_{m=1}^{\infty}\frac{A^m}{m!}\left[\frac{1+\textcircled{H}[z\sqrt{A/m}]}{2} + \frac{AC_4\phi^{(3)}[z\sqrt{A/m+1}]}{16\,(m+1)^2} - \frac{AC_6\phi^{(5)}[z\sqrt{A/m+1}]}{288\,(m+1)^3} + \ldots\right]\right\} \tag{5.16}$$

with the probability of exceeding the threshold z_o:

$$P(z \geq z_o)_P \simeq e^{-A}\left\{\int_{z_o}^{\infty}\delta(z-0)dz + \sum_{m=1}^{\infty}\frac{A^m}{m!}\left[\frac{1-\textcircled{H}[z_o\sqrt{A/m}]}{2} - \frac{AC_4\phi^{(3)}(z_o\sqrt{\frac{A}{m+1}})}{16(m+1)^2} + \frac{AC_6\phi^{(5)}(z_o\sqrt{\frac{A}{m+1}})}{288\,(m+1)^3} + \ldots\right]\right\}. \tag{5.17}$$

With $A \to \infty$ the correction terms drop out, as can be seen by returning to the c.f. (3.9) and developing the usual Edgeworth series, which yields asymptotically the expected normal forms, corresponding physically to the resulting indefinitely large number of overlapping, independent events (i.e., source emissions). Finally, we remark, in passing, that with non-vanishing mean values, including a possible desired signal, we simply replace X by $X-\overline{X}-S$, with corresponding modifications for z, cf., (5.6).

Figures (5.1) - (5.6) show some typical results for various selected values of the parameters (A, Γ'), and include modifications necessitated by the correction terms (C_4, C_6). The general behavior is as expected: for $\Gamma' = 0$ (no gaussian background) there are

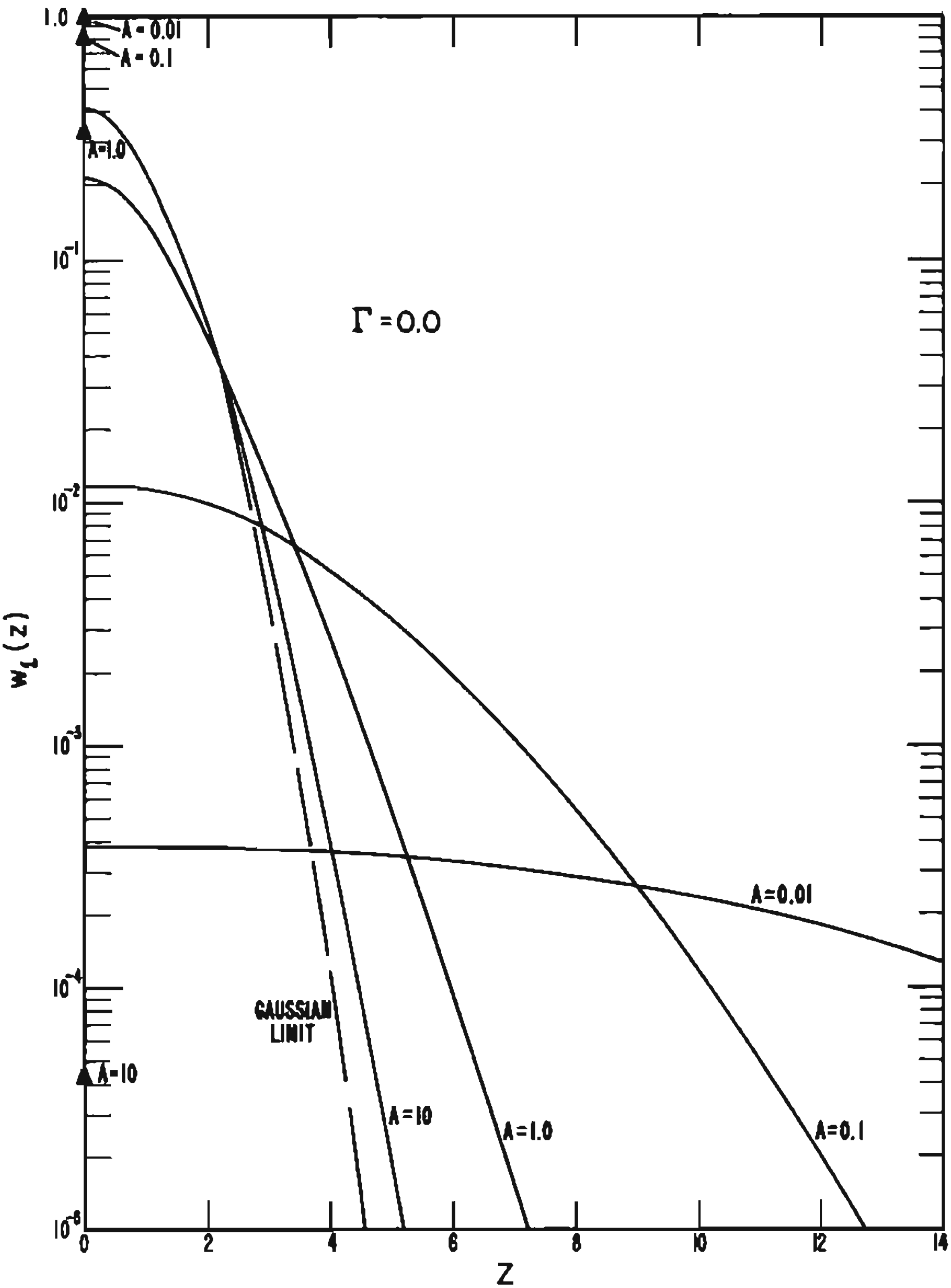

Figure 5.1. Normalized probability density for $\Gamma'=0$ (no gauss background component) [eq (5.15), with correction terms omitted]; ($A = 10^{-2}, 10^{-1}, 10^{0}, 10$).

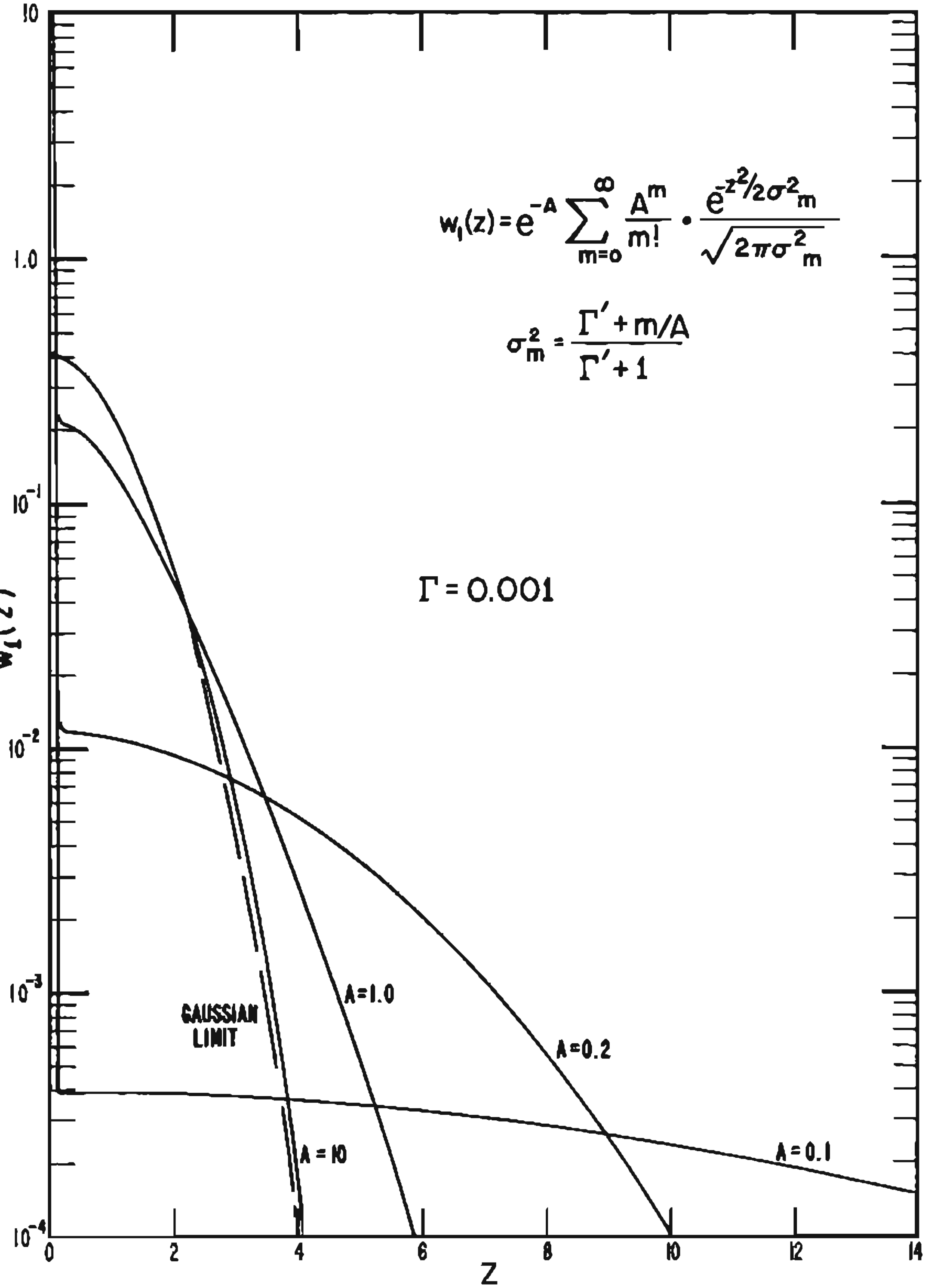

Figure 5.2. Normalized probability density for $\Gamma'=10^{-3}$ (a small gaussian component [eq (5.8) with correction terms omitted]; ($A = 10^{-1}$, $2 \cdot 10^{-1}$, 10^0, 10).

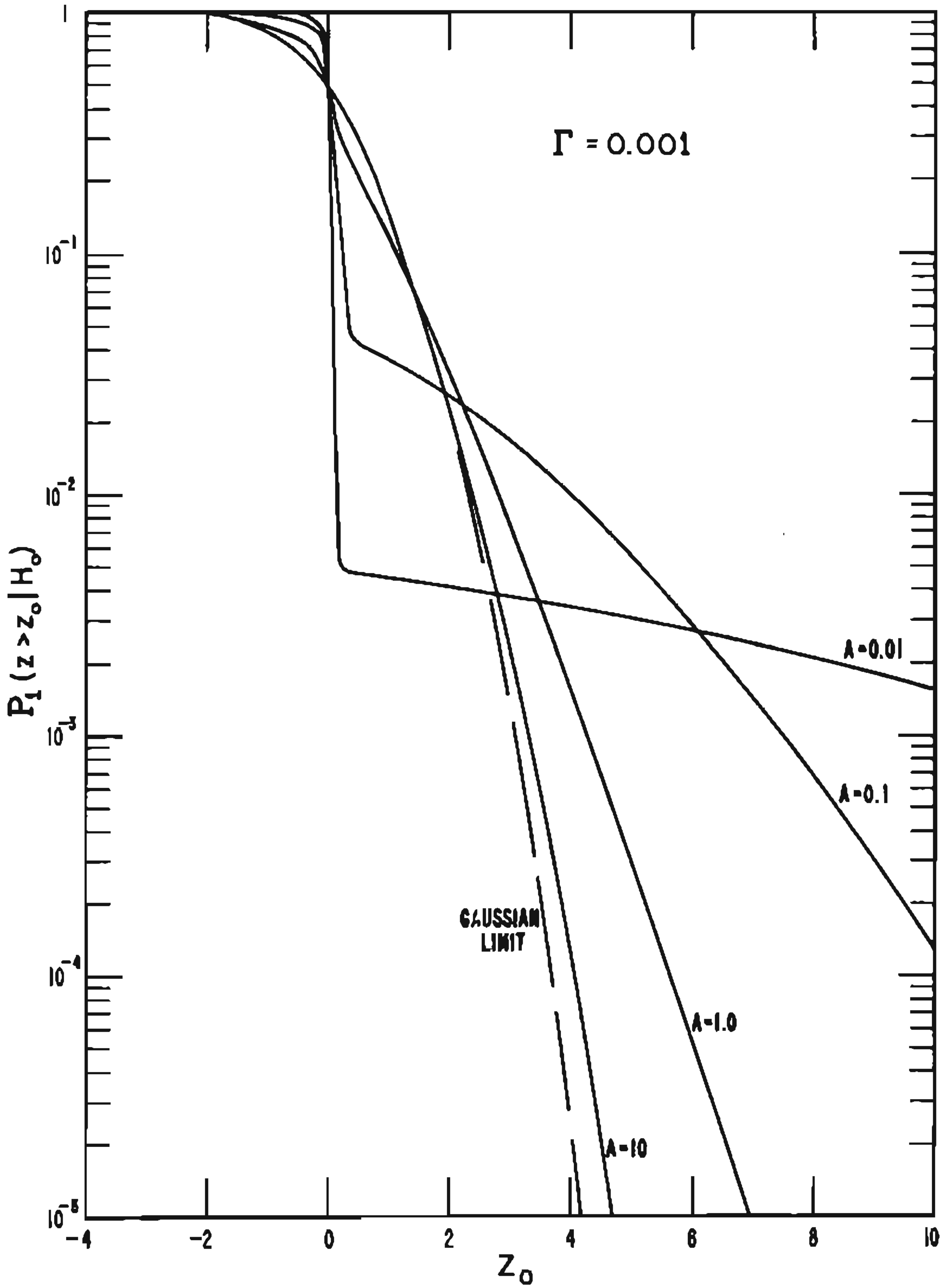

Figure 5.3. Probability of exceeding a threshold z_o, for $\Gamma' = 10^{-3}$ (a small gaussian background component) [eq (5.10), with correction terms omitted] ($A = 10^{-2}$, 10^{-1}, 10^{0}, 10).

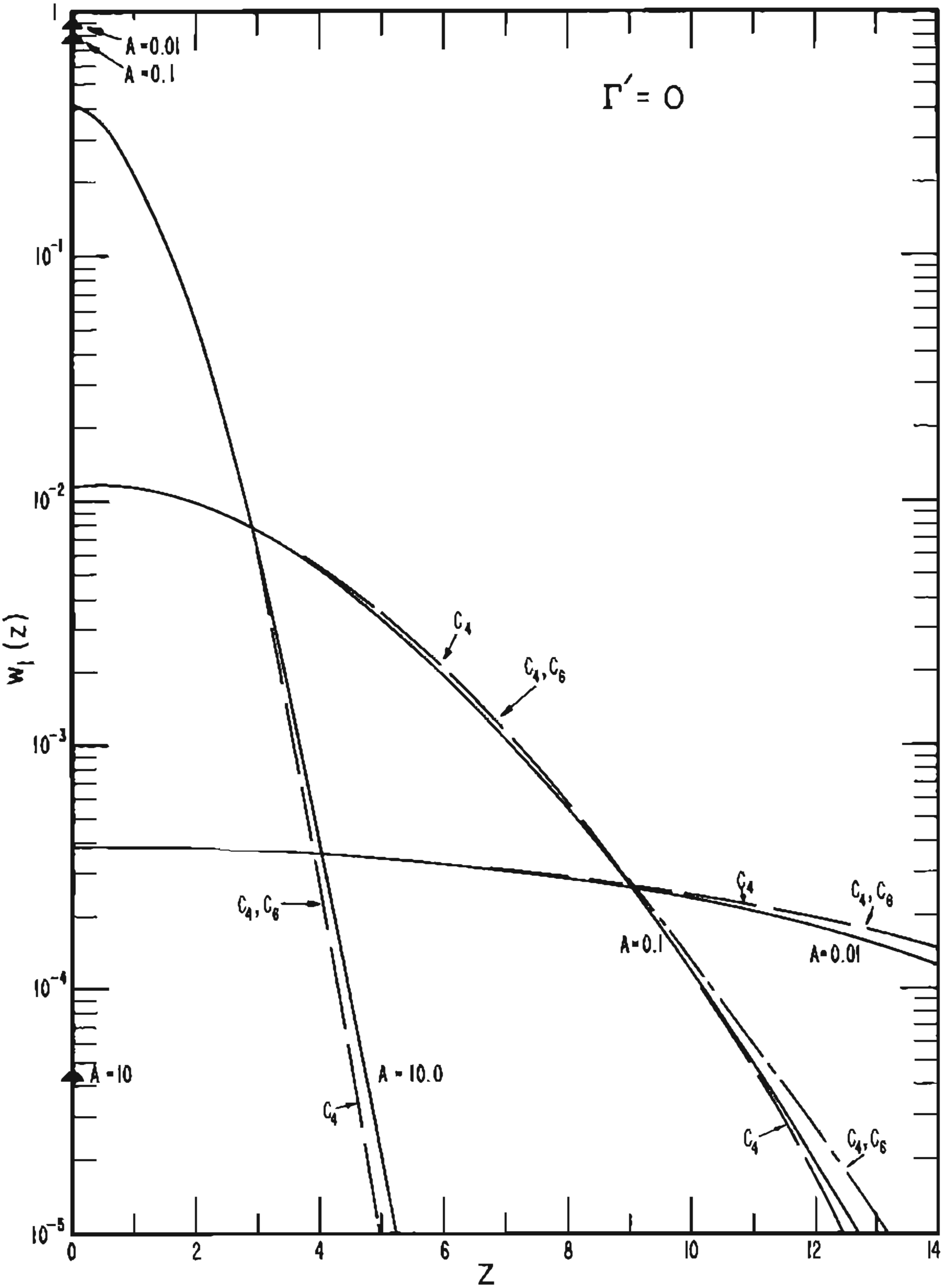

Figure 5.4. Same as Fig. (5.1), including now the correction terms in C_4, and (C_4, C_6). ($A = 10^{-2}, 10^{-1}$, 10 only).

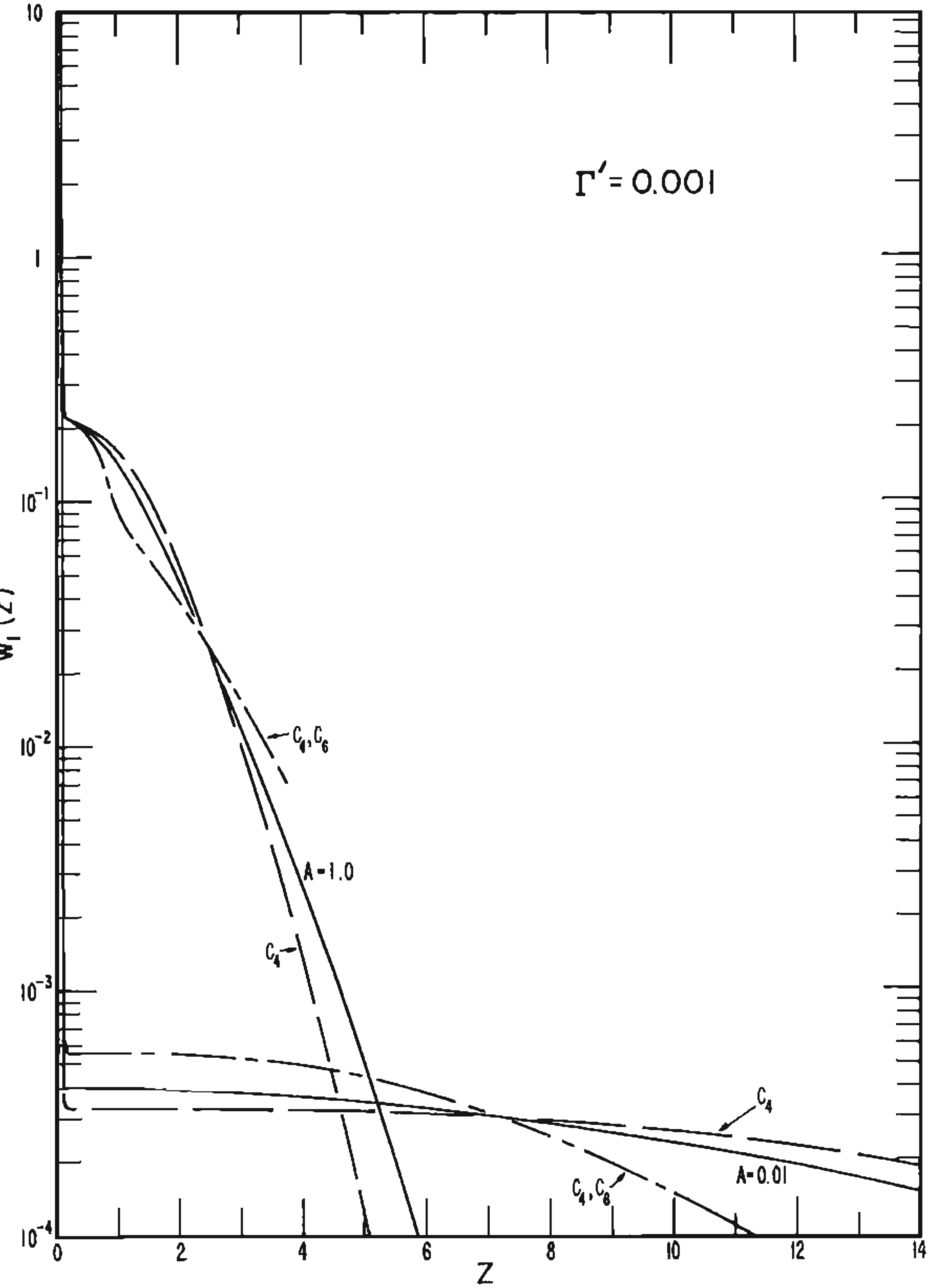

Figure 5.5. Same as Fig. (5.2), with correction terms in C_4, and (C_4, C_6). ($A = 10^{-2}$, 10^{0} only).

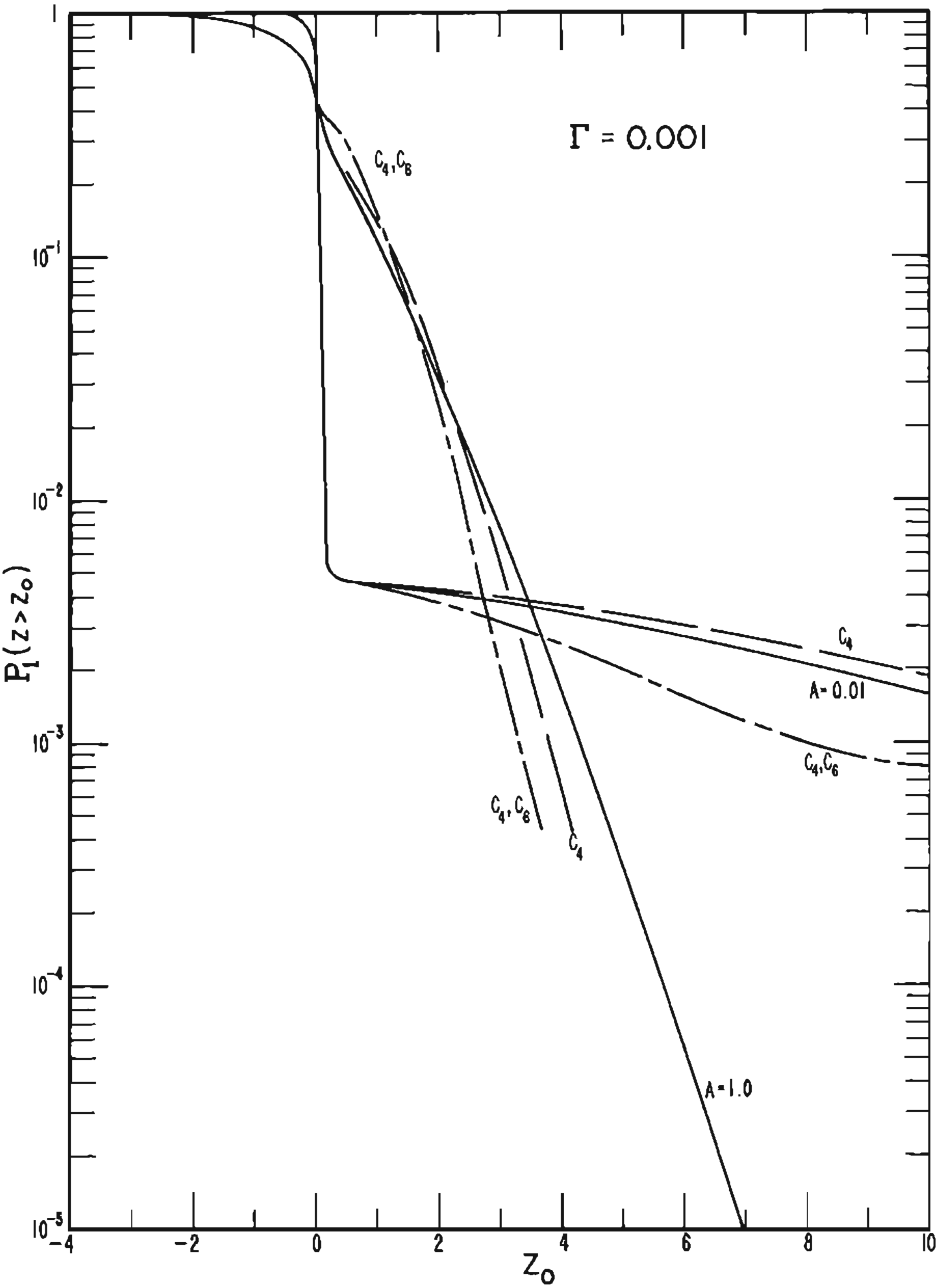

Figure 5.6. Same as Fig. (5.3), with correction terms, in C_4, and in (C_4, C_6). $(A = 10^{-2}, 10^{0})$.

"gaps in time", where there is a non-zero probability [exp (-A)] of zero amplitudes ($A < \infty$) cf., (5.12), (5.15). This occurs here because the "transient" sources comprising the received waveform, X, do not always overlap in time, particularly if this number is small (small Impulsive Index). As A gets larger, the gaps tend to disappear, until with ($A \to \infty$) we obtain an effectively gaussian interference.

Of particular importance is the physically anticipated slower fall-off with amplitude (as $|X| \to \infty$) vis-à-vis the limiting normal distribution in these situations, which occurs for both the purely "impulsive" ($\Gamma' = 0$) and mixed processes ($\Gamma' > 0$). When there is a gaussian background in addition ($\Gamma' > 0$), noticeable distortion of these curves occurs, as the figures indicate, in the regions (dependent on A) where one passes from a gaussianly-dominated background (small amplitudes, i.e., $|X| \approx 0$), to where the "impulsive" or more determinant character of the noise process takes over, toward the larger amplitudes ($|X| \to \infty$), out on the "tails" of the distributions and probability densities, which are more and more dominated by the typical waveforms of the impulsive (poisson) component. This is precisely what we expect physically: note from (3.12) that the coefficients C_4, C_6, etc., depend on progressively higher-order (even) moments of the generic envelope, B_o, of the typical impulsive source. Thus, C_4 is a function of $\langle B_o^4 \rangle$ as well as $\langle B_o^2 \rangle$; C_6 depends on $\langle B_o^6 \rangle$ and $\langle B_o^4 \rangle$, as well as $\langle B_o^2 \rangle$, and so on. We expect that such measures of signal waveform should become increasingly important as we go to regions of rarer and rarer "events", i.e., large amplitudes, where the source waveform is the essential determinant of instantaneous amplitude. Accordingly, for certain ranges of values of X (and A, Γ) the first correction term (C_4) will have an observable influence. At still larger values of X, the (C_6) term next becomes dominating, and so on. These effects are especially evident for small A (<1.0), and Γ'(<0.1), which represents the class of situations often encountered in communication practice, where the man-made (intelligent) interference, from comparatively few sources at any given instant, dominates, and there is a very small gaussian background of unresolveable emission "events". The effects of progressively including these correction terms is illustrated in the figures. A systematic, quantitative study of these effects including the analytical determination of C_4, C_6, etc., is reserved to a subsequent report in this series.

6. PRELIMINARY REMARKS ON PARAMETER MEASUREMENTS

In order to relate theory to experiment, and of perhaps even greater importance, to guide experiment toward the quantitative establishment of suitable models and quantative measures, we need to relate our analytical results (sec. 5) to observation. An essential step in this direction is the measurement of statistical model parameters, A, Ω_2, Γ', and C_4, C_6, etc.

We shall consider only the case here of very weak gaussian backgrounds ($\Gamma' \doteq 0$), which appears practically to be a common situation in the man-made noise environment, where also the impulsive index, A, is $O(1)$ or much less, say $O(0.1$ or $0.01)$. From (4.12) we see that we can estimate A in a straight-forward way, from our estimates of Ω_2 ($=\overline{X^2}$). Thus, we have the estimate

$$\hat{\Omega}_2 \equiv \frac{1}{n} \sum_{j=1}^{n} X_j^2 = \hat{X^2}; \quad \left(\frac{1}{n} \sum_j X_j = 0 \right) \tag{6.1}$$

and also

$$\hat{X^4} \equiv \frac{1}{n} \sum_{j=1}^{n} X_j^4 . \tag{6.2}$$

Accordingly, from (4.12), we have (theoretically)

$$A = \left[\frac{\overline{X^4}}{3\Omega_2^2} - 1 \right]^{-1} , \tag{6.3}$$

so that our estimate of A is

$$\hat{A} \doteq \left[\sum_j X_j^4 / n \Big/ 3 \left(\sum_j X_j^2 / n \right)^2 - 1 \right]^{-1} \; (>0) , \quad (\Gamma' = 0) \tag{6.4}$$

where the ($\doteq$) indicates that Γ' is treated as zero. Since $\langle B_o^2 \rangle = \frac{2}{A} \Omega_2$, cf. (4.3), a reasonable estimate of $\langle B_o^2 \rangle$ is

$$\left\langle B_o^2 \right\rangle = \frac{2}{\hat{A}} \frac{1}{n} \sum_j^n X_j^2 \qquad (6.5)$$

with $\hat{A}$ obtained from (6.4).

To get estimates of C_4, C_6, etc., ($\Gamma \doteq 0$), we need to rewrite (3.12) with the help of (4.9), (but not using (4.12): we must remember that $\Gamma' > 0$ actually). Thus, we use first the theoretical forms to get

$$\left\langle B_o^2 \right\rangle = \frac{2\Omega_2}{A} = \frac{2\overline{X^2}}{A} \qquad (6.6a)$$

$$\left\langle B_o^4 \right\rangle = \left(\frac{\overline{X^4}}{3\overline{X^2}^2} - 1 \right) \cdot \frac{8\overline{X^2}^2}{A} = \frac{8}{3} \frac{\overline{X^4}}{A} - \frac{8\overline{X^2}^2}{A} \qquad (6.6b)$$

$$\left\langle B_o^6 \right\rangle = \left(\frac{16}{5} \overline{X^6} - 48\, \overline{X^4}\, \overline{X^2} + 96\, \overline{X^2}^3 \right) A^{-1} \qquad (6.6c)$$

Putting these into (3.12) gives finally ($\Gamma' \doteq 0$)

$$C_4 = 2A\left(\overline{X^4} / 3\overline{X^2} - 1\right) - 2 \qquad (6.7a)$$

$$C_6 = A^2 \left[\frac{2}{5} \frac{\overline{X^6}}{\overline{X^2}^3} - \frac{6\overline{X^4}}{\overline{X^3}^2} + 12 \right] - 18A \left(\frac{\overline{X^4}}{3\overline{X^2}^2} - 1 \right) + 12. \qquad (6.7b)$$

Now we replace $\overline{X^2}$, $\overline{X^4}$, $\overline{X^6}$ by the experimental estimates, $\left\langle X^{2k} \right\rangle_{expt} = \sum_j X_j^{2k}/n$, in (6.7) and A by $\hat{A}$, (6.4). We thus obtain experimental values $\hat{C}_4$, $\hat{C}_6$, . . . of the correction parameters C_4, C_6, etc., based on our model.

Using these to estimate A, C_4, C_6, etc., in our various results (sec. 5) offers one form of comparison of theory with experiment. Another, somewhat more refined approach, is to employ the parameters as estimated above, to locate the neighborhood of values of A, C_4, C_6, ($\Gamma' \doteq 0$), etc., and then, by computation, find those that actually most closely fit the data. We shall, however, reserve to a later report these and other questions which arise in relating theory and experiment, including such topics as "goodness of fit", "best" estimates, sampling statistics, etc.

7. CONCLUSIONS; NEXT STEPS

In the preceeding sections we have developed a basic first-order mathematical model, including a gaussian background component, of man-made interference, where the bandwidth of this interference is comparable to or less than that of the receiver's input stages. Manageable analytical results, exact for the characteristic function and asymptotic for the desired probability densities, have been obtained. In addition to analytical tractability (sec. 5) is the critical fact that the parameters of the distribution are explicitly represented by the physical quantities which underlie the noise phenomenon in question, e.g., geometry, beam-patterns, propagation modes, doppler, source waveform, density of souces, etc. (secs. 2-4). Moreover, our model for this class of man-made noise is canonical, i.e., as long as the interference appears narrow-band at the receiver's first-stages, the form of the results is independent of the particular magnitudes of the physical parameters involved. This feature is specially important, because it allows us to apply the model to many practical situations, since reception is usually a narrow-band process. For these various reasons, we avoid the limitations of ad hoc distributions which are necessarily tailored to fit local and limited data, and where there can be no structural insight to, or derivation of, the postulated parameters. Our model is also intended as a guide to experimental study of these man-made interference phenomena.

Next steps, to be carried out in succeeding reports are:

I. A corresponding analytical model for the envelope and phase of the received, narrow-band noise. This will include representative calculations, similar to those of figs. (5.1) - (5.6) here, and an initial comparison with appropriate existing experimental data. An analytical study of the "correction" parameters $C_{2k} (k \geq 2)$, which like the other parameters of the model, are derivable from physical considerations, is also planned.

II. A report devoted primarily to comparisons with experiment and including possibly various model extensions, such as mixed types of interfering sources. Experimental estimation of model parameters will be a feature here.

III. Later stages of the investigation will include the effects of desired signals along with the interfering noise, higher-order distributions, modifications and extension of the model to incorporate multipath and scatter effects [Middleton (1972b)], "goodness of fit", and other statistical data analysis techniques [Middleton (1969)], sampling statistics, etc., as well as a continuing relation to the experimental environment. Our ultimate aim is to be able to predict, and quantify, from appropriate and limited measurement, at least the first-order statistical characteristics of man-made noise environments and their relation to the various physical mechanisms producing the the noise. With this we are technically empowered to describe and regulate the noise fields, as well as to evaluate the performance of communication systems embedded in such fields [sec. 1].

This is an extensive program, but one that appears mandatory, in the large, if we are successfully to quantify, predict, and measure these non-gaussian channels, which now rival in practical importance the familiar gaussian channel of previous decades of study. Indeed, we may say here, that predictive, tractable, verified analytical models of the nongaussian channel present one of the major technical challanges of this decade in Communication Theory.

8. GLOSSARY OF PRINCIPAL SYMBOLS

A.	a_m	an expansion coefficient
	A	the Impulsive Index
	$\hat{A}$	Sample value of A
	A_I	Typical source envelope, in receiver
	ARI	aperture - RF-IF
	α_Λ	time-constant of external source
	α_{AIR}	time constant of aperture-RF-IF stages
	$\mathcal{A}_T, \mathcal{A}_R$	beam patterns
	A_T	Source aperture
B.	β	A normalized doppler speed
	B_o	Envelope of interference in receiver
	$\langle B_o^2 \rangle, \langle B_o^4 \rangle, \ldots$	Even moments of received envelope
C.	c.f.	Characteristic function
	ξ	c.f. variable
	c	speed of propagation
	C_ℓ	"correction parameters" of p.d.f., p.d. development
	c_m, c_{m+1}	an expansion coefficient
D.	$D(X;t)$	p.d. (probability distribution)
E.	ϵ	Sum of dopplers (β)
	$\hat{\epsilon}$	receiver epoch (3.2)
F.	f_o	carrier frequency
	${}_1F_1$	confluent hypergeometric function
	$F_1(i\xi, t)$	1st-order characteristic function

	ΔF_{Λ}	bandwidth of external source
	ΔF_{ARI}	bandwidth of aperture-RF-IF stages
	$\mathcal{F}\{\ \}$	fourier transform
	$\mathcal{F}^{-1}\{\ \}$	inverse fourier transform
G.	G	gauss
	Γ'	ratio of gauss to impulsive noise powers
H.	h_R, h_{RI}, h_{ARI}	weighting functions of linear filters
	$H_1(i\xi, t)$	exponent of c.f.
I.	$\underset{\sim}{i}_R, \hat{\underset{\sim}{i}}_R$	unit vector
J.	$J_m(\xi, B_o)$	m<u>th</u> order Bessel function (1st kind)
K.	K_X	a covariance
L.	$\underset{\sim}{\lambda}$	(λ, θ, ϕ); coordinates
	Λ	source domain
M.	μ	1 + doppler
N.	$\hat{\Omega}_2$	sample moment
	Ω_2'	2<u>nd</u> moment (P + G)
O.	ω, ω_o	angular frequencies
	Ω_2, Ω_{2k}	moments distributions

P.	p.d.	probability distribution
	p.d.f.	probability density function
	P	poisson } as subscripts
	P+G	poisson plus gauss } as subscripts
	$P(X>X_o)$	false alarm probability
Q.	$\phi^{(k)}$	kth derivative of error function
R.	$\rho(\underset{\sim}{\lambda})$	process density
	ρ_{ARI}	covariance of ARI
S.	σ_S, σ_V	surface and volume source densities
	s	complex variable
	σ_G^2	mean intensity of gaussian noise
	S_I'	wave form, of signal or driving source
T.	$\underset{\sim}{\theta}_s, \underset{\sim}{\theta}$	set of source parameters
	T_o	path delay
	T_s	signal duration
	Ⓗ	error function
U.	$U(t, \underset{\sim}{\theta})$	basic waveform into or in receiver
V.	V_I	domain occupied by a typical source
W.	$w_1(\underset{\sim}{\lambda})$	p.d.f. of sources
	$W_1(X,t)$	p.d.f. of received wave, X

X.	$\langle X^{2k}\rangle_{exp}$, $\hat{\Omega}_2$, $\hat{A}$	sample moments
	$X(t)$	received interfering wave
	$\langle X^{2k}\rangle$, $\overline{X^{2k}}$	even moments of X
Y.	Y_I	time-varying frequency response
	$\mathcal{Y}_I$	bi-frequency function of source
Z.	z	normalized random variable

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STATISTICAL-PHYSICAL MODELS OF MAN-MADE RADIO NOISE

Part I. First-Order Probability Models of the Instantaneous Amplitude

David Middleton

Abstract

A general statistical-physical model of man-made radio noise processes appearing in the input stages of a typical receiver is described analytically. The first-order statistics of these random processes are developed in detail for narrow-band reception. These include, principally, the first-order probability densities and probability distributions for a) a purely impulsive (poisson) process, and b) an additive mixture of a gauss background noise and impulsive sources. Particular attention is given to the basic waveforms of the emissions, in the course of propagation, including such critical geometric and kinematic factors as the beam patterns of source and receiver, mutual location, doppler, far-field conditions, and the physical density of the sources, which are assumed independent and poisson distributed in space over a domain Λ.

Apart from specific analytic relations, the most important general results are that these first-order distributions are analytically tractable and canonical. They are not so complex as to be unusable in communication theory applications; they incorporate in an explicit way the controlling physical parameters and mechanisms which determine the actual radiated and received processes; and finally, they are formally invariant of the particular source location and density, waveform emission, propagation mode, etc., as long as the received disturbance is narrow-band, at least as it is passed by the initial stages of the typical receiver. The desired first-order distributions are represented by an asymptotic development, with additional terms dependent on the fourth and higher moments of the basic interference waveform, which in turn progressively affect the behavior at the larger amplitudes.

This first report constitutes an initial step in a program to provide workable analytical models of the general nongaussian channel ubiquitous in practical communications applications. Specifically treated here are the important classes of interference with bandwidths comparable to (or less than) the effective aperture-RF-IF bandwidth of the receiver, the common situation in the case of communication interference.

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