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A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM FOR RADIO WAVES IN THE IONOSPHERE

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PREFACE

This report documents the latest version of the three-dimensional ray tracing program originally described in "A Three-Dimensional Ray Tracing Computer Program," by R. M. Jones, ESSA Technical Report IER 17-ITSA 17, and later modified in "Modifications to the Three-Dimensional Ray Tracing Program Described in IER 17-ITSA 17," by R. M. Jones, ESSA Technical Memorandum ERLTM-ITS 134. This report replaces all of the material contained in the above two reports.

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A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM FOR RADIO WAVES IN THE IONOSPHERE

R. Michael Jones[*] and Judith J. Stephenson[**]

This report describes an accurate, versatile FORTRAN computer program for tracing rays through an anisotropic medium whose index of refraction varies continuously in three dimensions. Although developed to calculate the propagation of radio waves in the ionosphere, the program can be easily modified to do other types of ray tracing because of its organization into subroutines.

The program can represent the refractive index by either the Appleton-Hartree or the Sen-Wyller formula, and has several ionospheric models for electron density, perturbations to the electron density (irregularities), the earth's magnetic field, and electron collision frequency.

For each path, the program can calculate group path length, phase path length, absorption, Doppler shift due to a time-varying ionosphere, and geometrical path length. In addition to printing these parameters and the direction of the wave normal at various points along the ray path, the program can plot the projection of the ray path on any vertical plane or on the ground and punch the main characteristics of each ray path on cards.

The documentation includes equations, flow charts, program listings with comments, definitions of program variables, deck set-ups, descriptions of input and output, and a sample case.

Key words: Ray tracing, computer program, radio waves, ionosphere, three-dimensional, Appleton-Hartree formula, Sen-Wyller formula.

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1. INTRODUCTION

This report describes a three-dimensional ray tracing program written in FORTRAN language for the CDC-3800 computer. Copies of the program deck are available from the Institute for Telecommunication Sciences.

Earlier versions of this program have been in use now for over nine years, both by us and by people scattered all over the world. During that time we have improved and modified the program to the extent that we now need to document these changes so that the present program will be easier to use. We have included the input parameter forms that we use to request ray path calculations because they give nearly all the necessary input data and describe the electron density, collision frequency, and magnetic field models.

2. GENERAL DESCRIPTION

This computer program traces the path of radio wave through a user-specified model of the ionosphere when given the transmitter location (longitude, latitude, and height above the ground), the frequency of the wave, the direction of transmission (both elevation and azimuth), the receiver height, and the maximum number of hops wanted.

3. RAY TRACING EQUATIONS

The program calculates ray paths by numerically integrating Hamilton's equations. Lighthill (1965) gives Hamilton's equations in four dimensions (three spatial and one time) for Cartesian coordinates. Haselgrove (1954) gives Hamilton's equations in three dimensions for spherical polar coordinates. Combining the two gives Hamilton's equations in four dimensions in which the three spatial coordinates are spherical polar (see Table 1 for a definition of the symbols):

$$\frac{dr}{d\tau} = \frac{\partial H}{\partial k_r} , \qquad (1)$$

Table 1. List of the More Important Symbols

A	In section 3, absorption in decibels.
B_o	Magnetic induction of earth's magnetic field.
c	Speed of electromagnetic waves in free space.
C	Cosine of the angle of incidence on the ionosphere.
e	Charge of the electron (a negative number).
F(w)	$F(w) = w C_{3/2}(w) + i \frac{5}{2} C_{5/2}(w) = \frac{1}{3/2!} \int_0^\infty \frac{t^{3/2} \exp(-t) dt}{w - it}$. (Davies, 1965, p. 86)
f	Wave frequency.
Δf	Frequency shift of a wave due to a time varying ionosphere (sometimes called Doppler shift).
f_H	Gyro frequency for electrons, $\|e\| B_o/2\pi m$.
f_N	Plasma frequency, $(Ne^2/4\pi^2 \epsilon_o m)^{\frac{1}{2}}$.
$G(w)$	$wF(w)$.
H	Hamiltonian.
k_r, k_θ, k_φ	Components of the propagation vector in the r, θ, φ directions--a vector perpendicular to the wave front having a magnitude $2\pi/\lambda = \omega/v$.
m	Mass of electron.
N	Number of electrons per unit volume.
n	Phase refractive index (in general complex).
n'	Group refractive index (in general complex).
P	Phase path length, phase of wave divided by free space wave number $2\pi/\lambda_o$.
P'	Group path length ct.
r, θ, φ	Coordinates of a point in spherical polar coordinates.
s	Geometric ray path length.
S	Sine of the angle of incidence on the ionosphere.

Table 1. (Continued)

t	Time, travel time of a wave packet.
U	$1 - iZ$ in the Appleton-Hartree formula or $Z/F(1/Z) = 1/G(1/Z)$ in the Sen-Wyller formula.
V_r, V_θ, V_φ	Components of the wave normal direction in the r, θ, and φ directions, normalized so that $V_r^2 + V_\theta^2 + V_\varphi^2 = \text{Real}\,\{n^2\}$.
v	Phase velocity.
X	$\omega_N^2/\omega^2 = f_N^2/f^2 = Ne^2/(\epsilon_o m\omega^2)$.
Y	$\omega_H/\omega = f_H/f$.
Y_L	$Y \cos \psi$.
Y_T	$Y \sin \psi$.
Z	υ/ω or υ_m/ω.
ϵ_o	Electric permittivity of free space.
θ	Colatitude in spherical polar coordinates.
λ	Wavelength.
λ_o	Wavelength in free space.
υ	Electron collision frequency.
υ_m	Mean electron collision frequency.
ρ	Characteristic wave polarization (definition in Table 6).
ρ_L	Longitudinal polarization (definition in Table 6).
τ	Independent variable in Hamilton's equations.
φ	Longitude in spherical polar coordinates.
ψ	Angle between wave normal and $-B_o$.
ω	$2\pi f$, angular wave frequency.
$\Delta\omega$	$2\pi\Delta f$, angular frequency shift.
ω_H	$2\pi f_H = \|e\|B_o/m$, angular gyrofrequency.
ω_N	$2\pi f_N = (Ne^2/\epsilon_o m)^{\frac{1}{2}}$, angular plasma frequency.

$$\frac{d\theta}{d\tau} = \frac{1}{r}\frac{\partial H}{\partial k_\theta} \quad , \tag{2}$$

$$\frac{d\varphi}{d\tau} = \frac{1}{r\,\sin\theta}\frac{\partial H}{\partial k_\varphi} \quad , \tag{3}$$

$$\frac{dt}{d\tau} = -\frac{\partial H}{\partial \omega} \quad , \tag{4}$$

$$\frac{dk_r}{d\tau} = -\frac{\partial H}{\partial r} + k_\theta \frac{d\theta}{d\tau} + k_\varphi \sin\theta \frac{d\varphi}{d\tau} \quad , \tag{5}$$

$$\frac{dk_\theta}{d\tau} = \frac{1}{r}\left(-\frac{\partial H}{\partial \theta} - k_\theta \frac{dr}{d\tau} + k_\varphi\, r\cos\theta \frac{d\varphi}{d\tau}\right) \quad , \tag{6}$$

$$\frac{dk_\varphi}{d\tau} = \frac{1}{r\,\sin\theta}\left(-\frac{\partial H}{\partial \varphi} - k_\varphi \sin\theta \frac{dr}{d\tau} - k_\varphi\, r\cos\theta \frac{d\theta}{d\tau}\right) \quad , \tag{7}$$

$$\frac{d\omega}{d\tau} = \frac{\partial H}{\partial t} \quad , \tag{8}$$

The variables r, θ, φ are the spherical polar coordinates of a point on the ray path; k_r, k_θ, and k_φ are the components of the propagation vector (wave normal direction normalized so that in free space

$$k_r^2 + k_\theta^2 + k_\varphi^2 = \frac{\omega^2}{c^2} \quad , \tag{9}$$

where $\omega = 2\pi f$ is the angular frequency of the wave and c is the speed of propagation of electromagnetic waves in free space); t is time, in (4) it is the propagation time of a wave packet, in (8) it expresses the variation with time of a time varying medium; τ is a parameter whose value depends on the choice of the Hamiltonian H.

For actual calculation, the ray tracing program uses group path $P' = ct$ as the independent variable because the derivatives with respect to P' are independent of the choice of Hamiltonian, allowing the program to switch Hamiltonians in the middle of a path. This choice automatically causes the program to take smaller steps in real path length near reflection where the calculations are more critical. The resulting equations obtained by dividing (1) through (8) by c times (4) are:

$$\frac{dr}{dP'} = -\frac{1}{c}\,\frac{\partial H/\partial k_r}{\partial H/\partial \omega} \quad , \tag{9}$$

$$\frac{d\theta}{dP'} = -\frac{1}{rc}\,\frac{\partial H/\partial k_\theta}{\partial H/\partial \omega} \quad , \tag{10}$$

$$\frac{d\varphi}{dP'} = -\frac{1}{rc\ \sin\theta}\,\frac{\partial H/\partial k_\varphi}{\partial H/\partial \omega} \quad , \tag{11}$$

$$\frac{dk_r}{dP'} = \frac{1}{c}\,\frac{\partial H/\partial r}{\partial H/\partial \omega} + k_\theta \frac{d\theta}{dP'} + k_\varphi \sin\theta \frac{d\varphi}{dP'} \quad , \tag{12}$$

$$\frac{dk_\theta}{dP'} = \frac{1}{r}\left(\frac{1}{c}\,\frac{\partial H/\partial \theta}{\partial H/\partial \omega} - k_\theta \frac{dr}{dP'} + k_\varphi\ r \cos\theta \frac{d\varphi}{dP'}\right) \quad , \tag{13}$$

$$\frac{dk_\varphi}{dP'} = \frac{1}{r\ \sin\theta}\left(\frac{1}{c}\,\frac{\partial H/\partial \varphi}{\partial H/\partial \omega} - k_\varphi \sin\theta \frac{dr}{dP'} - k_\varphi\ r \cos\theta \frac{d\theta}{dP'}\right) \quad , \tag{14}$$

$$\frac{d(\Delta f)}{dP'} = \frac{1}{2\pi}\,\frac{d\Delta\omega}{dP'} = \frac{1}{2\pi}\,\frac{d\omega}{dP'} \tag{15}$$

$$= -\frac{1}{2\pi}\,\frac{\partial H/\partial t}{\partial H/\partial \omega} \quad .$$

Equation (15) for the frequency shift of a wave propagating through a time varying medium follows directly from Hamilton's equations (4) and (8). An alternative derivation is given by Bennett (1967). For large frequency shifts, the frequency shift should be accumulated along the ray path and the shifted frequency used in calculations at each point on the ray path. Equations (1) through (8) imply that all eight dependent variables vary along the path, and that at each point on the path the instantaneous value of all parameters (including frequency) is used in further evaluations of the equations. However, the time variation of the ionosphere due to natural causes (such as solar flares) is so slow that the resulting frequency shifts are small enough (less than one part in 10^5) to have negligible effect on the propagation. For this reason, the program calculates frequency shift to compare with frequency shift measurements, but does not adjust the carrier frequency of the wave used in the propagation calculations.

The first six differential equations (9) through (14) are always integrated. The user can choose whether to have the program integrate (15) to calculate the frequency shift.

There are three other quantities that can be calculated by integration along the ray path. The phase path P (phase divided by the free space wavenumber $2\pi/\lambda_o = \omega/c$) is calculated by integrating

$$\frac{dP}{dP'} = \frac{c}{\omega}\left(k_r \frac{dr}{dP'} + k_\theta\, r \frac{d\theta}{dP'} + k_\varphi\, r \sin\theta \frac{d\varphi}{dP'}\right)$$

$$= -\frac{1}{\omega}\,\frac{k_r \dfrac{\partial H}{\partial k_r} + k_\theta \dfrac{\partial H}{\partial k_\theta} + k_\varphi \dfrac{\partial H}{\partial k_\varphi}}{\partial H/\partial\omega} \quad . \tag{16}$$

If the absorption per wavelength is small (as it must be for this type of ray tracing to be valid), then an approximate formula can be integrated to give the absorption in decibels

$$\frac{dA}{dP'} = -\frac{10}{\log_e 10} \frac{\omega}{c} \frac{\mathrm{imag}\,(\frac{\omega^2}{c^2} n^2)}{k_r^2 + k_\theta^2 + k_\varphi^2} \frac{dP}{dP'}$$

$$= \frac{10}{\log_e 10} \frac{\mathrm{imag}\,(\frac{\omega^2}{c^2} n^2)}{k_r^2 + k_\theta^2 + k_\varphi^2} \frac{k_r \frac{\partial H}{\partial k_r} + k_\theta \frac{\partial H}{\partial k_\theta} + k_\varphi \frac{\partial H}{\partial k_\varphi}}{c\ \partial H/\partial \omega} , \qquad (17)$$

where n is the (complex) phase refractive index. The geometrical path length of the ray can be calculated by integrating

$$\frac{ds}{dP'} = \sqrt{\left(\frac{dr}{dP'}\right)^2 + r^2 \left(\frac{d\theta}{dP'}\right)^2 + r^2 \sin^2\theta \left(\frac{d\varphi}{dP'}\right)^2}$$

$$= -\frac{\sqrt{\left(\frac{\partial H}{\partial k_r}\right)^2 + \left(\frac{\partial H}{\partial k_\theta}\right)^2 + \left(\frac{\partial H}{\partial k_\varphi}\right)^2}}{c\ \partial H/\partial \omega} . \qquad (18)$$

The user can choose to have frequency shift, phase path, absorption, or path length calculated using equations (15), (16), (17), or (18) and printed by setting the appropriate value in the input W array. (W59, W57, W58, W60 in Table 2.)

If the user wants to add differential equations to the program, he can do so by modifying subroutine HAMLTN, which evaluates Hamilton's equations.

The Hamiltonian and its derivatives are calculated by one of the versions of subroutine RINDEX, which also calculates the phase refractive index and its derivatives.

4. CHOOSING AND CALCULATING THE HAMILTONIAN

Because Hamilton's equations guarantee that the Hamiltonian is constant along the ray path and because it is desirable to have the dispersion relation satisfied at each point on the ray path, it is usual to write the dispersion relation in the form H = constant and choose that H as the Hamiltonian. Two problems arise. First, in a lossy medium the dispersion relation is complex, so that the resulting complex Hamiltonian gives ray paths having complex coordinates when used in Hamilton's equations. Second, in some cases some forms of the dispersion relation have computational advantages over others when used as a Hamiltonian.

Allowing the coordinates of the ray path to assume complex values is called ray tracing in complex space (Budden and Jull, 1964; Jones, 1970; Budden and Terry, 1971) which is the extension to three dimensions of the phase integral method (Budden, 1961). Ray tracing in complex space is necessary to calculate the propagation of LF radio waves in the D region of the ionosphere (Jones, 1970), and it may also be needed for some medium frequencies.

However, the effect of losses on the ray path of HF radio waves in the ionosphere is probably small, so that the only effect of losses is to attenuate the signal. For this case, then, it is desirable to find a prescription for calculating ray paths having real coordinates. Several methods exist for doing this, and except for computational difficulties, one is probably as good as another. One should recognize that along the ray path:

(1) the dispersion cannot be exactly satisfied, or

(2) Hamilton's equations cannot be satisfied, or

(3) both of the above.

In our program, we have chosen to keep Hamilton's equations and require only the real part of the dispersion relation to be satisfied, neglecting the imaginary part. Another approach (Suchy, 1972) is to alter Hamilton's equations so that the full complex dispersion relation is still satisfied along a ray path having real coordinates. We are reasonably certain that for any situation in which Suchy's method gives significantly different answers from ours, neither method is valid; ray tracing in complex space or an equivalent method would then be required.

Three choices for the Hamiltonian illustrate the computational difficulties involved. Haselgrove (1954) used the following Hamiltonian

$$H = \frac{c}{\omega} \frac{(k_r^2 + k_\theta^2 + k_\varphi^2)^{\frac{1}{2}}}{\text{real}\,(n)} - 1 \;, \tag{19}$$

which, except for the effects of errors in the numerical integration and the value of the independent variable, is equivalent to

$$\begin{aligned} H &= 1 - \frac{\omega}{c} \frac{\text{real}\,(n)}{(k_r^2 + k_\theta^2 + k_\varphi^2)^{\frac{1}{2}}} \\ &= \text{real}\left\{1 - \frac{\omega}{c} \frac{n}{(k_r^2 + k_\theta^2 + k_\varphi^2)^{\frac{1}{2}}}\right\} \;. \end{aligned} \tag{20}$$

There are eight versions of the subroutine RINDEX which calculate the Hamiltonian and its partial derivatives. (Eight versions allow the user to choose the Appleton-Hartree formula or the Sen-Wyller formula, and to include or ignore the earth's magnetic field and collisions.) Six of these versions (subroutines AHWFWC, AHWFNC, AHNFWC, AHNFNC, SWWF, and SWNF) use the following Hamiltonian:

$$\begin{aligned} H &= \frac{1}{2} \left(\frac{c^2}{\omega^2} (k_r^2 + k_\theta^2 + k_\varphi^2) - \text{real}\,(n^2) \right) \\ &= \text{real}\left\{ \frac{1}{2} \left(\frac{c^2}{\omega^2} (k_r^2 + k_\theta^2 + k_\varphi^2) - n^2 \right) \right\} \;. \end{aligned} \tag{21}$$

The other two versions (subroutines BQWFWC and BQWFNC) use as a Hamiltonian the real part of the quadratic equation whose solution is the Appleton-Hartree formula (Budden, 1961)

$$H = \text{real}\left\{[(U-X)U^2 - Y^2U]\,c^4k^4 + X(k\cdot Y)^2\,c^4k^2 + \right.$$
$$+ [-2U(U-X)^2 + Y^2(2U-X)]\,c^2k^2\omega^2 - X(k\cdot Y)^2\,c^2\omega^2 +$$
$$\left. + [(U-X)^2 - Y^2](U-X)\,\omega^4\right\} \qquad (22)$$

except in or near free space (defined by $X < 0.1$) where they also use (21) as the Hamiltonian. In (22), $U = 1 - iZ$, and X, Y, and Z are the usual magnetoionic parameters.

In a lossy medium, the Hamiltonians in (20), (21), and (22) determine slightly different ray paths, but the differences are significant only when it is no longer valid to represent ray paths with coordinates that are real rather than complex. In fact, this is a weak criterion. The ray paths determined by these three Hamiltonians will become invalid before there are noticeable differences between the three ray paths. In a lossless medium, the above three Hamiltonians determine identical ray paths (except for integration errors).

For either a lossy or lossless medium, some of the above three Hamiltonians have computational difficulties. Special care must be taken in using (19) or (20) in an evanescent region (which is frequently necessary at or near vertical incidence because the numerical integration subroutine usually requires the evaluation of the differential equations not only on the ray path, but also at points near the ray path). For instance, in a lossless medium, real (n) is zero in an evanescent region, which leads to problems in (19) and (20). This problem will not arise in (21) because real (n^2) is well behaved in or at the boundary of an evanescent region, nor will it occur in using (22).

Neither (20) nor (21) (nor any other Hamiltonian based on the refractive index) will work for a ray passing through a spitze (Davies, 1965, p. 202) because the refractive index is indeterminate at a spitze, and some of the derivatives of n diverge. So far, we have had no problems using (22) to calculate ray paths through a spitze with or without collisions.

However, the Hamiltonian in (22) will not work in or near free space because all of its derivatives are zero in free space. This problem is related to (22) not being able to distinguish between ordinary and extraordinary waves. To get started, the program uses (21) until the electron density is large enough that X is equal or greater than 1/10.

As far as we can tell, the AHWFWC (Appleton-Hartree, with field, with collisions) version of subroutine RINDEX has been made obsolete by the BQWFWC (Booker quartic, with field, with collisions) version. The latter will do everything the AHWFWC version will do and in addition it will calculate rays through spitzes. A few trial runs, however, indicate that AHWFWC runs about 30 percent faster than BQWFWC.

Similarly, the AHWFNC (Appleton-Hartree, with field, no collisions) version has been made obsolete by the BQWFNC version, which apparently runs just as fast as the AHWFNC version. We are continuing to include the AHWFNC version just in case there are undiscovered problems with the BQWFNC version.

In addition to the Appleton-Hartree formula, which is based on a constant collision frequency, the program also includes the generalized formula of Sen and Wyller (1960), which assumes a Maxwell-Boltzman distribution of electron energy and a collision frequency proportional to energy. Two versions of subroutine RINDEX use the Sen-Wyller formula for calculating the refractive index and the resulting Hamiltonian with its derivatives. These are SWWF, which includes the effects of the earth's magnetic field, and SWNF, which neglects the Earth's magnetic field. The SWWF version will probably not work for calculating rays through a spitze. It would be possible to make a version which used as its Hamiltonian the quadratic equation whose solution is the Sen-Wyller formula for calculating rays through a spitze, but it is unlikely that we will ever do that.

The versions of subroutine RINDEX that use (21) for a Hamiltonian use the following formulas for calculating the derivatives of that Hamiltonian.

$$\frac{\partial H}{\partial t} = -n \frac{\partial n}{\partial t} \quad , \tag{23}$$

$$\frac{\partial H}{\partial r} = -n \frac{\partial n}{\partial r} \quad , \tag{24}$$

$$\frac{\partial H}{\partial \theta} = -n \frac{\partial n}{\partial \theta} \quad , \tag{25}$$

$$\frac{\partial H}{\partial \varphi} = -n \frac{\partial n}{\partial \varphi} \quad , \tag{26}$$

$$\frac{\partial H}{\partial \omega} = -\frac{n\, n'}{\omega} \quad , \tag{27}$$

$$\frac{\partial H}{\partial k_r} = \frac{c^2}{\omega^2} k_r - \frac{c}{\omega} n \frac{\partial n}{\partial V_r} \quad , \tag{28}$$

$$\frac{\partial H}{\partial k_\theta} = \frac{c^2}{\omega^2} k_\theta - \frac{c}{\omega} n \frac{\partial n}{\partial V_\theta} \quad , \tag{29}$$

$$\frac{\partial H}{\partial k_\varphi} = \frac{c^2}{\omega^2} k_\varphi - \frac{c}{\omega} n \frac{\partial n}{\partial V_\varphi} \quad , \tag{30}$$

$$\vec{k} \cdot \frac{\partial H}{\partial \vec{k}} = k_r \frac{\partial H}{\partial k_r} + k_\theta \frac{\partial H}{\partial k_\theta} + k_\varphi \frac{\partial H}{\partial k_\varphi} = n^2 \quad , \tag{31}$$

where n' is the group refractive index defined by

$$n' = n + f \frac{dn}{df} = n + \omega \frac{dn}{d\omega} \quad , \tag{32}$$

and V_r, V_θ, and V_φ are the components of the wave normal direction in the r, θ, and φ directions normalized so that

$$V_r^2 + V_\theta^2 + V_\varphi^2 = \text{Real}\ \{n^2\} \quad . \tag{33}$$

The derivatives of the Hamiltonian in (22) are given in section 5. 5.

5. REFRACTIVE INDEX EQUATIONS

The refractive index equations used in this ray tracing program are based either on the Appleton-Hartree formula (Budden, 1961) or on the generalized formula of Sen and Wyller (1960). There are eight versions of SUBROUTINE RINDEX, the subroutine that calculates the refractive index and its gradient:

(1) Appleton-Hartree formula with field, with collisions.
(2) Appleton-Hartree formula with field, no collisions.
(3) Appleton-Hartree formula with collisions, no field.
(4) Appleton-Hartree formula no field, no collisions.
(5) Booker quartic with field, with collisions.
(6) Booker quartic with field, no collisions.
(7) Sen-Wyller formula with field.
(8) Sen-Wyller formula, no field.

Each of these eight versions calculates n^2, nn', $n\ \partial n/\partial r$, $n\ \partial n/\partial\theta$, $n\ \partial n/\partial\varphi$, $n\ \partial n/\partial V_r$, $n\ \partial n/\partial V_\theta$, $n\ \partial n/\partial V_\varphi$, $n\ \partial n/\partial t$, and the polarization, where n is the complex phase refractive index; n' is the complex group refractive index; r, θ, and φ are the spherical polar coordinates of a point on the ray path, and V_r, V_θ, and V_φ are the components of the wave normal direction in the r, θ, and φ directions. The quantities

$$n' = n + f \frac{dn}{df} = n + \omega \frac{dn}{d\omega} \quad , \tag{32}$$

and V_r, V_θ, and V_φ are the components of the wave normal direction in the r, θ, and φ directions normalized so that

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(5) Booker quartic with field, with collisions.
(6) Booker quartic with field, no collisions.
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Each of these eight versions calculates n^2, nn', $n\ \partial n/\partial r$, $n\ \partial n/\partial \theta$, $n\ \partial n/\partial \varphi$, $n\ \partial n/\partial V_r$, $n\ \partial n/\partial V_\theta$, $n\ \partial n/\partial V_\varphi$, $n\ \partial n/\partial t$, and the polarization, where n is the complex phase refractive index; n' is the complex group refractive index; r, θ, and φ are the spherical polar coordinates of a point on the ray path, and V_r, V_θ, and V_φ are the components of the wave normal direction in the r, θ, and φ directions. The quantities

X, $\partial X/\partial r$, $\partial X/\partial \theta$, $\partial X/\partial \varphi$, and $\partial X/\partial t$ are supplied by one of the versions of subroutine ELECTX which defines the electron density model. The quantities Y, $\partial Y/\partial r$, $\partial Y/\partial \theta$, $\partial Y/\partial \varphi$, Y_r, $\partial Y_r/\partial r$, $\partial Y_r/\partial \theta$, $\partial Y_r/\partial \varphi$, Y_θ, $\partial Y_\theta/\partial r$, $\partial Y_\theta/\partial \theta$, $\partial Y_\varphi/\partial \varphi$, Y_φ, $\partial Y_\varphi/\partial r$, $\partial Y_\varphi/\partial \theta$, and $\partial Y_\varphi/\partial \varphi$ are supplied by one of the versions of subroutine MAGY which defines the magnetic field model. The quantities Z, $\partial Z/\partial r$, $\partial Z/\partial \theta$, and $\partial Z/\partial \varphi$ are supplied by one of the versions of subroutine COLFRZ which defines the collision frequency model.

In our formulation, we have tried to avoid using multivalued functions, such as the square root or $\cos^{-1}$, wherever possible. Only twice do we use the square root. One instance is the square root in the Appleton-Hartree formula, unavoidable without adding more differential equations to the system. The second instance is a square root used to calculate polarization. This latter use is unimportant because the polarization is not used in the ray tracing equations.

It is desirable to avoid multivalued functions because, unless extreme care is used, the value of such a function can change discontinuously from one point on the ray path to the next. A particularly troublesome case occurs at reflection for vertical incidence. At that point, the real part of n^2 goes through zero, and n changes from approximately purely real to approximately purely imaginary. Since the numerical integration subroutine usually requires the evaluation of the differential equations not only on the ray path, but also at points near the ray path, it is necessary to be able to evaluate the differential equations above the reflection height, that is, in an evanescent region.

We have found that it is possible to regroup the variables in the equations to avoid this problem: we calculate the real part of n^2 and its derivatives instead of the real part of n and its derivatives. And we calculate nn' instead of n'.

It was not easy to avoid using multivalued functions, however. Many of the usual parameters used to compute the refractive index require the use of multivalued functions in their calculation. Thus, we couldn't calculate

$$V = \sqrt{V_r^2 + V_\theta^2 + V_\varphi^2}$$

nor cos ψ, nor sin ψ, where ψ is the angle between the wave normal direction and the earth's magnetic field. Thus, we also could not calculate

$$Y_L = Y \cos \psi$$

nor

$$Y_T = Y \sin \psi \, .$$

The most difficult part of avoiding the use of multivalued functions was in calculating the derivatives.

The following is a list of the equations calculated by the eight versions of subroutine RINDEX.

5.1 Appleton-Hartree Formula with Field, with Collisions

The square of the complex phase refractive index is given by

$$n^2 = 1-2X \frac{1-iZ-X}{2(1-iZ)(1-iZ-X) - Y_T^2 \pm \sqrt{Y_T^4 + 4Y_L^2 (1-iZ-X)^2}} \quad , \tag{34}$$

where

$$X = \frac{f_N^2}{f^2} \quad , \tag{35}$$

$$Y = \frac{f_H}{f} \quad , \tag{36}$$

$$Z = \frac{\nu}{2\pi f} \quad , \tag{37}$$

$$Y_T = Y \sin \psi, \tag{38}$$

$$Y_L = Y \cos \psi, \tag{39}$$

f_N is the plasma frequency, f_H is the electron gyrofrequency, ν is the electron collision frequency, f is the wave frequency, and ψ is the angle between the wave normal direction and the earth's magnetic field.

The following equations parallel the formulas in this version of RINDEX.

$$V^2 = V_r^2 + V_\theta^2 + V_\varphi^2 \tag{40}$$

$$V \cdot Y = V_r Y_r + V_\theta Y_\theta + V_\varphi Y_\varphi \tag{41}$$

$$\frac{Y_L}{V} = \frac{V \cdot Y}{V^2} \tag{42}$$

$$Y_L^2 = \frac{(V \cdot Y)^2}{V^2} \tag{43}$$

$$Y_T^2 = Y^2 - Y_L^2 \tag{44}$$

$$Y_T^4 = \left(Y_T^2\right)^2 \tag{45}$$

$$U = 1 - iZ \tag{46}$$

$$RAD = \pm\sqrt{Y_T^4 + 4Y_L^2\ (U-X)^2} \tag{47}$$

$$D = 2U(U-X) - Y_T^2 + RAD \tag{48}$$

$$n^2 = 1 - \frac{2X(U-X)}{D} \tag{49}$$

$$\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi} = \frac{2X(U-X)\left(-1 + \dfrac{Y_T^2 - 2(U-X)^2}{RAD}\right)}{D^2} \tag{50}$$

$$Y_T Y_L \frac{\partial \psi}{\partial r} = \frac{Y_L^2}{Y} \frac{\partial Y}{\partial r} - \left(V_r \frac{\partial Y_r}{\partial r} + V_\theta \frac{\partial Y_\theta}{\partial r} + V_\varphi \frac{\partial Y_\varphi}{\partial r} \right) \left(\frac{Y_L}{V} \right) \tag{51}$$

$$Y_T Y_L \frac{\partial \psi}{\partial \theta} = \frac{Y_L^2}{Y} \frac{\partial Y}{\partial \theta} - \left(V_r \frac{\partial Y_r}{\partial \theta} + V_\theta \frac{\partial Y_\theta}{\partial \theta} + V_\varphi \frac{\partial Y_\varphi}{\partial \theta} \right) \left(\frac{Y_L}{V} \right) \tag{52}$$

$$Y_T Y_L \frac{\partial \psi}{\partial \varphi} = \frac{Y_L^2}{Y} \frac{\partial Y}{\partial \varphi} - \left(V_r \frac{\partial Y_r}{\partial \varphi} + V_\theta \frac{\partial Y_\theta}{\partial \varphi} + V_\varphi \frac{\partial Y_\varphi}{\partial \varphi} \right) \left(\frac{Y_L}{V} \right) \tag{53}$$

$$n \frac{\partial n}{\partial X} = - \frac{\left(2U(U-X)^2 - Y_T{}^2 (U-2X) + \dfrac{Y_T^4 (U-2X) + 4 Y_L^2 (U-X)^3}{RAD} \right)}{D^2} \tag{54}$$

$$n \frac{\partial n}{\partial Y} = \frac{2X(U-X)}{D^2 \; Y} \left(-Y_T^2 + \frac{Y_T^4 + 2 Y_L^2 \, (U-X)^2}{RAD} \right) \tag{55}$$

$$n \frac{\partial n}{\partial Z} = \frac{iX}{D^2} \left(-2(U-X)^2 - Y_T^2 + \frac{Y_T^4}{RAD} \right) \tag{56}$$

$$n \frac{\partial n}{\partial r} = n \frac{\partial n}{\partial X} \frac{\partial X}{\partial r} + n \frac{\partial n}{\partial Y} \frac{\partial Y}{\partial r} + n \frac{\partial n}{\partial Z} \frac{\partial Z}{\partial r} + \frac{n}{Y_L Y_T} \frac{\partial n}{\partial \psi} Y_L Y_T \frac{\partial \psi}{\partial r} \tag{57}$$

$$n \frac{\partial n}{\partial \theta} = n \frac{\partial n}{\partial X} \frac{\partial X}{\partial \theta} + n \frac{\partial n}{\partial Y} \frac{\partial Y}{\partial \theta} + n \frac{\partial n}{\partial Z} \frac{\partial Z}{\partial \theta} + \frac{n}{Y_L Y_T} \frac{\partial n}{\partial \psi} Y_T Y_L \frac{\partial \psi}{\partial \theta} \tag{58}$$

$$n \frac{\partial n}{\partial \varphi} = n \frac{\partial n}{\partial X} \frac{\partial X}{\partial \varphi} + n \frac{\partial n}{\partial Y} \frac{\partial Y}{\partial \varphi} + n \frac{\partial n}{\partial Z} \frac{\partial Z}{\partial \varphi} + \frac{n}{Y_L Y_T} \frac{\partial n}{\partial \psi} Y_T Y_L \frac{\partial \psi}{\partial \varphi} \tag{59}$$

$$n \frac{\partial n}{\partial V_r} = \frac{n}{Y_T Y_L} \frac{\partial n}{\partial \psi} \left(\frac{V_r Y_L^2}{V^2} - \left(\frac{Y_L}{V} \right) Y_r \right) \tag{60}$$

$$n\frac{\partial n}{\partial V_\theta} = \frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\left(\frac{V_\theta Y_L^2}{V^2} - \left(\frac{Y_L}{V}\right) Y_\theta\right) \tag{61}$$

$$n\frac{\partial n}{\partial V_\varphi} = \frac{n}{Y_L Y_T}\frac{\partial n}{\partial \psi}\left(\frac{V_\varphi Y_L^2}{V^2} - \left(\frac{Y_L}{V}\right) Y_\varphi\right) \tag{62}$$

$$nn' = n^2 - \left(2Xn\frac{\partial n}{\partial X} + Y\,n\frac{\partial n}{\partial Y} + Z\,n\frac{\partial n}{\partial Z}\right) \tag{63}$$

$$\text{Polarization} = \rho = -i\,\frac{\left(-Y_T^2 + \text{RAD}\right)\sqrt{V^2}}{2(U-X)\,V\cdot Y} \tag{64a}$$

(Budden, 1961, page 49).

$$\text{Longitudinal polarization} = \frac{iX\sqrt{Y_T^2}}{(U-X)\left(U+i\,\frac{V\cdot Y}{\sqrt{V^2}}\,\rho\right)} \tag{64b}$$

(Budden, 1961, page 54).

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial t} \tag{65}$$

5.2 Appleton-Hartree Formula with Field, no Collisions

The equations are the same as with field with collisions except for:

$$Z = \frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0, \tag{66}$$

$$U = 1. \tag{67}$$

5.3 Appleton-Hartree Formula no Field, With Collisions

$$U = 1 - iZ \tag{68}$$

$$n^2 = 1 - \frac{X}{U} \tag{69}$$

$$n\frac{\partial n}{\partial X} = -\frac{.5}{U} \tag{70}$$

$$n\frac{\partial n}{\partial Z} = -\frac{.5iX}{U^2} \tag{71}$$

$$nn' = n^2 - \left(2Xn\frac{\partial n}{\partial X} + Zn\frac{\partial n}{\partial Z}\right) \tag{72}$$

$$n\frac{\partial n}{\partial r} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial r} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial r} \tag{73}$$

$$n\frac{\partial n}{\partial \theta} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial \theta} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial \theta} \tag{74}$$

$$n\frac{\partial n}{\partial \varphi} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial \varphi} + n\frac{\partial n}{\partial Z}\frac{\partial Z}{\partial \varphi} \tag{75}$$

$$n\frac{\partial n}{\partial V_r} = 0 \tag{76}$$

$$n\frac{\partial n}{\partial V_\theta} = 0 \tag{77}$$

$$n\frac{\partial n}{\partial V_\varphi} = 0 \tag{78}$$

$$\text{Polarization} = i \tag{79a}$$

$$\text{Longitudinal polarization} = 0 \tag{79b}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial t} \tag{80}$$

5.4 Appleton-Hartree Formula no Field, no Collisions

$$n^2 = 1 - X \tag{81}$$

$$n\frac{\partial n}{\partial X} = -.5 \tag{82}$$

$$n\frac{\partial n}{\partial r} = \left(n\frac{\partial n}{\partial X}\right)\left(\frac{\partial X}{\partial r}\right) \tag{83}$$

$$n\frac{\partial n}{\partial \theta} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \theta} \tag{84}$$

$$n\frac{\partial n}{\partial \varphi} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \varphi} \tag{85}$$

$$n\frac{\partial n}{\partial V_r} = 0 \tag{86}$$

$$n\frac{\partial n}{\partial V_\theta} = 0 \tag{87}$$

$$n\frac{\partial n}{\partial V_\varphi} = 0 \tag{88}$$

$$nn' = 1 \tag{89}$$

$$\text{Polarization} = i \tag{90a}$$

$$\text{Longitudinal polarization} = 0 \tag{90b}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\,\frac{\partial X}{\partial t} \tag{91}$$

5.5 Booker Quartic with Field, with Collisions

The form of the dispersion relation used for the Hamiltonian in this version of subroutine RINDEX is the quadratic equation whose solution is the Appleton-Hartree formula. This Hamiltonian, given by

(22), is also a special case of the Booker quartic for S = 0 and C = 1 (Budden, 1961). This version uses the Hamiltonian in (22) only when the electron density is large enough that X is greater than or equal to 1/10. For X less than 1/10, the Hamiltonian in (21) is used.

Below are the equations for the derivatives of the Hamiltonian in (22). The equations for the derivatives of the Hamiltonian in (21) are the same as those in section 4.1.

$$k^2 = k_r^2 + k_\theta^2 + k_\varphi^2 \quad , \tag{92}$$

$$k \cdot Y = k_r Y_r + k_\theta Y_\theta + k_\varphi Y_\varphi \quad , \tag{93}$$

$$U = 1 - iZ \quad , \tag{94}$$

$$A = (U - X)\, U^2 - UY^2 \quad , \tag{95}$$

$$B = -2U(U - X)^2 + Y^2(2U - X) \quad , \tag{96}$$

$$\alpha = A\, c^4 k^4 + X(k \cdot Y)^2\, c^4 k^2 \quad , \tag{97}$$

$$\beta = B\, c^2 k^2 \omega^2 - X(k \cdot Y)^2\, c^2 \omega^2 \quad , \tag{98}$$

$$\gamma = ((U - X)^2 - Y^2)(U - X)\, \omega^4 \quad , \tag{99}$$

$$H = \alpha + \beta + \gamma \quad , \tag{100}$$

$$\frac{\partial H}{\partial X} = -U^2 c^4 k^4 + (k \cdot Y)^2 c^4 k^2 + (4U(U - X) - Y^2) c^2 k^2 \omega^2 +$$

$$-(k \cdot Y)^2 c^2 \omega^2 + (-3(U - X)^2 + Y^2)\, \omega^4 \quad , \tag{101}$$

$$\frac{\partial H}{\partial(Y^2)} = -U\ c^4k^4 + (2U - X)\ c^2k^2\omega^2 - (U - X)\ \omega^4 \quad , \tag{102}$$

$$\frac{\partial H}{\partial((k \cdot Y)^2)} = X\ c^2(c^2k^2 - \omega^2) \quad , \tag{103}$$

$$\begin{aligned} \frac{\partial H}{\partial U} = (2\ U(U - X) + U^2 - Y^2)\ c^4k^4 &+ \\ + 2(Y^2 - (U - X)^2 - 2U(U - X))\ c^2k^2\omega^2 &+ \\ + (3(U - X)^2 - Y^2)\ \omega^4 \quad , \end{aligned} \tag{104}$$

$$\frac{\partial H}{\partial Z} = -i\ \frac{\partial H}{\partial U} \quad , \tag{105}$$

$$\frac{\partial H}{\partial(k^2)} = 2\,A\ c^4k^2 + X(k \cdot Y)^2\ c^4 + Bc^2\omega^2 \quad , \tag{106}$$

$$\frac{\partial H}{\partial t} = \frac{\partial H}{\partial X}\ \frac{\partial X}{\partial t} \quad , \tag{107}$$

$$\begin{aligned} \frac{\partial H}{\partial r} = \frac{\partial H}{\partial X}\ \frac{\partial X}{\partial r} + 2\ \frac{\partial H}{\partial(Y^2)}\ \ Y\ \frac{\partial Y}{\partial r} + \frac{\partial H}{\partial Z}\ \frac{\partial Z}{\partial r} &+ \\ + 2\ \frac{\partial H}{\partial((k \cdot Y)^2)}\,(k \cdot Y)\left(k_r\ \frac{\partial Y_r}{\partial r} + k_\theta\ \frac{\partial Y_\theta}{\partial r} + k_\varphi\ \frac{\partial Y_\varphi}{\partial r}\right) \quad , \end{aligned} \tag{108}$$

$$\begin{aligned} \frac{\partial H}{\partial \theta} = \frac{\partial H}{\partial X}\ \frac{\partial X}{\partial \theta} + 2\ \frac{\partial H}{\partial(Y^2)}\ \ Y\ \frac{\partial Y}{\partial \theta} + \frac{\partial H}{\partial Z}\ \frac{\partial Z}{\partial \theta} &+ \\ + 2\ \frac{\partial H}{\partial((k \cdot Y)^2)}\,(k \cdot Y)\left(k_r\ \frac{\partial Y_r}{\partial \theta} + k_\theta\ \frac{\partial Y_\theta}{\partial \theta} + k_\varphi\ \frac{\partial Y_\varphi}{\partial \theta}\right) \quad , \end{aligned} \tag{109}$$

$$\frac{\partial H}{\partial \varphi} = \frac{\partial H}{\partial X}\,\frac{\partial X}{\partial \varphi} + 2\,\frac{\partial H}{\partial (Y^2)}\,Y\,\frac{\partial Y}{\partial \varphi} + \frac{\partial H}{\partial Z}\,\frac{\partial Z}{\partial \varphi} +$$

$$+ 2\,\frac{\partial H}{\partial ((k\cdot Y)^2)}\,(k\cdot Y)\left(k_r\,\frac{\partial Y_r}{\partial \varphi} + k_\theta\,\frac{\partial Y_\theta}{\partial \varphi} + k_\varphi\,\frac{\partial Y_\varphi}{\partial \varphi}\right) , \qquad (110)$$

$$\frac{\partial H}{\partial \omega} = (2\beta + 4\gamma)/\omega - 2\,\frac{\partial H}{\partial X}\,\frac{X}{\omega} - 2\,\frac{\partial H}{\partial (Y^2)}\,\frac{Y^2}{\omega} +$$

$$- 2\,\frac{\partial H}{\partial ((k\cdot Y)^2)}\,\frac{(k\cdot Y)^2}{\omega} - \frac{\partial H}{\partial Z}\,\frac{Z}{\omega} , \qquad (111)$$

$$\frac{\partial H}{\partial k_r} = 2\,\frac{\partial H}{\partial (k^2)}\,k_r + 2(k\cdot Y)\,\frac{\partial H}{\partial ((k\cdot Y)^2)}\,Y_r , \qquad (112)$$

$$\frac{\partial H}{\partial k_\theta} = 2\,\frac{\partial H}{\partial (k^2)}\,k_\theta + 2(k\cdot Y)\,\frac{\partial H}{\partial ((k\cdot Y)^2)}\,Y_\theta , \qquad (113)$$

$$\frac{\partial H}{\partial k_\varphi} = 2\,\frac{\partial H}{\partial (k^2)}\,k_\varphi + 2(k\cdot Y)\,\frac{\partial H}{\partial ((k\cdot Y)^2)}\,Y_\varphi , \qquad (114)$$

$$k^2(\text{calculated}) = k^2\,\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} , \qquad (115)$$

$$k\cdot\frac{\partial H}{\partial k} = k_r\,\frac{\partial H}{\partial k_r} + k_\theta\,\frac{\partial H}{\partial k_\theta} + k_\varphi\,\frac{\partial H}{\partial k_\varphi} = 4\alpha + 2\beta , \qquad (116)$$

$$\text{Polarization} = i\,\frac{\sqrt{k^2}}{k\cdot Y}\left(U + \frac{X\omega^2}{c^2 k^2_{\text{calculated}} - \omega^2}\right) , \qquad (117)$$

$$\begin{array}{l}\text{Longitudinal}\\ \text{Polarization}\end{array} = i\,\frac{\sqrt{Y^2 - \dfrac{(k\cdot Y)^2}{k^2}}}{U - X}\left(1 - \frac{c^2 k^2_{\text{calculated}}}{\omega^2}\right) . \qquad (118)$$

5.6 Booker Quartic with Field, no Collisions

All the equations here are the same as for the Booker quartic version with collisions (section 5.5) except for:

$$Z = \frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial \varphi} = 0 \quad , \tag{119}$$

$$U = 1. \tag{120}$$

All the variables except polarization are real; polarization is pure imaginary.

5.7 Sen-Wyller Formula with Field

It is possible to write the generalized Appleton-Hartree formula of Sen and Wyller (1960) in the following form:

$$n^2 = 1 - \frac{2X(U-X) + 2\ AUX\ \sin^2\psi}{2U(U-X)(1+A) + 2\ AUX\ \sin^2\psi - U((1-BC)U + A(U+X))\ \sin^2\psi + RAD} \quad , \tag{121}$$

where

$$RAD = \pm\sqrt{U^2((1-BC)\,U + A(U+X))^2\ \sin^4\psi + U^2(U-X)^2(C-B)^2\cos^2\psi} \quad , \tag{122}$$

$$A = \frac{C+B}{2} - 1 \ , \tag{123}$$

$$B = \frac{F\left(\frac{1}{Z}\right)}{F\left(\frac{1-Y}{Z}\right)} \ , \tag{124}$$

$$C = \frac{F\left(\frac{1}{Z}\right)}{F\left(\frac{1+Y}{Z}\right)} \; , \tag{125}$$

$$U = \frac{Z}{F\left(\frac{1}{Z}\right)} \tag{126}$$

$$F(w) = w\;C_{3/2}(w) + i\,\frac{5}{2}\;C_{5/2}\,(w) = \frac{1}{\frac{3}{2}\,!}\int_{o}^{\infty}\frac{t^{3/2}e^{-t}dt}{w-it} \; , \tag{127}$$

$$X = \frac{f_N^2}{f^2} \quad , \tag{128}$$

$$Y = \frac{f_H}{f} \quad , \tag{129}$$

$$Z = \frac{\nu_m}{2\pi f} \quad , \tag{130}$$

f_N is the plasma frequency, f_H is the electron gyrofrequency, ν_m is the mean electron collision frequency, f is the wave frequency, and ψ is the angle between the wave normal direction and the earth's magnetic field. (Note that if we would use

$$F(w) = \frac{1}{w-i} \quad , \tag{131}$$

then (121) would reduce to the usual Appleton-Hartree formula.)

This version of RINDEX calls subroutine FSW. The first argument in the calling sequence is the argument w of F(w). The second argument in the calling sequence is the value of the function F(w) calculated by FSW. The third argument in the calling sequence is the derivative of the function, F'(w).

The following equations parallel the formulas in this version of RINDEX.

$$\alpha = F\left(\frac{1}{Z}\right) \tag{132}$$

$$\alpha' = F'\left(\frac{1}{Z}\right) \tag{133}$$

$$\beta = F\left(\frac{1-Y}{Z}\right) \tag{134}$$

$$\beta' = F'\left(\frac{1-Y}{Z}\right) \tag{135}$$

$$\gamma = F\left(\frac{1+Y}{Z}\right) \tag{136}$$

$$\gamma' = F'\left(\frac{1+Y}{Z}\right) \tag{137}$$

$$U = \frac{Z}{\alpha} \tag{138}$$

$$\frac{\partial U}{\partial Z} = \left(1 + \frac{\alpha'}{\alpha Z}\right) \Big/ \alpha \tag{139}$$

$$B = \frac{\alpha}{\beta} \tag{140}$$

$$\frac{\partial B}{\partial Y} = \frac{B}{Z} \; \frac{\beta'}{\beta} \tag{141}$$

$$\frac{\partial B}{\partial Z} = -\frac{B}{Z^2}\left(\frac{\alpha'}{\alpha} - (1-Y)\,\frac{\beta'}{\beta}\right) \tag{142}$$

$$C = \frac{\alpha}{\gamma} \tag{143}$$

$$\frac{\partial C}{\partial Y} = -\frac{C}{Z}\ \frac{\gamma'}{\gamma} \tag{144}$$

$$\frac{\partial C}{\partial Z} = -\frac{C}{Z^2}\left(\frac{\alpha'}{\alpha} - (1+Y)\ \frac{\gamma'}{\gamma}\right) \tag{145}$$

$$A = .5(B+C) - 1 \tag{146}$$

$$\frac{\partial A}{\partial Y} = .5\left(\frac{\partial B}{\partial Y} + \frac{\partial C}{\partial Y}\right) \tag{147}$$

$$\frac{\partial A}{\partial Z} = .5\left(\frac{\partial B}{\partial Z} + \frac{\partial C}{\partial Z}\right) \tag{148}$$

$$V^2 = V_r^2 + V_\theta^2 + V_\varphi^2 \tag{149}$$

$$V \cdot Y = V_r\, Y_r + V_\theta\, Y_\theta + V_\varphi\, Y_\varphi \tag{150}$$

$$Y_L^2 = \frac{(V \cdot Y)^2}{V^2} \tag{151}$$

$$Y_T^2 = Y^2 - Y_L^2 \tag{152}$$

$$\sin^2 \psi = \frac{Y_T^2}{Y^2} \tag{153}$$

$$\cos^2 \psi = \frac{Y_L^2}{Y^2} \tag{154}$$

$$T_1 = \left[(1-BC)\,U^2 + A\,U(U+X)\right] \sin^2\psi \tag{155}$$

$$\frac{\partial T_1}{\partial X} = +AU \sin^2\psi \tag{156}$$

$$\frac{\partial T_1}{\partial Y} = \left(U(U+X)\frac{\partial A}{\partial Y} - U^2 \left(B\frac{\partial C}{\partial Y} + C\frac{\partial B}{\partial Y} \right) \right) \sin^2\psi \tag{157}$$

$$\begin{aligned} \frac{\partial T_1}{\partial Z} = \Bigg(& 2U\frac{\partial U}{\partial Z}(1-BC+A) + A\,X\frac{\partial U}{\partial Z} - U^2 \left(B\frac{\partial C}{\partial Z} + C\frac{\partial B}{\partial Z} \right) \\ & + U(U+X)\frac{\partial A}{\partial Z} \Bigg) \sin^2\psi \end{aligned} \tag{158}$$

$$\frac{1}{Y_L Y_T} \frac{\partial T_1}{\partial \psi} = \frac{2T_1}{Y_T^2} \tag{159}$$

$$T_2 = U^2 (C-B)^2 (U-X)^2 \cos^2\psi \tag{160}$$

$$\frac{\partial T_2}{\partial X} = -2(U-X)\,U^2 (C-B)^2 \cos^2\psi \tag{161}$$

$$\frac{\partial T_2}{\partial Y} = 2U^2 (U-X)^2 \cos^2\psi\,(C-B) \left(\frac{\partial C}{\partial Y} - \frac{\partial B}{\partial Y} \right) \tag{162}$$

$$\frac{\partial T_2}{\partial Z} = 2U^2 (U-X)^2 (C-B) \left(\frac{\partial C}{\partial Z} - \frac{\partial B}{\partial Z} \right) \cos^2\psi + 2T_2 \left(\frac{1}{U} + \frac{1}{U-X}\,\frac{\partial U}{\partial Z} \right) \tag{163}$$

$$\frac{1}{Y_L Y_T} \frac{\partial T_2}{\partial \psi} = -\frac{2T_2}{Y_L^2} \tag{164}$$

$$\mathrm{RAD} = \pm\sqrt{T_1^2 + T_2} \tag{165}$$

$$\frac{\partial RAD}{\partial X} = \frac{T_1 \frac{\partial T_1}{\partial X} + \frac{1}{2} \frac{\partial T_2}{\partial X}}{RAD} \tag{166}$$

$$\frac{\partial RAD}{\partial Y} = \frac{T_1 \frac{\partial T_1}{\partial Y} + \frac{1}{2} \frac{\partial T_2}{\partial Y}}{RAD} \tag{167}$$

$$\frac{\partial RAD}{\partial Z} = \frac{T_1 \frac{\partial T_1}{\partial Z} + \frac{1}{2} \frac{\partial T_2}{\partial Z}}{RAD} \tag{168}$$

$$\frac{1}{Y_L Y_T} \frac{\partial RAD}{\partial \psi} = \frac{T_1 \left(\frac{1}{Y_L Y_T} \frac{\partial T_1}{\partial \psi}\right) + \frac{1}{2} \left(\frac{1}{Y_L Y_T} \frac{\partial T_2}{\partial \psi}\right)}{RAD} \tag{169}$$

$$D = 2U(U-X)\,(1 + A) - T_1 + RAD + 2AUX \sin^2 \psi \tag{170}$$

$$\frac{\partial D}{\partial X} = -2U - \frac{\partial T_1}{\partial X} + \frac{\partial\, RAD}{\partial X} + 2\ AU \sin^2 \psi \tag{171}$$

$$\frac{\partial D}{\partial Y} = 2U(U-X) \frac{\partial A}{\partial Y} - \frac{\partial T_1}{\partial Y} + \frac{\partial\, RAD}{\partial Y} + 2\,U \sin^2 \psi \frac{\partial A}{\partial Y} \tag{172}$$

$$\frac{\partial D}{\partial Z} = 2(1+A) \frac{\partial U}{\partial Z} (2\,U-X) + 2U(U-X) \frac{\partial A}{\partial Z} - \frac{\partial T_1}{\partial Z} + \frac{\partial\, RAD}{\partial Z} + + 2AX \sin^2 \psi \frac{\partial U}{\partial Z} + 2\ UX \sin^2 \psi \frac{\partial A}{\partial Z} \tag{173}$$

$$\frac{1}{Y_L Y_T} \frac{\partial D}{\partial \psi} = -\left(\frac{1}{Y_L Y_T} \frac{\partial T_1}{\partial \psi}\right) + \left(\frac{1}{Y_L Y_T} \frac{\partial\, RAD}{\partial \psi}\right) + 2\,AUX/\,Y^2 \tag{174}$$

$$n^2 - 1 = \frac{-2X}{D}\left((U-X) + U A \sin^2\psi \right) \tag{175}$$

$$n^2 = 1 + (n^2 - 1) \tag{176}$$

$$n\frac{\partial n}{\partial X} = \frac{1}{2}\,(n^2 - 1)\left(\frac{1}{X} - \frac{1}{D}\frac{\partial D}{\partial X}\right) + \frac{X}{D} \tag{177}$$

$$n\frac{\partial n}{\partial Y} = -\frac{X U \sin^2\psi}{D}\,\frac{\partial A}{\partial Y} - \frac{(n^2-1)}{2D}\,\frac{\partial D}{\partial Y} \tag{178}$$

$$n\frac{\partial n}{\partial Z} = -\frac{X}{D}\,(1+A\sin^2\psi)\,\frac{\partial U}{\partial Z} - \frac{XU}{D}\,\sin^2\psi\,\frac{\partial A}{\partial Z} - \frac{(n^2-1)}{2D}\,\frac{\partial D}{\partial Z} \tag{179}$$

$$\frac{n}{Y_T Y_L}\,\frac{\partial n}{\partial\psi} = -\frac{XUA}{Y^2 D} - \frac{(n^2-1)}{2D}\left(\frac{1}{Y_L Y_T}\,\frac{\partial D}{\partial\psi}\right) \tag{180}$$

$$Y_L Y_T\,\frac{\partial\psi}{\partial r} = \frac{Y_L^2}{Y}\,\frac{\partial Y}{\partial r} - \left(V_r\,\frac{\partial Y_r}{\partial r} + V_\theta\,\frac{\partial Y_\theta}{\partial r} + V_\varphi\,\frac{\partial Y_\varphi}{\partial r}\right)\frac{V\cdot Y}{V^2} \tag{181}$$

$$Y_L Y_T\,\frac{\partial\psi}{\partial\theta} = \frac{Y_L^2}{Y}\,\frac{\partial Y}{\partial\theta} - \left(V_r\,\frac{\partial Y_r}{\partial\theta} + V_\theta\,\frac{\partial Y_\theta}{\partial\theta} + V_\varphi\,\frac{\partial Y_\varphi}{\partial\theta}\right)\frac{V\cdot Y}{V^2} \tag{182}$$

$$Y_L Y_T\,\frac{\partial\psi}{\partial\varphi} = \frac{Y_L^2}{Y}\,\frac{\partial Y}{\partial\varphi} - \left(V_r\,\frac{\partial Y_r}{\partial\varphi} + V_\theta\,\frac{\partial Y_\theta}{\partial\varphi} + V_\varphi\,\frac{\partial Y_\varphi}{\partial\varphi}\right)\frac{V\cdot Y}{V^2} \tag{183}$$

$$n\,\frac{\partial n}{\partial r} = n\frac{\partial n}{\partial X}\frac{\partial X}{\partial r} + n\frac{\partial n}{\partial Y}\,\frac{\partial Y}{\partial r} + n\frac{\partial n}{\partial Z}\,\frac{\partial Z}{\partial r} + \left(\frac{n}{Y_L Y_T}\,\frac{\partial n}{\partial\psi}\right)\left(Y_L Y_T\frac{\partial\psi}{\partial r}\right) \tag{184}$$

$$n\frac{\partial n}{\partial\theta} = n\frac{\partial n}{\partial X}\,\frac{\partial X}{\partial\theta} + n\frac{\partial n}{\partial Y}\,\frac{\partial Y}{\partial\theta} + n\frac{\partial n}{\partial Z}\,\frac{\partial Z}{\partial\theta} + \left(\frac{n}{Y_L Y_T}\,\frac{\partial n}{\partial\psi}\right)\left(Y_L Y_T\,\frac{\partial\psi}{\partial\theta}\right) \tag{185}$$

$$n\frac{\partial n}{\partial\varphi} = n\frac{\partial n}{\partial X}\,\frac{\partial X}{\partial\varphi} + n\frac{\partial n}{\partial Y}\,\frac{\partial Y}{\partial\varphi} + n\frac{\partial n}{\partial Z}\,\frac{\partial Z}{\partial\varphi} + \left(\frac{n}{Y_L Y_T}\,\frac{\partial n}{\partial\psi}\right)\left(Y_L Y_T\,\frac{\partial\psi}{\partial\varphi}\right) \tag{186}$$

$$n\frac{\partial n}{\partial V_r} = \left(\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi}\right)\left(V_r Y_L^2 - (V \cdot Y)Y_r\right) \Big/ V^2 \tag{187}$$

$$n\frac{\partial n}{\partial V_\theta} = \left(\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi}\right)\left(V_\theta Y_L^2 - (V \cdot Y)Y_\theta\right) \Big/ V^2 \tag{188}$$

$$n\frac{\partial n}{\partial V_\varphi} = \left(\frac{n}{Y_L Y_T}\ \frac{\partial n}{\partial \psi}\right)\left(V_\varphi Y_L^2 - (V \cdot Y)Y_\varphi\right) \Big/ V^2 \tag{189}$$

$$nn' = n^2 - \left(2X\left(n\frac{\partial n}{\partial X}\right) + Y\left(n\frac{\partial n}{\partial Y}\right) + Z\left(n\frac{\partial n}{\partial Z}\right)\right) \tag{190}$$

$$\text{polarization} = \rho = \frac{i\,(T_1 - \text{RAD})\,Y\sqrt{V^2}}{U(U-X)(C-B)(V \cdot Y)} \tag{191a}$$

$$\text{longitudinal polarization} = \frac{X(.5i\,(C-B)\,\rho + A\cos\psi\,)\sqrt{\sin^2\psi}}{\rho((U-X)(1+.5i(C-B)\rho\cos\psi)+A(U-X\cos^2\psi))} \tag{191b}$$

where

$$\cos\psi = \frac{V \cdot Y}{Y\sqrt{V^2}} \tag{192}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\ \frac{\partial X}{\partial t} \tag{193}$$

5.8 Sen-Wyller Formula no Field

This subroutine uses the function G which is related to the function F (defined in the previous section) as follows:

$$G(w) = wF(w)\ . \tag{194}$$

The following equations parallel the formulas used in this version of RINDEX.

$$F_1 = F\left(\frac{1}{Z}\right) \tag{195}$$

$$F_1' = F'\left(\frac{1}{Z}\right) \tag{196}$$

$$G_1 = G\left(\frac{1}{Z}\right) \tag{197}$$

$$G_1' = G'\left(\frac{1}{Z}\right) \tag{198}$$

$$n^2 = 1 - X\,G_1 \tag{199}$$

$$n\frac{\partial n}{\partial X} = -\frac{1}{2}\,G_1 \tag{200}$$

$$n\frac{\partial n}{\partial Z} = \frac{X\,G_1'}{2\,Z^2} \tag{201}$$

$$n\frac{\partial n}{\partial r} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial r} + \left(n\frac{\partial n}{\partial Z}\right)\frac{\partial Z}{\partial r} \tag{202}$$

$$n\frac{\partial n}{\partial \theta} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \theta} + \left(n\frac{\partial n}{\partial Z}\right)\frac{\partial Z}{\partial \theta} \tag{203}$$

$$n\frac{\partial n}{\partial \varphi} = \left(n\frac{\partial n}{\partial X}\right)\frac{\partial X}{\partial \varphi} + \left(n\frac{\partial n}{\partial Z}\right)\frac{\partial Z}{\partial \varphi} \tag{204}$$

$$nn' = n^2 - \left(2\,X\left(n\frac{\partial n}{\partial X}\right) + Z\left(n\frac{\partial n}{\partial Z}\right)\right) \tag{205}$$

$$\text{polarization} = i \tag{206a}$$

$$\text{longitudinal polarization} = 0 \tag{206b}$$

$$n\frac{\partial n}{\partial t} = n\frac{\partial n}{\partial X}\,\frac{\partial X}{\partial t} \tag{207}$$

6. IONOSPHERIC MODELS

When using the program, one must specify ionospheric models which define electron density, collision frequency (if the effect of collisions is being considered), and the earth's magnetic field (if its effects are being taken into account) as a function of position in space. Each of these three characteristics of the ionosphere is defined by a separate subroutine.

Appendices 3, 4, 5, and 6 contain descriptions, input parameter forms and listings of ionospheric models that now exist. These ionospheric models are not likely to cover the needs of everyone who wants to use the program. Anticipating this when we wrote the program, we made it possible to add models easily. The user may make up his own ionospheric models by simply writing subroutines to define electron density, collision frequency, and the earth's magnetic field (and their gradients) as a function of position in space in spherical polar coordinates, following the form of the subroutines in appendices 3, 4, 5, and 6.

Appendix 3 contains electron density models; appendix 4 contains models of irregularities which may be applied as perturbations to any of the electron density models; appendix 5 contains models of the earth's magnetic field; and appendix 6 contains collision frequency models.

Having several versions of the subroutines for refractive index, electron density, collision frequency, and the earth's magnetic field gives the user not only a wide choice among ionospheric models, but also a variety of compromises between cost and an accurate description of the ionosphere, while still keeping the program simple.

7. FINDING THE RAY PATHS THAT CONNECT A TRANSMITTER AND RECEIVER

The reason for using a ray tracing program is to find all important ray paths that connect a given transmitter and receiver (either or both of which may be on a satellite) on a particular frequency and such properties of these ray paths as group time delay, phase time delay, and absorption of the wave. "All important" ray paths include those that reflect from the various ionospheric layers (including multiple reflections) and that propagate off the great circle path.

Since basically all that a ray tracing program can do is to calculate the path of a ray when given the transmitter location, frequency, and direction of transmission, it cannot directly calculate those ray paths that arrive at a specified receiver. The problem is to know, before tracing the ray, in which directions to transmit the ray so that it will arrive at the receiver. Since there are no general solutions to this problem, the user of a ray tracing program must rely on some sort of trial and error technique to find those ray paths that connect the transmitter and receiver. This involves varying the direction of transmission until a ray is found that reaches the receiver. If a ray tracing program does this automatically, we say that it has a homing feature. This program does not have such a feature. To find all the paths connecting the transmitter with the receiver requires a very elaborate homing routine because "homing in" on a receiver takes more judgment and common sense than speed in performing massive calculations. Therefore, the person using the program is more fitted to this task than is the computer program itself.

As an aid, however, the program allows the user to specify the receiver height, the number of hops, and a range of azimuth and elevation angles-of-transmission that he thinks will include those rays that will arrive at the receiver. The program then calculates a ray path for each of the azimuth and elevation angles-of-transmission specified in the range. Usually only in the case of ionospheres with large horizontal gradients will the azimuth angle-of-transmission have to be varied. The program will calculate each ray path far enough to intersect or make a closest approach to the receiver height for the requested number of hops. The user can then interpolate between those rays which surround the receiver.

We define the point of "closest approach" as the point on the ray path where the wave normal direction is horizontal. It approximates an apogee if the receiver is above the apogee height and it approximates a perigee if the receiver is below the perigee height. The approximation is good for oblique propagation. When the earth's magnetic field is neglected, a point of "closest approach" is exactly an apogee or perigee.

We count one hop every time the ray crosses the receiver height. If the receiver is on the ground, a ground reflection counts as one hop for the downcoming ray before the ground reflection, and another hop for the upgoing ray after the ground reflection. We count two hops every time the ray passes through a point of "closest approach" to the receiver height. This procedure helps make rays that have the same hop number have a ground range that is a continuous function of the direction of transmission.

8. OUTPUT

8.1 Printout

Periodically and at selected points during a ray trace, the program will print information giving the position of the current ray path point, the direction of the wave normal, and the cumulative values of quantities being integrated along the ray path such as group path, phase path, absorption, and Doppler shift. Appendix 8c contains a sample of the printout.

8.2 Punched Cards

The program will punch a card at the beginning of each ray, a card at each ground reflection, a card at each crossing of the receiver height, two cards for each closest approach to the receiver height, and a card at the end of each ray to summarize the main results of the ray path calculations. These cards are explained in figures 1 and 2.

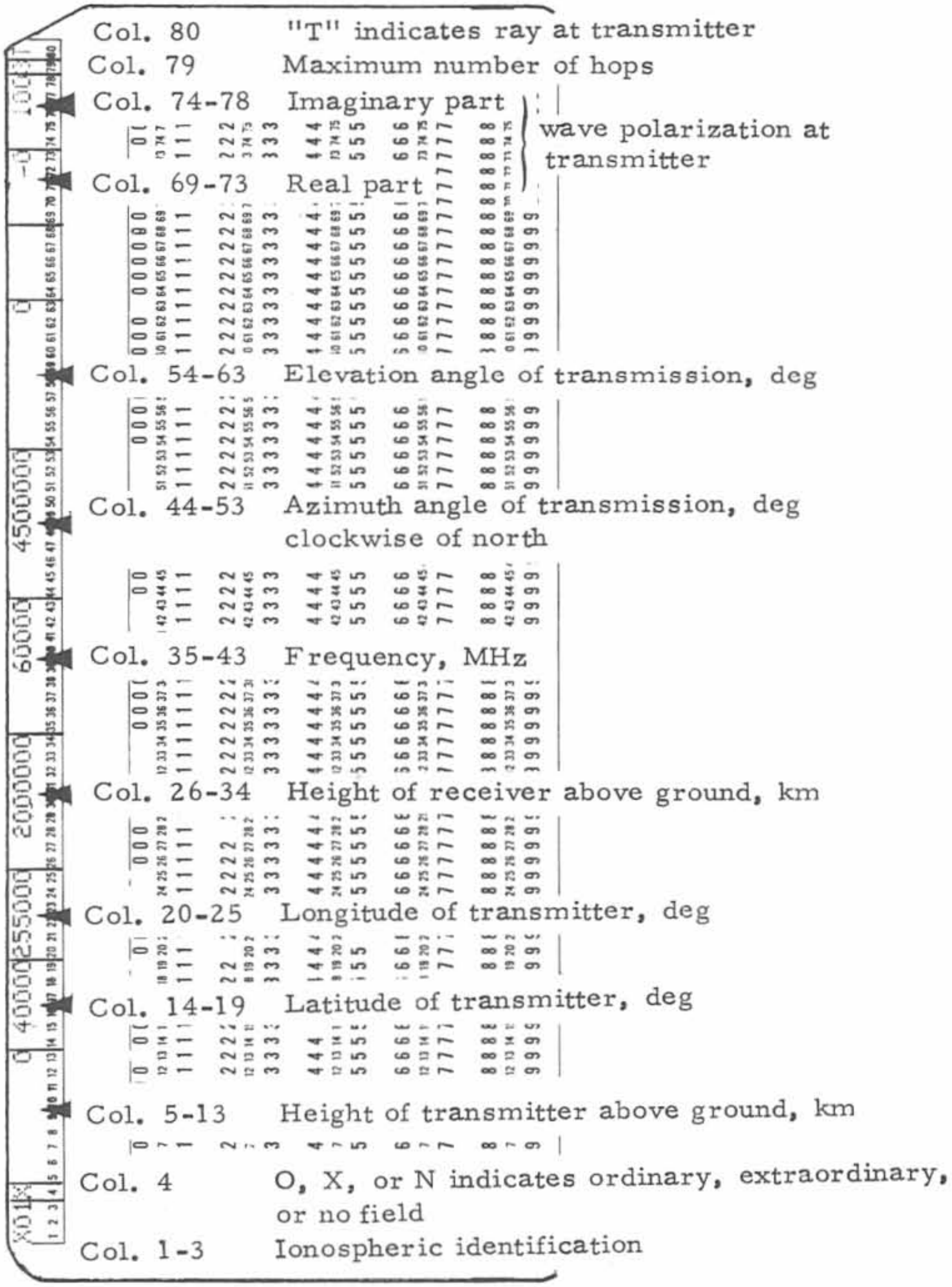
These cards are very useful as input data to other computer programs and for plotting the results of the ray tracing. In fact, these cards represent the most useful form of output for production ray path calculations. This method, called the rayset information-storage technique, was developed by Dr. T. A. Croft (Croft and Gregory, 1963) of Stanford University.

8.3 Plots of the Ray Path

A plot of the actual ray path, especially for very irregular ionospheres, can be helpful in understanding what sometimes seems like strange results in light of the input data. Thus, the program has an option for plotting, providing, of course, that the user has a plotter and plotting subroutines such as those described in appendix 7. The program can plot the projection of the ray path on any vertical plane or on the ground. The input parameter forms for plots of the ray path (appendix 1i) give more details. Appendix 8e contains sample plots of the raypath.

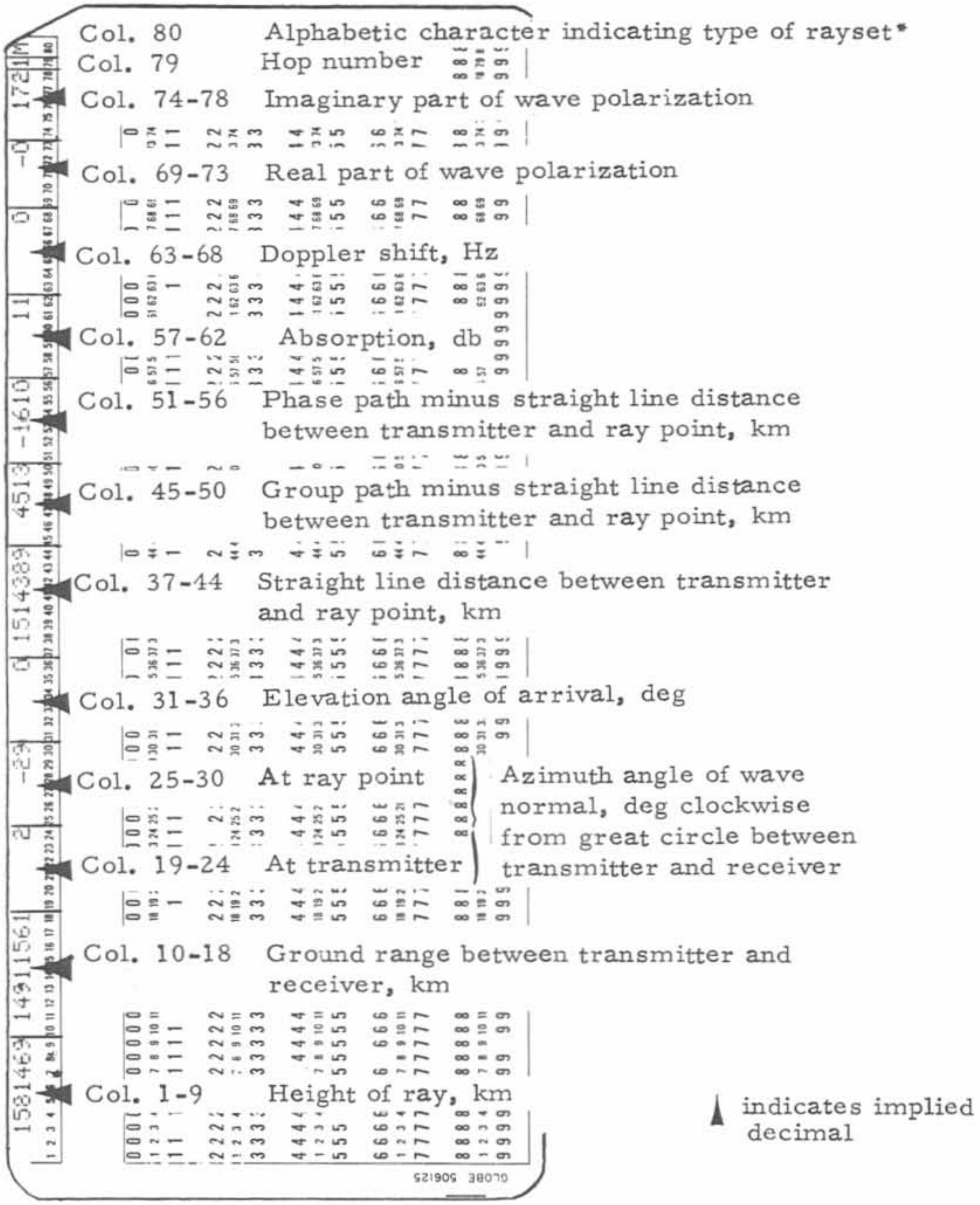
9. DECK SET UP

The versatility gained by having several versions of some of the subroutines is somewhat offset because the user must learn the deck set up in order to make necessary substitutions. Figure 3 shows the deck setup, including the subroutines that make up the main deck and those which are frequently exchanged with alternate versions. The order of the subroutines is unimportant.



▲ indicates implied decimal

Figure 1. Sample transmitter rayset.



*G ray ground reflected. The height punched out is the apogee height since the last ground reflect.

M ray made a closest approach to the receiver height

P ray penetrated

R ray at the receiver height. The height punched out is the height of the ray farthest from receiver height since last crossing of receiver height

S program reached maximum number of steps

Figure 2. Sample minimum distance rayset.

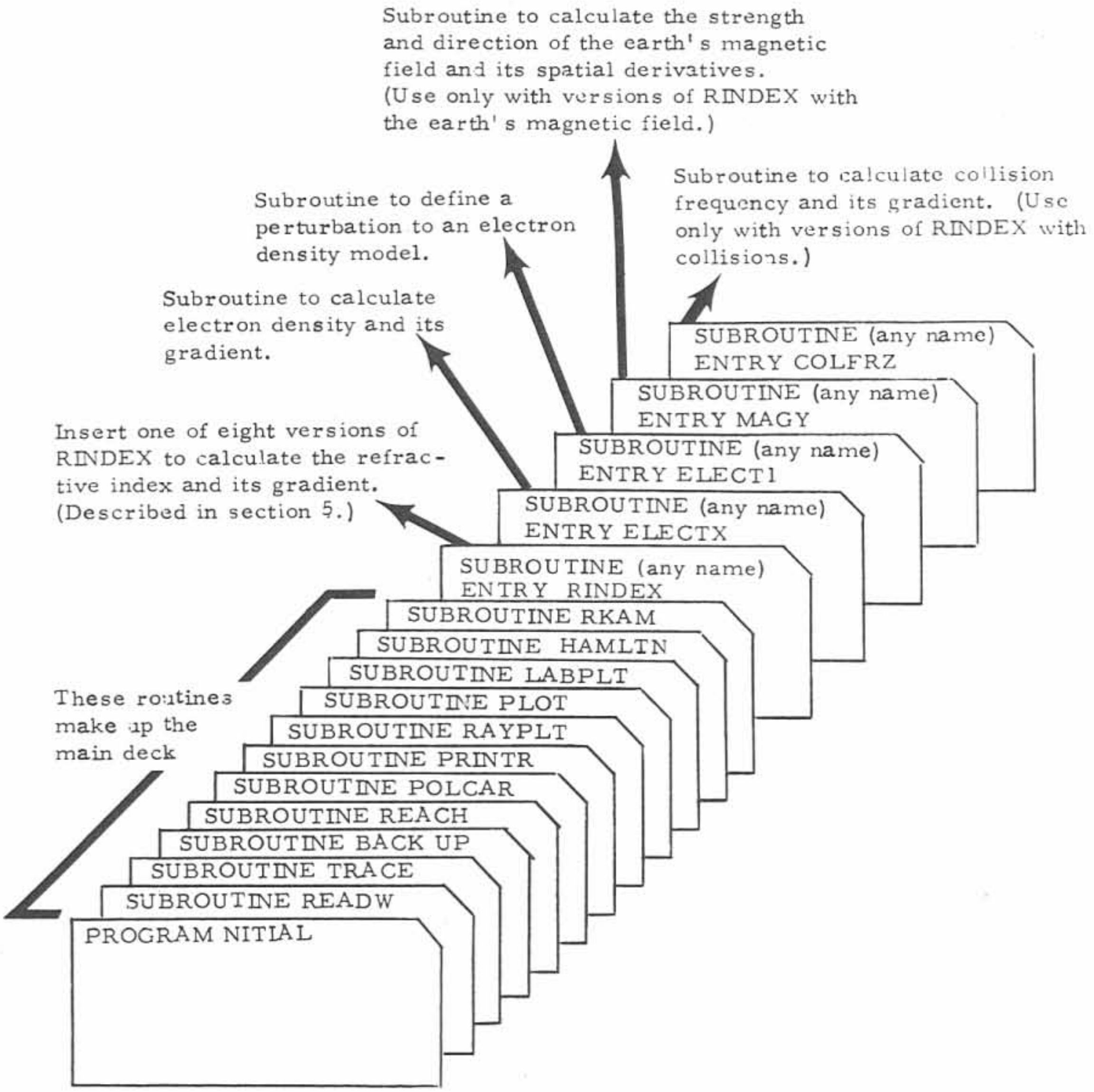


Figure 3. Program deck set-up

10. INPUT

The input data for a ray tracing program divide themselves naturally into two groups:

First, data that control the type of ray trace requested, such as the transmitter location and frequency, plus parameters describing analytic models of the ionosphere. Since there are few of these, efficiency in packing such data can be exchanged for versatility and ease of data handling. Therefore, by putting only one piece of data on each card, we gain the conveniences of reading in these data in any order and of having the program read in only those data that are different from those of the previous case. A number in the first three columns of each card identifies the data being read in. Table 2 defines the identifying numbers that are subscripts for a linear array, W. The last 56 columns of the card are available for comments.

We have also provided a method for conversion of units for input. The computer program needs angles in radians, whereas people usually like to use angles in degrees. The program is set up for angles in radians, but putting a "1" in column 18 allows the user to enter the angle in degrees and have the program make the conversion. A "1" in column 19 allows the user to enter central earth angles as the great circle distance along the ground in kilometers. (The program will calculate the latitude of a transmitter which is 500 km north of the equator, for instance.) The program expects distances in kilometers. A "1" in column 20 indicates a distance in nautical miles, and a "1" in column 21 indicates a distance in feet.

Appendix 8b contains a sample of how the cards are to be punched. If two or more cards have the same identifying number, the last one dominates. A card with the first three columns blank indicates the end of this type of data cards.

Table 2. Description of the Input Data for the W Array

W1	= 1. for ordinary ray = -1. for extraordinary ray
W2*	Radius of the earth in km
W3	Height of transmitter above the earth in km
W4	North geographic latitude of the transmitter
W5	East geographic longitude of the transmitter
W7	Initial frequency in MHz
W8	Final frequency in MHz
W9	Step in frequency in MHz (zero for a fixed frequency)
W11	Initial azimuth angle of transmission
W12	Final azimuth angle of transmission
W13	Step in azimuth angle of transmission (zero for a fixed azimuth)
W15	Initial elevation angle of transmission
W16	Final elevation angle of transmission
W17	Step in elevation angle of transmission (zero for a fixed elevation)
W20	Receiver height above the earth in km
W21	Nonzero to skip to the next frequency after the ray has penetrated the ionosphere
W22*	Maximum number of hops
W23*	Maximum number of steps per hop
W24*	North geographic latitude of the north geomagnetic pole
W25*	East geographic longitude of north geomagnetic pole
W41*	=1. for Runge-Kutta integration =2. for Adams-Moulton integration without error checking =3. for Adams-Moulton integration with relative error check =4. for Adams-Moulton integration with absolute error check
W42*	Maximum allowable single step error
W43*	Ratio of maximum single step error to minimum single step error
W44*	Initial integration step size in km (step in group path)
W45*	Maximum step length in km
W46*	Minimum step length in km
W47*	Factor by which to increase or decrease step length
W57	=1. to integrate, =2. to integrate and print phase path
W58	=1. to integrate, =2. to integrate and print absorption
W59	=1. to integrate, =2. to integrate and print doppler shift
w60	=1. to integrate, =2. to integrate and print path length
W71	Number of steps between periodic printout
W72	Nonzero to punch raysets on cards
W81	=0. to not plot ray path =1. to plot projection of ray path on a vertical plane =2. to plot projection of ray path on the ground
W82-88	Parameters used when plotting
W100-149	Parameters for analytic electron density models
W150-199	Parameters for perturbations to electron density models
W200-249	Parameters for analytic magnetic field models
W250-299	Parameters for analytic collision frequency models

*These values have been initialized in the main program but may be reset by reading them in. See Appendix 1b for the initial values.

A second group of input data cards are necessary if nonanalytic ionospheric models such as the electron density profile defined by subroutine TABLEX or the collision frequency profile defined by subroutine TABLEZ are used. Each subroutine defining a nonanalytic ionospheric model reads in data cards according to a format defined in that subroutine. An element in the W array controls the reading of these cards. (See table 2.) Figure 4 shows the order in which these data cards should be arranged.

11. ACCURACY

The numerical integration subroutine has a built-in mechanism to check errors and adjust the integration step length accordingly. If the errors get larger than a maximum specified by the user, the routine will decrease the step length in order to maintain the accuracy. On the other hand, if the accuracy is greater than that required by the user, the routine will increase the step length in order to reduce the computing cost. The user specifies the desired accuracy in W42 (see table 2). W42 is the maximum allowable relative error in any single step for any of the equations being integrated. To get a very accurate (but expensive) ray trace, one can use a small W42 (about 10^{-5} or 10^{-6}). For a cheap, approximate ray trace, one should use a large W42 (10^{-3} or even 10^{-2}). For cases in which all of the variables being integrated increase monotonically, the total relative error can be guaranteed to be less than W42. Otherwise, the total relative error cannot be easily estimated.

The far left column of the printout from the ray path calculation gives an indication of the integration error in the magnitude of the vector which points in the wave normal direction. Although the calculation of this error is made independently of the error calculation in the numerical integration routine, we have found that except near reflection for vertical or near vertical incidence this error is usually of the same order

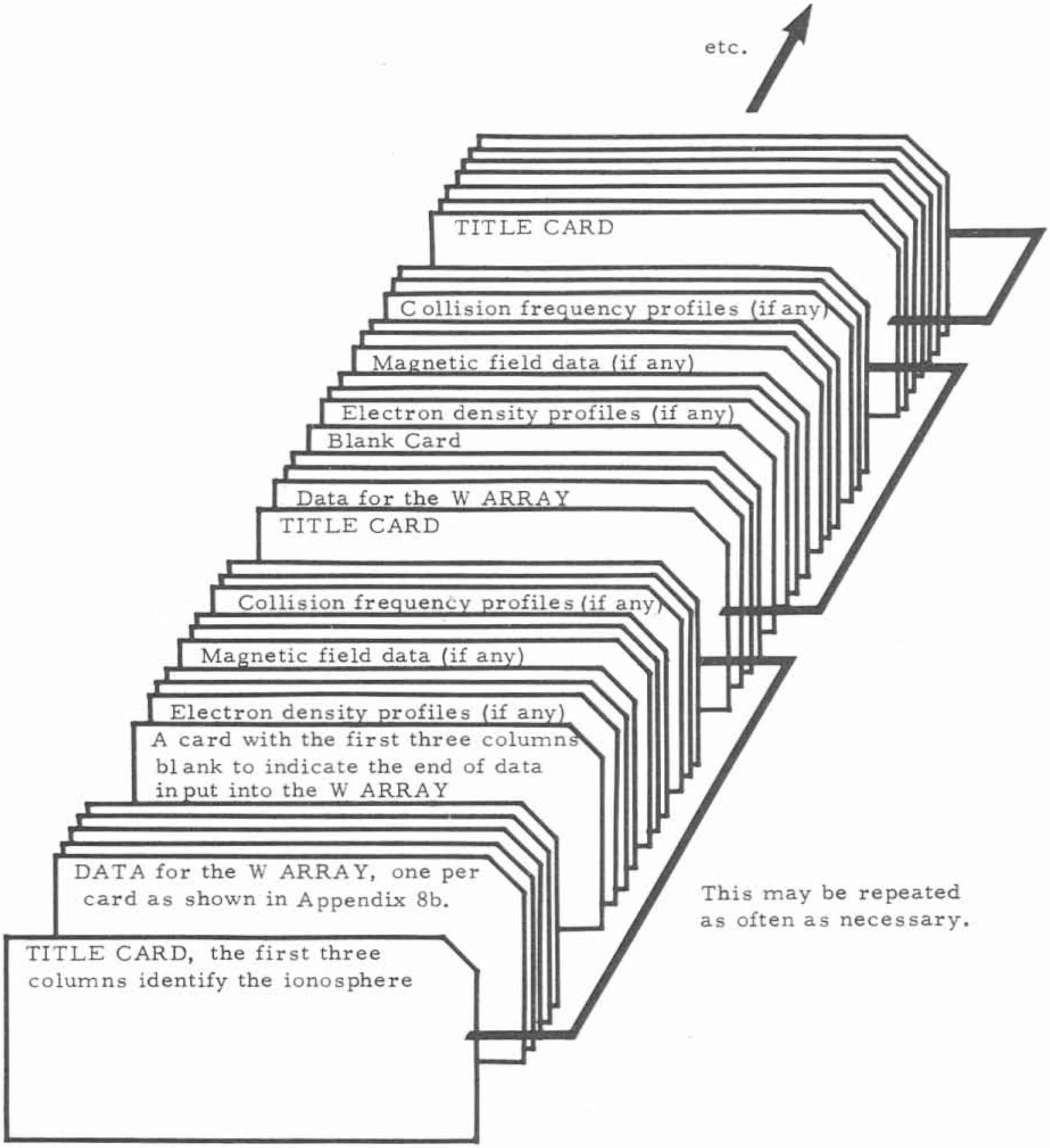


Figure 4. Data deck set-up.

of magnitude as that specified in W42. We have found that whenever this error has exceeded W42 by several orders of magnitude, the electron density subroutine we had written was calculating a gradient of electron density inconsistent with the spatial variation of electron density being calculated. See the general description of electron density models in Appendix 3a for more information.

12. COORDINATE SYSTEMS

The program uses two different spherical polar coordinate systems, namely, a geographic and a computational coordinate system. Input data for the coordinates of the transmitter (W4 and W5) and input data for the coordinates of the north pole of the computational coordinate system (W24 and W25) are entered in geographic coordinates. (Putting W25 equal to 0° and W24 equal to 90° would superimpose the two north poles and equate the two coordinate systems.)

When the two coordinate systems do not coincide, the three types of ionospheric models calculate electron density, the earth's magnetic field, and collision frequency in terms of the computational coordinate system. In particular, the dipole model of the earth's magnetic field uses the axis of the computational coordinate system as the axis for the dipole field. Thus, when using this dipole model, the computational coordinate system is a geomagnetic coordinate system, and both electron density and collision frequency must be defined in geomagnetic coordinates. Dudziak (1961) describes the transformations between these coordinate systems.

13. HOW THE PROGRAM WORKS

This ray tracing program consists of various subroutines that perform specific tasks in calculating ray paths. This division of labor facilitates modifying the program to solve specific problems. Often it may be necessary to change only one or two subroutines to convert the program to a different use.

The main program (NITIAL) sets up the initial conditions (transmitter location, wave frequency, and direction of transmission) for each ray trace. In setting up the initial conditions for each ray trace, the main program (NITIAL) steps frequency, azimuth angle of transmission, and elevation angle of transmission. The details of the workings of NITIAL can be found in the flow chart in figure 5. Then subroutine TRACE calculates one ray path for the requested number of crossings of the specified receiver height. Subroutine TRACE is the heart of the ray tracing program. It is the most complicated subroutine included, but also the most important to understand. The flow chart in figure 6 should help to explain TRACE.

Subroutine RKAM integrates the differential equations numerically using an Adams-Moulton predictor-corrector method with a Runge-Kutta starter. Subroutine HAMLTN evaluates the differential equations to be integrated. Subroutine RINDEX calculates the phase refractive index and its gradients, the group refractive index, and the polarization. (Eight versions of subroutine RINDEX are included.) Subroutines ELECTX, ELECT1, MAGY, and COLFRZ calculate the ionospheric electron density, perturbations to the electron density (irregularities), the earth's magnetic field, and the electron collision frequency, respectively. Several versions of these four subroutines are included and it is easy to add more. Subroutine REACH calculates a straight-line segment of a ray path in free space between the earth and the

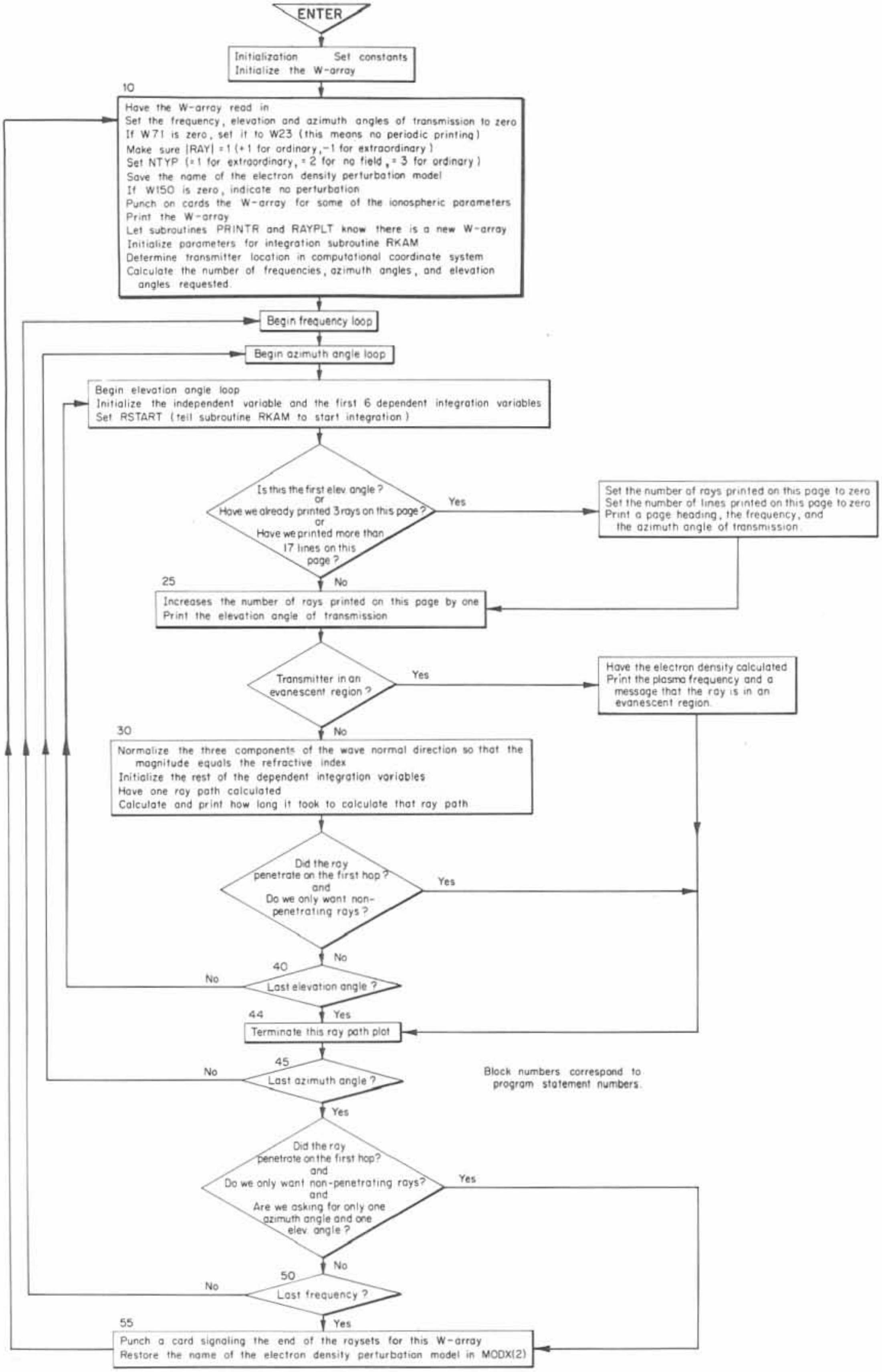
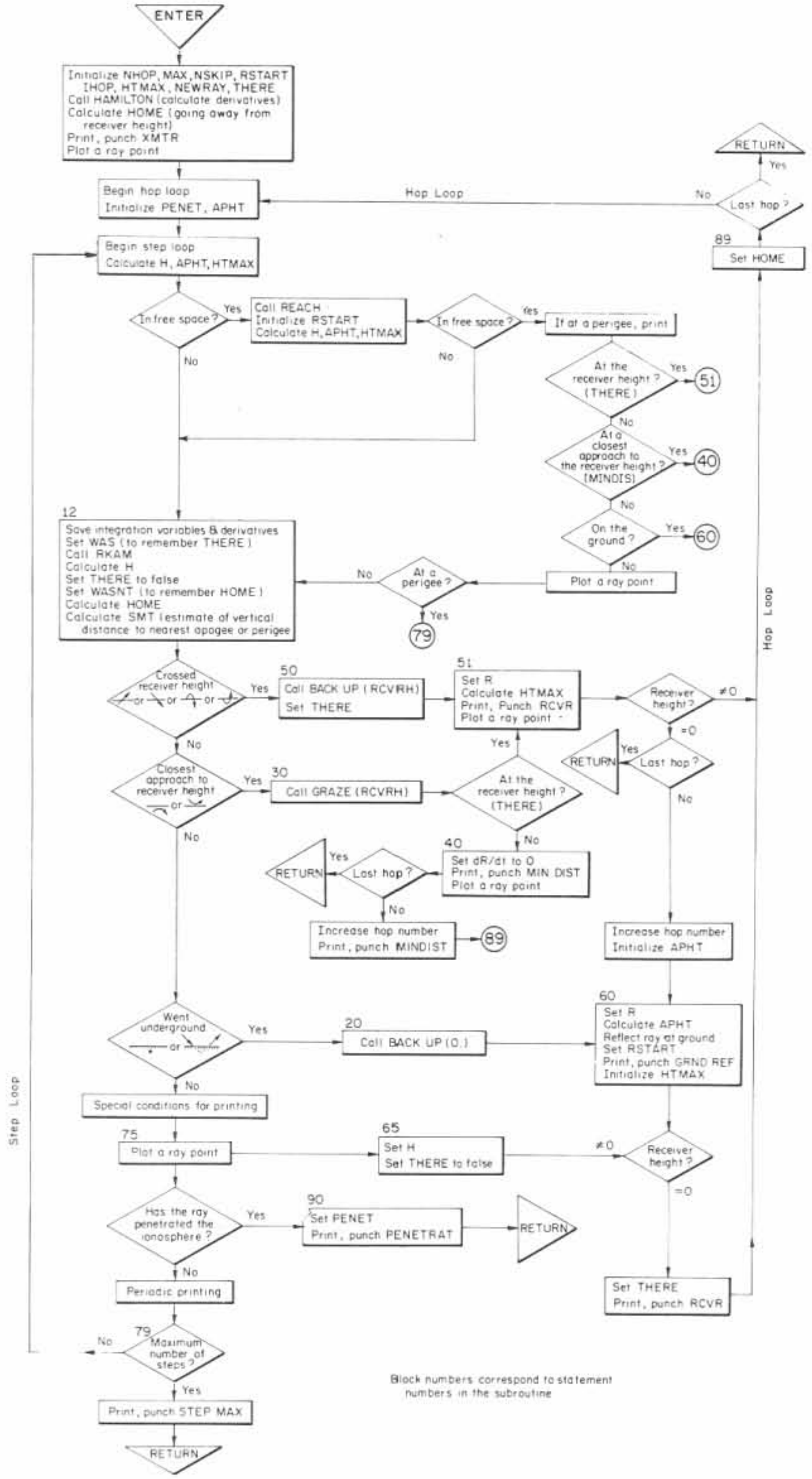


Figure 5. Flow chart for program NITIAL.



Block numbers correspond to statement numbers in the subroutine

Figure 6. Flow chart for subroutine TRACE.

The ray graphics illustrate the path of a ray during a single step.

ionosphere or between ionospheric layers. Subroutine BACK UP finds an intersection of the ray with the receiver height or with the ground. Subroutine PRINTR prints information describing the ray path and punches the results on cards (raysets). Subroutine RAYPLT plots the ray path. The block diagram in figure 7 shows the relationship among these (and other) subroutines.

The listings of most of the subroutines have comments that should help in understanding how they work. In addition, Tables 3 through 14 define the variables in the common blocks.

14. ACKNOWLEDGMENTS

Part of the organization of this program into subroutines follows that of the program of Dudziak (1961), in particular for subroutines RKAM, HAMLTN , RINDEX, ELECTX, MAGY, and COLFRZ. Also, the coordinate transformation in subroutine PRINTR and the method for data input via the W array are taken from the program of Dudziak (1961). The term "rayset," the idea of punching results of each hop for each ray trace onto cards, and the idea of automatically plotting ray paths come from the program of Croft and Gregory (1963). The quasi-parabolic layer electron density model QPARAB is taken from the paper by Croft and Hoogasian (1968). Notice that the quasi-parabolic layer that is now in the program is slightly different from the one in the program of Jones (1966). Subroutine RKAM is a modification of subroutine RKAMSUB, which was written by G. J. Lastman and is available through the CDC CO-OP library (the CO-OP identification is D2 UTEX RKAMSUB). Subroutine GAUSEL was written by L. David Lewis, Space Environment Laboratory, National Oceanic and Atmospheric Administration. Subroutine FSW was written in conjunction with Helmut Kopka of the Max-Planck-Institut für Aeronomie, Lindau/Harz, Germany.

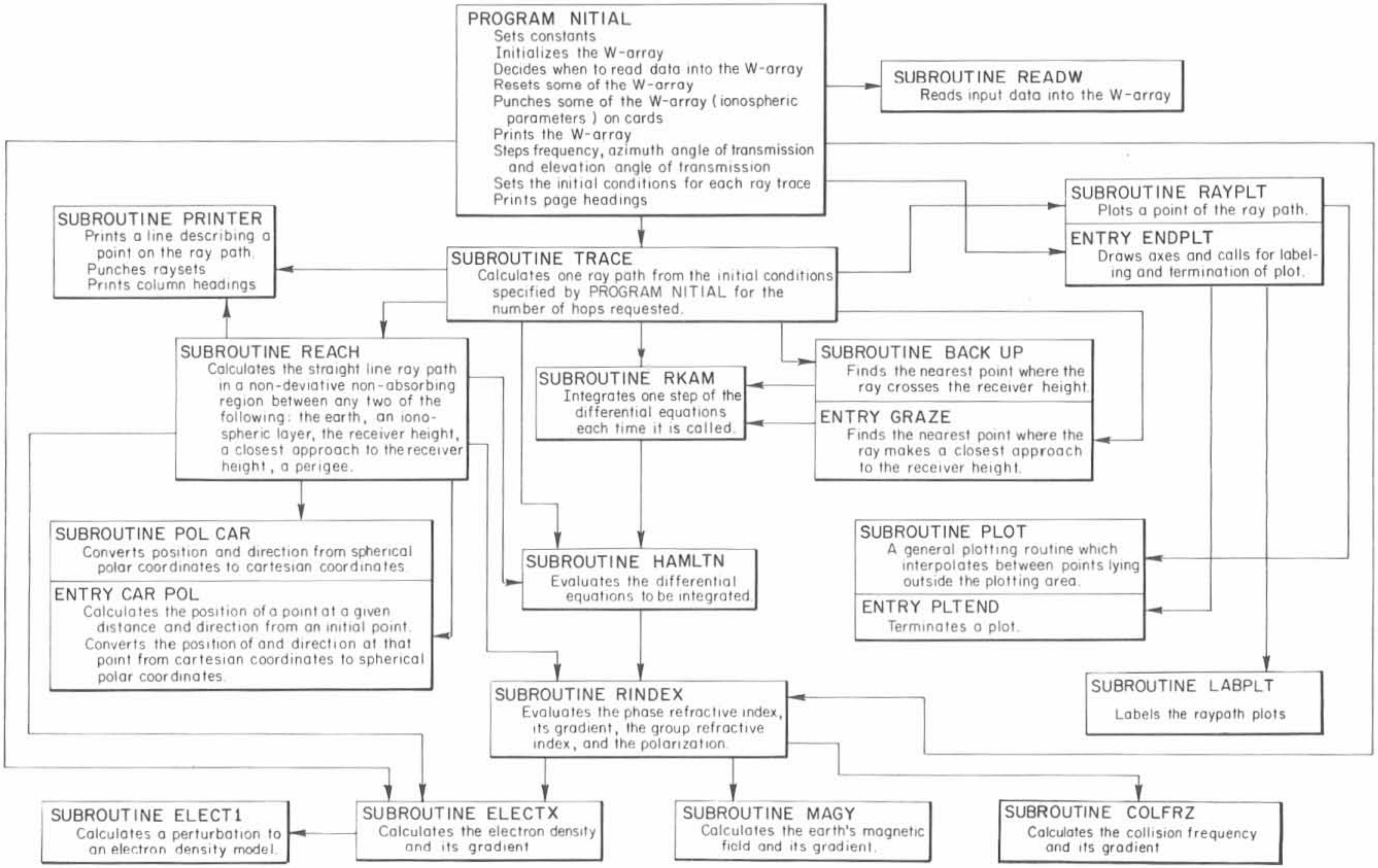


Figure 7. Block diagram for the ray tracing program.

Table 3. Definitions of the Parameters in Blank Common

Position in Common	Variable Name	Definition
1-20	R	The dependent variables in the differential equations being integrated--the definitions of the first six are fixed, but the others may be varied by the program user.
1	R(1)	r
2	R(2)	θ
3	R(3)	φ
4	R(4)	k_r
5	R(5)	k_θ
6	R(6)	k_φ
7-12	R(7)-R(12)	Those variables the user has chosen to integrate, taken in the following order: P -phase path in kilometers A -absorption in decibels Δf -Doppler shift in hertz s -geometrical path length in kilometers
13-20	R(13)-R(20)	Reserved for future expansion.
21	T	Group path in kilometers (the independent variable in the differential equations).
22	STP	Step length in group path.
23-42	DRDT	The derivatives of the dependent variables with respect to the independent variable T.

R and T are initialized in program NITIAL and changed in subroutines RKAM, REACH, and BACK UP.

STP is calculated in subroutine RKAM.

DRDT is calculated in subroutine HAMLTN and used in subroutine RKAM.

Table 4. Definitions of the Parameters in Common Block /CONST/

Position in Common	Variable Name	Definition
1	PI	π
2	PIT2	2π
3	PID2	$\pi/2$
4	DEGS	$180.0/\pi$
5	RAD	$\pi/180.0$
6	K	Ratio of the square of the plasma frequency to the electron density in $\mathrm{MHz}^2\mathrm{cm}^3 = r_o c^2/\pi = e^2/(4\pi^2 \varepsilon_o m)$, where r_o is the classical electron radius, c is the free space speed of light, e is the charge on the electron, m is the mass of the electron, and ε_o is the capacitivity of a vacuum.
7	C	Free space speed of light in km/sec.
8	LOGTEN	$\log_e 10$

These parameters are set in program NITIAL.

Table 5. Definitions of the Parameters in Common Block /RK/

Position in Common	Variable Name	Definition
1	N	The number of equations being integrated.
2	STEP	The initial step in group path in kilometers.
3	MODE	Defines type of integration used (same as W41), see Table 2.
4	E1MAX	Maximum allowable single step error (same as W42).
5	E1MIN	Minimum allowable single step error (= W42/W43).
6	E2MAX	Maximum step length (same as W45).
7	E2MIN	Minimum step·length (same as W46).
8	FACT	Factor by which to increase or decrease step length (same as W47).
9	RSTART	Nonzero to initialize numerical integration, zero to continue integration.

These parameters are calculated in program NITIAL (some are temporarily reset in subroutine BACK UP) and are used in subroutine RKAM.

Table 6. Definition of the Parameters in Common Block /RIN/

Postion in Common	Variable Name	Definition
1, 2, 3	MODRIN	Description of version of RINDEX in BCD.
4	COLL	= 1 if this version of RINDEX includes collisions, = 0 otherwise.
5	FIELD	= 1 if this version of RINDEX includes the earth's magnetic field, = 0 otherwise.
6	SPACE	TRUE, if the ray is in a nondeviative, nonabsorbing medium.
7, 8	KAY2	k^2, square of the complex phase refractive index times ω^2/c^2.
9, 10	H	Hamiltonian (complex)
11, 12	PHPT	$\partial H/\partial t$ (complex)
13, 14	PHPR	$\partial H/\partial r$ (complex)
15, 16	PHPTH	$\partial H/\partial \theta$ (complex)
17, 18	PHPPH	$\partial H/\partial \varphi$ (complex)
19, 20	PHPOM	$\partial H/\partial \omega$ (complex)
21, 22	PHPKR	$\partial H/\partial k_r$ (complex)
23, 24	PHPKTH	$\partial H/\partial k_\theta$ (complex)
25, 26	PHPKPH	$\partial H/\partial k_\varphi$ (complex)
27, 28	KPHPK	$\vec{k} \cdot \partial H/\partial \vec{k}$ (complex) $= k_r \partial H/\partial k_r + k_\theta \partial H/\partial k_\theta + k_\varphi \partial H/\partial k_\varphi$
29, 30	POLAR	Characteristic polarization of the wave; equal to the ratio of the component of the electric field perpendicular with the earth's magnetic field to the transverse component of the electric field parallel with the earth's magnetic field (complex) (Budden, 1961, p. 49, eq. (5.13)).

Table 6. (Continued)

Position in Common	Variable Name	Definition
31, 32	LPOLAR	Characteristic longitudinal polarization of the wave; equal to the ratio of the longitudinal component of the electric field to the component of the electric field perpendicular with the earth's magnetic field. (complex) Budden, 1961, p. 54, eq. (5. 38)).
33	SGN	= +1 or -1; used for ray tracing in complex space.

These parameters are calculated in subroutine RINDEX and used in subroutine HAMLTN.

Note: In some subroutines, the real and imaginary parts of the complex variables have separate names.

Table 7. Definitions of the Parameters in Common Block /FLG/
(See Subroutine TRACE)

Position in Common	Variable Name	Definition
1	NTYP	= 1 for extraordinary, = 2 for no field, = 3 for ordinary
2	NEWWR	Set equal to . TRUE. to tell subroutine RAYPLT there is a new W array.
3	NEWWP	Set equal to . TRUE. to tell subroutine PRINTR there is a new W array.
4	PENET	Set equal to . TRUE. if the ray just penetrated.
5	LINES	Number of lines printed on the current page.
6	IHOP	Hop number (at the beginning of each ray, subroutine TRACE sets this parameter to zero so that subroutine RAYPLT will begin a new line in plotting the ray path and subroutine PRINTR will print column headings and punch a transmitter rayset).
7	HPUNCH	The height to be punched on the raysets.

Table 8. Definitions of the Parameters in Common Block /XX/

Position in Common	Variable Name	Definition
1	MODX(1)	BCD name of the electron density subroutine.
2	MODX(2)	BCD name of the subroutine defining a perturbation to the electron density model.
3	X	X in Appleton-Hartree formula, square of the ratio of the plasma frequency to the wave frequency.
4	PXPR	$\frac{\partial X}{\partial r}$
5	PXPTH	$\frac{\partial X}{\partial \theta}$
6	PXPPH	$\frac{\partial X}{\partial \varphi}$
7	PXPT	$\frac{\partial X}{\partial t}$, where t is time; used for calculating Doppler shifts.
8	HMAX	Height of maximum electron density.

These parameters are calculated in subroutine ELECTX, possibly modified in subroutine ELECT1, and are mainly used in subroutine RINDEX.

Table 9. Definitions of the Parameters in Common Block /YY/

Position in Common	Variable Name	Definition
1	MODY	BCD name of the subroutine defining the earth's magnetic field.
2	Y	Y in the Appleton-Hartree formula, ratio of the electron gyrofrequency to the wave frequency.
3	PYPR	$\frac{\partial Y}{\partial r}$
4	PYPTH	$\frac{\partial Y}{\partial \theta}$
5	PYPPH	$\frac{\partial Y}{\partial \varphi}$
6	YR	Y_r, proportional to the component of the earth's magnetic field in the r direction.
7	PYRPR	$\frac{\partial Y_r}{\partial r}$
8	PYRPT	$\frac{\partial Y_r}{\partial \theta}$
9	PYRPP	$\frac{\partial Y_r}{\partial \varphi}$
10	YTH	Y_θ
11	PYTPR	$\frac{\partial Y_\theta}{\partial r}$
12	PYTPT	$\frac{\partial Y_\theta}{\partial \theta}$

Table 9. (Continued)

Position in Common	Variable Name	Definition
13	PYTPP	$\frac{\partial Y_\theta}{\partial \varphi}$
14	YPH	Y_φ
15	PYPPR	$\frac{\partial Y_\varphi}{\partial r}$
16	PYPPT	$\frac{\partial Y_\varphi}{\partial \theta}$
17	PYPPP	$\frac{\partial Y_\varphi}{\partial \varphi}$

These parameters are calculated in subroutine MAGY and are mainly used in subroutine RINDEX.

Table 10. Definitions of the Parameters in Common Block /ZZ/

Position in Common	Variable Name	Definition
1	MODZ	BCD name of the collision frequency subroutine.
2	Z	Z in the Appleton-Hartree formula, ratio of the electron-neutral collision frequency to the angular wave frequency.
3	PZPR	$\frac{\partial Z}{\partial r}$
4	PZPTH	$\frac{\partial Z}{\partial \theta}$
5	PZPPH	$\frac{\partial Z}{\partial \varphi}$

These parameters are calculated in subroutine COLFRZ and are mainly used in subroutine RINDEX.

Table 11. Definitions of the Parameters in Common Block /TRAC/

Position in Common	Variable Name	Definition
1	GROUND	.TRUE. if the ray is on the surface of the earth.
2	PERIGE	.TRUE. if the ray has just made a perigee.
3	THERE	.TRUE. if the ray is at the receiver height.
4	MINDIS	.TRUE. if the ray has just made a closest approach to the receiver height.
5	NEWRAY	Set equal to .TRUE. to tell subroutine REACH that this is a new ray.
6	SMT	An estimation of the vertical distance to an apogee or perigee of the ray.

These parameters are used for communication between subroutine TRACE and subroutines REACH and BACK UP.

Table 12. Definition of the Parameter in Common Block /COORD/

Position in Common	Variable Name	Definition
1	S	The straight line distance along the ray from the position of the ray where REACH was called to the present position.

This parameter is used for communication between subroutine REACH and subroutine POL CAR.

Table 13. Definitions of the Parameters in Common Block /PLT/

Position in Common	Variable Name	Definition
1	XMIN0, XL	The x coordinate of the left side of the plotting area in kilometers.
2	XMAX0, XR	The x coordinate of the right side of the plotting area in kilometers.
3	XMIN0, YB	The y coordinate of the bottom of the plotting area in kilometers.
4	YMAX0, YT	The y coordinate of the top of the plotting area in kilometers.
5	RESET	Set equal to one whenever the plotting area is changed.

These parameters are used for communication between subroutine RAYPLT and subroutine PLOT.

Table 14. Definitions of the Parameters in Common Block /DD/

Position in Common	Variable Name	Definition
1	IN	Intensity. IN = 0 specifies normal intensity. IN = 1 specifies high intensity.
2	IOR	Orientation. IOR = 0 specifies upright orientation. IOR = 1 specifies rotated orientation (90° counterclockwise).
3	IT	Italics (Font). IT = 0 specifies non-Italic (Roman) symbols. IT = 1 specifies Italic symbols.
4	IS	Symbol size. IS = 0 specifies miniature size. IS = 1 specifies small size. IS = 2 specifies medium size. IS = 3 specifies large size.
5	IC	Symbol case. IC = 0 specifies upper case. IC = 1 specifies lower case.
6	ICC	Character code, 0-63 (R1 format). ICC and IC together specify the symbol plotted.
7	IX	X-coordinate, 0-1023.
8	IY	Y-coordinate, 0-1023.

We also want to thank those who have used our program and have pointed out errors or made suggestions. In particular, we are grateful to Dr. T. M. Georges of the Wave Propagation Laboratory, National Oceanic and Atmospheric Administration, for his suggestions resulting from extensive use of the program, for development of some of the ionospheric models (DCHAPT, DTORUS, WAVE, WAVE2), and for financing part of the development of ray tracing through a spitze.

Examples of use of the ray tracing program are shown in the reports by Stephenson and Georges (1969) and Georges (1971).

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APPENDIX 1. LISTINGS OF THE MAIN PROGRAM AND SUBROUTINES IN THE MAIN DECK

This appendix contains listings of the main program and those subroutines that have only one version (with the exception of subroutine RAYPLT, which has a do-nothing version for users lacking a plotter to plot ray paths). Thus, the routines which form the contents of this appendix will be used in all ray path calculations.

Additionally, this appendix contains the main input parameter form for ray tracing and the input parameter forms for plotting. These forms are very useful when using the program because they indicate the input parameters needed for ray path calculations

The contents of this appendix are arranged as follows:

INPUT PARAMETER FORM
FOR THREE-DIMENSIONAL RAY PATHS

Name________________________ Project No.__________ Date__________

Ionospheric ID (3 characters) ________

Title (75 characters) ____

Models: Electron density ________
Perturbation ________
Magnetic field ________
Ordinary ________ (W1 = + 1.)
Extraordinary ________ (W1 = - 1.)
Collision frequency ________

Transmitter: Height ________ km, nautical miles, feet (W3)
Latitude ________ rad, deg, km (W4)
Longitude ________ rad, deg, km (W5)
Frequency, initial ________ MHz (W7)
final ________ (W8)
step ________ (W9)
Azimuth angle, initial ________ rad, deg clockwise of north (W11)
final ________ (W12)
step ________ (W13)
Elevation angle, initial ________ rad, deg (W15)
final ________ (W16)
step ________ (W17)

Receiver: Height ________ km, nautical miles, feet (W20)

Penetrating rays: Wanted ________ (W21 = 0.)
Not wanted ________ (W21 = 1.)

Maximum number of hops ________ (W22)

Maximum number of steps per hop ________ (W23)

Maximum allowable error per step ________ (W42)

Additional calculations: = 1. to integrate
= 2. to integrate and print

Phase path ________ (W57)
Absorption ________ (W58)
Doppler shift ________ (W59)
Path length ________ (W60)
Other ________
____
____

Printout: Every ________ steps of the ray trace (W71)

Punched cards (raysets): ________ (W72 = 1.)

```
      PROGRAM NITIAL                                                    NITI001
C           SETS THE INITIAL CONDITIONS FOR EACH RAY AND CALLS TRACE    NITI002
      DIMENSION MFLD(2)                                                 NITI003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,C,LOGTEN                   NITI004
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             NITI005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,N2I,PNP(10),POLAR,     NITI006
     1             LPOLAR,SGN                                           NITI007
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       NITI008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  NITI009
      COMMON /YY/ MODY,Y(16) /ZZ/ MODZ,Z(4)                             NITI010
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)                 NITI011
      EQUIVALENCE (RAY,W(1)),(EARTHR,W(2)),(XMTRH,W(3)),(TLAT,W(4)),    NITI012
     1 (TLON,W(5)),(F,W(6)),(FBEG,W(7)),(FEND,W(8)),(FSTEP,W(9)),       NITI013
     2 (AZ1,W(10)),(AZBEG,W(11)),(AZEND,W(12)),(AZSTEP,W(13)),          NITI014
     3 (BETA,W(14)),(ELBEG,W(15)),(ELEND,W(16)),(ELSTEP,W(17)),         NITI015
     4 (ONLY,W(21)),(HOP,W(22)),(MAXSTP,W(23)),(PLAT,W(24)),(PLON,W(25))NITI016
     5,(INTYP,W(41)),(MAXERR,W(42)),(ERATIO,W(43)),(STEP1,W(44)),       NITI017
     6 (STPMAX,W(45)),(STPMIN,W(46)),(FACTR,W(47)),(SKIP,W(71)),        NITI018
     7 (RAYSET,W(72)),(PLT,W(81)),(PERT,W(150))                         NITI019
      LOGICAL SPACE,NEWWR,NEWWP,PENET                                   NITI020
      REAL N2,N2I,LOGTEN,K,MAXSTP,INTYP,MAXERR,MU                       NITI021
      COMPLEX PNP,POLAR,LPOLAR                                          NITI022
      NDATE=IDATE(0)                                                    NITI025
      SECOND=KLOCK(0)*.001                                              NITI026
      KOLL=4H  NO                                                       NITI027
      IF (COLL.NE.0.) KOLL=4HWITH                                       NITI028
C********* CONSTANTS                                                    NITI029
      PI=3.1415926536                                                   NITI030
      PIT2=2.*PI                                                        NITI031
      PID2=PI/2.                                                        NITI032
      DEGS=180./PI                                                      NITI033
      RAD=PI/180.                                                       NITI034
      C=2.997925E5                                                      NITI035
      K=2.81785E-15*C**2/PI                                             NITI036
      LOGTEN=ALOG(10.)                                                  NITI037
C********* INITIALIZE SOME VARIABLES IN THE W ARRAY                     NITI038
      DO 5 NW=1,400                                                     NITI039
    5 W(NW)=0.                                                          NITI040
      PLON=0.                                                           NITI041
      PLAT=PID2                                                         NITI042
      EARTHR=6370.                                                      NITI043
      INTYP=3.                                                          NITI044
      MAXERR=1.E-4                                                      NITI045
      ERATIO=50.                                                        NITI046
      STEP1=1.                                                          NITI047
      STPMAX=100.                                                       NITI048
      STPMIN=1.E-8                                                      NITI049
      FACTR=0.5                                                         NITI050
      MAXSTP=1000.                                                      NITI051
      HOP=1.                                                            NITI052
C********* READ W ARRAY AND PRINT NON-ZERO VALUES                       NITI053
   10 CALL READ W                                                       NITI054
      F=BETA=AZ1=0.                                                     NITI055
      IF (SKIP.EQ.0.) SKIP=MAXSTP                                       NITI056
   12 RAY=SIGN(1.,RAY)                                                  NITI057
      NTYP=2.+FIELD*RAY                                                 NITI058
      GO TO (13,14,15), NTYP                                            NITI059
   13 MFLD(1)=8HEXTRAORD                                                NITI060
      MFLD(2)=5HINARY                                                   NITI061
      GO TO 16                                                          NITI062
   14 MFLD(1)=8HNO FIELD                                                NITI063
      MFLD(2)=1H                                                        NITI064
      GO TO 16                                                          NITI065
```

```
   15 MFLD(1)=8HORDINARY                                                NITI066
      MFLD(2)=8H                                                        NITI067
   16 MODSAV=MODX(2)                                                    NITI068
      IF (PERT.EQ.0.) MODX(2)=6H                                        NITI069
      IF (RAYSET.NE.0.) PUNCH 2000, ID,MODX(1),(W(NW),NW=101,107),      NITI070
     1 MODX(2),(W(NW),NW=151,157),MODY,(W(NW),NW=201,207),              NITI071
     2 MODZ,(W(NW),NW=251,257)                                          NITI072
 2000 FORMAT (10A8,4(/A8,2X,7E10.3))                                    NITI073
      PRINT 1000, ID,NDATE,MODX,MODY,MODZ,MODRIN,MFLD,KOLL              NITI074
 1000 FORMAT (1H1,10A8,25X,A8/4(1X,A8),24X,3A8,1X,A8,A5,1X,A4,          NITI075
     1 11H COLLISIONS/)                                                 NITI076
      PRINT 1050                                                        NITI077
 1050 FORMAT (85H INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIANNITI078
     1S, ONLY NONZERO VALUES PRINTED/)                                  NITI079
      DO 17 NW=1,400                                                    NITI080
      IF (W(NW).NE.0.) PRINT 1700, NW,W(NW)                             NITI081
 1700 FORMAT (I4,E19.11)                                                NITI082
   17 CONTINUE                                                          NITI083
C********* LET SUBROUTINES PRINTR AND RAYPLT KNOW THERE IS A NEW W ARRAYNITI084
      NEWWP=.TRUE.                                                      NITI085
      NEWWR=.TRUE.                                                      NITI086
C********* INITIALIZE PARAMETERS FOR INTEGRATION SUBROUTINE RKAM        NITI087
      N=6                                                               NITI088
      DO 20 NR=7,20                                                     NITI089
      IF (W(50+NR).NE.0.) N=N+1                                         NITI090
   20 CONTINUE                                                          NITI091
      MODE=INTYP                                                        NITI092
      STEP=STEP1                                                        NITI093
      E1MAX=MAXERR                                                      NITI094
      E1MIN=MAXERR/ERATIO                                               NITI095
      E2MAX=STPMAX                                                      NITI096
      E2MIN=STPMIN                                                      NITI097
      FACT=FACTR                                                        NITI098
C********* DETERMINE TRANSMITTER LOCATION IN COMPUTATIONAL COORDINATE   NITI099
C********* SYSTEM (GEOMAGNETIC COORDINATES IF DIPOLE FIELD IS USED)     NITI100
      R0=EARTHR+XMTRH                                                   NITI101
      SP=SIN (PLAT)                                                     NITI102
      CP=SIN (PID2-PLAT)                                                NITI103
      SDPH=SIN (TLON-PLON)                                              NITI104
      CDPH=SIN (PID2-(TLON-PLON))                                       NITI105
      SL=SIN (TLAT)                                                     NITI106
      CL=SIN (PID2-TLAT)                                                NITI107
      ALPHA=ATAN2(-SDPH*CP,-CDPH*CP*SL+SP*CL)                           NITI108
      TH0=ACOS (CDPH*CP*CL+SP*SL)                                       NITI109
      PH0=ATAN2(SDPH*CL,CDPH*SP*CL-CP*SL)                               NITI110
C********* LOOP ON FREQUENCY, AZIMUTH ANGLE, AND ELEVATION ANGLE        NITI111
      NFREQ=1                                                           NITI112
      IF (FSTEP.NE.0.) NFREQ=(FEND-FBEG)/FSTEP+1.5                      NITI113
      NAZ=1                                                             NITI114
      IF (AZSTEP.NE.0.) NAZ=(AZEND-AZBEG)/AZSTEP+1.5                    NITI115
      NBETA=1                                                           NITI116
      IF (ELSTEP.NE.0.) NBETA=(ELEND-ELBEG)/ELSTEP+1.5                  NITI117
      DO 50 NF=1,NFREQ                                                  NITI118
      F=FBEG+(NF-1)*FSTEP                                               NITI119
      DO 45 J=1,NAZ                                                     NITI120
      AZ1=AZBEG+(J-1)*AZSTEP                                            NITI121
      AZA=AZ1*DEGS                                                      NITI122
      GAMMA=PI-AZ1+ALPHA                                                NITI123
      SGAMMA=SIN (GAMMA)                                                NITI124
      CGAMMA=SIN (PID2-GAMMA)                                           NITI125
      DO 40 I=1,NBETA                                                   NITI126
      BETA=ELBEG+(I-1)*ELSTEP                                           NITI127
```

```
      EL=BETA*DEGS                                                      NITI128
      CBETA=SIN (PID2-BETA)                                             NITI129
      R(1)=R0                                                           NITI130
      R(2)=TH0                                                          NITI131
      R(3)=PH0                                                          NITI132
      R(4)=SIN (BETA)                                                   NITI133
      R(5)=CBETA*CGAMMA                                                 NITI134
      R(6)=CBETA*SGAMMA                                                 NITI135
      T=0.                                                              NITI136
      RSTART=1.                                                         NITI137
C     SGN=1.  (NEED FOR RAY TRACING IN COMPLEX SPACE.)                  NITI138
C********* ALLOW IONOSPHERIC MODEL SUBROUTINES TO READ AND PRINT DATA   NITI139
      CALL RINDEX                                                       NITI140
      IF (I.NE.1.AND.NPAGE.LT.3.AND.LINES.LE.17) GO TO 25               NITI141
      NPAGE=LINES=0                                                     NITI142
      PRINT 1000, ID,NDATE,MODX,MODY,MODZ,MODRIN,MFLD,KOLL              NITI143
      PRINT 2400, F,AZA                                                 NITI144
 2400 FORMAT (18X,11HFREQUENCY =,F12.6,37H MHZ, AZIMUTH ANGLE OF TRANSMINITI145
     1SSION =,F12.6,4H DEG)                                             NITI146
   25 NPAGE=NPAGE+1                                                     NITI147
      PRINT 2500, EL                                                    NITI148
 2500 FORMAT (/31X,33HELEVATION ANGLE OF TRANSMISSION =,F12.6,4H DEG/)  NITI149
      IF (N2.GT.0.) GO TO 30                                            NITI150
      CALL ELECTX                                                       NITI151
      FN=SIGN (SQRT (ABS (X))*F,X)                                      NITI152
      PRINT 2900, FN                                                    NITI153
 2900 FORMAT (58H0TRANSMITTER IN EVANESCENT REGION, TRANSMISSION IMPOSSINITI154
     1BLE/20H0PLASMA FREQUENCY = ,E17.10)                               NITI155
      GO TO 44                                                          NITI156
   30 MU=SQRT (N2/(R(4)**2+R(5)**2+R(6)**2))                            NITI157
      DO 34 NN=4,6                                                      NITI158
   34 R(NN)=R(NN)*MU                                                    NITI159
      DO 35 NN=7,N                                                      NITI160
   35 R(NN)=0.                                                          NITI161
      CALL TRACE                                                        NITI162
      OSEC=SECOND                                                       NITI163
      SECOND=KLOCK(0)*.001                                              NITI164
      DIFF=SECOND-OSEC                                                  NITI165
      PRINT 3500, DIFF                                                  NITI166
 3500 FORMAT (36X,26HTHIS RAY CALCULATION TOOK ,F8.3,4H SEC)            NITI167
      IF (PENET.AND.ONLY.NE.0..AND.IHOP.EQ.1) GO TO 44                  NITI168
   40 CONTINUE                                                          NITI169
   44 IF (PLT.NE.0.) CALL ENDPLT                                        NITI170
   45 CONTINUE                                                          NITI171
      IF(PENET.AND.ONLY.NE.0..AND.IHOP.EQ.1.AND.NAZ.EQ.1.AND.NBETA.EQ.1)NITI172
     1 GO TO 55                                                         NITI173
   50 CONTINUE                                                          NITI174
   55 IF (RAYSET.NE.0.) PUNCH 5000                                      NITI175
 5000 FORMAT (78X,1H-)                                                  NITI176
      MODX(2)=MODSAV                                                    NITI177
      GO TO 10                                                          NITI178
         END                                                            NITI179-
```

```
      SUBROUTINE READ W
C                          READS W ARRAY
C       A 1 IN THE FOLLOWING COLUMNS WILL MAKE THE DESCRIBED CONVERSIONS
COL 18   DEGREES TO RADIANS
COL 19   GREAT CIRCLE DISTANCE IN KM TO RADIANS
COL 20   NAUTICAL MILES TO KM
COL 21   FEET TO KM
C
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON /WW/ ID(10),W0,W(400)
      EQUIVALENCE (EARTHR,W(2))
      INTEGER DEG,FEET
      READ 1000, ID
 1000 FORMAT (10A8)
      IF (EOF,60) 3,4
    3 CALL EXIT
    4 READ 1100, NW,W(NW),DEG,KM,NM,FEET
 1100 FORMAT (I3,E14.7,5I1)
      IF (NW.EQ.0) GO TO 10
      IF (NW.GT.0.AND.NW.LE.400) GO TO 5
      PRINT 4000, NW
 4000 FORMAT(15H1THE SUBSCRIPT ,I3,77H ON THE W-ARRAY INPUT IS OUT OF BO
     1UNDS.  ALLOWABLE VALUES ARE 1 THROUGH 400. )
      CALL EXIT
    5 IF (DEG.NE.0.) W(NW)=W(NW)*RAD
      IF (KM.NE.0) W(NW)=W(NW)/EARTHR
      IF (NM.NE.0) W(NW)=W(NW)*1.852
      IF (FEET.NE.0) W(NW)=W(NW)*3.048006096E-4
      GO TO 4
   10 RETURN
      END
```

```
      SUBROUTINE TRACE
C          CALCULATES THE RAY PATH
      DIMENSION ROLD(20),DROLD(20)
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,NEWRAY,SMT
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,PNP(10),POLAR,LPOLAR
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)
      LOGICAL SPACE,HOME,WASNT,UNDRGD,GROUND,PERIGE,THERE,MINDIS,NEWWR,
     1        NEWWP,PENET,NEWRAY,WAS
      REAL MAXSTP
      COMPLEX N2,PNP,POLAR,LPOLAR
      EQUIVALENCE (EARTHR,W(2)),(RCVRH,W(20)),(HOP,W(22)),(MAXSTP,W(23))
     1,(SKIP,W(71)),(RAYSET,W(72)),(PLT,W(81))
      NHOP=HOP
      MAX=MAXSTP
      NSKIP=SKIP
      RSTART=1.
      CALL HAMLTN
      HOME=DRDT(1)*(R(1)-EARTHR-RCVRH).GE.0.
C********* IHOP=0 TELLS PRINTR TO PRINT HEADING AND PUNCH A TRANSMITTER
C********* RAYSET AND TELLS RAYPLT TO START A NEW RAY
      IHOP=0
      CALL PRINTR (8HXMTR    ,0.)
      IF (PLT.NE.0.) CALL RAYPLT
      HTMAX=0.
      NEWRAY=.TRUE.
      THERE=R(1)-EARTHR.EQ.RCVRH
```

```
C********* LOOP ON NUMBER OF HOPS                                        TRAC030
   10 IHOP=IHOP+1                                                        TRAC031
      IF (IHOP.GT.NHOP) RETURN                                           TRAC032
      PENET=.FALSE.                                                      TRAC033
      APHT=RCVRH                                                         TRAC034
C********* LOOP ON MAXIMUM NUMBER OF STEPS PER HOP                        TRAC035
      DO 79 J=1,MAX                                                      TRAC036
      H=R(1)-EARTHR                                                      TRAC037
      IF (ABS(H-RCVRH).GT.ABS(APHT-RCVRH)) APHT=H                        TRAC038
      HTMAX=AMAX1(H,HTMAX)                                               TRAC039
      IF (.NOT.SPACE) GO TO 12                                           TRAC040
      CALL REACH                                                         TRAC041
      RSTART=1.                                                          TRAC042
      H=R(1)-EARTHR                                                      TRAC043
      IF (ABS(H-RCVRH).GT.ABS(APHT-RCVRH)) APHT=H                        TRAC044
      HTMAX=AMAX1(H,HTMAX)                                               TRAC045
      IF (.NOT.SPACE) GO TO 12                                           TRAC046
      IF (PERIGE) CALL PRINTR (8HPERIGEE ,0.)                            TRAC047
      IF (THERE) GO TO 51                                                TRAC048
      IF (MINDIS) GO TO 40                                               TRAC049
      IF (GROUND) GO TO 60                                               TRAC050
      IF (PLT.NE.0.) CALL RAYPLT                                         TRAC051
      IF (PERIGE) GO TO 79                                               TRAC052
   12 DO 13 L=1,N                                                        TRAC053
      ROLD(L)=R(L)                                                       TRAC054
   13 DROLD(L)=DRDT(L)                                                   TRAC055
      TOLD=T                                                             TRAC056
      WAS=THERE                                                          TRAC057
      CALL RKAM                                                          TRAC058
      H=R(1)-EARTHR                                                      TRAC059
      THERE=.FALSE.                                                      TRAC060
      WASNT=.NOT.HOME                                                    TRAC061
      HOME=DRDT(1)*(H-RCVRH).GE.0.                                       TRAC062
      TMP=(DRDT(1)-DROLD(1))*(T-TOLD)                                    TRAC063
      SMT=0.                                                             TRAC064
      IF (TMP.NE.0.) SMT=0.5*(R(1)-ROLD(1)+0.5*TMP)**2/ABS(TMP)          TRAC065
      IF (((H-RCVRH)*(ROLD(1)-EARTHR-RCVRH).LT.0..AND..NOT.WAS).OR.      TRAC066
     1 (WAS.AND.DRDT(1)*DROLD(1).LT.0..AND.HOME)) GO TO 50               TRAC067
      IF (HOME.AND.WASNT) GO TO 30                                       TRAC068
      IF (H.LT.0..OR.DRDT(1).GT.0..AND.DROLD(1).LT.0..AND.SMT.GT.H)      TRAC069
     1 GO TO 20                                                          TRAC070
      IF (DROLD(1).LT.0..AND.DRDT(1).GT.0.) CALL PRINTR(8HPERIGEE ,0.)   TRAC071
      IF (DROLD(1).GT.0..AND.DRDT(1).LT.0.) CALL PRINTR(8HAPOGEE  ,0.)   TRAC072
      IF (DROLD(2)*DRDT(2).LT.0.) CALL PRINTR(8HMAX LAT ,0.)             TRAC073
      IF (DROLD(3)*DRDT(3).LT.0.) CALL PRINTR(8HMAX LONG,0.)             TRAC074
      DO 14 I=4,6                                                        TRAC075
      IF (ROLD(I)*R(I).LT.0.) CALL PRINTR(8HWAVE REV,0.)                 TRAC076
   14 CONTINUE                                                           TRAC077
      GO TO 75                                                           TRAC078
C********* RAY WENT UNDERGROUND                                           TRAC079
   20 CALL BACK UP(0.)                                                   TRAC080
      GO TO 60                                                           TRAC081
C********* RAY MAY HAVE MADE A CLOSEST APPROACH                           TRAC082
   30 CALL GRAZE(RCVRH)                                                  TRAC083
      IF (THERE) GO TO 51                                                TRAC084
   40 DRDT(1)=0.                                                         TRAC085
      HPUNCH=R(1)-EARTHR                                                 TRAC086
      CALL PRINTR(8HMIN DIST,RAYSET)                                     TRAC087
      IF (PLT.NE.0.) CALL RAYPLT                                         TRAC088
      IF (IHOP.GE.NHOP) RETURN                                           TRAC089
      IHOP=IHOP+1                                                        TRAC090
      CALL PRINTR (8HMIN DIST,RAYSET)                                    TRAC091
      GO TO 89                                                           TRAC092
```

```
C********* RAY CROSSED RECEIVER HEIGHT                                  TRAC093
   50 CALL BACK UP(RCVRH)                                               TRAC094
      THERE=.TRUE.                                                      TRAC095
   51 R(1)=EARTHR+RCVRH                                                 TRAC096
      HTMAX=AMAX1(RCVRH,HTMAX)                                          TRAC097
      HPUNCH=APHT                                                       TRAC098
      CALL PRINTR(8HRCVR    ,RAYSET)                                    TRAC099
      IF (PLT.NE.0.) CALL RAYPLT                                        TRAC100
      IF (RCVRH.NE.0.) GC TO 89                                         TRAC101
      IF (IHOP.GE.NHOP) RETURN                                          TRAC102
      IHOP=IHOP+1                                                       TRAC103
      APHT=RCVRH                                                        TRAC104
C********* GROUND REFLECT                                               TRAC105
   60 R(1)=EARTHR                                                       TRAC106
      IF (ABS(RCVRH).GT.ABS(APHT-RCVRH)) APHT=0.                        TRAC107
      R(4)=ABS (R(4))                                                   TRAC108
      DRDT(1)=ABS (DRDT(1))                                             TRAC109
      RSTART=1.                                                         TRAC110
      HPUNCH=HTMAX                                                      TRAC111
      CALL PRINTR(8HGRND REF,RAYSET)                                    TRAC112
      HTMAX=0.                                                          TRAC113
      IF (RCVRH.NE.0.) GO TO 65                                         TRAC114
      THERE=.TRUE.                                                      TRAC115
      HPUNCH=APHT                                                       TRAC116
      CALL PRINTR (8HRCVR    ,RAYSET)                                   TRAC117
      GO TO 89                                                          TRAC118
   65 H=0.                                                              TRAC119
      THERE=.FALSE.                                                     TRAC120
C*********                                                              TRAC121
   75 IF (PLT.NE.0.) CALL RAYPLT                                        TRAC122
      IF (H.GT.HMAX.AND.H.GT.RCVRH.AND.DRDT(1).GT.0.) GO TO 90          TRAC123
      IF (MOD(J,NSKIP).EQ.0) CALL PRINTR(8H        ,0.)                 TRAC124
   79 CONTINUE                                                          TRAC125
C********* EXCEEDED MAXIMUM NUMBER OF STEPS                             TRAC126
      HPUNCH=H                                                          TRAC127
      CALL PRINTR(8HSTEP MAX,RAYSET)                                    TRAC128
      RETURN                                                            TRAC129
C*********                                                              TRAC130
   89 HOME=.TRUE.                                                       TRAC131
      GO TO 10                                                          TRAC132
C********* RAY PENETRATED                                               TRAC133
   90 PENET=.TRUE.                                                      TRAC134
      HPUNCH=H                                                          TRAC135
      CALL PRINTR(8HPENETRAT,RAYSET)                                    TRAC136
      RETURN                                                            TRAC137
         END                                                            TRAC138
```

```
      SUBROUTINE BACK UP(HS)                                            BACK001
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       BACK002
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,NEWRAY,SMT               BACK003
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)                 BACK004
      EQUIVALENCE (EARTHR,W(2)),(INTYP,W(41)),(STEP1,W(44))             BACK005
      REAL INTYP                                                        BACK006
      LOGICAL GROUND,PERIGE,THERE,MINDIS,NEWRAY,HOME                    BACK007
C********* DIAGNOSTIC PRINTOUT                                          BACK008
C        CALL PRINTR (8HBACK UP0,0.)                                    BACK009
C********* GOING AWAY FROM THE HEIGHT HS                                BACK010
      HOME=DRDT(1)*(R(1)-EARTHR-HS).GE.0.                               BACK011
      IF (HS.GT.0..AND..NOT.HOME.OR.HS.EQ.0..AND.DRDT(1).GT.0.) GO TO 30BACK012
```

```
C********* FIND NEAREST INTERSECTION OF RAY WITH THE HEIGHT HS
      DO 10 I=1,10
      STEP=-(R(1)-EARTHR-HS)/DRDT(1)
      STEP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(1)-EARTHR-HS).LT..5E-4.AND.ABS(STEP).LT.1.) GO TO 60
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR(8HBACK UP1,0.)
      MODE=1
      RSTART=1.
      CALL RKAM
   10 RSTART=1.
C
C********* FIND NEAREST CLOSEST APPROACH OF RAY TO THE HEIGHT HS
      ENTRY GRAZE
      THERE=.FALSE.
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HGRAZE 0 ,0.)
      IF (SMT.GT.ABS(R(1)-EARTHR-HS)) GO TO 30
      DO 20 I=1,10
      STEP=-R(4)/DRDT(4)
      STEP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(4)).LE.1.E-6.AND.ABS(STEP).LT.1.) GO TO 60
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HGRAZE 1 ,0.)
      MODE=1
      RSTART=1.
      CALL RKAM
      RSTART=1.
      IF (DRDT(4)*(R(1)-EARTHR-HS).LT.0.) GO TO 30
      IF(R(5).EQ.0..AND.R(6).EQ.0.) GO TO 60
   20 CONTINUE
C********* IF A CLOSEST APPROACH COULD NOT BE FOUND IN 10 STEPS, IT
C********* PROBABLY MEANS THAT THE RAY INTERSECTS THE HEIGHT HS
   30 CONTINUE
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HBACK UP2,0.)
      MODE=1
C********* ESTIMATE DISTANCE TO NEAREST INTERSECTION OF RAY WITH HEIGHT
C********* HS BEHIND THE PRESENT RAY POINT
      STEP=(-R(4)-SQRT(R(4)**2-2.*(R(1)-EARTHR-HS)*DRDT(4)))/DRDT(4)
      RSTART=1.
      CALL RKAM
      RSTART=1.
C********* FIND NEAREST INTERSECTION OF RAY WITH HEIGHT HS
      DO 40 I=1,10
      STEP=-(R(1)-EARTHR-HS)/DRDT(1)
      STEP=SIGN(AMIN1(ABS(STP),ABS(STEP)),STEP)
      IF (ABS(R(1)-EARTHR-HS).LT..5E-4.AND.ABS(STEP).LT.1.) GO TO 60
C********* DIAGNOSTIC PRINTOUT
C         CALL PRINTR (8HBACK UP3,0.)
      MODE=1
      RSTART=1.
      CALL RKAM
   40 RSTART=1.
   50 THERE=.TRUE.
C********* RESET STANDARD MODE AND INTEGRATION TYPE
   60 MODE=INTYP
      STEP=STEP1
      RETURN
         END
```

```
      SUBROUTINE REACH
C           CALCULATES THE STRAIGHT LINE RAY PATH BETWEEN THE EARTH
C           AND THE IONOSPHERE OR BETWEEN IONOSPHERIC LAYERS
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      COMMON /TRAC/ GROUND,PERIGE,THERE,MINDIS,NEWRAY,SMT
      COMMON /COORD/ S
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,N2I,PNP(10),POLAR,
     1             LPOLAR
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (EARTHR,W(2)),(XMTRH,W(3)),(RCVRH,W(20))
      LOGICAL CROSS,CROSSG,CROSSR,SPACE,GROUND,PERIGE,THERE,MINDIS,
     1        NEWRAY,RSPACE
      REAL N2,N2I
      COMPLEX PNP,POLAR,LPOLAR
      DATA (NSTEP=500)
      CALL HAMLTN
      H=R(1)-EARTHR
      IF (.NOT.NEWRAY.AND..NOT.RSPACE) CALL PRINTR(8HEXIT ION,0.)
      NEWRAY=.FALSE.
      V=SQRT(R(4)**2+R(5)**2+R(6)**2)
C********* NORMALIZE THE WAVE NORMAL DIRECTION TO ONE
      R(4)=R(4)/V
      R(5)=R(5)/V
      R(6)=R(6)/V
C********* NEGATIVE OF DISTANCE ALONG RAY TO CLOSEST APPROACH TO CENTER
C********* OF EARTH
      UP=R(1)*R(4)
      RADG=EARTHR**2-R(1)**2*(R(5)**2+R(6)**2)
      DISTG=SQRT (AMAX1(0.,RADG))
C********* DISTANCE ALONG RAY TO FIRST INTERSECTION WITH OR CLOSEST
C********* APPROACH TO THE EARTH
      SG=-UP-DISTG
C********* CROSSG IS TRUE IF THE RAY WILL INTERSECT OR TOUCH THE EARTH
      CROSSG=UP.LT.0..AND.RADG.GE.0.
      RADR=(EARTHR+RCVRH)**2-R(1)**2*(R(5)**2+R(6)**2)
      DISTR=SQRT (AMAX1(0.,RADR))
C********* DISTANCE ALONG RAY TO THE FIRST INTERSECTION WITH OR CLOSEST
C********* APPROACH TO THE RECEIVER HEIGHT
      SR=DISTR-UP
      IF (UP.LT.0..AND.DISTR.LT.-UP.AND.R(1).NE.EARTHR+RCVRH) SR=-DISTR
     1 -UP
C********* CROSSR IS TRUE IF THE RAY WILL INTERSECT WITH OR MAKE A
C********* CLOSEST APPROACH TO THE RECEIVER HEIGHT
      CROSSR=R(4).LT.0..OR.R(1).LT.(EARTHR+RCVRH)
      CROSS=CROSSG.OR.CROSSR
C********* MAXIMUM DISTANCE IN WHICH TO LOOK FOR THE IONOSPHERE
      S1=AMIN1(SR,SG)
      IF(.NOT.CROSSG) S1=SR
      IF (UP.GE.0.) GO TO 15
      CROSS=.TRUE.
C********* IF RAY IS GOING DOWN, S1 IS AT MOST THE DISTANCE TO A PERIGEE
      S1=AMIN1(S1,-UP)
C********* CONVERT THE POSITION AND DIRECTION OF THE RAY TO CARTESIAN
C********* COORDINATES
   15 CALL POL CAR
      SSTEP=100.
      S=SSTEP
      DO 20 I=1,NSTEP
      IF ((S-SSTEP).GT.S1.AND.CROSS) GO TO 25
C********* CONVERT POSITION AND DIRECTION TO SPHERICAL POLAR COORDINATES
C********* AT A DISTANCE S ALONG THE RAY
      CALL CAR POL
      CALL ELECTX
C********* FREE SPACE
```

```
      IF (X.EQ.0.) GO TO 20                                             REAC 66
      CALL RINDEX                                                       REAC 67
C********* EFFECTIVELY FREE SPACE                                       REAC 68
      IF (SPACE) GO TO 20                                               REAC 69
      IF (SSTEP.LT.0.5E-4) GO TO 25                                     REAC 70
C********* RAY IN THE IONOSPHERE.  STEP BACK OUT                        REAC 71
      S=S-SSTEP                                                         REAC 72
C********* DECREASE STEP SIZE                                           REAC 73
      SSTEP=SSTEP/10.                                                   REAC 74
   20 S=S+SSTEP                                                         REAC 75
      PRINT 2000, NSTEP                                                 REAC 76
 2000 FORMAT (9H EXCEEDED,I5,26H STEPS IN SUBROUTINE REACH)             REAC 77
      CALL EXIT                                                         REAC 78
   25 IF(CROSS) S=AMIN1(S,S1)                                           REAC 79
C********* CONVERT POSITION AND DIRECTION TO SPHERICAL POLAR COORDINATESREAC 80
C********* AT A DISTANCE S ALONG THE RAY                                REAC 81
      CALL CAR POL                                                      REAC 82
C********* AVOID THE RAY BEING SLIGHTLY UNDERGROUND                     REAC 83
      R(1)=AMAX1(R(1),EARTHR)                                           REAC 84
C********* ONE STEP INTEGRATION                                         REAC 85
      IF (N.LT.7) GO TO 31                                              REAC 86
      DO 30 NN=7,N                                                      REAC 87
   30 R(NN)=R(NN)+S*DROT(NN)                                            REAC 88
   31 T=T+S                                                             REAC 89
      CALL RINDEX                                                       REAC 90
C********* AT A PERIGEE                                                 REAC 91
      PERIGE=S.EQ.(-UP)                                                 REAC 92
C********* CORRECT MINOR ERRORS                                         REAC 93
      IF (PERIGE) R(4)=0.                                               REAC 94
C********* KEEP CONSISTENCY AFTER CORRECTING MINOR ERRORS               REAC 95
      DROT(1)=R(4)                                                      REAC 95
C********* ON THE GROUND                                                REAC 97
      GROUND=S.EQ.SG.AND.CROSSG                                         REAC 98
C********* AT THE RECEIVER HEIGHT                                       REAC 99
      THERE=S.EQ.SR.AND.CROSSR.AND..NOT.PERIGE                          REAC100
C********* AT A CLOSEST APPROACH TO THE RECEIVER HEIGHT                 REAC101
      MINDIS=PERIGE.AND.S.EQ.SR.AND.CROSSR                              REAC102
      RSPACE=SPACE                                                      REAC103
      V=SQRT(N2/(R(4)**2+R(5)**2+R(6)**2))                              REAC104
C********* RENORMALIZE THE WAVE NORMAL DIRECTION TO = SQRT(REAL(N**2))  REAC105
      R(4)=R(4)*V                                                       REAC106
      R(5)=R(5)*V                                                       REAC107
      R(6)=R(6)*V                                                       REAC108
      RSTART=1.                                                         REAC109
      IF (.NOT.SPACE) CALL PRINTR (8HENTR ION,0.)                       REAC110
      RETURN                                                            REAC111
         END                                                            REAC112-


      SUBROUTINE POL CAR                                                POLC001
      DIMENSION XO(6),X(6),RO(4)                                        POLC002
      COMMON R(6) /COORD/ S                                             POLC003
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                POLC004
C                                                                       POLC005
C          CONVERTS SPHERICAL COORDINATES TO CARTESIAN                  POLC006
      IF (R(5).EQ.0..AND.R(6).EQ.0.) GO TO 1                            POLC007
      VERT=0.                                                           POLC008
      SINA=SIN(R(2))                                                    POLC009
      COSA=SIN(PID2-R(2))                                               POLC010
      SINP=SIN(R(3))                                                    POLC011
      COSP=SIN(PID2-R(3))                                               POLC012
      XO(1)=R(1)*SINA*COSP                                              POLC013
      XO(2)=R(1)*SINA*SINP                                              POLC014
      XO(3)=R(1)*COSA                                                   POLC015
```

```
      X(4)=R(4)*SINA*COSP+R(5)*COSA*COSP-R(6)*SINP
      X(5)=R(4)*SINA*SINP+R(5)*COSA*SINP+R(6)*COSP
      X(6)=R(4)*COSA-R(5)*SINA
      RETURN
C           VERTICAL INCIDENCE
    1 VERT=1.
      RO(1)=R(1)
      RO(2)=R(2)
      RO(3)=R(3)
      RO(4)=SIGN (1.,R(4))
      RETURN
C
C           STEPS THE RAY A DISTANCE S, AND THEN
C           CONVERTS CARTESIAN COORDINATES TO SPHERICAL COORDINATES
      ENTRY CAR POL
      IF (VERT.NE.0.) GO TO 2
      X(1)=XO(1)+S*X(4)
      X(2)=XO(2)+S*X(5)
      X(3)=XO(3)+S*X(6)
      TEMP=SQRT(X(1)**2+X(2)**2)
      R(1)=SQRT(X(1)**2+X(2)**2+X(3)**2)
      R(2)=ATAN2(TEMP,X(3))
      R(3)=ATAN2(X(2),X(1))
      R(4)=(X(1)*X(4)+X(2)*X(5)+X(3)*X(6))/R(1)
      R(5)=(X(3)*(X(1)*X(4)+X(2)*X(5))-(X(1)**2+X(2)**2)*X(6))/
     1 (R(1)*TEMP)
      R(6)=(X(1)*X(5)-X(2)*X(4))/TEMP
      RETURN
C           VERTICAL INCIDENCE
    2 R(1)=RO(1)+RO(4)*S
      R(2)=RO(2)
      R(3)=RO(3)
      R(4)=RO(4)
      R(5)=0.
      R(6)=0.
      RETURN
         END
```

```
      SUBROUTINE PRINTR(NWHY,CARD)
C           PRINTS OUTPUT AND PUNCHES RAYSETS WHEN REQUESTED
      DIMENSION G(3,3),G1(3,3),TYPE(3),HEADR1(20),HEADR2(20),UNITS(20),
     1          HEAD1(20),HEAD2(20),UNIT(20),RPRINT(20),NPR(20)
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,DUM(3)
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,N2,N2I,PNP(10),POLAR(2),
     1             LPOLAR(2)
      COMMON R(20),T /WW/ ID(10),W0,W(400)
      EQUIVALENCE (THETA,R(2)),(PHI,R(3))
      EQUIVALENCE (EARTHR,W(2)),(XMTRH,W(3)),(TLAT,W(4)),(TLON,W(5)),
     1 (F,W(6)),(AZ1,W(10)),(BETA,W(14)),(RCVRH,W(20)),(HOP,W(22)),
     2 (PLAT,W(24)),(PLON,W(25)),(RAYSET,W(72))
      LOGICAL SPACE,NEWWR,NEWWP,PENET
      REAL N2,N2I,LPOLAR
      COMPLEX PNP
      DATA (TYPE=1HX,1HN,1HO)
     2,(HEADR1(7)=6H  PHAS),(HEADR2(7)=6HE PATH),(UNITS(7)=6H   KM ),
     3 (HEADR1(8)=6H  ABSO),(HEADR2(8)=6HRPTION),(UNITS(8)=6H   DB ),
     4 (HEADR1(9)=6H   DOP),(HEADR2(9)=6HPLER  ),(UNITS(9)=6H  C/S ),
     5 (HEADR1(10)=5H PATH ),(HEADR2(10)=6HLENGTH),(UNITS(10)=6H   KM )
      CALL RINDEX
      IF (.NOT.NEWWP) GO TO 10
```

```
C********* NEW W ARRAY -- REINITIALIZE
      NEWWP=.FALSE.
      SPL=SIN (PLON-TLON)
      CPL=SIN (PID2-(PLON-TLON))
      SP=SIN (PLAT)
      CP=SIN (PID2-PLAT)
      SL=SIN (TLAT)
      CL=SIN (PID2-TLAT)
C********* MATRIX TO ROTATE COORDINATES
      G(1,1)=CPL*SP*CL-CP*SL
      G(1,2)=SPL*SP
      G(1,3)=-SL*SP*CPL-CL*CP
      G(2,1)=-SPL*CL
      G(2,2)=CPL
      G(2,3)=SL*SPL
      G(3,1)=CL*CP*CPL+SP*SL
      G(3,2)=CP*SPL
      G(3,3)=-SL*CP*CPL+SP*CL
      DENM=G(1,1)*G(2,2)*G(3,3)+G(1,2)*G(3,1)*G(2,3)+G(2,1)*G(3,2)*G(1,3
     1)-G(2,2)*G(3,1)*G(1,3)-G(1,2)*G(2,1)*G(3,3)-G(1,1)*G(3,2)*G(2,3)
C********* THE MATRIX G1 IS THE INVERSE OF THE MATRIX G
      G1(1,1)=(G(2,2)*G(3,3)-G(3,2)*G(2,3))/DENM
      G1(1,2)=(G(3,2)*G(1,3)-G(1,2)*G(3,3))/DENM
      G1(1,3)=(G(1,2)*G(2,3)-G(2,2)*G(1,3))/DENM
      G1(2,1)=(G(3,1)*G(2,3)-G(2,1)*G(3,3))/DENM
      G1(2,2)=(G(1,1)*G(3,3)-G(3,1)*G(1,3))/DENM
      G1(2,3)=(G(2,1)*G(1,3)-G(1,1)*G(2,3))/DENM
      G1(3,1)=(G(2,1)*G(3,2)-G(3,1)*G(2,2))/DENM
      G1(3,2)=(G(3,1)*G(1,2)-G(1,1)*G(3,2))/DENM
      G1(3,3)=(G(1,1)*G(2,2)-G(2,1)*G(1,2))/DENM
      R0=EARTHR+XMTRH
C********* CARTESIAN COORDINATES OF TRANSMITTER
      XR=R0*G(1,1)
      YR=R0*G(2,1)
      ZR=R0*G(3,1)
      CTHR=G(3,1)
      STHR=SIN (ACOS (CTHR))
      PHIR=ATAN2(YR,XR)
      ALPH=ATAN2(G(3,2),G(3,3))
C*********
      NR=6
      NP=0
      DO 7 NN=7,20
      IF (W(NN+50).EQ.0.) GO TO 7
C********* DEPENDENT VARIABLE NUMBER NN IS BEING INTEGRATED
C********* NR IS THE NUMBER OF DEPENDENT VARIABLES BEING INTEGRATED
      NR=NR+1
      IF (W(NN+50).NE.2.) GO TO 7
C********* DEPENDENT VARIABLE NUMBER NN IS BEING INTEGRATED AND PRINTED.
C********* NP IS THE NUMBER OF DEPENDENT VARIABLES BEING INTEGRATED AND
C********* PRINTED
      NP=NP+1
C********* SAVE THE INDEX OF THE DEPENDENT VARIABLE TO PRINT
      NPR(NP)=NR
      HEAD1(NP)=HEADR1(NN)
      HEAD2(NP)=HEADR2(NN)
      UNIT(NP)=UNITS(NN)
    7 CONTINUE
      NP1=MIN0(NP,3)
      PDEV=ABSORB=DOPP=0.
```

```
C********* PRINT COLUMN HEADINGS AT THE BEGINNING OF EACH RAY           PRIN086
   10 IF (IHOP.NE.0) GO TO 12                                           PRIN087
      PRINT 1100, (HEAD1(NN),HEAD2(NN),NN=1,NP1)                        PRIN088
 1100 FORMAT (44X,7HAZIMUTH/43X,9HDEVIATION,8X,9HELEVATION/             PRIN089
     1 19X,16HHEIGHT     RANGE,1X,2(5X,12HXMTR   LOCAL),5X,26HPOLARIZATIPRIN090
     2ON    GROUP PATH,5A6,A5)                                          PRIN091
      PRINT 1150, (UNIT(NN),NN=1,NP1)                                   PRIN092
 1150 FORMAT (13X,2(8X,2HKM),2X,2(6X,3HDEG,5X,3HDEG),6X,12HREAL    IMAG,PRIN093
     1 7X,2HKM,4X,3(4X,A6,2X))                                          PRIN094
      IF (RAYSET.EQ.0.) GO TO 12                                        PRIN095
C********* PUNCH A TRANSMITTER RAYSET                                   PRIN096
      TLOND=TLON*DEGS                                                   PRIN097
      IF (TLOND.LT.0.) TLOND=TLOND+360.                                 PRIN098
      TLATD=TLAT*DEGS                                                   PRIN099
      IF (TLATD.LT.0.) TLATD=TLATD+360.                                 PRIN100
      AZ=AZ1*DEGS                                                       PRIN101
      EL=BETA*DEGS                                                      PRIN102
      NHOP=HOP                                                          PRIN103
      PUNCH 1200, ID(1),TYPE(NTYP),XMTRH,TLATD,TLOND,RCVRH,F,AZ,EL,POLARPRIN104
     1,NHOP,1HT                                                         PRIN105
 1200 FORMAT (A3,A1,4PF9.0,3P2F6.0,4P2F9.0,5P2F10.0,5X,2P2F5.0,I1,A1)   PRIN106
C*********                                                              PRIN107
   12 V=0.                                                              PRIN108
      IF (N2.NE.0.) V=(R(4)**2+R(5)**2+R(6)**2)/N2-1.                   PRIN109
      H=R(1)-EARTHR                                                     PRIN110
      STH=SIN (THETA)                                                   PRIN111
      CTH=SIN (PID2-THETA)                                              PRIN112
C********* CARTESIAN COORDINATES OF RAY POINT, ORIGIN AT TRANSMITTER    PRIN113
      XP=R(1)*STH*SIN (PID2-PHI)-XR                                     PRIN114
      YP=R(1)*STH*SIN (PHI)-YR                                          PRIN115
      ZP=R(1)*CTH-ZR                                                    PRIN116
C********* CARTESIAN COORDINATES OF RAY POINT, ORIGIN AT TRANSMITTER ANDPRIN117
C********* ROTATED                                                      PRIN118
      EPS=XP*G1(1,1)+YP*G1(1,2)+ZP*G1(1,3)                              PRIN119
      ETA=XP*G1(2,1)+YP*G1(2,2)+ZP*G1(2,3)                              PRIN120
      ZETA=XP*G1(3,1)+YP*G1(3,2)+ZP*G1(3,3)                             PRIN121
      RCE2=ETA**2+ZETA**2                                               PRIN122
      RCE=SQRT (RCE2)                                                   PRIN123
C********* GROUND RANGE                                                 PRIN124
      RANGE=EARTHR*ATAN2(RCE,EARTHR+EPS)                                PRIN125
C********* ANGLE OF WAVE NORMAL WITH LOCAL HORIZONTAL                   PRIN126
      ELL=ATAN2(R(4),SQRT (R(5)**2+R(6)**2))*DEGS                       PRIN127
C********* STRAIGHT LINE DISTANCE FROM TRANSMITTER TO RAY POINT         PRIN128
      SR=SQRT (RCE2+EPS**2)                                             PRIN129
      IF (NP.LT.1) GO TO 16                                             PRIN130
      DO 15 I=1,NP                                                      PRIN131
      NN=NPR(I)                                                         PRIN132
   15 RPRINT(I)=R(NN)                                                   PRIN133
   16 IF (SR.GE.1.E-6) GO TO 20                                         PRIN134
C********* TOO CLOSE TO TRANSMITTER TO CALCULATE DIRECTION FROM         PRIN135
C********* TRANSMITTER                                                  PRIN136
      PRINT 1500, V,NWHY,H,RANGE,ELL,POLAR,T,(RPRINT(NN),NN=1,NP1)      PRIN137
 1500 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,26X,F8.3,F9.3,F8.3,4F12.4)      PRIN138
      GO TO 40                                                          PRIN139
C********* ELEVATION ANGLE OF RAY POINT FROM TRANSMITTER                PRIN140
   20 EL=ATAN2(EPS,RCE)*DEGS                                            PRIN141
      IF (RCE.GE.1.E-6) GO TO 30                                        PRIN142
C********* NEARLY DIRECTLY ABOVE OR BELOW TRANSMITTER. CAN NOT CALCULATEPRIN143
C********* AZIMUTH DIRECTION FROM TRANSMITTER ACCURATELY                PRIN144
      PRINT 2500, V,NWHY,H,RANGE,EL,ELL,POLAR,T,(RPRINT(NN),NN=1,NP1)   PRIN145
 2500 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,17X,F9.3,F8.3,F9.3,F8.3,        PRIN146
     1 4F12.4)                                                          PRIN147
      GO TO 40                                                          PRIN148
```

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C********* AZIMUTH ANGLE OF RAY POINT FROM TRANSMITTER                  PRIN149
   30 ANGA=ATAN2(ETA,ZETA)                                              PRIN150
      AZDEV=180.-AMOD(540.-(AZ1-ANGA)*DEGS,360.)                        PRIN151
      IF (R(5).NE.0..OR.R(6).NE.0.) GO TO 34                            PRIN152
C********* WAVE NORMAL IS VERTICAL, SO AZIMUTH DIRECTION CANNOT BE      PRIN153
C********* CALCULATED                                                   PRIN154
      PRINT 3000, V,NWHY,H,RANGE,AZDEV,EL,ELL,POLAR,T,(RPRINT(NN),NN=1, PRIN155
     1 NP1)                                                             PRIN156
 3000 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,F9.3,8X,F9.3,F8.3,F9.3,F8.3,    PRIN157
     1 4F12.4)                                                          PRIN158
      GO TO 40                                                          PRIN159
   34 ANA=ANGA-ALPH                                                     PRIN160
      SANA=SIN (ANA)                                                    PRIN161
      SPHI=SANA*STHR/STH                                                PRIN162
      CPHI=-SIN (PID2-ANA)*SIN (PID2-(PHI-PHIR))+SANA*SIN (PHI-PHIR)    PRIN163
     1 *CTHR                                                            PRIN164
      AZA=180.-AMOD (540.-(ATAN2(SPHI,CPHI)-ATAN2(R(6),R(5)))*DEGS,360.)PRIN165
      PRINT 3500, V,NWHY,H,RANGE,AZDEV,AZA,EL,ELL,POLAR,T,(RPRINT(NN),NNPRIN166
     1 =1,NP1)                                                          PRIN167
 3500 FORMAT (1X,E6.0,1X,A8,F10.4,F11.4,2(F9.3,F8.3),F9.3,F8.3,         PRIN168
     1 4F12.4)                                                          PRIN169
C*********                                                              PRIN170
   40 LINES=LINES+1                                                     PRIN171
      IF (NP.LE.3) GO TO 45                                             PRIN172
C********* ADDITIONAL LINE TO PRINT REMAINING DEPENDENT INTEGRATION     PRIN173
C********* VARIABLES                                                    PRIN174
      PRINT 4000, (RPRINT(NN),NN=4,NP)                                  PRIN175
 4000 FORMAT (99X,3F12.4)                                               PRIN176
      LINES=LINES+1                                                     PRIN177
   45 IF (CARD.EQ.0.) RETURN                                            PRIN178
C                                                                       PRIN179
C********* PUNCH A RAYSET                                               PRIN180
      IF (AZDEV.LT.-90.) AZDEV=AZDEV+360.                               PRIN181
      IF (AZA.LT.-90.) AZA=AZA+360.                                     PRIN182
      TDEV=T-SR                                                         PRIN183
      NR=6                                                              PRIN184
      IF (W(57).EQ.0.) GO TO 47                                         PRIN185
C********* PHASE PATH                                                   PRIN186
      NR=NR+1                                                           PRIN187
      PDEV=R(NR)-SR                                                     PRIN188
   47 IF (W(58).EQ.0.) GO TO 48                                         PRIN189
C********* ABSORPTION                                                   PRIN190
      NR=NR+1                                                           PRIN191
      ABSORB=R(NR)                                                      PRIN192
C********* DOPPLER SHIFT                                                PRIN193
   48 IF (W(59).NE.0.) DOPP=R(NR+1)                                     PRIN194
      PUNCH 4500, HPUNCH,RANGE,AZDEV,AZA,ELL,SR,TDEV,PDEV,ABSORB,DOPP,  PRIN195
     1 POLAR,IHOP,NWHY                                                  PRIN196
 4500 FORMAT (4P2F9.0,3P3F6.0,3PF8.0,3P4F6.0,2P2F5.0,I1,A1)             PRIN197
      RETURN                                                            PRIN198
         END                                                            PRIN199-
```

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON A VERTICAL PLANE

Coordinates of the left edge of the graph:

Latitude = ________________ rad / deg / km north (W83)

Longitude = ________________ rad / deg / km east (W84)

Coordinates of the right edge of the graph:

Latitude = ________________ rad / deg / km north (W85)

Longitude = ________________ rad / deg / km east (W86)

Height above the ground of the bottom of the graph = ________ km (W88)

Distance between tic marks = __________ rad / deg / km (W87)

(W81 = 1.)

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON THE GROUND

Coordinates of the left edge of the graph:

rad
Latitude = ______________ deg north (W83)
km

rad
Longitude = ______________ deg east (W84)
km

Coordinates of the right edge of the graph:

rad
Latitude = ______________ deg north (W85)
km

rad
Longitude = ______________ deg east (W86)
km

Factor to expand lateral deviation scale by = ____________ (W82)

rad
Distance between tic marks on range scale = ____________ deg (W87)
km

(W81 = 2.)

```
      SUBROUTINE RAYPLT                                                 YPLT001
C        REPLACES SUBROUTINES RAYPLT,PLOT, AND LABPLT IF PLOTS ARE      YPLT002
C        NOT WANTED OR IF A PLOTTER IS NOT AVAILABLE                    YPLT003
      COMMON /WW/ ID(10),W0,W(400)                                      YPLT004
      EQUIVALENCE (PLT,W(81))                                           YPLT005
      PLT=0.                                                            YPLT006
      ENTRY ENDPLT                                                      YPLT007
      RETURN                                                            YPLT008
         END                                                            YPLT  9-
```

```
      SUBROUTINE RAYPLT                                                 RAYP001
C        W(81)=1. PLOTS PROJECTION OF RAYPATH ON VERTICAL PLANE         RAYP002
C              =2. PLOTS PROJECTION OF RAYPATH ON GROUND                RAYP003
      COMMON /PLT/ XL,XR,YB,YT,RESET                                    RAYP004
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                RAYP005
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH             RAYP006
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 RAYP007
      EQUIVALENCE (TH,R(2)),(PH,R(3))                                   RAYP008
      EQUIVALENCE (EARTHR,W(2)),(PLAT,W(24)),(PLON,W(25)),(PLT,W(81)),  RAYP009
     1 (FACTR,W(82)),(LLAT,W(83)),(LLON,W(84)),(RLAT,W(85)),(RLON,W(86))RAYP010
     2,(TIC,W(87)),(HB,W(88))                                           RAYP011
      REAL LLAT,LLON,LTIC                                               RAYP012
      LOGICAL NEWWR,NEWWP,PENET                                         RAYP013
      IF (.NOT.NEWWR) GO TO 5                                           RAYP014
C                                                                       RAYP015
C          NEW W ARRAY -- REINITIALIZE                                  RAYP016
      NEWWR=.FALSE.                                                     RAYP017
      RESET=1.                                                          RAYP018
CONVERT COORDINATES OF VERTICAL PLANE FROM GEOGRAPHIC TO GEOMAGNETIC    RAYP019
      SW=SIN (PLAT)                                                     RAYP020
      CW=SIN (PID2-PLAT)                                                RAYP021
      SLM=SIN (LLAT)                                                    RAYP022
      CLM=SIN (PID2-LLAT)                                               RAYP023
      SRM=SIN (RLAT)                                                    RAYP024
      CRM=SIN (PID2-RLAT)                                               RAYP025
      CDPHI=SIN (PID2-(LLON-PLON))                                      RAYP026
      PHL=ATAN2(SIN (LLON-PLON)*CLM,CDPHI*SW*CLM-CW*SLM)                RAYP027
      CTHL=CDPHI*CW*CLM+SW*SLM                                          RAYP028
      STHL=SIN (ACOS (CTHL))                                            RAYP029
      CDPHI=SIN (PID2-(RLON-PLON))                                      RAYP030
      PHR=ATAN2(SIN (RLON-PLON)*CRM,CDPHI*SW*CRM-CW*SRM)                RAYP031
      CTHR=CDPHI*CW*CRM+SW*SRM                                          RAYP032
      STHR=SIN (ACOS (CTHR))                                            RAYP033
      CLR=CTHL*CTHR+STHL*STHR*SIN (PID2-(PHL-PHR))                      RAYP034
      SLR=SQRT (1.-CLR**2)                                              RAYP035
      IF (PLT.EQ.2.) GO TO 3                                            RAYP036
      FACTR=1.                                                          RAYP037
      R0=EARTHR+HB                                                      RAYP038
      ALPHA=.5*ACOS (CLR)                                               RAYP039
      XR=R0*SIN (ALPHA)                                                 RAYP040
      XL=-XR                                                            RAYP041
      YB=R0*SIN (PID2-ALPHA)                                            RAYP042
      YT=YB+2.*XR                                                       RAYP043
      GO TO 5                                                           RAYP044
    3 IF (FACTR.EQ.0.) FACTR=1.                                         RAYP045
      ALPH1=ATAN2(STHR*SIN (PHR-PHL),(CTHR-CTHL*CLR)/STHL)              RAYP046
      XL=0.                                                             RAYP047
      XR=EARTHR*ACOS (CLR)                                              RAYP048
      YT=0.5*XR/FACTR                                                   RAYP049
      YB=-YT                                                            RAYP050
```

```
C
    5 STH=SIN (TH)
      CTH=SIN (PID2-TH)
      CR=CTHR*CTH+STHR*STH*SIN (PID2-(PHR-PH))
      CL=CTHL*CTH+STHL*STH*SIN (PID2-(PHL-PH))
      CEA=ATAN2(CR-CL*CLR,CL*SLR)
      NEW=1
      IF (IHOP.NE.0) NEW=0
      IF (PLT.EQ.2.) GO TO 10
      CALL PLOT (R(1)*SIN(CEA-ALPHA),R(1)*SIN(PID2-(CEA-ALPHA)),NEW)
      RETURN
   10 SL=SQRT (1.-CL**2)
      TMP1=STH*SIN (PH-PHL)
      TMP2=(CTH-CTHL*CL)/STHL
      ALPH2=0.
      IF (TMP1.NE.0..OR.TMP2.NE.0.) ALPH2=ATAN2(TMP1,TMP2)
      CALL PLOT (EARTHR*CEA,EARTHR*ASIN(SL*SIN (ALPH1-ALPH2)),NEW)
      RETURN
C
C           DRAW AXES AND CALL FOR LABELING AND TERMINATION OF THIS PLOT
      ENTRY ENDPLT
      TICKX=0.01*(YT-YB)
      IF (PLT.EQ.2.) GO TO 25
      R1=EARTHR-TICKX
      X=XL
      Y=YB
      CALL PLOT (X,Y,1)
      NTIC=2
      IF (TIC.NE.0.) NTIC=NTIC+2.*ALPHA/TIC
      NLINE=MAX0 (1,100/NTIC)
      DO 20 I=1,NTIC
      ANG=-ALPHA+(I-1)*TIC
      CALL PLOT (R1*SIN (ANG),R1*SIN (PID2-ANG),0)
      CALL PLOT (X,Y,0)
      DO 20 J=1,NLINE
      ANG=ANG+TIC/NLINE
      X=EARTHR*SIN (ANG)
      Y=EARTHR*SIN (PID2-ANG)
   20 CALL PLOT (X,Y,0)
      CALL PLOT (XR,YB,0)
      GO TO 50
   25 DTIC=TIC*EARTHR
      LTIC=DTIC/FACTR
      TICY=XL+0.01*(XR-XL)
      NTIC=YT/LTIC
      TIC1=-LTIC*NTIC
      CALL PLOT (XL,YB,1)
      NTIC=2*NTIC+1
      DO 30 I=1,NTIC
      Y=TIC1+(I-1)*LTIC
      CALL PLOT (XL,Y,0)
      CALL PLOT (TICY,Y,0)
   30 CALL PLOT (XL,Y,0)
      CALL PLOT (XL,YT,0)
      CALL PLOT (XL,0.,1)
      NTIC=(XR-XL)/DTIC
      DO 40 I=1,NTIC
      X=I*DTIC
      CALL PLOT (X,0.,0)
      CALL PLOT (X,TICKX,0)
   40 CALL PLOT (X,0.,0)
      CALL PLOT (XR,0.,0)
   50 CALL LABPLT
      CALL PLTEND
      RETURN
         END
```

```
      SUBROUTINE PLOT (X,Y,NEW)                                         PLOT001
      COMMON /PLT/ XMIN0,XMAX0,YMIN0,YMAX0,RESET                        PLOT002
      COMMON /DD/ INT,IOR,IT,IS,IC,ICC,IX,IY                            PLOT003
      DATA (INITAL=1),(MINX=0),(MINY=0),(MAXX=1023),(MAXY=1023),        PLOT004
     1 (MINX0=23),(MINY0=23),(MAXX0=1023),(MAXY0=1023)                  PLOT005
C                                                                       PLOT006
C           INITIALIZE LIBRARY PLOTTING ROUTINES                        PLOT007
      IF (INITAL.EQ.0) GO TO 1                                          PLOT008
      INITAL=0                                                          PLOT009
      CALL DDINIT (1,1H )                                               PLOT010
C                                                                       PLOT011
C           COMPUTE SCALE FACTORS                                       PLOT012
    1 IF (RESET.EQ.0.) GO TO 5                                          PLOT013
      RESET=0.                                                          PLOT014
      XSCALE=(MAXX0-MINX0)/(XMAX0-XMIN0)                                PLOT015
      YSCALE=(MAXY0-MINY0)/(YMAX0-YMIN0)                                PLOT016
      XMIN=XMIN0-(MINX0-MINX)/XSCALE                                    PLOT017
      YMIN=YMIN0-(MINY0-MINY)/YSCALE                                    PLOT018
      XMAX=XMAX0+(MAXX-MAXX0)/XSCALE                                    PLOT019
      YMAX=YMAX0+(MAXY-MAXY0)/YSCALE                                    PLOT020
C                                                                       PLOT021
C           START A NEW LINE                                            PLOT022
    5 IF (NEW.EQ.0) GO TO 10                                            PLOT023
      IX=MINX0+(X-XMIN0)*XSCALE                                         PLOT024
      IY=MINY0+(Y-YMIN0)*YSCALE                                         PLOT025
      IF (IX.GE.MINX.AND.IX.LE.MAXX.AND.IY.GE.MINY.AND.IY.LE.MAXY)      PLOT026
     1 CALL DDBP                                                        PLOT027
      GO TO 50                                                          PLOT028
C                                                                       PLOT029
C           HORIZONTAL DISPLACEMENT                                     PLOT030
   10 XS=X-XOLD                                                         PLOT031
      YS=Y-YOLD                                                         PLOT032
      IF (XS) 11,12,16                                                  PLOT033
C           NEGATIVE                                                    PLOT034
   11 X1=XMAX                                                           PLOT035
      X2=XMIN                                                           PLOT036
      GO TO 20                                                          PLOT037
C           ZERO                                                        PLOT038
   12 IF (YS) 13,50,14                                                  PLOT039
   13 S1=(YMAX-YOLD)/YS                                                 PLOT040
      S2=(YMIN-YOLD)/YS                                                 PLOT041
      GO TO 40                                                          PLOT042
   14 S1=(YMIN-YOLD)/YS                                                 PLOT043
      S2=(YMAX-YOLD)/YS                                                 PLOT044
      GO TO 40                                                          PLOT045
C           POSITIVE                                                    PLOT046
   16 X1=XMIN                                                           PLOT047
      X2=XMAX                                                           PLOT048
C                                                                       PLOT049
C           VERTICAL DISPLACEMENT                                       PLOT050
   20 IF (YS) 21,22,26                                                  PLOT051
C           NEGATIVE                                                    PLOT052
   21 Y1=YMAX                                                           PLOT053
      Y2=YMIN                                                           PLOT054
      GO TO 30                                                          PLOT055
C           ZERO                                                        PLOT056
   22 S1=(X1-XOLD)/XS                                                   PLOT057
      S2=(X2-XOLD)/XS                                                   PLOT058
      GO TO 40                                                          PLOT059
C           POSITIVE                                                    PLOT060
   26 Y1=YMIN                                                           PLOT061
      Y2=YMAX                                                           PLOT062
C                                                                       PLOT063
   30 S1=AMAX1((X1-XOLD)/XS,(Y1-YOLD)/YS)                               PLOT064
      S2=AMIN1((X2-XOLD)/XS,(Y2-YOLD)/YS)                               PLOT065
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C                                                                       PLOT066
C         PLOT LINE -- CHECKING FOR BORDER CROSSINGS                    PLOT067
   40 S=SQRT(XS**2+YS**2)                                               PLOT068
      IF (S2.LT.0..OR.S*S1-S.GT.0.) GO TO 50                            PLOT069
      IF (S1.LT.0.) GO TO 42                                            PLOT070
C         PREVIOUS POINT OFF GRAPH                                      PLOT071
      IX=MINXO+(XOLD+XS*S1-XMINO)*XSCALE+0.5                            PLOT072
      IY=MINYO+(YOLD+YS*S1-YMINO)*YSCALE+0.5                            PLOT073
      CALL DDBP                                                         PLOT074
   42 IF (S*S2-S.GT.0.) GO TO 44                                        PLOT075
C         CURRENT POINT OFF GRAPH                                       PLOT076
      IX=MINXO+(XOLD+XS*S2-XMINO)*XSCALE+0.5                            PLOT077
      IY=MINYO+(YOLD+YS*S2-YMINO)*YSCALE+0.5                            PLOT078
      CALL DDVC                                                         PLOT079
      GO TO 50                                                          PLOT080
C         CURRENT POINT ON GRAPH                                        PLOT081
   44 IX=MINXO+(X-XMINO)*XSCALE+0.5                                     PLOT082
      IY=MINYO+(Y-YMINO)*YSCALE+0.5                                     PLOT083
      CALL DDVC                                                         PLOT084
C                                                                       PLOT085
C         EXIT ROUTINE                                                  PLOT086
   50 XOLD=X                                                            PLOT087
      YOLD=Y                                                            PLOT088
      RETURN                                                            PLOT089
C                                                                       PLOT090
C         TERMINATE THE CURRENT PLOT                                    PLOT091
      ENTRY PLTEND                                                      PLOT092
      CALL DDFR                                                         PLOT093
      RETURN                                                            PLOT094
         END                                                            PLOT 95-
```

```
      SUBROUTINE LABPLT                                                 LABP001
C         LABEL THE CURRENT PLOT                                        LABP002
      DIMENSION LABEL(9),TYPE(3)                                        LABP003
      COMMON /DD/ INT,IOR,IT,IS,IC,ICC,IX,IY                            LABP004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,DUM(4)                           LABP005
      COMMON /FLG/ NTYP,NEWWP,NEWWP,PENET,LINES,IHOP,HPUNCH             LABP006
      COMMON /WW/ ID(10),W0,W(400)                                      LABP007
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(AZ1,W(10)),(PLT,W(81)),       LABP008
     1 (FACTR,W(82)),(TIC,W(87))                                        LABP009
      LOGICAL NEWWR,NEWWP,PENET                                         LABP010
      REAL LTIC                                                         LABP011
      DATA (TYPE=8HEXTRAORD,8HNO FIELD,8HORDINARY)                      LABP012
      IOR=IT=0                                                          LABP013
      IS=2                                                              LABP014
      IX=0     $  IY=1023  $  CALL DDTAB  $  CALL DDTEXT (7,ID)         LABP015
      NDATE=IDATE(0)                                                    LABP016
      CALL DDTEXT (1,NDATE)                                             LABP017
      AZA=AZ1*DEGS                                                      LABP018
      DTIC=TIC*EARTHR                                                   LABP019
      ENCODE (72,1000,LABEL)  F,AZA,TYPE(NTYP),DTIC                     LABP020
 1000 FORMAT (3HF =,F7.3,6H, AZ =,F7.2,2H, ,A8,2H, ,F7.2,24H KM BETWEEN LABP021
     1TICK MARKS≠.)                                                     LABP022
      IX=0     $  IY=991   $  CALL DDTAB  $  CALL DDTEXT (9,LABEL)      LABP023
      IF (PLT.EQ.1.) RETURN                                             LABP024
      LTIC=DTIC/FACTR                                                   LABP025
      ENCODE (32,2000,LABEL)  LTIC                                      LABP026
 2000 FORMAT (F7.2,24H KM BETWEEN TICK MARKS≠.)                         LABP027
      IOR=1                                                             LABP028
      IX=0     $  IY=0     $  CALL DDTAB  $  CALL DDTEXT (4,LABEL)      LABP029
      IOR=0                                                             LABP030
      RETURN                                                            LABP031
         END                                                            LABP 32-
```

```
      SUBROUTINE RKAM                                                   RKAM001
C        NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS                RKAM002
      COMMON /RK/ NN,SPACE,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART     RKAM003
      COMMON Y(20),T,STEP,DYDT(20)                                      RKAM004
      DIMENSION DELY(4,20),BET(4),XV(5),FV(4,20),YU(5,20)               RKAM005
      TYPE DOUBLE YU                                                    RKAM006
      IF (RSTART.EQ.0.) GO TO 1000                                      RKAM007
      LL=MM=1                                                           RKAM008
      IF (MODE.EQ.1) MM=4                                               RKAM009
      ALPHA=T                                                           RKAM010
      EPM=0.0                                                           RKAM011
      BET(1)=BET(2)=0.5                                                 RKAM012
      BET(3)=1.0                                                        RKAM013
      BET(4)=0.0                                                        RKAM014
      STEP=SPACE                                                        RKAM015
      R=19.0/270.0                                                      RKAM016
      XV(MM)=T                                                          RKAM017
      IF (E1MIN.LE.0.) E1MIN=E1MAX/55.                                  RKAM018
      IF (FACT.LE.0.) FACT=0.5                                          RKAM019
      CALL HAMLTN                                                       RKAM020
      DO 320 I=1,NN                                                     RKAM021
      FV(MM,I)=DYDT(I)                                                  RKAM022
  320 YU(MM,I)=Y(I)                                                     RKAM023
      RSTART=0.                                                         RKAM024
      GO TO 1001                                                        RKAM025
 1000 IF (MODE.NE.1) GO TO 2000                                         RKAM026
C                                                                       RKAM027
C           RUNGE-KUTTA                                                 RKAM028
 1001 DO 1034 K=1,4                                                     RKAM029
      DO 1350 I=1,NN                                                    RKAM030
      DELY(K,I)=STEP*FV(MM,I)                                           RKAM031
      Z=YU(MM,I)                                                        RKAM032
 1350 Y(I)=Z+BET(K)*DELY(K,I)                                           RKAM033
      T=BET(K)*STEP+XV(MM)                                              RKAM034
      CALL HAMLTN                                                       RKAM035
      DO 1034 I=1,NN                                                    RKAM036
 1034 FV(MM,I)=DYDT(I)                                                  RKAM037
      DO 1039 I=1,NN                                                    RKAM038
      DEL=(DELY(1,I)+2.0*DELY(2,I)+2.0*DELY(3,I)+DELY(4,I))/6.0         RKAM039
 1039 YU(MM+1,I)=YU(MM,I)+DEL                                           RKAM040
      MM=MM+1                                                           RKAM041
      XV(MM)=XV(MM-1)+STEP                                              RKAM042
      DO 1400 I=1,NN                                                    RKAM043
 1400 Y(I)=YU(MM,I)                                                     RKAM044
      T=XV(MM)                                                          RKAM045
      CALL HAMLTN                                                       RKAM046
      IF (MODE.EQ.1) GO TO 42                                           RKAM047
      DO 150 I=1,NN                                                     RKAM048
  150 FV(MM,I)=DYDT(I)                                                  RKAM049
      IF (MM.LE.3) GO TO 1001                                           RKAM050
C                                                                       RKAM051
C           ADAMS-MOULTON                                               RKAM052
 2000 DO 2048 I=1,NN                                                    RKAM053
      DEL=STEP*(55.*FV(4,I)-59.*FV(3,I)+37.*FV(2,I)-9.*FV(1,I))/24.     RKAM054
      Y(I)=YU(4,I)+DEL                                                  RKAM055
 2048 DELY(1,I)=Y(I)                                                    RKAM056
      T=XV(4)+STEP                                                      RKAM057
      CALL HAMLTN                                                       RKAM058
      XV(5)=T                                                           RKAM059
      DO 2051 I=1,NN                                                    RKAM060
      DEL=STEP*(9.*DYDT(I)+19.*FV(4,I)-5.*FV(3,I)+FV(2,I))/24.          RKAM061
      YU(5,I)=YU(4,I)+DEL                                               RKAM062
 2051 Y(I)=YU(5,I)                                                      RKAM063
      CALL HAMLTN                                                       RKAM064
      IF (MODE.LE.2) GO TO 42                                           RKAM065
```

```
C
C         ERROR ANALYSIS
      SSE=0.0
      DO 3033 I=1,NN
      EPSIL=R*ABS(Y(I)-DELY(1,I))
      IF (MODE.EQ.3.AND.Y(I).NE.0.) EPSIL=EPSIL/ABS(Y(I))
      IF (SSE.LT.EPSIL) SSE=EPSIL
 3033 CONTINUE
      IF (E1MAX.GT.SSE) GO TO 3035
      IF (ABS(STEP).LE.E2MIN) GO TO 42
      LL=MM=1
      STEP=STEP*FACT
      GO TO 1001
 3035 IF (LL.LE.1.OR.SSE.GE.E1MIN.OR.E2MAX.LE.ABS(STEP)) GO TO 42
      LL=2
      MM=3
      XV(2)=XV(3)
      XV(3)=XV(5)
      DO 5363 I=1,NN
      FV(2,I)=FV(3,I)
      FV(3,I)=DYDT(I)
      YU(2,I)=YU(3,I)
 5363 YU(3,I)=YU(5,I)
      STEP=2.0*STEP
      GO TO 1001
C
C         EXIT ROUTINE
   42 LL=2
      MM=4
      DO 12 K=1,3
      XV(K)=XV(K+1)
      DO 12 I=1,NN
      FV(K,I)=FV(K+1,I)
   12 YU(K,I)=YU(K+1,I)
      XV(4)=XV(5)
      DO 52 I=1,NN
      FV(4,I)=DYDT(I)
   52 YU(4,I)=YU(5,I)
      IF (MODE.LE.2) RETURN
      E=ABS(XV(4)-ALPHA)
      IF (E.LE.EPM) GO TO 2000
      EPM=E
      RETURN
         END
```

```
      SUBROUTINE HAMLTN
C********* CALCULATES HAMILTONS EQUATIONS FOR RAY TRACING
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,C,LOGTEN
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,
     1              H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI
     2, PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI, PHPKPH,PHPKPI
     3 ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI
      COMMON R(20),T,STP,DRDT(20) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (TH,R(2)),(PH,R(3)),(KR,R(4)),(KTH,R(5)),(KPH,R(6)),
     1 (DTHDT,DRDT(2)),(DPHDT,DRDT(3)),(DKRDT,DRDT(4)),(DKTHDT,DRDT(5)),
     2 (DKPHDT,DRDT(6)),(F,W(6))
      REAL KR,KTH,KPH,KPHPK,KPHPKI,LPOLAR,LPOLRI,LOGTEN,K,KAY2,KAY2I
      OM=PIT2*1.E6*F
      STH=SIN(TH)
      CTH=SIN(PID2-TH)
      RSTH=R(1)*STH
      RCTH=R(1)*CTH
      CALL RINDEX
      DRDT=-PHPKR/(PHPOM*C)
      DTHDT=-PHPKTH/(PHPOM*R(1)*C)
      DPHDT=-PHPKPH/(PHPOM*RSTH*C)
      DKRDT=PHPR/(PHPOM*C)+KTH*DTHDT+KPH*STH*DPHDT
      DKTHDT=(PHPTH/(PHPOM*C)-KTH*DRDT+KPH*RCTH*DPHDT)/R(1)
      DKPHDT=(PHPPH/(PHPOM*C)-KPH*STH*DRDT-KPH*RCTH*DTHDT)/RSTH
      NR=6
C********* PHASE PATH
      IF (W(57).EQ.0.) GO TO 10
      NR=NR+1
      DRDT(NR)=-     KPHPK/PHPOM/OM
C********* ABSORPTION
   10 IF (W(58).EQ.0.) GO TO 15
      NR=NR+1
      DRDT(NR)= 10./LOGTEN*KPHPK*KAY2I/(KR*KR+KTH*KTH+KPH*KPH)/PHPOM/C
C********* DOPPLER SHIFT
   15 IF (W(59).EQ.0.) GO TO 20
      NR=NR+1
      DRDT(NR)=-PHPT/PHPOM/C/PIT2
C********* GEOMETRICAL PATH LENGTH
   20 IF (W(60).EQ.0.) GO TO 25
      NR=NR+1
      DRDT(NR)=-SQRT(PHPKR**2+PHPKTH**2+PHPKPH**2)/PHPOM   /C
C********* OTHER CALCULATIONS
   25 CONTINUE
      RETURN
         END
```

APPENDIX 2. VERSIONS OF THE REFRACTIVE INDEX SUBROUTINE (RINDEX)

This ray tracing program gains versatility without sacrificing speed by having several versions of some of the subroutines. For example, the 8 versions of the refractive index subroutine allow the user to decide for each ray path calculation whether to include or ignore various aspects of the propagation medium such as the earth's magnetic field or collisions between electrons and neutral air molecules.

If collisions are included, the user has the option of using the Appleton-Hartree formula (which assumes a constant collision frequency) or the Sen-Wyller formula (which assumes a Maxwell distribution of electron energies and a collision frequency proportional to energy). The Sen-Wyller formula is generally assumed to be more accurate, especially in the lower ionosphere, but the Appleton-Hartree formula can often be used with an effective collision frequency profile to save computer time.

When the effect of the earth's magnetic field is included and ray paths are calculated near vertical incidence, a spitze (Davies, 1965, p. 202) often occurs in the ray path. (At a spitze, the usual formulas for refractive index become indeterminate because the wave normal is parallel with the earth's magnetic field and the wave frequency equals the local plasma frequency.) Two versions of the refractive index subroutine have been developed to calculate ray paths through a spitze. These two versions will also work in the absence of a spitze, but the standard versions are much faster.

The input to the refractive index subroutines is through blank common and common blocks /XX/, /YY/, and /ZZ/. Output is through common block /RIN/. The refractive index subroutine is called through the entry RINDEX. The subroutine names are used only for user identification. The following 8 versions of the refractive index subroutine are listed in this appendix:

```
      SUBROUTINE AHWFWC                                                 WFWC001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   WFWC002
C          APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS         WFWC003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                WFWC004
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   WFWC005
     1              PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR  WFWC006
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WFWC007
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRWFWC008
     1            ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                    WFWC009
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               WFWC010
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                WFWC011
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       WFWC012
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   WFWC013
      LOGICAL SPACE                                                     WFWC014
      REAL KR,KTH,KPH,K2                                                WFWC015
      COMPLEX N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT,         WFWC016
     1        POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,    WFWC017
     2        KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   WFWC018
     3        KPHPK                                                     WFWC019
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),         WFWC020
     1     (FIELD=1.),                                                  WFWC021
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            WFWC022
     3     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   WFWC023
     4     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        WFWC024
     5     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)         WFWC025
     6    ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      WFWC026
     7     (I=(0.,1.)),(ABSLIM=1.E-5)                                   WFWC027
      ENTRY RINDEX                                                      WFWC028
      OM=PIT2*1.E6*F                                                    WFWC029
      C2=C*C                                                            WFWC030
      K2=KR*KR+KTH*KTH+KPH*KPH                                          WFWC031
      OM2=OM*OM                                                         WFWC032
      VR =C/OM*KR                                                       WFWC033
      VTH=C/OM*KTH                                                      WFWC034
      VPH=C/OM*KPH                                                      WFWC035
      CALL ELECTX                                                       WFWC036
      CALL MAGY                                                         WFWC037
      V2=VR**2+VTH**2+VPH**2                                            WFWC038
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       WFWC039
      YLV=VDOTY/V2                                                      WFWC040
      YL2=VDOTY**2/V2                                                   WFWC041
      YT2=Y**2-YL2                                                      WFWC042
      YT4=YT2*YT2                                                       WFWC043
      CALL COLFRZ                                                       WFWC044
      U=CMPLX(1.,-Z)                                                    WFWC045
      UX=U-X                                                            WFWC046
      UX2=UX*UX                                                         WFWC047
      RAD=RAY*CSQRT(YT4+4.*YL2*UX2)                                     WFWC048
      D=2.*U*UX-YT2+RAD                                                 WFWC049
      D2=D*D                                                            WFWC050
      N2=1.-2.*X*UX/D                                                   WFWC051
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           WFWC052
      PPSPR =YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             WFWC053
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             WFWC054
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             WFWC055
      PNPX=-(2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2 WFWC056
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   WFWC057
      PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2                                 WFWC058
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR                WFWC059
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH               WFWC060
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH               WFWC061
      PNPVR =PNPPS*(VR *YL2/V2-YLV*YR )                                 WFWC062
      PNPVTH=PNPPS*(VTH*YL2/V2-YLV*YTH)                                 WFWC063
      PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)                                 WFWC064
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)                                  WFWC065
```

```
      PNPT=PNPX*PXPT                                                    WFWC066
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 WFWC067
      POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VDOTY*UX)                        WFWC068
      GAM=(-YT2+RAD)/(2.*UX)                                            WFWC069
      LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))                                 WFWC070
      KAY2=OM2/C2*N2                                                    WFWC071
      IF(RSTART.EQ.0.) GO TO 1                                          WFWC072
      SCALE=SQRT(REAL(KAY2)/K2)                                         WFWC073
      KR =SCALE*KR                                                      WFWC074
      KTH=SCALE*KTH                                                     WFWC075
      KPH=SCALE*KPH                                                     WFWC076
    1 CONTINUE                                                          WFWC077
C********* CALCULATES A HAMILTONIAN H                                   WFWC078
      H=.5*(C2*K2/OM2-N2)                                               WFWC079
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  WFWC080
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            WFWC081
      PHPT =-PNPT                                                       WFWC082
      PHPR =-PNPR                                                       WFWC083
      PHPTH=-PNPTH                                                      WFWC084
      PHPPH=-PNPPH                                                      WFWC085
      PHPOM=-NNP/OM                                                     WFWC086
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      WFWC087
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     WFWC088
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     WFWC089
      KPHPK=N2                                                          WFWC090
      RETURN                                                            WFWC091
         END                                                            WFWC092
```

```
      SUBROUTINE AHWFNC                                                 WFNC001
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE  WFNC002
C           APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS          WFNC003
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                WFNC004
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               WFNC005
     1              H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHIWFNC006
     2           ,PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPIWFNC007
     3           ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN           WFNC008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WFNC009
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRWFNC010
     1          ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                      WFNC011
      COMMON /ZZ/ MODZ,Z(4)                                             WFNC012
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       WFNC013
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)                 WFNC014
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   WFNC015
      LOGICAL SPACE                                                     WFNC016
      REAL KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,N2,NNP,LPOLAR,LPOLRI   WFNC017
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),         WFNC018
     1     (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),         WFNC019
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), WFNC020
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),              WFNC021
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            WFNC022
     5     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   WFNC023
     6     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        WFNC024
     7     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),        WFNC025
     8     (MODZ=1H ),(U=1.)                                            WFNC026
      ENTRY RINDEX                                                      WFNC027
      OM=PIT2*1.E6*F                                                    WFNC028
      C2=C*C                                                            WFNC029
      K2=KR*KR+KTH*KTH+KPH*KPH                                          WFNC030
      OM2=OM*OM                                                         WFNC031
      VR =C/OM*KR                                                       WFNC032
      VTH=C/OM*KTH                                                      WFNC033
      VPH=C/OM*KPH                                                      WFNC034
```

```
      CALL ELECTX                                                       WFNC035
      CALL MAGY                                                         WFNC036
      V2=VR**2+VTH**2+VPH**2                                            WFNC037
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       WFNC038
      YLV=VDOTY/V2                                                      WFNC039
      YL2=VDOTY**2/V2                                                   WFNC040
      YT2=Y**2-YL2                                                      WFNC041
      YT4=YT2*YT2                                                       WFNC042
      UX=U-X                                                            WFNC043
      UX2=UX*UX                                                         WFNC044
      RAD=RAY*SQRT(YT4+4.*YL2*UX2)                                      WFNC045
      D=2.*UX-YT2+RAD                                                   WFNC046
      D2=D*D                                                            WFNC047
      N2=1.-2.*X*UX/D                                                   WFNC048
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           WFNC049
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             WFNC050
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             WFNC051
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             WFNC052
      PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2   WFNC053
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   WFNC054
      NNP=N2-(2.*X*PNPX+Y*PNPY)                                         WFNC055
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPPS*PPSPR                           WFNC056
      PNPTH=PNPK*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH                          WFNC057
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH                          WFNC058
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               WFNC059
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               WFNC060
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               WFNC061
      PNPT=PNPX*PXPT                                                    WFNC062
      SPACE=N2.EQ.1.                                                    WFNC063
      POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)                           WFNC064
      GAM=(-YT2+RAD)/(2.*UX)                                            WFNC065
      LPOLRI=X*SQRT(YT2)/(UX*(U+GAM))                                   WFNC066
      KAY2=OM2/C2*N2                                                    WFNC067
      IF(RSTART.EQ.0.) GO TO 1                                          WFNC068
      SCALE=SQRT(KAY2/K2)                                               WFNC069
      KR =SCALE*KR                                                      WFNC070
      KTH=SCALE*KTH                                                     WFNC071
      KPH=SCALE*KPH                                                     WFNC072
 1    CONTINUE                                                          WFNC073
C********** CALCULATES A HAMILTONIAN H                                  WFNC074
      H=.5*(C2*K2/OM2-N2)                                               WFNC075
C********** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                 WFNC076
C********** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.           WFNC077
      PHPT =-PNPT                                                       WFNC078
      PHPR =-PNPR                                                       WFNC079
      PHPTH=-PNPTH                                                      WFNC080
      PHPPH=-PNPPH                                                      WFNC081
      PHPOM=-NNP/OM                                                     WFNC082
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      WFNC083
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     WFNC084
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     WFNC085
      KPHPK=N2                                                          WFNC086
      RETURN                                                            WFNC087
         END                                                            WFNC088-
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      SUBROUTINE AHNFWC                                                 NFWC010
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   NFWC011
C          APPLETON-HARTREE FORMULA -- NO FIELD, WITH COLLISIONS        NFWC012
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                NFWC013
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   NFWC014
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  NFWC015
     2             SGN                                                  NFWC016
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  NFWC017
      COMMON /YY/ MODY,Y(16)                                            NFWC018
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               NFWC019
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       NFWC020
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ ID(10),W0,W(400)                NFWC021
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   NFWC022
      LOGICAL SPACE                                                     NFWC023
      REAL KR,KTH,KPH,K2                                                NFWC024
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   NFWC025
     1        KPHPK,POLAR,LPOLAR,U,I,PNPX,PNPZ,                         NFWC026
     2        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          NFWC027
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=1.),         NFWC028
     1     (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),                 NFWC029
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            NFWC030
     3     (MODY=1H ),                                                  NFWC031
     4     (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      NFWC032
     5     (I=(0.,1.)),(ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)NFWC033
      ENTRY RINDEX                                                      NFWC034
      OM=PIT2*1.E6*F                                                    NFWC035
      C2=C*C                                                            NFWC036
      K2=KR*KR+KTH*KTH+KPH*KPH                                          NFWC037
      OM2=OM*OM                                                         NFWC038
      VR =C/OM*KR                                                       NFWC039
      VTH=C/OM*KTH                                                      NFWC040
      VPH=C/OM*KPH                                                      NFWC041
      CALL ELECTX                                                       NFWC042
      CALL COLFRZ                                                       NFWC043
      U=1.-I*Z                                                          NFWC044
      N2=1.-X/U                                                         NFWC045
      PNPX=-1./(2.*U)                                                   NFWC046
      PNPZ=-I*X/(2.*U**2)                                               NFWC047
      NNP=N2-(2.*X*PNPX+Z*PNPZ)                                         NFWC048
      PNPR =PNPX*PXPR +PNPZ*PZPR                                        NFWC049
      PNPTH=PNPX*PXPTH+PNPZ*PZPTH                                       NFWC050
      PNPPH=PNPX*PXPPH+PNPZ*PZPPH                                       NFWC051
      PNPT=PNPX*PXPT                                                    NFWC052
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 NFWC053
      KAY2=OM2/C2*N2                                                    NFWC054
      IF(RSTART.EQ.0.) GO TO 1                                          NFWC055
      SCALE=SQRT(REAL(KAY2)/K2)                                         NFWC056
      KR =SCALE*KR                                                      NFWC057
      KTH=SCALE*KTH                                                     NFWC058
      KPH=SCALE*KPH                                                     NFWC059
    1 CONTINUE                                                          NFWC060
C********* CALCULATES A HAMILTONIAN H                                   NFWC061
      H=.5*(C2*K2/OM2-N2)                                               NFWC062
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  NFWC063
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            NFWC064
      PHPT =-PNPT                                                       NFWC065
      PHPR =-PNPR                                                       NFWC066
      PHPTH=-PNPTH                                                      NFWC067
      PHPPH=-PNPPH                                                      NFWC068
      PHPOM=-NNP/OM                                                     NFWC069
      PHPKR =C2/OM2*KR                                                  NFWC070
      PHPKTH=C2/OM2*KTH                                                 NFWC071
      PHPKPH=C2/OM2*KPH                                                 NFWC072
      KPHPK=N2                                                          NFWC073
      RETURN                                                            NFWC074
         END                                                            NFWC075-
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      SUBROUTINE AHNFNC
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE
C          APPLETON-HARTREE FORMULA -- NO FIELD, NO COLLISIONS
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,
     1             H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,
     2             PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPI
     3            ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON /YY/ MODY,Y(16)  /ZZ/ MODZ,Z(4)
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)
      EQUIVALENCE (RAY,W(1)),(F,W(6))
      LOGICAL SPACE
      REAL N2,NNP,KR,KTH,KPH,K2,KPHPK,KPHPKI,KAY2,KAY2I,LPOLAR,LPOLRI
      DATA (MODRIN=8HAPPLETON,8H-HARTREE,8H FORMULA),(COLL=0.),
     1     (FIELD=0.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.),
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(POLARI=1.),(LPOLAR=0.),
     4     (LPOLRI=1.),
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
     5     (MODY=1H ),(MODZ=1H ),
     6     (NNP=1.),(PNPX=-0.5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)
      ENTRY RINDEX
      OM=PIT2*1.E6*F
      C2=C*C
      K2=KR*KR+KTH*KTH+KPH*KPH
      OM2=OM*OM
      VR =C/OM*KR
      VTH=C/OM*KTH
      VPH=C/OM*KPH
      CALL ELECTX
      PNPR =PNPX*PXPR
      PNPTH=PNPX*PXPTH
      PNPPH=PNPX*PXPPH
      PNPT=PNPX*PXPT
      N2=1.-X
      SPACE=N2.EQ.1.
      KAY2=OM2/C2*N2
      IF(RSTART.EQ.0.) GO TO 1
      SCALE=SQRT(KAY2/K2)
      KR =SCALE*KR
      KTH=SCALE*KTH
      KPH=SCALE*KPH
    1 CONTINUE
C********* CALCULATES A HAMILTONIAN H
      H=.5*(C2*K2/OM2-N2)
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
      PHPT =-PNPT
      PHPR =-PNPR
      PHPTH=-PNPTH
      PHPPH=-PNPPH
      PHPOM=-NNP/OM
      PHPKR =C2/OM2*KR
      PHPKTH=C2/OM2*KTH
      PHPKPH=C2/OM2*KPH
      KPHPK=N2
      RETURN
         END
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      SUBROUTINE BQWFWC
C********** CALCULATES A HAMILTONIAN H
C********** (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)
C********** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO
C********** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.
C********** WITH FIELD, WITH COLLISIONS
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,
     2             SGN
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART
      COMMON /FLG/ NTYP,NEWWR,NEWWP,PENET,LINES,IHOP,HPUNCH
      EQUIVALENCE (RAY,W(1)),(F,W(6))
      LOGICAL SPACE
      REAL KR,KTH,KPH,K2,KDOTY,K4,KDOTY2
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,
     1        POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2,D2,
     2        KPHPK,U2,A,B,ALPHA,BETA,GAMMA,PHPX,PHPY2,PHPK2,PHPU,PHPZ,
     3        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT
      DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=1.),
     1     (FIELD=1.),
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),
     3     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),
     4     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),
     5     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.)
     6   ,(Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),
     7     (I=(0.,1.)),(ABSLIM=1.E-5),(SGN=1.)
      ENTRY RINDEX
      OM=PIT2*1.E6*F
      C2=C*C
      K2=KR*KR+KTH*KTH+KPH*KPH
      OM2=OM*OM
      CALL ELECTX
      IF(X.LT..1) GO TO 2
      K4=K2*K2
      OM4=OM2*OM2
      C4=C2*C2
      CALL MAGY
      Y2=Y*Y
      KDOTY=KR*YR+KTH*YTH+KPH*YPH
      KDOTY2=KDOTY*KDOTY
      CALL COLFRZ
      U=CMPLX(1.,-Z)
      U2=U*U
      UX=U-X
      UX2=UX*UX
      A=UX*U2-U*Y2
      B=-2.*U*UX2+Y2*(2.*U-X)
      ALPHA=A*C4*K4+X*KDOTY2*C4*K2
      BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2
      GAMMA=(UX2-Y2)*UX*OM4
      H=ALPHA+BETA+GAMMA
      PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+
     1 (-3.*UX2+Y2)*OM4
      PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4
      PHPKY2   =X*C2*(C2*K2-OM2)
      PHPU=(2.*U*UX+U2-Y2)*C4*K4+2.*(Y2-UX2-2.*U*UX)*C2*K2*OM2+(3.*UX2
     1 -Y2)*OM4
      PHPZ=-I*PHPU
      PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2
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```
      PHPT=PHPX*PXPT                                                    BQWC066
      PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2  *2.*KDOTY*              BQWC067
     1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR) +PHPZ*PZPR                        BQWC068
      PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2  *2.*KDOTY*              BQWC069
     1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT) +PHPZ*PZPTH                       BQWC070
      PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2  *2.*KDOTY*              BQWC071
     1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP) +PHPZ*PZPPH                       BQWC072
      PHPOM=(2.*BETA+4.*GAMMA)/OM                                       BQWC073
     1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2  *KDOTY2/OM -PHPZ*Z/OM    BQWC074
      PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2  *YR                          BQWC075
      PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2  *YTH                         BQWC076
      PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2  *YPH                         BQWC077
      KAY2=K2*(-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)  BQWC078
C                                                                       BQWC079
      IF(RSTART.EQ.0.) GO TO 1                                          BQWC080
      SCALE=SQRT((-REAL(BETA)+SGN*RAY*SQRT(REAL(BETA)**2                BQWC081
     1 -4.*REAL(ALPHA)*REAL(GAMMA)))/(2.*REAL(ALPHA)))                  BQWC082
      KR =SCALE*KR                                                      BQWC083
      KTH=SCALE*KTH                                                     BQWC084
      KPH=SCALE*KPH                                                     BQWC085
    1 CONTINUE                                                          BQWC086
C********* THE FOLLOWING 3 CARDS USED FOR RAY TRACING IN COMPLEX SPACE  BQWC087
C     IF(CABS((-BETA-SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.).  BQWC088
C    1LT.CABS((-BETA+SGN*RAY*CSQRT(BETA**2-4.*ALPHA*GAMMA))/ALPHA-2.)   BQWC089
C    2 .AND.RSTART.EQ.0.) SGN=-SGN                                      BQWC090
      KPHPK=4.*ALPHA+2.*BETA                                            BQWC091
      SPACE=CABS(C2*KAY2/OM2-1.).LT.ABSLIM                              BQWC092
      POLAR =SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY*I                   BQWC093
      LPOLR = SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)*I                  BQWC094
      RETURN                                                            BQWC095
C           CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE  BQWC096
C           APPLETON-HARTREE FORMULA WITH FIELD, WITH COLLISIONS        BQWC097
    2 CONTINUE                                                          BQWC098
      VR =C/OM*KR                                                       BQWC099
      VTH=C/OM*KTH                                                      BQWC100
      VPH=C/OM*KPH                                                      BQWC101
      CALL MAGY                                                         BQWC102
      V2=VR**2+VTH**2+VPH**2                                            BQWC103
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       BQWC104
      YLV=VDOTY/V2                                                      BQWC105
      YL2=VDOTY**2/V2                                                   BQWC106
      YT2=Y**2-YL2                                                      BQWC107
      YT4=YT2*YT2                                                       BQWC108
      CALL COLFRZ                                                       BQWC109
      U=CMPLX(1.,-Z)                                                    BQWC110
      UX=U-X                                                            BQWC111
      UX2=UX*UX                                                         BQWC112
      RAD=SGN*RAY*CSQRT(YT4+4.*YL2*UX2)                                 BQWC113
      D=2.*U*UX-YT2+RAD                                                 BQWC114
      D2=D*D                                                            BQWC115
      N2=1.-2.*X*UX/D                                                   BQWC116
      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           BQWC117
      PPSPR= YL2/Y*PYPR -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV             BQWC118
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             BQWC119
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             BQWC120
      PNPX=-(2.*U*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2 BQWC121
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   BQWC122
      PNPZ=I*X*(-2.*UX2-YT2+YT4/RAD)/D2                                 BQWC123
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)                                  BQWC124
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPZ*PZPR +PNPPS*PPSPR                BQWC125
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH               BQWC126
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH               BQWC127
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               BQWC128
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               BQWC129
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               BQWC130
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      PNPT=PNPX*PXPT                                                    BQWC131
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 BQWC132
      POLAR=-I*SQRT(V2)*(-YT2+RAD)/(2.*VDOTY*UX)                        BQWC133
      GAM=(-YT2+RAD)/(2.*UX)                                            BQWC134
      LPOLAR=I*X*SQRT(YT2)/(UX*(U+GAM))                                 BQWC135
      KAY2=OM2/C2*N2                                                    BQWC136
      IF(RSTART.EQ.0.) GO TO 3                                          BQWC137
      SCALE=SQRT(REAL(KAY2)/K2)                                         BQWC138
      KR =SCALE*KR                                                      BQWC139
      KTH=SCALE*KTH                                                     BQWC140
      KPH=SCALE*KPH                                                     BQWC141
    3 CONTINUE                                                          BQWC142
      H=.5*(C2*K2/OM2-N2)                                               BQWC143
      PHPT =-PNPT                                                       BQWC144
      PHPR =-PNPR                                                       BQWC145
      PHPTH=-PNPTH                                                      BQWC146
      PHPPH=-PNPPH                                                      BQWC147
      PHPOM=-NNP/OM                                                     BQWC148
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      BQWC149
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     BQWC150
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     BQWC151
      KPHPK=N2                                                          BQWC152
      RETURN                                                            BQWC153
         END                                                            BQWC154
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      SUBROUTINE BQWFNC                                                 BQNC001
C********** CALCULATES A HAMILTONIAN H                                   BQNC002
C********** (= BOOKER QUARTIC FOR VERTICAL INCIDENCE, S=0, C=1)          BQNC003
C********** AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  BQNC004
C********** TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            BQNC005
C********** WITH FIELD, NO COLLISIONS                                    BQNC006
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                BQNC007
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,KAY2I,               BQNC008
     1             H,HI,PHPT,PHPTI,PHPR,PHPRI,PHPTH,PHPTHI,PHPPH,PHPPHI,BQNC009
     2             PHPOM,PHPOMI,PHPKR,PHPKRI,PHPKTH,PHPKTI,PHPKPH,PHPKPIBQNC010
     3            ,KPHPK,KPHPKI,POLAR,POLARI,LPOLAR,LPOLRI,SGN          BQNC011
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  BQNC012
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRBQNC013
     1           ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                     BQNC014
      COMMON /ZZ/ MODZ,Z(4)                                             BQNC015
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       BQNC016
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)                 BQNC017
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   BQNC018
      LOGICAL SPACE                                                     BQNC019
      REAL N2,NNP,LPOLAR,LPOLARI,KR,KTH,KPH,K2,KDOTY,K4,KDOTY2,         BQNC020
     1     KPHPK,KPHPKI,KAY2,KAY2I                                      BQNC021
      DATA (MODRIN=8HBOOKER Q,8HUARTIC, ,8HS=0, C=1),(COLL=0.),         BQNC022
     1     (FIELD=1.),(KAY2I=0.),(HI=0.),(PHPTI=0.),(PHPRI=0.),         BQNC023
     2     (PHPTHI=0.),(PHPPHI=0.),(PHPOMI=0.),(PHPKRI=0.),(PHPKTI=0.), BQNC024
     3     (PHPKPI=0.),(KPHPKI=0.),(POLAR=0.),(LPOLAR=0.),              BQNC025
     4     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            BQNC026
     5     (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),   BQNC027
     6     (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),        BQNC028
     7     (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),        BQNC029
     8     (MODZ=1H ),(U=1.),(U2=1.)                                    BQNC030
      ENTRY RINDEX                                                      BQNC031
      OM=PIT2*1.E6*F                                                    BQNC032
      C2=C*C                                                            BQNC033
      K2=KR*KR+KTH*KTH+KPH*KPH                                          BQNC034
      OM2=OM*OM                                                         BQNC035
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      CALL ELECTX                                                       BQNC036
      IF(X.LT..1) GO TO 2                                               BQNC037
      K4=K2*K2                                                          BQNC038
      OM4=OM2*OM2                                                       BQNC039
      C4=C2*C2                                                          BQNC040
      CALL MAGY                                                         BQNC041
      Y2=Y*Y                                                            BQNC042
      KDOTY=KR*YR+KTH*YTH+KPH*YPH                                       BQNC043
      KDOTY2=KDOTY*KDOTY                                                BQNC044
      UX=U-X                                                            BQNC045
      UX2=UX*UX                                                         BQNC046
      A=UX*U2-U*Y2                                                      BQNC047
      B=-2.*U*UX2+Y2*(2.*U-X)                                           BQNC048
      ALPHA=A*C4*K4+X*KDOTY2*C4*K2                                      BQNC049
      BETA=B*C2*OM2*K2-X*KDOTY2*C2*OM2                                  BQNC050
      GAMMA=(UX2-Y2)*UX*OM4                                             BQNC051
      H=ALPHA+BETA+GAMMA                                                BQNC052
      PHPX=-U2*C4*K4+KDOTY2*C4*K2+(4.*U*UX-Y2)*C2*OM2*K2-KDOTY2*C2*OM2+ BQNC053
     1 (-3.*UX2+Y2)*OM4                                                 BQNC054
      PHPY2=-U*C4*K4+(2.*U-X)*C2*OM2*K2-UX*OM4                          BQNC055
      PHPKY2   =X*C2*(C2*K2-OM2)                                        BQNC056
      PHPK2=2.*A*C4*K2+X*KDOTY2*C4+B*C2*OM2                             BQNC057
      PHPT=PHPX*PXPT                                                    BQNC058
      PHPR =PHPX*PXPR +PHPY2*2.*Y*PYPR +PHPKY2  *2.*KDOTY*              BQNC059
     1 (KR*PYRPR+KTH*PYTPR+KPH*PYPPR)                                   BQNC060
      PHPTH=PHPX*PXPTH+PHPY2*2.*Y*PYPTH+PHPKY2  *2.*KDOTY*              BQNC061
     1 (KR*PYRPT+KTH*PYTPT+KPH*PYPPT)                                   BQNC062
      PHPPH=PHPX*PXPPH+PHPY2*2.*Y*PYPPH+PHPKY2  *2.*KDOTY*              BQNC063
     1 (KR*PYRPP+KTH*PYTPP+KPH*PYPPP)                                   BQNC064
      PHPOM=(2.*BETA+4.*GAMMA)/OM                                       BQNC065
     1 -2.*PHPX*X/OM-2.*PHPY2*Y2/OM-2.*PHPKY2  *KDOTY2/OM               BQNC066
      PHPKR= 2.*PHPK2*KR +2.*KDOTY*PHPKY2  *YR                          BQNC067
      PHPKTH=2.*PHPK2*KTH+2.*KDOTY*PHPKY2  *YTH                         BQNC068
      PHPKPH=2.*PHPK2*KPH+2.*KDOTY*PHPKY2  *YPH                         BQNC069
      KAY2 = K2   *(-BETA+RAY*SQRT(BETA**2-4.*ALPHA*GAMMA))/(2.*ALPHA)  BQNC070
      IF(RSTART.EQ.0.) GO TO 1                                          BQNC071
      SCALE=SQRT(KAY2/K2)                                               BQNC072
      KR =SCALE*KR                                                      BQNC073
      KTH=SCALE*KTH                                                     BQNC074
      KPH=SCALE*KPH                                                     BQNC075
  1   CONTINUE                                                          BQNC076
      KPHPK=4.*ALPHA+2.*BETA                                            BQNC077
      SPACE=KAY2.EQ.OM2/C2                                              BQNC078
      POLARI=SQRT(K2)*(U+X*OM2/(C2*KAY2-OM2))/KDOTY                     BQNC079
      LPOLRI= SQRT(Y2-KDOTY2/K2)/UX*(1.-C2*KAY2/OM2)                    BQNC080
      RETURN                                                            BQNC081
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   BQNC082
C          APPLETON-HARTREE FORMULA WITH FIELD, NO COLLISIONS           BQNC083
  2   CONTINUE                                                          BQNC084
      VR =C/OM*KR                                                       BQNC085
      VTH=C/OM*KTH                                                      BQNC086
      VPH=C/OM*KPH                                                      BQNC087
      CALL MAGY                                                         BQNC088
      V2=VR**2+VTH**2+VPH**2                                            BQNC089
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                       BQNC090
      YLV=VDOTY/V2                                                      BQNC091
      YL2=VDOTY**2/V2                                                   BQNC092
      YT2=Y**2-YL2                                                      BQNC093
      YT4=YT2*YT2                                                       BQNC094
      UX=U-X                                                            BQNC095
      UX2=UX*UX                                                         BQNC096
      RAD=RAY*SQRT(YT4+4.*YL2*UX2)                                      BQNC097
      D=2.*UX-YT2+RAD                                                   BQNC098
      D2=D*D                                                            BQNC099
      N2=1.-2.*X*UX/D                                                   BQNC100
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      PNPPS=2.*X*UX*(-1.+(YT2-2.*UX2)/RAD)/D2                           BQNC101
      PPSPR= YL2/Y*PYPR  -(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV            BQNC102
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             BQNC103
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             BQNC104
      PNPX=-(2.*UX2-YT2*(U-2.*X)+(YT4*(U-2.*X)+4.*YL2*UX*UX2)/RAD)/D2   BQNC105
      PNPY=2.*X*UX*(-YT2+(YT4+2.*YL2*UX2)/RAD)/(D2*Y)                   BQNC106
      NNP=N2-(2.*X*PNPX+Y*PNPY)                                         BQNC107
      PNPR =PNPX*PXPR +PNPY*PYPR +PNPPS*PPSPR                           BQNC108
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPPS*PPSPTH                          BQNC109
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPPS*PPSPPH                          BQNC110
      PNPVR =PNPPS*(VR *YL2-VDOTY*YR )/V2                               BQNC111
      PNPVTH=PNPPS*(VTH*YL2-VDOTY*YTH)/V2                               BQNC112
      PNPVPH=PNPPS*(VPH*YL2-VDOTY*YPH)/V2                               BQNC113
      PNPT=PNPX*PXPT                                                    BQNC114
      SPACE=N2.EQ.1.                                                    BQNC115
      POLARI=SQRT(V2)*(YT2-RAD)/(2.*VDOTY*UX)                           BQNC116
      GAM=(-YT2+RAD)/(2.*UX)                                            BQNC117
      LPOLRI=K*SQRT(YT2)/(UX*(J+GAM))                                   BQNC118
      KAY2=OM2/C2*N2                                                    BQNC119
      IF(RSTART.EQ.0.) GO TO 3                                          BQNC120
      SCALE=SQRT(KAY2/K2)                                               BQNC121
      KR =SCALE*KR                                                      BQNC122
      KTH=SCALE*KTH                                                     BQNC123
      KPH=SCALE*KPH                                                     BQNC124
    3 CONTINUE                                                          BQNC125
      H=.5*(C2*K2/OM2-N2)                                               BQNC126
      PHPT =-PNPT                                                       BQNC127
      PHPR =-PNPR                                                       BQNC128
      PHPTH=-PNPTH                                                      BQNC129
      PHPPH=-PNPPH                                                      BQNC130
      PHPOM=-NNP/OM                                                     BQNC131
      PHPKR =C2/OM2*KR -C/OM*PNPVR                                      BQNC132
      PHPKTH=C2/OM2*KTH-C/OM*PNPVTH                                     BQNC133
      PHPKPH=C2/OM2*KPH-C/OM*PNPVPH                                     BQNC134
      KPHPK=N2                                                          BQNC135
      RETURN                                                            BQNC136
         END                                                            BQNC137-
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      SUBROUTINE SWWF                                                   SWWF001
C         CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE    SWWF002
C         SEN-WYLLER FORMULA -- WITH FIELD                              SWWF003
C         NEEDS SUBROUTINE FSW AND FUNCTIONS C AND S.                   SWWF004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,SEA,LOGTEN              SWWF005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   SWWF006
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  SWWF007
     2             SGN                                                  SWWF008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SWWF009
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRSWWF010
     1            ,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                    SWWF011
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               SWWF012
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       SWWF013
      COMMON R,TH,PH,KR,KTH,KPH   /WW/ ID(10),W0,W(400)                 SWWF014
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   SWWF015
      LOGICAL SPACE                                                     SWWF016
      REAL KR,KTH,KPH,K2                                                SWWF017
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   SWWF018
     1        KPHPK,POLAR,LPOLAR,I,U,RAD,D,PNPPS,PNPX,PNPY,PNPZ,UX,UX2, SWWF019
     2        ALPHA,BETA,GAMMA,A,B,C,TEMP1,TEMP2,TEMP3,ALPOAL,BEPOBE,   SWWF020
     3        GAPOGA,CB2,N2M1,J2,D2GA,DAL,DBET,DGAM,DADY,DADZ,DBDY,DBDZ,SWWF021
     4        DCDY,DCDZ,DUDZ,DT1DX,DT1DY,DT1DZ,DT1DPS,DT2DX,DT2DY,DT2DZ,SWWF022
     5        DT2DPS,DRADDX,DRADDY,DRADDZ,DRODPS,DODX,DODY,DODZ,DODPX,  SWWF023
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     6         UPX,N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT             SWWF024
      DATA (MODRIN=8H      SE,8HN-WYLLER,8H FORMULA),(COLL=1.),               SWWF025
     1         (FIELD=1.),(LPOLAR=(0.,0.)),                                   SWWF026
     2         (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),              SWWF027
     3         (Y=0.),(PYPR=0.),(PYPTH=0.),(PYPPH=0.),(YR=0.),(PYRPR=0.),     SWWF028
     4         (PYRPT=0.),(PYRPP=0.),(YTH=0.),(PYTPR=0.),(PYTPT=0.),          SWWF029
     5         (PYTPP=0.),(YPH=0.),(PYPPR=0.),(PYPPT=0.),(PYPPP=0.),          SWWF030
     6         (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                        SWWF031
     7         (I=(0.,1.)),(ABSLIM=1.E-5)                                     SWWF032
      ENTRY RINDEX                                                            SWWF033
      OM=PIT2*1.E6*F                                                          SWWF034
      C2=SEA*SEA                                                              SWWF035
      K2=KR*KR+KTH*KTH+KPH*KPH                                                SWWF036
      OM2=OM*OM                                                               SWWF037
      VR =SEA/OM*KR                                                           SWWF038
      VTH=SEA/OM*KTH                                                          SWWF039
      VPH=SEA/OM*KPH                                                          SWWF040
      CALL ELECTX                                                             SWWF041
      CALL MAGY                                                               SWWF042
      OPY=1.+Y                                                                SWWF043
      OMY=1.-Y                                                                SWWF044
      CALL COLFRZ                                                             SWWF045
      Z2=Z*Z                                                                  SWWF046
      CALL FSW(1./Z,ALPHA,DAL)                                                SWWF047
      ALPOAL=DAL/ALPHA                                                        SWWF048
      CALL FSW(OMY/Z,BETA,DBET)                                               SWWF049
      BEPOBE=DBET/BETA                                                        SWWF050
      CALL FSW(OPY/Z,GAMMA,DGAM)                                              SWWF051
      GAPOGA=DGAM/GAMMA                                                       SWWF052
      U=Z/ALPHA                                                               SWWF053
      DUDZ=(1.+ALPOAL/Z)/ALPHA                                                SWWF054
      U2=U*U                                                                  SWWF055
      UX=U-X                                                                  SWWF056
      UPX=U+X                                                                 SWWF057
      B=ALPHA/BETA                                                            SWWF058
      DBDY=B*BEPOBE/Z                                                         SWWF059
      DBDZ=-B*(ALPOAL-OMY*BEPOBE)/Z2                                          SWWF060
      C=ALPHA/GAMMA                                                           SWWF061
      DCDY=-C*GAPOGA/Z                                                        SWWF062
      DCDZ=-C*(ALPOAL-OPY*GAPOGA)/Z2                                          SWWF063
      A=.5*(B+C)-1.                                                           SWWF064
      DADY=.5*(DBDY+DCDY)                                                     SWWF065
      DADZ=.5*(DBDZ+DCDZ)                                                     SWWF066
      TEMP3=(1.-B*C)*U2+A*U*UPX                                               SWWF067
      V2=     VR**2+VTH**2+VPH**2                                             SWWF068
      VDOTY=VR*YR+VTH*YTH+VPH*YPH                                             SWWF069
      YL2=VDOTY**2/V2                                                         SWWF070
      YT2=Y**2-YL2                                                            SWWF071
      Y2=Y*Y                                                                  SWWF072
      S2PSI=YT2/Y2                                                            SWWF073
      C2PSI=YL2/Y2                                                            SWWF074
      UX2=UX*UX                                                               SWWF075
      CB2=(C-B)**2                                                            SWWF076
      TEMP1=TEMP3*S2PSI                                                       SWWF077
      DT1DX= A*U*S2PSI                                                        SWWF078
      DT1DY=(U*UPX*DADY-U2*(B*DCDY+C*DBDY))*S2PSI                             SWWF079
      DT1DZ=(2.*U*DUDZ*(1.-B*C+A)+A*X*DUDZ-U2*(B*DCDZ+C*DBDZ)+U*UPX*DADZSWWF080
     1 )*S2PSI                                                                SWWF081
C              (1/YLYT) D/DPSI(TEMP1)                                         SWWF082
      DT1DPS=2.*TEMP1/YT2                                                     SWWF083
      TEMP2=U2*CB2*UX2*C2PSI                                                  SWWF084
      DT2DX=-2.*UX*U2*CB2*C2PSI                                               SWWF085
      DT2DY=2.*U2*UX2*C2PSI*(C-B)*(DCDY-DBDY)                                 SWWF086
      DT2DZ=2.*U2*UX2*C2PSI*(C-B)*(DCDZ-DBDZ)+2.*TEMP2*(1./U+1./UX)*DUDZSWWF087
C              (1/YLYT) D/DPSI(TEMP2)                                         SWWF088
      DT2DPS=-2.*TEMP2/YL2                                                    SWWF089
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      RAD=RAY*CSQRT(TEMP1**2+TEMP2)                                     SWHF090
      DRADDX=(TEMP1*DT1DX +.5   *DT2DX )/RAD                            SWHF091
      DRADDY=(TEMP1*DT1DY +   .5*DT2DY )/RAD                            SWHF092
      DRADDZ=(TEMP1*DT1DZ +   .5*DT2DZ )/RAD                            SWHF093
C           (1/YLYT) D/DPSI(RAD)                                        SWHF094
      DRDDPS=(TEMP1*DT1DPS+   .5*DT2DPS)/RAD                            SWHF095
      D=2.*U*UX*(1.+A)-TEMP1+RAD+2.*A*U*X*S2PSI                         SWHF096
      DDDX=-2.*U-DT1D X+DRADDX+2.*A*U*S2PSI                             SWHF097
      DDDY= 2.*U*UX*DADY-DT1DY+DRADDY+2.*U*S2PSI*DADY                   SWHF098
      DDDZ=2.*(1.+A)*DUDZ*(U+UX)+2.*U*UX*DADZ-DT1DZ+DRADDZ+2.*X*S2PSI*  SWHF099
     1 (A*DUDZ+U*DADZ)                                                  SWHF100
C         (1/YLYT) D/DPSI(D)                                            SWHF101
      DDDPS=-DT1DPS+DRDDPS+2.*A*U*X/Y2                                  SWHF102
      N2M1=-2.*X*(UX+U*A*S2PSI)/D                                       SWHF103
      N2=1.+N2M1                                                        SWHF104
C              N D/DX(N)                                                SWHF105
      PNPX=-(UX+U*A*S2PSI)*(1.-X*DDDX/D)/D+X/D                          SWHF106
C              N D/DY(N)                                                SWHF107
      PNPY=-X*U*S2PSI/D*DADY-.5*N2M1/D*DDDY                             SWHF108
C              N D/DZ(N)                                                SWHF109
      PNPZ=-X*(1.+A*S2PSI)/D*DUDZ-X*U*S2PSI/D*DADZ-.5*N2M1/D*DDDZ       SWHF110
C           (N/YLYT) D/DPSI(N)                                          SWHF111
      PNPPS=-X*U*A/(D*Y2)     -.5*N2M1/D*DDDPS                          SWHF112
      YLV=VDOTY/V2                                                      SWHF113
C         (YLYT) D/DR(PSI)                                              SWHF114
      PPSPR=YL2/Y*PYPR-(VR*PYRPR+VTH*PYTPR+VPH*PYPPR)*YLV               SWHF115
C         (YLYT) D/DTHETA(PSI)                                          SWHF116
      PPSPTH=YL2/Y*PYPTH-(VR*PYRPT+VTH*PYTPT+VPH*PYPPT)*YLV             SWHF117
C         (YLYT) D/DPHI(PSI)                                            SWHF118
      PPSPPH=YL2/Y*PYPPH-(VR*PYRPP+VTH*PYTPP+VPH*PYPPP)*YLV             SWHF119
      PNPR=PNPX*PXPR+PNPY*PYPR+PNPZ*PZPR+PNPPS*PPSPR                    SWHF120
      PNPTH=PNPX*PXPTH+PNPY*PYPTH+PNPZ*PZPTH+PNPPS*PPSPTH               SWHF121
      PNPPH=PNPX*PXPPH+PNPY*PYPPH+PNPZ*PZPPH+PNPPS*PPSPPH               SWHF122
      PNPVR=PNPPS*(VR*YL2/V2-YLV*YR)                                    SWHF123
      PNPVTH=PNPPS*(V TH*YL2/V2-YLV*YTH)                                SWHF124
      PNPVPH=PNPPS*(VPH*YL2/V2-YLV*YPH)                                 SWHF125
      NNP=N2-(2.*X*PNPX+Y*PNPY+Z*PNPZ)                                  SWHF126
      PNPT=PNPX*PXPT                                                    SWHF127
      POLAR=I*(TEMP1-RAD)*Y*SQRT(V2)/(U*UX*(C-B)*VDOTY)                 SWHF128
      COSPSI=VDOTY/(Y*SQRT(V2))                                         SWHF129
      LPOLAR=(.5*I*(C-B)*POLAR+A*COSPSI)*SQRT(S2PSI)/                   SWHF130
     1 (POLAR*(UX*(1.+.5*I*(C-B)*COSPSI*POLAR)+A*(U-X*C2PSI)))          SWHF131
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 SWHF132
      KAY2=OM2/C2*N2                                                    SWHF133
      IF(RSTART.EQ.0.) GO TO 1                                          SWHF134
      SCALE=SQRT(REAL(KAY2)/K2)                                         SWHF135
      KR =SCALE*KR                                                      SWHF136
      KTH=SCALE*KTH                                                     SWHF137
      KPH=SCALE*KPH                                                     SWHF138
  1   CONTINUE                                                          SWHF139
C********* CALCULATES A HAMILTONIAN H                                   SWHF140
      H=.5*(C2*K2/OM2-N2)                                               SWHF141
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  SWHF142
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            SWHF143
      PHPT =-PNPT                                                       SWHF144
      PHPR =-PNPR                                                       SWHF145
      PHPTH=-PNPTH                                                      SWHF146
      PHPPH=-PNPPH                                                      SWHF147
      PHPOM=-NNP/OM                                                     SWHF148
      PHPKR =C2/OM2*KR -SEA/OM*PNPVR                                    SWHF149
      PHPKTH=C2/OM2*KTH-SEA/OM*PNPVTH                                   SWHF150
      PHPKPH=C2/OM2*KPH-SEA/OM*PNPVPH                                   SWHF151
      KPHPK=N2                                                          SWHF152
      RETURN                                                            SWHF153
         END                                                            SWHF154-
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      SUBROUTINE SWNF                                                   SWNF001
C          CALCULATES THE REFRACTIVE INDEX AND ITS GRADIENT USING THE   SWNF002
C          SEN-WYLLER FORMULA -- NO FIELD                               SWNF003
C          NEEDS SUBROUTINES FGSW AND FSW AND FUNCTIONS C AND S.        SWNF004
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RADIAN,K,C,LOGTEN                SWNF005
      COMMON /RIN/ MODRIN(3),COLL,FIELD,SPACE,KAY2,H,PHPT,PHPR,PHPTH,   SWNF006
     1             PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,KPHPK,POLAR,LPOLAR,  SWNF007
     2             SGN                                                  SWNF008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SWNF009
      COMMON /YY/ MODY,Y(16)                                            SWNF010
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               SWNF011
      COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART       SWNF012
      COMMON R,TH,PH,KR,KTH,KPH    /WW/ IO(10),W0,W(400)                SWNF013
      EQUIVALENCE (RAY,W(1)),(F,W(6))                                   SWNF014
      LOGICAL SPACE                                                     SWNF015
      REAL KR,KTH,KPH,K2                                                SWNF016
      COMPLEX KAY2,H,PHPT,PHPR,PHPTH,PHPPH,PHPOM,PHPKR,PHPKTH,PHPKPH,   SWNF017
     1        KPHPK,POLAR,LPOLAR,PNPX,PNPZ,F1,DF,G1,DG1,                SWNF018
     2        N2,PNPR,PNPTH,PNPPH,PNPVR,PNPVTH,PNPVPH,NNP,PNPT          SWNF019
      DATA (MODRIN=8H       SE,8HN-WYLLER,8H FORMULA),(COLL=1.),        SWNF020
     1     (FIELD=0.),(POLAR=(0.,1.)),(LPOLAR=(0.,0.)),                 SWNF021
     2     (X=0.),(PXPR=0.),(PXPTH=0.),(PXPPH=0.),(PXPT=0.),            SWNF022
     3     (MODY=1H ),                                                  SWNF023
     4     (Z=0.),(PZPR=0.),(PZPTH=0.),(PZPPH=0.),                      SWNF024
     5     (ABSLIM=1.E-5),(PNPVR=0.),(PNPVTH=0.),(PNPVPH=0.)            SWNF025
      ENTRY RINDEX                                                      SWNF026
      OM=PIT2*1.E6*F                                                    SWNF027
      C2=C*C                                                            SWNF028
      K2=KR*KR+KTH*KTH+KPH*KPH                                          SWNF029
      OM2=OM*OM                                                         SWNF030
      VR =C/OM*KR                                                       SWNF031
      VTH=C/OM*KTH                                                      SWNF032
      VPH=C/OM*KPH                                                      SWNF033
      CALL ELECTX                                                       SWNF034
      CALL COLFRZ                                                       SWNF035
      CALL FGSW(1./Z,F1,DF1,G1,DG1)                                     SWNF036
      N2=1.-X*G1                                                        SWNF037
      PNPX=-.5*G1                                                       SWNF038
      PNPZ=.5*X*DG1/Z**2                                                SWNF039
      PNPR=PNPX*PXPR+PNPZ*PZPR                                          SWNF040
      PNPTH=PNPX*PXPTH+PNPZ*PZPTH                                       SWNF041
      PNPPH=PNPX*PXPPH+PNPZ*PZPPH                                       SWNF042
      NNP=N2-(2.*X*PNPX+Z*PNPZ)                                         SWNF043
      PNPT=PNPX*PXPT                                                    SWNF044
      SPACE=REAL(N2).EQ.1..AND.ABS(AIMAG(N2)).LT.ABSLIM                 SWNF045
      KAY2=OM2/C2*N2                                                    SWNF046
      IF(RSTART.EQ.0.) GO TO 1                                          SWNF047
      SCALE=SQRT(REAL(KAY2)/K2)                                         SWNF048
      KR =SCALE*KR                                                      SWNF049
      KTH=SCALE*KTH                                                     SWNF050
      KPH=SCALE*KPH                                                     SWNF051
    1 CONTINUE                                                          SWNF052
C********* CALCULATES A HAMILTONIAN H                                   SWNF053
      H=.5*(C2*K2/OM2-N2)                                               SWNF054
C********* AND ITS PARTIAL DERIVATIVES WITH RESPECT TO                  SWNF055
C********* TIME, R, THETA, PHI, OMEGA, KR, KTHETA, AND KPHI.            SWNF056
      PHPT =-PNPT                                                       SWNF057
      PHPR =-PNPR                                                       SWNF058
      PHPTH=-PNPTH                                                      SWNF059
      PHPPH=-PNPPH                                                      SWNF060
      PHPOM=-NNP/OM                                                     SWNF061
      PHPKR =C2/OM2*KR                                                  SWNF062
      PHPKTH=C2/OM2*KTH                                                 SWNF063
      PHPKPH=C2/OM2*KPH                                                 SWNF064
      KPHPK=N2                                                          SWNF065
      RETURN                                                            SWNF066
         END                                                            SWNF067-
```

```
      SUBROUTINE FGSW (X,F,DF,G,DG)                                     FGSW001
      COMPLEX F,DF,G,DG                                                 FGSW002
      CALL FSW (X,F,DF)                                                 FGSW003
      IF(ABS(X).GT.50.) GO TO 1                                         FGSW004
      G=X*F                                                             FGSW005
      DG=F+X*DF                                                         FGSW006
      RETURN                                                            FGSW007
    1 X2=X*X                                                            FGSW008
      X3=X2*X                                                           FGSW009
      T2=2.*X2                                                          FGSW010
      T3=3.*X2                                                          FGSW011
      T4=4.*X2                                                          FGSW012
      T8=8.*X2                                                          FGSW013
      T12=12.*X2                                                        FGSW014
      T16=16.*X2                                                        FGSW015
      G=CMPLX(1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4)))/T4,       FGSW016
     12.5*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/X)         FGSW017
      DG=.5*CMPLX(35.*(1.-99./T2*(1.-585./T8*(1.-323./T3*(1.-2415./T16))FGSW018
     1 ))/X3,                                                           FGSW019
     2-5.*(1.-189./T4*(1.-715./T12*(1.-357./T4*(1.-513./T4))))/X2)      FGSW020
      RETURN                                                            FGSW021
          END                                                           FGSW 22-
```

```
      SUBROUTINE FSW (Z,F,DF)                                           FSW 001
C        F(Z) = Z*C3/2(Z) + 2.5*I*C5/2(Z)  AND  DF(Z) = DF/DZ           FSW 002
C        WHERE THE INPUT Z IS REAL AND THE OUTPUT F AND DF ARE COMPLEX. FSW 003
C       NEEDS THE SUBPROGRAMS FOR THE FRESNEL INTEGRAL FUNCTIONS S AND CFSW 004
      DIMENSION A(10),B(10),D(10)                                       FSW 005
      COMPLEX F,DF,C1,C2,C3,C8,W,TEMP,I                                 FSW 006
      DATA (I=(0.,1.)),(PI=3.1415926536),(A3=1.333333333)               FSW 007
      DATA (C2=(1.,1.)),(C3=(1.,-1.)),(C4=.79788456  ),(C6=1.333333333) FSW 008
C        C4=SQRT(2./PI)                                                 FSW 009
      DATA(A=.36230845E-02,.29579186E+00,.23193588E+01,.91355870E+01,   FSW 010
     1.25856287E+02,.60488560E+02,.12562218E+03,.24214980E+03,          FSW 011
     2.44918106E+03,.84244774E+03),                                     FSW 012
     3(B=.16747479E-02,.84796280E-01,.25285001E+00,.22665867E+00,       FSW 013
     4.83871933E-01,.13811875E-01,.98017417E-03,.26299148E-04,          FSW 014
     5.19761006E-06,.18781476E-09),                                     FSW 015
     6(D=.10080653E-03,.46117941E-01,.38507643E+00,.68507885E+00,       FSW 016
     7.42648105E+00,.10742102E+00,.10985920E-01,.40924533E-03,          FSW 017
     8.41881263E-05,.54513142E-08),(G=1.5045055)                        FSW 018
      C1=2./3.*I                                                        FSW 019
      C8=C2*A3*SQRT(PI/2.)                                              FSW 020
      X=Z                                                               FSW 021
      X2=X*X                                                            FSW 022
      X3=X2*X                                                           FSW 023
      IF(ABS(X).GT.50.) GO TO 500                                       FSW 024
      IF(ABS(X).GT.6.) GO TO 1                                          FSW 025
      IF(ABS(X).LT..05)GO TO 200                                        FSW 026
C                                     FRESNEL                           FSW 027
      IF(X.GT.0.) GO TO 300                                             FSW 028
100    Y=C4*SQRT(-X)                                                    FSW 029
      X2=X*X                                                            FSW 030
      W=(COS(X)+I*SIN(X))*(1.-C3*(C(Y)+I*S(Y)))                         FSW 031
      F   =C1+C6*(X+C3*X*X/Y*W)                                         FSW 032
      DF=A3*CMPLX(1.,X)+CMPLX(1.5,X)*A3*C3*X/Y*W                        FSW 033
       RETURN                                                           FSW 034
300   Y=C4*SQRT(X)                                                      FSW 035
      X2=X*X                                                            FSW 036
      W=(COS(X)+I*SIN(X))*(1.-C2*(C(Y)-I*S(Y)))                         FSW 037
      F   =C1+C6*(X-C2*X*X/Y*W)                                         FSW 038
      DF=A3*CMPLX(1.,X)-CMPLX(1.5,X)*A3*C2*X/Y*W                        FSW 039
      RETURN                                                            FSW 040
```

```
C                          POWER SERIES                                 FSW 041
  200 X=ABS(Z)                                                          FSW 042
      X2=X*X                                                            FSW 043
      X3=X2*X                                                           FSW 044
      X4=X*X3                                                           FSW 045
      X5=X*X4                                                           FSW 046
      TEMP=-C8* SQRT(X)*CEXP(I*X)                                       FSW 047
      F=CMPLX(4./3.*X-16./9.*X3+64./315.*X5,2./3.+8./3.*X2-32./45.*X4)  FSW 048
     1   +TEMP*X                                                        FSW 049
      DF=CMPLX(4./3.-16./3.*X2+64./63.*X4,16./3.*X-128./45.*X3          FSW 050
     1   +256./945.*X5)                                                 FSW 051
     2   +TEMP*CMPLX(1.5,X)                                             FSW 052
      IF(Z.GE.0.) RETURN                                                FSW 053
      F=-CONJG(F)                                                       FSW 054
      DF=CONJG(DF)                                                      FSW 055
      RETURN                                                            FSW 056
C                               HERMITE                                 FSW 057
    1 XQ = X**2                                                         FSW 058
      X2=XQ                                                             FSW 059
      FR = 0.                                                           FSW 060
      FI = 0.                                                           FSW 061
      DFR = 0.                                                          FSW 062
      DFI = 0.                                                          FSW 063
      DO 2 J = 1,10                                                     FSW 064
      SS = A(J) + XQ                                                    FSW 065
      SB = B(J)/SS                                                      FSW 066
      SD = D(J)/SS                                                      FSW 067
      FR = FR + SB                                                      FSW 068
      FI = FI + SD                                                      FSW 069
      DFR = DFR + SB/SS                                                 FSW 070
    2 DFI = DFI + SD/SS                                                 FSW 071
      F = CMPLX(X*FR,FI)*G                                              FSW 072
      DF = G*(FR - 2.*X* CMPLX(X*DFR,DFI))                              FSW 073
      RETURN                                                            FSW 074
C                            ASYMPTOTIC                                 FSW 075
  500 X2=X*X                                                            FSW 076
      X3=X2*X                                                           FSW 077
      X4=X3*X                                                           FSW 078
      X5=X4*X                                                           FSW 079
      T2=2.*X2                                                          FSW 080
      T3=3.*X2                                                          FSW 081
      T4=4.*X2                                                          FSW 082
      T8=8.*X2                                                          FSW 083
      T16=16.*X2                                                        FSW 084
      T28=28.*X2                                                        FSW 085
      F=CMPLX((1.-35./T4*(1.-99./T4*(1.-195./T4*(1.-323./T4))))/X       FSW 086
     1,5.*(1.-63./T4*(1.-143./T4*(1.-255./T4*(1.-399./T4))))/T2)        FSW 087
      DF=-CMPLX((1.-105./T4*(1.-165./T4*(1.-273./T4*(1.-2907./T28))))/X2FSW 088
     1,5.*(1.-63./T2*(1.-429./T8*(1.-255./T3*(1.-1995./T16))))/X3)      FSW 089
      RETURN                                                            FSW 090
         END                                                            FSW  91-
```

```
      FUNCTION C(X)
      DOUBLEPRECISION   PIH, XD, Y, V, A, QZ, QN, Q, Z
      DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)
      PIH = 1.570796326794897
      XA = ABS(X)
      IF (XA.GT.4.)   GOTO 20
C
      XD = X
      Y = PIH*XD*XD
      V = Y*Y
      A = 1.D0
      Z = A
      M = 15.*(XA + 1.)
      DO 10   I = 1, M
      KZ=2*(I-1)
      KV=4*(I-1)
      QZ = KV + 1
      QN = (KZ + 1)*(KZ + 2)*(KV + 5)
       Q = QZ/QN
      A = -A*Q*V
   10 Z = Z + A
      Z = Z*XD
      C = Z
      RETURN
C
   20  W = PIH*X*X
      XV=XA**4
      C=0.5+(A1-B1/XV)*SIN(W)/XA-(A2-B2/XV)*COS(W)/XA**3
      IF (X.LT.0.)   C = -C
      RETURN
         END
```

```
      FUNCTION S(X)
      DOUBLEPRECISION   PIH, XD, Y, V, A, QZ, QN, Q, Z
      DATA (A1=0.3183099),(A2=0.10132),(B1=0.0968),(B2=0.154)
      PIH = 1.570796326794897
C
      XA = ABS(X)
      IF (XA.GT.4.)   GOTO 20
C
      XD = X
      Y = PIH*XD*XD
      V = Y*Y
      A = Y/3.D0
      Z = A
      M = 15.*(XA + 1.)
      DO 10   I = 1, M
      KZ=2*(I-1)
      KV=4*(I-1)
      QZ = KV + 3
      QN = (KZ + 2)*(KZ + 3)*(KV + 7)
       Q = QZ/QN
       A = -A*Q*V
   10 Z = Z + A
      Z = Z*XD
      S = Z
      RETURN
C
   20  W = PIH*X*X
      XV=XA**4
      S=0.5-(A1-B1/XV)*COS(W)/XA-(A2-B2/XV)*SIN(W)/XA**3
      IF (X.LT.0.)   S = -S
      RETURN
         END
```

APPENDIX 3. ELECTRON DENSITY SUBROUTINES WITH INPUT PARAMETER FORMS

The following electron density models are available. The input parameter forms, which describe the model, and the subroutine listings are given on the pages shown.

A further source of versatility in this ray tracing program is the ease with which specific ionospheric models, suited to the users needs, may be introduced. To add electron density models not included in the program, the user must write a subroutine that calculates the normalized electron density (X) and its gradient ($\partial X/\partial r$, $\partial X/\partial \theta$, $\partial X/\partial \varphi$) as a function of position in spherical coordinates (r, θ, φ). ($X = 80.5 \times 10^{-6} N/f^2$, where N is the electron density in cm^{-3} and f is the wave frequency in MHz.)

Both X and its gradient must be continuous functions of position. The formulas for $\partial X/\partial r$, $\partial X/\partial \theta$, and $\partial X/\partial \varphi$ must be consistent with the variation of X with r, θ, and φ. Otherwise, the program will run slowly and give incorrect results.

The coordinates r, θ, φ refer to the computational coordinate system, which may not be the same as geographic coordinates. In particular, they are geomagnetic coordinates when the earth-centered dipole model of the earth's magnetic field is used.

The input to the subroutine (r, θ, ϕ) is through blank common. (See

Table 3.) The output is through common block /XX/. (See Table 8.) It is useful if the name of the subroutine suggests the model to which it corresponds. The subroutine should have an entry point ELECTX so that other subroutines in the program can call it. Any parameters needed by the subroutine should be input into W101 through W149 of the W array. (See Table 2.) If the model needs massive amounts of data, these should be read in by the subroutine following the example of TABLEX. As in the already existing electron density subroutines, provision should be made for perturbations to the electron density model (irregularities) by having the statement

```
IF(PERT.NE.0.) CALL ELECT1
```

before the RETURN statement at the end of the subroutine.

INPUT PARAMETER FORM FOR SUBROUTINE TABLEX

IONOSPHERIC ELECTRON DENSITY PROFILE

First card tells how many profile points in I4 format. The cards following the first card give the height and electron density of the profile points one point per card in F8. 2, E12. 4 format. The heights must be in increasing order. Set W100 = 1. 0 to read in a new profile. After the cards are read, TABLEX will reset W100 = 0. 0. This subroutine makes an exponential extrapolation down using the bottom 2 points in the profile.

1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16 17 18 19 20	1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16 17 18 19 20
HEIGHT h km	ELECTRON DENSITY N ELECTRONS/cm³	HEIGHT h km	ELECTRON DENSITY N ELECTRONS/cm³

```
      SUBROUTINE TABLEX                                                 TABX001
C        CALCULATES ELECTRON DENSITY AND GRADIENT FROM PROFILES HAVING  TABX002
C        THE SAME FORM AS THOSE USED BY CROFTS RAY TRACING PROGRAM      TABX003
C MAKES AN EXPONENTIAL EXTRAPOLATION DOWN USING THE BOTTOM TWO POINTS   TABX004
C        NEEDS SUBROUTINE GAUSEL                                        TABX005
      DIMENSION HPC(250),FN2C(250),ALPHA(250),BETA(250),GAMMA(250),     TABX006
     1 DELTA(250),SLOPE(250),MAT(4,5)                                   TABX007
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     TABX008
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  TABX009
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 TABX010
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READFN,W(100)),(PERT,W(150))  TABX011
      REAL MAT,K                                                        TABX012
      DATA (MODX(1)=6HTABLEX)                                           TABX013
      ENTRY ELECTX                                                      TABX014
      IF (READFN.EQ.0.) GO TO 10                                        TABX015
      READFN=0.                                                         TABX016
      READ 1000, NOC,(HPC(I),FN2C(I),I=1,NOC)                           TABX017
 1000 FORMAT (I4/(F8.2,E12.4))                                          TABX018
      PRINT 1200, (HPC(I),FN2C(I),I=1,NOC)                              TABX019
 1200 FORMAT(1H1,14X,6HHEIGHT,4X,16HELECTRON DENSITY/(1X,F20.10,E20.10))TABX020
      A=0.                                                              TABX021
      IF(FN2C(1).NE.0.) A=ALOG(FN2C(2)/FN2C(1))/(HPC(2)-HPC(1))         TABX022
      FN2C(1)=K*FN2C(1)                                                 TABX023
      FN2C(2)=K*FN2C(2)                                                 TABX024
      SLOPE(1)=A*FN2C(1)                                                TABX025
      SLOPE(NOC)=0.                                                     TABX026
      NMAX=1                                                            TABX027
      DO 6 I=2,NOC                                                      TABX028
      IF (FN2C(I).GT.FN2C(NMAX)) NMAX=I                                 TABX029
      IF (I.EQ.NOC) GO TO 4                                             TABX030
      FN2C(I+1)=K*FN2C(I+1)                                             TABX031
      DO 3 J=1,3                                                        TABX032
      M=I+J-2                                                           TABX033
      MAT(J,1)=1.                                                       TABX034
      MAT(J,2)=HPC(M)                                                   TABX035
      MAT(J,3)=HPC(M)**2                                                TABX036
    3 MAT(J,4)=FN2C(M)                                                  TABX037
      CALL GAUSEL (MAT,4,3,4,NRANK)                                     TABX038
      IF (NRANK.LT.3) GO TO 60                                          TABX039
      SLOPE(I)=MAT(2,4)+2.*MAT(3,4)*HPC(I)                              TABX040
    4 DO 5 J=1,2                                                        TABX041
      M=I+J-2                                                           TABX042
      MAT(J,1)=1.                                                       TABX043
      MAT(J,2)=HPC(M)                                                   TABX044
      MAT(J,3)=HPC(M)**2                                                TABX045
      MAT(J,4)=HPC(M)**3                                                TABX046
      MAT(J,5)=FN2C(M)                                                  TABX047
      L=J+2                                                             TABX048
      MAT(L,1)=0.                                                       TABX049
      MAT(L,2)=1.                                                       TABX050
      MAT(L,3)=2.*HPC(M)                                                TABX051
      MAT(L,4)=3.*HPC(M)**2                                             TABX052
    5 MAT(L,5)=SLOPE(M)                                                 TABX053
      CALL GAUSEL (MAT,4,4,5,NRANK)                                     TABX054
      IF (NRANK.LT.4) GO TO 60                                          TABX055
      ALPHA(I)=MAT(1,5)                                                 TABX056
      BETA(I)=MAT(2,5)                                                  TABX057
      GAMMA(I)=MAT(3,5)                                                 TABX058
    6 DELTA(I)=MAT(4,5)                                                 TABX059
      HMAX=HPC(NMAX)                                                    TABX060
      NH=2                                                              TABX061
   10 H=R(1)-EARTHR                                                     TABX062
      F2=F*F                                                            TABX063
      PXPR=0.                                                           TABX064
      IF (H.GE.HPC(1)) GO TO 12                                         TABX065
   11 NH=2                                                              TABX066
      X=0.                                                              TABX067
```

```
      IF(FN2C(1).EQ.0.) GO TO 50                                        TABX068
      X=FN2C(1)*EXP(A*(H-HPC(1)))/F2                                    TABX069
      PXPR=A*X                                                          TABX070
      GO TO 50                                                          TABX071
   12 IF (H.GE.HPC(NOC)) GO TO 18                                       TABX072
      NSTEP=1                                                           TABX073
      IF (H.LT.HPC(NH-1)) NSTEP=-1                                      TABX074
   15 IF (HPC(NH-1).LE.H.AND.H.LT.HPC(NH)) GO TO 16                     TABX075
      NH=NH+NSTEP                                                       TABX076
      GO TO 15                                                          TABX077
   16 X=(ALPHA(NH)+H*(BETA(NH)+H*(GAMMA(NH)+H*DELTA(NH))))/F2           TABX078
      PXPR=(BETA(NH)+H*(2.*GAMMA(NH)+H*3.*DELTA(NH)))/F2                TABX079
      GO TO 50                                                          TABX080
   18 X=FN2C(NOC)/F2                                                    TABX081
   50 IF (PERT.NE.0.) CALL ELECT1                                       TABX082
      RETURN                                                            TABX083
   60 PRINT 6000, I,HPC(I)                                              TABX084
 6000 FORMAT(4H THE,I4,55HTH POINT IN THE ELECTRON DENSITY PROFILE HAS TTABX085
     1HE HEIGHT,F8.2,40H KM, WHICH IS THE SAME AS ANOTHER POINT.)       TABX086
      CALL EXIT                                                         TABX087
         END                                                            TABX088
```

```
      SUBROUTINE GAUSEL (C,NRD,NRR,NCC,NSF)                             GAUS001
C********* SAME AS SUBROUTINE GAUSSEL WRITTEN BY L. DAVID LEWIS ********GAUS002
      DIMENSION C(NRD,NCC),L(128,2)                                     GAUS003
C         BITS = 2.**-18                                                GAUS004
      DATA (BITS=3.8146972656E-6)                                       GAUS005
      NR=NRR                                                            GAUS006
      NC=NCC                                                            GAUS007
      IF(NC.LT.NR.OR.NR.GT.128.OR.NR.LE.0) CALL EXIT                    GAUS008
C                                                                       GAUS009
C         INITIALIZE.                                                   GAUS010
      NSF=0                                                             GAUS011
      NRM=NR-1                                                          GAUS012
      NRP=NR+1                                                          GAUS013
      D=1.                                                              GAUS014
      LSD=1                                                             GAUS015
      DO 1 KR=1,NR                                                      GAUS016
      L(KR,1)=KR                                                        GAUS017
1     L(KR,2)=0                                                         GAUS018
      IF(NR.EQ.1) GO TO 42                                              GAUS019
C                                                                       GAUS020
C         ELIMINATION PHASE.                                            GAUS021
      DO 41 KP=1,NRM                                                    GAUS022
      KPP=KP+1                                                          GAUS023
      PM=0.                                                             GAUS024
      MPN=0                                                             GAUS025
C                                                                       GAUS026
C         SEARCH COLUMN KP FROM DIAGONAL DOWN FOR MAX PIVOT.            GAUS027
      DO 2 KR=KP,NR                                                     GAUS028
      LKR=L(KR,1)                                                       GAUS029
      PT=ABS(C(LKR,KP))                                                 GAUS030
      IF(PT.LE.PM) GO TO 2                                              GAUS031
      PM=PT                                                             GAUS032
      MPN=KR                                                            GAUS033
      LMP=LKR                                                           GAUS034
2     CONTINUE                                                          GAUS035
C                                                                       GAUS036
C         IF MAX PIVOT IS ZERO, MATRIX IS SINGULAR.                     GAUS037
      IF(MPN.EQ.0) GO TO 9                                              GAUS038
      NSF=NSF+1                                                         GAUS039
      IF(MPN.EQ.KP) GO TO 3                                             GAUS040
```

```
C                                                                       GAUS041
C          NEW ROW NUMBER KP HAS MAX PIVOT.                             GAUS042
      LSD=-LSD                                                          GAUS043
      L(KP,2)=L(MPN,1)=L(KP,1)                                          GAUS044
      L(KP,1)=LMP                                                       GAUS045
C                                                                       GAUS046
C          ROW OPERATIONS TO ZERO COLUMN KP BELOW DIAGONAL.             GAUS047
3     MKP=L(KP,1)                                                       GAUS048
      P=C(MKP,KP)                                                       GAUS049
      D=D*P                                                             GAUS050
      DO 41 KR=KPP,NR                                                   GAUS051
      MKR=L(KR,1)                                                       GAUS052
      Q=C(MKR,KP)/P                                                     GAUS053
      IF(Q.EQ.0.) GO TO 41                                              GAUS054
C                                                                       GAUS055
C          SUBTRACT Q * PIVOT ROW FROM ROW KR.                          GAUS056
      DO 4 LC=KPP,NC                                                    GAUS057
      R=Q*C(MKP,LC)                                                     GAUS058
      C(MKR,LC)=C(MKR,LC)-R                                             GAUS059
4     IF(ABS(C(MKR,LC)).LT.ABS(R)*BITS) C(MKR,LC)=0.                    GAUS060
41    CONTINUE                                                          GAUS061
C                                                                       GAUS062
C          LOWER RIGHT HAND CORNER.                                     GAUS063
42    LNR=L(NR,1)                                                       GAUS064
      P=C(LNR,NR)                                                       GAUS065
      IF(P.EQ.0.) GO TO 9                                               GAUS066
      NSF=NSF+1                                                         GAUS067
      D=D*P*LSD                                                         GAUS068
      IF(NR.EQ.NC) GO TO 8                                              GAUS069
C                                                                       GAUS070
C          BACK SOLUTION PHASE.                                         GAUS071
      DO 61 MC=NRP,NC                                                   GAUS072
      C(LNR,MC)=C(LNR,MC)/P                                             GAUS073
      IF(NR.EQ.1) GO TO 61                                              GAUS074
      DO 6 LL=1,NRM                                                     GAUS075
      KR=NR-LL                                                          GAUS076
      MR=L(KR,1)                                                        GAUS077
      KRP=KR+1                                                          GAUS078
      DO 5 MS=KRP,NR                                                    GAUS079
      LMS=L(MS,1)                                                       GAUS080
      R=C(MR,MS)*C(LMS,MC)                                              GAUS081
      C(MR,MC)=C(MR,MC)-R                                               GAUS082
5     IF(ABS(C(MR,MC)).LT.ABS(R)*BITS) C(MR,MC)=0.                      GAUS083
6     C(MR,MC)=C(MR,MC)/C(MR,KR)                                        GAUS084
61    CONTINUE                                                          GAUS085
C                                                                       GAUS086
C          SHUFFLE SOLUTION ROWS BACK TO NATURAL ORDER.                 GAUS087
      DO 71 LL=1,NRM                                                    GAUS088
      KR=NR-LL                                                          GAUS089
      MKR=L(KR,2)                                                       GAUS090
      IF(MKR.EQ.0) GO TO 71                                             GAUS091
      MKP=L(KR,1)                                                       GAUS092
      DO 7 LC=NRP,NC                                                    GAUS093
      Q=C(MKR,LC)                                                       GAUS094
      C(MKR,LC)=C(MKP,LC)                                               GAUS095
7     C(MKP,LC)=Q                                                       GAUS096
71    CONTINUE                                                          GAUS097
C                                                                       GAUS098
C          NORMAL AND SINGULAR RETURNS.  GOOD SOLUTION COULD HAVE D=0.  GAUS099
8     C(1,1)=D                                                          GAUS100
      GO TO 91                                                          GAUS101
9     C(1,1)=0.                                                         GAUS102
   91 RETURN                                                            GAUS103
         END                                                            GAUS104-
```

INPUT PARAMETER FORM FOR SUBROUTINE CHAPX

An ionospheric electron density model consisting of a Chapman layer with tilts, ripples, and gradients

$$f_N^2 = f_c^2 \exp\left(\alpha (1 - z - e^{-z}) \right)$$

$$z = \frac{h - h_{max}}{H}$$

$$f_c^2 = f_{co}^2 \left(1 + A \sin\left(2\pi \left(\theta - \frac{\pi}{2}\right)/B \right) + C\left(\theta - \frac{\pi}{2}\right)\right)$$

$$h_{max} = h_{max_o} + E\left(\theta - \frac{\pi}{2} \right) R_o$$

f_N is the plasma frequency

h is the height above the ground

R_o is the radius of the earth in km

and θ is the colatitude in radians.

Specify:

Critical frequency at the equator, f_{c_o} = ______________ MHz (W101)

Height of the maximum electron density at the equator, h_{max_o} = ____ km (W102)

Scale height, H = ______________ km (W103)

α = __________ (W104, 0.5 for an α Chapman layer, 1.0 for a β Chapman layer)

Amplitude of periodic variation of f_c^2 with latitude, A = __________ (W105)

Period of variation of f_c^2 with latitude, B = __________ rad / deg / km (W106)

Coefficient of linear variation of f_c^2 with latitude, C = _______ rad^{-1} (W107)

Tilt of the layer, E = ______________ rad / deg (W108)

```
      SUBROUTINE CHAPX
C           CHAPMAN LAYER WITH TILTS, RIPPLES, AND GRADIENTS
C           W(104) = 0.5 FOR AN ALPHA-CHAPMAN LAYER
C                  = 1.0 FOR A BETA-CHAPMAN LAYER
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON R(6) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (THETA,R(2))
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC,W(101)),(HM,W(102)),
     1 (SH,W(103)),(ALPHA,W(104)),(A,W(105)),(B,W(106)),(C,W(107)),
     2 (E,W(108)),(PERT,W(150))
      DATA (MODX(1)=6H CHAPX)
      ENTRY ELECTX
      THETA2=THETA-PID2
      HMAX=HM+EARTHR*E*THETA2
      H=R(1)-EARTHR
      Z=(H-HMAX)/SH
      D=0.
      IF (B.NE.0.) D=PIT2/B
      TEMP=1.+A*SIN(D*THETA2)+C*THETA2
      EXZ=1.-EXP(-Z)
      X=(FC/F)**2*TEMP*EXP(ALPHA*(EXZ-Z))
      PXPR=-ALPHA*X*EXZ/SH
      PXPTH=X*(D*A*SIN(PID2-D*THETA2)+C)/TEMP-PXPR*EARTHR*E
      IF (PERT.NE.0.) CALL ELECT1
      RETURN
         END
```

INPUT PARAMETER FORM FOR SUBROUTINE VCHAPX

An ionospheric electron density model consisting of a Chapman layer with variable scale height

$$f_N^2 = f_c^2 \; \tau^{\frac{1}{2}} \; e^{\frac{1}{2}(1 - \tau)}$$

$$\tau = \left(\frac{h_{max}}{h}\right)^{\chi}$$

h is the height above the ground.

Specify:

critical frequency, fc = __________ MHz (W101)

height of maximum electron density, h_{max} = __________ km (W102)

χ = __________ (W103)

```
      SUBROUTINE VCHAPX                                                 VCHA001
C          CHAPMAN LAYER WITH VARIABLE SCALE HEIGHT                     VCHA002
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  VCHA003
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 VCHA004
      EQUIVALENCF (EARTHR,W(2)),(F,W(6)),(FC,W(101)),(HM,W(102)),       VCHA005
     1 (CHI,W(103)),(PERT,W(150))                                       VCHA006
      DATA (MODX(1)=6HVCHAPX)                                           VCHA007
      ENTRY ELECTX                                                      VCHA008
      HMAX=HM                                                           VCHA009
      X=PXPR=0.                                                         VCHA010
      H=R(1)-EARTHR                                                     VCHA011
      IF (H.LE.0.) GO TO 50                                             VCHA012
      TAU=(HM/H)**CHI                                                   VCHA013
      X=(FC/F)**2*SQRT(TAU)*EXP(0.5*(1.-TAU))                           VCHA014
      PXPR=.5*X*(TAU-1.)*CHI/H                                          VCHA015
   50 IF (PERT.NE.0.) CALL ELECT1                                       VCHA016
      RETURN                                                            VCHA017
         END                                                            VCHA018-
```

INPUT PARAMETER FORM FOR SUBROUTINE DCHAPT

An ionospheric electron density model consisting of a double, tilted α-Chapman layer

$$f_N^2 = f_{c1}^2 \exp \frac{1}{2} \left(1 - z_1 - e^{-z_1}\right) + f_{c2}^2 \exp \frac{1}{2} \left(1 - z_2 - e^{-z_2}\right)$$

$$z_1 = \frac{h - h_{m1}}{H_1} \quad ; \quad z_2 = \frac{h - h_{m2}}{H_2}$$

$$f_{c1}^2 = f_{c10}^2 \quad C(\theta - \pi/2)$$

$$f_{c2}^2 = f_{c20}^2 \quad C(\theta - \pi/2)$$

$$h_{m1} = h_{m10} + R_o E \left(\frac{\pi}{180}\right) \left(\theta - \frac{\pi}{2}\right)$$

$$h_{m2} = h_{m20} + R_o E \left(\frac{\pi}{180}\right) \left(\theta - \frac{\pi}{2}\right)$$

Specify:

f_{c10} = ____________ MHz (f_{c1} at equator) (W101)

h_{m10} = ____________ Km (h_{m1} at equator) (W102)

H_1 = ____________ Km (W103)

f_{c20} = ____________ MHz (f_{c2} at equator) (W104)

h_{m20} = ____________ Km (h_{m2} at equator) (W105)

H_2 = ____________ Km (W106)

C = ____________ rad^{-1} (fractional change in f_{c1}, f_{c2}, position for increases southward) (W107)

E = ____________ deg (positive for upward tilt to the south) (W108)

```
      SUBROUTINE DCHAPT
C          TWO CHAPMAN LAYERS WITH TILTS
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON R(6) /WW/ ID(10),WO,W(400)
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC1,W(101)),(HM1,W(102)),
     1 (SH1,W(103)),(FC2,W(104)),(HM2,W(105)),(SH2,W(106)),(C,W(107)),
     2 (E,W(108)),(PERT,W(150))
      DATA (MODX(1)=6HDCHAPT)
      ENTRY ELECTX
      EARTHE=EARTHR*E
      THETA2=R(2)-PID2
      HMAX=HM1+EARTHE*THETA2
      X=PXPR=PXPTH=0.
      H=R(1)-EARTHR
      IF (H.LT.0.) GO TO 50
      Z1=(H-HMAX)/SH1
      EXPZ1=1.-FXP(-Z1)
      TEMP=1.+C*THETA2
      X=(FC1/F)**2*TEMP*EXP(.5*(EXPZ1-Z1))
      PXPR=-0.5*X*EXPZ1/SH1
      PXPTH=X*C/TEMP-PXPR*EARTHE
      IF (FC2.EQ.0.) GO TO 50
      Z2=(H-HM2-EARTHE*THETA2)/SH2
      EXPZ2=1.-EXP(-Z2)
      X2=(FC2/F)**2*TEMP*EXP(.5*(EXPZ2-Z2))
      X=X+X2
      PXPR2=-0.5*X2*EXPZ2/SH2
      PXPR=PXPR+PXPR2
      PXPTH=PXPTH+X2*C/TEMP-PXPR2*EARTHE
   50 IF (PERT.NE.0.) CALL ELECT1
      RETURN
         END
```

INPUT PARAMETER FORM FOR SUBROUTINE LINEAR

An ionospheric electron density model consisting of a linear layer

$N = 0$ for $h \leq h_{min}$

$N = A(h - h_{min})$ for $h > h_{min}$

The ray will penetrate if $h > h_{max}$.

Specify:

$A =$ __________ electrons/cm^3/ km (W101)

$h_{max} =$ __________ km (W102)

$h_{min} =$ __________ km (W103)

```
      SUBROUTINE LINEAR                                                 LINE001
C        LINEAR ELECTRON DENSITY MODEL                                  LINE002
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     LINE003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  LINE004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 LINE005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FACT,W(101)),(HM,W(102)),     LINE006
     1 (HMIN,W(103)),(PERT,W(150))                                      LINE007
      REAL K                                                            LINE008
      DATA (MODX(1)=6HLINEAR)                                           LINE009
      ENTRY ELECTX                                                      LINE010
      H=R(1)-EARTHR                                                     LINE011
      HMAX=HM                                                           LINE012
      X=PXPR=0.                                                         LINE013
      IF (H.LE.HMIN) GO TO 50                                           LINE014
      PXPR=K*FACT/F**2                                                  LINE015
      X=PXPR*(H-HMIN)                                                   LINE016
   50 IF (PERT.NE.0.) CALL ELECT1                                       LINE017
      RETURN                                                            LINE018
         END                                                            LINE019-
```

INPUT PARAMETER FORM FOR SUBROUTINE QPARAB

An ionospheric electron density model consisting of a parabolic or a quasi-parabolic layer (concentric)

$$f_N^2 = f_c^2 \; [1. - \frac{h-h_{max}}{Y_m} \cdot C \;{}^2] \qquad \text{if } f_N^2 > 0.$$

$f_N^2 = 0.$ otherwise.

$C = 1.$ for a parabolic layer

$$C = \frac{R_o + h_{max} - Y_m}{R_o + h} \quad \text{for a quasi-parabolic layer}$$

where R_o is the radius of the earth.

Specify:

Critical frequency,. f_c = __________Mc/s (W101)

Height of maximum electron density, h_{max} = __________km. (W102)

Semi-thickness, Y_m = __________km. (W103)

Type of profile:

Plain parabolic______________(W104 = 0.)

Quasi-parabolic______________(W104 = 1.)

```
      SUBROUTINE QPARAB                                                 PARA001
C        PLAIN PARABOLIC OR QUASI-PARABOLIC PROFILE                     PARA002
C        W(104) = 0. FOR A PLAIN PARABOLIC PROFILE                      PARA003
C               = 1. FOR A QUASI-PARABOLIC PROFILE                      PARA004
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  PARA005
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 PARA006
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FC,W(101)),(HM,W(102)),       PARA007
     1 (YM,W(103)),(QUASI,W(104)),(PERT,W(150))                         PARA008
      DATA (MODX(1)=6HQPARAB)                                           PARA009
      ENTRY ELECTX                                                      PARA010
      HMAX=HM                                                           PARA011
      PXPR=0.                                                           PARA012
      H=R(1)-EARTHR                                                     PARA013
      FCF2=(FC/F)**2                                                    PARA014
      CONST=1.                                                          PARA015
      IF (QUASI.EQ.1.) CONST=(EARTHR+HM-YM)/R(1)                        PARA016
      Z=(H-HM)/YM*CONST                                                 PARA017
      X=MAX1F(0.,FCF2*(1.-Z*Z))                                         PARA018
      IF (X.EQ.0.) GO TO 50                                             PARA019
      IF (QUASI.EQ.1.) CONST=(EARTHR+HM)*(EARTHR+HM-YM)/R(1)**2         PARA020
      PXPR=-2.*Z*FCF2/YM*CONST                                          PARA021
   50 IF (PERT.NE.0.) CALL ELECT1                                       PARA022
      RETURN                                                            PARA023
         END                                                            PARA024-
```

INPUT PARAMETER FORM FOR SUBROUTINE BULGE

An analytic ionospheric electron density model which represents the general latitude variation of the equatorial ionosphere (afternoon, equinox, sunspot maximum) - see the center panel of figure 3.18b, page 133 of Davies (1965).

The model is an alpha Chapman layer with parameters which vary with geomagnetic latitude.

$$f_N^2 = f_c^2 \, e^{\frac{1}{2}(1 - z - e^{-z})}$$

where $z = \frac{h - h_{max}}{H}$

f_N is the plasma frequency

f_c is the critical frequency

h_{max} is the height of the maximum electron density

H is the scale height

h is height

---

f_c, h_{max}, H vary with geomagnetic latitude in the following way:

if $h < 100$ km, $h_{max} = 350$ km, $f_c = 15$ Mc/s

For $h \geq 100$ km,

$h_{max} = 350$ if $\lambda \geq 24°$

$h_{max} = 430 + 80 \cos\left(\frac{180}{24}\lambda\right)$ if $\lambda < 24°$

λ is the geomagnetic latitude in degrees

$$f_c = \sqrt{50\left(\frac{\lambda}{8}\right)^2 \exp\left(2 - \left|\frac{\lambda}{8}\right|\right) + 40}$$

In all cases H is determined by the constraint that

$f_N = 2$ Mc/s at 100 km.

```
      SUBROUTINE BULGE                                                  BULG001
C        ANALYTICAL MODEL OF THE VARIATION OF THE EQUATORIAL F2 LAYER   BULG002
C        IN GEOMAGNETIC LATITUDE  (EQUATORIAL BULGE AND ANOMALY)        BULG003
C        SEE FIGURE 3.18B, PAGE 133 IN DAVIES (1965).                   BULG004
C        THIS MODEL HAS NO VARIATION IN GEOMAGNETIC LONGITUDE.          BULG005
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                BULG006
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  BULG007
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 BULG008
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(PERT,W(150))                  BULG009
      DATA (MODX(1)=6H BULGE)                                           BULG010
      ENTRY ELECTX                                                      BULG011
      H=R(1)-EARTHR                                                     BULG012
      PHMPTH=PFC2PTH=0.                                                 BULG013
      HMAX=350.                                                         BULG014
      FC2=225.                                                          BULG015
      IF(H.LT.100.) GO TO 2                                             BULG016
C        EQUATORIAL BULGE                                               BULG017
      BULLAT=7.5*(PID2-R(2))                                            BULG018
      IF(ABS(BUL LAT).GE.PI)  GO TO 1                                   BULG019
      HMAX=430.+80.*COS(BULLAT)                                         BULG020
      PHMPTH=600.*SIN(BUL LAT)                                          BULG021
C        EQUATORIAL ANOMALY                                             BULG022
    1 ANMLAT=22.5*(PID2-R(2))/PI                                        BULG023
      POW=2.-ABS(ANM LAT)                                               BULG024
      FC2=50.*ANM LAT**2*EXP(  POW     ) + 40.                          BULG025
      PFC2PTH=-1125./PI*POW*ANMLAT*EXP(POW)                             BULG026
C     FORCING PLASMA FREQ AT 100 KM TO BE 2 MHZ IN ORDER TO CALCULATE SH BULG027
    2 ALPHA=2.*ALOG(FC2/4.)+1.                                          BULG028
      Z100=-ALOG(ALPHA)                                                 BULG029
      DO 3  I=1,5                                                       BULG030
    3 Z100=-ALOG(ALPHA-Z100)                                            BULG031
      SH=(100.-HMAX)/Z100                                               BULG032
      Z=(H-HMAX)/SH                                                     BULG033
      EXZ=1.-EXP(-Z)                                                    BULG034
      X=FC2*EXP(.5*(EXZ-Z))/F**2                                        BULG035
      PXPR=-0.5*X*EXZ/SH                                                BULG036
      PXPTH=-PXPR*(1.-Z/Z100)*PHMPTH+(1.-Z*EXZ/(Z100*(1.-EXP(-Z100))))  BULG037
     1       *X/FC2*PFC2PTH                                             BULG038
      IF (PERT.NE.0.) CALL ELECT1                                       BULG039
      RETURN                                                            BULG040
         END                                                            BULG041-
```

INPUT PARAMETER FORM FOR SUBROUTINE EXPX

An exponential electron density profile

$$N = N_o \, e^{a(h-h_o)}$$

h is the height above the ground.

Specify:

the electron density at the height h_o, N_o = ____________ cm^{-3} (W101)

the reference height, h_o = ____________ km (W102)

the exponential increase of N with height, a = __________ km^{-1} (W103)

```
      SUBROUTINE EXPX                                                   EXPX001
C        EXPONENTIAL ELECTRON DENSITY MODEL                             EXPX002
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     EXPX003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  EXPX004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 EXPX005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),                               EXPX006
     1 (N0,W(101)),(H0,W(102)),(A,W(103)),(PERT,W(150))                 EXPX007
      REAL N, N0 ,K                                                     EXPX008
      DATA (MODX(1)=4HEXPX),(HMAX=350.)                                 EXPX009
      ENTRY ELECTX                                                      EXPX010
      H=R(1)-EARTHR                                                     EXPX011
      N =N0 * EXP(A*(H-H0))                                             EXPX012
      X=K*N/F**2                                                        EXPX013
      PXPR=A*X                                                          EXPX014
      IF (PERT.NE.0.) CALL ELECT1                                       EXPX015
      RETURN                                                            EXPX016
         END                                                            EXPX017-
```

APPENDIX 4. PERTURBATIONS TO ELECTRON DENSITY MODELS WITH INPUT PARAMETER FORMS

The following perturbations to electron density models (irregularities) are available. The input parameter forms, which describe the perturbation, and the subroutine listings are given on the pages shown.

To add other perturbations to electron density models the user must write a subroutine to modify the normalized electron density (X) and its gradient ($\partial X/\partial r$, $\partial X/\partial\theta$, $\partial X/\partial\varphi$) as a function of position in spherical polar coordinates (r, θ, φ).

The restrictions on electron density models also apply to perturbations. Again, the coordinates r, θ, φ refer to the computational coordinate system, which may not be the same as geographic coordinates. In particular, they are geomagnetic coordinates when the earth-centered dipole model of the earth's magnetic field is used.

The input to the subroutine is through blank common (see Table 3) for the position (r, θ, φ) and through common block /XX/ (see Table 8) for the unperturbed electron density and its gradient. The output is through common block /XX/. It is useful if the name of the subroutine suggests the perturbation model to which it corresponds. It should have an entry point ELECT1 so that it may be called by an electron density subroutine. Any parameters needed by the subroutine should be input into W151 through W199 of the W array. (See Table 2.)

If no perturbation is wanted, the following subroutine should be used.

```
      SUBROUTINE ELECT1                                                 ELEC001
C           USE WHEN AN ELECTRON DENSITY PERTURBATION IS NOT WANTED     ELEC002
      COMMON /XX/ MODX(2),X(6)                                          ELEC003
      COMMON /WW/ ID(10),W0,W(400)                                      ELEC004
      EQUIVALENCE (PERT,W(150))                                         ELEC005
      DATA (MODX(2)=6H NONE )                                           ELEC006
      PERT=0.                                                           ELEC007
      RETURN                                                            ELEC008
         END                                                            ELEC009-
```

INPUT PARAMETER FORM FOR SUBROUTINE TORUS

A perturbation to an ionospheric electron density model consisting of an East-West irregularity with an elliptical cross section above the equator

$$N = N_o (1 + \Delta)$$

$$\Delta = C_o \exp \left\{ - \left[\frac{(R_o + H_o)(\theta - \pi/2) \cos \beta + (R - R_o - H_o) \sin \beta}{A} \right]^2 - \left[\frac{(R - R_o - H_o) \cos \beta - (R_o + H_o)(\theta - \pi/2) \sin \beta}{B} \right]^2 \right\}$$

R_o is the radius of the earth.

R, θ, φ give the position in spherical polar coordinates.

$N_o(R, \theta, \varphi)$ is any ionospheric electron density model.

Specify:

C_o = __________. (W151)

Semi-major axis of ellipse, A = __________km (W152)

Semi-minor axis of ellipse, B = __________km (W153)

Tilt of ellipse, β = __________ degrees (W154)

Height of torus from ground, H_o = __________ km (W155)

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE TORUS
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX
      COMMON R(6) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (EARTHR,W(2)),(C0,W(151)),(A,W(152)),(B,W(153)),
     1 (BETA,W(154)),(H0,W(155))
      REAL LAMBDA
      DATA (PDPP=0.),(MODX(2)=6H TORUS)
      ENTRY ELECT1
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURN
      IF (C0.EQ.0.) RETURN
      R0=EARTHR+H0
      Z=R(1)-R0
      LAMBDA=R0*(R(2)-PID2)
      SINBET=SIN(BETA)
      COSBET=COS(BETA)
      P=LAMBDA*COSBET+Z*SINBET
      Y=Z*COSBET-LAMBDA*SINBET
      DELTA=C0*EXP(-(P/A)**2-(Y/B)**2)
      DEL1=DELTA+1.
      PDPR=-2.*DELTA*(P*SINBET/A**2+Y*COSBET/B**2)
      PDPT=-2.*DELTA*(P*R0*COSBET/A**2-Y*R0*SINBET/B**2)
      PXPR=PXPR*DEL1+X*PDPR
      PXPTH=PXPTH*DEL1+X*PDPT
      PXPPH=PXPPH*DEL1+X*PDPP
      X=X*DEL1
      RETURN
         END
```

INPUT PARAMETER FORM FOR SUBROUTINE DTORUS

A perturbation to an ionospheric electron density model consisting of two east-west irregularities with elliptical cross sections above the equator. Since the model is expressed in spherical coordinates and does not depend on longitude, the perturbation is actually a torus circling the earth above the equator.

$$N = N_0 \ (1 + \Delta)$$

$$\Delta = C_1 \exp \left\{ - \left[\frac{(r_0 + H_1)(\theta - \pi / 2) \cos \beta + (r - r_0 - H_1) \sin \beta}{A_1} \right]^2 \right.$$

$$\left. - \left[\frac{(r - r_0 - H_1) \cos \beta - (r_0 + H_1)(\theta - \pi / 2) \sin \beta}{B_1} \right]^2 \right\}$$

$$+ \ C_2 \ \exp \left\{ - \left[\frac{(r_0 + H_2)(\theta - \pi / 2 + \delta\theta) \cos \beta + (r - r_0 - H_2) \sin \beta}{A_2} \right] \right.$$

$$\left. - \left[\frac{(r - r_0 - H_2) \cos \beta - (r_0 + H_2)(\theta - \pi / 2 + \delta\theta) \sin \beta}{B_2} \right]^2 \right\}$$

$\delta\theta$ = Northward angular displacement of the lower blob from the upper one

$$= \frac{H_1 - H_2}{\tan \beta \ (r_0 + H_2)}$$

r_0 is the radius of the earth.

r, θ, φ are spherical (earth-centered) polar coordinates.

$N_0(r, \theta, \varphi)$ is any electron density model.

Specify:

use perturbation__________(W150 = 1.)
ignore perturbation_________(W150 = 0.)

Fractional perturbation electron density at the center of the upper blob, C_1 = _____________(W151).

That of the lower blob, C_2 = _____________(W156).

Height (above ground) of the center of the upper blob, H_1 = ____________km (W155).

That of the lower blob, H_2 = ____________ km (W159).

Angle (with a horizontal southward vector) of the line joining the blob centers, θ = ______________ rad ~~deg~~ (W154).

Semi-axis of the upper blob, to the 1/e perturbation contour, in the direction of the line joining the blobs, A_1 = __________ km (W152).

That of the lower blob, A_2 = ______________ km (W157).

Semi-axis of the upper blob in the direction normal to the line joining the blobs, B_1 = ____________ km (W153).

That of the lower blob, B_2 = ____________ km (W158).

```
      SUBROUTINE DTORUS
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT
      COMMON R(6) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (EARTHR,W(2)),(C1,W(151)),(A1,W(152)),(B1,W(153)),
     1 (BETA,W(154)),(H1,W(155)),(C2,W(156)),(A2,W(157)),(B2,W(158)),
     2 (H2,W(159))
      REAL LAMBDA1,LAMBDA2
      DATA (MODX(2)=6HDTORUS),(PDPP=0.)
      ENTRY ELECT1
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURN
      IF (C1.EQ.0.) RETURN
      R1=EARTHR+H1
      R2=EARTHR+H2
      Z1=R(1)-R1
      Z2=R(1)-R2
      LAMBDA1=R1*(R(2)-PID2)
      LAMBDA2=R2*(R(2)-PID2+(H1-H2)/R2/TANF(BETA))
      SINBET=SIN(BETA)
      COSBET=COS(BETA)
      P1=LAMBDA1*COSBET+Z1*SINBET
      P2=LAMBDA2*COSBET+Z2*SINBET
      Y1=Z1*COSBET-LAMBDA1*SINBET
      Y2=Z2*COSBET-LAMBDA2*SINBET
      DELTA1=C1*EXP(-(P1/A1)**2-(Y1/B1)**2)
      DELTA2=C2*EXP(-(P2/A2)**2-(Y2/B2)**2)
      DEL1=1.+DELTA1+DELTA2
      PDPR1=-2.*DELTA1*(P1*SINBET/A1**2+Y1*COSBET/B1**2)
      PDPR2=-2.*DELTA2*(P2*SINBET/A2**2+Y2*COSBET/B2**2)
      PDPT1=-2.*DELTA1*(P1*R1*COSBET/A1**2-Y1*R1*SINBET/B1**2)
      PDPT2=-2.*DELTA2*(P2*R2*COSBET/A2**2-Y2*R2*SINBET/B2**2)
      PXPR=PXPR*DEL1+X*(PDPR1+PDPR2)
      PXPTH=PXPTH*DEL1+X*(PDPT1+PDPT2)
      PXPPH=PXPPH*DEL1*PDPP
      X=X*DEL1
      RETURN
         END
```

INPUT PARAMETER FORM FOR SUBROUTINE TROUGH

A perturbation to an ionospheric electron density model consisting of an increase in electron density near any latitude

$N = (1 + \Delta)\ N_0\ (R,\ \theta,\ \varphi)$ $\qquad$ $W = B \text{ for } \frac{\pi}{2} - \theta - \lambda \geq 0$

$\Delta = A \exp\left(-\left(\frac{\pi/2 - \theta - \lambda}{W}\right)^2\right)$ $\qquad$ $W = B \times C \text{ for } \frac{\pi}{2} - \theta - \lambda < 0$

$N_0\ (R,\ \theta,\ \varphi)$ is any ionospheric electron density model.

$R,\ \theta,\ \varphi$ give the position in spherical polar coordinates.

Specify:

Amplitude of the perturbation, A = __________ (W151)

half width of the perturbation, B = __________ degrees (W152)

latitude of the perturbation, λ = __________ degrees (W153)

width factor for South of trough, C = __________ (W154)

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE TROUGH                                                 TROU001
C             A PERTURBATION TO AN ELECTRON DENSITY MODEL               TROU002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                TROU003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  TROU004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 TROU005
      EQUIVALENCE (A,W(151)),(E,W(152)),(ALAT,W(153)),(FACTOR,W(154))   TROU006
      DATA (MODX(2)=6HTROUGH)                                           TROU007
      ENTRY ELECT1                                                      TROU008
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNTROU009
      IF (A.EQ.0.) RETURN                                               TROU010
      ANGLE=R(2)+ALAT-PID2                                              TROU011
      WIDTH=B                                                           TROU012
      IF (ANGLE.GT.0.)   WIDTH=FACTOR*B                                 TROU013
      ANGLE=ANGLE/WIDTH                                                 TROU014
      DELTA=A*EXP(-ANGLE**2)                                            TROU015
      DEL1=DELTA+1.                                                     TROU016
      PXPR=PXPR*DEL1                                                    TROU017
      PXPTH=PXPTH*DEL1-2.*X*ANGLE*DELTA/WIDTH                           TROU018
      PXPPH=PXPPH*DEL1                                                  TROU019
      X=X*DEL1                                                          TROU020
      RETURN                                                            TROU021
         END                                                            TROU022-
```

INPUT PARAMETER FORM FOR SUBROUTINE SHOCK

A perturbation to an ionospheric electron density model consisting of an increase in electron density produced by a shock wave

$$N(R,\ \theta,\ \varphi) = N_0(R,\ \theta,\ \varphi)\left[1 + P\exp\left(-9\left(\frac{\rho_c - \rho}{w}\right)^2\right)\right]$$

$$\rho_c = s(h - h_0) - w$$

$$\rho = R\left|\cos^{-1}[\cos(\varphi - \varphi_0)\cos(\lambda - \lambda_0)]\right|$$

$N_0(R,\ \theta,\ \varphi)$ is the ambient electron density specified by any electron density model.

$R,\ \theta,\ \varphi$ give the position in spherical polar coordinates.

$h = R - a$ is the height above the surface of the earth.

a is the radius of the earth.

$\lambda = \frac{\pi}{2} - \theta$ is the latitude.

Specify:

Relative increase in electron density, $P =$ ________ (W151).

Width of the disturbance, $w =$ ________ km (W152).

Latitude of the center of the disturbance, $\lambda_0 =$ ________ radians or degrees (W153).

Longitude of the center of the disturbance, $\varphi_0 =$ ________ radians or degrees (W154).

Slope measured from vertical - rate of increase of ρ_c with height, $s =$ ________ (W155).

Height to the bottom of the disturbance, $h_0 =$ ________ km (W156).

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE SHOCK                                                  SHOC001
C   A PERTURBATION TO AN ELECTRON DENSITY MODEL SIMULATING A SHOCK WAVE SHOC002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                SHOC003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  SHOC004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 SHOC005
      EQUIVALENCE (EARTHR,W(2)),(P,W(151)),(WW,W(152)),(ALAT,W(153)),   SHOC006
     1 (ALON,W(154)),(S,W(155)),(H0,W(156))                             SHOC007
      REAL LAT,LON                                                      SHOC008
      DATA (MODX(2)=6H SHOCK)                                           SHOC009
      ENTRY ELECT1                                                      SHOC010
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNSHOC011
      IF (P.EQ.0..OR.WW.EQ.0.)  RETURN                                  SHOC012
      H=R(1)-EARTHR                                                     SHOC013
      RHOC=S*(H-H0)-WW                                                  SHOC014
      LON=R(3)-ALON                                                     SHOC015
      LAT=PID2-R(2)-ALAT                                                SHOC016
      COSLON=COS(LON)                                                   SHOC017
      COSLAT=COS(LAT)                                                   SHOC018
      U=COSLON*COSLAT                                                   SHOC019
      RHO=R(1)*ACOS(U)                                                  SHOC020
      DIF=RHOC-RHO                                                      SHOC021
      CON=-9./WW**2                                                     SHOC022
      CONS=P*EXP(CON*DIF**2)                                            SHOC023
      CONST=1.+CONS                                                     SHOC024
      CON=2.*CON*CONS*DIF                                               SHOC025
      PXPR=PXPR*CONST+X*CON*(S-RHO/R(1))                                SHOC026
      CONS=R(1)*(1./SQRT(1.-U**2))                                      SHOC027
      PXPTH=PXPTH*CONST+X*CON*CONS*COSLON*SIN(LAT)                      SHOC028
      PXPPH=PXPPH*CONST-X*CON*CONS*COSLAT*SIN(LON)                      SHOC029
      X=X*CONST                                                         SHOC030
      RETURN                                                            SHOC031
          END                                                           SHOC032-
```

INPUT PARAMETER FORM FOR SUBROUTINE WAVE

A perturbation to an ionospheric electron density model consisting of a "gravity-wave" irregularity traveling from north pole to south pole

$$N = N_o(1 + \Delta)$$

$$\Delta = \delta \exp\left\{- [(R - R_o - z_o)/H]^2\right\} \cdot$$

$$\cos\left\{2\pi \left[t' + (\pi/2 - \theta)\frac{R_o}{\lambda_x} + (R - R_o)/\lambda_z\right]\right\}$$

$$\frac{\partial N}{\partial t} = \frac{-2\pi}{\lambda_x} V_x N_o \delta \exp - [(R - R_o - z_o)/H]^2 \cdot$$

$$\sin 2\pi \left[t' + (\pi/2 - \theta)\frac{R_o}{\lambda_x} + (R - R_o)/\lambda z\right]$$

R_o is the radius of the earth.

R, θ, φ are the spherical (earth-centered) polar coordinates (Δ is independent of φ).

N_o (R, θ, φ) is any electron density model.

Specify:

the height of maximum wave amplitude, z_o = ______ km (W151)

wave-amplitude "scale height," H = ______ km (W152)

wave perturbation amplitude, δ = ______ [0. to 1.] (W153)

horizontal trace velocity, V_x = ______ km/sec (W154)
(needed only if Doppler shift is calculated)

horizontal wavelength, λ_x = ______ km (W155)

vertical wavelength, λ_z = ______ km (W156)

time in wave periods, t' = ______ [0. to 1.] (W157)

(W150: = 1. to use perturbation, = 0. to ignore perturbation)

```
      SUBROUTINE WAVE                                                   WAVE001
C          PERTURBATION TO AN ALPHA-CHAPMAN ELECTRON DENSITY MODEL      WAVE002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                WAVE003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,HMAX                  WAVE004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 WAVE005
      EQUIVALENCE (EARTHR,W(2)),(Z0,W(151)),(SH,W(152)),(DELTA,W(153)), WAVE006
     1 (VSUBX,W(154)),(LAMBDAX,W(155)),(LAMBDAZ,W(156)),(TP,W(157))     WAVE007
      REAL LAMBDAX,LAMBDAZ                                              WAVE008
      DATA (MODX(2)=6H WAVE )                                           WAVE009
      ENTRY ELECT1                                                      WAVE010
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNWAVE011
      IF (DELTA.EQ.0..OR.SH.EQ.0.)   RETURN                             WAVE012
      H=R(1)-EARTHR                                                     WAVE013
      EXPQ=EXP(-((H-Z0)/SH)**2)                                         WAVE014
      TMP=PIT2*(TP+(PID2-R(2))*EARTHR/LAMBDAX+H/LAMBDAZ)                WAVE015
      SINW=SIN(TMP)                                                     WAVE016
      COSW=SIN(PID2-TMP)                                                WAVE017
      CONS=1.0+DELTA*EXPQ*COSW                                          WAVE018
      IF (H.NE.0.) PXPR=PXPR*CONS-X*DELTA*EXPQ*(2.0/SH**2*(H-Z0)*COSW   WAVE019
     1 +PIT2/LAMBDAZ*SINW)                                              WAVE020
      PXPTH=PXPTH*CONS+X*DELTA*PIT2*EARTHR/LAMBDAX*SINW*EXPQ            WAVE021
      PXPPH=PXPPH*CONS                                                  WAVE022
      PXPT=0.                                                           WAVE023
      IF (VSUBX.NE.0.) PXPT=-PIT2*VSUBX/LAMBDAX*X*DELTA*EXPQ*SINW       WAVE024
      X=X*CONS                                                          WAVE025
      RETURN                                                            WAVE026
         END                                                            WAVE027-
```

INPUT PARAMETER FORM FOR SUBROUTINE WAVE 2

PERTURBATION TO AN IONOSPHERIC ELECTRON DENSITY MODEL

A "gravity-wave" irregularity traveling from north pole to south pole - same as WAVE 1, but with Gaussian amplitude variations in latitude and longitude, and provision for a horizontal "group velocity "

$N = N_0 \ (1 + AC)$

$$A = \delta \exp -\left(\frac{r - r_0 - z_\ominus}{H}\right)^2 \cdot \exp -\left(\frac{\theta - \theta_\ominus(t)}{\Theta}\right)^2 \cdot \exp -\left(\frac{\varphi}{\Phi}\right)^2$$

$$C = \cos 2\pi \left[t' + (\pi / 2 - \theta) \frac{r_0}{\lambda_x} + (r - r_\ominus)/\lambda_z \right]$$

$$\theta_0 = \theta_\infty + V_g t / r_\ominus$$

$r_\ominus$ is the radius of the earth.

r, θ, φ are spherical (earth-centered) polar coordinates.

$N_0(r, \theta, \varphi)$ is any electron density model.

Specify:

use perturbation____________(W150 = 1.)

ignore perturbation____________(W150 = 0.)

the height of maximum wave amplitude, $z_\ominus$ = __________km (W151)

wave-amplitude "scale height," H = __________km (W152)

wave perturbation amplitude, δ = __________(0 to 1) (W153)

horizontal trace velocity, V_x = __________km/sec. (W154)
(needed only if Doppler shift is calculated)

horizontal wavelength, λ_x = __________km (W155)

vertical wavelength, λ_z = __________km (W156)

time in wave periods, t' = __________ (W157)

amplitude "scale distance" in latitude, Θ = __________degrees (W159)

amplitude "scale distance" in longitude, Φ = __________degrees (W160)

latitude of maximum amplitude at t = 0, θ_∞ = __________degrees (W158)

southward group velocity, V_g = __________km/sec (W161)
(needed even if Doppler shift is not calculated)

```
      SUBROUTINE WAVE2                                                  WAV2001
C          PERTURBATION TO AN ANY ELECTRON DENSITY MODEL                WAV2002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                WAV2003
      COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT                       WAV2004
      COMMON R(6) /WW/ ID(10),WO,W(400)                                 WAV2005
      EQUIVALENCE (EARTHR,W(2)),(Z0,W(151)),(SH,W(152)),(DELTA,W(153)), WAV2006
     1 (VSUBX,W(154)),(LAMBDAX,W(155)),(LAMBDAZ,W(156)),(TP,W(157)),    WAV2007
     2 (TH00,W(158)),(THC,W(159)),(PHIC,W(160)),(VGX,W(161))            WAV2008
      REAL LAMBDAX,LAMBDAZ                                              WAV2009
      DATA (MODX(2)=6H WAVE2)                                           WAV2010
      ENTRY ELECT1                                                      WAV2011
      IF (X.EQ.0..AND.PXPR.EQ.0..AND.PXPTH.EQ.0..AND.PXPPH.EQ.0.) RETURNWAV2012
      IF (DELTA.EQ.0..OR.SH.EQ.0.)   RETURN                             WAV2013
      H=R(1)-EARTHR                                                     WAV2014
      TH0=TH00+LAMBDAX*TP*VGX/VSUBX/EARTHR                              WAV2015
      EXPR=EXP(-((H-Z0)/SH)**2)                                         WAV2016
      EXPTH=EXP(-((R(2)-TH0)/THC)**2)                                   WAV2017
      EXPPHI=EXP(-(R(3)/PHIC)**2)                                       WAV2018
      WW=PIT2*(TP+(PID2-R(2))*EARTHR/LAMBDAX+H/LAMBDAZ)                 WAV2019
      SINW=SIN(WW)                                                      WAV2020
      COSW=COS(WW)                                                      WAV2021
      E=DELTA*EXPR*EXPTH*EXPPHI                                         WAV2022
      CONS=1.0+E*COSW                                                   WAV2023
      PXPR=PXPR*CONS-X*E*2.*(COSW*(H-Z0)/SH**2+PI/LAMBDAZ*SINW)         WAV2024
      PXPTH=PXPTH*CONS+2.*E*(X*PI*EARTHR*SINW/LAMBDAX-(R(2)-TH0)/       WAV2025
     1 THC**2*COSW)                                                     WAV2026
      PXPPH=PXPPH*CONS-X*2.*E*R(3)/PHIC**2*COSW                         WAV2027
      PXPT=-PIT2*VSUBX*E/LAMBDAX*SINW+2.0*E*VGX/EARTHR*COSW*(R(2)-TH0   WAV2028
     1 -LAMBDAX*TP/EARTHR)/THC                                          WAV2029
      X=X*CONS                                                          WAV2030
      RETURN                                                            WAV2031
         END                                                            WAV2032-
```

INPUT PARAMETER FORM FOR SUBROUTINE DOPPLER

HEIGHT PROFILE OF dN/dt

(A perturbation to an ionospheric electron density model which calculates the time derivative of electron density for calculating Doppler shifts)

First card tells how many profile points in I4 format. The cards following the first card give the height and dN/dt of the profile points one point per card in F8.2, E12.4 format. The heights must be in increasing order. Set W151 =1.0 to read in a new profile. After the cards are read, DOPPLER will reset W151=0. This subroutine makes an exponential extrapolation down using the bottom 2 points in the profile.

1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16 17 18 19 20	1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16 17 18 19 20
HEIGHT h km	TIME DERIVATIVE OF ELECTRON DENSITY dN/dt ELECTRONS/cm^3/sec	HEIGHT h km	TIME DERIVATIVE OF ELECTRON DENSITY dN/dt ELECTRONS/cm^3/sec

```
      SUBROUTINE DOPPLER                                                DOPP001
C       COMPUTES DN/DT                    FROM PROFILES HAVING THE SAME FORMDOPP002
C       AS THOSE USED BY SUBROUTINE TABLE X                             DOPP003
C MAKES AN EXPONENTIAL EXTRAPOLATION DOWN USING THE BOTTOM TWO POINTS   DOPP004
C          NEEDS SUBROUTINE GAUSEL                                      DOPP005
      DIMENSION HPC(250),FN2C(250),ALPHA(250),BETA(250),GAMMA(250),     DOPP006
     1 DELTA(250),SLOPE(250),MAT(4,5)                                   DOPP007
      COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,DUM(2)                     DOPP008
      COMMON /XX/ MODX(2),XDUM,PXPR,PXPTH,PXPPH,X,HMAX                  DOPP009
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 DOPP010
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READFN,W(151))                DOPP011
      REAL MAT,K                                                        DOPP012
      DATA (MODX(2)=7HDOPPLER)                                          DOPP013
      ENTRY ELECT1                                                      DOPP014
      IF (READFN.EQ.0.) GO TO 10                                        DOPP015
      READFN=0.                                                         DOPP016
      READ 1000, NOC,(HPC(I),FN2C(I),I=1,NOC)                           DOPP017
 1000 FORMAT (I4/(F8.2,E12.4))                                          DOPP018
      PRINT 1200, (HPC(I),FN2C(I),I=1,NOC)                              DOPP019
 1200 FORMAT(1H1,14X,6HHEIGHT,4X,16H      DN/DT      /(1X,F20.10,E20.10))DOPP020
      A=0.                                                              DOPP021
      IF(FN2C(1).NE.0.) A=ALOG(FN2C(2)/FN2C(1))/(HPC(2)-HPC(1))         DOPP022
      FN2C(1)=K*FN2C(1)                                                 DOPP023
      FN2C(2)=K*FN2C(2)                                                 DOPP024
      SLOPE(1)=A*FN2C(1)                                                DOPP025
      SLOPE(NOC)=0.                                                     DOPP026
      NMAX=1                                                            DOPP027
      DO 6 I=2,NOC                                                      DOPP028
      IF (FN2C(I).GT.FN2C(NMAX)) NMAX=I                                 DOPP029
      IF (I.EQ.NOC) GO TO 4                                             DOPP030
      FN2C(I+1)=K*FN2C(I+1)                                             DOPP031
      DO 3 J=1,3                                                        DOPP032
      M=I+J-2                                                           DOPP033
      MAT(J,1)=1.                                                       DOPP034
      MAT(J,2)=HPC(M)                                                   DOPP035
      MAT(J,3)=HPC(M)**2                                                DOPP036
    3 MAT(J,4)=FN2C(M)                                                  DOPP037
      CALL GAUSEL (MAT,4,3,4,NRANK)                                     DOPP038
      IF (NRANK.LT.3) GO TO 60                                          DOPP039
      SLOPE(I)=MAT(2,4)+2.*MAT(3,4)*HPC(I)                              DOPP040
    4 DO 5 J=1,2                                                        DOPP041
      M=I+J-2                                                           DOPP042
      MAT(J,1)=1.                                                       DOPP043
      MAT(J,2)=HPC(M)                                                   DOPP044
      MAT(J,3)=HPC(M)**2                                                DOPP045
      MAT(J,4)=HPC(M)**3                                                DOPP046
      MAT(J,5)=FN2C(M)                                                  DOPP047
      L=J+2                                                             DOPP048
      MAT(L,1)=0.                                                       DOPP049
      MAT(L,2)=1.                                                       DOPP050
      MAT(L,3)=2.*HPC(M)                                                DOPP051
      MAT(L,4)=3.*HPC(M)**2                                             DOPP052
    5 MAT(L,5)=SLOPE(M)                                                 DOPP053
      CALL GAUSEL (MAT,4,4,5,NRANK)                                     DOPP054
      IF (NRANK.LT.4) GO TO 60                                          DOPP055
      ALPHA(I)=MAT(1,5)                                                 DOPP056
      BETA(I)=MAT(2,5)                                                  DOPP057
      GAMMA(I)=MAT(3,5)                                                 DOPP058
    6 DELTA(I)=MAT(4,5)                                                 DOPP059
      HMAX=AMAX1(HMAX,HPC(NMAX))                                        DOPP060
      NH=2                                                              DOPP061
   10 H=R(1)-EARTHR                                                     DOPP062
      F2=F*F                                                            DOPP063
      IF (H.GE.HPC(1)) GO TO 12                                         DOPP064
   11 NH=2                                                              DOPP065
```

```
                                                                        DOPP066
      IF(FN2C(1).EQ.0.) GO TO 50                                        DOPP067
      X=FN2C(1)*EXP(A*(H-HPC(1)))/F2                                    DOPP068
      GO TO 50                                                          DOPP069
   12 IF (H.GE.HPC(NOC)) GO TO 18                                       DOPP070
      NSTEP=1                                                           DOPP071
      IF (H.LT.HPC(NH-1)) NSTEP=-1                                      DOPP072
   15 IF (HPC(NH-1).LE.H.AND.H.LT.HPC(NH)) GO TO 16                     DOPP073
      NH=NH+NSTEP                                                       DOPP074
      GO TO 15                                                          DOPP075
   16 X=(ALPHA(NH)+H*(BETA(NH)+H*(GAMMA(NH)+H*DELTA(NH))))/F2           DOPP076
      GO TO 50                                                          DOPP077
   18 X=FN2C(NOC)/F2                                                    DOPP078
   50 CONTINUE                                                          DOPP079
      RETURN                                                            DOPP080
   60 PRINT 6000, I,HPC(I)                                              DOPP081
 6000 FORMAT(4H THE,I4,55HTH POINT IN THE   DN/DT          PROFILE HAS TDOPP082
     1HE HEIGHT,F8.2,40H KM, WHICH IS THE SAME AS ANOTHER POINT.)       DOPP083
      CALL EXIT                                                         DOPP084
         END                                                            DOPP085
```

APPENDIX 5. MODELS OF THE EARTH'S MAGNETIC FIELD WITH INPUT PARAMETER FORMS

The following models of the earth's magnetic field are available. The input parameter forms, which describe the model, and the subroutine listings are given on the pages shown.

To add other models of the earth's magnetic field the user must write a subroutine that will calculate the normalized strength and direction of the earth's magnetic field ($Y, Y_r, Y_\theta, Y_\varphi$) and their gradients ($\partial Y/\partial r$, $\partial Y/\partial \theta$, $\partial Y/\partial \varphi$, $\partial Y_r/\partial r$, $\partial Y_r/\partial \theta$, $\partial Y_r/\partial \varphi$, $\partial Y_\theta/\partial r$, $\partial Y_\theta/\partial \theta$, $\partial Y_\theta/\partial \varphi$, $\partial Y_\varphi/\partial r$, $\partial Y_\varphi/\partial \theta$, $\partial Y_\varphi/\partial \varphi$) as a function of position in spherical polar coordinates (r, θ, φ). ($Y = f_H/f$, where f_H is the electron gyrofrequency and f is the wave frequency.)

The restrictions on electron density models also apply to models of the earth's magnetic field. The coordinates r, θ, φ refer to the computational coordinate system, which is not necessarily the same as geographic coordinates. W24 and W25 give the geographic latitude and longitude of the north pole of the computational coordinate system.

The input to the subroutine (r, θ, ϕ) is through blank common. (See Table 3.) The output is through common block /YY/. (See Table 9.) It is useful if the name of the subroutine suggests the model to which it corresponds. It should have an entry point MAGY so that other subroutines in the program can call it. Any parameters needed by the subroutine should be input into W201 through W249 of the W array. (See Table 2.) If the subroutine needs massive amounts of data, these should be read in by the subroutine following the example of subroutine HARMONY.

INPUT PARAMETER FORM FOR SUBROUTINE CONSTY

An ionospheric model of the earth's magnetic field consisting of constant dip and gyrofrequency

Specify:

gyrofrequency, f_H = __________ MHz (W201)

dip, I = __________ degrees (W202)
radians

The magnetic meridian is defined by the geographic coordinates of the north magnetic pole:

radians
latitude = __________ degrees north (W24)

radians
longitude = __________ degrees east (W25)

```
      SUBROUTINE CONSTY                                                 CONY001
C         CONSTANT DIP AND GYROFREQUENCY                                CONY002
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRCONY003
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                CONY004
      COMMON /WW/ ID(10),W0,W(400)                                      CONY005
      EQUIVALENCE (F,W(6)),(FH,W(201)),(DIP,W(202))                     CONY006
      DATA (MODY=6HCONSTY)                                              CONY007
      ENTRY MAGY                                                        CONY008
      Y=FH/F                                                            CONY009
      YR=Y*SIN(DIP)                                                     CONY010
      YTH=Y*COS(DIP)                                                    CONY011
      RETURN                                                            CONY012
        END                                                             CONY013-
```

INPUT PARAMETER FORM FOR SUBROUTINE DIPOLY

An ionospheric model of the earth's magnetic field consisting of an earth centered dipole

The gyrofrequency is given by:

$$f_H = f_{H_0} \left(\frac{R_0+h}{R_0}\right)^3 \left(1 + 3\cos^2\lambda\right)^{\frac{1}{2}}$$

The magnetic dip angle, I, is given by

$\tan I = 2 \cot \lambda$

h is the height above the ground

R_0 is the radius of the earth

λ is the geomagnetic colatitude

Specify:

the gyrofrequency at the equator on the ground, f_{H_0} = __________ MHz (W201)

the geographic coordinates of the north magnetic pole

latitude = __________ radians / degrees north (W24)

longitude = __________ radians / degrees east (W25)

```
      SUBROUTINE DIPOLY                                                 DIP0001
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                DIP0002
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRDIP0003
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                DIP0004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 DIP0005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FH,W(201))                    DIP0006
      DATA (MODY=6HDIPOLY)                                              DIP0007
      ENTRY MAGY                                                        DIP0008
      SINTH=SIN(R(2))                                                   DIP0009
      COSTH=SIN(PID2-R(2))                                              DIP0010
      TERM9=SQRT(1.+3.*COSTH**2)                                        DIP0011
      T1=FH*(EARTHR/R(1))**3/F                                          DIP0012
      Y=T1*TERM9                                                        DIP0013
      YR= 2.*T1*COSTH                                                   DIP0014
      YTH= T1*SINTH                                                     DIP0015
      PYRPR=-3.*YR/R(1)                                                 DIP0016
      PYRPT=-2.*YTH                                                     DIP0017
      PYTPR=-3.*YTH/R(1)                                                DIP0018
      PYTPT=.5*YR                                                       DIP0019
      PYPR=-3.*Y/R(1)                                                   DIP0020
      PYPTH=-3.*Y*SINTH*COSTH/TERM9**2                                  DIP0021
      RETURN                                                            DIP0022
        END                                                             DIP0023-
```

INPUT PARAMETER FORM FOR SUBROUTINE CUBEY

A model of the earth's magnetic field consisting of a constant dip and a gyrofrequency which varies as the inverse cube of the distance from the center of the earth

This model has the same height variation as a dipole magnetic field.

The gyrofrequency is given by:

$$f_H = f_{Ho}\left(\frac{a}{r}\right)^3$$

a is the radius of the earth.

r is the distance from the center of the earth.

Specify:

gyrofrequency at the ground, f_{Ho} = ____________ MHz (W201)

dip, I = ____________ radians / degrees (W202)

The magnetic meridian is defined by the geographic coordinates of the north magnetic pole:

latitude = ____________ radians / degrees north / km (W24)

longitude = ____________ radians / degrees east / km (W25)

```
      SUBROUTINE CUBEY                                                  CUBE001
C        CONSTANT DIP.                                                  CUBE002
C        GYROFREQ DECREASES AS CUBE OF DISTANCE FROM CENTER OF EARTH.   CUBE003
C        THIS MODEL HAS SAME HEIGHT VARIATION AS A DIPOLE FIELD.        CUBE004
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPRCUBE005
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP                                CUBE006
      COMMON 'R /WW/ ID(10),W0,W(400)                                   CUBE007
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(FH,W(201)),(DIP,W(202))       CUBE008
      DATA(MODY=5HCUBEY)                                                CUBE009
      ENTRY MAGY                                                        CUBE010
       Y=(EARTHR/R)**3 *FH/F                                            CUBE011
      YR= Y*SIN(DIP)                                                    CUBE012
      YTH= Y*COS(DIP)                                                   CUBE013
      PYPR=-3.*Y/R                                                      CUBE014
      PYRPR=-3.*YR/R                                                    CUBE015
      PYTPR=-3.*YTH/R                                                   CUBE016
      RETURN                                                            CUBE017
         END                                                            CUBE018-
```

INPUT PARAMETER FORM FOR SUBROUTINE HARMONY

A model of the earth's magnetic field based on a spherical harmonic expansion

The upward, southerly, and easterly components of the earth's magneti field are given by:

$$H_r = - \sum_{n=o}^{6} (n+1)\left(\frac{a}{r}\right)^{n+2} \sum_{m=o}^{n} H_n{}^m(\theta)\left(g_n^m \cos m\varphi + h_n^m \sin m\varphi\right)$$

$$H_\theta = - \frac{1}{\sin\theta} \sum_{n=o}^{6} \left(\frac{a}{r}\right)^{n+2} \sum_{m=o}^{n} G_n{}^m(\theta)\left(g_n{}^m \cos m\varphi + h_n^m \sin m\varphi\right)$$

$$H_\varphi = \frac{1}{\sin\theta} \sum_{n=o}^{6} \left(\frac{a}{r}\right)^{n+2} \sum_{m=o}^{n} m\, H_n{}^m(\theta)\left(h_n{}^m \cos m\varphi - g_n{}^m \sin m\varphi\right)$$

where

a is the radius of the earth.

r, θ, φ are spherical (earth-centered) polar coordinates.

$H_o{}^o(\theta) = 1$

$H_1{}^o(\theta) = \cos\theta$

$H_1{}^1(\theta) = \sin\theta$

$H_{m+1}{}^m(\theta) = H_m{}^m(\theta) \cos\theta$

$H_{m+1}{}^{m+1}(\theta) = H_m{}^m(\theta) \sin\theta$

$$H_{n+2}{}^m(\theta) = H_{n+1}{}^m(\theta)\cos\theta - \frac{(n+m+1)(n-m+1)}{(2n+3)\quad(2n+1)} H_n{}^m(\theta)$$

$$G_n{}^m(\theta) = - \frac{d}{d\theta} H_n{}^m(\theta) \sin\theta$$

$$G_m^{\ m}(\theta) = -m\,H_m^{\ m}(\theta)\cos\theta$$

$$G_{n+1}^{\ m}(\theta) = -(n+1)\,H_{n+1}^{\ m}(\theta)\cos\theta + \frac{(n+m+1)(n-m+1)}{2n+1}\,H_n^{\ m}(\theta)$$

The recursion formulas for calculating $H_n^{\ m}(\theta)$ and $G_n^{\ m}(\theta)$ are from Eckhouse (1964).

This subroutine uses coefficients $g_n^{\ m}$ and $h_n^{\ m}$ for Gauss normalization. Some coefficients are now being published for Schmidt normalization (e. g. Cain and Sweeney, 1970). The factors $S_{n,\,m}$ used for converting the "Schmidt normalized" coefficients to the "Gauss normalized" coefficients are as follows (Cain, et. al., 1968, Chapman and Bartels, 1940):

$$S_{o,\,o} = -1$$

$$S_{n,\,o} = S_{n-1,\,o}\left[\frac{2n-1}{n}\right]$$

$$S_{n,\,1} = S_{n,\,o}\sqrt{\frac{2n}{n+1}}$$

$$S_{n,\,m} = S_{n,\,m-1}\sqrt{\frac{(n-m+1)}{n+m}} \qquad \text{for } m>1$$

By convention, the "Gauss normalized" coefficient $g_1^{\ o}$ is positive, whereas the "Schmidt normalized" coefficient $g_1^{\ o}$ is negative. Coefficients based on more recent data on the earth's magnetic field including more satellite data are in the POGO 8/69 model.

Specify below the Gauss coefficients $g_n^{\ m}$ and $h_n^{\ m}$ in gauss.

	columns 2 → 10	columns 11 → 20	columns 21 → 30	columns 31 → 40	columns 41 → 50	columns 51 → 60	columns 61 → 70
1st card	$g_o^{\ o}$ = ______						
2nd card	$g_1^{\ o}$ = ______	g_1^1=_____					
3rd card	$g_2^{\ o}$ = ______	g_2^1=_____	g_2^2=_____				
4th card	$g_3^{\ o}$ = ______	g_3^1=_____	g_3^2=_____	g_3^3=_____			
5th card	$g_4^{\ o}$ = ______	g_4^1=_____	g_4^2=_____	g_4^3=_____	g_4^4=_____		
6th card	$g_5^{\ o}$ = ______	g_5^1=_____	g_5^2=_____	g_5^3=_____	g_5^4=_____	g_5^5=_____	
7th card	$g_6^{\ o}$ = ______	g_6^1=_____	g_6^2=_____	g_6^3=_____	g_6^4=_____	g_6^5=_____	g_6^6=_____

8th card $h_o^{o} =$ ______

9th card $h_1^{o} =$ ______ $h_1^1 =$ ______

10th card $h_2^{o} =$ ______ $h_2^1 =$ ______ $h_2^2 =$ ______

11th card $h_3^{o} =$ ______ $h_3^1 =$ ______ $h_3^2 =$ ______ $h_3^3 =$ ______

12th card $h_4^{o} =$ ______ $h_4^1 =$ ______ $h_4^2 =$ ______ $h_4^3 =$ ______ $h_4^4 =$ ______

13th card $h_5^{o} =$ ______ $h_5^1 =$ ______ $h_5^2 =$ ______ $h_5^3 =$ ______ $h_5^4 =$ ______ $h_5^5 =$ ______

14th card $h_6^{o} =$ ______ $h_6^1 =$ ______ $h_6^2 =$ ______ $h_6^3 =$ ______ $h_6^4 =$ ______ $h_6^5 =$ ______ $h_6^6 =$ ______

Set W200 = 1. to read in a set of coefficients.

This subroutine represents:

$H_n^{m}(\theta)$ by H(m+1, n+1)

$G_n^{m}(\theta)$ by G(m+1, n+1)

g_n^{m} by GG(m+1, n+1)

h_n^{m} by HH(m+1, n+1)

```
      SUBROUTINE HARMONY
C     MODEL OF THE EARTH S MAGNETIC FIELD BASED ON A HARMONIC ANALYSIS
      DIMENSION PHPTH(7,7),PGPTH(7,7),A1(7,7),B1(7,7)
      DIMENSION H(7,7),G(7,7),GG(7,7),HH(7,7),SINP(7),COSP(7)
      COMMON /YY/ MODY,Y,PYPR,PYPTH,PYPPH,YR,PYRPR,PYRPT,PYRPP,YTH,PYTPR
     1,PYTPT,PYTPP,YPH,PYPPR,PYPPT,PYPPP
      COMMON R(6) /WW/ ID(10),W0,W(400)
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)
      EQUIVALENCE (THETA,R(2)),(PHI,R(3))
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READFH,W(200))
C     RATIO OF CHARGE TO MASS FOR FLECTRON
      DATA(EOM=1.7589E7)
      DATA (SET=0.),(H=1.,48(0.)),(G=49(0.)),(PHPTH=49(0.))
     1 ,(PGPTH=49(0.)),(MODY=7HHARMONY)
      ENTRY MAGY
      IF(SET) GO TO 2
      DO 1 M=1,7
      DO 1 N=1,7
      B1(M,N)=(N+M-1)*(N-M+1)/(2*N-1.)
    1 A1(M,N)=B1(M,N)/(2*N+1)
      SET=1.
```

```
   2 IF(READFH.EQ.0.) GO TO 3                                          HARM022
     READ 2000,GG,HH                                                   HARM023
2000 FORMAT(1X,F9.4,6F10.4)                                            HARM024
     PRINT 2100,GG                                                     HARM025
2100 FORMAT(1H1,10X,1H0,14X,1H1,14X,1H2,14X,1H3,14X,1H4,14X,1H5,14X,1H6HARM026
    1 /9X,7(1HG,14X)/10X,7(1HN,14X)//(1X,7F15.6))                      HARM027
     PRINT 2200,HH                                                     HARM028
2200 FORMAT(//  11X,1H0,14X,1H1,14X,1H2,14X,1H3,14X,1H4,14X,1H5,14X,1H6HARM029
    1 /9X,7(1HH,14X)/10X,7(1HN,14X)//(1X,7F15.6))                      HARM030
     READFH=0.                                                         HARM031
 3   COSTHE=COS(THETA)                                                 HARM032
     SINTHE=SIN(THETA)                                                 HARM033
     AOR=EARTHR/R(1)                                                   HARM034
     PAORPR=-AOR/R(1)                                                  HARM035
     CNST2=AOR                                                         HARM036
     PCNSPR=PAORPR                                                     HARM037
     FIN1=PFIN1R=PFIN1T=PFIN1P=0.                                      HARM038
     FIN2=PFIN2R=PFIN2T=PFIN2P=0.                                      HARM039
     FIN3=PFIN3R=PFIN3T=PFIN3P=0.                                      HARM040
     DO 4 M=1,7                                                        HARM041
     SINP(M)=SIN((M-1)*PHI)                                            HARM042
 4   COSP(M)=COS((M-1)*PHI)                                            HARM043
     H(1,2)=COSTHE                                                     HARM044
     H(2,2)=SINTHE                                                     HARM045
     DO 5 M=1,5                                                        HARM046
     H(M+1,M+2)=COSTHE*H(M+1,M+1)                                      HARM047
     H(M+2,M+2)=SINTHE*H(M+1,M+1)                                      HARM048
     DO 5 N=M,5                                                        HARM049
 5   H(M,N+2)=COSTHE*H(M,N+1)-A1(M,N)*H(M,N)                           HARM050
     DO 6 M=1,6                                                        HARM051
     G(M+1,M+1)=-M*COSTHE*H(M+1,M+1)                                   HARM052
     PHPTH(M+1,M+1)=-G(M+1,M+1)/SINTHE                                 HARM053
     PGPTH(M+1,M+1)=M*SINTHE*H(M+1,M+1)-M*COSTHE*PHPTH(M+1,M+1)        HARM054
     DO 6 N=M,6                                                        HARM055
     G(M,N+1)=-N*COSTHE*H(M,N+1)+B1(M,N)*H(M,N)                        HARM056
     PHPTH(M,N+1)=-G(M,N+1)/SINTHE                                     HARM057
 6   PGPTH(M,N+1)=N*SINTHE*H(M,N+1)-N*COSTHE*PHPTH(M,N+1)+B1(M,N)*PHPTHHARM058
    1 (M,N)                                                            HARM059
     DO 8 N=1,7                                                        HARM060
     CR=PCRPTH=PCRPPH=0.                                               HARM061
     CTH=PCTHPT=PCTHPP=0.                                              HARM062
     CPH=PCPHPT=PCPHPP=0.                                              HARM063
     DO 7 M=1,N                                                        HARM064
     TEMP1=GG(M,N)*COSP(M)+HH(M,N)*SINP(M)                             HARM065
     TEMP2=(M-1)*(HH(M,N)*COSP(M)-GG(M,N)*SINP(M))                     HARM066
     CR    =CR    +H(M,N)*TEMP1                                        HARM067
     PCRPTH=PCRPTH+PHPTH(M,N)*TEMP1                                    HARM068
     PCRPPH=PCRPPH+H(M,N)*TEMP2                                        HARM069
     CTH   =CTH   +G(M,N)*TEMP1                                        HARM070
     PCTHPT=PCTHPT+PGPTH(M,N)*TEMP1                                    HARM071
     PCTHPP=PCTHPP+G(M,N)*TEMP2                                        HARM072
     CPH   =CPH   +H(M,N)*TEMP2                                        HARM073
     PCPHPT=PCPHPT+PHPTH(M,N)*TEMP2                                    HARM074
 7   PCPHPP=PCPHPP-H(M,N)*(M-1)**2*TEMP1                               HARM075
     CNST2=CNST2*AOR                                                   HARM076
     PCNSPR=CNST2*PAORPR+AOR*PCNSPR                                    HARM077
     FIN1=FIN1+N*CNST2*CR                                              HARM078
     PFIN1R=PFIN1R+N*PCNSPR*CR                                         HARM079
     PFIN1T=PFIN1T+N*CNST2*PCRPTH                                      HARM080
     PFIN1P=PFIN1P+N*CNST2*PCRPPH                                      HARM081
     FIN2=FIN2+CNST2*CTH                                               HARM082
     PFIN2R=PFIN2R+PCNSPR*CTH                                          HARM083
     PFIN2T=PFIN2T+CNST2*PCTHPT                                        HARM084
     PFIN2P=PFIN2P+CNST2*PCTHPP                                        HARM085
     FIN3=FIN3+CNST2*CPH                                               HARM086
```

```
      PFIN3R=PFIN3R+PCNSPR*CPH
      PFIN3T=PFIN3T+CNST2*PCPHPT
    8 PFIN3P=PFIN3P+CNST2*PCPHPP
      HTHETA=-FIN2/SINTHE
      HPHI=FIN3/SINTHE
C********* CONVERT FROM MAG FIELD IN GAUSS TO GYROFREQ IN MHZ
      CONST=-EOM/PIT2*1.E-6/F
      YR=-CONST*FIN1
      YTH=CONST*HTHETA
      YPH=CONST*HPHI
      Y=SQRT(YR**2+YTH**2+YPH**2)
      PYRPR=-CONST*PFIN1R
      PYTPR=-CONST*PFIN2R/SINTHE
      PYPPR=CONST*PFIN3R/SINTHE
      PYPR=(YR*PYRPR+YTH*PYTPR+YPH*PYPPR)/Y
      PYRPT=-CONST*PFIN1T
      PYTPT=-CONST*(PFIN2T/SINTHE+HTHETA*COSTHE/SINTHE)
      PYPPT=CONST*(PFIN3T/SINTHE-HPHI*COSTHE/SINTHE)
      PYPTH=(YR*PYRPT+YTH*PYTPT+YPH*PYPPT)/Y
      PYRPP=-CONST*PFIN1P
      PYTPP=-CONST*PFIN2P/SINTHE
      PYPPP=CONST*PFIN3P/SINTHE
      PYPPH=(YR*PYRPP+YTH*PYTPP+YPH*PYPPP)/Y
      RETURN
C   COEFFICIENTS IN GAUSSIAN UNITS FROM JONES AND MELOTTE (1953).
C   THE FOLLOWING 14 CARDS CAN BE USED AS DATA CARDS FOR THIS SUBROUTINE
C   0.
C.3039    .0218
C.0176   -.0509    -.0135
C-.0255    .0515   -.0236    -.0074
C-.0398   -.0397   -.0238     .0087    -.0018
C.0293   -.0329    -.0130     .0031     .0034     .0005
C-.0211   -.0073   -.0007     .0210     .0017    -.0004     .0006
C 0.
C        -.0555
C         .0260   -.0044
C         .0190   -.0033    -.0001
C        -.0139    .0076     .0019     .0010
C         .0057   -.0018     .0003     .0032    -.0004
C        -.0026   -.0204     .0018     .0009     .0004     .0002
C   THE FOLLOWING SET OF GAUSS NORMALIZED COEFFICIENTS WERE CONVERTED
C   FROM THE SCHMIDT NORMALIZED COEFFICIENTS CALCULATED BY LINEARLY
C   EXTRAPOLATING TO EPOCH 1974 THE COEFFICIENTS PUBLISHED FOR EPOCH
C  1960 BY CAIN AND SWEENEY (1970).  (USES EARTH RADIUS = 6371.2)
C .000000
C+.300953 +.020298
C+.028106 -.05214   -.014435
C-.0308   +.06560   -.025252  -.006952
C-.041243 -.043956  -.016897  +.008021  -.002525
C+.014742 -.037078  -.018906  +.002819  +.003656  +.000036
C-.006713 -.012234  -.004364  +.02137   +.001593  -.000072  +.00068
C .000000
C .000000 -.057886
C .000000 +.035942  +.001129
C .000000 +.011084  -.004421  +.001180
C .000000 -.010299  +.008794  -.000086  +.002256
C .000000 -.003849  -.012615  +.007845  +.002207  -.000328
C .000000 +.003157  -.012670  -.009281  +.002286  -.000135  +.000243
         END
```

APPENDIX 6. COLLISION FREQUENCY MODELS WITH INPUT PARAMETER FORMS

The following collision frequency models are available. The input parameter forms, which describe the model, and the subroutine listings are given on the pages shown.

To add other collision frequency models the user must write a subroutine that will calculate the normalized collision frequency (Z) and its gradient ($\partial Z/\partial r$, $\partial Z/\partial \theta$, $\partial Z/\partial \varphi$) as a function of position in spherical polar coordinates (r, θ, φ). ($Z = \nu/2\pi f$, where ν is the collision frequency between electrons and neutral air molecules and f is the wave frequency. If the Sen-Wyller formula for refractive index is used, then $Z = \nu_m/2\pi f$, where ν_m is the mean collision frequency.)

The restrictions on electron density models also apply to collision frequency models. The coordinates r, θ, φ refer to the computational coordinates system, which may not be the same as geographic coordinates. In particular, they are geomagnetic coordinates when the earth-centered dipole model of the earth's magnetic field is used.

The input to the subroutine (r, θ, ϕ) is through blank common. (See Table 3.) The output is through common block /ZZ/. (See Table 10.) It is useful if the name of the subroutine suggests the model to which it corresponds. It should have an entry point COLFRZ so that other subroutines in the program can call it. Any parameter needed by the subroutine should be input into W251 through W299 of the W array. (See Table 2.) If the model needs massive amounts of data, these should be read in by the subroutine following the example of subroutine TABLEZ.

INPUT PARAMETER FORM FOR SUBROUTINE TABLEZ

IONOSPHERIC COLLISION FREQUENCY PROFILE

The first card tells how many profile points in I4 format. The cards following the first card give the height and collision frequency of the profile points one point per card in F8. 2, E12. 4 format. The heights must be in increasing order. Set W250 = 1. 0 to read in a new profile. After the cards are read, TABLEZ will reset W250 = 0. 0. This subroutine makes an exponential extrapolation down using the bottom 2 points in the profile.

1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16 17 18 19 20	1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16 17 18 19 20
HEIGHT h km	COLLISION FREQUENCY ν COLLISIONS/sec.	HEIGHT h km	COLLISION FREQUENCY ν COLLISIONS/sec.

```
      SUBROUTINE TABLEZ
C     CALCULATES COLLISION FREQUENCY AND ITS GRADIENT FROM PROFILES
C     HAVING THE SAME FORM AS THOSE USED BY CROFTS RAY TRACING PROGRAM
C MAKES AN EXPONENTIAL EXTRAPOLATION DOWN USING THE BOTTOM TWO POINTS
C         NEEDS SUBROUTINE GAUSEL
      DIMENSION HPC(100),FN2C(100),ALPHA(100),BETA(100),
     1 GAMMA(100),DELTA(100),MAT(4,5),SLOPE(100)
      COMMON /CONST/ PI,PIT2,PID2,DJM(5)
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH
      COMMON R(6) /WW/ ID(10),W0,W(400)
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(READNU,W(250))
      REAL MAT
      DATA (MODZ=6HTABLEZ)
      ENTRY COLFRZ
      IF (.NOT.READNU) GO TO 10
      READNU=0.
      READ 2, NOC,(HPC(I),FN2C(I),I=1,NOC)
  2   FORMAT(I4/(F8.2,E12.4))
      PRINT 1200, (HPC(I),FN2C(I), I=1,NOC)
 1200 FORMAT(1H1,14X,6HHEIGHT,4X,20HCOLLISION FREQUENCY
     1(1X,F20.10,E20.10))
      A=0.
      IF(FN2C(1).NE.0.) A=ALOG(FN2C(2)/FN2C(1))/(HPC(2)-HPC(1))
      FN2C(1)=FN2C(1)/PIT2*1.E-6
      FN2C(2)=FN2C(2)/PIT2*1.E-6
      SLOPE(1)=A*FN2C(1)
      SLOPE(NOC)=0.
      DO 5 I=2,NOC
      IF(I.EQ.NOC) GO TO 6
      FN2C(I+1)= FN2C(I+1)/PIT2*1.E-6
      DO 3 J=1,3
      M=I+J-2
      MAT(J,1)=1.
      MAT(J,2)=HPC(M)
      MAT(J,3)=HPC(M)**2
  3   MAT(J,4)=FN2C(M)
      CALL GAUSEL (MAT,4,3,4,NRANK)
      IF (NRANK.LT.3) GO TO 20
      SLOPE(I)=MAT(2,4)+2.*MAT(3,4)*HPC(I)
  6   CONTINUE
      DO 4 J=1,2
      M=I+J-2
      MAT(J,1)=1.
      MAT(J,2)=HPC(M)
      MAT(J,3)=HPC(M)**2
      MAT(J,4)=HPC(M)**3
      MAT(J,5)=FN2C(M)
      L=J+2
      MAT(L,1)=0.
      MAT(L,2)=1.
      MAT(L,3)=2.*HPC(M)
      MAT(L,4)=3.*HPC(M)**2
  4   MAT(L,5)=SLOPE(M)
      CALL GAUSEL (MAT,4,4,5,NRANK)
      IF (NRANK.LT.4) GO TO 20
      ALPHA(I)=MAT(1,5)
      BETA(I)=MAT(2,5)
      GAMMA(I)=MAT(3,5)
  5   DELTA(I)=MAT(4,5)
      JUP=2
   10 H=R(1)-EARTHR
      IF (H.GE.HPC(1)) GO TO 12
   11 JUP=2
      Z=FN2C(1)*EXP(A*(H-HPC(1)))/F
      PZPR=A*Z
```

```
      RETURN                                                            TABZ066
   12 IF (H.GE.HPC(NOC)) GO TO 18                                       TABZ067
      NSTEP=1                                                           TABZ068
      IF (H.LT.HPC(JUP-1)) NSTEP=-1                                     TABZ069
   15 IF (HPC(JUP-1).GT.H.OR.H.GE.HPC(JUP)) GO TO 16                    TABZ070
      Z=(ALPHA(JUP)+H*(BETA(JUP)+H*(GAMMA(JUP)+H*DELTA(JUP))))/F        TABZ071
      PZPR=(BETA(JUP)+H*(2.*GAMMA(JUP)+H*3.*DELTA(JUP)))/F              TABZ072
      RETURN                                                            TABZ073
   16 JUP=JUP+NSTEP                                                     TABZ074
      IF (JUP.LT.2) GO TO 11                                            TABZ075
      IF (JUP.LT.NOC) GO TO 15                                          TABZ076
   18 JUP=NOC                                                           TABZ077
      Z=FN2C(NOC)/F                                                     TABZ078
      PZPR=0.                                                           TABZ079
      RETURN                                                            TABZ080
   20 PRINT 21, I,HPC(I)                                                TABZ081
21    FORMAT(4H THE,I4,58HTH POINT IN THE COLLISION FREQUENCY PROFILE HATABZ082
     1S THE HEIGHT,F8.2,40H KM, WHICH IS THE SAME AS ANOTHER POINT.)    TABZ083
      CALL EXIT                                                         TABZ084
          END                                                           TABZ085
```

INPUT PARAMETER FORM FOR SUBROUTINE CONSTZ

An ionospheric collision frequency model consisting of a constant collision frequency

$\nu = 0$ for $h \leq h_{min}$

$\nu = \nu_0$ for $h > h_{min}$

Specify:

ν_0 = __________ collisions per second (W251)

h_{min} = __________ km (W252)

```
      SUBROUTINE CONSTZ                                                 CONZ001
C     CONSTANT COLLISION FREQUENCY                                      CONZ002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                CONZ003
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               CONZ004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 CONZ005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(NU,W(251)),(HMIN,W(252))      CONZ006
      REAL NU                                                           CONZ007
      DATA (MODZ=6HCONSTZ)                                              CONZ008
      ENTRY COLFRZ                                                      CONZ009
      H=R(1)-EARTHR                                                     CONZ010
      Z=0.                                                              CONZ011
      IF (H.GT.HMIN) Z=NU/(PIT2*F)*1.E-6                                CONZ012
      RETURN                                                            CONZ013
         END                                                            CONZ014-
```

INPUT PARAMETER FORM FOR SUBROUTINE EXPZ

An ionospheric collision frequency model consisting of an exponential profile

$$\nu = \nu_o e^{-a(h-h_o)}$$

h is the height above the ground

Specify:

The collision frequency at the height h_o, ν_o = ________________ collisions per second (W251)

The reference height, h_o = ________________ km (W252)

The exponential decrease of ν with height, a = ________________ km^{-1} (W253)

```
      SUBROUTINE EXPZ                                                   EXPZ001
C        EXPONENTIAL COLLISION FREQUENCY MODEL                          EXPZ002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                EXPZ003
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               EXPZ004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 EXPZ005
      REAL NU,NU0                                                       EXPZ006
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(NU0,W(251)),(H0,W(252)),      EXPZ007
     1 (A,W(253))                                                       EXPZ008
      DATA (MODZ=6H EXPZ )                                              EXPZ009
      ENTRY COLFRZ                                                      EXPZ010
      H=R(1)-EARTHR                                                     EXPZ011
      NU=NU0/EXP (A*(H-H0))                                             EXPZ012
      Z=NU/(PIT2*F*1.E6)                                                EXPZ013
      PZPR   =-A*Z                                                      EXPZ014
      RETURN                                                            EXPZ015
         END                                                            EXPZ016-
```

INPUT PARAMETER FORM FOR SUBROUTINE EXPZ2

An ionospheric collision frequency model consisting of a combination of two exponential profiles

$$\nu = \nu_1 e^{-a_1(h-h_1)} + \nu_2 e^{-a_2(h-h_2)}$$

where h is the height above the ground.

Specify for the first exponential:

Collision frequency at height h_1, ν_1 = ____________________ collisions per second (W251)

Reference height, h_1 = ______________ km (W252)

Exponential decrease of ν with height, a_1 = __________ km^{-1} (W253)

Specify for the second exponential:

Collision frequency at height h_2, ν_2 = ____________________ collisions per second (W254)

Reference height, h_2 = ______________ km (W255)

Exponential decrease of ν with height, a_2 = __________ km^{-1} (W256)

```
      SUBROUTINE EXPZ2                                                  XPZ2001
C          COLLISION FREQUENCY PROFILE FROM TWO EXPONENTIALS            XPZ2002
      COMMON /CONST/ PI,PIT2,PID2,DUM(5)                                XPZ2003
      COMMON /ZZ/ MODZ,Z,PZPR,PZPTH,PZPPH                               XPZ2004
      COMMON R(6) /WW/ ID(10),W0,W(400)                                 XPZ2005
      EQUIVALENCE (EARTHR,W(2)),(F,W(6)),(NU1,W(251)),(H1,W(252)),      XPZ2006
     1 (A1,W(253)),(NU2,W(254)),(H2,W(255)),(A2,W(256))                 XPZ2007
      REAL NU1,NU2                                                      XPZ2008
      DATA (MODZ=6H EXPZ2)                                              XPZ2009
      ENTRY COLFRZ                                                      XPZ2010
      H=R(1)-EARTHR                                                     XPZ2011
      EXP1= NU1* EXP(-A1*(H-H1))                                        XPZ2012
      EXP2= NU2* EXP(-A2*(H-H2))                                        XPZ2013
      Z=(EXP1+EXP2)/(PIT2*F*1.E6)                                       XPZ2014
      PZPR=(-A1*EXP1-A2*EXP2)/(PIT2*F*1.E6)                             XPZ2015
      RETURN                                                            XPZ2016
         END                                                            XPZ2017-
```

APPENDIX 7. CDC 250 PLOT PACKAGE

This appendix describes the plotting routines used by the Three-Dimensional Ray Tracing Program. The information was taken from "User's Guide to Cathode Ray Plotter Subroutines," ESSA Technical Memorandum ERLTM-ORSS 5, by L. David Lewis, January, 1970, and is printed with the permission of the author.

If you have access to a plotter, you may obtain plots by converting the following plotting commands to comparable commands on your system.

The CDC-250 Microfilm Recorder, under control of the NOAA Boulder CDC-3800 computer, plots data on the face of a high resolution cathode ray tube, which is photographed onto standard sized perforated, 35 mm film.

The plotting area, called a frame, is a square. Plotting positions are described in rectangular coordinates. Coordinate values are integers in the range 0 - 1023; (0, 0) is the "lower left hand corner".

Plotting specifications are transmitted to the plot routines via the following COMMON.

```
COMMON /DD/ IN, IOR, IT, IS, IC, ICC, IX, IY
```

The usage of each of the eight variables is listed below, followed by an explanation of the subroutine calls.

IN	Intensity. IN=0 specifies normal intensity. IN=1 specifies high intensity.
IOR	Orientation. IOR=0 specifies upright orientation. IOR=1 specifies rotated orientation (90° counter-clockwise).
IT	Italics (Font). IT=0 specifies non-Italic (Roman) symbols. IT=1 specifies Italic symbols.

IS	Symbol size. IS=0 specifies miniature size. IS=1 specifies small size. IS=2 specifies medium size. IS=3 specifies large size.
IC	Symbol case. IC=0 specifies upper case. IC=1 specifies lower case.
ICC	Character code, 0-63 (R1 format). ICC and IC together specify the symbol plotted.
IX	X-coordinate, 0-1023.
IY	Y-coordinate, 0-1023.

CALL DDINIT (N,ID)	is required to initialize the plotting process.
CALL DDBP	defines a vector origin at position IX, IY.
CALL DDVC	plots a vector (straight line), with intensity IN, from the vector origin defined by the previous DDBP or DDVC call, to the vector end position at IX, IY. A single call to DDBP followed by successive calls to DDVC (with changing IX and IY) plots connected vectors.
CALL DDTAB	initializes tabular plotting.
CALL DDTEXT (N,NT)	plots a given array in a tabular mode, after initiating tabular plotting via DDTAB, as described above. NT is an array of length N, containing "text" for tabular plotting. Text consists of character codes, packed 8 per word (A8 Format). Text characters are plotted as tabular symbols until the command character ≠ (octal code 14, card code 4, 8, or the alphabetic shift counterpart of the = on the keypunch) occurs. The command character is not plotted. DDTEXT interprets the next character as a command; and after the command is processed, tabular plotting resumes until ≠ is again encountered. ≠ . means end of text: DDTEXT returns to the calling routine.
CALL DDFR	causes a frame advance operation. Plotting on the current frame is completed, and the film advances to the next frame.

APPENDIX 8. SAMPLE CASE

A sample case is included with the description of the program for three reasons. First, it demonstrates the use of the program. Second, it illustrates the three types of output available (printout, punched cards, and ray path plots). Finally, it serves as a test case to verify that the user's copy of the program is running correctly. This last point is especially important if the user has had to make many modifications in converting the program to run on a computer other than a CDC 3800.

Although the ionospheric models in the sample case demonstrate the use of the program, they don't give realistic absorption for the radio waves. The absorption in the sample case is too low for two reasons. First, although the Chapman layer has a realistic electron density for the F region, it has much too low an electron density for the D region, where most of the absorption occurs. Second, the collision frequency profile in the sample case is designed for use with the Sen-Wyller formula for refractive index rather than the Appleton-Hartree formula used in the sample case. Multiplying the collision frequency profile in the sample case by 2.5 gives an effective collision frequency profile for use with the Appleton-Hartree formula that will give nearly the correct absorption for HF radio waves (Davies, 1965, p. 89).

Appendix 8a. Input Parameter Forms for the Sample Case

Filled-out input parameter forms are included to describe the sample case (i. e., show what ray paths are requested for which ionospheric models and what type of output is wanted). Furthermore, comparing them with Appendix 8b illustrates the relationship between the forms and the input data cards.

INPUT PARAMETER FORM FOR THREE-DIMENSIONAL RAY PATHS

Name ________ Project No. ________ Date ________

Ionospheric ID (3 characters) X01

Title (75 characters) Test Case

Models: Electron density CHAPX
Perturbation WAVE
Magnetic field DIPOLY
Ordinary ________ (W1 = + 1.)
Extraordinary ✓ (W1 = - 1.)
Collision frequency EXPZ2

Transmitter: Height 0 km, nautical miles, feet (W3)
Latitude 40 rad, [✓ deg] km (W4)
Longitude -105 rad, [✓ deg] km (W5)
Frequency, initial 6 MHz (W7)
final ________ (W8)
step ________ (W9)
Azimuth angle, initial 45 rad, [✓ deg] clockwise of north (W11)
final ________ (W12)
step ________ (W13)
Elevation angle, initial 0 rad, [✓ deg] (W15)
final 90 (W16)
step 15 (W17)

Receiver: Height 200 [✓ km] nautical miles, feet (W20)

Penetrating rays: Wanted ✓ (W21 = 0.)
Not wanted ________ (W21 = 1.)

Maximum number of hops 3 (W22)

Maximum number of steps per hop 1000 (W23)

Maximum allowable error per step 10^{-4} (W42)

Additional calculations: = 1. to integrate
= 2. to integrate and print

Phase path 2 (W57)
Absorption 2 (W58)
Doppler shift ________ (W59)
Path length ________ (W60)
Other ________

Printout: Every 5 steps of the ray trace (W71)

Punched cards (raysets): ✓ (W72 = 1.)

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON A VERTICAL PLANE

Coordinates of the left edge of the graph:

Latitude = 40. rad [✓ deg] km north (W83)

Longitude = - 105. rad [✓ deg] km east (W84)

Coordinates of the right edge of the graph:

Latitude = 52.12 rad [✓ deg] km north (W85)

Longitude = - 81.8 rad [✓ deg] km east (W86)

Height above the ground of the bottom of the graph = 0. km (W88)

Distance between tic marks = 100. rad deg [✓ km] (W87)

(W81 = 1.)

INPUT PARAMETER FORM FOR PLOTTING THE PROJECTION OF THE RAY PATH ON THE GROUND

Coordinates of the left edge of the graph:

Latitude = 40. rad [✓ deg] km north (W83)

Longitude = − 105. rad [✓ deg] km east (W84)

Coordinates of the right edge of the graph:

Latitude = 52.12 rad [✓ deg] km north (W85)

Longitude = − 81.8 rad [✓ deg] km east (W86)

Factor to expand lateral deviation scale by = 200. (W82)

Distance between tic marks on range scale = 100. rad deg [✓ km] (W87)

(W81 = 2.)

INPUT PARAMETER FORM FOR SUBROUTINE CHAPX

An ionospheric electron density model consisting of a Chapman layer with tilts, ripples, and gradients

$$f_N^2 = f_c^2 \exp\left(\alpha \left(1 - z - e^{-z}\right) \right)$$

$$z = \frac{h - h_{max}}{H}$$

$$f_c^2 = f_{co}^2 \left(1 + A \sin\left(2\pi\left(\theta - \frac{\pi}{2}\right)/B \right) + C\left(\theta - \frac{\pi}{2}\right)\right)$$

$$h_{max} = h_{max_o} + E\left(\theta - \frac{\pi}{2}\right) R_o$$

f_N is the plasma frequency

h is the height above the ground

R_o is the radius of the earth in km

and θ is the colatitude in radians.

Specify:

Critical frequency at the equator, f_{c_o} = 6.5 MHz (W101)

Height of the maximum electron density at the equator, h_{max_o} = 300 km (W102)

Scale height, H = 62. km (W103)

α = 0.5 (W104, 0.5 for an α Chapman layer, 1.0 for a β Chapman layer)

Amplitude of periodic variation of f_c^2 with latitude, A = 0. (W105)

Period of variation of f_c^2 with latitude, B = 0. rad deg km (W106)

Coefficient of linear variation of f_c^2 with latitude, C = 0. rad^{-1} (W107)

Tilt of the layer, E = 0. rad deg (W108)

INPUT PARAMETER FORM FOR SUBROUTINE WAVE

A perturbation to an ionospheric electron density model consisting of a "gravity-wave" irregularity traveling from north pole to south pole

$$N = N_o(1 + \Delta)$$

$$\Delta = \delta \exp - [(R - R_o - z_o)/H]^2 \cdot$$

$$\cos 2\pi \left[t' + (\pi/2 - \theta)\frac{R_o}{\lambda_x} + (R - R_o)/\lambda_z\right]$$

$$\frac{\partial N}{\partial t} = \frac{-2\pi}{\lambda_x} V_x N_o \delta \exp - [(R - R_o - z_o)/H]^2 \cdot$$

$$\sin 2\pi \left[t' + (\pi/2 - \theta)\frac{R_o}{\lambda_x} + (R - R_o)/\lambda z\right]$$

R_o is the radius of the earth.

R, θ, φ are the spherical (earth-centered) polar coordinates (Δ is independent of φ).

N_o (R, θ, φ) is any electron density model.

Specify:

the height of maximum wave amplitude, z_o = 250. km (W151)
wave-amplitude "scale height," H = 100. km (W152)
wave perturbation amplitude, δ = 0.1 [0. to 1.] (W153)
horizontal trace velocity, V_x = — km/sec (W154)
(needed only if Doppler shift is calculated)
horizontal wavelength, λ_x = 100. km (W155)
vertical wavelength, λ_z = 100. km (W156)
time in wave periods, t' = 0. [0. to 1.] (W157)

INPUT PARAMETER FORM FOR SUBROUTINE DIPOLY

An ionospheric model of the earth's magnetic field consisting of an earth centered dipole

The gyrofrequency is given by:

$$f_H = f_{H_0} \left(\frac{R_0 + h}{R_0}\right)^3 \left(1 + 3 \cos^2 \lambda\right)^{\frac{1}{2}}$$

The magnetic dip angle, I, is given by

$$\tan I = 2 \cot \lambda$$

h is the height above the ground

R_0 is the radius of the earth

λ is the geomagnetic colatitude

Specify:

the gyrofrequency at the equator on the ground, f_{H_0} = 0.8 MHz (W201)

the geographic coordinates of the north magnetic pole

latitude = 78.5 radians [✓ degrees] north (W24)

longitude = 291. radians [✓ degrees] east (W25)

INPUT PARAMETER FORM FOR SUBROUTINE EXPZ 2

An ionospheric collision frequency model consisting of a double exponential profile

$$\nu = \nu_1 e^{-a_1(h-h_1)} + \nu_2 e^{-a_2(h-h_2)}$$

where h is the height above the ground.

Specify for the first exponential:

Collision frequency at height h_1, ν_1 = 3.65 x 10^4 collisions per second (W251)

Reference height, h_1 = 100 km (W252)

Exponential decrease of ν with height, a_1 = 0.148 km^{-1} (W253)

Specify for the second exponential:

Collision frequency at height h_2, ν_2 = 30 collisions per second (W254)

Reference height, h_2 = 140 km (W255)

Exponential decrease of ν with height, a_2 = 0.0183 km^{-1} (W256)

Appendix 8b. Listing of Input Cards for the Sample Case

```
X01  TEST CASE
  1  0.                     OF DUPLICATE W CARDS, THE LAST ONE DOMINATES
  1  -1.                    EXTRAORDINARY RAY
  3  0.                     TRANSMITTER HEIGHT, KM
  4  40.           1        TRANSMITTER LATITUDE, DEG NORTH
  5  -105.         1        TRANSMITTER LONGITUDE, DEG EAST
  7  6.0                    INITIAL FREQUENCY, MC/S
  9  0.                     DONT STEP FREQUENCY
 11  45.0          1        INITIAL AZIMUTH ANGLE, DEGS CLOCKWISE FROM NORTH POLE
 13  0.                     DONT STEP AZIMUTH ANGLE
 15  0.            1        INITIAL ELEVATION ANGLE, DEG
 16  90.0          1        FINAL ELEVATION ANGLE, DEG
 17  15.0          1        STEP IN ELEVATION ANGLE, DEG
 20   200.                  RECEIVER HEIGHT ABOVE THE EARTH, KM
 22  3.                     NUMBER OF HOPS
 57  2.                     INTEGRATE AND PRINT PHASE PATH
 58  2.                     INTEGRATE AND PRINT ABSORPTION
 71  5.0                    NUMBER OF STEPS FOR EACH PRINTING
 72  1.                     PUNCH RAYSETS
 81  1.                     PLOT PROJECTION OF RAY PATH ON A VERTICAL PLANE
 83  40.0          1        LEFT LATITUDE OF PLOT, DEG
 84  -105.         1        LEFT LONGITUDE OF PLOT, DEG
 85  52.12         1        RIGHT LATITUDE OF PLOT, DEG
 86  -81.8         1        RIGHT LONGITUDE OF PLOT, DEG
 87  100.0          1       DISTANCE BETWEEN TIC MARKS, KM
101  6.5                    CRITICAL FREQUENCY, MC/S
102  300.0                  HMAX, KM
103  62.                    SCALE HEIGHT, KM
104  0.5                         ALPHA CHAPMAN LAYER
150  1.                     CALL PERTURBATION SUBROUTINE
151  250.                   Z0, KM
152 100.                    SH, SCALE HEIGHT, KM
153  0.1                    DELTA
155  100.                   LAMBDAX, HORIZONTAL WAVELENGTH, KM
156  100.                   LAMBDAZ, VERTICAL WAVELENGTH, KM
201  0.8                    GYROFREQUENCY ON THE GROUND AT THE EQUATOR, MHZ
 24  78.5          1        ACCEPTED STANDARD LAT. OF NORTH MAGNETIC POLE, DEG NORTH
 25  291.          1        ACCEPTED STANDARD LONG. OF NORTH MAGNETIC POLE, DEG EAST
251  3.65         E4        COLLISION FREQUENCY AT H1, /SEC
252  100.0                  H1, REFERENCE HEIGHT, KM
253  .148                   A1, EXPONENTIAL DECREASE OF NU WITH HEIGHT, /KM
254  30.                    COLLISION FREQUENCY AT H2, /SEC
255  140.                   H2, REFERENCE HEIGHT, KM
256  .0183                  A2, EXPONENTIAL DECREASE OF NU WITH HEIGHT, /KM
                            (A BLANK IN COL. 1-3 ENDS THE CURRENT  W ARRAY)
X01  TEST CASE
 71   0.                    NO PERIODIC PRINTOUT
 72  0.                     DO NOT PUNCH RAYSETS
 81  2.                     PLOT PROJECTION OF RAY PATH ON THE GROUND
 82  10.0                   LATERAL DEVIATION EXPANSION FACTOR
                            (A BLANK IN COL. 1-3 ENDS THE CURRENT  W ARRAY)
```

Col. 1-3 Identification number
 Col. 4-17 Data in E14.6 format
   Col. 18 A 1 indicates an angle in degrees
    Col. 19 A 1 indicates a central earth angle in kilometers
     Col. 20 A 1 indicates a distance in nautical miles
      Col. 21 A 1 indicates a distance in feet
       Col. 22-24 Left for other conversions
        Col. 25-80 Description of the data

```
X01  TEST CASE                                                                                              11/05/74
 CHAPX     WAVE     01POLY    EXPZ2                              APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIANS, ONLY NONZERO VALUES PRINTED

  1 -1.00000000000+000
  2  6.37000000000+003
  4  6.98131700803-001
  5 -1.83259571460+000
  7  6.00000000000+000
 11  7.85398163390-001
 16  1.57079632679+000
 17  2.61799387791-001
 20  2.00000000000+002
 22  3.00000000000+000
 23  1.00000000000+003
 24  1.37008346280+000
 25  5.07890812331+000
 41  3.00000000000+000
 42  1.00000000000-004
 43  5.00000000000+001
 44  1.00000000000+000
 45  1.00000000000+002
 46  1.00000000000-008
 47  5.00000000000-001
 57  2.00000000000+000
 58  2.00000000000+000
 71  5.00000000000+000
 72  1.00000000000+000
 81  1.00000000000+000
 83  6.98131700803-001
 84 -1.83259571460+000
 85  9.09665606124-001
 86 -1.42767932613+000
 87  1.56985871273-002
101  6.50000000000+000
102  3.00000000000+002
103  6.13999999995+001
104  5.00000000000-001
150  1.00000000000+000
151  2.50000000000+002
152  1.00000000000+002
153  1.00000000000-001
155  1.00000000000+002
156  1.00000000000+002
201  8.00000000000-001
251  3.65000000002+004
252  1.00000000000+002
253  1.48000000001-001
254  3.00000000000+001
255  1.39999999999+002
256  1.83000000002-002
```

Perturbation to the electron density model

Model of the earth's magnetic field

Collision frequency model

Electron density model

```
X01  TEST CASE                                                                             11/05/74
CHAPX    WAVE    DIPOLY  EXPZ2            APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

            FREQUENCY =      6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                      ELEVATION ANGLE OF TRANSMISSION =    3.000000 DEG

                                        AZIMUTH
                                       DEVIATION       ELEVATION
                     HEIGHT    RANGE   XMTR   LOCAL    XMTR   LOCAL    POLARIZATION    GROUP PATH  PHASE PATH  ABSORPTION
                      KM        KM     DEG     DEG     DEG     DEG     REAL    IMAG       KM          KM          DB
 0+000 XMTR          0.0000    0.0000                          0.000   0.000  -1.000     0.0000      0.0000      0.0000
-3-011 ENTR ION     52.9787  818.7210  0.000   0.000  -0.000   7.364   0.597  -1.427   823.2593    823.2593      0.0000
 0+000              59.4429  866.8660 -0.000  -0.000  -0.000   7.797   0.513  -2.672   872.2593    872.2593      0.0000
-3-011              64.9916  906.0962 -0.000  -0.000  -0.000   8.150   0.293  -2.386   912.2593    912.2593      0.0000
-3-011              73.7639  945.2593  0.000   0.000  -0.000   8.502   0.141  -2.599   952.2593    952.2593      0.0000
-2-009              85.6755 1038.9470  0.000  -0.000  -0.000   9.345   0.021  -3.200  1048.2593   1048.2592      0.0004
-2-008              99.1450 1116.6700  0.000   0.000  -0.000  10.039   0.004  -4.060  1128.2593   1128.2584      0.0014
-9-008             113.5441 1194.0737  0.000   0.000  -0.002  10.683   0.001  -5.656  1208.2593   1208.2467      0.0034
-1-006             128.6479 1271.1132  0.000  -0.001  -0.014  10.999   0.001  -7.942  1288.2593   1288.1390      0.0058
-4-006             143.3758 1347.7703  0.000   0.002  -0.078   9.958   0.000  -5.844  1368.2593   1367.5112      0.0076
-7-006             153.8536 1416.4564  0.001   0.012  -0.269   6.649   0.000  -2.873  1440.2593   1437.7718      0.0089
-9-006             157.2033 1454.4623  0.001   0.051  -0.473   3.399   0.000  -2.083  1480.2593   1476.0613      0.0097
-3-011 MIN DIST    158.1469 1491.1561  0.002  -0.029  -0.753   0.000   0.000  -1.720  1518.9022   1512.7786      0.0106
-3-011 MIN DIST    158.1469 1491.1561  0.002  -0.029  -0.753   0.000   0.000  -1.720  1518.9022   1512.7786      0.0106
-6-011 WAVE REV    158.1177 1494.9559  0.003  -0.039  -0.736  -0.341   0.000  -1.695  1522.9022   1516.5809      0.0107
-2-006             151.2539 1583.3795  0.006   0.028  -1.756  -7.550   0.000  -1.385  1615.9022   1605.6686      0.0126
 3-006             133.2039 1659.7028  0.007   0.024  -2.782 -10.588   0.000  -1.318  1695.9022   1684.0907      0.0144
 6-006             108.1392 1813.6452  0.006   0.016  -4.796 -10.483   0.000  -1.341  1855.9022   1843.6645      0.0191
 8-006              80.8887 1969.0039  0.004   0.015  -6.536  -9.113   0.008  -1.403  2015.9022   2003.6590      0.0218
 8-005 EXIT ION     49.2239 2186.5251  0.005   0.013  -8.572  -7.138   0.205  -1.093  2239.9022   2227.6590      0.0220
 0+000 GRND REF      0.0000 2900.0452 -0.006   0.010 -13.042   0.738   0.001  -1.000  2955.4929   2943.2497      0.0220
 0+000 ENTR ION     52.9830 3640.6051 -0.003   0.008 -15,566   7.401  -0.933   1.636  3700.7907   3688.5475      0.0220
 0+000              53.4995 3644.7388 -0.003   0.008 -15.577   7.436  -0.936   1.705  3704.7907   3692.5475      0.0220
-9-008              91.1164 3892.1351 -0.003   0.008 -16.214   9.661  -0.004   1.984  3957.7907   3945.5474      0.0227
-1-006 MAX LAT     107.9072 3985.1998 -0.003   0.008 -16.435  10.475  -0.000   1.835  4053.7907   4041.5426      0.0247
-1-006 WAVE REV    107.9072 3985.1998 -0.003   0.008 -16.435  10.475  -0.000   1.835  4053.7907   4041.5426      0.0247
-4-006             119.7979 4046.9652 -0.003   0.008 -16.578  10.907  -0.000   1.765  4117.7907   4105.5117      0.0266
-5-006             140.3669 4185.0294 -0.004   0.031 -16.914   9.243  -0.000   1.868  4261.7907   4248.4412      0.0303
-5-005             156.1166 4261.1682 -0.006   0.047 -17.169   4.188  -0.000   2.901  4341.7907   4325.8067      1.0320
 0+000 MIN DIST    157.9616 4305.0842 -0.007   0.076 -17.366  -0.000  -0.000   9.317  4388.0431   4369.8069      0.0330
                                  THIS RAY CALCULATION TOOK    12.382 SEC
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V^2/Real part $(n^2)-1$ where V is the magnitude of the wave normal vector, and n is the complex phase refractive index. This quantity would be zero if there were no errors in the numerical integration.

Height of ray point above the ground.

Great circle distance along the ground between the ray point and the transmitter.

Azimuth angle of the direction of transmission in degrees clockwise from great circle between transmitter and ray point.

Azimuth angle of the wave normal in degrees clockwise from great circle between transmitter and ray point.

Elevation angle of current ray path point at the transmitter.

Angle of the wave normal with the local horizontal.

Polarization = +i means the electric field* vector is rotating counter clockwise when looking along the ray.

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X01  TEST CASE                                                                                   11/05/74
CHAPX    WAVE    DIPOLY     EXPZ2                        APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                 FREQUENCY =     6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =    45.000000 DEG

                            ELEVATION ANGLE OF TRANSMISSION =   15.000000 DEG

                                          AZIMUTH
                                         DEVIATION          ELEVATION
                   HEIGHT      RANGE     XMTR   LOCAL      XMTR   LOCAL      POLARIZATION    GROUP PATH   PHASE PATH   ABSORPTION
                     KM          KM      DEG     DEG        DEG    DEG       REAL    IMAG        KM           KM           DB
 0+000 XMTR        0.0000     0.0000                              15.000     0.001  -1.000     0.0000       C.0000       0.0000
 0+000 ENTR ION   52.9852   186.0044   -0.000  -0.000     15.000  16.673     3.113  -3.368   194.1401     194.1401       0.0000
 0+000            60.7838   211.6249   -0.000  -0.000     15.000  16.903     2.323  -7.474   221.1401     221.1401       0.0000
 0+000            66.6275   230.5627   -0.000  -0.000     15.000  17.074     1.297  -9.793   241.1401     241.1401       0.0000
-5-010            79.6832   272.1042   -0.000  -0.000     15.000  17.447     0.368 -18.918   285.1401     285.1401       0.0001
-7-009            91.7890   309.7220   -0.000   0.000     15.000  17.785     0.359-110.019   325.1401     325.1400       0.0004
-2-008           104.1175   347.1979   -0.000  -0.000     15.000  18.116    -0.015  29.112   365.1401     365.1388       0.0011
 1-006           126.7922   414.2887   -0.000   0.001     14.990  18.525    -0.000  10.310   437.1401     437.0846       0.0031
 3-005           151.4725   483.2463   -0.000   0.012     14.841  16.907    -0.001 459.192   517.1401     516.1469       0.0048
-9-006           161.8111   524.9617   -0.001  -0.002     14.559  13.612     0.000  -4.495   557.1401     554.2934       0.0057
-1-005           169.1102   561.6061    0.006  -0.078     14.015   8.649     0.000  -2.160   597.1401     591.1301       0.0067
-7-005           172.1418   598.0253    0.015   0.155     13.155   1.378     0.000  -1.537   637.1401     626.6153       0.0079
-3-011 MIN DIST  172.1392   604.1034    0.015   0.184     12.975   0.000     0.000  -1.481   643.8528     632.4672       0.0081
-3-011 MIN DIST  172.1392   604.1034    0.015   0.184     12.975   0.000     0.000  -1.481   643.8528     632.4672       0.0081
 2-010 WAVE REV  172.0655   607.7222    0.015   0.196     12.864  -0.320     0.000  -1.452   647.8528     635.9500       0.0083
-3-005           153.8276   706.9590    0.026  -0.078      8.942 -15.755     0.000  -1.173   756.8528     735.2639       0.0114
-3-005           130.0051   750.9017    0.033  -0.036      5.835 -18.205     0.000  -1.154   836.8528     813.9074       0.0133
-3-005           105.1516   855.5089    0.037  -0.032      3.033 -17.862     0.000  -1.162   916.8528     893.8080       0.0160
-3-005            81.0547   930.6930    0.039  -0.030      0.752 -17.194     0.003  -1.174   996.8528     973.8062       0.0173
-3-005 EXIT ION   48.3333  1036.6769    0.042  -0.027     -1.983 -16.239     0.076  -1.037  1108.8528    1085.8062       0.0174
 0+000 GRND REF    0.0000  1212.9251    0.046  -0.023     -5.455  14.656    -0.002   1.000  1292.1934    1269.1468       0.0174
-3-011 ENTR ION   52.9855  1403.0408    0.049  -0.020     -4.165  16.366    -1.306   1.885  1490.3081    1467.2614       0.0174
-3-011            54.1138  1406.8464    0.049  -0.020     -4.142  16.400    -1.305   2.105  1494.3081    1471.2614       0.0174
-9-007           110.0072  1565.1016    0.052  -0.018     -3.213  17.984    -0.000   2.342  1683.3081    1660.2574       0.0191
-1-005           134.9372  1659.5945    0.052  -0.018     -2.890  18.138    -0.000   2.221  1763.3081    1740.0923       0.0214
-5-005           156.1139  1726.0642    0.052   0.021     -2.688  15.432    -0.000   3.024  1835.3081    1810.6175       0.0228
-1-005           165.2114  1762.7028    0.051  -0.021     -2.675  11.173    -0.000  14.789  1875.3081    1848.1145       0.0237
-7-006           170.7131  1799.2998    0.054  -0.203     -2.779   5.562     0.000  -3.471  1915.3081    1884.3821       0.0248
 0+000 MIN DIST  171.9566  1828.6204    0.059  -0.065     -2.959  -0.000     0.000  -1.952  1947.4107    1912.8923       0.0258
                                 THIS RAY CALCULATION TOOK    11.247 SEC
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X01  TEST CASE                                                                                        11/05/74
 CHAPX    WAVE    DIPOLY    EXPZ2                          APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                  FREQUENCY =    6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                              ELEVATION ANGLE OF TRANSMISSION =   30.000000 DEG

                                           AZIMUTH
                                          DEVIATION        ELEVATION
                     HEIGHT      RANGE    XMTR    LOCAL    XMTR   LOCAL      POLARIZATION     GROUP PATH  PHASE PATH  ABSORPTION
                       KM         KM      DEG      DEG     DEG     DEG      REAL     IMAG        KM          KM           DB
 3-011 XMTR           0.0000      0.0000                           30.000  -0.000    1.000      0.0000      0.0000       0.0000
 0+000 ENTR ION      52.9739     89.9010  -0.000   -0.000  30.000  30.809  -0.332    1.259    104.6686    104.6686       0.0000
-3-011               60.6634    102.6627  -0.000   -0.000  30.000  30.923  -0.218    1.562    119.6686    119.6686       0.0000
 0+000               65.8141    111.1535  -0.000   -0.000  30.000  31.000  -0.110    1.614    129.6686    129.6686       0.0000
-2-010               78.2077    131.4761  -0.000   -0.000  30.000  31.183  -0.017    1.609    153.6686    153.6686       0.0000
-2-009               88.5857    148.3519  -0.000   -0.000  30.000  31.334  -0.004    1.592    173.6686    173.6685       0.0002
-1-007              107.3766    178.5914  -0.000    0.000  30.000  31.600  -0.000    1.562    209.6686    209.6671       0.0008
-1-006              128.3954    211.9816  -0.000   -0.001  29.993  31.773  -0.000    1.539    249.6686    249.6257       0.0020
 5-006              149.2460    245.1205  -0.002    0.009  29.935  31.083  -0.000    1.574    289.6686    289.1825       0.0029
-2-005              168.5912    277.9003  -0.011   -0.068  29.657  27.599  -0.000    1.895    329.6686    326.8860       0.0038
 2-005              184.0395    310.3408  -0.016    0.029  28.909  19.956  -0.000    7.331    369.6686    360.9884       0.0052
 1-006              191.3569    342.2014  -0.027    0.444  27.306   5.952   0.000   -1.911    409.6686    390.0049       0.0073
 0+000 MIN DIST     191.5641    354.9408  -0.023    0.219  26.398   0.000   0.000   -1.530    425.7924    400.9614       0.0082
 0+000 MIN DIST     191.5641    354.9408  -0.023    0.219  26.398   0.000   0.000   -1.530    425.7924    400.9614       0.0082
 6-010 WAVE REV     191.4066    358.1065  -0.020    0.137  26.154  -1.395   0.000   -1.475    429.7924    403.6919       0.0085
 4-006              172.2684    426.8860   0.135   -0.735  19.783 -23.513   0.000   -1.113    514.7924    468.9299       0.0125
 2-005              135.6960    493.7148   0.235   -0.521  12.986 -29.961   0.000   -1.076    594.7924    544.6800       0.0147
 2-005               95.7861    561.7275   0.298   -0.455   7.074 -29.718   0.000   -1.080    674.7924    624.5459       0.0172
 2-005               56.5019    630.5940   0.348   -0.405   2.257 -29.100   0.039   -1.062    754.7924    704.5457       0.0177
 2-005 EXIT ION      46.7358    644.4661   0.357   -0.397   1.406 -28.975   0.035   -1.018    770.7924    720.5457       0.0177
-3-011 GRND REF       0.0000    733.6060   0.405   -0.349  -3.299  28.173  -0.000    1.000    872.6850    822.4383       0.0177
 0+000 ENTR ION      52.9781    830.3382   0.446   -0.308  -0.104  29.043  -0.277    1.223    983.3256    933.0789       0.0177
-4-007               92.6830    930.1376   0.470   -0.284   1.779  29.671  -0.002    1.497   1064.3256   1014.0788       0.0180
-2-005              132.5904    968.2234   0.490   -0.266   3.349  30.046  -0.000    1.461   1144.3256   1093.9984       0.0201
-1-005              163.3870   1021.9170   0.501   -0.223   4.356  26.978  -0.000    1.628   1208.3256   1156.1125       0.0215
-2-005              179.0552   1055.0139   0.507   -0.452   4.735  20.718  -0.000    2.556   1248.3256   1191.4144       0.0228
-9-006              188.6742   1087.7704   0.516    0.102   4.784   9.433   0.000   -4.108   1288.3256   1222.7505       0.0245
 0+000 MIN DIST     189.8217   1107.5272   0.514    0.454   4.581   0.000   0.000   -1.771   1312.8736   1240.1070       0.0259
                                        THIS RAY CALCULATION TOOK     8.642 SEC
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X01  TEST CASE                                                                                                           11/05/74
 CHAPX     WAVE     DIPOLY     EXPZ2                                       APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                    FREQUENCY =     6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                                ELEVATION ANGLE OF TRANSMISSION =   45.000000 DEG

                                            AZIMUTH
                                          DEVIATION          ELEVATION
                    HEIGHT      RANGE     XMTR     LOCAL    XMTR     LOCAL     POLARIZATION    GROUP PATH  PHASE PATH  ABSORPTION
                       KM         KM       DEG       DEG     DEG       DEG      REAL     IMAG      KM          KM          DB
 0+000 XMTR         0.0000     0.0000                               45.000    -0.000    1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION    52.9759    52.3246    -0.000   -0.000   45.000   45.471    -0.113    1.100     74.6128     74.6128      0.0000
 3-011             62.2499    61.3530    -0.000   -0.000   45.000   45.552    -0.059    1.207     87.6128     87.6128      0.0000
 0+000             69.3926    68.2802     0.000    0.000   45.000   45.614    -0.021    1.219     97.6128     97.6128      0.0000
-3-009             85.1331    83.4659    -0.000   -0.000   45.000   45.751    -0.002    1.217    119.6128    119.6128      0.0001
-4-008             99.4743    97.2070    -0.000   -0.000   45.000   45.873    -0.000    1.213    139.6128    139.6125      0.0004
-1-007            113.8433   110.8868    -0.000   -0.000   45.000   45.986    -0.000    1.209    159.6128    159.6091      0.0009
 5-005            142.5566   138.0458    -0.002    0.003   44.980   45.857    -0.000    1.209    199.6128    199.4233      0.0021
-2-005            170.1037   164.7826    -0.015   -0.106   44.789   43.268    -0.000    1.260    239.6128    237.3102      0.0030
 6-005            193.2703   190.6257    -0.071    0.369   44.104   34.673    -0.000    1.601    279.6128    268.9418      0.0048
-3-011 RCVR       200.0000   200.2014    -0.132    0.607   43.622   28.480    -0.000    2.293    295.1649    278.4398      0.0058
 2-006            209.6843   233.2599    -0.369   -1.590   40.436    4.780     0.000   -1.976    348.1649    304.3962      0.0095
 2-006 APOGEE     209.6834   235.8629    -0.040   -1.746   40.109    3.110     0.000   -1.831    352.1649    306.3131      0.0098
 3-006 WAVE REV   209.4360   241.0969     0.021   -1.982   39.431   -0.266     0.000   -1.623    360.1649    310.1892      0.0103
 3-007            205.9618   259.5199     0.244   -2.001   36.819  -12.892     0.000   -1.268    388.1649    324.3166      0.0122
 3-011 RCVR       200.0000   274.3768     0.389   -1.415   34.427  -23.456     0.000   -1.142    410.8543    336.8372      0.0137
 1-005            172.6329   311.8736     0.547   -0.183   27.233  -41.318     0.000   -1.041    467.8543    377.6491      0.0169
 4-006            117.8520   366.4800     0.568   -0.033   16.021  -45.154     0.000   -1.031    547.8543    454.5212      0.0193
 6-006             89.5472   394.2849     0.573   -0.030   10.932  -44.927     0.000   -1.032    587.8543    494.5131      0.0204
 6-005             38.3028   444.9524     0.574   -0.027    2.978  -44.472     0.004   -1.000    659.8543    566.5130      0.0206
 6-006 EXIT ION    38.3028   444.9524     0.574   -0.027    2.978  -44.472     0.004   -1.000    659.8543    566.5130      0.0206
 0+000 GRND REF     0.0000   484.7060     0.576   -0.025   -2.180   44.114    -0.000    1.000    715.5626    622.2214      0.0206
 0+000 ENTR ION    52.9777   538.6621     0.578   -0.022    3.168   44.599    -0.105    1.094    791.3392    697.9980      0.0206
-6-007            110.1068   595.3574     0.581   -0.020    7.705   45.103    -0.000    1.196    872.3392    778.9960      0.0214
-6-007            138.4297   622.9739     0.581   -0.016    9.586   45.095    -0.000    1.194    912.3392    818.8774      0.0226
-4-006            165.8750   650.1889     0.579   -0.068   11.199   42.989    -0.000    1.230    952.3392    857.2445      0.0236
 7-005            183.5935   676.6832     0.572    0.057   12.376   35.811    -0.000    1.434    992.3392    890.3367      0.0252
-3-011 RCVR       200.0000   691.5163     0.546    0.796   12.769   26.353    -0.000    2.159   1015.8617    905.3752      0.0266
                                      THIS RAY CALCULATION TOOK   10.310 SEC
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XJ1  TEST CASE                                                                                         11/05/74
 CHAPX    WAVE    DIPOLY   EXPZ2                    APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                  FREQUENCY =    6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                                 ELEVATION ANGLE OF TRANSMISSION =   60.000000 DEG

                                          AZIMUTH
                                         DEVIATION         ELEVATION
                  HEIGHT      RANGE      XMTR    LOCAL     XMTR    LOCAL      POLARIZATION      GROUP PATH  PHASE PATH  ABSORPTION
                    KM          KM       DEG      DEG      DEG      DEG       REAL     IMAG        KM          KM          DB
 0+000 XMTR        0.0000      0.0000                               60.000   -0.000    1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION   52.9788     30.2936   -0.000   -0.000   60.000   60.272   -0.053    1.049     61.0908     61.0908      0.0000
 0+000            59.9272     34.2236   -0.000   -0.000   60.000   60.308   -0.035    1.092     69.0908     69.0908      0.0000
 0+000            67.7470     38.6347   -0.000   -0.000   60.000   60.348   -0.012    1.103     78.0908     78.0908      0.0000
-7-009            85.1354     46.3990   -0.000   -0.000   60.000   60.435   -0.001    1.103     98.0908     98.0908      0.0001
-1-007           102.5387     58.1107   -0.000   -0.000   60.000   60.522   -0.000    1.101    118.0908    118.0904      0.0004
-4-007           119.9525     67.7696   -0.000   -0.000   59.999   60.592   -0.000    1.100    138.0908    138.0820      0.0011
-4-006           154.5342     86.6836   -0.007   -0.005   59.967   60.149   -0.000    1.103    178.0908    177.5563      0.0022
 4-005           136.1656    105.2175   -0.094    0.088   59.697   55.830   -0.000    1.158    218.0908    211.9329      0.0037
 0+000 RCVR      200.0000    114.4162   -0.157   -0.851   59.325   50.578   -0.000    1.250    239.2552    225.6343      0.0050
 9-005           224.6062    137.5936   -0.234    0.797   57.441   24.698   -0.000    5.656    296.2552    248.7596      0.0090
-1-004 APOGEE    225.8382    146.6930   -0.354    5.402   55.872   -5.298    0.000   -2.002    324.2552    253.2681      0.0114
-1-004 WAVE REV  225.8382    146.6930   -0.354    5.402   55.872   -5.298    0.000   -2.002    324.2552    253.2681      0.0114
-1-004           221.4155    153.9714   -0.522    9.624   54.028  -33.727    0.000   -1.188    348.2552    257.0925      0.0135
-3-005           204.2335    166.0800   -1.210   16.099   49.687  -60.227    0.000   -1.030    388.2552    269.8039      0.0168
 0+000 RCVR      200.0000    166.1639   -1.383   16.376   48.744  -62.499    0.000   -1.025    395.1225    273.3158      0.0173
-1-005           153.9530    185.8715   -3.075   16.994   38.460  -69.419    0.000   -1.013    452.1225    318.1391      0.0201
-4-005           116.7265    198.8446   -4.206   15.804   29.230  -69.701    0.000   -1.013    492.1225    357.6180      0.0213
-2-005            79.2276    212.0602   -5.215   14.796   19.415  -69.586    0.000   -1.013    532.1225    397.6124      0.0223
-2-005 EXIT ION   26.7751    230.9196   -6.447   13.564    5.561  -69.411    0.000   -1.000    588.1225    453.6124      0.0223
 0+000 GRND REF    0.0000    240.7118   -7.005   13.003   -1.083   69.320   -0.000    1.000    616.7330    482.2229      0.0223
-3-011 ENTR ION   52.9836    260.0662   -7.992   12.021   10.239   69.499   -0.026    1.025    673.3376    538.8275      0.0223
-3-011            56.7354    261.4245   -8.056   11.957   11.015   69.511   -0.023    1.038    677.3376    542.8275      0.0223
-3-005           113.9101    281.9769   -8.339   11.075   20.549   69.695   -0.000    1.050    738.3376    603.8243      0.0232
 6-005           151.2131    295.2591   -8.446   10.601   25.514   69.477   -0.000    1.051    778.3376    643.4278      0.0244
-4-005           185.7563    308.0424   -9.310    9.231   29.335   66.600   -0.000    1.071    818.3376    678.2033      0.0259
 0+000 RCVR      200.0000    313.9111  -10.115    8.555   30.682   63.564   -0.000    1.099    837.7013    691.0808      0.0271
                                    THIS RAY CALCULATION TOOK     8.004 SEC
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X01  TEST CASE                                                                                   11/05/74
 CHAPX    WAVE     DIFOLY    EXPZ2                     APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                   FREQUENCY =    6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                                  ELEVATION ANGLE OF TRANSMISSION =   75.000000 DEG

                                            AZIMUTH
                                           DEVIATION         ELEVATION
                     HEIGHT      RANGE     XMTR   LOCAL     XMTR   LOCAL     POLARIZATION    GROUP PATH  PHASE PATH  ABSORPTION
                      KM          KM       DEG     DEG      DEG     DEG     REAL    IMAG        KM          KM          DB
 0+000 XMTR          0.0000      0.0000                            75.000  -0.000   1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION     52.9646     14.0759   -0.000  -0.000   75.000  75.127  -0.024   1.023     54.8374     54.8374      0.0000
-3-011              60.7166     16.1100   -0.000  -0.000   75.000  75.145  -0.014   1.044     62.8374     62.8374      0.0000
-3-011              65.5499     17.3768   -0.000  -0.000   75.000  75.156  -0.008   1.047     67.8374     67.8374      0.0000
-2-006              87.7850     23.1909   -0.000  -0.000   75.000  75.209  -0.000   1.048     90.8374     90.8374      0.0001
-1-007             107.1237     28.2124   -0.000  -0.000   75.000  75.253  -0.000   1.047    110.8374    110.8365      0.0005
-9-007             126.4556     33.2025   -0.001   0.000   74.999  75.279  -0.000   1.046    130.8374    130.8161      0.0013
 4-007             145.6921     38.1498   -0.007  -0.012   74.934  75.188  -0.000   1.047    150.8374    150.6143      0.0019
 3-005             178.7737     46.6081   -0.062  -0.321   74.920  73.849  -0.000   1.057    186.8374    183.5011      0.0030
 0+000 RCVR        200.0000     52.6875   -0.302   1.334   74.783  71.357  -0.000   1.087    213.9181    202.1284      0.0046
 6-005             224.9440     60.6695   -1.677   1.370   74.379  52.074  -0.000   1.568    266.9181    217.2922      0.0091
 1-004             230.7996     66.1625   -1.711  -7.396   73.432  15.569   0.000  -9.825    306.9181    220.0213      0.0127
 1-004 APOGEE      230.9183     67.5564   -1.511  -8.853   73.111   8.237   0.000  -3.876    314.9181    220.4673      0.0134
 9-005 WAVE REV    230.2593     70.7641   -0.922 -11.181   72.307  -4.474   0.000  -1.950    330.9181    221.5782      0.0147
 5-005             222.0226     84.2620    1.343 -14.206   68.508 -29.337   0.000  -1.143    378.9181    228.5798      0.0187
 0+000 RCVR        200.0000    106.6297    5.511 -13.543   61.084 -46.475   0.000  -1.024    434.9673    249.2747      0.0232
 2-005             161.6693    134.6131    8.542 -11.278   49.256 -55.293   0.000  -1.007    491.9673    291.7804      0.0263
-2-005             129.0155    155.6964   10.092  -9.754   38.661 -56.407   0.000  -1.006    531.9673    330.5977      0.0275
-3-006              95.7312    177.1805   11.285  -8.562   27.405 -56.282   0.000  -1.006    571.9673    370.5601      0.0289
-1-006              62.4974    196.9508   12.226  -7.620   16.463 -56.084   0.001  -1.006    611.9673    410.5600      0.0292
-1-006 EXIT ION     29.3420    220.9908   12.392  -6.356    6.551 -55.885   0.000  -1.000    651.9673    450.5600      0.0292
 0+000 GRND REF      0.0000    240.7569   13.557  -6.291   -1.083  55.706  -0.000   1.000    687.4459    486.0386      0.0292
 0+000 ENTR ION     52.9836    276.3463   14.370  -5.479    9.565  56.027  -0.069   1.064    751.4563    550.0489      0.0292
-8-009              87.0257    296.8465   14.784  -5.065   14.785  56.231  -0.001   1.134    792.4563    591.0489      0.0293
-6-006             120.3097    320.5884   15.128  -4.721   18.948  56.406  -0.000   1.130    832.4563    631.0396      0.0303
 2-005             153.3778    342.0624   15.423  -4.425   22.353  55.904  -0.000   1.135    872.4563    670.5260      0.0314
 4-005             183.3935    362.9372   15.675  -4.214   24.915  51.894  -0.000   1.195    912.4563    705.8795      0.0327
-3-011 RCVR        200.0000    375.3507   15.794  -3.644   25.987  45.376  -0.000   1.374    937.9577    722.8911      0.0343
                                         THIS RAY CALCULATION TOOK     9.624 SEC
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K01  TEST CASE                                                                                          11/05/74
 CHAPX     WAVE     DIPOLY     EXPZ2                   APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

              FREQUENCY =     6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                                  ELEVATION ANGLE OF TRANSMISSION =   90.000000 DEG

                                            AZIMUTH
                                           DEVIATION          ELEVATION
                     HEIGHT      RANGE     XMTR    LOCAL     XMTR    LOCAL      POLARIZATION     GROUP PATH   PHASE PATH   ABSORPTION
                       KM         KM       DEG      DEG      DEG      DEG      REAL     IMAG        KM           KM           DB
 0+000 XMTR          0.0000     0.0000                                90.000   -0.000   1.000       0.0000       0.0000      0.0000
-3-011 ENTR ION     52.9908     0.0000                       90.000   90.000   -0.003   1.009      52.9908      52.9908      0.0000
-3-011 MAX LAT      54.9908     0.0000                       90.000   90.000   -0.003   1.012      54.9908      54.9908      0.0000
-3-011              56.9908     0.0000                       90.000   90.000   -0.003   1.014      56.9908      56.9908      0.0000
-6-011              62.4908     0.0000                       90.000   90.000   -0.004   1.018      62.4908      62.4908      0.0000
-9-011              64.9908     0.0000                       90.000   90.000   -0.003   1.018      64.9908      64.9908      0.0000
-6-011              67.4908     0.0000                       90.000   90.000   -0.002   1.018      67.4908      67.4908      0.0000
-6-011 WAVE REV     69.2408     0.0000                       90.000   90.000   -0.002   1.019      69.2408      69.2408      0.0000
-6-011              69.2408     0.0000                       90.000   90.000   -0.002   1.019      69.2408      69.2408      0.0000
-6-011              71.7408     0.0000                       90.000   90.000   -0.001   1.019      71.7408      71.7408      0.0000
-3-011              74.2408     0.0000                       90.000   90.000   -0.001   1.019      74.2408      74.2408      0.0000
-3-011              76.7408     0.0000                       90.000   90.000   -0.001   1.019      76.7408      76.7408      0.0000
-6-011              79.2408     0.0000                       90.000   90.000   -0.000   1.019      79.2408      79.2408      0.0000
-9-011              84.2408     0.0000                       90.000   90.000   -0.000   1.019      84.2408      84.2408      0.0001
-2-010              89.2408     0.0000                       90.000   90.000   -0.000   1.019      89.2408      89.2408      0.0001
-3-010              94.2408     0.0000 -143.742-178.476      90.000   90.000   -0.000   1.018      94.2408      94.2407      0.0002
-1-009             105.2405     0.0000 -144.362-179.696      90.000   90.000   -0.000   1.018     105.2408     105.2402      0.0005
-2-009             110.2339     0.0001 -145.186-179.920      90.000   90.000   -0.000   1.018     110.2408     110.2392      0.0007
-3-009             115.2387     0.0001 -145.276 179.991      90.000   90.000   -0.000   1.018     115.2408     115.2370      0.0009
-3-009 WAVE REV    119.7365     0.0003 -145.262   0.004      90.000   90.000   -0.000   1.018     119.7408     119.7331      0.0010
-3-009             120.7358     0.0003 -145.260   0.006      90.000   90.000   -0.000   1.018     120.7408     120.7318      0.0011
-4-009             125.7304     0.0007 -145.269  -0.003      90.000   90.000   -0.000   1.018     125.7408     125.7220      0.0013
-4-009             138.6856     0.0039 -145.266   0.001      89.998   89.997   -0.000   1.018     138.7408     138.6413      0.0017
 9-008             148.5779     0.0121 -145.266   0.000      89.995   89.987   -0.000   1.018     148.7408     148.4476      0.0019
 3-007             158.3272     0.0318 -145.266  -0.000      89.988   89.970   -0.000   1.019     158.7408     157.9965      0.0022
 3-007             167.8234     0.0700 -145.266   0.000      89.975   89.964   -0.000   1.020     168.7408     167.1041      0.0025
 2-007 WAVE REV    175.1532     0.1157 -145.266-180.000      89.961   89.993   -0.000   1.021     176.7408     173.9276      0.0029
 2-007             176.9406     0.1289 -145.266-180.000      89.957   89.970   -0.000   1.021     178.7408     175.5573      0.0029
-1-006             192.0775     0.2533 -145.266 180.000      89.922   89.409   -0.000   1.025     196.7408     188.6768      0.0040
 0+000 RCVR        200.0000     0.3028 -145.266-180.000      89.911   88.779   -0.000   1.030     207.1658     194.9665      0.0047
 4-006             218.5537     0.3932 -145.266-180.000      89.893   87.446   -0.000   1.047     236.1658     207.6856      0.0069
 3-006             227.9866     0.7709 -145.266-180.000      89.799   89.319   -0.000   1.054     256.1658     212.5942      0.0084
 4-006 WAVE REV    230.2098     0.9818 -145.266   0.000      89.747   89.207   -0.000   1.052     262.1658     213.5202      0.0089
-4-004             237.3654     2.6623 -145.266   0.000      89.333   55.360   -0.000   1.017     294.1658     215.5935      0.0115
-3-003 WAVE REV    238.2305     3.6109 -145.266   0.000      89.099  -11.160   -0.000   4.505     310.1658     215.7522      0.0127
-2-003 APOGEE      238.0027     4.0933 -145.266  -0.000      88.978  -33.291    0.000  -5.606     318.1658     215.8346      0.0134
-4-004             236.0571     5.1869 -145.266  -0.000      88.696  -51.733    0.000  -1.866     334.1658     216.3578      0.0146
 2-005             221.9860     9.8361 -145.266   0.000      87.375  -66.363    0.000  -1.198     374.1658     223.1020      0.0177
-6-011 RCVR        200.0000    15.3973 -145.266   0.000      85.460  -75.213    0.000  -1.080     411.0570     238.1693      0.0205
 2-005             151.1549    24.7823 -145.266   0.000      80.469  -79.011    0.000  -1.042     468.0570     282.8996      0.0232
-1-005             112.0674    32.1365 -145.266   0.000      73.720  -79.110    0.000  -1.042     508.0570     322.5389      0.0245
 2-005              72.7936    39.6085 -145.266   0.000      61.133  -79.045    0.002  -1.043     548.0570     362.5365      0.0252
 2-006 EXIT ION     25.6746    48.6937 -145.266   0.000      27.535  -78.963    0.001  -1.000     596.0570     410.5365      0.0252
-3-011 GRND REF      0.0000    53.7019 -145.266   0.000      -0.242   78.918   -0.000   1.000     622.2176     436.6971      0.0252
-3-011 ENTR ION     52.9956    63.9944 -145.266   0.000      39.224   79.011   -0.003   1.003     676.2116     490.6911      0.0252
-1-007              85.3935    70.2016 -145.266   0.000      50.073   79.067   -0.000   1.006     709.2116     523.6910      0.0253
-2-005             124.0623    77.6423 -145.266   0.000      57.484   79.121   -0.000   1.006     749.2116     563.6739      0.0265
 3-005             163.2840    84.9939 -145.266   0.000      61.819   78.758   -0.000   1.006     789.2116     602.5108      0.0277
-3-006             197.0532    92.0417 -145.266   0.000      64.210   77.455   -0.000   1.006     829.2116     633.2002      0.0297
 0+000 RCVR        200.0000    92.7180 -145.266   0.000      64.368   77.233   -0.000   1.006     833.2790     635.5670      0.0300
                                       THIS RAY CALCULATION TOOK    13.937 SEC
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X01  TEST CASE                                                                              11/05/74
 CHAPX    WAVE    DIFOLY    FXPZ2                    APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

INITIAL VALUES FOR THE W ARRAY -- ALL ANGLES IN RADIANS, ONLY NONZERO VALUES PRINTED

  1 -1.00000000000+000
  2  5.37000000000+003
  4  6.98131700803-001
  5 -1.83259571460+000
  7  5.00000000000+000
 11  7.85398163390-001
 16  1.57079632679+000
 17  2.61799387791-001
 20  2.00000000000+002
 22  3.00000000000+000
 23  1.00000000000+003
 24  1.37008346280+000
 25  5.07890812331+003
 41  3.00000000000+000
 42  1.00000000000-004
 43  5.00000000000+001
 44  1.00000000000+000
 45  1.00000000000+002
 46  1.00000000000-008
 47  5.00000000000-001
 57  2.00000000000+000
 58  2.00000000000+000
 71  1.00000000000+003
 81  2.00000000000+000
 82  1.00000000000+001
 83  6.98131700803-001
 84 -1.83259571460+000
 85  9.09665606124-001
 86 -1.42767932813+000
 87  1.56945671273-002
101  6.50000000000+000
102  3.00000000000+002
103  6.19999999995+001
104  5.00000000000-001
150  1.00000000000+000
151  2.50000000000+002
152  1.00000000000+002
153  1.00000000000-001
155  1.00000000000+002
156  1.00000000000+002
201  8.00000000000-001
251  3.65000000002+004
252  1.00000000000+002
253  1.48000000001-001
254  3.00000000000+001
255  1.33999999999+002
256  1.93000000002-002
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X01  TEST CASE                                                                                                   11/05/74
CHAPX     WAVE     DIFOLY     EXPZ2                          APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                   FREQUENCY =     6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =    45.000000 DEG

                             ELEVATION ANGLE OF TRANSMISSION =     0.000000 DEG

                                              AZIMUTH
                                             DEVIATION           ELEVATION
                      HEIGHT      RANGE      XMTR    LOCAL      XMTR    LOCAL      POLARIZATION      GROUP PATH   PHASE PATH   ABSORPTION
                        KM          KM       DEG      DEG        DEG     DEG       REAL     IMAG        KM           KM            DB
 0+000 XMTR           0.0000      0.0000                                  0.000    0.000  -1.000      0.0000       0.0000        0.0000
-3-011 ENTR ION      52.9737    818.7210    0.000    0.000    -0.000    7.364    0.597  -1.427    823.2593     823.2593        0.0000
-3-011 MIN DIST     158.1469   1491.1561    0.002   -0.029    -0.753    0.000    0.000  -1.720   1518.9022    1512.7786        0.0106
-3-011 MIN DIST     158.1469   1491.1561    0.002   -0.029    -0.753    0.000    0.000  -1.720   1518.9022    1512.7786        0.0106
-6-011 WAVE REV     158.1177   1494.9559    0.003   -0.039    -0.736   -0.341    0.000  -1.695   1522.9022    1516.5809        0.0107
 8-006 EXIT ION      49.2239   2186.5251    0.003    0.013    -8.572   -7.138    0.205  -1.093   2239.9022    2227.6590        0.0220
 0+000 GRND REF       0.0000   2900.0482   -0.000    0.010   -13.042    0.738    0.001  -1.000   2955.4929    2943.2497        0.0220
 0+000 ENTR ION      52.9830   3640.8051   -0.003    0.008   -15.566    7.401   -0.933   1.636   3700.7907    3688.5475        0.0220
-1-006 MAX LAT      107.9072   3985.1998   -0.003    0.008   -16.435   10.475   -0.000   1.835   4053.7907    4041.5426        0.0247
-1-006 WAVE REV     107.9072   3985.1998   -0.003    0.008   -16.435   10.475   -0.000   1.835   4053.7907    4041.5426        0.0247
 0+000 MIN DIST     157.9016   4305.0842   -0.007    0.076   -17.366   -0.000   -0.000   9.317   4388.0431    4369.8069        0.0330
                                        THIS RAY CALCULATION TOOK    10.723 SEC

                             ELEVATION ANGLE OF TRANSMISSION =    15.000000 DEG

                                              AZIMUTH
                                             DEVIATION           ELEVATION
                      HEIGHT      RANGE      XMTR    LOCAL      XMTR    LOCAL      POLARIZATION      GROUP PATH   PHASE PATH   ABSORPTION
                        KM          KM       DEG      DEG        DEG     DEG       REAL     IMAG        KM           KM            DB
 0+000 XMTR           0.0000      0.0000                                 15.000    0.001  -1.000      0.0000       0.0000        0.0000
 0+000 ENTR ION      52.9852    186.0044   -0.000   -0.000    15.000   16.673    3.119  -3.368    194.1401     194.1401        0.0000
-3-011 MIN DIST     172.1332    604.1034    0.015    0.184    12.975    0.000    0.000  -1.481    643.8528     632.4672        0.0081
-3-011 MIN DIST     172.1332    604.1034    0.015    0.184    12.975    0.000    0.000  -1.481    643.8528     632.4672        0.0081
 2-010 WAVE REV     172.0659    607.7222    0.015    0.196    12.864   -0.820    0.000  -1.452    647.8528     635.9500        0.0083
-3-005 EXIT ION      48.8383   1036.8769    0.042   -0.027    -1.983  -16.239    0.076  -1.037   1108.8528    1085.8062        0.0174
 0+000 GRND REF       0.0000   1212.9251    0.046   -0.023    -5.455   14.656   -0.002   1.000   1292.1934    1269.1468        0.0174
-3-011 ENTR ION      52.9855   1403.0468    0.049   -0.020    -4.165   16.366   -1.306   1.885   1490.3081    1467.2614        0.0174
 0+000 MIN DIST     171.9566   1622.6204    0.059   -0.065    -2.959   -0.000    0.000  -1.952   1947.4107    1912.8923        0.0258
                                        THIS RAY CALCULATION TOOK    10.736 SEC
```

```
X01  TEST CASE                                                                                                   11/05/74
 CHAPX    WAVE     DIPOLY    EXPZ2                        APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                   FREQUENCY =     6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                              ELEVATION ANGLE OF TRANSMISSION =   30.000000 DEG

                                           AZIMUTH
                                          DEVIATION          ELEVATION
                    HEIGHT      RANGE     XMTR   LOCAL     XMTR   LOCAL      POLARIZATION     GROUP PATH  PHASE PATH  ABSORPTION
                      KM          KM       DEG    DEG       DEG    DEG       REAL    IMAG        KM          KM          DB
 3-011 XMTR          0.0000      0.0000                            30.000   -0.000   1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION     52.9739     89.9010  -0.000  -0.000   30.000   30.609   -0.332   1.259    104.6686    104.6686      0.0000
 0+000 MIN DIST    191.5641    354.9408  -0.023   0.219   26.398    0.000    0.000  -1.530    425.7924    400.9614      0.0082
 0+000 MIN DIST    191.5641    354.9408  -0.023   0.219   26.398    0.000    0.000  -1.530    425.7924    400.9614      0.0082
 6-010 WAVE REV    191.4066    358.1065  -0.020   0.137   26.154   -1.395    0.000  -1.475    429.7924    403.6919      0.0085
 2-005 EXIT ION     48.7358    644.4661   0.357  -0.397    1.406  -28.975    0.035  -1.018    770.7924    720.5457      0.0177
-3-011 GRND REF      0.0000    733.6060   0.405  -0.349   -3.299   28.173   -0.000   1.000    872.6850    822.4383      0.0177
 0+000 ENTR ION     52.9731    830.3362   0.446  -0.308   -0.104   29.043   -0.277   1.223    983.3256    933.0789      0.0177
 0+000 MIN DIST    139.8217   1107.5272   0.514   0.454    4.581    0.000    0.000  -1.771   1312.8736   1240.1070      0.0259
                                     THIS RAY CALCULATION TOOK     8.113 SEC

                              ELEVATION ANGLE OF TRANSMISSION =   45.000000 DEG

                                           AZIMUTH
                                          DEVIATION          ELEVATION
                    HEIGHT      RANGE     XMTR   LOCAL     XMTR   LOCAL      POLARIZATION     GROUP PATH  PHASE PATH  ABSORPTION
                      KM          KM       DEG    DEG       DEG    DEG       REAL    IMAG        KM          KM          DB
 0+000 XMTR          0.0000      0.0000                            45.000   -0.000   1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION     52.9759     52.3246  -0.000  -0.000   45.000   45.471   -0.113   1.100     74.6128     74.6128      0.0000
-3-011 RCVR        200.0000    200.2014  -0.132   0.607   43.622   28.480   -0.000   2.293    295.1649    278.4398      0.0058
 2-005 APOGEE      209.6834    235.6629  -0.040  -1.746   40.109    3.110    0.000  -1.831    352.1649    306.3131      0.0098
 3-006 WAVE REV    209.4360    241.0969   0.021  -1.382   39.431   -0.266    0.000  -1.623    360.1649    310.1892      0.0103
 3-011 RCVR        200.0000    274.3788   0.389  -1.415   34.427  -23.456    0.000  -1.142    410.8543    336.8372      0.0137
 6-005 EXIT ION     38.9028    444.9524   0.574  -0.027    2.978  -44.472    0.004  -1.000    659.8543    566.5130      0.0206
 0+000 GRND REF      0.0000    484.7060   0.576  -0.025   -2.130   44.114   -0.000   1.000    715.5626    622.2214      0.0206
 0+000 ENTR ION     52.9777    634.6621   0.576  -0.022    3.168   44.599   -0.105   1.094    791.3392    697.9980      0.0206
-3-011 RCVR        200.0000    691.5163   0.546   0.796   12.769   26.953   -0.000   2.159   1015.3617    905.3752      0.0266
                                     THIS RAY CALCULATION TOOK     9.813 SEC
```

```
X01  TEST CASE                                                                                              11/05/74
 CHAPX    WAVE    DIPCLY    EXPZ2                     APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                  FREQUENCY =    6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                             ELEVATION ANGLE OF TRANSMISSION =   60.000000 DEG

                                            AZIMUTH
                                           DEVIATION        ELEVATION
                     HEIGHT      RANGE    XMTR   LOCAL     XMTR   LOCAL      POLARIZATION     GROUP PATH  PHASE PATH  ABSORPTION
                       KM         KM      DEG     DEG      DEG     DEG      REAL    IMAG         KM          KM          DB
 0+000 XMTR          0.0000     0.0000                            60.000   -0.000   1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION     52.9786    30.2936   -0.000  -0.000   60.000  60.272   -0.053   1.049     61.0908     61.0908      0.0000
 0+000 RCVR        200.0000   114.4182   -0.157  -0.851   59.325  50.578   -0.000   1.250    239.2552    225.6343      0.0050
-1-004 APOGEE      225.8382   146.6930   -0.354   5.402   55.872  -5.298    0.000  -2.002    324.2552    253.2681      0.0114
-1-004 WAVE REV    225.8382   146.6930   -0.354   5.402   55.872  -5.298    0.000  -2.002    324.2552    253.2681      0.0114
 0+000 RCVR        200.0000   168.1639   -1.383  16.876   48.744 -62.499    0.000  -1.025    395.1225    273.3158      0.0173
-2-005 EXIT ION     26.7751   230.9196   -6.447  13.564    5.561 -59.411    0.000  -1.000    588.1225    453.6124      0.0223
 0+000 GRND REF      0.0000   240.7118   -7.009  13.003   -1.083  69.320   -0.000   1.000    616.7330    482.2229      0.0223
-3-011 ENTR ION     52.9836   260.0662   -7.992  12.021   10.299  69.499   -0.026   1.025    673.3376    538.8275      0.0223
 0+000 RCVR        200.0000   313.9111  -10.115   8.555   30.682  63.564   -0.000   1.099    837.7013    691.0808      0.0271
                                   THIS RAY CALCULATION TOOK     7.518 SEC

                             ELEVATION ANGLE OF TRANSMISSION =   75.000000 DEG

                                            AZIMUTH
                                           DEVIATION        ELEVATION
                     HEIGHT      RANGE    XMTR   LOCAL     XMTR   LOCAL      POLARIZATION     GROUP PATH  PHASE PATH  ABSORPTION
                       KM         KM      DEG     DEG      DEG     DEG      REAL    IMAG         KM          KM          DB
 0+000 XMTR          0.0000     0.0000                            75.000   -0.000   1.000      0.0000      0.0000      0.0000
 0+000 ENTR ION     52.9846    14.0759   -0.000  -0.000   75.000  75.127   -0.024   1.023     54.8374     54.8374      0.0000
 0+000 RCVR        200.0000    52.6875   -0.302   1.334   74.783  71.357   -0.000   1.087    213.9181    202.1284      0.0046
 1-004 APOGEE      230.9183    67.5504   -1.511  -8.853   73.111   8.237    0.000  -3.876    314.9181    220.4673      0.0134
 9-005 WAVE REV    230.2593    70.7691   -0.322 -11.181   72.307  -4.474    0.000  -1.950    330.9181    221.5782      0.0147
 0+000 RCVR        200.0000   106.6297    5.511 -13.543   61.084 -46.475    0.000  -1.024    434.9673    249.2747      0.0232
-1-006 EXIT ION     29.3420   220.9908   12.392  -6.856    6.551 -55.885    0.000  -1.000    651.9673    450.5600      0.0292
 0+000 GRND REF      0.0000   240.7589   13.557  -6.291   -1.083  55.706   -0.000   1.000    687.4459    486.0386      0.0292
 0+000 ENTR ION     52.9836   276.3403   14.370  -5.479    9.565  56.027   -0.069   1.064    751.4563    550.0489      0.0292
-3-011 RCVR        200.0000   375.3507   15.794  -3.644   25.987  45.376   -0.000   1.374    937.9577    722.8911      0.0343
                                   THIS RAY CALCULATION TOOK     9.049 SEC
```

```
X01  TEST CASE                                                                                          11/05/74
 CHAPX     WAVE    DIPOLY    EXPZZ                          APPLETON-HARTREE FORMULA EXTRAORDINARY WITH COLLISIONS

                  FREQUENCY =    6.000000 MHZ, AZIMUTH ANGLE OF TRANSMISSION =   45.000000 DEG

                           ELEVATION ANGLE OF TRANSMISSION =   90.000000 DEG

                                          AZIMUTH
                                         DEVIATION          ELEVATION
                      HEIGHT     RANGE    XMTR    LOCAL     XMTR   LOCAL     POLARIZATION     GROUP PATH  PHASE PATH  ABSORPTION
                        KM        KM      DEG      DEG      DEG     DEG      REAL    IMAG        KM          KM           DB
 0+000 XMTR           0.0000    0.0000                              90.000  -0.000   1.000      0.0000      0.0000      0.0000
-3-011 ENTR ION      52.9908    0.0000                     90.000   90.000  -0.009   1.009     52.9908     52.9908      0.0000
-3-011 MAX LAT       54.9908    0.0000                     90.000   90.000  -0.003   1.012     54.9908     54.9908      0.0000
-6-011 WAVE REV      69.2408    0.0000                     90.000   90.000  -0.002   1.019     69.2408     69.2408      0.0000
-3-009 WAVE REV     119.7355    0.0003 -145.262    0.004   90.000   90.000  -0.000   1.018    119.7408    119.7331      0.0010
 2-007 WAVE REV     175.1532    0.1157 -145.266-180.000    89.961   89.993  -0.000   1.021    176.7408    173.9276      0.0029
 0+000 RCVR         200.0000    0.3028 -145.266-180.000    89.911   88.779  -0.000   1.030    207.1658    194.9665      0.0047
 4-006 WAVE REV     230.2036    0.9818 -145.266    0.000   89.747   89.207  -0.000   1.052    262.1658    213.5202      0.0089
-3-003 WAVE REV     235.2305    3.6109 -145.266    0.000   89.039  -11.160  -0.000   4.505    310.1658    215.7522      0.0127
-2-003 APOGEE       238.0027    4.0933 -145.265   -0.000   88.978  -33.291   0.000  -5.606    318.1658    215.8346      0.0134
-6-011 RCVR         200.0000   15.3973 -145.266    0.000   85.450  -75.213   0.000  -1.080    411.0570    238.1693      0.0205
 2-005 EXIT ION      25.6746   48.6937 -145.266    0.000   27.535  -78.963   0.001  -1.000    596.0570    410.5365      0.0252
-3-011 GRND REF       0.0000   53.7019 -145.266    0.000   -0.242   78.918  -0.003   1.009    622.2176    436.6971      0.0252
-3-011 ENTR ION      52.9956   63.9944 -145.266    0.000   39.224   79.011  -0.003   1.003    676.2116    490.6911      0.0252
 0+000 RCVR         200.0000   92.7183 -145.266    0.000   64.368   77.233  -0.000   1.006    833.2790    635.5670      0.0300
                                   THIS RAY CALCULATION TOOK    13.161 SEC
```

Appendix 8d. Listing of Punched Card Output (ray sets) for Sample Case

```
X01  TEST CASE
 CHAPX     6.500+000 3.000+002 6.200+001 5.000-001 0.000+000 0.000+000 0.000+000
 WAVE      2.500+002 1.000+002 1.000-001 0.000+000 1.000+002 1.000+002 0.000+000
DIPOLY     8.000-001 0.000+000 0.000+000 0.000+000 0.000+000 0.000+000 0.000+000
 EXPZ2     3.650+004 1.000+002 1.480-001 3.000+001 1.400+002 1.830-002 0.000+000
X01X         0 40000255000 2000000     60000   4500000         0        0 -1003T
  1581469 14911561     2    -29     0 1514389  4513 -1610   11     0    0 -1721M
  1581469 14911561     2    -29     0 1514389  4513 -1610   11     0    0 -1722M
  1581469 29000482    -0     10   738 2875068 80425 68182   22     0    0 -1003G
  1579016 43050842    -7     76    -0 4278561109482 91246   33     0   -0  9323M
X01X         0 40000255000 2000000     60000   4500000   1500000        0 -1003T
  1721392  6041034    15    184     0  635731  8122 -3264    8     0    0 -1481M
  1721392  6041034    15    184     0  635731  8122 -3264    8     0    0 -1482M
  1721418 12129251    46    -23 14656 1211094 81100 58053   17     0   -0  1003G
  1719566 18286204    59    -65    -0 1854769 92641 58123   26     0    0 -1953M
X01X         0 40000255000 2000000     60000   4500000   3000000       -0  1003T
  1915641  3549408   -23    219     0  407964 17828 -7003    8     0    0 -1531M
  1915641  3549408   -23    219     0  407964 17828 -7003    8     0    0 -1532M
  1916346  7336080   405   -349 28173  733203139482 89236   18     0   -0  1003G
  1898217 11075272   514    454     0 1138430174444101677   26     0    0 -1773M
X01X         0 40000255000 2000000     60000   4500000   4500000       -0  1003T
        0  2002014  -132    607 28480  285194  9971 -6754    6     0   -0  2291R
  2096843  2743788   389  -1415-23456  342980 67874 -6143   14     0    0 -1142R
  2096843  4847060   576    -25 44114  484589230974137632   21     0   -0  1003G
        0  6915163   548    796 26953  729880285982175495   27     0   -0  2163R
X01X         0 40000255000 2000000     60000   4500000   6000000       -0  1003T
        0  1144182  -157   -851 50578  231305  7950 -5671    5     0   -0  1251R
  2258382  1681639 -1383  16876-62499  262993132130 10323   17     0    0 -1032R
  2258382  2407118 -7009  13003 69320  240697376036241525   22     0   -0  1003G
        0  3139111-10115   8555 63564  376316461386314765   27     0   -0  1103R
X01X         0 40000255000 2000000     60000   4500000   7500000       -0  1003T
        0   526875  -302   1334 71357  207034  6884 -4906    5     0   -0  1091R
  2309183  1066297  5511-13543-46475  227435207533 21840   23     0    0 -1022R
  2309183  2407589 13557  -6291 55706  240745446701245294   29     0   -0  1003G
        0  3753507 15794  -3644 45376  430430507528292462   34     0   -0  1373R
X01X         0 40000255000 2000000     60000   4500000   9000000       -0  1003T
        0      3028214734180000 88779  200000  7166 -5034    5     0   -0  1031R
  2382305   153973214734      0-75213  200610210447 37559   20     0    0 -1082R
  2382305   537019214734      0 78918   53702568516382995   25     0   -0  1003G
        0   927180214734      0 77233  221057612222414510   30     0   -0  1013R
```

The first card is the title card.

The second card contains the name of the electron density model plus parameters W101-W107.

The third card contains the name of the perturbation model plus parameters W151-W157.

The fourth card contains the name of the magnetic field model plus parameters W201-W207.

The fifth card contains the name of the collision frequency model plus parameters W251-W257.

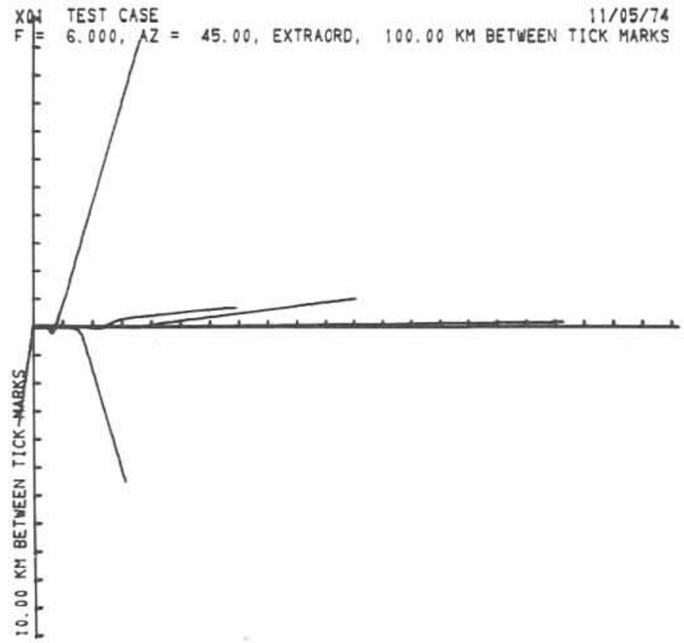
For description of remaining cards, see figures 1 and 2.

Appendix 8e. Ray Path Plots for Sample Case

Projection of raypath on vertical plane

```
X01  TEST CASE                                                     11/05/74
F =  6.000, AZ =  45.00, EXTRAORD,  100.00 KM BETWEEN TICK MARKS
```

Projection of raypath on ground for sample case



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14. SUPPLEMENTARY NOTES

15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This report describes an accurate, versatile FORTRAN computer program for tracing rays through an anisotropic medium whose index of refraction varies continuously in three dimensions. Although developed to calculate the propagation of radio waves in the ionosphere, the program can be easily modified to do other types of ray tracing because of its organization into subroutine.

The program can represent the refractive index by either the Appleton-Hartree or the Sen-Wyller formula, and has several ionospheric models for electron density perturbations to the electron density (irregularities), the earth's magnetic field and electron collision frequency.

For each path, the program can calculate group path length, phase path length, absorption, Doppler shift due to a time-varying ionosphere, and geometrical path length. In addition to printing these parameters and the direction of the wave normal at various points along the ray path, the program can plot the projection of the ray path on any vertical plane or on the ground and punch the main characteristics of each ray path on cards.

The documentation includes equations, flow charts, program listings with comments, definitions of program variables, deck set-ups, description of input and output, and a sample case.

KEY WORDS:
Appleton-Hartree formula; computer program; ionosphere; radio waves; ray tracing; Sen-Wyller formula; three-dimensional.

17. AVAILABILITY STATEMENT

☒ UNLIMITED.

☐ FOR OFFICIAL DISTRIBUTION.

18. Security Class *(This report)*
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OT REPORT 75-76

A VERSATILE THREE-DIMENSIONAL RAY TRACING COMPUTER PROGRAM FOR RADIO WAVES IN THE IONOSPHERE

by R. Michael Jones and Judith J. Stephenson

The 8th line from the bottom of page 9 should read:

(1) the dispersion relation cannot be exactly satisfied, or

The first line in the 3rd complete paragraph on page 12 should read:

Similarly, the AHWFNC (Appleton-Hartree, with field, no colli-

Line PRIN125 in SUBROUTINE PRINTR on page 80 should read:

```
      RANGE=EARTHR*ATAN2(RCE,EARTHR+EPS+XMTRH)                   PRIN125
```

Line BQNC020 in SUBROUTINE BQWFNC on page 100 should read:

```
      REAL N2,NNP,LPOLAR,LPOLRI,KR,KTH,KPH,K2,KDOTY,K4,KDOTY2,   BQNC020
```

Line TABX064 in SUBROUTINE TABLEX on page 112 should read:

```
      PXPR=PXPTH=PXPPH=0.                                        TABX064
```

Following line CHAP024 in SUBROUTINE CHAPX on page 116, insert the line:

```
      PXPPH=0.                                                   CHAP0245
```

Line VCHA010 in SUBROUTINE VCHAPX on page 117 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                      VCHA010
```

Line DCHA014 in SUBROUTINE DCHAPT on page 119 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                      DCHA014
```

Line LINE013 in SUBROUTINE LINEAR on page 120 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                      LINE013
```

Line PARA012 in SUBROUTINE QPARAB on page 121 should read:

```
      X=PXPR=PXPTH=PXPPH=0.                                      PARA012
```

Following line BULG038 in SUBROUTINE BULGE on page 123, insert the line:

```
      PXPPH=0.                                                   BULG0385
```

Following line EXPX014 in SUBROUTINE EXPX on page 124, insert the line:

```
      PXPTH=PXPPH=0.                                             EXPX0145
```

The equation for the gyrofrequency near the top of page 143 should read:

$$F_H = F_{H_o}(R_o/(R_o + h))^3(1 + 3\cos^2\theta)^{1/2}$$

where θ is the geomagnetic colatitude.

Line TABZ015 in SUBROUTINE TABLEZ on page 153 should read:

```
      IF (READNU.EQ.0.) GO TO 10                                 TABZ015
```