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# Spectrum Efficiency for Multiple Independent Spread-Spectrum Land Mobile Radio Systems

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# PREFACE

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## TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

# SPECTRUM EFFICIENCY FOR MULTIPLE INDEPENDENT SPREAD-SPECTRUM LAND MOBILE RADIO SYSTEMS

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The use of spread-spectrum modulation in a common type of land-mobile radio service is studied. The radio service contains many networks operated independently, similar to the business land-mobile radio service. Network independence implies (among other things) that there is no central control of a transmitter's radiated power. A congested urban environment is assumed.

If a band is allocated exclusively to spread-spectrum modulation for such a service, base stations and mobiles must operate on different channels, and all base station antennas must be located within a relatively small area. In this case, the worst interference is in the mobile-to-base channel.

An explicit formulation of the "far-near" problem and a powerful computer calculation both show that in such a service spread-spectrum modulation is less spectrum efficient than conventional FM modulation. In fact, the spectrum efficiency of the spread-spectrum systems is inversely proportional to the cube root of the processing gain. However, overlaying a spread-spectrum system on conventional bands shows some promise.

Key words: Land mobile radio; spectrum efficiency; spread spectrum.

## 1. INTRODUCTION

With increased demand for land-mobile radio (LMR) services, the Federal Communications Commission (FCC) has had to increase the communications capacity of the LMR services. They have split channels and added new spectrum bands. (The newest band is the one between 806 and 947 MHz added by Docket #18262.) The FCC is now investigating the spectrum efficiency of different technologies and modulation techniques for delivering the service. An example is the "narrowband technology" described by Lusignan (1978) and discussed in more detail by Wilmotte and Lusignan (1978).

Another technique is suggested by Cooper and Nettleton (1977), by Utlaut (1978), and by the CCIR (1978a). This technique uses spread-spectrum (SS) modulation--a technique often used to overcome jamming and to hide covert signals below conventional signals or noise (Dixon, 1976).

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Appendix B of this report gives a short introduction to spread-spectrum techniques in the context of land-mobile radio service. Readers unfamiliar with spread-spectrum techniques should read Appendix B first. More complete and general discussions are given by Dixon (1976), Utlaut (1978), and the CCIR (1978a). Haakinson (1978) gives an annotated bibliography on the subject.

Utlaut quotes Costas (1959) as showing that, under certain assumptions, wideband systems should be more spectrum efficient than narrowband systems. One assumption is that each user transmits only a small fraction of the time--a situation qualitatively similar to some business LMR services. In principle, each SS user would have instant access to the shared, wideband channel without disabling interference.

This exciting possibility was investigated in the study reported here. The study is limited to services containing multiple independent networks in an urban environment. Network independence implies that there is no dynamic central control of any transmitter's radiated power and that the transmission times of different networks are independent. We assume that the processing gain of a spread-spectrum system is a reasonable upper bound for the actual performance gain. This is equivalent to assuming that potential gains from nearly orthogonal codes or (for frequency hoppers) good filters are approximately balanced by implementation losses. To the extent that this assumption is valid, the conclusions apply to all spread-spectrum systems, but the validity is easier to demonstrate for direct sequence systems. A probabilistic analysis of an LMR service using SS modulation exclusively was reinforced by a more realistic computer calculation. This analysis is shown in Sections 2 and 3. For these conditions, we conclude that spread-spectrum modulation would not allow as many users with a specified reliability as conventional FM modulation in the same total bandwidth.

Another possible use of spread-spectrum systems, suggested by their resistance to jamming and the low power density of their emissions, is to overlay them on conventional service bands (Utlaut, 1978). This possibility is investigated in Section 4. There is some potential for this application.

# 2. INTERFERENCE AND SPECTRUM EFFICIENCY OF MULTIPLE INDEPENDENT NETWORKS IN AN EXCLUSIVE SPREAD-SPECTRUM LAND-MOBILE RADIO BAND

In this section we analyze the mutual interference between independent LMR networks operating in a band allocated exclusively to similar spread-spectrum radio systems. The paradigm is the business land-mobile service. We assume that each network has a base station and several (or many) mobiles communicating with that base station, and that the networks operate independently of each other. The spectrum efficiency of such networks is compared with the spectrum efficiency for similar networks using conventional FM LMR systems.

The general form of the definition of spectrum efficiency is (Berry, 1977b; CCIR, 1978b)

$$\frac{\text{communications achieved}}{\text{spectrum space used}} \; .$$

For land-mobile radio, we take this to mean

$$\frac{\text{number of users achieving specified service quality and reliability}}{\text{bandwidth x area x time denied to other uses}} \; .$$

In actual calculations, we fix the area and time denied to others. In some comparisons, we also fix the specified service quality and the bandwidth and compare the number of users achieving the desired service. In this case, the ratio of the efficiencies of two services is the ratio of the number of users in the two services. In other comparisons, we fix all variables except bandwidth and compare the bandwidths necessary to achieve the services. In this case the ratio of the efficiencies is the ratio of the bandwidths.

First, we derive simple analytical formulas for the interference in spread-spectrum bands. The implications of the formulas allow us to limit the number of ways that the service might be configured to duplex systems with base stations located in close proximity to each other. Then the spectrum efficiency of spread-spectrum systems relative to conventional FM duplex systems is calculated with a computer program implementing a probabilistic model of radio system interaction. For all cases evaluated, the FM systems are more spectrum efficient than the spread-spectrum systems.

We assume that the LMR systems operate in an urban environment because that is where the greatest need for spectrum efficiency is. We assume that the radio frequencies are between about 150 MHz and 900 MHz; so our propagation model is an approximation to that suggested by Okumura et al. (1968) and quoted by Jakes (1974). The model assumes that the base station antenna is high (greater than 30 m), and the mobile antenna is low--generally less than 4 m.

The formula for median received power is

$$P_r(\text{dBW}) = \text{ERP}(\text{dBW}) - [33.2 + 20 \log f(\text{MHz}) + 20 \log d(\text{km}) - G_r] \qquad (1)$$
$$- [G + H \log d(\text{km})] \quad ,$$

where ERP is the effective radiated power of the transmitter and $G_r$ is the receiving antenna gain.

The quantity in the first brackets is the free space transmission loss and the term in the second brackets is the "urban loss." The constants G and H depend on frequency, antenna heights, and path lengths. Table 1 shows these constants for several values of these parameters.

Table 1. Constants for Urban Propagation Loss Given in Equation 1
(Base station antenna height is 200 m; mobile antenna height is 1.5 m.)

| | Frequency | G | H |
|---|---|---|---|
| $d \leq 30$ km | 150 MHz | 19 | 10 |
| | 450 MHz | 20 | 10 |
| | 900 MHz | 23 | 10 |
| $30 < d \leq 100$ km | 150 MHz | -25.6 | 40.3 |
| | 450 MHz | -31.7 | 45.35 |
| | 900 MHz | -37.75 | 50.35 |

For a fixed pair of antenna heights, fixed antenna gains, and fixed frequency, note that (1) has the form

$$P_r(\text{dBW}) = A - B \log d \quad , \qquad (2)$$

where B = 20 + H. For path lengths less than 30 km, which includes most paths of interest in a metropolitan area, B = 30. This form is useful for approximate analytical analysis.

The distribution of the received power around the median is assumed to be log normal. Actually, as explained in Appendix B, the short-term fading has a Rayleigh distribution, and the long-term fading is log normal. To simplify initial analysis, the distributions are combined and assumed to have a log normal distribution.

### 2.1 Why Multiple Independent SS Networks Must Be Duplex

The following derivations apply generally to co-channel operation, but the specific parameter values assumed to get numerical results are appropriate for SS systems in urban areas.

In a half-duplex channel (used for transmitting and receiving by both mobile and base stations), the worst interference comes from a base station to another base station trying to receive a transmission from a mobile. This situation is illustrated in Figure 1, where mobile radio $M_1$ is transmitting the desired signal, S, to its base station $T_1$, which is $d_1$ km away. At the same time, another base station, $T_2$, $d_2$ km from $T_1$, is transmitting to its network. Transmitter $T_2$'s signal, I, is interference to the $M_1$ - $T_1$ link.

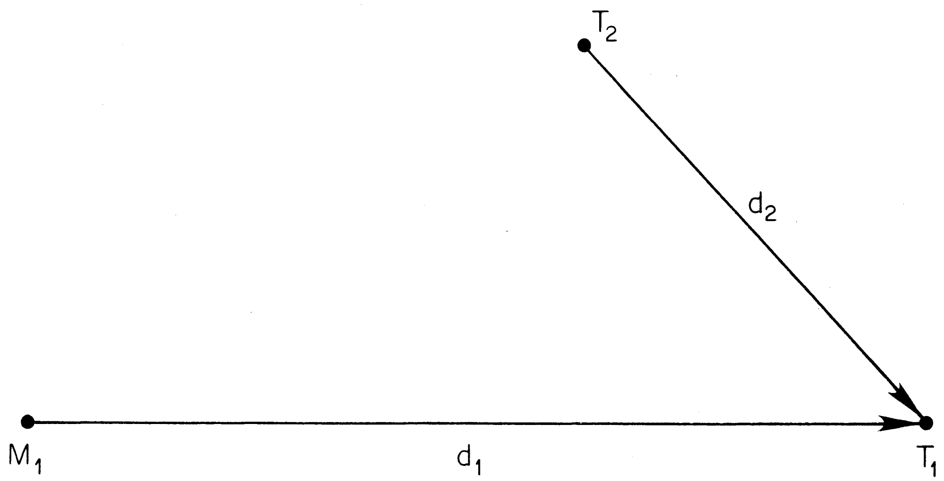

Figure 1. Geometry for analysis of base-to-base station interference to a mobile-to-base transmission. A base station at $T_1$ is trying to receive a mobile station at $M_1$ while a base station at $T_2$ is transmitting.

Now, using (2)

$$S = A_1 - B_1 \log d_1, \text{ dBW}, \tag{3}$$

and

$$I = A_2 - B_2 \log d_2, \text{ dBW} . \tag{4}$$

In general, $A_1 \neq A_2$ because of different ERP of mobile and base stations and different antenna heights, and possibly $B_1 \neq B_2$, because of different path characteristics. Let C be the signal-to-interference (S/I) ratio (in decibels) required at the terminals of $T_1$'s receiving antenna in order to produce the desired output S/I. That is, we require that

$$S - I \geq C \quad . \tag{5}$$

Substituting from (3) and (4) yields

$$(A_1 - A_2) - B_1 \log d_1 + B_2 \log d_2 \geq C \quad ,$$

or

$$\log d_2 - \frac{B_1}{B_2} \log d_1 \geq \frac{C+A_2-A_1}{B_2} \quad , \quad \text{dBW} \quad . \tag{6}$$

Transform from decibels above a watt to watts and rearrange to get

$$d_2 \geq \left(d_1^{\,B_1/B_2}\right) 10^{(C+A_2-A_1)/B_2} \quad . \tag{7}$$

Equation (7) is an explicit formula for the (deterministic) "near-far problem" (Dixon, 1976).

The distance $d_2$ is the separation required between two independent base stations if $T_1$ is to be able to hear its mobiles at distance $d_1$ in an interference-limited environment. For conventional LMR systems, this distance is sometimes called the frequency re-use distance.

Suppose that an SS LMR system requires an output S/I of 10 dB and has 30 dB processing gain. Then C = -20 dB. Assume the bases and the mobile have equal effective radiated power and equal antenna gains, that the mobile antenna is 1.5 m above ground, and the base station antennas are 200 m above ground. Because the antennas of $T_1$ and $T_2$ are so high, we can assume free-space transmission between them; that is, $G = H = 0$, so $A_2 = \text{ERP} + G_r - 33.2$ and $B_2 = 20$. Using the constants in Table 1 for a frequency of 450 MHz,

$A_1 = ERP + G_r - 33.2 + 20$, and $A_2 - A_1 = 20$. Also, $B_1 = 20 + 10 = 30$. Substituting these values into (7) yields

$$d_2 \geq d_1^{\frac{3}{2}} \cdot 10^{(-20+20)/20} = d_1^{\frac{3}{2}} \quad . \tag{8}$$

If $T_1$ wants to receive messages from its mobile when the mobile is 10 km away, then $T_2$ must be 31.6 km away. In most metropolitan areas, this would mean that not even two base stations could operate simultaneously. But 30 dB of processing gain for a 3 kHz voice signal takes at least 3 MHz bandwidth. One LMR net that requires 3 MHz of exclusive bandwidth in a city is certainly not spectrum efficient!

So increase the bandwidth to 30 MHz and assume the processing gain is 40 dB. Then

$$d_2 \geq d_1^{\frac{3}{2}} \cdot 10^{-\frac{1}{2}} \approx .32\, d_1^{\frac{3}{2}} \quad .$$

If we set $d_1 = 10$ km, then $d_2 \geq 10$ km. For this case, there could possibly be several base stations (LMR networks) in the same metropolitan area. But there could be $\frac{30\ 000}{25} = 1200$ conventional LMR networks (with 25 kHz channel spacing); each with an exclusive channel. If intermodulation and adjacent channel interference allowed only half of these channels to be used, there could still be 600 FM networks. The spread-spectrum half-duplex systems clearly would be very inefficient spectrum users.

Consider the effect of different assumptions. It is likely that the transmitter power plus antenna gain of base stations will be greater than that of mobiles. This would increase $A_2 - A_1$ and hence would increase the required separation $d_2$. On the other hand, it is possible that base-to-base transmission is not quite as good as free space. As a limiting condition, assume that base-to-base signals attenuate as much with distance as mobile-to-base signals do; that is, that $B_2 = 30$.

Then, for 30 dB processing gain,

$$d_2 \geq d_1 \quad .$$

If $d_1 = 10$ km, only a few SS networks could operate simultaneously in a city compared to 120 conventional LMR networks; each with an exclusive channel.

The discussion above assumes that an interfering base station is on nearly all the time. Base stations do transmit a substantial fraction of the time. If there are several base stations, the probability of none transmitting at a given time (no interference) is unacceptably small; so the analysis above still applies.

The conclusion is that if LMR mobile and base stations share the same spread-spectrum band, no more than a few independently controlled networks can operate satisfactorily in a city, even though they occupy enough bandwidth for hundreds of conventional LMR networks. Therefore, independent SS LMR systems must have separate frequency bands for base and mobile stations. From here on, we assume SS LMR systems are duplex and compare them with conventional duplex FM systems.

Notice that spread-spectrum systems have been represented entirely by their processing gain. We have ignored implementation losses and gains achieved with nearly orthogonal codes or filters. Because these two factors work in opposite directions, they partially cancel each other, and the conclusion above is probably valid for both direct sequence and frequency hopping SS systems.

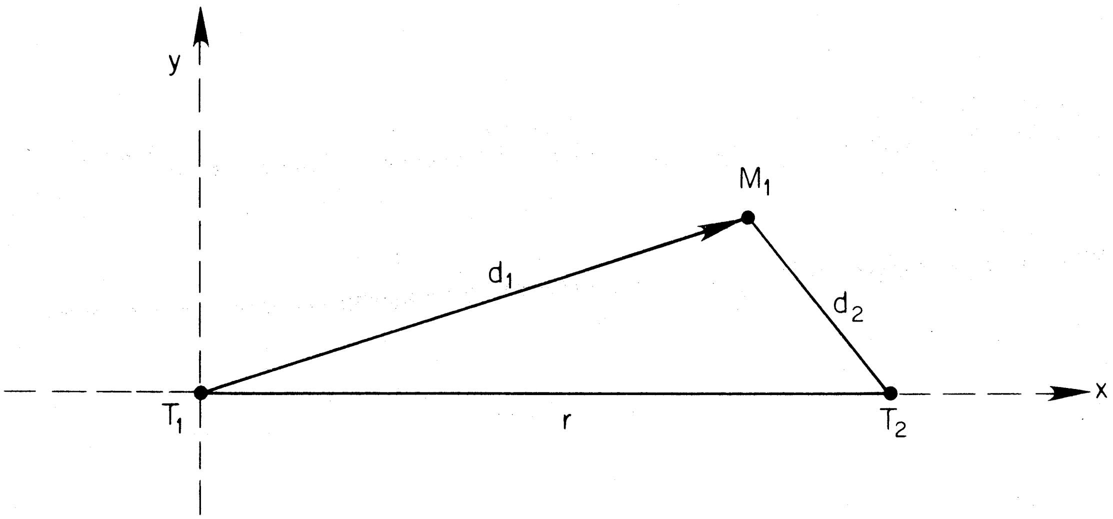

Figure 2. Geometry for analysis of base station interference to a base-to-mobile transmission. A mobile at $M_1$ is trying to receive base station $T_1$ while base station $T_2$ is transmitting.

## 2.2 Why SS LMR Base Stations Must Be Located Close Together

Consider a wanted, base-to-mobile transmission in a duplex system. Interference comes from other bases sharing the same channel, as illustrated in Figure 2, where the two base stations, $T_1$ and $T_2$, are r km apart. The wanted signal from $T_1$ to $M_1$ has power

$$S = A_1 - B \log d_1 \quad . \qquad \text{dBW}$$

The interference is

$$I = A_2 - B \log d_2 \quad . \qquad \text{dBW}$$

As before, assume that at the antenna terminals of $M_1$ we require that $S - I \geq C$. Then

$$A_1 - A_2 - B \log d_1 + B \log d_2 \geq C$$

or

$$\frac{d_1}{d_2} \geq 10^{(C + A_2 - A_1)/B} = R \quad . \qquad (9)$$

If $R = 1$, $d_1/d_2 = 1$ is the perpendicular bisector of $\overline{T_1T_2}$. Performance is unacceptable ($S - I < C$) to the right of this locus. As Freeman (1972) shows, if $R \neq 1$, $d_1/d_2 = R$ defines a circle with radius

$$r \left| R/(R^2 - 1) \right| \quad . \qquad (10)$$

If $T_1$ and $T_2$ are on the x-axis of an (x,y) coordinate system with $T_1$ at the origin, the center of the circle is at $(r/(1 - R^2), 0)$. If $R^2 < 1$, as it usually is for SS systems, the circle contains $T_2$, and the region of unacceptable interference ($S - I < C$) is inside the circle. If $R^2 > 1$, the circle contains $T_1$ and $S - I < C$ outside the circle. This is the conventional LMR case in an interference-limited environment ($I >> N$).

Assume identical base stations so that $A_1 = A_2$. For paths less than 30 km, B = 30 (from Table 1). If the processing gain is 30 dB and the required output S/I is 10 dB, then C = -20 dB and $R = 10^{-2/3} \approx 0.2154$. Then the radius of the denied circle is 0.226 r. If $T_2$ is 10 km from $T_1$, then mobile stations cannot receive messages from $T_1$ inside a circle containing

$T_2$ with radius 2.26 km. Station $T_2$ denies a significant area in the city to $T_1$'s mobiles.

An exclusive spread-spectrum LMR band would have many networks and many base stations sharing the band, each one denying an area to $T_1$. If these bases are scattered randomly over the city, the total area denied to the $T_1$ network is clearly unacceptable.

However, suppose that $r = 0.1$ km. Then the radius of the denied area is only 23 m--a small spot very near $T_1$. If all the base station antennas are located on the same tall building or tower, then all the denied areas would overlap the area in proximity to the tower. This situation might be acceptable. We evaluate this possibility in Section 3.

Under the same conditions, but with a processing gain of 40 dB, the radius of the denied circle is only 0.0226 r and the denied area is $5.11 (10^{-4})\ r^2\ \mathrm{km}^2$. This might be acceptable even for $r = 10$ km.

However, there must be many networks assigned to the band to use this much bandwidth ($\geq$ 30 MHz) efficiently. A large number of stations like $T_2$ randomly distributed over the area causes these effects:

(a) Interference from many sources at the mobile receiver increases the denied area and distorts its shape.

(b) Some of the denied areas overlap and should not be counted twice in a calculation of total denied area.

These two effects work in opposite directions; so for a crude estimate of total denied area, assume that they cancel each other. Then the denied area is about $K \cdot 5.11 (10^{-4})\ \bar{r}^2\ \mathrm{km}^2$, where K is the number of interfering stations and $\bar{r}$ is the average separation of the stations. As shown in the previous section, $K > 1000$ for efficient use of 30 MHz of bandwidth. For points randomly distributed in a circle with radius 20 km, $\bar{r} \approx 18$ km (Crow, 1972), and the denied area is about 166 $\mathrm{km}^2$ or 13 percent of the area of the circle. This is probably unacceptable.

However, if all base station antennas are confined to a small area, $\bar{r}$ is much smaller, and the denied areas effectively coincide. The conclusion is that in an exclusive SS LMR band with multiple independent networks, base station antennas must be effectively co-located.

## 2.3 Approximate Analysis of Interference When Base Station Antennas Are Co-located

### 2.3.1 Base-to-Mobile Transmissions

Fading of a signal received at a mobile from a high base-station antenna is due primarily to scattering from objects near the mobile (Jakes, 1974). If the antenna for the wanted transmitter is sufficiently near the antenna(s) for the interfering transmitter(s), the signal and interference will fade together. If all transmitters have the same power and radiate from the same height, and if there are N interferers, then the signal-to-interference ratio is 1/N, or $-10 \log_{10} N$ dB.

If a spread-spectrum system has 30 dB processing gain and requires 10 dB output S/I, then the minimum input S/I = -20 dB. This implies that 100 base stations could be transmitting simultaneously from the same location without causing unacceptable interference at a mobile. Because each base station would be silent part of the time (when listening to a mobile), an even greater number of networks could operate satisfactorily.

By comparison, the 3 MHz of bandwidth necessary (at least) for a processing gain of 30 dB would hold 120 25-kHz FM channels. For this situation, the number of networks is roughly comparable. A more refined analysis that accounts for the channel usage statistics would be necessary to compute which modulation is more spectrally efficient. This analysis will not be pursued because mobile-to-base transmissions limit the number of SS networks that can operate.

### 2.3.2 Mobile-to-Base Transmissions

In duplex systems, a mobile-to-base transmission is interfered with by simultaneous transmissions from other mobiles. If all base stations are co-located or proximate, then it is likely that some of the interfering mobiles will be closer than the wanted mobile to the base stations. Interferers close to the receiver have a disproportionate effect on signal quality because of the rapid attenuation of signal strength with distance. This is called the "near-far problem" (Dixon, 1976). An approximate analysis of how this interference affects the spectrum efficiency of spread-spectrum systems follows.

Assume that:

(a) All base stations are located at the center of a circle of radius a that encloses the metropolitan service area.

(b) All antennas are omnidirectional and are at the same height, and all transmitters have the same power.

(c) Mobile units of all networks are located randomly in the circle, so that the probability that an area contains a mobile is proportional to the size of the area.

(d) The propagation law is $P_r = K/d^{\alpha}$, where $P_r$ is the received power in watts, K is a constant, d is the distance from transmitter to receiver, and $\alpha$ is a constant. (This is the power form of equation (2).) For transmission between one high (base) antenna and one low (mobile) antenna in a builtup urban area, $\alpha \approx 3$ (Jakes, 1974).

(e) The total interference from n interferers is caused primarily by the nearest interferer, so that the total interference can be approximated by a constant times the interference from the nearest interferer. (See Appendix A for justification of this assumption.)

(f) Networks operate independently.

Assumptions (a) and (c) imply that the expected value of the distance to the nearest of N interferers is $Q/\sqrt{N}$, where Q is a constant (see Appendix A). Combining this with assumption (d), we find that the interference from the expected distance to the nearest interferer is

$$I_1 \approx \frac{K}{(Q/\sqrt{N})^{\alpha}} = \frac{K}{Q^{\alpha}} N^{\frac{\alpha}{2}} . \tag{11}$$

Equation (11) and assumption (e) imply that the total interference is just a constant times this;

$$I_{total} = V N^{\frac{\alpha}{2}} , \tag{12}$$

where V is the combination of all constants.

Now assume that in situation a we have an exclusively spread-spectrum LMR service that is fully loaded--there are so many users that performance is at the threshold of unacceptability. The users achieve the required input signal-to-interference ratio $(S/I_a)$ the required percentage of the time, but no more. In this situation, the SS systems have rf bandwidth $B_a$, and there are an average of $N_a$ simultaneous interferers. (The total number of users is assumed to be proportional to $N_a$.)

We now compute the relative spectrum efficiency of a spread-spectrum service with greater bandwidth, $B_b$. Let

$$B_b = F\,B_a \quad , \tag{13}$$

where $F > 1$, and increase the number of users to the threshold of unacceptability. Because the processing gain of spread-spectrum systems increases linearly with bandwidth, this will happen when

$$(S/I_b) = \frac{B_a}{B_B}\,(S/I_a) = (S/I_a)/F \quad . \tag{14}$$

Here $I_b$ is the total interference from $N_b$ simultaneous interferers in situation b. We have assumed that the average signal strength, S, is the same in both cases, because the coverage area and the transmitter power remain constant. Substituting from (12) into (14) and simplifying yields

$$\frac{1}{N_b^{\alpha/2}} = \frac{1}{F\,N_a^{\alpha/2}} \quad , \tag{15}$$

or

$$\frac{N_b}{N_a} = F^{\frac{2}{\alpha}} \quad . \tag{16}$$

Because the service area and time have been kept constant, the relative spectrum efficiency is given by the relative number of users per unit bandwidth. Rewrite (16) as

$$\frac{\left(\frac{N_b}{B_b}\right)}{\left(\frac{N_a}{B_a}\right)} = \frac{B_a}{B_b}\,F^{\frac{2}{\alpha}} = \frac{F^{2/\alpha}}{F} = \frac{1}{F^{1-2/\alpha}} \quad . \tag{17}$$

Since $F > 1$ and $\alpha > 2$, $F^{1-2/\alpha} > 1$. Therefore,

$$\left(\frac{N_b}{B_b}\right) = \left(\frac{N_a}{B_a}\right)\frac{1}{F^{1-2/\alpha}} < \left(\frac{N_a}{B_a}\right) \quad . \tag{18}$$

Equation (18) says that the number of users per megahertz getting a specified reliability decreases as the bandwidth of the spread-spectrum system increases.

This result applies only under the listed assumptions. The assumptions approximate an LMR service with multiple independent networks operating in a terrestrial area of fixed size. The result says nothing about LMR services with central control of mobile transmitter power or about geostationary satellite-to-earth systems (in which all paths are effectively the same length). In addition, some of the assumptions are over-simplified. For example, assumption (d) gives a single value for the power received from a given distance, when in fact, that power has a statistical distribution. Also, not all transmitters will have the same power. The next section describes more realistic computer calculations of the spectrum efficiency of SS LMR systems.

## 3. COMPUTER CALCULATION OF SPECTRUM EFFICIENCY FOR SS LMR

In this section, we calculate more realistically the performance of a duplex SS system with all base stations near the center of the service area. As shown in the previous section, interference will be worse in the mobile-to-base channel; so we concentrate on it. We include the effects of probabilistic traffic intensity and the statistical distribution of equipment characteristics and transmission loss. The resulting model is so complicated that computer evaluation is necessary.

### 3.1 Analysis of Communications Probability

In a multi-network, spread-spectrum LMR service each network will have its own assigned code. A user will not be able to hear (decode) transmissions from other networks, and so he will transmit whenever he has a message without waiting for a clear channel. This "instant access" is one of the appealing features of SS land mobile radio service.

In a city there will be hundreds or thousands of mobile radios; so there may be several of them transmitting simultaneously at any time, even if the average individual usage is low. We must calculate the probability that each user will achieve communications in the presence of interference from the others.

We assume that communications are satisfactory if

$$S/(I + N) \geq R \quad ,$$

where S is the desired signal, I is the total interference, N is the noise, and R is a specified value.

There can be no communications failure unless someone is trying to communicate; so we calculate

$$P(\text{communications}) = P(S/(I + N) \geq R \,|\, \text{at least one user is transmitting}) \quad ,$$

where P(A|B) means "the probability of A, given B."

Now (Davenport, 1970),

$$P(\text{communications}) = P(S/(I+N) \geq R \,|\, \text{at least one } T_x \text{ on})$$

$$= \frac{P([S/(I+N) \geq R] \text{ and } [\text{at least one } T_x \text{ on}])}{P\ (\text{at least one } T_x \text{ on})}$$

$$= \frac{P([S/(I+N) \geq R \text{ and } 1 \text{ on}] \text{ or } [S/(I+N) \geq R \text{ and } 2 \text{ on}] \text{or} \ldots \text{or} [S(I+N) \geq R \text{ and } m \text{ on}])}{P(1 \text{ on or } 2 \text{ on or } \ldots \text{ or } m \text{ on})} \qquad (19)$$

where there are m licensed transmitters in the channel.

Because the event "$k_1$ transmitters are on" and the event "$k_2$ transmitters are on" are mutually exclusive when $k_1 \neq k_2$,

$$P(\text{communications}) = \frac{\sum_{k=1}^{m} P([S/(N+I) \geq R] \text{ and } [k \text{ transmitters are on}])}{1 - P\ (\text{no transmitters are on})}$$

$$= \sum_{k=1}^{m} \frac{P(S/(N+I) \geq R \,|\, k \text{ transmitters on})\ p_k}{1 - p_o} \quad , \qquad (20)$$

where $p_k \equiv P(k \text{ transmitters are on})$.

Let $U = Jm\ell/60$, where J is the average number of transmissions per hour by each transmitter, m is the total number of transmitters assigned to the channel, and $\ell$ is the average length of a transmission in minutes. Then U is the total number of message hours per hour and (Kleinrock, 1975, p. 105)

$$p_o = \left\{ \sum_{k=0}^{m} \frac{u^k}{k!} \right\}^{-1} , \tag{21}$$

$$\approx e^{-u}, \text{ if } U^{m+1}/(m+1)! << 1 . \tag{22}$$

Notice the U must be less than or equal to m.

For $k > 0$,

$$p_k = p_o \frac{u^k}{k!} . \tag{23}$$

These formulas are based on the usual assumption that the distribution of transmitter turn-ons is exponential, and the distribution of message lengths is poisson (Kleinrock, 1975). Table 2 shows $p_k$ as a function of k for various N.

Berry (1977a) describes a computer program that calculates the other factor in (20), $P(S/(N+I) \geq R|k$ transmitters are on), when given the statistical distributions of system parameters, path lengths, and propagation loss. This program was used with the input described in the next section.

## 3.2 Distributions of Input Parameters

As a result of the analysis in Section 2.2, we assume that all base station antennas are located in close proximity near the center of a metropolitan area. Mobile stations are located randomly inside a circle with radius 30 km, which is large enough to enclose most metropolitan areas. Other calculations show that the results are not very sensitive to the size of the circle, as long as it encloses most stations of interest (Berry, 1977a). The probability density function (pdf) of the distance from the base station to a random mobile is then

$$f_d(d) = \frac{2}{(30)^2} d . \tag{24}$$

This distribution applies to both wanted and interfering signal paths.

Table 2. Probability that i (Out of Possible m) Transmitters Assigned to a Channel are Transmitting

(U is the total channel utilization, $U = (J\ell/60)\ m$, where J is number of transmissions per hour by a transmitter and $\ell$ is average transmission length in minutes.)

| i | U, Channel Utilization | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.25 | 0.5 | 1 | 2 | 3 | 4 | 5 | 10 |
| 0 | .7788 | .6065 | .3679 | .1353 | .0498 | .0183 | .0067 | $4.54(10^{-5})$ |
| 1 | .1947 | .3033 | .3679 | .2707 | .1494 | .0733 | .0337 | .0005 |
| 2 | .0243 | .0758 | .1839 | .2707 | .2240 | .1465 | .0842 | .0023 |
| 3 | .0020 | .0126 | .0613 | .1804 | .2240 | .1954 | .1404 | .0076 |
| 4 | .0001 | .0016 | .0153 | .0902 | .1680 | .1954 | .1755 | .0189 |
| 5 | .0000 | .0002 | .0031 | .0361 | .1008 | .1563 | .1755 | .0378 |
| 6 | | .0000 | .0005 | .0120 | .0504 | .1042 | .1462 | .0631 |
| 7 | | | .0001 | .0034 | .0216 | .0595 | .1044 | .0901 |
| 8 | | | .0000 | .0009 | .0081 | .0298 | .0653 | .1126 |
| 9 | | | | .0002 | .0027 | .0132 | .0363 | .1251 |
| 10 | | | | .0000 | .0008 | .0053 | .0181 | .1251 |
| 11 | | | | | .0002 | .0019 | .0082 | .1137 |
| 12 | | | | | .0001 | .0006 | .0034 | .0948 |
| 13 | | | | | .0000 | .0002 | .0013 | .0729 |
| 14 | | | | | | .0001 | .0005 | .0521 |
| 15 | | | | | | .0000 | .0002 | .0347 |
| 16 | | | | | | | .0000 | .0217 |
| 17 | | | | | | | | .0128 |
| 18 | | | | | | | (m>15) | .0071 |
| 19 | | | | | | | | .0037 |
| 20 | | | | | | | | .0019 |
| 21 | | | | | | | | .0009 |
| 22 | | | | | | | | .0004 |
| 23 | | | | | | | | .0002 |
| 24 | | | | | | | | .0001 |
| 25 | | | | | | | | .0000 |
| | | | | | | | | (m>30) |

In an interference-limited environment, such as a fully used SS LMR service, the mean power radiated by transmitters is not important as long as it is sufficiently greater than the ambient noise. The variation of power about the mean is more important. The variation results from transmitters with different power ratings, installation, maintenance, and antenna characteristics. The pdf of effective radiated power (ERP) assumed in this study is shown in Figure 3. The nominal power of 80 percent of the transmitters is 100 W, and most stations ERP is within 3 dB of this value. A few users buy 20 W transmitters, and poor installation and maintenance cause a small percentage to radiate less than 10 dBW.

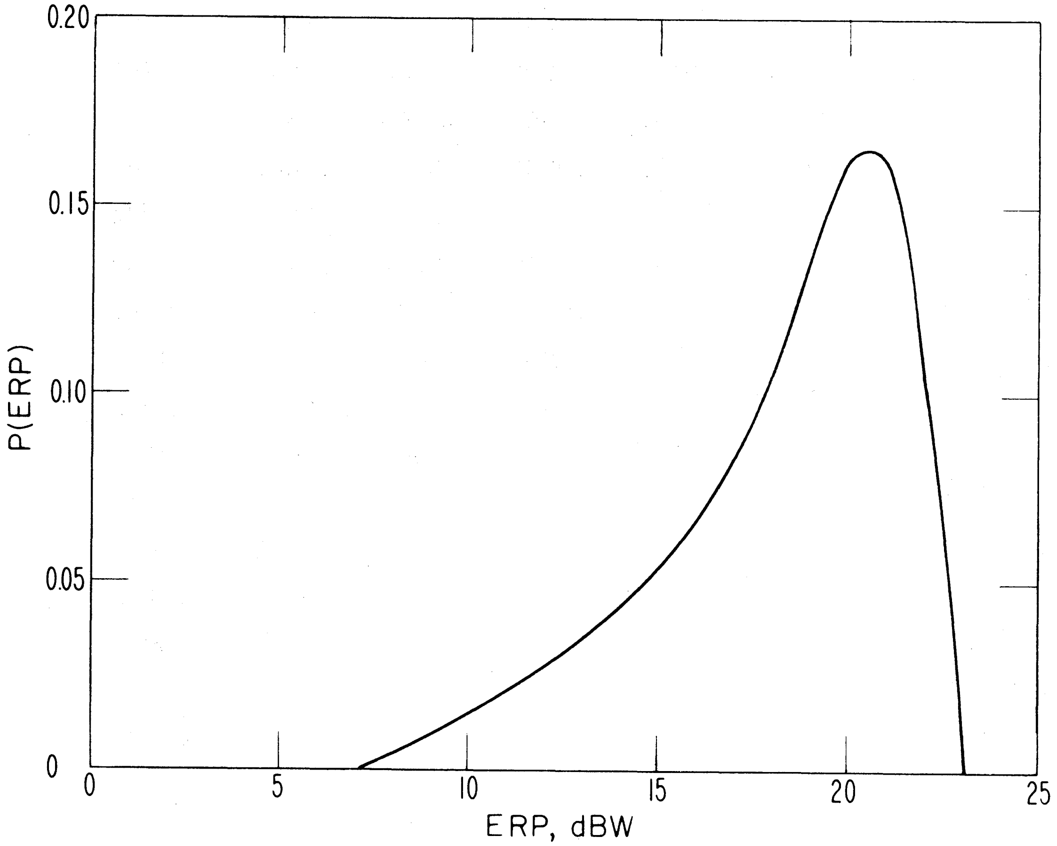

Figure 3. Probability density function for the effective radiated power of transmitters used in the service calculation. The ordinate is the probability that a transmitter emits the power shown on the abcissa.

The mean transmission loss assumed is an empirical fit to the curves given by Okumura et al. (1968):

$$L_b = 33.2 + 20 \log f_{MHz} + 20 \log_{10} d + G + H \log d \quad , \qquad (25)$$

where G and H are given in Table 1 of Section 2. For a fixed frequency, and for paths less than 30 km, (25) reduces to a constant plus 30 log d. It is the variation with distance that is important.

The transmission loss is assumed to be log normally distributed with a standard deviation of 9 dB (Okumura et al., 1968). This combines the fast and slow fading.

Although we are interested in the interference-limited case, ambient urban noise was added to the interference to provide a natural limit to performance when there are only a few interferers. We assumed urban, man-made noise is log normally distributed at 150 MHz with the mean noise power equal to -184 dBW +10 log b, where b is the receiver bandwidth in hertz. The standard deviation of the noise is 6 dB.

### 3.3 Computed Probability of Communications

The number of interferers, $N_i$, was varied to compute $P(S/(I+N) \geq R|N_i$ interferers). (Notice that the number of transmitters on, $k = N_i + 1$.) Figure 4 shows this probability as a function of $N_i$ every 3 dB from R = -30 dB to R = 6 dB.

The small and negative input S/I ratios are appropriate for spread-spectrum systems. Although the probabilities are actually discrete, those for a given R have been connected with a smooth line to identify them.

Figure 4 and Table 2 [or equations (21) and (23)] provide the data to evaluate (20)--the probability of communications. Table 3 contains this probability for the conditions shown. To simplify calculation and use of Table 3, we assumed that communications were certain if there were no interferers. Although this is not true, the error in the entries in Table 3 are correct within a few percent for practical LMR systems.

Notice that the probability of communication gets quite low for $N_i \geq 10$, even for very small S/I ratios. For example, the probability that $S/I \geq -24$ dB is less than 0.5 when there are 20 simultaneous interferers.

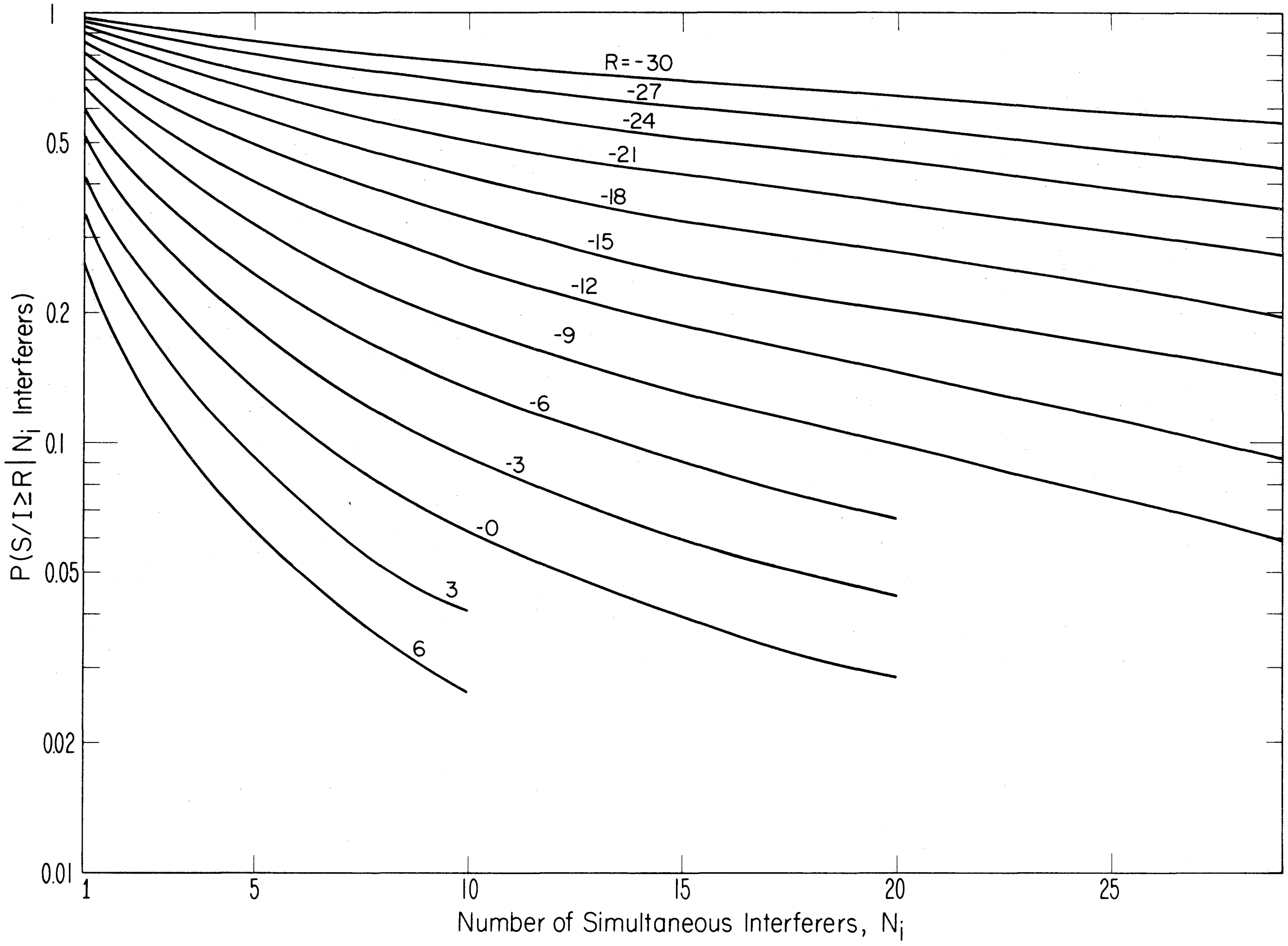

Figure 4. The probability that the signal-to-interference ratio exceeds a specified value, R, as a function of the number of simultaneous interferers. The system parameters and environment are those given in the text.

Table 3. Probability that S/I R for Different Traffic Intensities, Given that at Least One Transmitter Is on. Mobile-to-Base Transmission in Urban Areas

|  | U, Traffic Intensity |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|
| R | .25 | .5 | 1 | 2 | 3 | 4 | 5 | 10 | 20 |
| -30 | .995 | .991 | .981 | .959 | .937 | .910 | .886 | .784 | .64 |
| -27 | .993 | .987 | .973 | .941 | .908 | .874 | .843 | .711 | .55 |
| -24 | .990 | .981 | .961 | .916 | .871 | .827 | .786 | .633 | .46 |
| -21 | .987 | .973 | .945 | .886 | .814 | .773 | .723 | .544 | .37 |
| -18 | .982 | .964 | .926 | .849 | .776 | .710 | .652 | .456 | .29 |
| -15 | .976 | .951 | .902 | .805 | .717 | .640 | .574 | .372 | .21 |
| -12 | .968 | .937 | .875 | .758 | .654 | .567 | .496 | .294 | .15 |
| - 9 | .960 | .921 | .845 | .705 | .587 | .492 | .417 | .223 | .11 |
| - 6 | .950 | .902 | .810 | .648 | .517 | .421 | .342 | .164 |  |
| - 3 | .940 | .883 | .776 | .592 | .452 | .350 | .276 | .118 |  |
| 0 | .930 | .864 | .741 | .537 | .390 | .288 | .218 | .082 |  |

Notes: Assumes that the probability that $S/I \geq R$ is one if exactly one transmitter is on.

All bases near the center of a circle with radius 30 km.

Mobiles are randomly located in the circle.

Table was computed for radio frequency of 150 MHz, base antenna height = 200 m, mobile antenna height = 1.5 m; but should be correct within a few percent for frequencies between 100 and 1000 MHz; base antenna heights from 50 to 400 m, and mobile antenna heights from 1 to 3 m.

$U = (N\ell/60)\,m$ erlangs, where N is average number of transmissions per hour by an individual transmitter, $\ell$ is the average length of a transmission in minutes, and m is the total number of transmitters.

Values in the table assume that $U^m/m! \ll 1$. This is true if $m > 3U$.

### 3.4 Spectrum Efficiency of SS Systems

Table 3 can be used as a trade-off table to compare the efficiency of spread-spectrum LMR systems to conventional FM LMR systems. The approach is to assume both kinds of systems operate in the same metropolitan area and handle the same amount and type of traffic. If we require that they do so with the same reliability, the relative spectrum efficiency will be the ratio of the total radio bandwidth required to provide the service. Alternatively, we can give both systems the same bandwidth and compare their communications reliability.

In all cases, we assume a voice signal with 3 kHz bandwidth. We assume that FM channels are 25 kHz wide, and the systems have sufficient power so that reliability is nearly perfect in the absence of interference. This is the same assumption made in constructing Table 3. We do not count transmissions from mobiles in the same network as interference because we assume circuit discipline is maintained by the base station. We assume that both systems require an output S/I of 10 dB.

We also assume that the performance gain of spread-spectrum systems is equal to the processing gain, (total bandwidth)/(3 kHz). This is equivalent to assuming that the implementation loss is balanced by the gain realized from use of nearly orthogonal codes or (in frequency hopping systems) good filters. It turns out that the difference in efficiency of SS and conventional systems is very large; so a more accurate assumption is probably not required.

Relative spectrum efficiency will now be computed for several examples. We begin with an example with very small total utilization because Costas (1959) concluded that wideband systems would be more efficient in this case.

<u>Example 1</u>: Assume 10 networks, each with a base station and 6 mobiles. Each mobile transmits 1 minute per hour. Each FM network could be given an exclusive channel, and hence "perfect" reliability under the assumptions, in a total of 25 kHz × 10 = 250 kHz total bandwidth.

On the other hand, Table 3 shows that the SS system gets comparable reliability (0.98) if R = -30. (For this traffic, U = (60•1)/60 = 1.) This requires a processing gain of 40 dB--or a system bandwidth of $10^4$ (3 kHz) = 30 MHz! Comparing another way, if the SS bandwidth is 250 kHz, the processing gain is 19 dB; so the required input S/I is R = -9. The reliability is only 0.845 for this case.

It should be pointed out that intermodulation and adjacent channel interference prevent assignment of all FM channels in a practical LMR environment. Suppose that only one-half of all FM channels could be assigned because of these problems. Then if each FM network is assigned one channel, the total required bandwidth would be 500 kHz--one-sixtieth of that required for SS networks with comparable reliability.

Example 2: There are 120 networks with an average of 10 mobiles per network; each mobile transmits an average of 1 min/hr. Thus, U = 20.

(a) If the processing gain of an SS system is 40 dB and the required output S/I is 10 dB, then the required input S/I is -30 dB. The table shows that the probability of getting this S/I is 0.64. For 3 kHz voice, this system would require at least 30 MHz bandwidth.

(b) If the processing gain is only 30 dB so that the required input S/I is -20 dB, the probability of success is only 0.34. This system would require at least 3 MHz bandwidth.

(c) On the other hand, assume 25-kHz FM channels. Each network can have its own channel, with "perfect" reliability in 3 MHz of bandwidth. Compare this with the reliability of 0.34 in (b), which requires the same bandwidth, or the reliability of 0.64 in (a), which requires ten times as much bandwidth.

As in Example 1, the total bandwidth required for the FM networks may need to be increased to prevent intermodulation and adjacent channel interference. However, even if the bandwidth is doubled, SS still requires more for comparable reliability.

Example 3: Assume 120 25-kHz FM channels in a 3 MHz band. Assign 600 networks, each with 5 mobiles that transmit 1 min/hr. This example represents a well-loaded urban LMR band. Then for the FM networks on each channel, the average channel utilization is 25/60 = 0.417, and the probability of finding the channel free is only 0.61 (Kleinrock, 1975). The average waiting time is about 0.71 $\ell$. After waiting (if necessary), the user transmits with nearly perfect reliability.

For an SS system, the utilization factor U = 50. This isn't in the table, but we can make estimates. If we confine the SS system to the same 3 MHz of bandwidth, so R = -20, the probability of communicating is certainly less than 0.34 (the value for U = 20) and is probably less than 0.2. Even if the bandwidth is 30 MHz, the reliability must be much less than 0.64, which is about the probability of getting on the FM channel immediately. The reliability is probably less than 0.4.

In each of these examples, the conventional FM systems get comparable reliability with less total bandwidth than the SS systems or better reliability in the same bandwidth. So FM is more spectrum efficient than SS in these examples.

Of course, a general rule cannot be proven with examples. But these examples, using realistic assumptions about the systems and the environment, agree with the analytical result derived in Section 2.3.2 for a simpler model. The two results together provide a convincing argument for the general conclusion: for a land-mobile radio service consisting of many independent networks, spread-spectrum modulation is less spectrum efficient than conventional channelized frequency modulation.

## 4. OVERLAYING A SPREAD-SPECTRUM SYSTEM ON AN EXISTING SERVICE

Because SS systems use low spectral power density and are resistant to narrowband interference (Dixon, 1976), it has been conjectured that one or more SS systems could operate in a service band already assigned to conventional LMR without unacceptable interference to any user (Utlaut, 1977). This section contains a first-order analysis of the probability of intersystem interference for this possibility.

### 4.1 SS Interference to FM

Assume a single SS system overlaid on an occupied FM LMR service and assume that the base stations of both systems are near the center of the city. Assume that both FM and SS transmitters have equal power, $P_t$. Assume that the bandwidth of the SS system is 3 MHz so that its average spectral power density is $10^{-6}\ P_t/3$, W/Hz. The bandwidth of an FM receiver in 25-kHz channels is about 16 kHz; so it receives

$$\frac{16 \cdot 10^{3}}{L_i} \frac{10^{-6}\ P_t}{3} \simeq \frac{5 \cdot 10^{-3}\ P_t}{L_i} \quad W \ , \tag{26}$$

where $L_i$ is the transmission loss on the interfering path. Assuming no other interference and that wanted and interfering paths have the same pdf, the mean input S/I is $1/(5 \cdot 10^{-3}) = 200$, or 23 dB. This is well above the desired FM threshold of 10 dB; so the FM receiver will not be bothered by the interference. If S/I is log-normally distributed (normal in decibels) with a standard deviation of 10 dB, then S/I > 10 dB 99 percent of the time. It is calculations like this that have led to the suggestion that an overlaid SS system would not interfere with FM systems.

However, an SS mobile transmitter near an FM base station trying to receive a remote FM mobile might cause unacceptable interference. If the propagation law is given by $P_{received} = K\, P_t/d^3$, then the input S/I is [using (26)]

$$S/I = \frac{\dfrac{K\, P_t}{d_w^3}}{\dfrac{K \cdot P_t \cdot 5 \cdot 10^{-3}}{d_i^3}} = \left(\frac{d_i}{d_w}\right)^3 200 \quad , \tag{27}$$

where $d_w$ is the distance to the wanted FM mobile and $d_i$ is the distance to the interfering SS mobile. Then S/I is unacceptable (less than 10 dB) if

$$\left(\frac{d_i}{d_w}\right)^3 200 < 10$$

or

$$\frac{d_i}{d_w} < .05^{\frac{1}{3}} \approx 0.37 \tag{28}$$

provided both transmitters are on.

Assume that both networks have the same number of mobiles randomly located in a circle of radius r so that the pdf of path lengths is

$$f_{d_i}(d_i) = \frac{2}{r^2}\, d_i \text{ and } f_{d_w}(d_w) = \frac{2}{r^2}\, d_w \quad .$$

Then the probability that $d_i/d_w \leq q$ can be shown to be (Zehna, 1970, p. 213)

$$P\left(\frac{d_i}{d_w} \leq q\right) = \begin{cases} \dfrac{q^2}{2} & , \text{ if } q \leq 1 \\ 1 - \dfrac{1}{2q^2}, & \text{if } q \geq 1 \end{cases} \quad . \tag{29}$$

Substituting q = 0.37 yields

$$P\left(\frac{d_i}{d_w} \leq 0.37\right) \approx 0.07 \qquad (30)$$

if both are transmitting.

Assume each SS mobile transmits t fraction of the time; then the probability of an SS mobile interfering with an FM mobile can be derived as follows:

We take the conservative view that interference occurs when two transmissions overlap totally or partially and $d_i/d_w \leq 0.37$. We further assume that the average message lengths in both systems are approximately equal and short compared to the statistical interval--e.g., 1-minute message lengths and 1-hour observation intervals. Thus, to a first approximation, if mobile transmissions in one network occur t fraction of the time, overlap with a mobile transmission of another network occurs approximately 2t fraction of the time. For small t's, the probability of overlap $\approx$ 2t; for larger t's, the relationship is more complex, and the probability of overlap is less than 2t. Then, an FM mobile transmitting can expect interference with a probability of

$$P(\text{interference}) = P\left(\text{SS transmission overlaps and } \frac{d_i}{d_w} \leq 0.37\right)$$

$$\approx (2t)\ (0.07) \approx 0.14\ t \quad . \qquad (31)$$

For t = 0.25, which might represent a busy hour, the probability of interference is about 3½ percent; for t = 0.1, it is about 1½ percent. For large t, the maximum probability of interference is, or course,

$$P(\text{interference})_{\text{max}} = P\left(\frac{d_i}{d_w} \leq 0.37\right) \approx .07 \quad . \qquad (32)$$

The analysis for several overlaid SS networks is more complicated. It seems likely, however, that one SS network (say a digital, semi-secure police network) could be overlaid on conventional FM services without undue probability of interference to the FM service.

## 4.2 FM Interference to SS

Assume FM power is $P_t$ and that there is a transmission in an average channel w fraction of the time. So the time average power in a channel is w w $P_t$. As before, assume an SS system with 3 MHz bandwidth overlaid on the FM service. There could be $\frac{3\text{ MHz}}{25\text{ kHz}} = 120$ FM channels, and the average interference power in the 3 MHz bandwidth is 120 w $P_t$. If the pdf of path lengths is the same, and the SS transmitter also has $P_t$ watts power, then the mean signal-to-interference power input to the SS receiver is 1/(120 w). If w is 1/10, input S/I $\approx$ -11 dB. <u>If the SS system realizes all 30 dB of its processing gain</u>, the output S/I of 19 dB is satisfactory. An implementation loss as high as 9 dB would be acceptable.

As before, there is the possibility that an FM transmitter very near the desired receiver could interfere with the signal from a remote SS mobile. It is difficult to analyze the probability of this happening because there would probably be many more FM mobiles than SS mobiles. The situation would probably need to be resolved by actual field testing or computer simulation. The potential seems sufficiently promising to encourage further investigation.

## 5. CONCLUSIONS

We have analyzed the feasibility and spectrum efficiency of using spread-spectrum modulation in a land-mobile radio service containing many independent networks. The cases analyzed are shown in Table 4. In a band allocated exclusively to spread-spectrum systems, base stations and mobile stations would have to operate on separate channels so that the total bandwidth required by the service would be twice that of the SS system. Base stations all would have to be located near each other. In this configuration, the worst interference occurs in the mobile-to-base channel.

Analysis of a simplified model of the service and computer calculations for a more realistic model both show that such a spread-spectrum service would not be as spectrum-efficient as conventional FM land-mobile radio. This result is a particular example of the "near-far problem." The calculations did not include the possibility of central control of all mobiles transmitted power because "central control" is the antithesis of "independence."

However, one or more spread-spectrum systems probably could be overlaid on an existing FM band without causing unacceptable interference to the

## Table 4. Configurations of Spread-Spectrum LMR Networks Considered in This Report and Conclusions Reached

Assumptions: A service band is allocated exclusively to land-mobile radio networks that use only spread-spectrum modulation. There are many networks in the service band, and they operate without dynamic central control of transmitter power or transmission time.

1. Bases and Mobiles on Same Channel

   Conclusion: Base stations cause unacceptable interference to mobile-to-base signals.

2. Bases on One Channel; Mobiles on Another

   A. Arbitrary location of base stations, base-to-mobile channel.

      Conclusion: A mobile receives unacceptable interference in the vicinity of a base station of another network. Thus, the areas around the base stations of all other networks are denied to it, and the total denied area is unacceptably large.

   B. All base stations located close together.

      (i) Base-to-mobile channel

         Conclusions: Spectrum efficiency of SS systems comparable to that of conventional FM systems.

      (ii) Mobile-to-base channel

         Conclusions: Interference from mobiles near the bases to signals from mobiles further away limits the number of networks achieving a specified reliability. This number is less than the number of conventional FM LMR networks that could achieve comparable reliability.

existing service. The SS service probably could operate satisfactorily with occasional interuptions from FM transmitters very near the desired receiver.

## 6. ACKNOWLEDGMENTS

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# APPENDIX A. THE SPECTRUM EFFICIENCY OF MULTIPLE, INDEPENDENT, SS LMR NETWORKS

This appendix contains a more rigorous and detailed derivation of the result given in Section 2.3 of the main body of this report. It is assumed that the wanted signal is from a mobile to a base station. Interfering signals come from n other mobiles randomly distributed in the metropolitan area and transmitting simultaneously. It is understood that the total number of mobiles in the area is much larger, but proportional to the number transmitting at any time.

## A.1 THE EXPECTED DISTANCES TO THE INTERFERERS

There are n points randomly distributed in a circle with radius r. Then the distance x from the center of the circle to a point is a random variable with probability density function (pdf)

$$f_x(x) = \frac{2}{r^2} \quad x, \quad 0 \leq x \leq r \quad , \tag{A-1}$$

and probability distribution function

$$F_x(x) = x^2/r^2 \quad . \tag{A-2}$$

Order the distances to the n points and label them $x_1, x_2, \ldots, x_n$, so that

$$0 \leq x_1 \leq x_2 \leq \ldots \leq x_n \leq r \quad .$$

Then the pdf of $x_m$, m = 1, n is (Gumbel, 1958, p. 43)

$$h(x_m) = \frac{n!2}{(n-m)!(m-1)!r} \left(\frac{x_m^2}{r^2}\right)^{m-\frac{1}{2}} \left(1 - \frac{x_m^2}{r^2}\right)^{n-m} \quad . \tag{A-3}$$

The expected value of $x_m$,

$$E(x_m) = \frac{2n!}{(n-m)!(m-1)!r} \int_0^r x\left(\frac{x^2}{r^2}\right)^{m-\frac{1}{2}} \left(1 - \frac{x^2}{r^2}\right)^{n-m} dx \quad . \tag{A-3}$$

Using the transformation $U = x^2/r^2$,

$$E(x_m) = \frac{n!r}{(n-m)!(m-1)!} \int_0^1 U^{m-\frac{1}{2}}(1-U)^{n-m} \, dU \quad . \tag{A-4}$$

Section 6.2 of Abramovitz and Stegun (1964) shows that

$$E(x_m) = \frac{n!\ r}{(n-m)!\,(m-1)!}\ \frac{\Gamma(m+\frac{1}{2})\ \Gamma(n-m+1)}{\Gamma(n+\frac{3}{2})} \quad . \tag{A-5}$$

Because $k! = \Gamma(k+1)$,

$$E(x_m) = r\ \frac{\Gamma(n+1)\ \Gamma(m+\frac{1}{2})}{\Gamma(n+\frac{3}{2})\ \Gamma(m)} \quad . \tag{A-6}$$

The expected values can be calculated recursively beginning with the largest by noting that for m=n, (A-6) reduces to

$$E(x_n) = \frac{n}{n+\frac{1}{2}}\ r \quad , \tag{A-7}$$

and that

$$E(x_m) = \frac{m}{m+\frac{1}{2}}\ E(x_{m+1}), \quad \text{for } m < n \quad . \tag{A-8}$$

If n is large (say, 10 or more), $E(x_1)$ can be approximated with the first term of (6.1.36) from Abramowitz and Stegun (1964):

$$E(x_1) \approx \frac{r}{2}\sqrt{\frac{\pi}{n+1}} \quad . \tag{A-9}$$

The relative error is of the order of $\frac{1}{12\ n}$ .

When $n >> 1$, $E(x_1)$ is inversely proportional to $\sqrt{n}$ as assumed in the main report. The expected distance to the other points in order is

$$E(x_{m+1}) = \frac{m+\frac{1}{2}}{m}\ E(x_m), \ m \geq 1 \quad . \tag{A-10}$$

## A.2 THE TOTAL INTERFERENCE POWER

The analysis of the relative efficiency of spread-spectrum LMR systems in the main body of this report requires expressing the total interference that arrives at a point in terms of the interference from the nearest interferer. Assume that the power from the $m^{th}$ nearest interferer is inversely proportional to some power of the expected distance to the $m^{th}$ interferer; that is,

$$I_m = \frac{K}{[E(X_m)]^{\alpha}} \quad , \qquad \text{W} \quad , \tag{A-11}$$

where $I_m$ is the interfering power from the $m^{th}$ nearest interferer and K is a constant that depends on frequency and the transmitter power. We assume all transmitters have the same power. In the LMR bands, for paths less than 30 km, $\alpha \approx 3$ for transmission from a high base station to a mobile in an urban environment; $\alpha \approx 4$ for transmission over smooth land between two low antennas. Assume that the total interfering power is the sum of the powers from all interferers:

$$I_T = \sum_{m=1}^{n} \frac{K}{[E(x_m)]^{\alpha}} = \frac{K}{[E(x_1)]^{\alpha}} \left\{ 1 + \sum_{m=2}^{n} \left[\frac{E(x_1)}{E(x_m)}\right]^{\alpha} \right\}$$

$$= I_1 \left\{ 1 + \sum_{m=2}^{n} \left[\frac{E(x_1)}{E(x_m)}\right]^{\alpha} \right\} . \qquad \text{(A-12)}$$

Recursive application of (A-10) shows that $E(x_1)/E(x_m)$ is independent of r and of n. Table A-1 shows values of $E(x_1)/E(x_m)$ and the factor in brackets in (A-12) for m = 1--10 and $\alpha$ = 3 and 4.

Table A.1. Ratios of the Expected Values of Ordered Distances from Random Points to the Center of a Circle
$0 \le x_1 \le x_2 \le x_3 \le \ldots \le x_m \le \ldots$

| m | $\frac{E(x_1)}{E(x_m)}$ | $\left(\frac{E(x_1)}{E(x_m)}\right)^3$ | $\sum_{i}^{m}\left(\frac{E(x_1)}{E(x_i)}\right)^3$ | $\left(\frac{E(x_1)}{E(x_m)}\right)^4$ | $\sum_{i}^{m}\left(\frac{E(x_1)}{E(x_i)}\right)^4$ |
|---|---|---|---|---|---|
| 1 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 2 | 0.6667 | 0.2963 | 1.2963 | 0.1975 | 1.1975 |
| 3 | 0.5333 | 0.1517 | 1.4480 | 0.0809 | 1.2784 |
| 4 | 0.4571 | 0.0955 | 1.5435 | 0.0437 | 1.3221 |
| 5 | 0.4063 | 0.0671 | 1.6106 | 0.0273 | 1.3494 |
| 6 | 0.3694 | 0.0504 | 1.6610 | 0.0186 | 1.3688 |
| 7 | 0.3410 | 0.0396 | 1.7007 | 0.0135 | 1.3815 |
| 8 | 0.3183 | 0.0322 | 1.7329 | 0.0103 | 1.3918 |
| 9 | 0.2995 | 0.0269 | 1.7598 | 0.0081 | 1.3998 |
| 10 | 0.2838 | 0.0229 | 1.7827 | 0.0065 | 1.4063 |

The first three or four interferers contribute most of the interference. For $\alpha = 3$, interference from the expected location of the second closest interferer is 5 dB below that from the nearest interferer, the third interferer is 8 dB down, and the fourth is 10 dB down. The approximation, $I_T \approx 2 \cdot I_1$, is accurate enough for crude analyses like that in Section 2.3.2 of this report.

However, a more rigorous and accurate derivation of (17) in the main report is possible. First, an accurate approximation to (A-12) is derived. The idea is to approximate the sum of higher-order terms in (A-12) by multiplying the average value of the terms by the number of terms. Table A-1 shows that $E(x_1)/E(x_m)$ is a slowly varying function of m for $m > 5$. Also, combination of equations (A-7) and (A-9) yields an approximation for $E(x_1)/E(x_n)$:

$$E(x_1)/E(x_n) \approx \frac{1}{2}\sqrt{\frac{\pi}{n+1}} \quad . \tag{A-13}$$

Trial shows that the geometric mean,

$$\left(\frac{E(x_1)}{E(x_k)} \cdot \frac{E(x_1)}{E(x_n)}\right)^{\frac{1}{2}} \quad , \tag{A-14}$$

is a better approximation to the average of terms with index higher than k than the arithmetic mean. So

$$I_t \approx I_1 \left\{ 1 + \sum_{m=2}^{k} \left(\frac{E(x_1)}{E(x_n)}\right)^{\alpha} + (n-k)\left(\frac{E(x_1)}{E(x_k)} \frac{1}{2}\sqrt{\frac{\pi}{n+1}}\right)^{\frac{\alpha}{2}} \right\} \tag{A-15}$$

for $k < n$. The sum of the first k terms and $E(x_1)/E(x_k)$ can be found in Table 1. For $\alpha = 3$, $n = 10$, and $k = 6$, the error in (A-15) is only two units in the fourth significant digit. For $k = 3$, (A-15) is correct to two digits.

## A.3 THE SPECTRUM EFFICIENCY AS A FUNCTION OF BANDWIDTH

Now, follow the outline of the analysis in Section 2.3.2 of the report. The assumptions are as follows:

(1) interferers are randomly located in a circle with radius r around the base station;

(2) there are n interferers transmitting simultaneously;

(3) all transmitters have equal radiated power; and

(4) the propagation law is $P_r = K/d^{\alpha}$, where $P_r$ is the power received; K is a constant incorporating common radiated power, antenna gain, and frequency; d is the transmission path length; and $\alpha$ is a constant.

Assume a fully loaded spread-spectrum LMR service; that is, the networks just barely achieve the required signal-to-interference ratio the required percent of the time, and addition of another network would lower the reliability below the standard. Variables for this situation will be subscripted with a. In situation a, there are $N_a$ interferers at a given time. The total interference is [using (A-15)]

$$I_{ta} \approx I_{1a} \left\{ 1 + \sum_{m=1}^{k} \left( \frac{E(x_1)}{E(x_m)} \right) + (N_a - k) \left( \frac{E(x_1)}{E(x_k)} \frac{1}{2} \sqrt{\frac{\pi}{N_a+1}} \right)^{\frac{\alpha}{2}} \right\} , \qquad \text{(A-16)}$$

and

$$I_{1a} = K \left\{ \frac{r}{2} \sqrt{\frac{\pi}{N_a+1}} \right\}^{-\alpha} . \qquad \text{(A-17)}$$

Putting (A-17) into (A-16) and combining constants results in

$$I_{ta} = H(N_a+1)^{\frac{\alpha}{2}} \left\{ Q + T \frac{(N_a - k)}{(N_a+1)^{\alpha/4}} \right\} , \qquad \text{(A-18)}$$

where Q and T are positive numbers independent of $N_a$.

We are going to increase the bandwidth of the SS systems and see whether we get a commensurate increase in the number of users. In situation b, increase the bandwidth of the spread-spectrum systems so that

$$B_b = F\, B_a \quad , \qquad \text{(A-19)}$$

where F > 1.

Increase the number of users until performance is again at the threshold of unacceptability. In this case, there are $N_b$ simultaneous interferers. The total interference for case b is

$$I_{tb} = H(N_b+1)^{\frac{\alpha}{2}} \left\{ Q + T \frac{N_b - k}{(N_b+1)^{\alpha/4}} \right\} . \qquad \text{(A-20)}$$

Assume that the wanted signal has the same value in both cases. Then the output S/I is the same provided that the input S/I is related by the ratio of processing gains; that is

$$B_a(S/I_{ta}) = B_b(S/I_{tb}) \quad ,$$

or

$$\frac{I_{tb}}{I_{ta}} = \frac{B_b}{B_a} = F \quad . \tag{A-21}$$

Substitute (A-18) and (A-20) into (A-21) and simplify:

$$\left(\frac{N_b+1}{N_a+1}\right)^{\frac{\alpha}{2}} \left\{\frac{Q + T\,\dfrac{N_b-k}{(N_b+1)^{\alpha/4}}}{Q + T\,\dfrac{N_a-k}{(N_a+1)^{\alpha/4}}}\right\} = F \quad . \tag{A-22}$$

Consider

$$f(x) = Q + T\,\frac{(x-k)}{(x+1)^{\alpha/4}} \quad , \quad (x > k > 1) \quad .$$

The derivative,

$$f'(x) = \frac{T}{(x+1)^{\alpha/4}} \left(1 - \frac{\alpha}{4}\,\frac{x-k}{x+1}\right) \quad ,$$

is positive provided

$$x > \frac{\frac{\alpha}{4}k - 1}{1 + \frac{\alpha}{4}} \quad . \tag{A-23}$$

But

$$k > \frac{\alpha}{\alpha+4}\,k > \frac{\alpha k-4}{\alpha+4} = \frac{\frac{\alpha}{4}k-1}{1+\frac{\alpha}{4}}$$

for all $\alpha > 0$ and $k > 1$; so (A-23) is always true since $x > k$. Therefore, $f(x)$ is an increasing function of $x$, and, because $N_b \geq N_a$,

$$Q + T\,\frac{N_b-k}{(N_b+1)^{\alpha/4}} \geq Q + T\,\frac{N_a-k}{(N_a+1)^{\alpha/4}} \quad . \tag{A-24}$$

Comparing (A-24) with (A-22) shows that

$$\left(\frac{N_b+1}{N_a+1}\right)^{\frac{\alpha}{2}} \leq F \quad ,$$

$$\frac{N_b+1}{N_a+1} \leq F^{2/\alpha} \quad . \tag{A-25}$$

Both situations involve the same geographic area and the same time; so the spectrum efficiency is proportional to the number of users per unit bandwidth. Multiplying both sides of (A-25) by $B_a/B_b$ and rearranging does not change the inequality:

$$\frac{\left(\frac{N_b+1}{B_b}\right)}{\left(\frac{N_a+1}{B_a}\right)} \leq \frac{B_a}{B_b} F^{2/\alpha} = \frac{F^{2/\alpha}}{F} = F^{2/\alpha-1} \leq 1 \tag{A-26}$$

provided that $\alpha \geq 2$.

Finally,

$$\frac{N_b+1}{B_b} \leq \frac{N_a+1}{B_a} \quad ; \tag{A-27}$$

that is, there are fewer successful users per unit bandwidth in situation b than there are in situation a. Increasing the bandwidth of the spread-spectrum systems decreased the spectrum efficiency.

To further appreciate the information in (A-26), assume that $B_a$ is the bandwidth of the information to be sent. (Then the system in case a is not strictly a spread-spectrum system; it is the limiting case of a spread-spectrum system with the processing gain, $G_p = 1$.) Then $F = G_p$ for situation b, and (A-26) can be written as follows:

$$\frac{\frac{N_b+1}{B_b}}{\frac{N_a+1}{B_a}} \leq \frac{1}{G_p^{1-2/\alpha}} \quad . \tag{A-28}$$

The exponent of $G_p$ is 1/3 for $\alpha = 3$. This can be interpreted to mean that the spectrum efficiency for these kinds of spread-spectrum systems is inversely proportional to the cube root of the processing gain.

Recall that it was assumed that the interfering transmitters were randomly located in a circle and had equal power. Therefore, this result says nothing about geostationary satellite-to-earth systems (in which all paths are the same length) or about systems in which the base station equalizes the power from mobiles.

## A.4 REFERENCES

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# APPENDIX B. CONVENTIONAL LMR AND SPREAD-SPECTRUM BACKGROUND

## B.1 THE LAND-MOBILE RADIO CHANNEL

### B.1.1 Land-Mobile Radio Frequency Bands

Land-mobile radio (LMR) serves both nongovernment and federal government users; however, the LMR frequency bands have been divided to allocate separate spectrum space to nongovernment needs and government requirements. The LMR bands are given in Table B.1.

Table B.1. LMR Frequency Bands

| Government | Non-Government |
|---|---|
| 30-50 MHz | 25-50 MHz |
| 162-174 MHz | 150-174 MHz |
| 406-420 MHz | 450-470 MHz |
| | 470-512 MHz (some cities only) |
| | 806-947 MHz |

The Federal Communications Commission (FCC) has divided each nongovernment band into smaller groups called Services such as Public Safety, Industrial, etc. The nongovernment bands, as they are divided into the various services, are shown in Table B.2.

Specific channel assignments within each band are made for both nongovernment and government LMR. The channel widths are normally from 25 kHz to 50 kHz, depending upon the band and the service, with allowed emission bandwidths less than the channel widths. Attempts have been made to reduce the channel widths to 12.5 kHz, which would double the number of available channels in the already congested LMR bands. However, since frequency modulation (FM) is the most widely used form of modulation in LMR, the 12.5 kHz channel width would lead to less allowable frequency deviation for the FM systems. This results in a lower FM improvement which, in turn, means that the performance of the communication system is reduced; i.e., the LMR user would have to tolerate more noise and more interference for a given level of

Table B.2. Service Groups Within the LMR Bands

| Frequency (MHz) | Service Groups |
|---|---|
| 25.01 - 25.33 | I |
| 26.1 - 26.48 | --- |
| 27.28 - 27.54 | I |
| 29.7 - 29.8 | I |
| 30.56 - 32 | I,LT,PS |
| 33 - 33.01 | LT |
| 33.01 - 33.11 | PS |
| 33.11 - 33.41 | I |
| 33.41 - 34 | PS |
| 35 - 35.19 | I |
| 35.19 - 35.69 | DP,I,PS |
| 35.69 - 36 | I |
| 37 - 37.01 | I |
| 37.01 - 37.43 | PS |
| 37.43 - 37.89 | I |
| 37.89 - 38 | PS |
| 39 - 40 | PS |
| 42 - 42.95 | PS |
| 42.95 - 43.19 | I |
| 43.19 - 43.69 | DP,I,PS |
| 43.69 - 44.61 | LT |
| 44.61 - 46.6 | PS |
| 47 - 47.43 | PS |
| 47.43 - 47.69 | PS,I |
| 47.69 - 49.6 | I |
| | |
| 150.8 - 150.98 | LT |
| 150.98 - 151.4825 | PS |
| 151.4825 - 151.4975 | I,PS |
| 151.4975 - 152 | I |
| 152 - 152.255 | DP |
| 152.255 - 152.465 | LT |
| 152.465 - 152.495 | I |
| 152.495 - 152.855 | DP |
| 152.855 - 153.7325 | I |

---

DP - Domestic Public
I - Industrial
LT - Land Transportation
PS - Public Safety

Table B.2. Service Groups Within the LMR Bands (continued)

| | |
|---|---|
| 151.4975 - 152 | I |
| 152 - 152.255 | DP |
| 152.255 - 152.465 | LT |
| 152.465 - 152.495 | I |
| 152.495 - 152.855 | DP |
| 152.855 - 153.7325 | I |
| 153.7325 - 154.46 | PS |
| 154.46 - 154.6375 | I |
| 154.6375 - 156.25 | PS |
| 157.45 - 157.725 | LT |
| 157.725 - 157.755 | I |
| 157.755 - 158.115 | DP |
| 158.115 - 158.475 | I |
| 158.475 - 158.715 | DP |
| 158.715 - 159.48 | PS |
| 159.48 - 161.575 | LT |
| 161.625 - 161.775 | -- |
| 173.2 - 173.4 | PS,I |
| | |
| 450 - 451 | -- |
| 451 - 454 | PS,I,LT |
| 454 - 455 | DP |
| 455 - 456 | -- |
| 456 - 459 | PS,I,LT |
| 459 - 460 | DP |
| 460 - 462.5375 | PS,I,LT |
| 462.5375 - 462.7375 | -- |
| 462.7375 - 465.0125 | PS,I,LT |
| 465.0125 - 467.5375 | PS,I,LT |
| 467.5375 - 467.7375 | -- |
| 467.7375 - 470 | PS,I,LT |
| | |
| 806 - 821 | -- |
| 821 - 825 | -- |
| 825 - 845 | -- |
| 845 - 851 | -- |
| 851 - 866 | -- |
| 866 - 870 | -- |
| 870 - 890 | -- |
| 890 - 902 | -- |
| 928 - 947 | -- |

input signal-to-noise ratio. Thus, it is expected that FM channel width will remain at 25 kHz or greater. As we shall see later, spread-spectrum communications in land-mobile applications will require bandwidths much greater than 25 kHz.

In the next section, we will consider the effects of propagation on the LMR channel, and following that we will consider noise and interference.

### B.1.2 Propagation Effects on the Land-Mobile Channel

In the typical land-mobile radio application, one end of the communication system is fixed in location with its antenna high above the average terrain, on top of a building or hill, with as few obstructions close to the antenna as are physically controllable. The other end of the system is mobile, with its antenna usually about 2-3 m (6-10 ft) above the local terrain and with all sorts of surrounding obstructions such as other vehicles, tall buildings, and irregular terrain.

Whether the system is to be used in the city or in the country, the signal path between the mobile and its fixed base station is rarely ever line-of-sight and usually is a composite of many scatter paths. Thus, as the mobile unit moves along a particular street or road, the geometry of the scatter paths from the mobile back to the base station change. The phase and amplitude of each scatter path signal contribute to the composite signal in such a way that the received signal amplitude is not constant but instead is rising or falling continually. This propagation effect is called fading (Arredondo and Smith, 1977; Hansen and Meno, 1977).

The signal level fading has two definite features: one feature is short-term or short-distance and is called Rayleigh fading; the other is apparent over the long-term or long-distance and is log-normal fading. Figure B.1 shows a recorded received mobile signal level which demonstrates both fading features; the rapid, deep fading occurring over short distances and the slow, shallow fading over long distances. For each half-wavelength (approximately) of forward movement of the mobile, the received signal goes through a Rayleigh fade, which occasionally is as much as 30 to 40 dB. This fading is due to the many signals, transmitted by the base station, which arrive at the mobile antenna out of phase with respect to each other, and either enhance or cancel each other. The amplitude distribution of the received signal level is

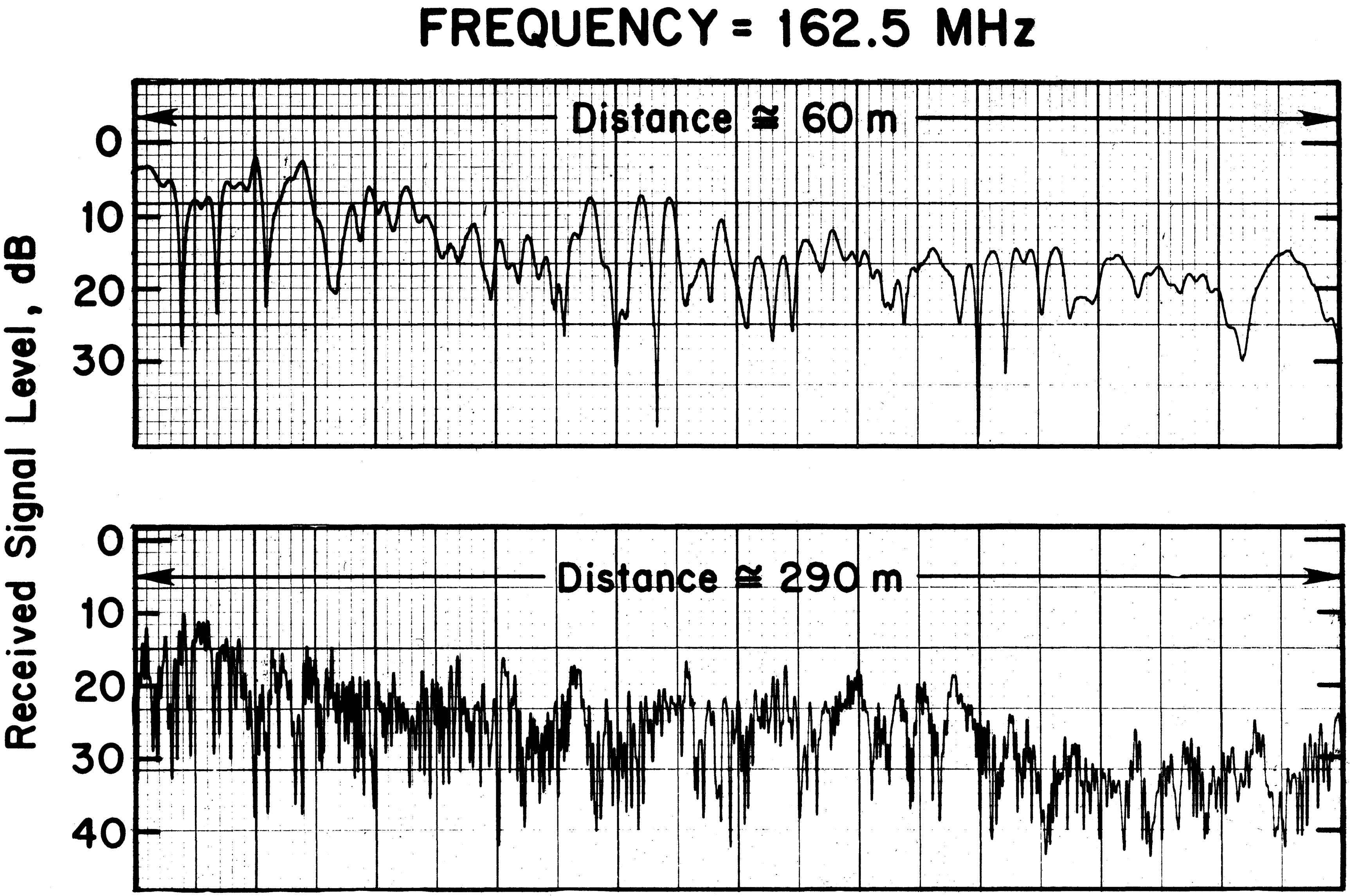


Figure B.1. Signal level at a mobile receiver.

Rayleigh distributed, hence the name Rayleigh fading. Figure B.2 shows the expected received signal strength versus percent of time for a Rayleigh distribution. What the conventional FM LMR user actually hears in a multipath environment has been described aptly by Arrendo and Smith (1977) for land-mobile operations at 900 MHz:

> "As a vehicle moves through the fading signal pattern, interruptions of the voice modulation can be heard, noise capturing the FM receiver during the interruptions. These interruptions have a different subjective effect as a function of the vehicle speed. At low speeds, less than 5 mi/h, the rate of speech interruptions is not as important as their duration. The effect is much like the swishing sound heard when driving slowly with the window open close to parked cars or closely spaced trees. At higher speeds, up to 20 mi/h, the interruptions are heard as a series of 'pops' or 'clicks', much like the sound made running down the street hitting a picket fence with a stick. At speeds higher than 20 mi/h, the interruptions blend into a low-pitched noise, the pitch increasing with speed."

The long-term fading, which is apparent over a distance of 100 wavelengths or more, is due to natural or man-made obstacles. The log of the received signal amplitude has a normal distribution; hence the fading is called log-normal fading. Figure B.3 shows an example received signal level with log-normal fading. The mean and the standard deviation of the received signal level depend upon frequency; antenna heights; terrain irregularity; and size, construction, and density of buildings. There are methods to predict the log-normal fading due to irregular terrain (Longley and Rice, 1968) or the amount of urbanization (Jakes, 1974; Okumura et al., 1968; Longley, 1978). Okumura's measurements (1968) show that the signal mean is about 5 dB (at 150 MHz) to 10 dB (at 900 MHz) higher in a suburban environment (mostly one and two story buildings) as compared to an urban area (mostly multiple story buildings) for the same antenna heights and separation distances. The log normal signal's standard deviation is about 5-6 dB in urban locations and 7-8 dB in suburban areas.

Remedies exist to lessen the effects of Rayleigh fading; for example, space or frequency diversity (Jakes, 1974; Brennan, 1959). However, we will show later that spread-spectrum techniques also can help to lessen Rayleigh fading effects.

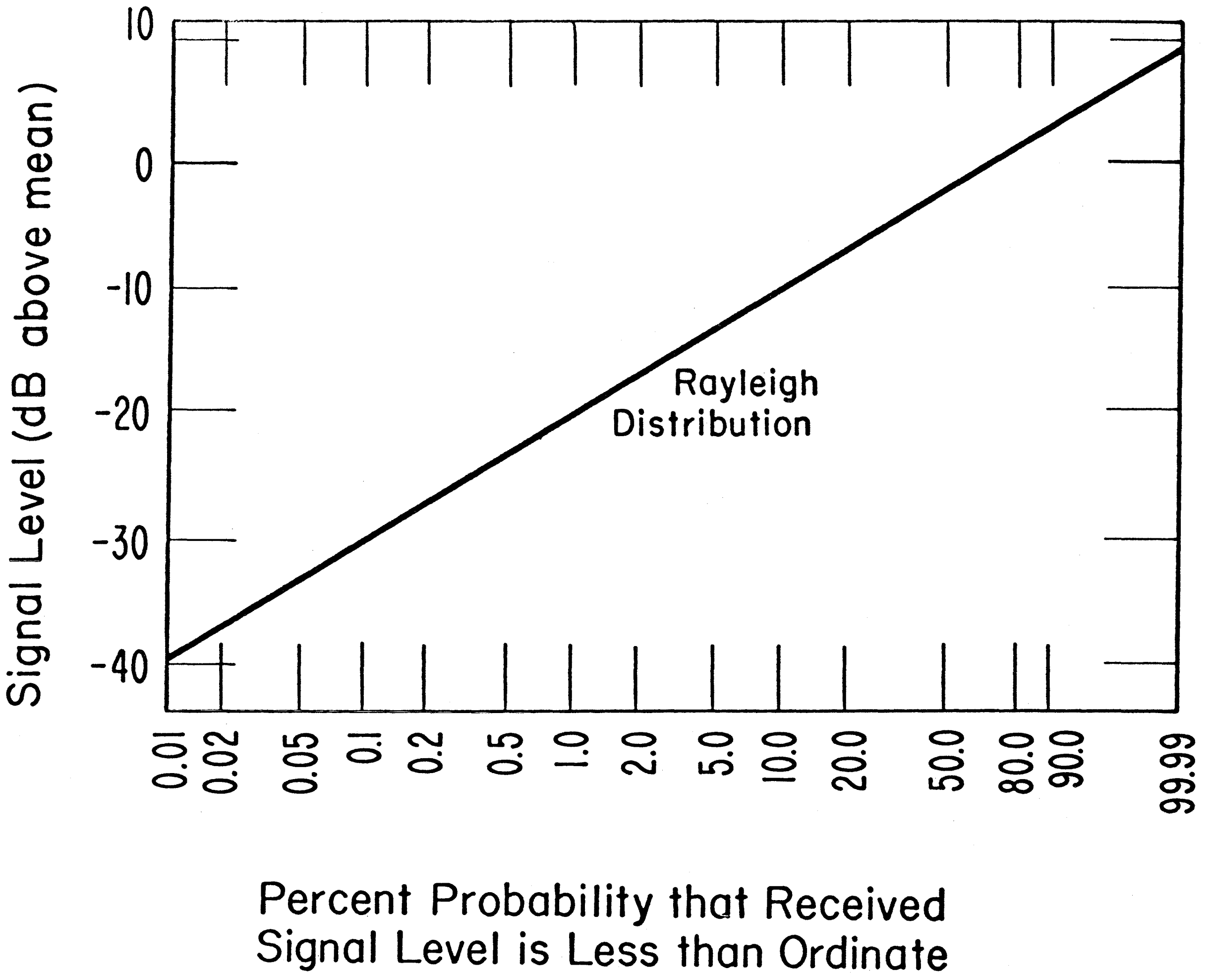


Figure B.2. Signal level statistics for a Rayleigh distributed received signal.

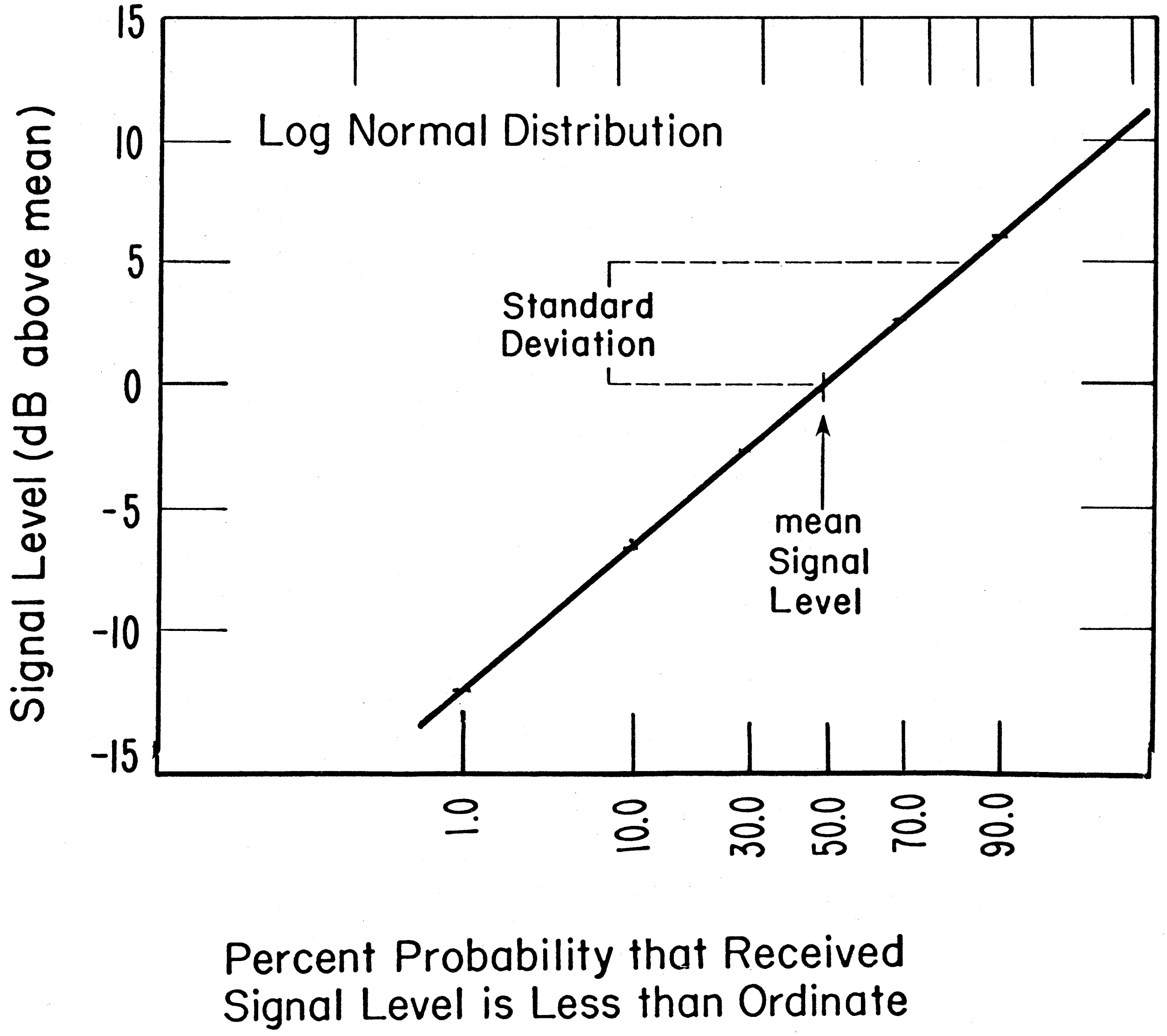


Figure B.3. Signal level statistics for a log normal distributed received signal.

### B.1.3 Noise and Interference in the LMR Channel

The noise experienced by a land-mobile receiver comes from sources which are both internal to the receiver and external. However, the dominance of one noise source over the other depends upon the receiver's operating frequency. Figure B.4 shows this dependence of the received noise power on frequency. At frequencies above 2 GHz, the receiver's own internal noise usually dominates. Below 2 GHz in the urban environment and below 500 MHz in the suburban environment, man-made impulsive noise dominates; and below 500 MHz in the quiet rural areas, atmospheric and galactic noise sources are predominant.

Cline (1973) shows that spread-spectrum systems provide no noise power reduction when only wideband Gaussian noise is present; i.e., the noise power at the input to the receiver's IF filter is the same regardless of whether or not spread-spectrum modulation is used ahead of the filter. We shall show later that spread-spectrum systems do show an improvement when the noise is narrower than the spread signal bandwidth. Thus, in most cases, the spread-spectrum systems offer no improvement with impulsive noise present since impulsive noise, both man-made and atmospheric noise, is very wide bandwidth. For example, automotive ignition sparks are about 4 ns in duration, which relates to a 3 dB bandwidth of about 500 MHz. This is certainly wider than conceivable land-mobile spread-spectrum bandwidths.

In many cases, land-mobile radio systems are not noise limited, but they are interference limited. In large metropolitan areas, there may be thousands of users all trying to share the same few multiple-user LMR channels. As a result, channel discipline is tossed aside and it becomes "everyone for himself." In this environment, the user learns to tolerate the "party-line" type of co-channel interference and to recognize those messages directed toward him. There are methods to reduce the interference to which the user has to listen, such as tone-coded squelch; however, if two co-channel users attempt to transmit simultaneously, the receiver will detect both signals with a possible result of a garbled message. On the other hand, spread-spectrum systems result in signal-to-interference ratio improvements, if the interference is narrowband.

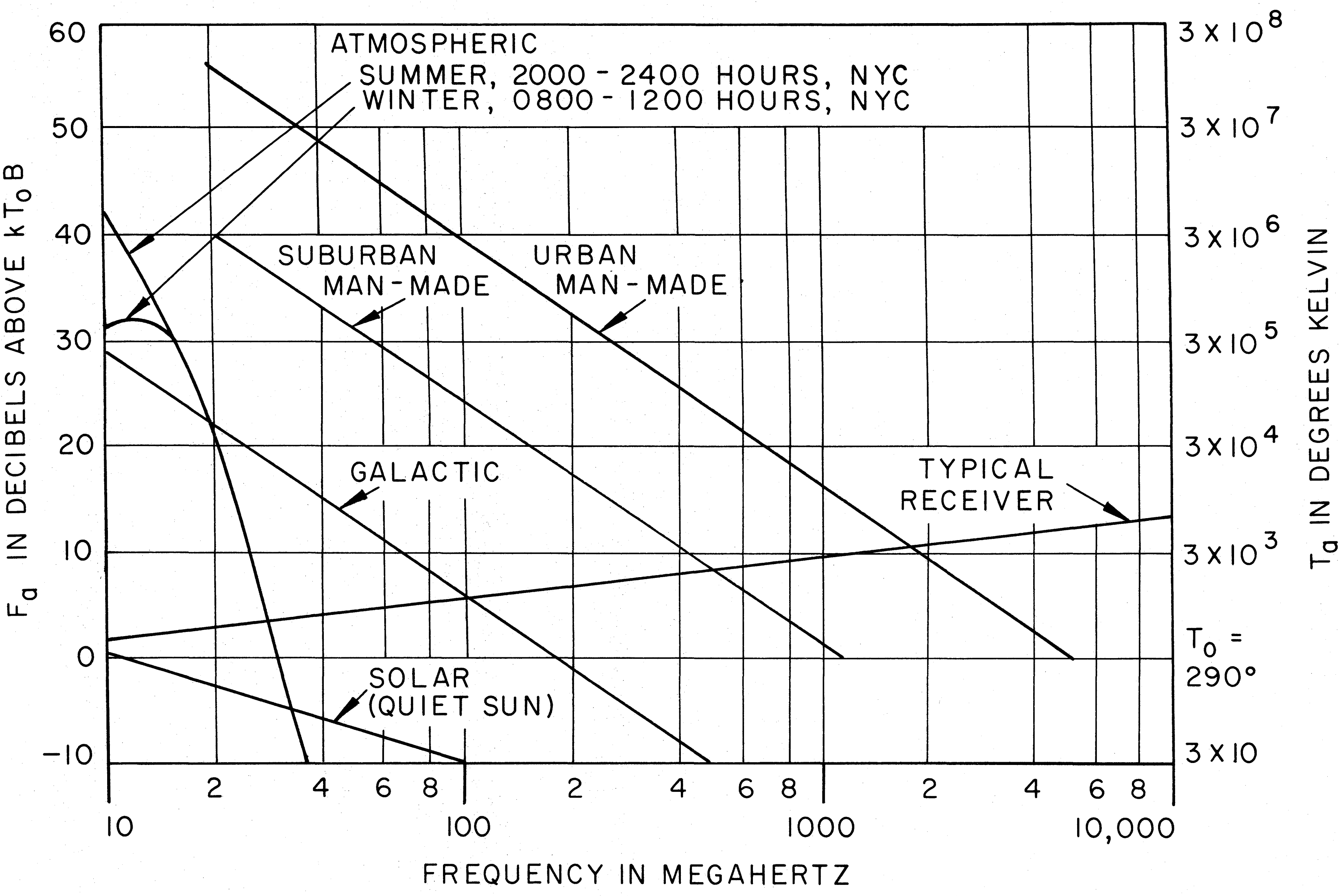


Figure B.4. Sources of receiver noise versus frequency (ITT, 1973).

## B.2 LAND-MOBILE RADIO SIGNAL CHARACTERISTICS

### B.2.1 Conventional LMR Modulation-FM

In the past, land-mobile radio has been used primarily for the transmission of analog voice, which has a bandwidth of about 3 kHz at baseband. The FCC rules for nongovernment land mobile allow F3 (FM) modulation to be used with 5 kHz frequency deviation in the 25 kHz authorized channels and 15 kHz deviation in the 50 kHz channels (FCC, 1974). Frequency modulation is preferred because of the FM improvement factor, but A3 (telephony) modulation also is allowed.

For either F3 or A3, analog voice is the signal that modulates the carrier. However, for spread-spectrum systems, digital signals are preferred over analog as the source signal; thus digitized voice and digitized data signal characteristics are discussed next.

### B.2.2 Digitized Voice

To digitize voice, the voice signal must be sampled at a sufficient sampling rate, and a sufficient number of quantization bits must be used so that the message is understandable when the digital signal is converted back to analog voice. However, listener intelligibility or tolerance allows quite a large range in data rates. Rates from 10000 bps to 64000 bps have been suggested, with a rate of about 30000 bps giving acceptable performance (assuming 8 bits of quantization per sample and 6000 samples per second).

### B.2.3 Digitized Data

For some applications of LMR such as in the land transportation (taxis) and public safety (police and fire protection) services, the use of hard-copy terminals is desired. This allows the user to have a permanent copy of the message, but it also requires that digital data be transmitted. Although information can be transmitted faster as digital data than as analog voice, the digital data must be coded carefully to detect and correct errors which result when the digital signal fades into the noise. A listener can adapt his comprehension of a voice message as the voice signal fades; i.e., as the signal fades, the listener usually can guess what was lost in the message because of the message context on either side of the fade. Or if the listener fails to comprehend because of the noise, he can ask for a repeat of the critical parts of the message.

Kelly and Ward (1973) suggest that data rates of 2000-2400 words per minute are a reasonable goal for law enforcement digital communications with vehicle printers; normal voice is about 150 words per minute. They also recommend that the digital rates be from 2000-10000 bps to achieve the desired data rate and to improve the channel utilization. Since the digital data rates (up to 10 kbps) are less than the digital voice rates (about 30 kbps), we only need to consider the spread-spectrum system implications when digitized voice is to be transmitted.

## B.3 SPREAD SPECTRUM IN LM RADIO

### B.3.1 Spread-Spectrum Concept

In most instances, use of more spectrum than is really necessary to transmit information is wasteful. For example, voice occupies about 3 kHz of spectrum and can be transmitted as single sideband amplitude modulation (AM) in about 3 kHz bandwidth or as double sideband AM in about 6 kHz. An exception is made to "the transmission bandwidth should equal the information bandwidth" rule when something is gained by going to wider transmission bandwidths. For example, wideband frequency modulation (WBFM) typically uses transmission bandwidths which are much wider than that of the voice or information source bandwidth. In this case, the gain for using the wider bandwidth is in the signal-to-noise ratio improvement, called the FM improvement. Another benefit is closer frequency re-use distances resulting from signal-to-interference improvement (captive effect).

Spread spectrum is another exception. In fact, the definition of a spread-spectrum system is one which meets two criteria (Dixon, 1976):

(1) the transmitted bandwidth is much greater than the bandwidth of the information being sent, and

(2) some function other than the information being transmitted is employed to determine the resulting modulated rf bandwidth.

Although WBFM meets the first of these criteria, it doesn't meet the second. The function used by most spread-spectrum systems is a code sequence that periodically repeats itself.

The desired properties of spread-spectrum systems listed by Dixon (1976) include the following:

(1) selective addressing,
(2) code division multiplexing,
(3) low power spectral densities,
(4) message privacy,
(5) high resolution ranging, and
(6) interference rejection.

Items (1), (2), and (6) are the primary advantages to be gained by using spread-spectrum techniques in land-mobile radio, along with instant access to the channel.

### B.3.2 Types of SS Techniques and Modulation

The types of techniques used in spread spectrum are as follows:

(1) direct sequence,
(2) frequency hopping,
(3) chirp, and
(4) hybrid of the above methods.

The direct sequence technique usually uses PSK (phase-shift-key) or FSK (frequency-shift-key) modulation of the carrier. Other spectrum conserving modulation methods such as MSK (minimum-shift-key) also are used and, in fact, are sometimes preferred.

The frequency hopping technique has available m discrete carrier frequencies, each separated by the width of the information bandwidth. Thus, the total bandwidth is m times the information bandwidth. The transmitter's carrier frequency jumps from one of the m frequencies to another in a prescribed manner known to both the transmitter and the receiver.

Chirp spread-spectrum systems have primarily radar and navigation applications. Chirp signals are generated by sweeping the carrier from a low frequency to a high frequency, delta F, in a certain time period, T. The resultant signal is spread over the bandwidth of delta F, or more, for the time duration T. In the receiver, the chirp signal is processed by a filter with the characteristic that the lower frequencies in the chirp signal are time delayed longer than the higher frequencies. The result is that all of the frequencies try to exit from the filter at the same instant. The collapsed signal is like an impulse at the filter's output.

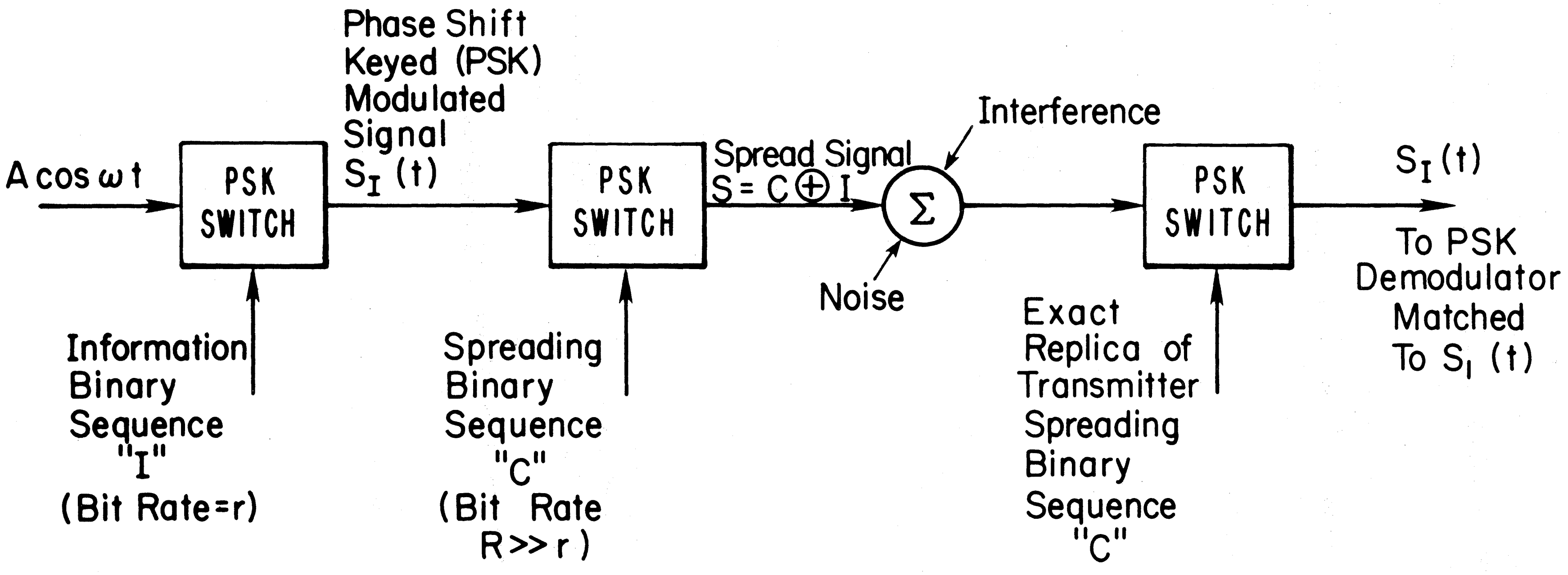
Frequency synthesizers with the range and speed required are probably too expensive to compete in the business LMR market. So in our discussions for land-mobile applications, we will concentrate on the direct sequence methods and, in particular, the PSK and FSK forms of modulation.

### B.3.3 Spread-Spectrum Processing Gain

A direct sequence spread-spectrum system transmits and receives a code whose sequence is known to both the transmitter and receiver. Because the receiver does not initially know exactly where in the code sequence the transmitter will start and because of the time lags in propagation and component transit, the receiver must first acquire, synchronize, and track the transmitted code. Once the receiver is "locked" to the transmitter, it can decode the information.

The code occupies a wide bandwidth relative to the information's bandwidth. For each information bit, there will be 100 to 1000 (or more) code bits or chips that are transmitted and received. The system can achieve a processing gain if the information is superimposed on the code. Because of this superposition, the narrow information bandwidth is spread out to the wide code bandwidth. The receiver correlates the received signal with its own replica of the code. If the receiver code is locked to the transmitter code, then the output from the receiver correlator is the narrow bandwidth information. This signal passes through a narrowband filter which is a matched filter to the information. It is the correlating and filtering processes which give the receiver its processing gain to interference.

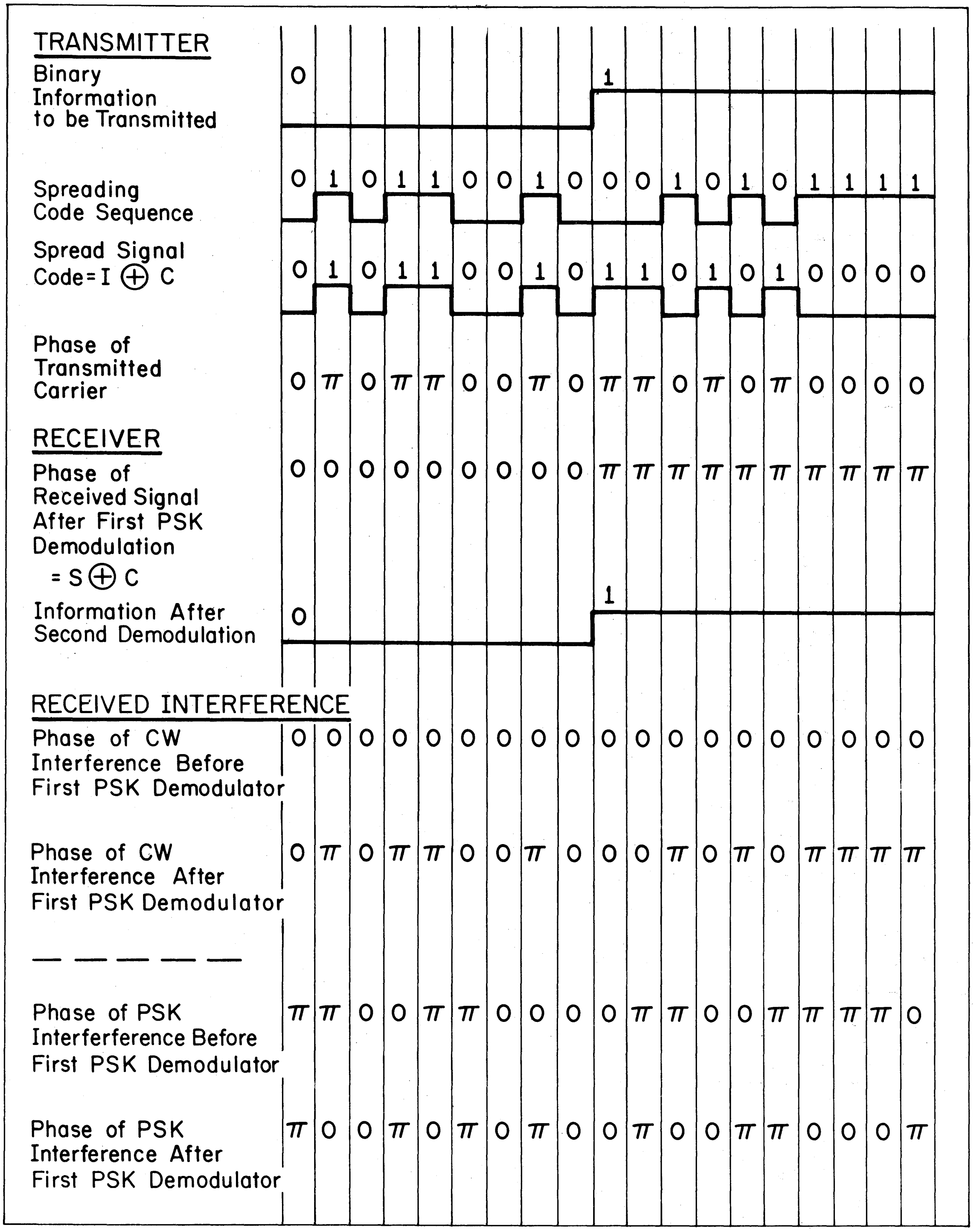
As an example, consider a spread-spectrum system where both the information and code are used to PSK (phase-shift-key) modulate the carrier (see Figure B.5). The information is a binary sequence of 1's and 0's at a bit rate of r bits per second. The code is also a binary sequence, but at a much higher bit rate of R chips per second. The information bit stream phase modulates the carrier at the first PSK modulator, and the code bit sequence further modulates the carrier at the second modulator. An example of the bit sequences and the output carrier phase sequences is shown in Figure B.6. Note the long period of the information bit relative to the code bit. After



$$S_I(t) = \begin{cases} A\cos(\omega t) & \text{if } I = 0 \\ A\cos(\omega t + \pi) & \text{if } I = 1 \end{cases}$$

$$S = C \oplus I = \begin{cases} A\cos(\omega t) & \text{if } I = 0,\ C = 0 \\ A\cos(\omega t + \pi + 0) & \text{if } I = 1,\ C = 0 \\ A\cos(\omega t + 0 + \pi) & \text{if } I = 0,\ C = 1 \\ A\cos(\omega t + \pi + \pi) = A\cos(\omega t) & \text{if } I = 1,\ C = 1 \end{cases}$$

Figure B.5. An example spread-spectrum system using direct sequence phase-shift-keying modulation.



| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **TRANSMITTER** | | | | | | | | | | | | | | | | | | | |
| Binary Information to be Transmitted | 0 | | | | | | | | | 1 | | | | | | | | | |
| Spreading Code Sequence | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| Spread Signal Code = I ⊕ C | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Phase of Transmitted Carrier | 0 | $\pi$ | 0 | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | 0 | $\pi$ | $\pi$ | 0 | $\pi$ | 0 | $\pi$ | 0 | 0 | 0 | 0 |
| **RECEIVER** | | | | | | | | | | | | | | | | | | | |
| Phase of Received Signal After First PSK Demodulation = S ⊕ C | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\pi$ | $\pi$ | $\pi$ | $\pi$ | $\pi$ | $\pi$ | $\pi$ | $\pi$ | $\pi$ | $\pi$ |
| Information After Second Demodulation | 0 | | | | | | | | | 1 | | | | | | | | | |
| **RECEIVED INTERFERENCE** | | | | | | | | | | | | | | | | | | | |
| Phase of CW Interference Before First PSK Demodulator | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Phase of CW Interference After First PSK Demodulator | 0 | $\pi$ | 0 | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | 0 | 0 | 0 | $\pi$ | 0 | $\pi$ | 0 | $\pi$ | $\pi$ | $\pi$ | $\pi$ |
| — — — — — | | | | | | | | | | | | | | | | | | | |
| Phase of PSK Interferference Before First PSK Demodulator | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | $\pi$ | 0 | 0 | 0 | 0 | $\pi$ | $\pi$ | 0 | 0 | $\pi$ | $\pi$ | $\pi$ | $\pi$ | 0 |
| Phase of PSK Interference After First PSK Demodulator | $\pi$ | 0 | 0 | $\pi$ | 0 | $\pi$ | 0 | $\pi$ | 0 | 0 | $\pi$ | 0 | 0 | $\pi$ | $\pi$ | 0 | 0 | 0 | $\pi$ |

Figure B.6. Examples of transmitted SS codes, received SS signals, and received interference.

passing through the first PSK demodulator, the received signal is stripped of the rapidly changing phase due to the code, and all that remains is the slowly varying phase due to the information. This signal passes through a narrowband filter, and the information bit sequence is recovered.

Note, in Figure B.6, the results when cw interference or an unmatched PSK interference signal is received. The correlator switches the constant phase of the cw signal at a rate equal to the code sequence. The result is that the cw interference spreads from an infinitesimally small bandwidth to a bandwidth equal to the code's bandwidth. Also, the PSK interference has its phase altered by the correlator, and its resulting bandwidth is either equal to or greater than the correlator code bandwidth. Thus only the desired signal becomes narrowband after passing through the receiver correlator.

Figure B.7 shows the spreading and despreading of the signals and interference. The information, with its relatively slow bit rate, has a narrow $[(\sin x)/x]^2$ amplitude. The code has a wide $[(\sin x)/x]^2$ amplitude spectral shape due to its fast bit rate. When the information is superimposed on the code, no noticeable change can be noted in the spectral shape. Then, when the receiver correlates its code with the signal, the information spectral shape is recovered. Notice, in Figure B.7, what happens to the spectral shapes of the cw interference and the PSK interference as they pass through the receiver correlator.

The spread-spectrum receiver's processing gain is defined by

$$G_p = \frac{B_{rf}}{B_{info}} \quad ,$$

where $B_{rf}$ is the rf bandwidth of the transmitted spread-spectrum signal and $B_{info}$ is the information bandwidth (Dixon, 1976). For the PSK example given, the signal-to-noise ratio output of the receiver's second PSK demodulator is

$$S/N_{out} = \left(S/N_{in}\right) G_p \quad ,$$

where $S/N_{in}$ is the signal-to-noise ratio at the input to the receiver's first PSK demodulator. Suppose the required output S/N is 12 dB and the processing is 20 dB; then the input S/N can be as low as -8 dB and the system will perform as required.

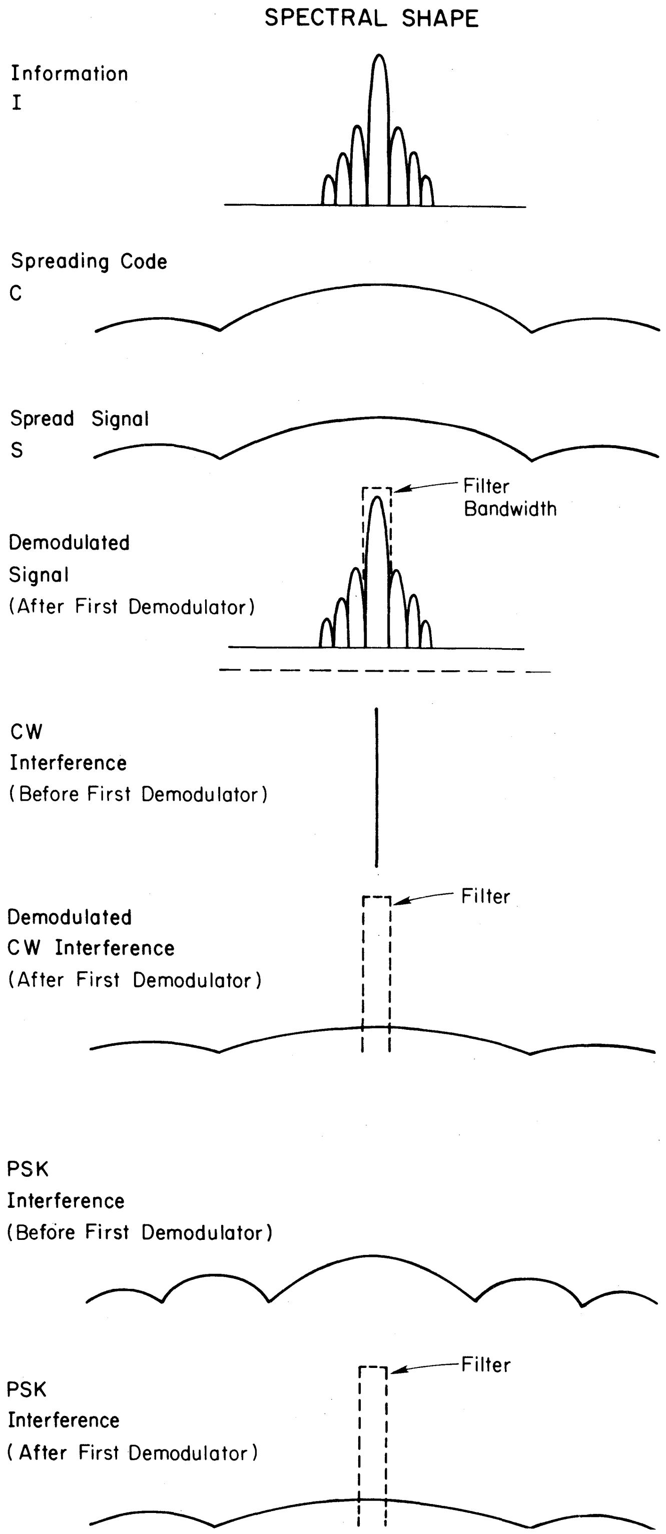


Figure B.7. Spectral shapes of the information, SS codes, and processed SS signals and interference.

### B.3.4 Required Bandwidth for Desired Processing Gain

For land-mobile spread-spectrum systems, the processing gain should be at least 20 dB, or a 100 to 1 code rate to information rate ratio. Previously, we suggested that digitized voice required from 10000 bps to 64000 bps. Assuming this is the range of the information rate, the required rf bandwidth would have to be from 1 to 6.4 MHz.

For illustration, we choose a digitizing rate of 30 kbps. Although this is not strictly a part of the spread-spectrum process, by digitizing the voice we have spread its spectrum from 3 kHz to 30 kHz. In demodulating and converting the digital 30 kHz signal back to the analog 3 kHz signal, we have an available processing gain of 10 dB. This can be added to $G_p$, less any realization losses.

### B.3.5 Spread-Spectrum System Synchronization Problems

Code synchronization has to be accomplished before any information can be received, and synchronization is probably the most difficult function that any SS system has to perform. In addition, code acquisition and synchronization problems are compounded when interference is present. Conceptually, the receiver must slide its replica of the code along with the incoming code until they are matched in time or correlated. The shorter the code, the faster the correlation can be made. Here then is the trade-off. Shorter codes result in shorter acquisition and synchronization times, but longer codes result in much better cross-correlation characteristics. We would expect that acquisition times longer than 1 sec would be unacceptable in LMR. The SS LMR receiver must be synchronized within 1 sec after the user at the transmitter end has pushed his microphone button to talk.

Two possible solutions to the synchronization problem are suggested. The first is a synchronization preamble; the second is the use of passive spread-spectrum devices.

Synchronization preambles are described by Dixon (1976) as an effective technique for establishing synchronization between push-to-talk systems. At the beginning of each transmission, a well-chosen code sequence (preamble) is transmitted. The receiver's acquisition time depends upon the preamble's length. Consider the following example. A spread-spectrum system has 3 MHz rf bandwidth, a 30 kHz information bandwidth, and a 4095 bit synchronization preamble. What is the minimum synchronization time for this system? If the synch recognition rise time is assumed to be $0.35/(30 \times 10^3) = 11.67$ µs and if the 4095 bits are searched at 1/2 bit per search, then at least $4095 \times 2 \times 11.67 \times 10^{-6}$ seconds or 0.095 seconds would be required for acquisition. Without a technique like synchronization preambles, the acquisition time for the same system could take several seconds.

Passive spread-spectrum devices include Surface Acoustic Wave (SAW) devices and Charge Coupled Devices (CCD). For either device, the code is programmed by the user who chooses the tap selection, or it is fixed at the time of the device's fabrication. Because SAW's and CCD's are passive, the SS receiver doesn't have to search the received code to establish correlation. The SAW's and CCD's are matched filters only to one code (usually from a matching SAW or CCD) and whenever that code is transmitted, the receiver is able immediately to despread the total code with no acquisition time requirements or synchronization constraints. These devices have the most promise for SS LMR applications, but have limited processing gain.

### B.3.6 Spread-Spectrum LMR in Multipath

The SS LMR system is a digital system; thus we need to investigate what happens to the bit error rate or probability of a bit error, $P_e$, when the receiver operates with Rayleigh fading. Figure B.8(a) (from Schwartz et al., 1966) shows the effects of a nonfading channel and a Rayleigh fading channel on both PSK and FSK. The probability of a bit error is plotted against the energy per bit-to-noise power density ratio, $E_b/N_o$. The effects of Rayleigh fading are dramatic; an $E_b/N_o$ of 9 dB in the nonfading channel gives a very satisfactory bit error probability of $10^{-5}$, but in the fading channel gives an unacceptable bit error probability of $5 \times 10^{-2}$.

Figures B.8(b) through B.8(h) (developed by Cline, 1973) demonstrate the improvement of using a PSK spread-spectrum system in multipath. Using Cline's notation, G represents the SS processing gain, $K_1$ is the power division factor between the direct and reflected signal paths (e.g., $K_1 = 0$ means there is no multipath signal and $K_1 = 1$ means the reflected and direct signals are equal in power), $K_2$ is the multipath time spreading factor relating the spreading code bit or chip length to the average multipath time spread, and $\rho$ is the correlation coefficient between the direct and reflected signals

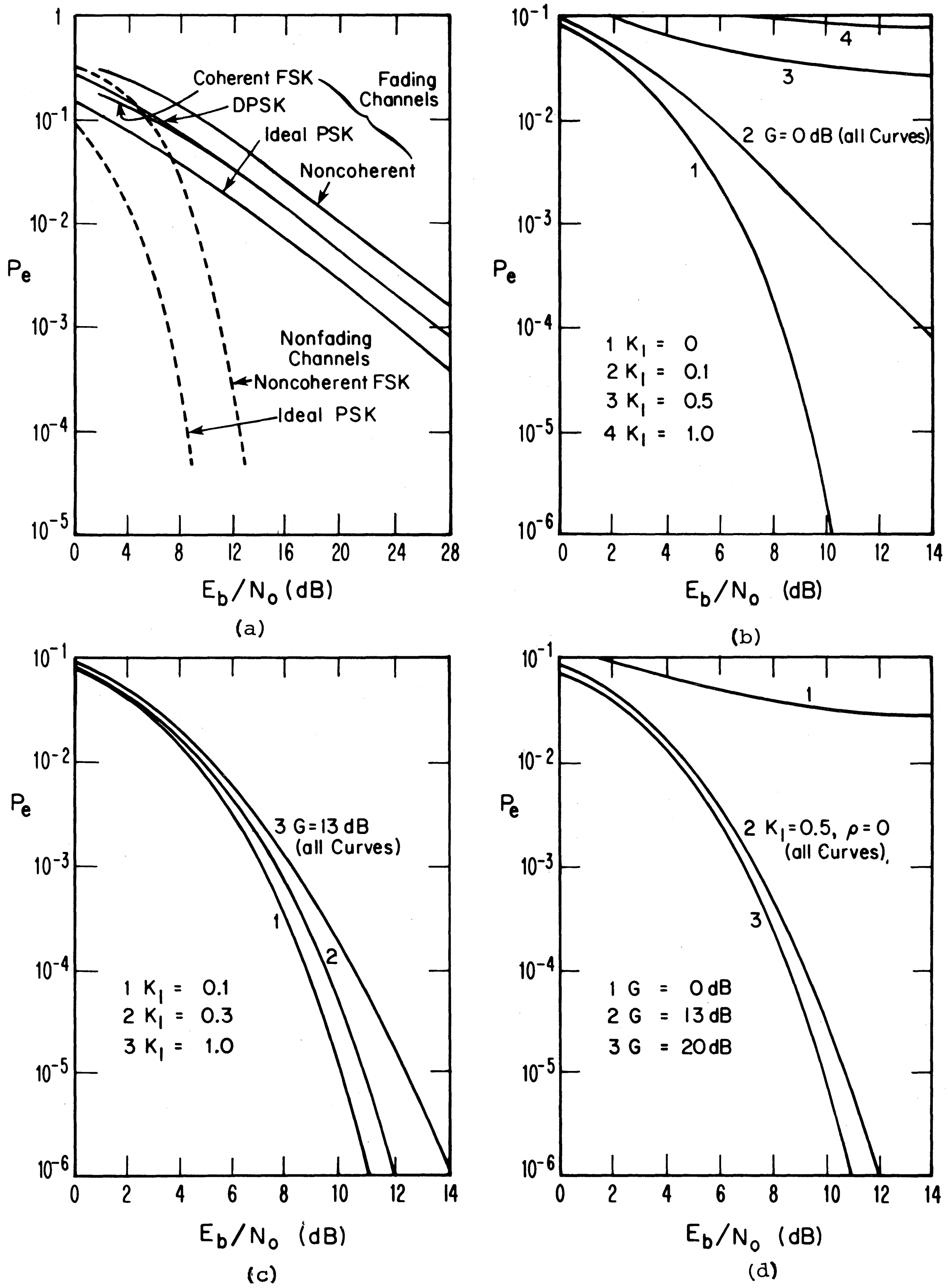


Figure B.8. Comparison of FSK, PSK, and SS system performance in Rayleigh fading.

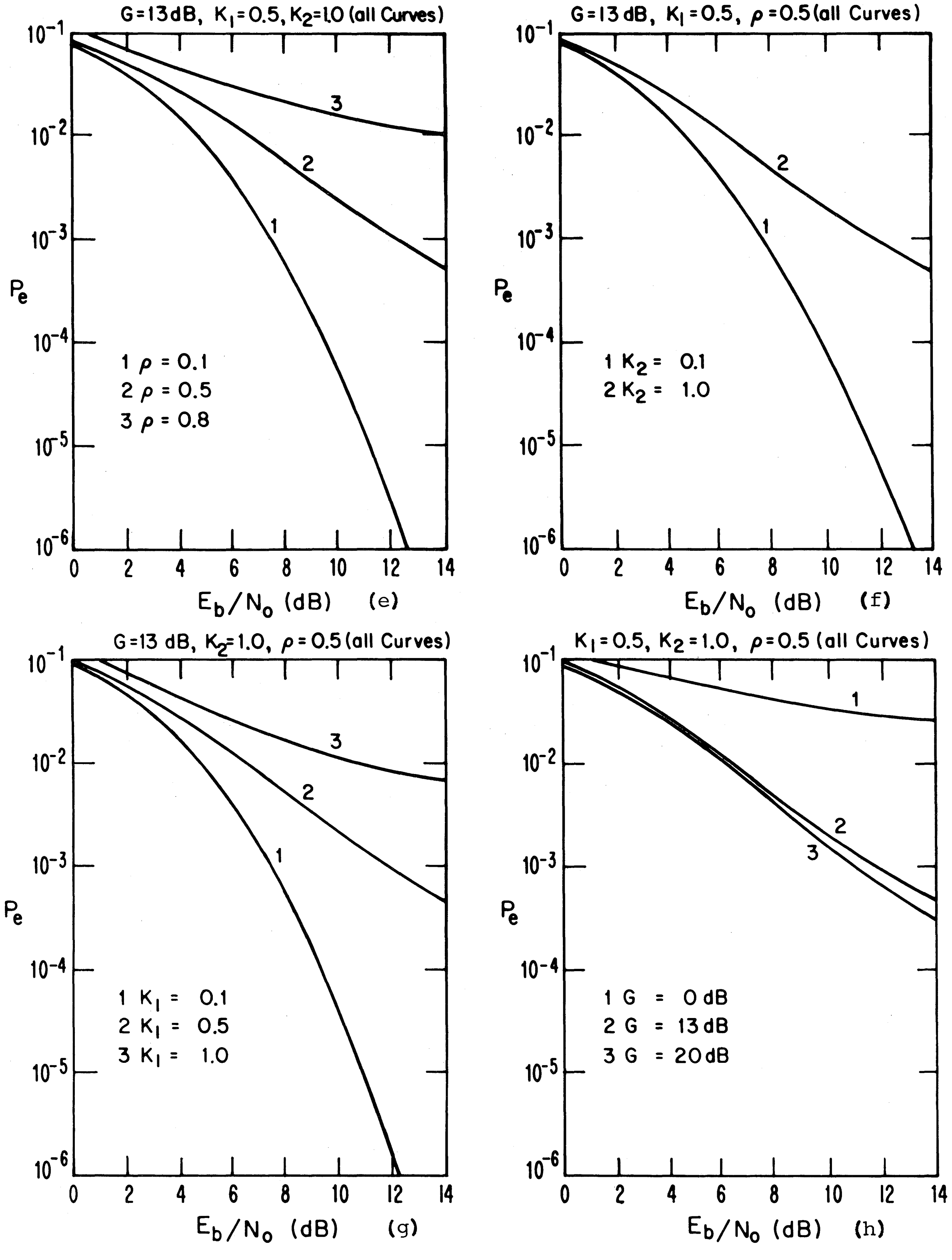


Figure B.8. (continued) Comparison of FSK, PSK, and SS system performance in Rayleigh fading.

(for $\rho = 1$ the reflected signal is specular, whereas when $\rho = 0$ it is diffuse).

Compare Figures B.8(a) and (b). Note that the curve labeled 1 in Figure B.8(b) is identical to the curve in Figure B.8(a) labeled ideal PSK in a nonfading channel. Curves 2, 3, and 4 are to be compared with the ideal PSK in a Rayleigh fading channel curve. Curves 2, 3, and 4 consider static relationships between the direct and reflected signal whereas the fading PSK curve of Figure B.8(a) is dynamic (see Figure B.2).

Figure B.8(c) shows the dramatic improvement available with PSK spread spectrum using a processing gain of 13 dB. Increasing the gain offers little improvement as shown in Figure B.8(d).

Figures B.8(e) through B.8(h) show the effects of multipath when it isn't entirely diffuse. Figure B.8(e) considers the case when the multipath energy is within the SS receiver's correlation filter window (i.e., the direct signal and the multipath signal are separated by less than one chip's time length). Note that as the multipath gets more diffuse, the spread-spectrum system's multipath rejection improves. Figure B.8(f) shows that as more of the multipath energy is outside of the correlation window (i.e., $K_2$ approaches 0), the SS system's multipath rejection again improves. Finally, Figures B.8(g) and (h) demonstrate how the SS system behaves with nondiffuse multipath that is within the correlation filter window as a function of $K_1$ and G.

Cox (1972 and 1977) shows measurements of the average multipath time delay spreads in urban and suburban environments.

In all cases, spread-spectrum system performance is markedly better than nonspread-spectrum system performance. Of course, this improvement has been bought with the additional bandwidth.

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The use of spread-spectrum modulation in a common type of land-mobile radio service is studied. The radio service contains many networks operated independently, similar to the business land-mobile radio service. Network independence implies (among other things) that there is no central control of a transmitter's radiated power. A congested urban environment is assumed.

If a band is allocated exclusively to spread-spectrum modulation for such a service, base stations and mobiles must operate on different channels, and all base station antennas must be located within a relatively small area. In this case, the worst interference is in the mobile-to-base channel.

An explicit formulation of the "far-near" problem and a powerful computer calculation both show that in such a service spread-spectrum modulation is less spectrum efficient than conventional FM modulation. In fact, the spectrum efficiency of the spread-spectrum systems is inversely proportional to the cube root of the processing gain. However, overlaying a spread-spectrum system on conventional bands shows some promise.

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