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Three Phases of Teletraffic Congestion in Military Access Areas

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The views, opinions, and/or findings contained in this report are those of the authors and should not be construed as an official Department of the Army position, policy, or decision unless designated by other official documentation.

PREFACE

The study reported here was performed for the U. S. Army's Communications Systems Agency (CSA), Fort Monmouth, NJ, by the Institute for Telecommunication Sciences (ITS), Boulder, CO. It is part of a continuing program in support of that agency's Access Area Digital Switching System (AADSS) program on project orders 501-RD, 804-RD, 809-RD, and 906-RD.

Previous related study reports are as follows:

"Parametric Cost Alternatives for Local Digital Distribution Systems," M. Nesenbergs and R. F. Linfield, OT Report 76-95, March 1976.

"Preliminary Evaluation of Hub Alternatives for Access Area Digital Switching," J. C. Blair, Special unpublished ITS report to CSA, October 1977.

"Access Area Switching and Signaling: Concepts, Issues, and Alternatives," R. F. Linfield and M. Nesenbergs, NTIA Report 78-2, August 1978.

"Control Signaling in a Military Switching Environment," R. F. Linfield, NTIA Report 79-13, January 1979.

"Switch Element Capacities in Access Area Digital Switching Systems," R. F. Linfield and M. Nesenbergs, NTIA Report 79-26, September 1979.

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THREE PHASES OF TELETRAFFIC CONGESTION IN MILITARY ACCESS AREAS

Martin Nesenbergs and Robert F. Linfield*

In military base access areas, akin to many commercial installations, the telecommunications traffic passes through several concentration and switching stages. Loads to server facilities are formed through mergers and branch outs of offered traffic substreams. Blocking of calls is known to occur in many ways throughout the existing access area networks.

In this report, an effort is made to represent the access area grades of service (i.e., the probabilities of blocking for different substreams) in more realistic ways than before. The message flow process is structured into three representative contention phases. The three phases are realistic and occur often in military networks. All three phases apparently possess queueing models and analytical properties distinct from the conventional Engset, Erlang, and other classical models. Their blocking probabilities also differ significantly.

One of the three models appears tractable only through bounds and asymptotically tight approximations. The other two models are shown to permit formal solutions. Given an access area network, the three blocking probabilities may be applied individually or in a variety of combinations. The paper demonstrates several applications to access area telephony.

1. INTRODUCTION

The military access area telecommunications traffic possesses numerous characteristics. Perhaps of most visibility, are the impressive types of communication services provided to various terminal mixes of end-users. Of comparable importance, if not always to the system designers and large network managers, but certainly to individual users, is the quality of service that the individual user perceives.

This paper is concerned with one dimension of service quality, the so-called "grade of service". In the present context, grade of service is interpreted as a probability level of being blocked by the system. Thus, busy called parties and malfunctioning message receivers are ignored. This study further assumes that at least one end user of the communication transaction resides in a military access area.

The access area concept is illustrated in Figure 1 (Krevsky, et al., 1972; Levine, 1976). This figure stresses three interacting parts: the military access area (which is of primary interest to this report), the Defense Communications

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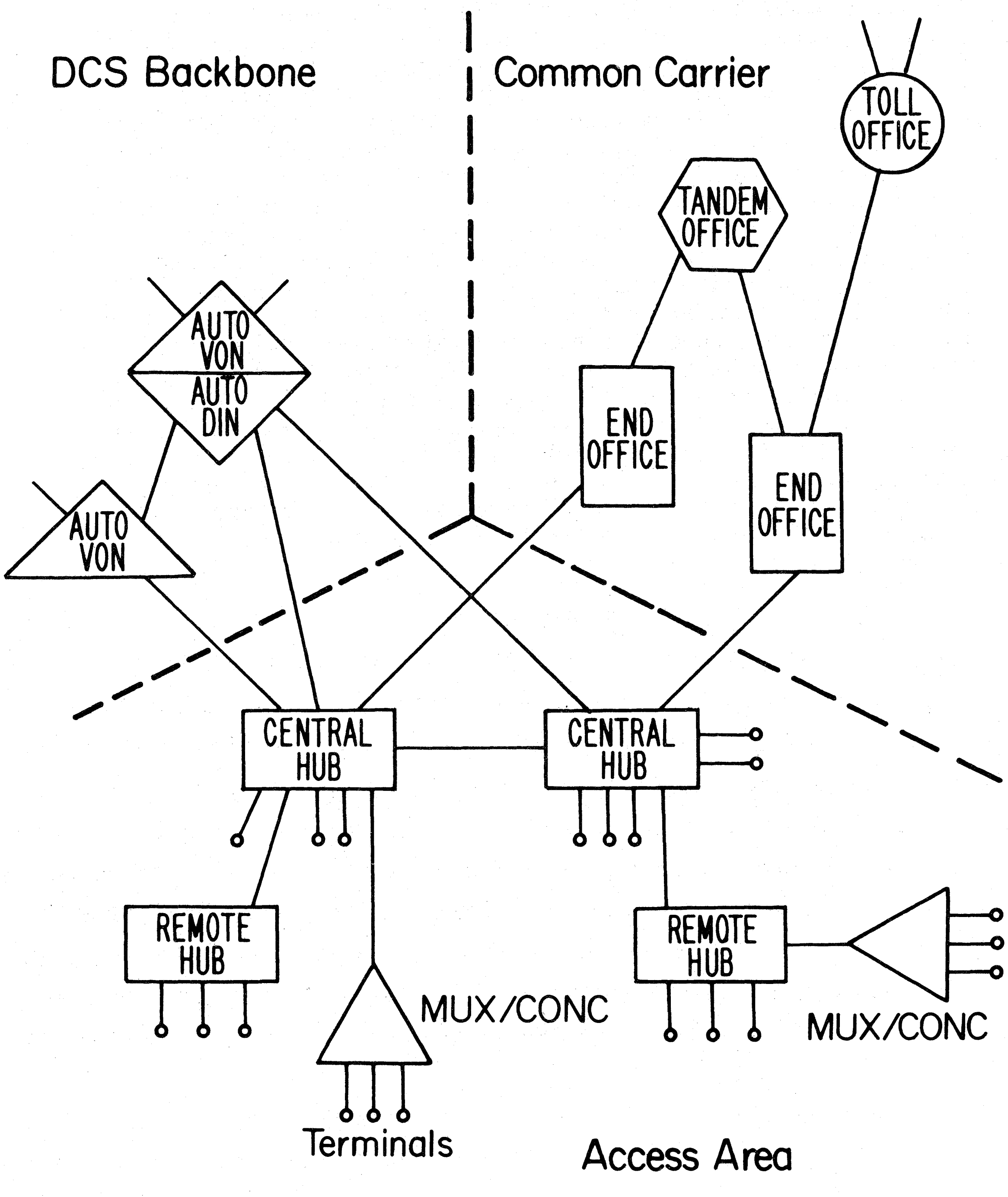


Figure 1. Basic deployment concept of a military access area.

System (DCS) backbone network, and the domestic common carriers (Linfield and Nesenbergs, 1978; Linfield, 1979).

The access area proper may be comprised of one or two, but not many more, military bases. The access area bases share parts of a common telecommunications network (Nesenbergs and Linfield, 1976). In addition to transmission facilities, the access area may contain central switching hubs (or switches, or hubs, for short), lesser switches (such as PABX's or PBX's), plus various concentration elements. Both multiplexers (MUX) and intelligent or statistical concentrators (CONC) are remotable devices found in cost effective designs of access area networks.

Externally, the access area interfaces with the previously mentioned DCS backbone and with the common carriers. The access area telecommunications traffic is either generated by the user terminals in that area or is destined for the terminals in the area. Seldom, if ever, does a military access area network play a tandem role for through traffic.

The access area traffic may be partly local, i.e., going to the same or to a geographically adjacent site, or it may be destined for some distant military region. In the latter case, the traffic is switched outward from or inward to the access area, either on the DCS backbone or via the common carriers. At the other end, the terminating address could represent a military terminal or a civilian (government, commercial, household, or institutional) installation.

When a circuit path is established, it can take one of several routes through the cooperating networks. The route may pass through the DCS, but not more than once. It may pass through the toll or local facilities of a common carrier, but also with high probability, not more than once. And, to be complete, at least once but certainly not more than twice, the route must traverse local access areas.

Given that individual military communications must take one of such paths, several traffic engineering questions arise immediately. Does the existing traffic engineering theory provide needed tools for access area system design of the 1980's? The classic work of Erlang (Brockmeyer, et al., 1948), Molina (1927), Fry (1928), and others provides a basic background. However, it hardly extends into the specifics of modern switched networks. More realism is needed. That realism is partly found in the neo-classic models of the fifties and sixties (Jacobeaus, 1950; Lee, 1955; Rönnblom, 1959; Lotze, 1963; Wolman, 1965; Lee and Brzozowski, 1966). Unfortunately, the complexity encountered by these writers is quite formidable. Their results appear difficult to interpret and to apply. It is therefore not surprising that recent workers have generated a resurgence of useful switched network characterization (Kuczura and Bajaj, 1977; Lin, et al., 1978; Nesenbergs, 1979; Manfield and Downs, 1979; Haugen, 1979). The present study follows in their wake by seeking reasonable accuracy, simplicity, and applicability to the access area.

Succinctly put, this report addresses the teletraffic blocking problem for a circuit switched access area. A simple and apparently effective sequential circuit flow description is developed. Starting with the next section, a set of three congestion or blocking phases are introduced. The intent is to represent the access area blocking events as uniquely identifiable, separate members of these blocking phases.

Thereafter, in later sections, probability of blocking formulas and curves are developed for the three phases (see Sections 3, 4, and 5). Section 6 summarizes these analytical and numerical tools by applying them to representative access area scenarios.

2. THREE PHASES OF BLOCKING IN ACCESS AREA

The previous section has alluded to the variety of routes that access area traffic may take. If so, then network blocking may occur in a number of ways and in a number of places.

Let us be more specific. Assume that the DCS backbone and common carrier nets have sufficient capacity to be essentially nonblocking. Blocking - if it occurs - must then occur within an access area. But there, as suggested in Figure 1, the route may encounter three kinds of nodal elements: the concentrator (CONC), the remote hub (or the small customized switch, or PABX), and the larger central hub or the dial central office (DCO). The latter is also variously referred to as the switch, the end office (EO), the local telephone exchange, or the Class 5 switching center.

In the originating access area, the route may pass through various combinations of CONC, PABX, and switches. The same is true at the destination access area, if a separate such area is involved. All total, some forty different route profiles may occur. They are tabulated in Table 1. Each row in this table represents an access area communications path. For instance, path #1 is one routed through a concentrator, followed by a PABX, plus the switch, all in the initial access area. In the final access area, as shown, it again passes through another switch, another PABX, and another concentrator.

Table 1. Various Route Profiles

Path #	Initial			Additional		
	CONC	PABX	Switch	Switch	PABX	CONC
1	X	X	X	X	X	X
2	X	X	X	X	X	
3	X	X	X	X		X
4	X	X	X	X		
5	X	X	X		X	X
6	X	X	X		X	
7	X	X	X			X
8	X	X	X			
9		X	X	X	X	X
10		X	X	X	X	
11		X	X	X		X
12		X	X	X		
13		X	X		X	X
14		X	X		X	
15		X	X			X
16		X	X			
17	X		X	X	X	X
18	X		X	X	X	
19	X		X	X		X
20	X		X	X		
21	X		X		X	X
22	X		X		X	
23	X		X			X
24	X		X			
25			X	X	X	X
26			X	X	X	
27			X	X		X
28			X	X		
29			X		X	X
30			X		X	
31			X			X
32			X			
33	X	X			X	X
34	X	X			X	
35	X	X				X
36	X	X				
37		X			X	X
38		X			X	
39		X				X
40		X				

The paths in Table 1 ignore everything that happens in the DCS backbone or in the commercial common carriers. For simplicity sake, they also ignore several local elements, such as operator consoles (often blocking), multiplexers (non-blocking), and others of lesser significance.

Nevertheless, the number of route profiles is considerable. And for any offered load distribution for different services, each of the forty paths is apt to have different grade of service characteristics. The task is considerably simplified if one can assign a unique probability of blocking feature to the three elements: the CONC, the PABX and the switch. In what follows, such a three-step approach is introduced.

Consider Figure 2. It defines the three phases of circuit blocking. The first phase of blocking occurs at a concentrator. Analytically, this will turn out to be the most unwieldy of the blocking phases. The complete problem setting of blocking Phase 1 and its approximate solution are presented in Section 3.

The second phase of blocking in Figure 2 is associated with a PABX. Looking from the user terminal side, the small PABX switch could be the first switching entity encountered by a station line. Lines can pass through concentrators or they can by-pass the concentrators and be directly patched to the PABX. The probability of blocking for Phase 2 is treated in Section 4.

The third and last phase of blocking is shown (Figure 2) to take place at a central switch. User terminals can be connected directly to the switch. Or they can be routed either through the concentrator, or the PABX, or both. The switch also has a long distance or toll side. For example, the right hand side of the switch in Figure 2 can be viewed as the collection of trunks to other DCS or common carrier switches. Section 5 discusses the Phase 3 blocking that occurs at such a switch.

3. BLOCKING PHASE 1

3.1. Problem Definition

Assume the m-server facility of Figure 3, where two types of calls contend for the services of a concentrator. In a nutshell, the two types are distinguished by the number of servers (e.g., one or two) that they require per call.

The users or sources are the M terminals. Typically, $M>m$. To simplify matters, assume exponential distributions for both the interarrival and service (holding) times. Let blocked calls be lost or cleared without any aftereffect. Let the average service times be unity for both call categories, denoted as (α)

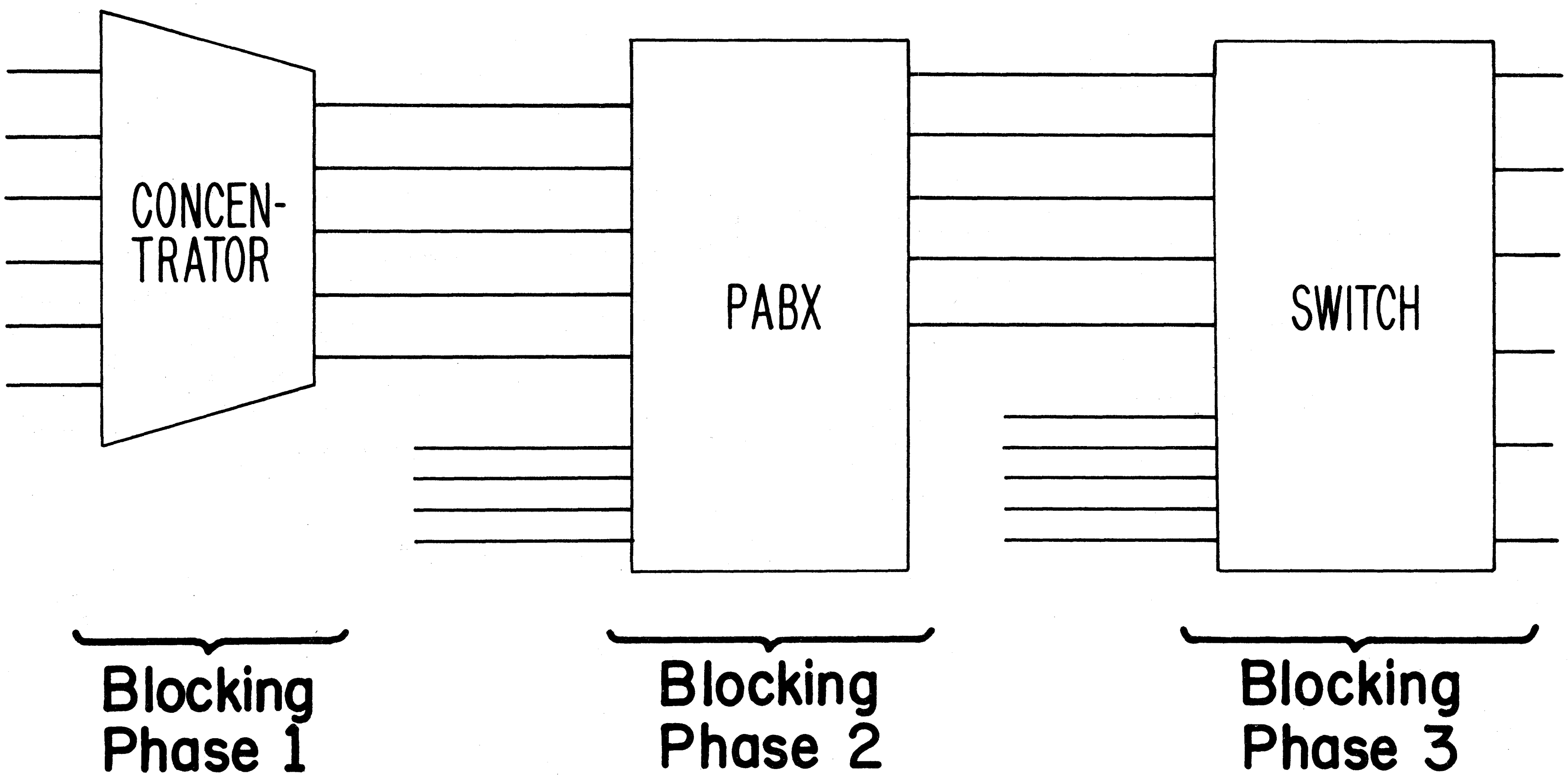


Figure 2. Three phases of blocking.

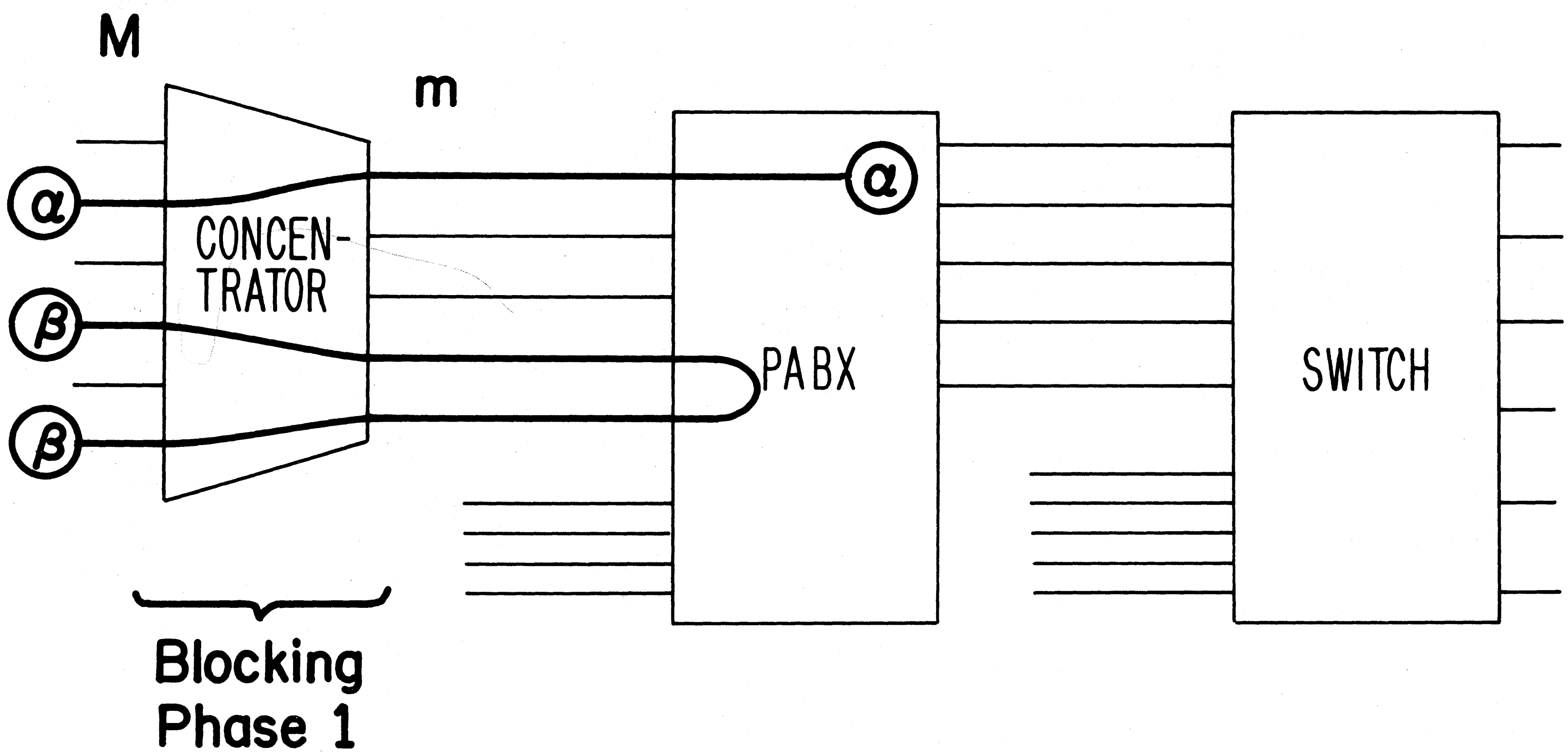


Figure 3. Phase 1 - blocking at a concentrator.

and (β) in Figure 3. In an informal way, one may refer to the (α) calls as "distant" calls, and to (β) calls as "local". However, one should note that (α) represents a set of paths with common properties. In Table 1, (α) is identified with any path that passes through a concentrator, except for #36. Path (β) depicts only #36.

Assume that the average arrival rates are different for the distant and local calls. Let "a" be the arrival rate per idle terminal for distant or (α) calls, and let "b" be the corresponding rate for local or (β) calls. Finally, assume that the concentrator and the PABX have full availability and that the PABX is effectively nonblocking, at least as far as the present Phase 1 blocking is concerned.

The problem here is to derive the blocking probabilities for the two types of calls. The given parameters are M, m, a and b. This is a queueing system or a birth-death problem of the M/M/m/0/M kind (Kleinrock, 1975). The next-to-last 0 refers to the lack of buffer storage in the assumed loss system, and the final M represents the finite user population. Systems of this type are known to have the desired steady state or equilibrium properties (Kleinrock, 1975; Saaty, 1961; Riordan, 1962). All state probabilities, including blocking probabilities, do exist. The usual derivation of state probabilities employs the flow rate conservation or statistical equilibrium difference equations. The same approach will be pursued here.

Let the integer pair (i,j) stand for the joint event or state, that i distant (α) and j local (β) calls are in progress. Let the equilibrium probability of state (i,j) be p(i,j). Then

$$\sum_{\text{all } i,j} p(i,j) = 1. \tag{1}$$

To get rid of trivial terms, require that p(i,j) vanishes for all $i<0$, all $j<0$, as well as for all $i+2j>m$.

Blocking probabilities are of interest here. Whenever $i+2j=m$, outgoing and incoming calls of type (α) are blocked. Likewise, the local return or (β) calls are blocked if and only if $i+2j\geq m-1$. One defines the respective blocking probabilities as,

$$P_o = \sum_{i+2j=m} p(i,j),$$

$$P_r = \sum_{i+2j\geq m-1} p(i,j), \tag{2}$$

where subscript o depicts the "out/in" nature of the (α) category, and r stands for the "return" or (β) calls.

For each (i,j), define the following column vector of state probabilities:

$$\underline{P} = \begin{bmatrix} p(i-1,j) \\ p(i,j-1) \\ p(i,j) \\ p(i+1,j) \\ p(i,j+1) \end{bmatrix}. \tag{3}$$

Vectors $\underline{P}$ play the role of unknowns. They must be solved using flow or transition-rate conservation equations. There are many such equations, applicable to different (i,j), and all of the form

$$\underline{C}^T \cdot \underline{P} = 0. \tag{4}$$

Here, $\underline{C}^T$ is the transpose of the column vector $\underline{C}$. Vector $\underline{C}$ consists of the appropriate flow rate or birth-death coefficients needed to conserve the outward and inward flows at the state (i,j). The dot ($\cdot$) in (4) stands for the ordinary inner product of two five-dimensional vectors.

The coefficient vector $\underline{C}$ is given in Table 2. The columns in the table are the column vectors. They depend on the system state (i,j) as shown by the three different columns. For non-vanishing p(i,j), one considers only (i,j) that satisfy $\min\{i,j,m-i-2j\}\geq 0$, as noted earlier. This obviates the need for special i=0 and j=0 columns, where the first and the second rows vanish, respectively and by definition.

Table 2 is the initial coefficient listing of this report. Others, such as Tables 3, 4 and 7, will follow. It seems helpful, at least initially, to explain the structure of a typical vector $\underline{C}$. Assume $i+2j\leq m-2$, which corresponds to the left column of Table 2. The easiest place to begin may be the third row. It depicts, see (3), the state (i,j) itself.

Given that the system is in the state (i,j), outward flows may be caused by several mechanisms. First, any of the i active (α) calls may hang up. Due to unity holding time, this happens with average rate i. Second, the j (β) calls cease with average rate j. Next, there are the two types of call arrivals. Since for the (i,j) state M-i-2j users are idle, they contribute (M-i-2j)a (α) service requests per unit time. Likewise, on the average, they generate (M-i-2j)b calls

Table 2. Column Vector $\underline{C}$ for Different (i,j) States

$i \geq 0 \quad , \quad j \geq 0$		
$i+2j \leq m-2$	$i+2j = m-1$	$i+2j = m$
$-(M-i-2j+1)a$	$-(M-m+2)a$	$-(M-m+1)a$
$-(M-i-2j+2)b$	$-(M-m+3)b$	$-(M-m+2)b$
$i+j+(M-i-2j)(a+b)$	$i+j+(M-m+1)a$	$i+j$
$-(i+1)$	$-(i+1)$	0
$-(j+1)$	0	0

of the (β) type. The total average outflow from the (i,j) state is their sum, namely $i+j+(M-i-2j)(a+b)$. It is the coefficient entry in the third row, first column, of Table 2.

The first row shows the flow into state (i,j) from state (i-1,j), see (3). The i increment is generated by (α) traffic from $M-i-2j+1$ idle sources. Their total average contribution is thus $(M-i-2j+1)a$. In Table 2 this entry carries a negative sign, because inflow counteracts the outflow.

The second row of Table 2 represents the (i,j) inflow from the state (i,j-1). It is a (β) flow from $M-i-2j+2$ idle users. The resultant coefficient in the second row is thus $-(M-i-2j+2)b$.

The fourth and fifth rows are similar. They give the inflows to state (i,j) from states (i+1,j) and (i,j+1), respectively. In both cases, the assumed unity average service time dictates that the coefficients are $-(i+1)$ and $-(j+1)$.

The net sum of the positive and negative products must vanish, see (4), to ensure flow equilibrium. That is the steady state solution p(i,j) for all i and j (Kleinrock, 1975, Chap. 3).

The solution to the equation set (3), (4), plus Table 2, appears unknown and difficult. To simplify, assume with no great loss of generality that both M and m are even numbers. Then one has $(\frac{m}{2}+1)^2$ unknowns, p(i,j), as well as the same number of equations (4). The set of equations turns out to be linearly dependent. Fortunately, that is a minor problem. Any equation may be replaced by the normalization condition (1).

A bigger problem is the number of equations. Consider a typical T1 or T2 span between the concentrator and the PABX. The T1 accommodates 24 PCM channels, the T2 up to 96. In the first case, the number of equations is 169, in the second case it is 2401. Inversion of matrices of such high dimensionality is not easy, even with the latest computer resources. Moreover, a single matrix inversion provides numerical answers only for the selected values of M, m, a and b. To depict a comprehensive grade of service picture, many tedious computer runs may be needed. To proceed, a different approach is tried here. An approximate model is proposed for Phase 1. The model provides the desired tight asymptotic fits plus bounds on blocking probability. The model is tractable and is given next.

3.2. The Approximating Model

Instead of the previous exact coefficient vector $\underline{C}$, consider two approximate vectors, $\underline{C}_\nu$, where $\nu=1,2$. Let the $\underline{C}_\nu$ vectors be defined by the state dependent columns of Table 3. As in Table 2, the first row vanishes when i=0, the second vanishes when j=0.

Table 3. Column Vectors $\underline{C}_\nu$ (ν=1,2) for Different (i,j) States

$i \geq 0 \quad , j \geq 0$		
$i+2j \leq m-2$	$i+2j = m-1$	$i+2j = m$
$-(M-\nu i-\nu j+\nu)a$	$-(M-\nu i-\nu j+\nu)a$	$-(M-\nu i-\nu j+\nu)a$
$-(M-\nu i-\nu j+\nu)b$	$-(M-\nu i-\nu j+\nu)b$	$-(M-\nu i-\nu j+\nu)b$
$i+j+(M-\nu i-\nu j)(a+b)$	$i+j+(M-\nu i-\nu j)a$	$i+j$
$-(i+1)$	$-(i+1)$	0
$-(j+1)$	0	0

The solutions generated by the approximate flow coefficient model, $\underline{C}_\nu$, will be denoted as $p_\nu(i,j)$. In general, state probabilities $p_\nu(i,j)$ must differ from the exact $p(i,j)$. However, there are at least two advantages for looking at the $p_\nu(i,j)$'s. First, one obtains closed form expressions. And second, in asymptotic and limit cases of interest the expressions agree with the true $p(i,j)$.

The solution to equations (1), (3), (4), and Table 3 is given by

$$p_\nu(i,j) = \frac{\binom{M/\nu}{i+j}\binom{i+j}{i}(\nu a)^i(\nu b)^j}{\sum_{\text{all } s,t}\binom{M/\nu}{s+t}\binom{s+t}{s}(\nu a)^s(\nu b)^t}, \tag{5}$$

where ν=1,2, and $\min\{i,j,m-i-2j\}\geq 0$. The proof is omitted here for reasons of brevity. It can be carried out by substituting (5), for all three cases of Table 3, into (1), (3) and (4). The procedure is long, but straightforward.

The asymptotic behavior of the approximating solutions (5) turns out to be related to the Engset distribution (Riordan, 1962, Chap. 5):

$$E_k(m;M,a) = \frac{\binom{M}{k}a^k}{\sum_{s=0}^{m}\binom{M}{s}a^s}, \tag{6}$$

where $k=0,1,2,\ldots,m$.

Consider first the case when the distant (α) traffic becomes negligible. Then the only nonvanishing terms occur at $i=0$, and

$$p(0,j) \xrightarrow[a\to 0]{} E_j(\tfrac{m}{2};\tfrac{M}{2},2b),$$

$$p_\nu(0,j) \xrightarrow[a\to 0]{} E_j(\tfrac{m}{\nu};\tfrac{M}{\nu},\nu b). \tag{7}$$

Thus, ν=2 offers the correct asymptotic fit for $a\to 0$. Next, assume that the local (β) traffic vanishes. Then $j=0$ is of interest and

$$p(i,0) \xrightarrow[b\to 0]{} E_i(m;M,a),$$

$$p_\nu(i,0) \xrightarrow[b\to 0]{} E_i(\tfrac{m}{\nu};\tfrac{M}{\nu},\nu a). \tag{8}$$

Now, contrary to the previous case, ν=1 gives the correct fit to the true $p(i,j)$.

Finally, one must not exclude the practically significant limit of infinitely many sources. One lets $M \to \infty$, while insisting that $Ma \to A$ and $Mb \to B$. Then, as is easily shown (Rönnblom, 1959), all solutions tend to the same limit:

$$\left\{ \begin{matrix} p(i,j) \\ p_\nu(i,j) \end{matrix} \right\} \xrightarrow[M\to\infty]{} \frac{\frac{A^i B^j}{i!j!}}{\sum\limits_{\text{all } s,t} \frac{A^s B^t}{s!t!}} . \tag{9}$$

This last, $M \to \infty$, limit distribution is not Engset either. However, it does provide a common correct limit for both $\nu=1$ and $\nu=2$.

From the asymptotic properties, (7) to (9), one feels justified to suggest a "realistic" approximation,

$$\tilde{p}(i,j) = \frac{a}{a+b} p_1(i,j) + \frac{b}{a+b} p_2(i,j). \tag{10}$$

Clearly, $\tilde{p}(i,j) \to p(i,j)$ for all $a \to 0$, $b \to 0$, and $M \to \infty$ cases.

The behavior of this approximation, plus possible bounds, will be explored in the next section on numerical results.

3.3. Numerical Results

Based on equations (2) to (10), the out/in and return call probabilities of blocking are approximately,

$$\tilde{P}_o = \sum_{i+2j=m} \tilde{p}(i,j),$$

$$\tilde{P}_r = \sum_{i+2j \geq m-1} \tilde{p}(i,j). \tag{11}$$

In this approximation, the quantities $\tilde{p}(i,j)$ are given in terms of $p_\nu(i,j)$ (10). The latter $p_\nu(i,j)$ are defined in (5). With the aid of these definitions, $\tilde{P}_o$ and $\tilde{P}_r$ have been computed for selected values of parameters M, m, a and b. The results are presented in Figures 4 to 8.

All of these figures consist of four parts. The four parts show constant blocking probabilities, such as $\tilde{P}_o$=0.002 or $\tilde{P}_o$=0.010 for distant out/in calls, and $\tilde{P}_r$=0.010 or $\tilde{P}_r$=0.050 for local return calls. The abscissa in all cases is m, the number of server channels. Notice that the m scale may be different for the different figures, but it is consistently the same for the four parts of each figure. Similarly, a common ordinate M(a+b) applies for every figure.

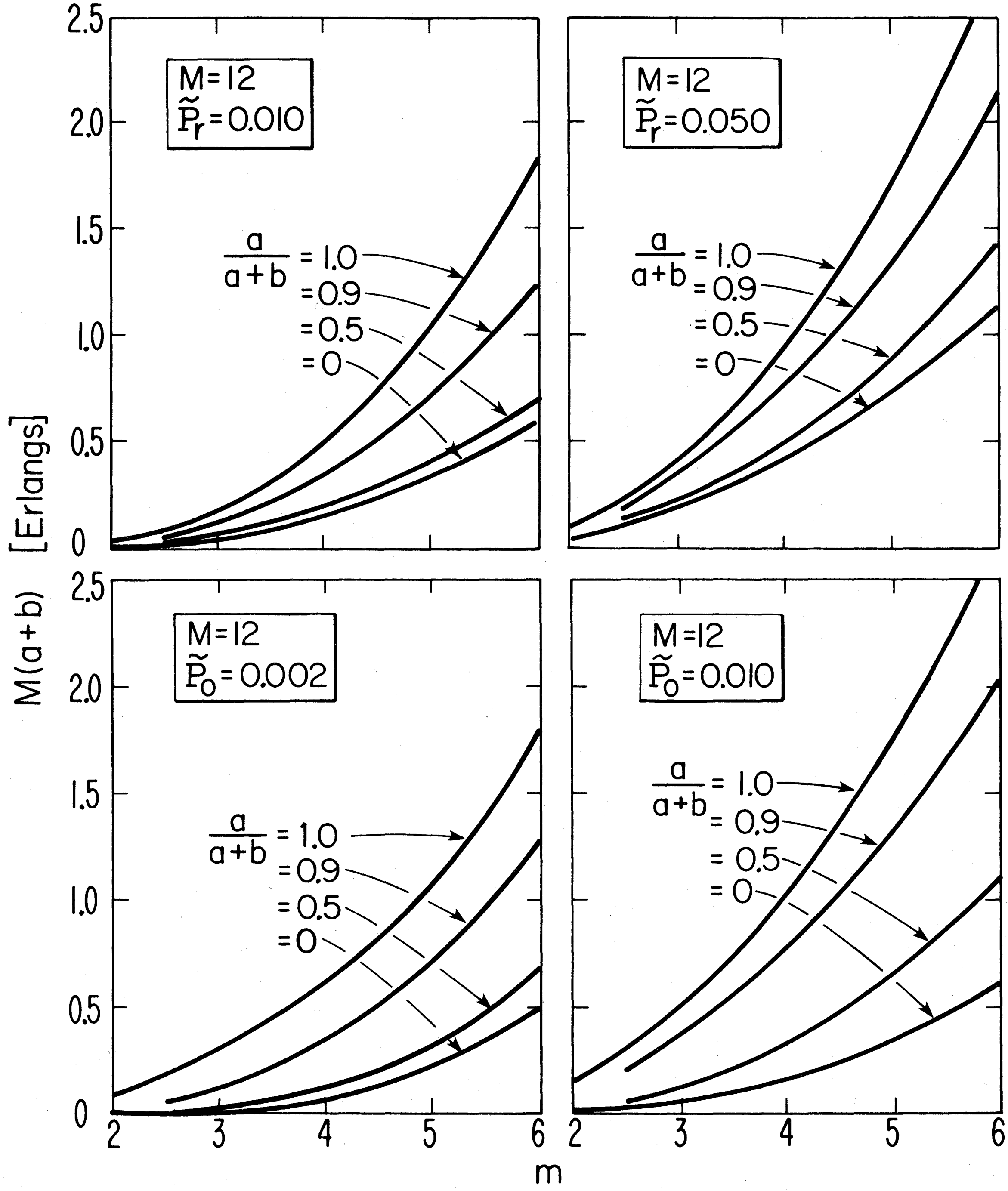


Figure 4. Approximate Phase 1 blocking probabilities for M=12 sources.

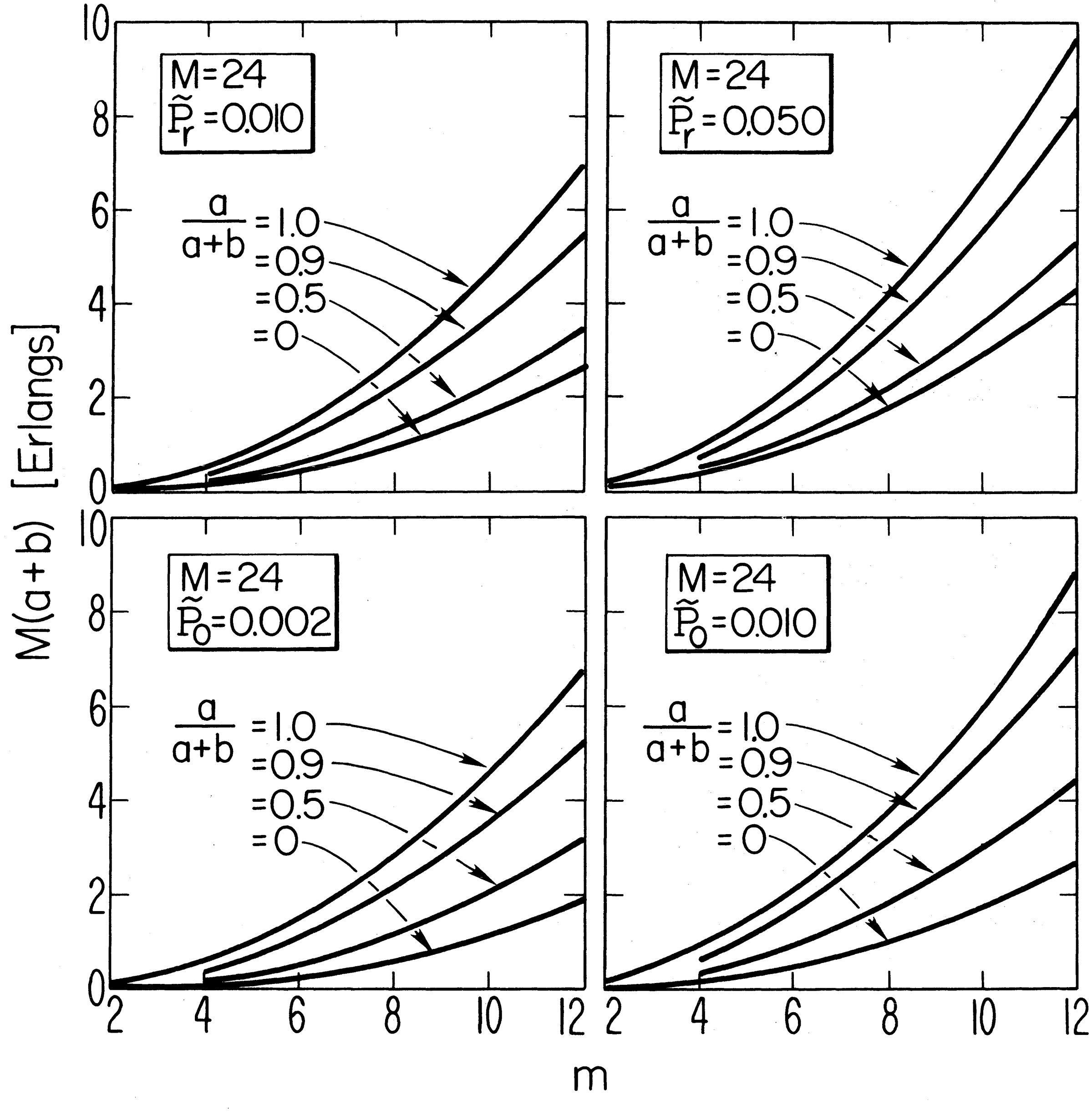


Figure 5. Approximate Phase 1 blocking probabilities for M=24 sources.

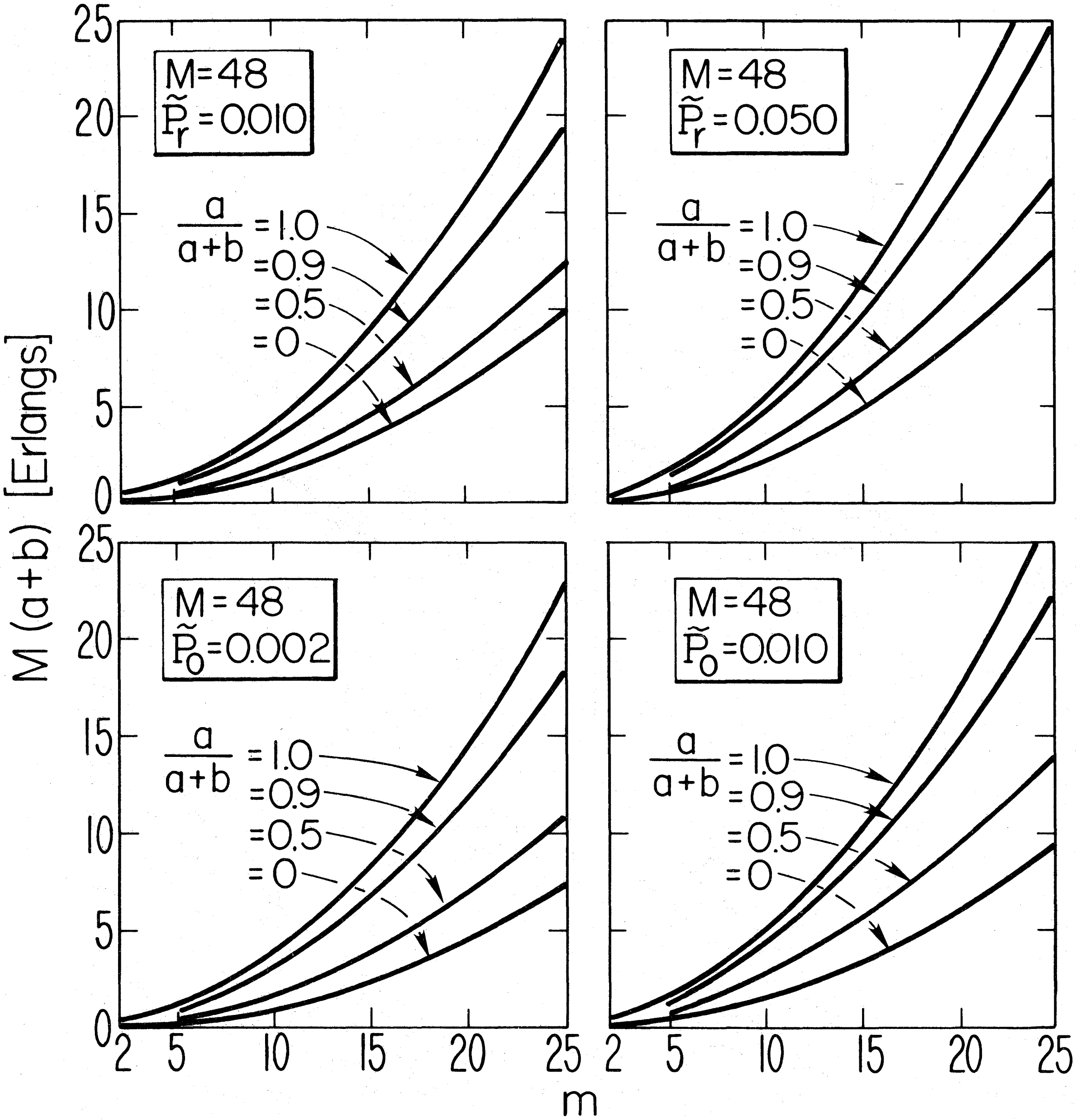


Figure 6. Approximate Phase 1 blocking probabilities for M=48 sources.

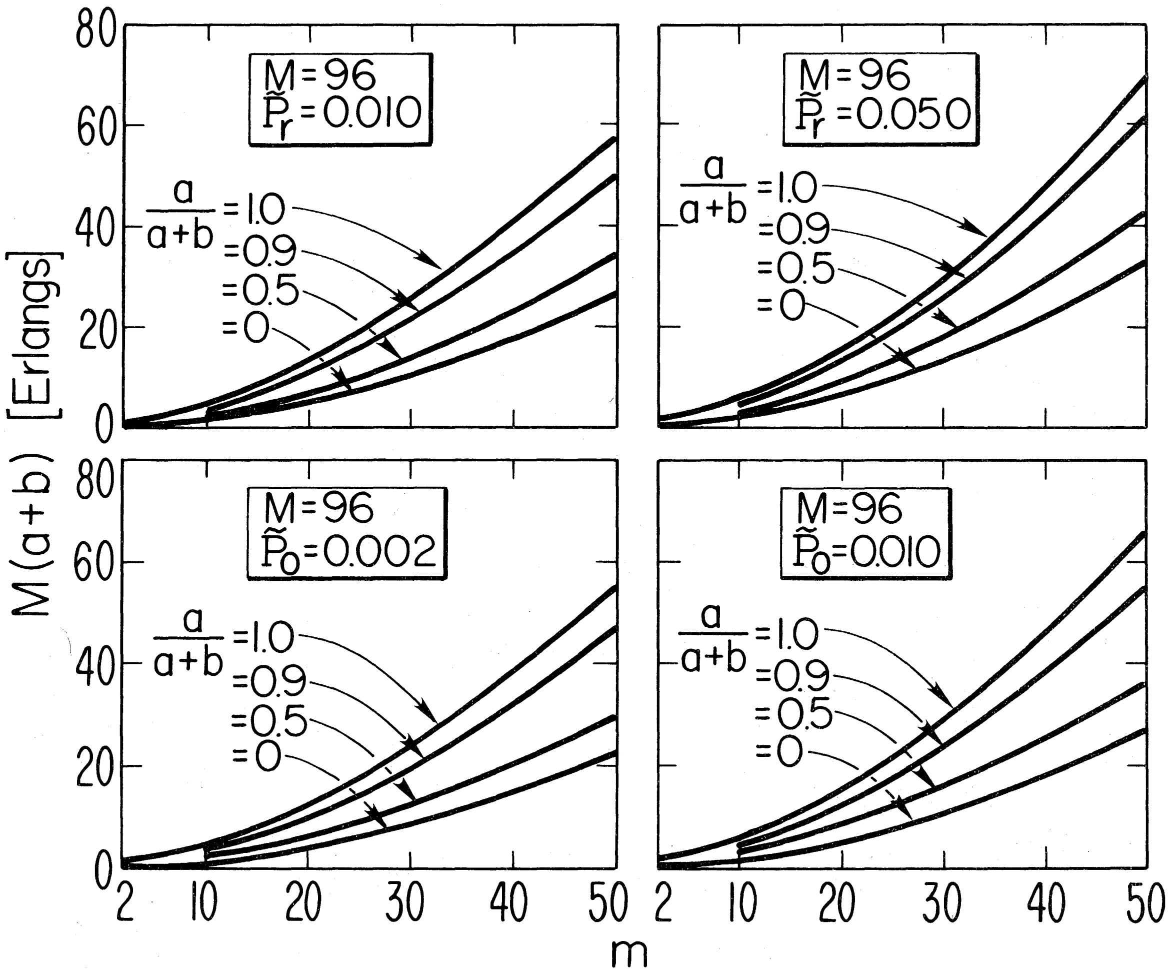


Figure 7. Approximate Phase 1 blocking probabilities for M=96 sources.

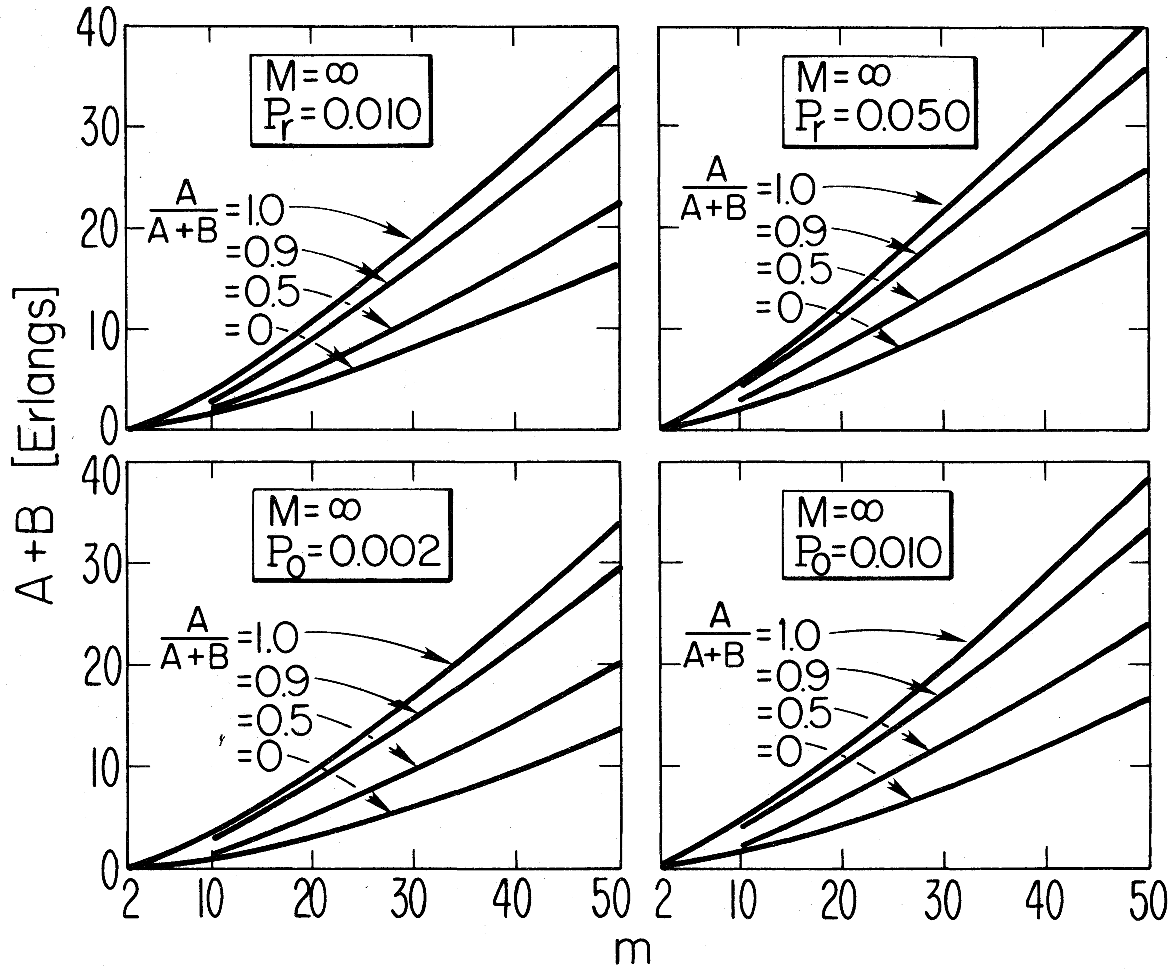


Figure 8. Exact Phase 1 blocking probabilities for an infinite number of sources.

The ordinate, $M(a+b)$, deserves a comment. It is called the "initial" offered load, as it is strictly observed only when $i=j=0$. To each initial load, there corresponds the effective or measurable offered load. The effective offered load may be defined as

$$L = (a+b)[M - \sum_{\text{all } i,j} (i+2j)p(i,j)]. \tag{12}$$

The effective load is always less than the initial load, but at the same time it is slightly higher than the actual carried load (Bear, 1976; Siemens, 1974).

If one prefers to present the blocking probabilities as functions of the effective load, one has to carry out the corresponding additional calculation (12). That calculation is skipped here.

Every quadrant part of each figure, Figures 4 to 8, contains four curves. These curves are selected for constant values of parameter $a/(a+b)$. The latter is the long term average fraction of distant calls made through the service facility. One should comment on the adjective "approximate" in the captions of Figures 4 to 7. It is valid only for $0<a/(a+b)<1$. When either $a\to 0$ or $b\to 0$, the curves become exact, see equations (7), (8), and (10). The latter cases are also useful as upper and lower performance bounds.

As the fraction of distant calls is increased, the same number of m servers can handle more traffic in the sense of the initial load $M(\underset{\sim}{a}+b)$. Such a statement, of course, presumes a fixed grade of service, for instance $P_r=0.010$.

As the number of users, M, increases, a clear cut limit behavior is achieved. In the limit, $M\to\infty$, the two types of initial load tend to their respective limits, $Ma\to A$ and $Mb\to B$. The initial and effective loads become indistinguishable. All the approximations converge to the correct values, and Figure 8 applies. Note that the ordinate is the total load $A+B$.

In the computation of these blocking probability curves, certain cases can be verified with the aid of published tables. Thus, the $a/(a+b)=0$ and $a/(a+b)=1$ cases correspond to two distinct Engset distributions; see earlier equations (7) and (8). In addition to the Engset traffic tables (Siemens, 1974), one can use binomial tables, such as the NBS binomial tables (1949). When $M\to\infty$, the ordinary Engset distributions tend to Erlang B. There are several Erlang B charts and tables available (Siemens, 1974; AT&T, 1961), as well as the related rather extensive Poisson tables (General Electric, 1962).

Figures 4 to 8 can be used in various ways. Given user groups and their traffic statistics, the concentration networks can be economically designed. Or given m, the number of concentrated channels, acceptable size terminal groups can be configured. Likewise, the grades of service for out/in and return calls (i.e., blocking probabilities P_o and P_r) can be variously engineered. More on this subject will be said in Section 6. To emphasize the role of specific parameters, the curves can be redrawn as desired. An example is shown in Figure 9.

Figure 9 assumes constants P_o=0.010 and a/(a+b)=0.9. The 1% blocking loss for distant outgoing and incoming calls may be a typical service objective. Likewise, the premise that 10% of all calls are turned around locally and 90% are not, may be true in many military installations. Given these ground rules, plus a fixed initial offered load M(a+b), one sees how many more servers are needed for larger M. Most servers, m, are required when $M=\infty$. This is not surprising, as finite user populations tend to offer less load when a significant fraction of the terminals are occupied.

Other graphs, similar to Figure 9, can be deduced from the basic curves (Figures 4 to 8) as needed. One must, however, be careful with the scales of the variables involved. For instance, both M(a+b) and m for M=12 are limited to small regions in Figure 9.

4. BLOCKING PHASE 2

4.1. Problem Definition

Assume the N-server facility of Figure 10, which is depicted to be a PABX. Two groups of users, of arbitrary sizes m and n and with different traffic, contend for the N server channels shown.

Consider the two user classes. In accordance with the notation of Figure 3, the upper class offers traffic of type (α). The m users here are concentrator or multiplex (MUX) "loops." The lower class of n users supplies traffic of the type (γ). The latter may be loosely called "lines" although there is nothing to forbid these from being another class of concentrated loops. In practice, $m \leq n$. We assume that $\min\{m,n\} \geq N$. According to the tabulation of path profiles (see Table 1), both classes (α) and (γ) may have many members. Thus, in Phase 2, class (α) contains paths numbered 1, 2, 3, ... Class (γ) consists of 9, 10, 11, ...

Assume exponential distributions for both the interarrival and service (holding) times. Let the average arrival rate be λ_1 on every loop and let $1/\mu_1$ be its average service time. Likewise, assume that λ_2 and μ_2 are the average arrival and service rates for the lines, respectively.

The reason for choosing different λ_1 and λ_2 is clear. Concentrated loops are apt to carry more traffic intensity than individual station lines. The reason for

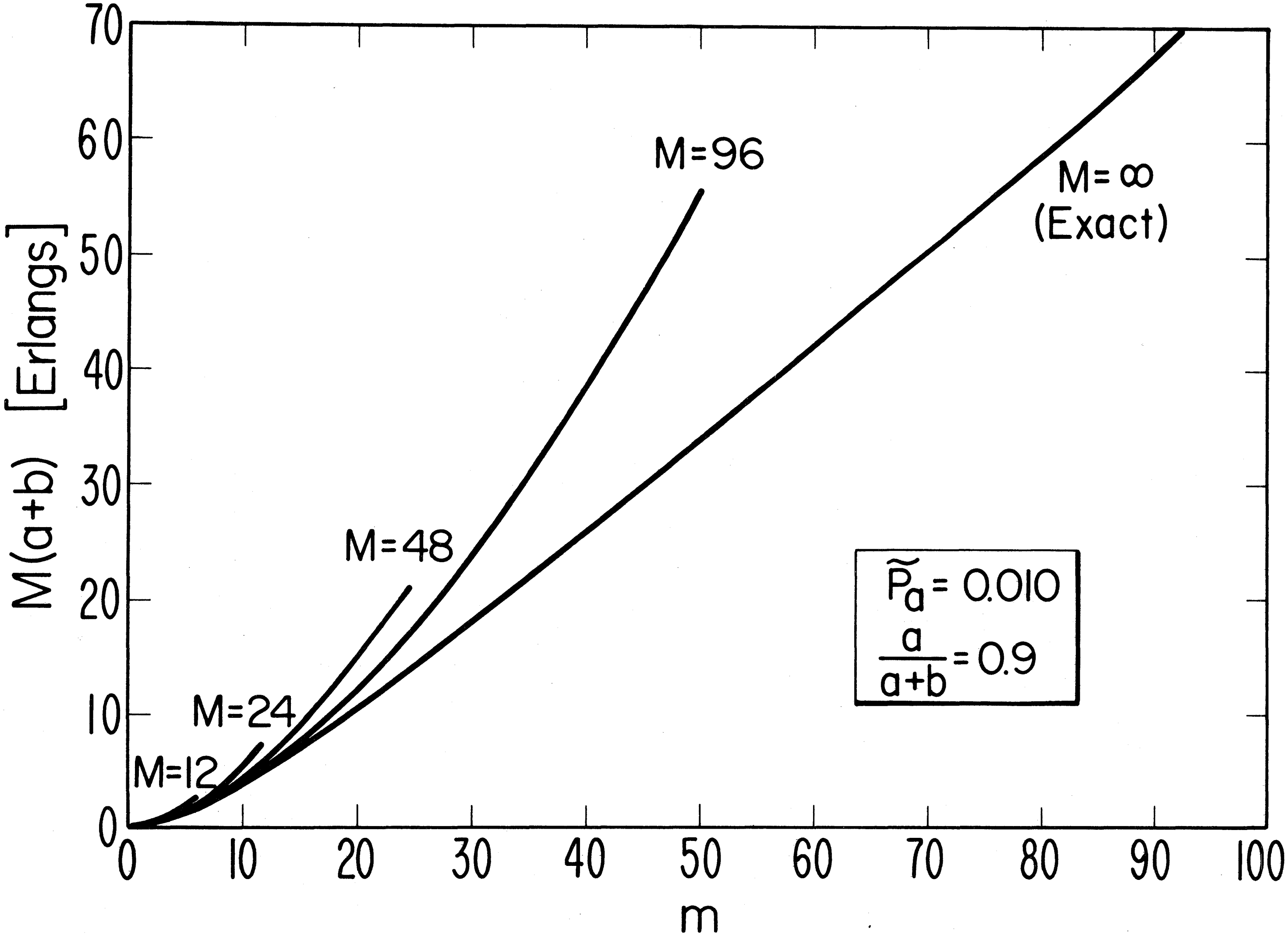


Figure 9. The effect of the number of users, M, on 1% out/in call blocking.

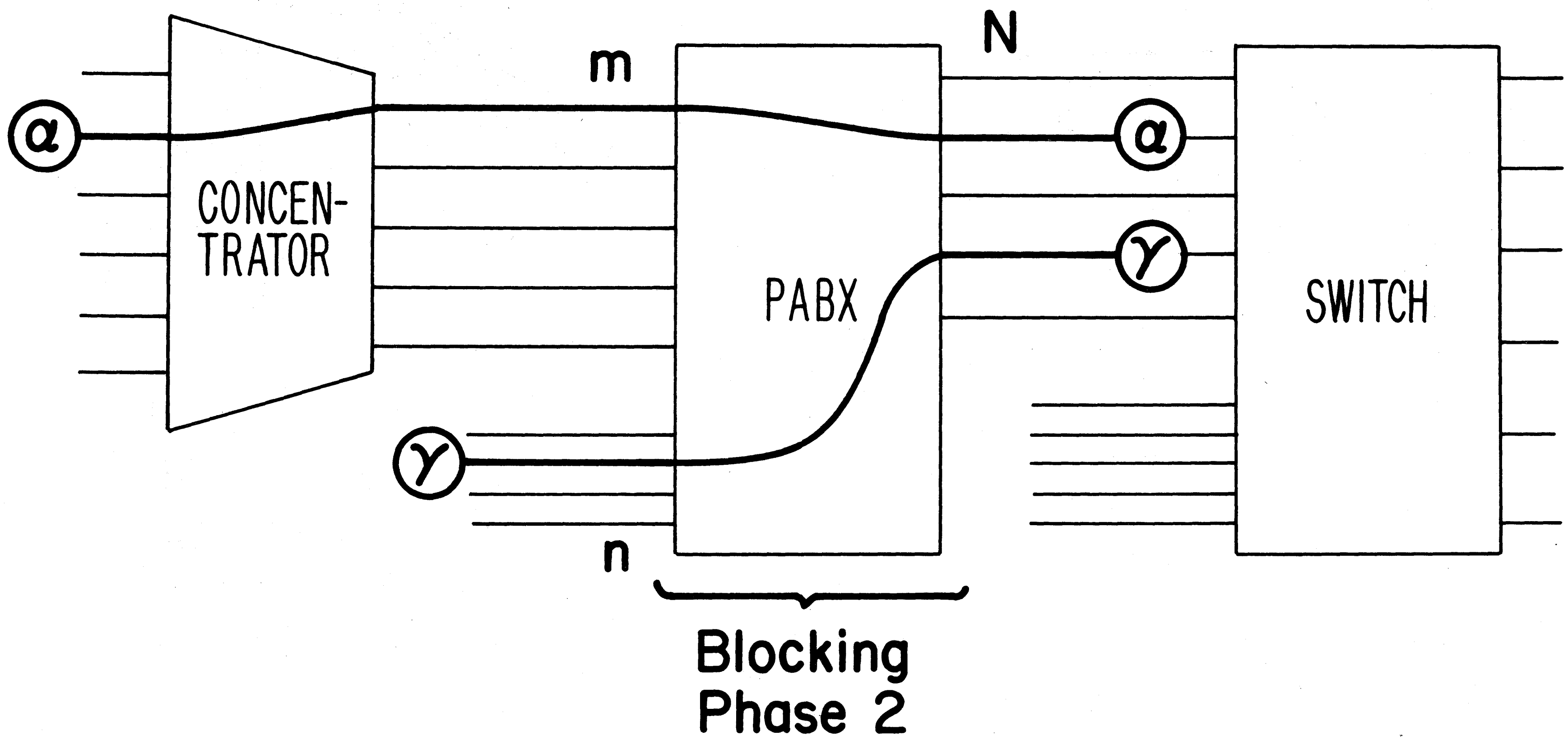


Figure 10. Phase 2 - blocking at a PABX.

permitting $\mu_1 \neq \mu_2$ has to do with potential generalization. One can envision many uses and extensions of this problem in military switching network applications. Two possibilities are shown in Figure 11. Part (A) of the figure depicts a common pool of N service units, perhaps control and signaling units. Two types of links, identified as lines and trunks, seek service from the common pool. The statistics of the two service processes are likely to differ.

Part (B) of Figure 11 illustrates a version of the recently proposed voice and data integration. Both actual and virtual circuit-switched voice may be concentrated on N common channels with data packets. Military applications have been discussed by many (Vena and Coviello, 1975; Fischer and Harris, 1976; Weinstein, et al., 1978; Schutzer, 1979; Fischer, 1979). There are also other nonmilitary applications and issues (Sherman, 1971; Arthurs and Stuck, 1979; Tsuda, et al., 1979; Schneider, 1979). Data packets are much shorter than a phone conversation or any of its vocal segments. In practice, this model may have to be both further generalized, or restricted, by incorporating military requirements, such as preemptive interrupts.

The generalization of the model for Phase 2 could also include alternate options for blocked calls. That is, given that a service request finds all N servers occupied, the request could be either lost (dropped), or cleared (rerouted), or entered into a queue to wait its turn for available servers. The latter assuredly would be the case for the data packets of Figure 11 (B). Access area PABX's, for reasons of survivability, occassionally provide direct plus alternate routes, at least between the busiest hubs.

The general topic is a mix of loss and delay system features. It leads to blocking probabilities that are hybrids of Erlang B and C formulas, or variants thereof (Nesenbergs, 1979). For the time being, however, assume that all blocked Phase 2 calls are simply lost at the PABX. Finally, assume full availability for all m loops and all n lines to the N service channels.

As done in Section 3, one next seeks the probability of blocking. It will be seen shortly that the situation is rather fortuitous. A closed form solution will follow.

Let (i,j) represent the event that i loops and j lines are receiving service. Then $0 \leq i \leq m$, $0 \leq j \leq n$, and $0 \leq i+j \leq N$ must hold. The steady state probability of event (i,j) is again denoted as p(i,j). These probabilities satisfy the same normalization condition of equation (1). Note, that there is a single, common blocking event here. It applies for both (α) and (γ) traffic and occurs when i+j=N. Thus,

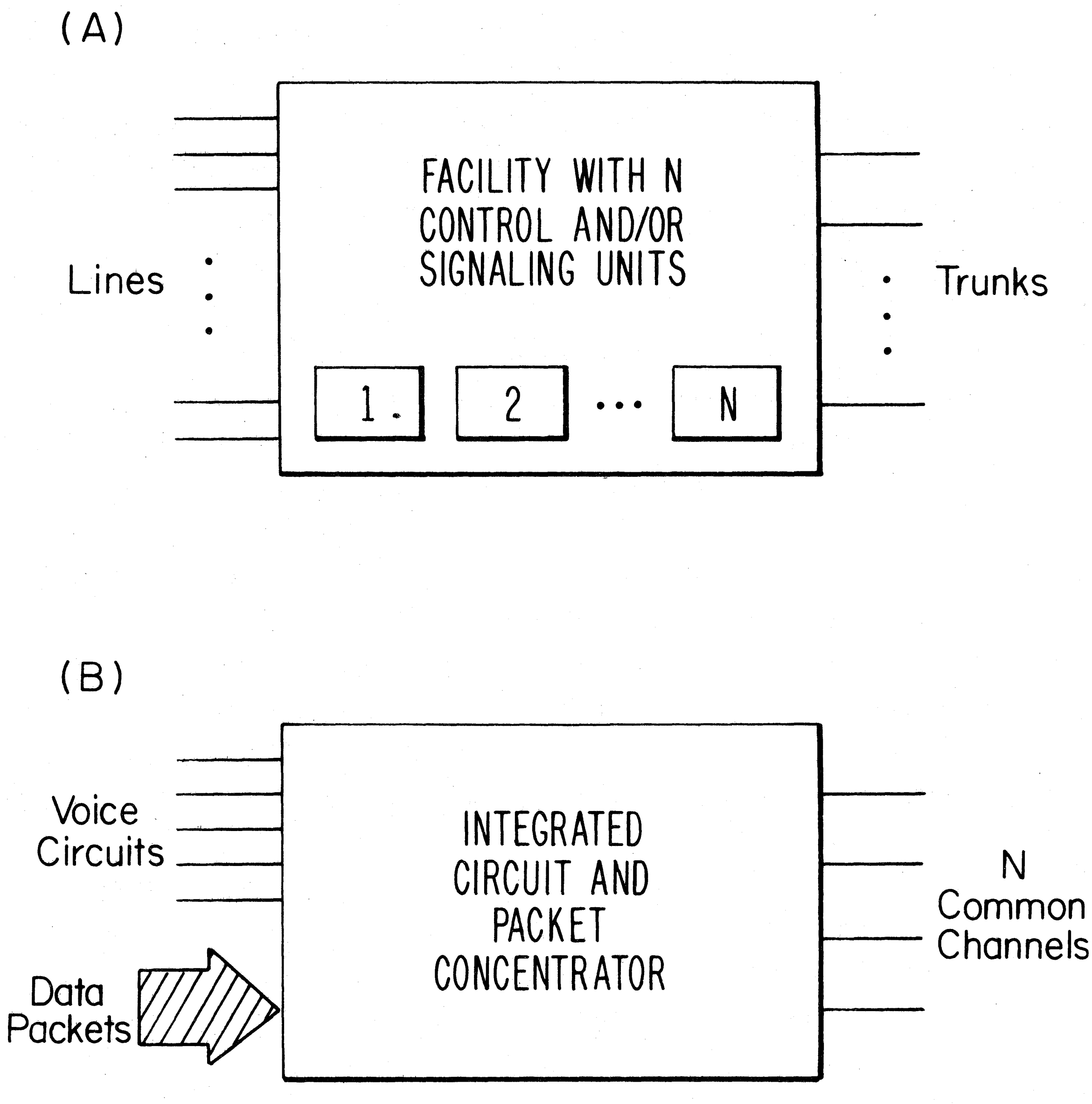


Figure 11. Application examples of the Phase 2 model.

the probability of blocking, P_x for Phase 2 (subscript x to emphasize PABX) is given by

$$P_x = \sum_{i+j=N} p(i,j). \tag{13}$$

To deduce the new state probabilities, p(i,j), one uses the same $\underline{P}$ vector definition of equation (3), as well as the conservation of flow principle represented in (4). However, the coefficient vector $\underline{C}$ takes a different form now. The new state dependent column vector $\underline{C}$ is given in Table 4, including its specific i=0 and j=0 realizations.

4.2. The Solution

The solution p(i,j) to the equations (1), (3), (4), and Table 4, follows from the separation of variables or product method (Cooper, 1972). Assume the proportionality,

$$p(i,j) \propto p_1(i)p_2(j), \tag{14}$$

for all nonnegative i and j, such that $0 \leq i+j \leq N$. Upon substitution, this leads to separate equations for the unknowns $p_1(i)$ and $p_2(j)$.

Without going into the details, one obtains seven equations for the seven columns of Table 4.

For i=0 and j=0:

$$[m\lambda_1 p_1(0) - \mu_1 p_1(1)]/p_1(0)$$
$$+ \; [n\lambda_2 p_2(0) - \mu_2 p_2(1)]/p_2(0) = 0.$$

For i=0 and 0<j<N:

$$[m\lambda_1 p_1(0) - \mu_1 p_1(1)]/p_1(0)$$
$$- \; [(n-j+1)\lambda_2 p_2(j-1) - j\mu_2 p_2(j)]/p_2(j)$$
$$+ \; [(n-j)\lambda_2 p_2(j) - (j+1)\mu_2 p_2(j+1)]/p_2(j) = 0.$$

For i=0 and j=N:

$$- \; [(n-N+1)\lambda_2 p_2(N-1) - N\mu_2 p_2(N)]/p_2(N) = 0.$$

Table 4. Column Vector $\underline{C}$ for the (i,j) States of Blocking Phase 2

i=0			0<i<N			i=N
j=0	0<j<N	j=N	j=0	0<j<N-i	j=N-i	j=0
0	0	0	$-(m-i+1)\lambda_1$	$-(m-i+1)\lambda_1$	$-(m-i+1)\lambda_1$	$-(m-N+1)\lambda_1$
0	$-(n-j+1)\lambda_2$	$-(n-N+1)\lambda_2$	0	$-(n-j+1)\lambda_2$	$-(n-j+1)\lambda_2$	0
$m\lambda_1$ $+n\lambda_2$	$m\lambda_1$ $+j\mu_2+(n-j)\lambda_2$	$N\mu_2$	$i\mu_1+(m-i)\lambda_1$ $+n\lambda_2$	$i\mu_1+(m-i)\lambda_1$ $+j\mu_2+(n-j)\lambda_2$	$i\mu_1$ $+j\mu_2$	$N\mu_1$
$-\mu_1$	$-\mu_1$	0	$-(i+1)\mu_1$	$-(i+1)\mu_1$	0	0
$-\mu_2$	$-(j+1)\mu_2$	0	$-\mu_2$	$-(j+1)\mu_2$	0	0

For $0<i<N$ and $j=0$:

$$
\begin{aligned}
&- [(m-i+1)\lambda_1 p_1(i-1) - i\mu_1 p_1(i)]/p_1(i) \\
&+ [(m-i)\lambda_1 p_1(i) - (i+1)\mu_1 p_1(i+1)]/p_1(i) \\
&+ [n\lambda_2 p_2(0) - \mu_2 p_2(1)]/p_2(0) = 0.
\end{aligned}
\tag{15}
$$

For $0<i<N$ and $0<j<N-i$:

$$
\begin{aligned}
&- [(m-i+1)\lambda_1 p_1(i-1) - i\mu_1 p_1(i)]/p_1(i) \\
&+ [(m-i)\lambda_1 p_1(i) - (i+1)\mu_1 p_1(i+1)]/p_1(i) \\
&- [(n-j+1)\lambda_2 p_2(j-1) - j\mu_2 p_2(j)]/p_2(j) \\
&+ [(n-j)\lambda_2 p_2(j) - (j+1)\mu_2 p_2(j+1)]/p_2(j) = 0.
\end{aligned}
$$

For $0<i<N$ and $j=N-i$:

$$
\begin{aligned}
&- [(m-i+1)\lambda_1 p_1(i-1) - i\mu_1 p_1(i)]/p_1(i) \\
&- [(n-j+1)\lambda_2 p_2(j-1) - j\mu_2 p_2(j)]/p_2(j) = 0.
\end{aligned}
$$

For $i=N$ and $j=0$:

$$
- [(m-N+1)\lambda_1 p_1(N-1) - N\mu_1 p_1(N)]/p_1(N) = 0.
$$

The key point to observe in (15) is that all the sixteen square brackets are structurally the same. One simply has for $0 \leq i < N$ and $0 \leq j < N$, a pair of symmetric conditions:

$$
\begin{aligned}
&(m-i)\lambda_1 p_1(i) - (i+1)\mu_1 p_1(i+1) = 0, \\
&(n-j)\lambda_2 p_2(j) - (j+1)\mu_2 p_2(j+1) = 0.
\end{aligned}
\tag{16}
$$

If these equations hold, one has a solution. The separated equations for $p_1(i)$ and $p_2(j)$ are both of the Engset type (Kleinrock, 1975, Chap. 3). Except for the missing proportionality factor, their solutions are known,

$$p_1(i) \propto \binom{m}{i}\left(\frac{\lambda_1}{\mu_1}\right)^i,$$

$$p_2(j) \propto \binom{n}{j}\left(\frac{\lambda_2}{\mu_2}\right)^j. \tag{17}$$

The resultant state probabilities are therefore

$$p(i,j) = \frac{\binom{m}{i}\binom{n}{j}a_1{}^i a_2{}^j}{\sum\limits_{\text{all } s,t} \binom{m}{s}\binom{n}{t}a_1{}^s a_2{}^t}, \tag{18}$$

where $a_1=\lambda_1/\mu_1$ and $a_2=\lambda_2/\mu_2$ are the individual loop and line loads in Erlangs, respectively. The nontrivial domain for (i,j) is, of course, $\min\{i,j,m-i,n-j,N-i-j\}\geq 0$.

The state probability solution for Phase 2 differs from the approximate solution for Phase 1. Compare (5) and (18). However, their limiting properties are similar. Thus, for Phase 2 one gets,

$$\begin{aligned}
p(i,j) &\xrightarrow[a_1\to 0]{} E_j(N;n,a_2), && \text{if } i=0,\\
&\xrightarrow[a_1\to 0]{} 0, && \text{if } i>0,\\
&\xrightarrow[a_2\to 0]{} E_i(N;m,a_1), && \text{if } j=0,\\
&\xrightarrow[a_2\to 0]{} 0, && \text{if } j>0,\\
&\xrightarrow[m,n\to\infty]{} \frac{\dfrac{A_1{}^i A_2{}^j}{i!j!}}{\sum\limits_{\text{all } s,t} \dfrac{A_1{}^s A_2{}^t}{s!t!}}, && \text{all } i,\ j,
\end{aligned} \tag{19}$$

where $A_1=ma_1$, and $A_2=na_2$ are assumed to be constants in the last limit. These limits are formally the same as (7), (8), and (9) for the blocking problem in Phase 1. The last of the limits covers the case of both user groups being infinite. This asymptotic result has been previously derived by Cooper (1972). For Phase 2 it implies the simplification,

$$\Pr\{i+j=k\} \xrightarrow[m,n\to\infty]{} \frac{\frac{(A_1+A_2)^k}{k!}}{\sum_{s=0}^{N}\frac{(A_1+A_2)^s}{s!}} . \tag{20}$$

It leads to the Erlang B blocking formula, as noted later.

The general blocking probability for Phase 2 results when (18) is inserted into (13). The P_x expression is a symmetric function of the two user groups. If one assumes $N\le\min\{m,n\}$, then probability P_x can be written as

$$P_x = \frac{\binom{n}{N} a_2^{\,N} F(-m,-N;n-N+1;\frac{a_1}{a_2})}{\sum_{i=0}^{N}\binom{n}{i} a_2^{\,i} F(-m,-i;n-i+1;\frac{a_1}{a_2})} . \tag{21}$$

Here, the F(.,.;.;.) function is the Gauss hypergeometric series (Abramowitz and Stegun, 1964).

$$F(i,j;k;x) = \sum_{n=0}^{\infty}\frac{(i)_n (j)_n}{(k)_n}\frac{x^n}{n!} , \tag{22}$$

where $(i)_n=i(i+1)\ldots(i+n-1)$ and $(i)_0=1$. For negative i or j, the Gauss series becomes a polynomial. That clearly is the case in (21).

4.3. Numerical Results

The numerical evaluation of blocking probability for the Phase 2 problem follows from the formal solution, see equations (13) to (21). The task involves five independent parameters: m, n, N, a_1 and a_2. Thus, like in earlier Phase 1, it is the mechanics of presentation that causes difficulties, perhaps in excess of the computational effort. In this section, a simplified summary of the numerical results is presented.

As before, let $A_1=ma_1$ and $A_2=na_2$ be the total loads offered by the two user classes. Let the total initial load to the service facility of Phase 2 be their sum, $A=A_1+A_2$. One expects, quite reasonably, that said loads must play a key part in determining the eventual grade of service. This parallels the rationale for the Engset distribution, and the fact that -- at least marginally -- the present solution coincides with the Engset formulas. See equations (19) and (20).

Special marginal cases for the PABX blocking probability are:

$$
\begin{aligned}
P_x &= E_N(N;n,\tfrac{A}{n}) && \text{if } A_1=0, \\
&= E_N(N;m,\tfrac{A}{m}) && \text{if } A_2=0, \\
&= E_N(N;m+n,\tfrac{A}{m+n}) && \text{if } \frac{A_1}{m}=\frac{A_2}{n}, \\
&= \lim_{k\to\infty} E_N(N;k,\tfrac{A}{k})=B(N,A) && \text{if both } m=\infty,\ n=\infty.
\end{aligned} \tag{23}
$$

The final form B(N,A) is the Erlang B formula (Bear, 1976). The notation used in (23) is the same as defined in (6).

The Engset distribution is known to possess several monotonic properties. Thus, $E_N(N;k,\frac{A}{k})$ decreases as N increases, and increases as either k or A increases (Siemens, 1974). These properties can be variously extended and applied to the Phase 2 solution. The typical characteristic of Figure 12 results.

Figure 12 shows the blocking probability, P_x, for Phase 2 as a function of the relative total loop load A_1/A. This does not imply that the total load A is an insignificant parameter. Quite the contrary, the magnitude of P_x is bounded and determined by A, perhaps more than by anything else.

In the interpretation of Figure 12, assume that all of m, n, N and A are fixed. Then A_1 is the only variable. As a function of this variable, P_x obeys simple monotonic rules. Its maximum value or worst case occurs when all loop and line loads are the same. This happens when

$$\frac{A_1}{A} = \frac{m}{m+n} \quad . \tag{24}$$

At the other extreme, the function has two local minima. They occur at the end points $A_1/A=0$ or 1, as shown. Between the maximum and the minima, the function has a monotonic gradually increasing slope. The function is symmetric around the mid-point when m=n.

In access area applications, one expects fewer loops than lines, but with higher loop traffic. Then $m\leq n$ and $A_1/A\geq m/(m+n)$. It follows that the system operation should be characterized by the right side of Figure 12, where

$$E_N(N;m,\tfrac{A}{m})\leq P_x\leq E_N(N;m+n,\tfrac{A}{m+n}). \tag{25}$$

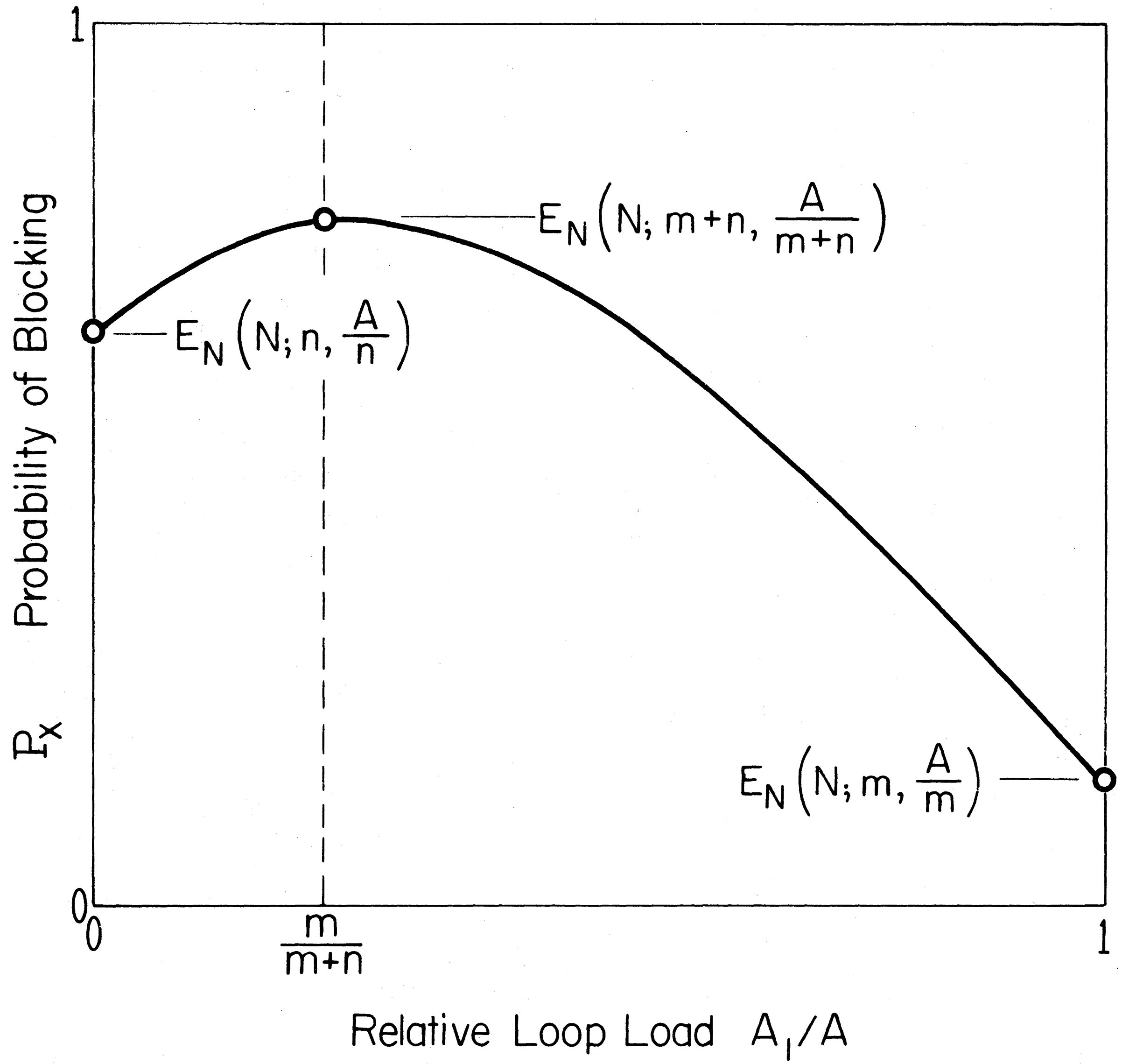


Figure 12. The general effect of the Engset distribution on the blocking probability for Phase 2.

One should emphasize that two things happen as m and n are jointly increased. First, the P_x curve is raised. And second, it is flattened out, eventually approaching the Erlang B(N,A) limit (Beneš, 1965; Siemens, 1974; Kleinrock, 1975), that was indicated in (23).

Several curves of interest have been computed. They are presented in three parts of Figure 13. All the plots show P_x as a function of the relative total loop load A_1/A. All plots also assume twice as many lines as loops, i.e., m/n=1/2. This implies, see (24), that the maximum probability of blocking occurs at the indicated $A_1/A=1/3$. The maximum value is the worst case bound for given m, n, N and A.

The three parts of Figure 13 differ in the N (i.e., number of servers) and A (i.e., total initial load) values assumed. For other values, two choices are open to the system analyst. One can perform exact calculations on the formal solution, (13), (18) and (21), or one may resort to some, quick but rough, pseudo-parabolic interpolation between the Engset extremes (25).

Further exploration of the complete parameter ranges for m, n, N, A_1 and A_2 appears to be a sizable effort. It is beyond the scope of this paper.

5. BLOCKING PHASE 3

5.1. Problem Definition

Consider the service facility labeled "switch" in Figure 14. It is distinguished from other previously analyzed facilities by the fact that it provides two kinds of servers for three types of service requests (calls).

On the left side of the switch, there are a total of L channel ports, to be loosely called "lines." On the right side there are T "trunks." There are three types of calls through the switch. The distinction is illustrated by the (α), (δ) and (ε) paths in Figure 14. (Note that the distinction between previous (α) and (γ) calls becomes blurred here.) For lack of better names, the (α)-type traffic will be referred to as "distant;" the (δ)-type as "local;" and the (ε)-type as "tandem." The route profiles of Table 1 again show that many paths belong to these categories. For instance, all the paths that go through switches more than once, such as 1, 2, ..., 27, 28, belong to type (α) at Phase 3. If a switch is encountered once in Table 1, as for paths 5, 6, ..., 31, 32, then one has category (δ). Finally, of course, every member path of (α) may well be a member of (ε) class, somewhere down the route.

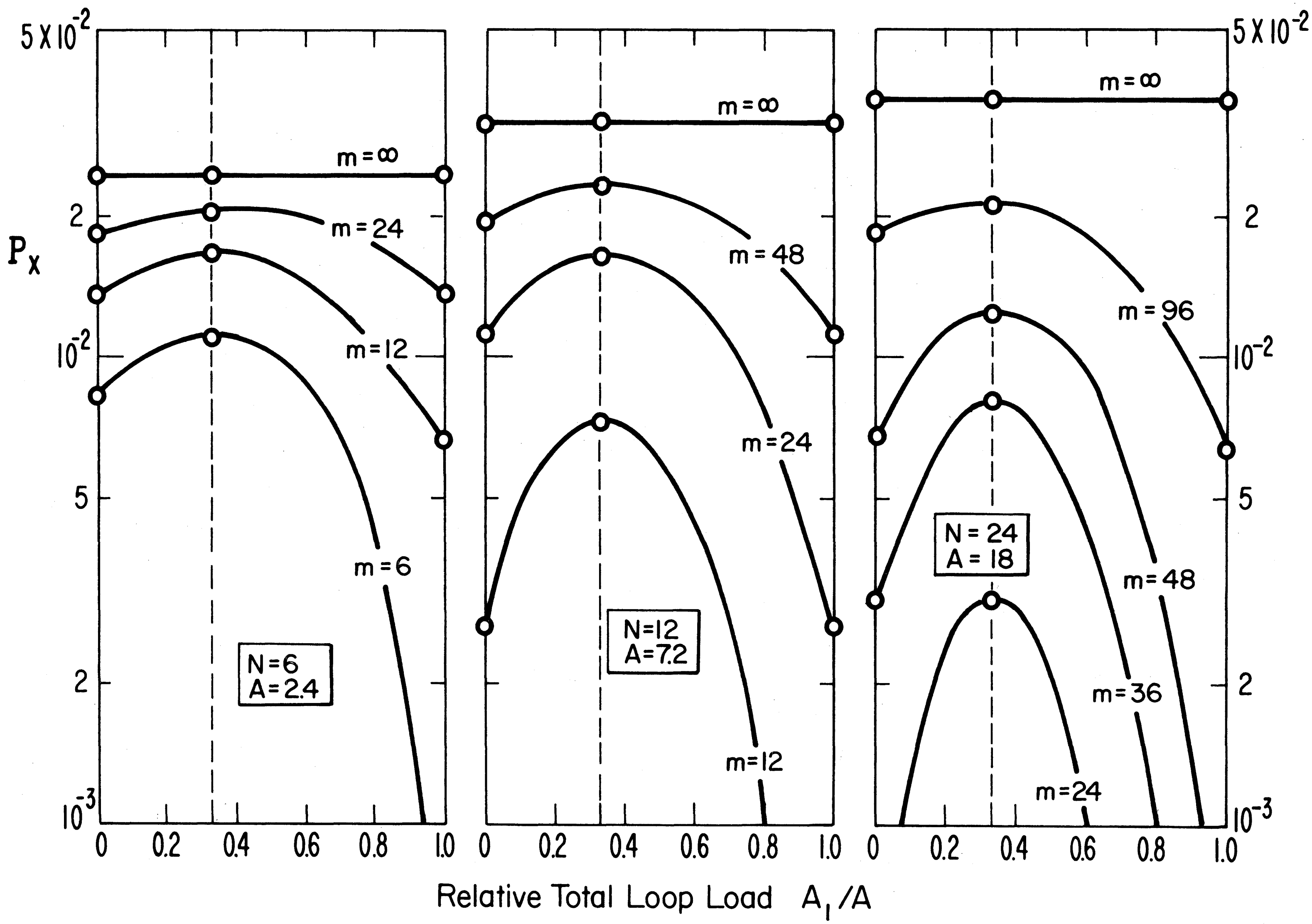


Figure 13. Phase 2 blocking probabilities for loop-to-line ratio m/n=1/2.

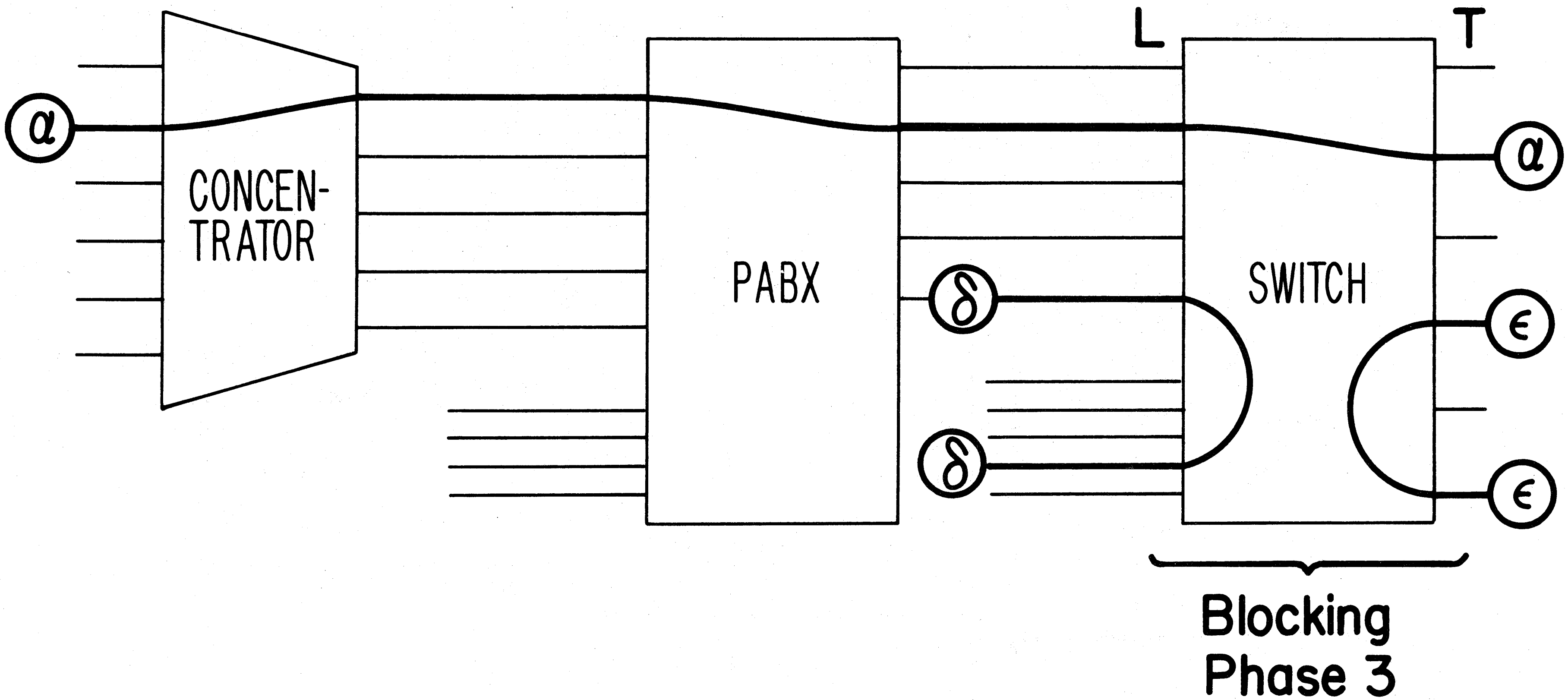


Figure 14. Phase 3 - blocking at a switch.

This section derives blocking probabilities for all three kinds of calls. The probabilities are inherently different, because distant calls require one idle line plus one idle trunk, local calls require two idle lines, and tandem calls need two idle trunks.

It is recognized that blocking for Phase 3 is a generalization of Phase 1 (see Section 3) and therefore apparently untractable. However, after invoking a few reasonable assumptions, the problem becomes simpler. That is done in this section. Formal closed form expressions are next derived for both the state and blocking probabilities subject to simplifying assumptions.

The simplifying assumptions are:

(i) The interarrival time distributions for the three kinds of service requests are exponential, all possibly distinct. The service times are also exponentially distributed, but with a common mean of unity. Their Erlang loads may differ.

(ii) All service requests are generated by infinite user populations. This assumption appears reasonable for switches that serve several concentrators, several PABX's, and a large number of terminals. The three arrival intensities thus stay constant, regardless of system state.

(iii) The number of servers, i.e., the L lines and T trunks of Figure 14, are given constants.

(iv) All blocked calls are lost without any aftereffect to the system.

(v) The switching network is intrinsically nonblocking. By that one means that blocking events can only be caused by a shortage of lines, trunks or both.

(vi) The switching network provides full availability from all requesting lines and trunks, to all idle lines and trunks on the network.

(vii) The network model in Figure 14 is equivalent to two (i.e., $k=2$) channel groups with $n_1=L$ and $n_2=T$ servers, respectively. A more representative model should permit $k\geq 2$ channel groups with $\{n_1, n_2, \ldots, n_k\}$ servers each. The total number of traffic streams would then amount to $k(k+1)/2$, instead of the present three. Solely for reasons of simplicity, the general model is not pursued here. One suspects, nevertheless, that the solution for general k is tractable and in fact similar to $k=2$ in structure. This general topic should be best explored in a separate in-depth study.

To be specific, assume the notation summarized in Table 5. When d distant calls, ℓ local calls, and t tandem calls are in progress, one says that the system is in state (d,ℓ,t). The steady state probability of state (d,ℓ,t) is $p(d,\ell,t)$. Quantities, P_d, P_ℓ and P_t, denote the probabilities of blocking for the distant, local and tandem calls, respectively. The total offered loads for the three traffic types will be A, B and C Erlangs. To obtain service, a distant call requires that at least one line and trunk is idle. As shown, local calls need two lines and tandem calls need two trunks.

Nonzero probabilities can occur only when $\min\{d,\ell,t,L-d-2\ell,T-d-2t\}\geq 0$ is true. Impossible (d,ℓ,t) states have $p(d,\ell,t)=0$, by definition. The blocking probabilities are sums of $p(d,\ell,t)$ over appropriate index sets (d,ℓ,t). To this end, one next derives the individual state probabilities.

The derivation of the $p(d,\ell,t)$ formula, to be given next, parallels the procedure for blocking Phases 1 and 2. One constructs and solves the steady-state flow conservation equations for unknowns $p(d,\ell,t)$. As before [see (4)] two column vectors are involved. First, let the following seven-dimensional vector $\underline{P}$ represent the unknown state probabilities,

$$\underline{P} = \begin{bmatrix} p(d-1,\ell,t) \\ p(d,\ell-1,t) \\ p(d,\ell,t-1) \\ p(d,\ell,t) \\ p(d+1,\ell,t) \\ p(d,\ell+1,t) \\ p(d,\ell,t+1) \end{bmatrix} . \tag{26}$$

The second seven-dimensional vector, $\underline{C}$, is comprised of the flow rate or birth-death coefficients necessary for flow conservation at every state (d,ℓ,t). Because of the network complexity, one must now distinguish at least seven separate (d,ℓ,t) regions, each with its own coefficients. These seven state regions are defined in Table 6. Each region $(k=1,2,\ldots,7)$ has a unique coefficient vector $\underline{C}_k$. The blocking events, be they for distant, local or tandem calls, are also different. The blocking occurrences are indicated in Table 6. Note that region k=1 blocks all calls. Region k=7 blocks no calls. The other regions, 2 to 6, have different individual blocking characteristics.

Table 5. Notation for the Three Kinds of Service Requests from Two Classes of Server Channels

Type of Traffic	State Index	Prob. of Blocking	Offered Load	Required Number of Idle	
				Lines	Trunks
Distant (α)	(d,.,.)	P_d	A	1	1
Local (δ)	(.,ℓ,.)	P_ℓ	B	2	0
Tandem (ε)	(.,.,t)	P_t	C	0	2

Table 6. The Seven State Regions and Their Blocking Roles

	$d+2t=T$	$d+2t=T-1$	$d+2t<T-1$
$d+2\ell=L$	k=1 Distant Local Tandem		k=4 Distant Local --
$d+2\ell=L-1$		k=2 -- Local Tandem	k=5 -- Local --
$d+2\ell<L-1$	k=3 Distant -- Tandem	k=6 -- -- Tandem	k=7 -- -- --

The seven-dimensional column vectors $\underline{C}_k$ are given in Table 7, for all seven regions k=1,2,...,7. For reasons of simplicity, just like in Tables 2 and 3 the trivial $\min\{d,\ell,t\}=0$ cases are avoided here. The flow conservation identities, see earlier equation (4), consist of products of all $\underline{C}_k$ (i.e., the columns of Table 7) with $\underline{P}$ (26). The solution must again be normalized,

$$\sum_{\text{all } d,\ell,t} p(d,\ell,t) = 1. \tag{27}$$

5.2. Formal Solution

As for Phase 2, one tries a separable or product solution (Cooper, 1972),

$$p(d,\ell,t) \propto p_1(d)p_2(\ell)p_3(t). \tag{28}$$

The substitution of (28) into (26), followed by a second substitution of (26) and Table 7 into (4), produces a long string of vanishing sums. But, just as in (15) and (16), the sums are very simply constituted. They all turn out to be variously weighted sums of three elementary equations. The elementary equations are

$$\begin{aligned} dp_1(d) - Ap_1(d-1) &= 0, \\ \ell p_2(\ell) - Bp_2(\ell-1) &= 0, \\ tp_3(t) - Cp_3(t-1) &= 0. \end{aligned} \tag{29}$$

They are easily solved. After all, identities of type (29) have been the basis for the classical Erlang formulas (Riordan, 1962, Chap. 5; Kleinrock, 1975, Chap. 3). The separated solutions are thus known to within a multiplicative constant:

$$\begin{aligned} p_1(d) &\propto \frac{A^d}{d!}, \\ p_2(\ell) &\propto \frac{B^\ell}{\ell!}, \\ p_3(t) &\propto \frac{C^t}{t!}. \end{aligned} \tag{30}$$

Equations (27), (28) and (30) uniquely determine the state probabilities for the Phase 3 blocking problem. For all nonnegative d, ℓ, and t values that satisfy $d+2\ell\leq L$ and $d+2t\leq T$, one obtains

Table 7. The Column Vectors $\underline{C}_k$ for the Seven State Regions k=1,2,...,7

$\underline{C}_1$	$\underline{C}_2$	$\underline{C}_3$	$\underline{C}_4$	$\underline{C}_5$	$\underline{C}_6$	$\underline{C}_7$
-A	-A	-A	-A	-A	-A	-A
-B	-B	-B	-B	-B	-B	-B
-C	-C	-C	-C	-C	-C	-C
$d+\ell+t$	$d+\ell+t+A$	$d+\ell+t+B$	$d+\ell+t+C$	$d+\ell+t+A+C$	$d+\ell+t+A+B$	$d+\ell+t+A+B+C$
0	-(d+1)	0	0	-(d+1)	-(d+1)	-(d+1)
0	0	$-(\ell+1)$	0	0	$-(\ell+1)$	$-(\ell+1)$
0	0	0	-(t+1)	-(t+1)	0	-(t+1)

$$p(d,\ell,t) = \frac{\frac{A^d B^\ell C^t}{d!\ell!t!}}{\sum\limits_{\text{all } i,j,k} \frac{A^i B^j C^k}{i!j!k!}} . \tag{31}$$

Three distinct blocking probabilities are encountered here. See Tables 5 and 6 for notation. The probability of distant call blocking, P_d, is the sum of all state probabilities in the index set k=1,3,4. The probability of local call blocking, P_ℓ, is aggregated over the index set k=1,2,4,5. And, finally, the tandem call blocking, P_t, comes from k=1,2,3,6.

5.3. Numerical Results

Expressions for three kinds of blocking probabilities, P_d, P_ℓ, and P_t, have been formally derived. It remains to evaluate them numerically for the appropriate parameters. The three probabilities of blocking, P_d, P_ℓ, and P_t, are functions of five independent parameters: L, T, A, B, and C. The computation and presentation of results appears again to be rather involved.

In Figure 15 one finds typical and slightly oversimplified numerical results. Figure 15 has three parts. All show the three blocking probabilities as ordinates, versus the tandem call load C as abscissa. Furthermore, all three parts assume that the number of lines and trunks are L=12 and T=6, respectively. The three parts differ in the distant (A) and local (B) call loads. The first part depicts A=0.5, B=1 Erlangs. The second has A=0.75 and B=1.5, and the third A=1, B=2 Erlangs. Thus, all nine curves of Figure 15 correspond to fixed ratios L/T=B/A=2.

The tandem load C is seen to have a marked effect on the tandem call blocking probability, P_t. At C=0, P_t has its lowest value. As C increases, P_t grows in a monotonic fashion. A somewhat lower rate of increase is encountered by P_d, the probability of blocking for distant calls. With respect to C, P_d also has a minimum at C=0. At that point, P_d is a special case of the distant call blocking P_o, discussed under Phase 1 blocking (see equations (2) to (11), and Figures 4 to 9). In the cases considered so far, one finds $P_d > P_t$ at C=0. However, because of their different rates of increase as functions of load C, the roles are soon reversed for relatively modest C>0. One finds in Figure 15, that $P_d < P_t$ for all C>0.12 Erlangs.

Blocking probability for local calls, P_ℓ, is nearly independent of C for the cases considered. This is expected. However, there is a slight yet discernible paradox here. The highest value of P_ℓ occurs at C=0, as can be noted from a careful scrutiny of the curves. Increases of C, which happen to block trunks needed for distant traffic, may occasionally augment the idle available lines.

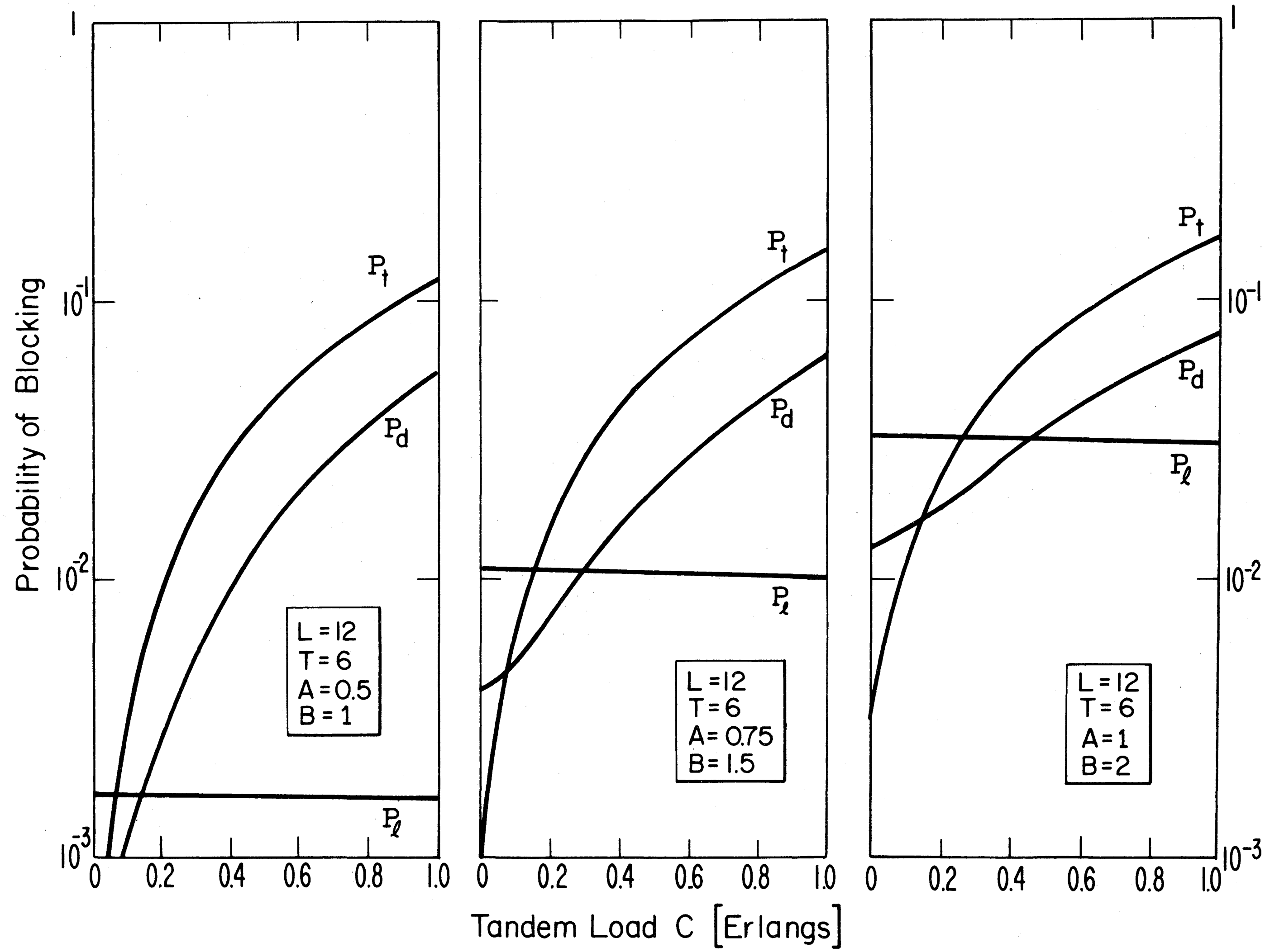


Figure 15. The three blocking probabilities of Phase 3.

Figure 15 is an initial step towards a full Phase 3 representation of a military access area switch. Larger sizes and more channel groups must be addressed, as mentioned earlier. The number of lines, L, and the number of trunks, T, are usually in hundreds, if not thousands. The loads, offered and carried, must also be magnified accordingly. Unfortunately, however, the computation of the five parameter (i.e., L, T, A, B, C) effects on the three blocking probabilities (i.e., P_d, P_ℓ, P_t) is already quite complex. A more comprehensive numerical effort may be warranted, but was not undertaken as part of this study.

A simplified symmetric case has been evaluated. It assumes that the local and tandem loads are the same, and so are the number of lines and trunks provided. An example is given in Figure 16. It postulates modest offered loads B=C=5 Erlangs on both sides of the switch. One also assumes that a T1 line, or its equivalent 24-channel set, serves each side of the switching network. As abscissa, Figure 16 varies the distant call load A. Because of the inherent symmetries (i.e., L=T and B=C), the local and tandem call blocking probabilities are identical in the figure. The second curve depicts the distant call blocking probability.

When A=0, the Erlang B formula (Cooper, 1972; Bear, 1976) applies. Then

$$P_d \cong 2P_\ell = 2P_t = 2B(12,B). \tag{32}$$

However, as A increases, the roles of P_d and $P_\ell = P_t$ become reversed. While $P_\ell = P_t$ increases in a monotonic way, the distant call blocking probability, P_d, first decreases, passes through a minimum, and finally also increases as a function of load A. For the symmetric case L=T=24 and B=C=5 Erlangs, $P_\ell = P_t > P_d$ holds for A>0.5 Erlangs.

6. APPLICATIONS OF THE THREE BLOCKING RESULTS

6.1. Interpretation and Combination of Results

For the purpose of estimating access area grade of service, previous sections have structured the access area network operation in a special way. At key nodes, the communication paths are partitioned in any of three ways. These ways are called Phases 1, 2 and 3. Probabilities of blocking are derived, and to a limited extent, computed and graphed for these phases.

A total of six blocking probabilities have been identified in Sections 3, 4 and 5. A given network path can encounter various sequences of these six blocking events. Early in this report, Table 1 outlined forty potential route profiles for the access area teletraffic. Every one of said routes passes either zero times,

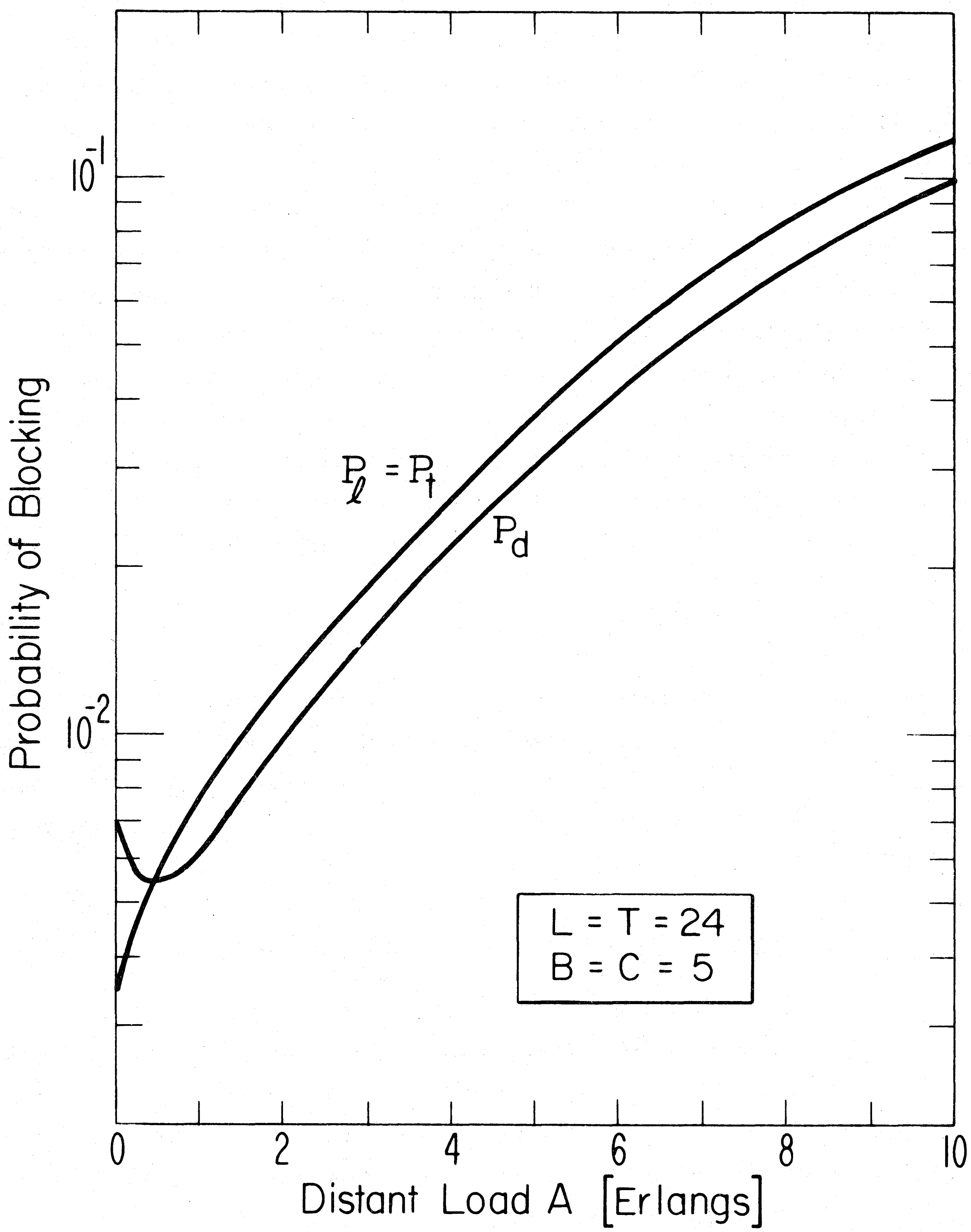


Figure 16. Phase 3 blocking for a symmetric switch (L=T) with a symmetric load (B=C).

or once, or twice through the blocking phases. In doing so, the routes encounter blocking possibilities of the various types, and with different multiplicity per each path. Table 8 brings this all together. It shows for each of the forty paths, the incidences (i.e., the number of occurrences) of the six blocking elements.

The header of Table 8 identifies the three phases of blocking. Phase 1, which takes place at a concentrator, offers two probabilities of blocking:

P_o - for outgoing/incoming through-traffic,

P_r - for return traffic to the same concentrator.

As seen from Table 8, most of the forty paths encounter P_o either once or twice. On the other hand, P_r occurs only for one path, namely for #36. The properties and numerical values for both P_o and P_r are presented in Section 3. They both depend on:

M - the number of concentrated user terminals,

m - the server channels,

a - the out/in load per terminal in Erlangs,

b - the return load per terminal, also in Erlangs.

Of the two probabilities, as one expects, P_r is the largest.

Phase 2 blocking occurs at a PABX. Its probability is denoted in Table 8 by P_x. One notes that the majority of the forty path types face P_x either once or twice. Section 4 presents the P_x material. It depends on such parameters as:

m - the number of concentrated loops,

n - the number of lines,

N - the number of server channels,

a_1 - the offered load per loop in Erlangs,

a_2 - the offered load per line in Erlangs.

The header of Table 8 shows that blocking at Phase 3 can appear in three forms. First, there is the distant call probability of blocking, P_d. It occurs in pairs, on 16 out of the 40 paths. Next, the local call blocking, P_ℓ, occurs only in singles, but on another set of 16 path types. Finally, there is the tandem call blocking probability, P_t. While not of immediate interest to the access area

Table 8. Blocking Probability Structures for the Forty Access Area Circuit Paths

Path #	Blocking Probability Elements					
	Phase 1		Phase 2	Phase 3		
	P_o (Out/In)	P_r (Return)	P_x (PABX)	P_d (Distant)	P_ℓ (Local)	P_t (Tandem)
1	2		2	2		...
2	1		2	2		...
3	2		1	2		...
4	1		1	2		...
5	2		2		1	
6	1		2		1	
7	2		1		1	
8	1		1		1	
9	1		2	2		...
10			2	2		...
11	1		1	2		...
12			1	2		...
13	1		2		1	
14			2		1	
15	1		1		1	
16			1		1	
17	2		1	2		...
18	1		1	2		...
19	2			2		...
20	1			2		...
21	2		1		1	
22	1		1		1	
23	2				1	
24	1				1	
25	1		1	2		...
26			1	2		...
27	1			2		...
28				2		...
29	1		1		1	
30			1		1	
31	1				1	
32					1	
33	2		2			
34	1		2			
35	2		1			
36		1				
37	1		2			
38			2			
39	1		1			
40			1			

study, P_t seems to have potential of occurring any number of 0,1,2,..., times for all those 16 paths that encounter P_d twice.

The three Phase 3 blocking probabilities, P_d, P_ℓ and P_t, depend on:

L - the number of serving lines,

T - the number of serving trunks,

A - the total offered distant load in Erlangs,

B - the total offered local load in Erlangs,

C - the total offered tandem load in Erlangs.

The derivation of Phase 3 formulas, as well as the related P_d, P_ℓ and P_t curves, is presented in Section 5.

Table 8 offers a prescription on how to add the various blocking probabilities for various teletraffic profiles. We shall demonstrate this later by application to specific access area implementation.

6.1.1. Worst and best cases

A glance at Table 8 tells that certain paths' blocking probabilities are bounded by others. Some are simple ordered upper bounds and others are lower bounds. There are also cases where direct comparison is impossible without additional information.

Let P(k) be the effective access area end-to-end blocking probability for path k(k=1,2,...,40). As is appropriate for access area, exclude the tandem blocking, P_t, from this total. If the blocking events are roughly independent, P(k) must be approximately equal to the sum of the probabilities listed in Table 8 for path k. Under such a premise a partial ordering of P(k)'s is possible. Without going into details, one realizes that the highest probability of blocking must occur on either path 1, 5, or 36. The least amount of blocking must occur on either path 28, 32, 36, or 40.

Formally, the worst and best case bounds must therefore be

$$\begin{aligned} P(k) &\leq \max\{P(1), P(5), P(36)\} \\ &\geq \min\{P(28), P(32), P(36), P(40)\}. \end{aligned} \tag{33}$$

And if the individual probabilities are realistically small, additive, and consistent, then

$$P(k) \leq \max\{P_r,\ 2P_o + 2P_x + \max(2P_d,\ P_\ell)\}$$
$$\geq \min\{P_r,\ P_x,\ 2P_d,\ P_\ell\} \qquad (34)$$

holds for all access area paths listed in Table 8, k=1,2,...,40. (Note: P_o is said to be consistent or uniform in an access area, if it has roughly the same value throughout that area. The same consistency definition holds for P_r, P_x, P_d, and P_ℓ, as well.)

The first line of (34) is suited for conservative access area end-to-end grade of service estimation. Thus, by omitting tandem blocking effects at switches, it is quite easy to estimate the worst case bound for blocking on all forty paths. Examples:

If $P_o = P_x = P_\ell = 0.002$,
$P_r = 0.010$,
$P_d = 0.001$,

then the overall blocking cannot exceed 0.01 or 1%.

If $P_o = P_d = 0.010$,
$P_r = 0.050$,
$P_x = 0.005$,
$P_\ell = 0.020$,

then the probability of blocking must be lower than or equal to 5%.

6.2. Specific Application Examples

Teletraffic engineering parameters developed in the previous sections have bearing on the design of new, and upgrading of existing, networks for military access areas. The relationships between the parameters tell one whether the sizes or structures of certain network elements meet the grade of service requirements.

This section demonstrates the procedure. The teletraffic profile of an unspecified access area is used to test the implementation. The profile is typical of a large access area. It contains a main post, plus several somewhat isolated camps and military stations nearby. The implementation concept is perhaps unique. Digital technology is used for concentration, switching and transmission. Both analog and digital services may be integrated on the network.

6.2.1. Access area and its communications outline

The posts, camps and military stations of an access area share the local telecommunication facilities. Many existing areas include tactical units, military reservations, and other entities whose communications systems planning and implementation is the responsibility of a single military department. To overview them briefly, note the following.

There are approximately 1500 military access areas worldwide. Based on the number of user terminals in each area, the areas range from small (<300 terminals), to medium (300 to 3000 terminals), to large (>3000 terminals). The majority of terminal types are telephones, but they also include computer, teletypewriters, facsimile, and a myriad of other terminals. Terminal densities vary from less than 10 per km^2 to over 10,000 per km^2. The higher density cases offer a variety of line concentration alternatives.

Table 9 summarizes the military communications environment. In addition to sizes and population estimates, Table 9 lists the number of various communications terminals which might be found on a medium size post; in an entire relatively large access area; a large military region (consisting of multiple access areas); the continental United States (CONUS); and the global arena of the entire outside world (OCONUS).

Local access area traffic destined for other areas uses the military long haul transmission facilities. There are interfaces between the access area networks and the long haul networks (see Figure 1). The future long haul facilities may interconnect local access areas on either a smaller or larger area basis by a hierarchical network structure. A possible structure is illustrated in Figure 17. It shows that terminals in an office complex first access a remote switch hub, or PABX, in the office complex. Through that PABX they gain access to the central switch hub or the dial central office on the base, followed by the regional hub or access area switch, and finally to the backbone switch on the Defense Communication Systems (DCS) Global Network.

At the same time, each local area may access common carriers for domestic services. While not highlighted in Figure 17, this fact was illustrated earlier in Figure 1.

Let us return to a typical, medium size, local access area. As stated, it consists of a main post, plus several camps and military stations within its immediate environs, some 1 to 20 km away. Let one station be a large office and laboratory complex outside the main post's reservation boundary.

Table 9. Estimates of Military Communications Terminals, Local to Worldwide

	Medium Post	Access Area	Military Region	CONUS	OCONUS
Area (km^2)	5	50	5,000	15M	500M (Worldwide)
Population (Military & civilian)	5,000	15,000	50,000	1.5M	0.5M
Telephones					
Mainline	1,500	6,000	15,000	300,000	50,000
Extensions	1,000	4,000	10,000	200,000	30,000
Sub-totals	2,500	10,000	25,000	500,000	80,000
Data Terminals					
Interactive	10	30	100	1,000	400
Computer	2	6	30	200	50
Narrative	5	15	50	500	100
Facsimile	3	8	25	250	50
Data	2	6	30	300	50
Sub-totals	22	65	235	2,250	650
Secure Terminals					
AUTOSEVOCOM	1	3	10	1,000	500

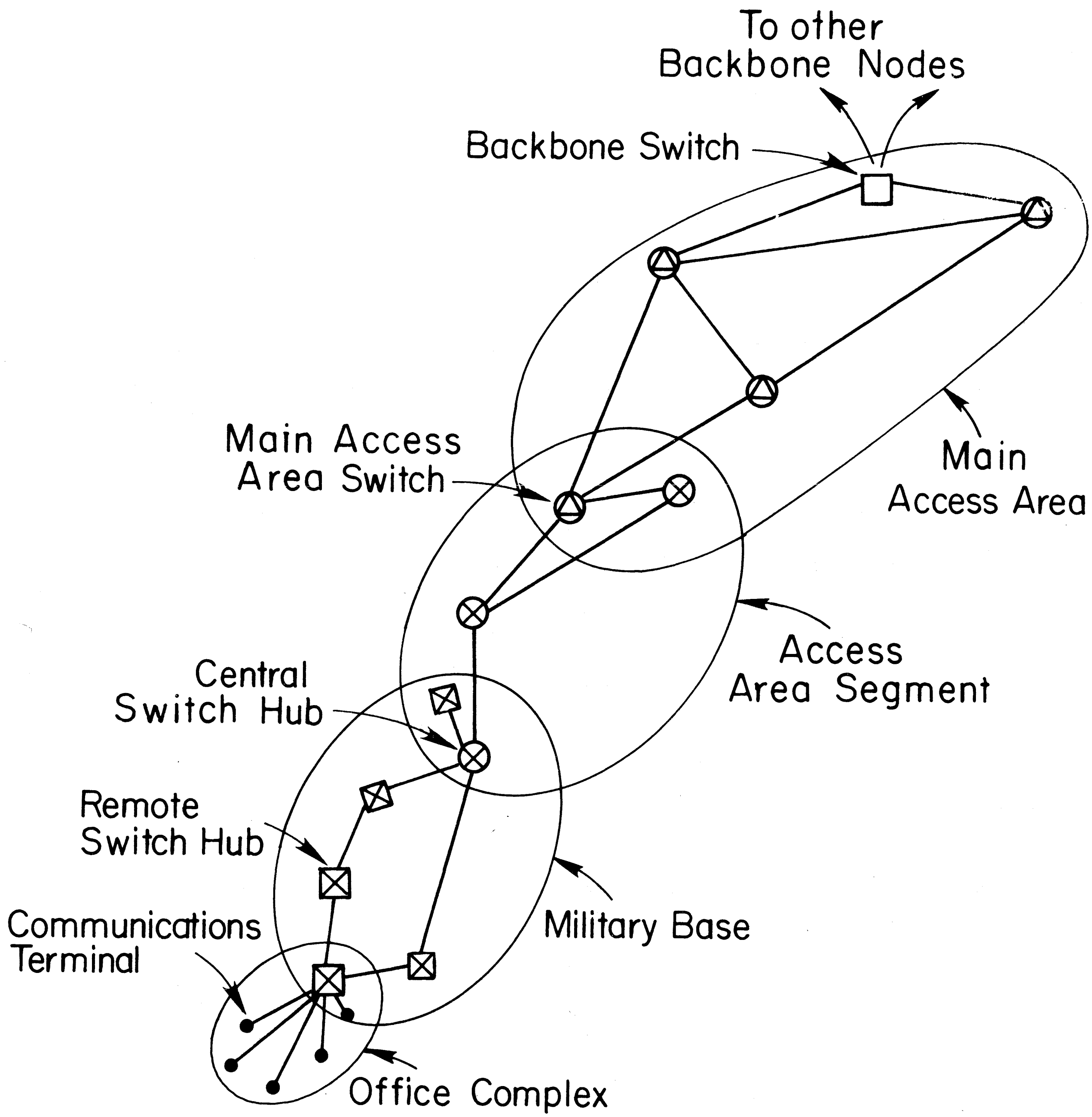


Figure 17. Hierarchical network configurations for interregional access.

A telephone terminal density profile for this illustrative area is postulated in Table 10. Each post, camp and station is divided into subareas corresponding to the average neighborhood terminal densities. This division determines the numbers and the locations of terminal clusters, concentration points, remote switching hubs (e.g., PABX's) and central offices.

A further result from the telephone density breakout is the estimate of the number of mainlines and bridged lines required to serve the area. Table 11 lists such a telephone dispersal versus eight subarea types, from high density offices to low density suburban. Since the distinction between various office and residential service areas tends to be subjective, the ratios of mainlines to total terminal number are always somewhat arbitrary.

6.2.2. Network implementation concept

Next, one must configure the network. An implementation concept for an all-digital access area network is shown in Figure 18. Terminals interface with switching hubs via concentrators whose outputs are time division multiplex (TDM) loops. Concentrators may be remoted from the switches themselves to sites at or near the terminal clusters. Repeatered trunk circuits are used for such remoted units whenever the span is sufficiently large. Trunk circuits may be multiplexed to higher MUX levels, to relatively standard 24 channels (T1), or 48 channels (T1C), or 96 channels (T2) -- all digital trunks.

The remote switch hubs (RSH), such as PABX's, are located in office complexes. Local concentration units (LCU) provide terminations at the switch itself. Remote concentration units (RCU) are some distance from the RSH, at strategic clusters of terminals.

The switching is performed by time slot interchange on the concentrator output loops, or by time multiplexed gates on space division multiplexed (SDM) highways (Linfield and Nesenbergs, 1978).

The local and remoted components of the switching system are shown in Figure 19. The size of the switch is partly determined by the RCU's and LCU's that home on the switch (Linfield and Nesenbergs, 1979). At the design level, the concentration ratio depends on the projected traffic intensity.

All inputs and outputs of the switching matrix are assumed to consist of multiplexed 12 time slot units. A time slot contains one 8-bit byte of a standard 64 kb/s pulse code modulated (PCM) signal. Switching occurs 8000 times per second for each such voice-equivalent channel.

Table 10. Telephone Terminal Density Profile

Subarea Type	Density Term/km^2	Subarea in km^2				Total Terminals
		Main Post	Remote Camps	Remote Stations	Total Area	
Large Office	10,000	0.15	0.2	0.3	0.65	6,500
Medium Office	5,000	0.2	0.2	--	0.40	2,000
Small Office	2,000	0.4	--	0.02	0.42	820
Barracks	1,000	0.15	--	--	0.15	150
High Residential	500	0.5	0.4	--	0.90	450
Low Residential	200	0.2	0.1	--	0.30	60
High Suburban	100	0.2	0.05	--	0.25	25
Low Suburban	50	0.1	--	--	0.10	5
Total Terminals		3765	3225	3040		10,000

Table 11. Main Line and Bridged Telephone Distribution Profile

Subarea Type	Main Post		Remote Camps		Remote Stations	
	Main Lines + Bridged	Main Line	Main Lines + Bridged	Main Line	Main Lines + Bridged	Main Line
Large Office	1,500	700	2,000	1,300	3,000	1,600
Medium Office	1,000	400	1,000	600	--	--
Small Office	800	300	--	--	40	25
Barracks	150	80	--	--	--	--
High Residential	250	90	200	140	--	--
Low Residential	40	30	20	10	--	--
High Suburban	20	20	5	5	--	--
Low Suburban	5	5	--	--	--	--
Totals	3,765	1,625	3,225	2,065	3,040	1,625

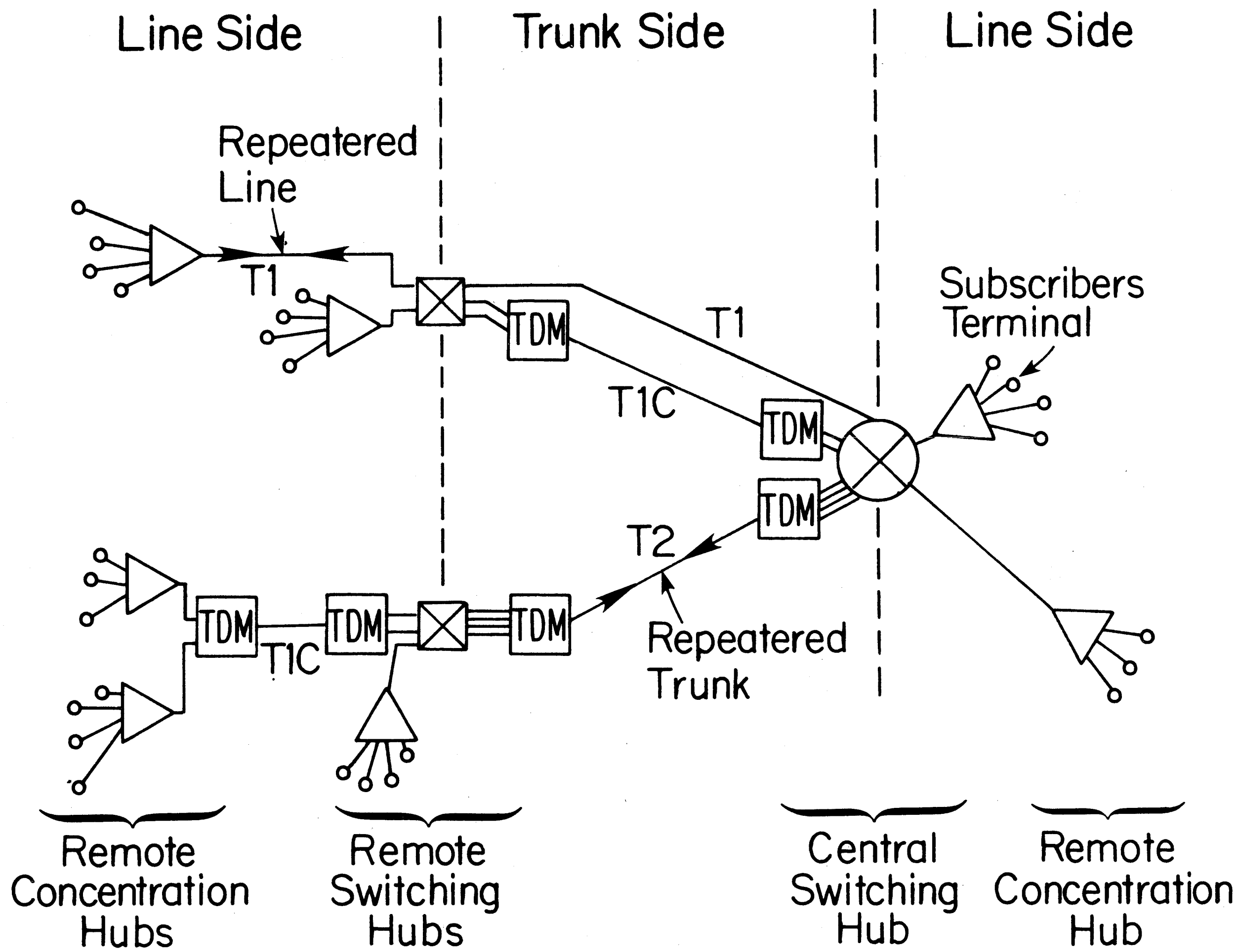
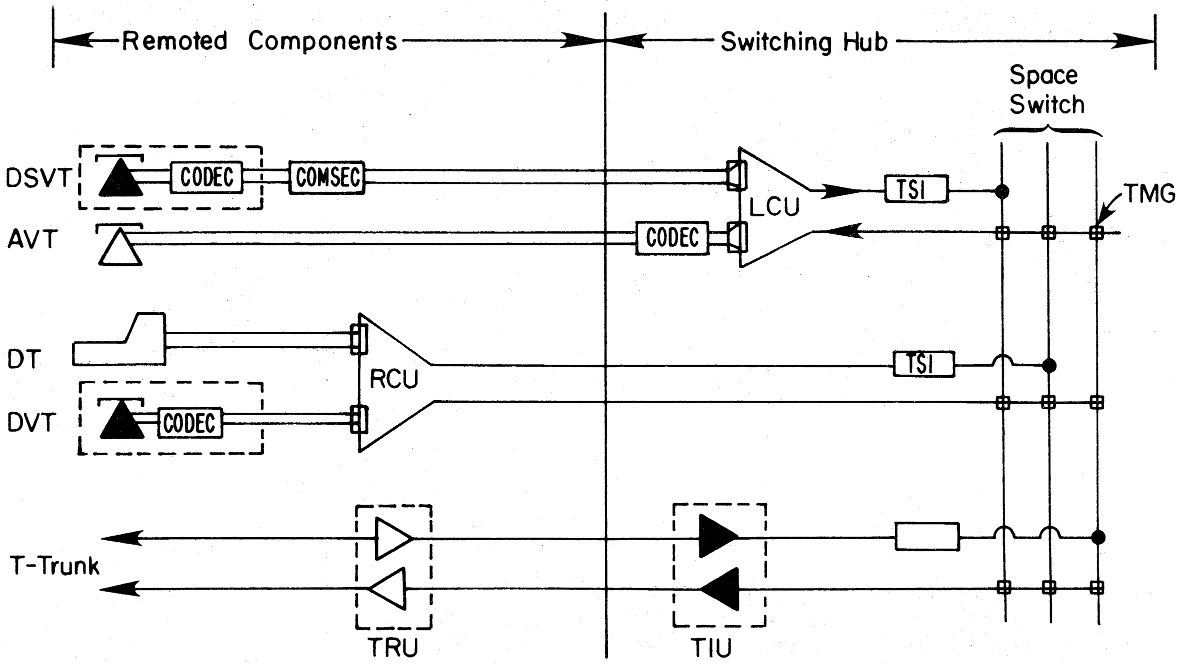


Figure 18. Implementation concept for digital concentration, multiplexing, switching and transmission in the access area.



Legend

DSVT:	Digital Secure Voice Terminal	RCU:	Remote Concentration Unit
AVT:	Analog Voice Terminal	TRU:	Trunk Repeater Unit
DT:	Digital Terminal	TIU:	Trunk Interface Unit
DVT:	Digital Voice Terminal	TSI:	Time Slot Interchange
LCU:	Local Concentration Unit	TMG:	Time Multiplexed Gate

Figure 19. Local and remoted components of a switching hub.

The above implementation concept has been applied to the access area with population, terminal and mainline density profiles given in Tables 9, 10, and 11, respectively. The result is the topological structure of Figure 20. In this figure, the numbers next to a terminal symbol denote the number of such terminals that are concentrated at the next higher stage, such as at RCU, LCU, RSH, and so forth.

The remaining traffic engineering job is to ascertain the blocking probability levels for the family of forty potential paths (see Tables 1 and 8), as they traverse the network of Figure 20 in all conceivable directions. Note that two of the forty paths, namely #5 and #36, are drawn with broken lines in Figure 20. The following paragraphs of this section estimate the blocking probabilities. The key tools will be the five blocking probabilities (i.e., see Table 8 and ignore P_t), that result from the three phases of access area traffic flow (see Sections 2, 3, 4 and 5).

6.2.3. Grade of service estimates

In this section, the estimation of numerical values is demonstrated. The example applies to the network of Figure 20, and the various paths through it (see Table 1). The blocking probabilities are determined in terms of the three blocking phases.

First, however, one additional piece of information is needed. That is the amount of traffic offered and/or carried by the scenarios of Tables 9, 10, 11, and Figure 20. For simplicity, assume that the loss is so small as to be negligible to the gross flow. Then the carried load is practically indistinguishable from the offered load, and one can refer to them both as the "load." As before, let all loads be expressed in Erlangs.

To expedite matters, assume the average loads per line to be as given in Table 12. There are three number columns in this table. The first column shows how the traffic from a station that is tied to a concentrator is distributed to six different terminal groups. Finer resolutions of the substreams are possible, but not attempted here. The key point is that such a concentrated station carries 0.1 Erlangs, of which half returns through the same CONC.

The second column depicts an access area terminal that is tied directly to the PABX. Its load, perhaps as an anomaly, is shown to be less than that of the first column. Finally, the third column applies to a station that has a direct local loop to the end office switch. Its load is typically the largest and, as

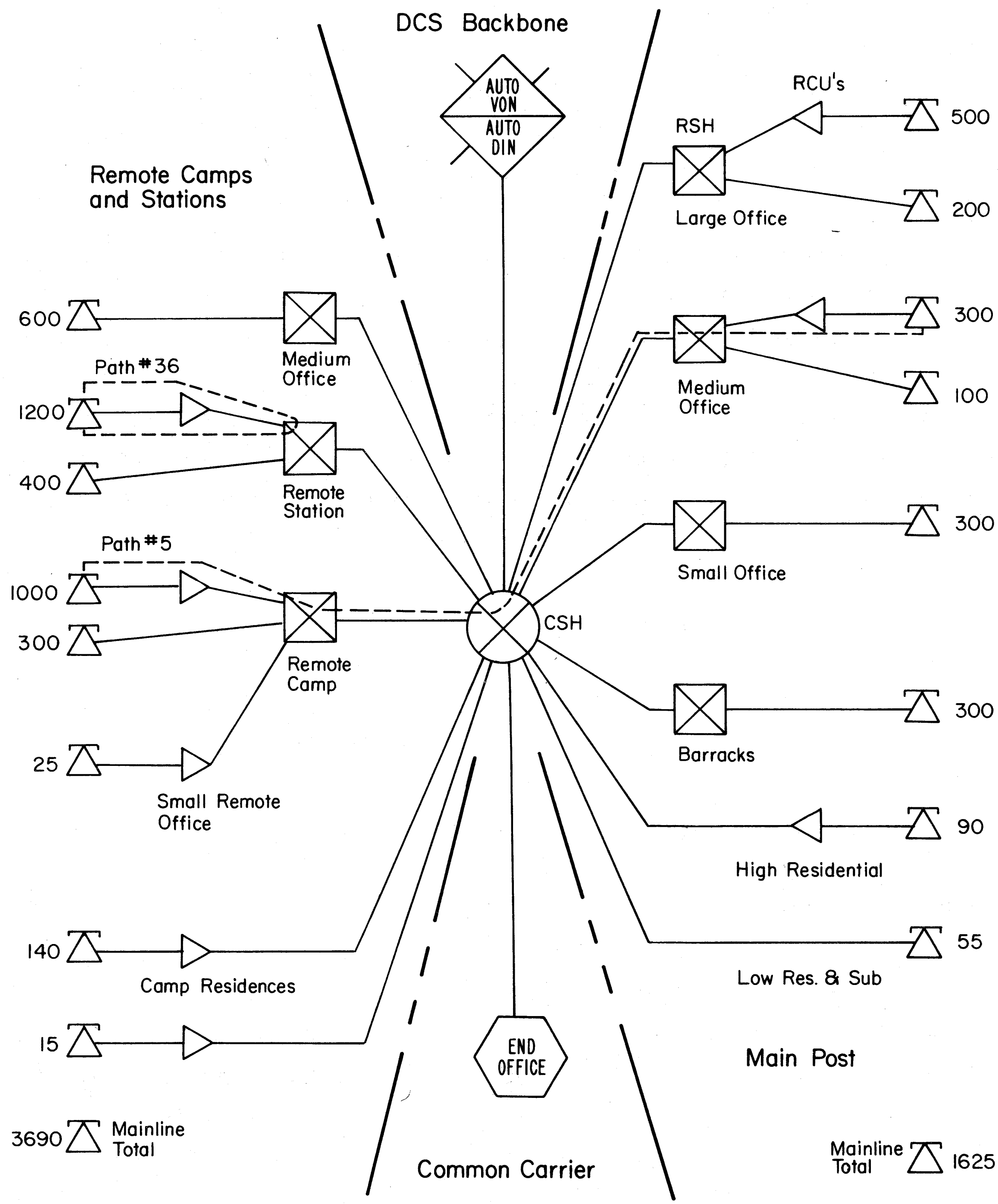


Figure 20. Illustrative example of an access area communications network.

Table 12. Assumed Loads Per Line (In Erlangs) for Selected Terminal-to-Terminal Group Substreams

To a Terminal Group that is Directly Homed to: CONC	PABX	SWITCH	From a Terminal that is Directly Homed to: CONC	PABX	SWITCH
Same	Same	Same	.050	.022	.042
Diff	None, Same or Diff	Same	.022	.010	.042
None	Same	Same	.010	.008	.017
None	Diff	Same	.005	.006	.017
None	None	Same	.010	.008	.025
None or Diff	None or Diff	Diff	.003	.006	.017
Total Per Line Load at a Terminal			.100	.060	.160

seen from Table 12, it is assumed in this sample to be equal to the sum of the previous two.

Incidentally, the general distribution of per line intensities is roughly comparable in the three columns. All substreams, and there are 18 such in Table 12, are identifiable in the access area network (Figure 20), with groupings of circuit paths (Table 1), as well as with elements of the three-phase blocking model (Figure 2).

The quantitative information provided about the access area teletraffic, plus the three-phase approach to blocking, suffices for estimating grade of service for any given service line and trunk group provision. That means that any network configuration, such as that given in Figure 20, may be specified in terms of blocking probabilities P(k) for all forty paths k=1,2,...,40 (see Tables 1 and 8).

To show one way of estimating grade of service quickly, assume that all network elements in the present access area example (Figure 20) are grouped in modules equivalent to or better than the module in Figure 21. The concentrators compress M=96 user lines into m=24 CONC loops. These, in turn, are grouped with n=48 direct lines at PABX line side. On the other side of the PABX, a tie-line group of N=12 provides access to the central switch hub. Here, the switch is depicted as a rather uncommonly small one. The choice of L=T=24 (lines and trunks, respectively) is strictly for illustration sake. It enables one to get along with the very limited graphs computed in Section 5. (For most switches, with larger L and T groups, additional computation of P_d, P_ℓ and P_t formulas appears necessary.)

Table 12 establishes the loads at the concentrator:

$$M = 96,$$

$$m = 24,$$

$$M(a+b) = 9.6 \text{ Erlangs},$$

$$\frac{a}{a+b} = 0.5,$$

and thus from Figure 7, the out/in and return call blocking probabilities are

$$P_o \cong .005,$$

$$P_r \cong .010. \qquad \text{(Phase 1)}$$

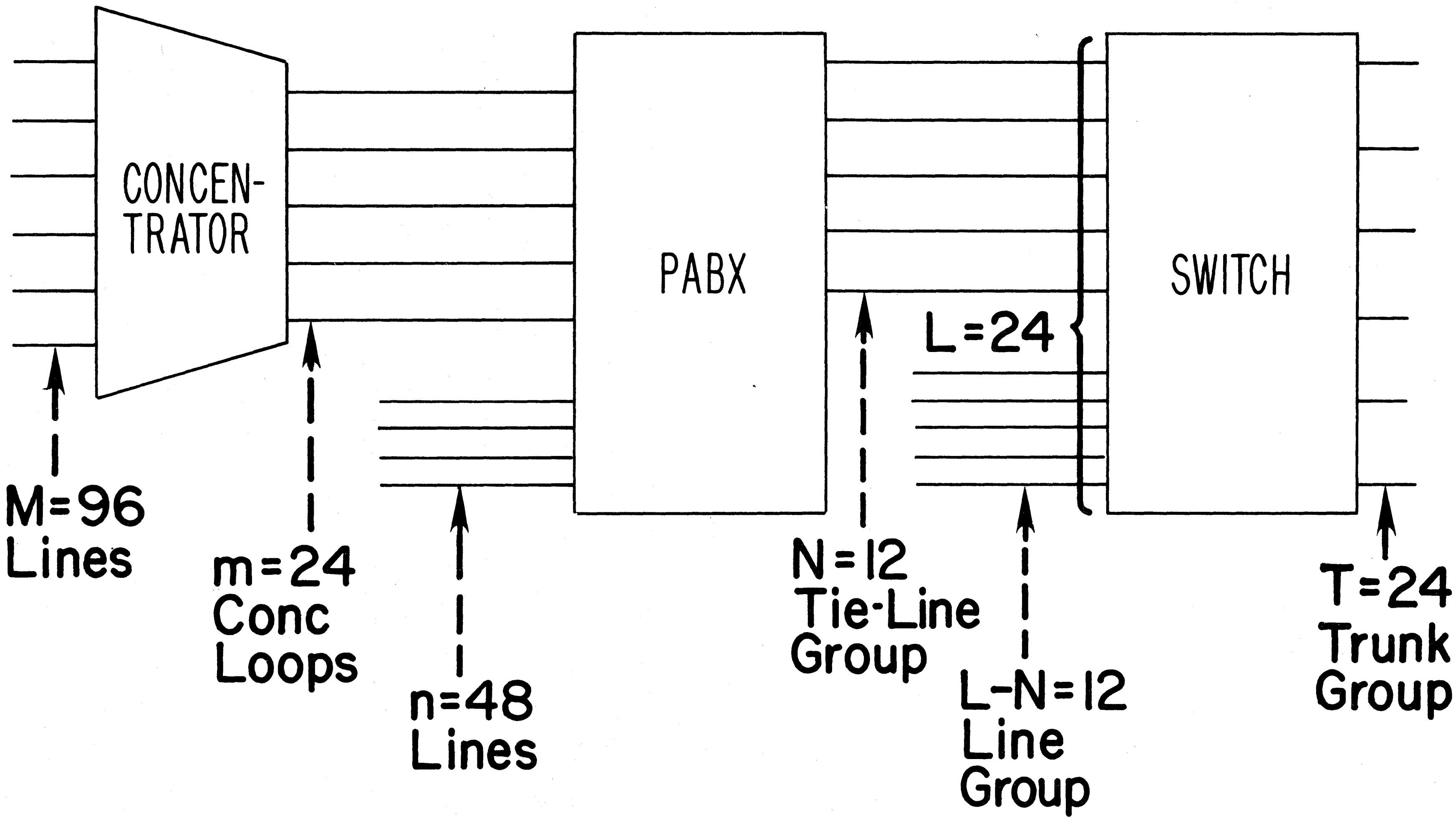


Figure 21. Sizing of line and trunk groups in the three phase format.

At the PABX:

$m = 24,$

$n = 48,$

$A_1 = 4.7$ Erlangs,

$A_2 = 2.5$ Erlangs.

From Figure 13 then, the PABX blocking loss is

$P_x \cong .012$ (Phase 2)

Finally, at the switch:

$L = T = 24,$

$A = 1.6$ Erlangs,

$B = C = 5.0$ Erlangs.

Here Figure 16 applies, and the distant, local and tandem call probabilities are appoximately

$P_d \cong .008,$

$P_\ell = P_t \cong .010.$ (Phase 3)

The probabilities of blocking for the above phases must be combined to estimate the end-to-end blocking for the forty circuit paths (see Tables 1 and 8). When P_o, P_x or P_d occur twice, one assumes that multiplication by two is justified in view of the uniform modular layout (Figure 21). The individual P_o, P_r,..., probabilities have all been shown above to be of the order of 1%. Hence intersections of second and higher orders, as well as statistical correlations can be realistically neglected. Sums of the appropriate multiples of the element probabilities provide an adequate total probability estimate.

Table 13 is the result. It lists the blocking probabilities for all forty paths under the above access area assumptions made. In particular, this table reflects the implementation of the illustrative network (Figure 20) with the concentrator-PABX-switch modules of Figure 21, or their equivalents. The table does ignore, as stated earlier, all tandem blocking in the long haul networks.

Table 13 reveals a five-to-one range in P(k) values. The largest value of 5% loss is seen for path #1. The least value of loss is 1%. It arises for two paths, #32 and #36. These extremes agree with the ranges described in equations (33) and

Table 13. Blocking Probabilities for All Forty Paths for the Assumed Access Area

Path Number k	Blocking Probability P(k)
1	.050 (max)
2	.045
3	.038
4	.033
5	.044
6	.039
7	.032
8	.027
9	.045
10	.040
11	.033
12	.028
13	.039
14	.034
15	.027
16	.022
17	.038
18	.033
19	.026
20	.021
21	.032
22	.027
23	.020
24	.015
25	.033
26	.028
27	.021
28	.016
29	.027
30	.022
31	.015
32	.010 (min)
33	.034
34	.029
35	.022
36	.010 (min)
37	.029
38	.024
39	.017
40	.012

(34). Of the two, the largest represents the worst overall case. It appears more useful in access area teletraffic engineering.

A mean, $\overline{P}$, averaged with the relative weights of the appropriate substreams (as summarized in Table 12) is straightforward to calculate. One gets $\overline{P}$=.0408, or roughly a 4% loss, for the access area model assumed.

7. CONCLUSION

Tools have been developed for analysis of military access area circuit switching grade of service. The tools consist of structuring the network flow into three distinct congestion phases. These phases (see Sections 3, 4, and 5) are individually quite distinct. From a switching point of view, they represent new problems that warrant new solutions.

This report has derived solutions for all three phases. The Phase 1 solution is perhaps most succinctly stated in equation (10). It is a linear weighted sum of two apparently reasonable approximations (5). The latter also provide valid bounds. As spelled out in text, the solution for Phase 1 is exact for many parametric conditions of interest. Unfortunately, the general role of the Phase 1 solution is that of an approximation. Its goodness of fit has not been formally established, but is expected to be quite sufficient for access area applications. The goodness of fit could be a topic for further work. Another extension of Phase 1 should address the existing multiplicity of loads. Equation (12), or its equivalent, can be used to map initial loads into offered, as well as into effective or measured, loads at the concentrator facility.

The general state probability solution for Phase 2 is given in equation (18). It is an exact result for two user classes. Extensions to more user classes at a PABX may be of interest. If so, they could be addressed in future studies. But a far more significant generalization than the number of classes seems to be the SENET type generalization. As mentioned in this report and reviewed elsewhere (Vena and Coviello, 1975; Weinstein, et al., 1978; Blackman 1979; Gallagher, 1979), integrated voice and data, AUTOVON and AUTODIN, message and packet, transmission and switching, rerouting and queueing, plus other complex issues, enter into the DCS transition thrust from the present analog toward the all-digital system of the 1990's. The Phase 2 model should accordingly be extended to include priority structures and pre-emption interrupts. This may alter the state and blocking probabilities to a considerable degree. One should also incorporate store-and-forward queueing aspects. In addition to the hybrid formulas for blocking (Nesenbergs, 1979), this elicits a statistical framework for random delays.

Such random delays are apt to occur at all storage nodes in the network (Baran, 1964; Kleinrock, 1976; Fischer and Harris, 1976), in a statistically dependent manner, and with peculiar joint distributions. Future studies of local access areas should take a close look at the priority delays generated by digital message or packet switched nodes. After all, that may be the future function of access area switching hubs.

The formal solution for Phase 3 state probabilities is given in (31). The formula is exact for the assumptions stated. Representing a switch, the model merits all the comments and extensions directed to the Phase 2 PABX. That, of course, includes storage delays and priority aspects. But, more significantly, a larger number of channel groups - instead of the two treated in text - should be analyzed. From all preliminary indications, this task appears tractable whenever an infinite number of traffic sources may be postulated. Other issues at the switch should address limited or partial availability, as well as effects induced by control and network structures.

Blocking probability curves have been computed for selected cases. While these cases have turned out to be quite useful here (see Section 6) to demonstrate the applications, additional computations seem needed for more realistic parameter ranges of interest.

Both in the early parts (Sections 1 and 2) and in the later part (Section 6), the definitions and the parametric features of the military access areas have been emphasized. This appears natural, as the three phase approach has been motivated to a large extent by traffic engineering in the local access area. This does not mean, however, that generalizations and extensions to other areas of military or civilian teletraffic are impossible. The opposite seems to be true. As indicated in the description of Phase 2 (see Figure 11, for instance), the latter can be applied to several situations outside the access area proper. It is hoped that all three blocking phases and their formulas, as well as their potential future extensions - perhaps in ways not immediately obvious to these writers - may be useful to communications network and traffic planners.

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In military base access areas, akin to many commercial installations, the telecommunications traffic passes through several concentration and switching stages. Loads to server facilities are formed through mergers and branch outs of offered traffic substreams. Blocking of calls is known to occur in many ways throughout the existing access area networks.

In this report, an effort is made to represent the access area grades of service (i.e., the probabilities of blocking for different substreams) in more realistic ways than before. The message flow process is structured into three representative contention phases. The three phases are realistic and occur often in military networks. All three phases apparently possess queueing models and analytical properties distinct from the conventional Engset, Erlang, and other classical models. Their blocking probabilities also differ significantly.

One of the three models appears tractable only through bounds and asymptotically tight approximations. The other two models are shown to permit formal solutions. Given an access area network, the three blocking probabilities may be applied individually or in a variety of combinations. The paper demonstrates several applications to access area telephony.

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