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A Method for Determining the Minimum Elliptical Beam of a Satellite Antenna

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The 1977 World Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service decided that elliptical antenna beams should be used for satellite antennas for planning purposes. Since it is desirable to minimize the antenna beam solid angle, we have developed a method for determining the minimum elliptical beam of a satellite antenna. The method is based on a straightforward and exhaustive search for a minimum elliptical beam over all possible ranges of all necessary parameters that specify the elliptical beam. Although it may sound contrary, this method is more efficient than an existing method based on the advanced technique of nonlinear programming. This report describes the method and the computer program that implements the method. It also gives some examples.

Key words: BSS (broadcasting-satellite service), elliptical antenna beam, minimum elliptical antenna beam, satellite antenna

1. INTRODUCTION

The 1977 WARC-BS (World Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service) decided that elliptical antenna beams should be used for planning purposes (ITU, 1977, Annex 8, par. 3.13.1). To specify the elliptical beam of a satellite antenna, it used the following five parameters: the longitude and latitude of the boresight point of the antenna beam on the surface of the earth, the major-axis and minor-axis beamwidths of the elliptical beam, and the orientation angle of the ellipse. The orientation angle of the ellipse is defined as the angle measured anti-clockwise (counterclockwise), in a plane normal to the beam axis, from a line parallel to the equatorial plane to the major axis of the ellipse to the nearest degree (ITU, 177, art. 11).

It is advantageous for the operator of a system in the BSS (broadcasting-satellite service) who wants to use an elliptical-beam antenna to minimize the satellite transmitter power by using an elliptical antenna beam of minimum solid angle (or minimum area or, equivalently, minimum value of product of major-axis beamwidth by minor-axis beamwidth) that covers the service area.

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For the planner of the channel allotments and orbital positions plan for the BSS, therefore, it is desirable to have a computer program that determines the minimum antenna beam for a service area defined by a set of earth station locations at the polygon vertex points.

Such a computer program has been in existence in the United States for some time. It was programmed by a company under a NASA (National Aeronautical and Space Administration) contract.[1] It uses an advanced technique called the nonlinear programming described by Himmelblau (1972). Unfortunately, however, running the program takes a long time. It might be inappropriate for the U.S. delegation to use at the conference site during the forthcoming 1983 RARC-BS (Regional Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service) of Region 2 (i.e., North, Central, and South America, and Greenland).

We have developed another method for determining the minimum elliptical beam of a satellite antenna. This method is a very primitive one based on a straightforward and exhaustive search for the minimum elliptical beam, i.e., it makes a thorough search, in effect, over all possible ranges of all necessary parameters that specify the elliptical beam. It may sound awkward but, as shown later, it proves to be very efficient.

Section 2 of this report describes the method we have developed for determining the minimum elliptical beam of a satellite antenna. Section 3 describes the computer subroutine package that implements the method, with some examples. Some mathematical details associated with the development of the method are given in Appendix A, and the computer subroutine package is listed in Appendix B.

2. THE METHOD

2.1. Outline

A series of searches for the minimum elliptical beam is performed in sufficiently wide ranges of the longitude and latitude of the boresight point. We assume that the western boundary of the boresight-point longitude for the search is the more western of the following two longitudes: (i) the longitude of the western-most earth station, and (ii) the midvalue of (i) and the satellite longitude. A similar assumption applies to the eastern boundary of the boresight-

[1]This program is outlined in a draft amendment to CCIR Report 812 proposed by the United States, but no published report is available on this program. For completeness, note that the program to be described in this report is also outlined in the same CCIR input document.

point longitude as well as the northern and southern boundaries of the boresight-point latitude.

For each boresight-point location, a series of searches is performed in a range of the orientation angle of the ellipse from 0° to 180°.

For each boresight-point location and an orientation angle of the ellipse, a series of searches is performed in a range of axial ratio of the ellipse from a positive near-zero value to unity. The axial ratio of the ellipse is defined here as the ratio of the minor axis to the major axis of the ellipse; the axial ratio never exceeds unity.

For each boresight-point location, an orientation angle of the ellipse, and an axial ratio of the ellipse, the minimum elliptical beam is determined by an element procedure described in the following subsections. In Subsection 2.2., we will describe the element procedure applicable when there is no pointing error nor orientation-angle error (i.e., rotation) of the satellite antenna beam. In Subsection 2.3., we will modify the element procedure by taking these errors into consideration.

2.2. Element Procedure

We will establish a procedure for determining the minimum elliptical beam of a satellite antenna that covers a given set of earth stations from a given orbital position of the satellite when the boresight point of the satellite antenna, the orientation angle of the ellipse, and the axial ratio of the ellipse are all fixed. This procedure is a basic element of the whole procedure for determining the minimum elliptical beam for a set of earth stations and an orbital position. It consists of the following four steps.

Step 1 of the procedure is to calculate the off-axis angle α and orientation angle β of each earth station from the given orbital position of the satellite and the locations of the boresight point and earth station. This calculation can be done with the procedure described in Appendix A. We denote the resulting angles for the ith earth station by α_i and β_i, $i = 1, 2, \ldots, n_e$, where n_e is the total number of earth stations.

For a given orientation angle of the ellipse, denoted by β_a, we next calculate

$$\left.\begin{aligned} y_i'' &= \alpha_i \cos(\beta_i - \beta_a) \quad , \\ z_i'' &= \alpha_i \sin(\beta_i - \beta_a) \quad , \end{aligned}\right\} \quad i = 1, 2, \ldots, n_e \quad , \tag{1}$$

for each earth station as Step 2. We can plot point (y''_i, z''_i) in a plane with y'' and z'' as the abscissa and ordinate, respectively. Our problem is now to determine, in the y''-z'' plane, a minimum ellipse with the following properties: a) it encloses every point (y''_i, z''_i), b) its center is at the origin of the y''-z'' coordinate system, c) its major axis is in the y''-axis direction, and d) its axial ratio equals an assigned value.

Since an ellipse becomes a circle when it is stretched uniformly in the direction of its minor axis (in the direction of z'' axis, in our case) by the ratio equal to the reciprocal of the axial ratio, our problem reduces to a simpler problem of determining a minimum circle that encloses every point $(y''_i, z''_i/\rho)$, where ρ is the axial ratio, and with its center at the origin of the y''-z'' coordinate system. The square of the radius of such a circle, R^2, can be simply calculated by taking the maximum of $y''^2_i + (z''_i/\rho)^2$ over all possible value of i; it is expressed mathematically by

$$R^2 = \max_i \{y''^2_i + (z''_i/\rho)^2\} \quad . \tag{2}$$

This calculation is done as Step 3.

Step 4 of the element procedure is to translate the result obtained in the stretched x-y/ρ plane to the original x-y plane. From the radius R thus determined and the assigned value of ρ, we can calculate the major and minor axes, B_1 and B_2, of the ellipse by

$$\begin{aligned} B_1 &= 2R \quad , \\ B_2 &= 2\rho R \quad . \end{aligned} \tag{3}$$

2.3. Modifications to the Element Procedure

The element procedure consisting of the above four steps is applicable only when there is no pointing error nor orientation-angle error of the satellite antenna beam. In reality, however, these errors exist, and some of the steps must be modified accordingly. As a matter of fact, the Final Acts of the 1977 WARC-BS allowed a pointing error of 0.1° and an orientation-angle error of 2° (ITU, 1977, Annex 8, par. 3.14.1).

We assume that the effect of the orientation-angle error can be included in the procedure by using, as β_a in (1), the following three values, $\beta_{a0} - \beta_r$, β_{a0}, and $\beta_{a0} + \beta_r$, where β_{a0} is the given value of the orientation angle of the ellipse and β_r is the orientation-angle error. Taking the maximum in (2) must be done also over these three values of β_a. The inclusion of the orientation-angle error requires some additional computation time but does not create any problem concerning the principle.

Although the pointing error changes the boresight point in Step 1, its effect can equivalently be treated by changing, in Step 2, each point (y_i'', z_i'') corresponding to each earth station to a circle having a radius equal to the pointing error and centered at the point. Each circle becomes an ellipse in Step 3, and a rigorous calculation of the radius of the minimum circle, R, becomes almost prohibitive in computation time. To reduce computation time, we replace each ellipse by a closed curve that encloses the ellipse and can be easily handled mathematically. The curve we have chosen here consists of two semicircles having their diameters equal to the minor axis of the ellipse connected by two line segments parallel to the major axis, i.e., a closed curve of a shape similar to an athletic racetrack. Equation (2) is replaced by

$$(R - \varepsilon)^2 = \max \{y_i''^2 + [(z_i''^2 + \varepsilon)/\rho - \varepsilon]^2\} , \tag{4}$$

where ε is the pointing error. The inclusion of the pointing error with this replacement requires very little additional computation time. Since the pointing error is relatively small, the modification with this replacement yields a reasonably tight upper bound to R and, therefore, to the area of the antenna beam.

3. COMPUTER PROGRAM

3.1. Programming Considerations

In implementing the method described in Section 2 in an efficient computer algorithm, we have divided the whole procedure into two stages. In the first stage, a series of preliminary searches is performed over all possible ranges of the longitude and latitude of the boresight point, orientation angle of the ellipse, and axial ratio of the ellipse, without taking into account the pointing and orientation-angle errors. In the second stage, a series of close searches is performed over limited ranges of values of the above four parameters, with the two errors taken into consideration.

In the first stage, a series of two-dimensional binary searches with respect to the longitude and latitude of boresight point is performed with the initial interval of 12.8° and the final interval of 0.4° for both the longitude and latitude. For each boresight location, a series of binary searches with respect to the orientation angle of the ellipse is performed, starting with the initial interval of 22.5° and ending with the final interval of approximately 0.35°. For each orientation-angle value, a series of binary searches with respect to the axial ratio is performed, starting with the initial interval of 12.5% and ending with the final interval of approximately 0.8%. We have confirmed through numerous test runs that truncating the searches with respect to the orientation angle and the axial ratio at the above values does not lead to local minima of the solid angle of the elliptical beam.

In the second stage, a series of two-dimensional binary searches is performed over a possible maximum range of 12.6° each of boresight longitude and latitude, centered at the values obtained in the first stage. The searches start with the initial interval of 0.8° and end with the final interval of 0.1°. This precision (final interval) is considered sufficient since it corresponds to the off-axis angle of the antenna beam of less than 0.02° at the satellite antenna. For each boresight location, a series of searches is performed over a maximum of 20 consecutive integer values of orientation angle in degrees, starting with two consecutive integer values that embrace the value obtained in the first stage. For each orientation-angle value, a series of binary searches with respect to the axial ratio is performed over a possible maximum range of approximately 25% of the axial-ratio value, centered at the value obtained in the first stage. The searches start with the initial interval of 6.25% and end with the final interval of approximately 0.2%. This precision (final interval) is again considered sufficient since it corresponds, in most cases, to the precision of beamwidth of 0.01° or less.

3.2. Computer Subroutine Package

The method described in Section 2 has been implemented in a Fortran computer subroutine package, named MNEBSA. The MNEBSA subroutine package consists of five subroutines. Two subroutines, MNEBSA and MNEBTL, have interfaces with the user, while the remaining three subroutines are supporting subroutines called by MNEBSA and/or MNEBTL.

The MNEBSA subroutine is the master subroutine of the package. It determines the minimum elliptical beam of a geostationary satellite antenna. Although the 1977 WARC-BS adopted a minimum value of 0.6° for the beamwidth of a satellite transmitting antenna (ITU, 1977, Annex 8, par. 3.13.2) as well as a pointing error of 0.1° and an orientation-angle error of 2° for a satellite antenna beam, this subroutine is programmed in such a way that the user can give any values to these parameters for possible application to other problems. It includes a simple calculation of the area (or solid angle) of the beam for the convenience for possible comparison with other results.

The input parameters of the MNEBSA subroutine are:

OLON = longitude of the satellite orbital position,

NE = number of earth stations,

ELON = array of dimension NE that contains the longitude of the earth stations,

ELAT = array of dimension NE that contains the latitude of the earth stations,

BWMN = minimum beamwidth,

PTER = pointing error,

OAEA = orientation-angle error,

all angles in degrees. The output parameters of this subroutine are:

BLON = longitude of the boresight point on the surface of the earth,

BLAT = latitude of the boresight point on the surface of the earth,

BWMA = major-axis beamwidth,

BWMI = minor-axis beamwidth,

ORAN = orientation angle of the ellipse,

AREA = area (or solid angle) of the beam (in degrees squared),

all angles in degrees.

The MNEBTL subroutine accepts all the input data to MNEBSA and all the output data from MNEBSA, except BWMN and AREA, as the input data and calculates the tolerance for each earth station, TLRC; the tolerance is defined here as the distance between the earth station and the circumference of the ellipse, expressed in the angle seen from the satellite antenna. The call to the MNEBTL subroutine is optional.

The MNEBSA package arbitrarily restricts the maximum number of earth stations to 50. The package occupies approximately 1700 locations on the central computer of the U.S. Department of Commerce Boulder Laboratories, that stores up to four instructions (or machine codes) in a location. It is written in ANSI Standard Fortran (ANSI, 1966).

3.3. Examples

Tables 1 and 2 illustrate two examples of test runs with the ETZ (Eastern Time Zone) and PTZ (Pacific Time Zone) of the United States and with the orbital positions of 115°W (OLON = -115.0) and 175°W (OLON = -175.0), respectively. The

Table 1. Example of Minimum Elliptical Beam Calculation
(All angles are in degrees.)

Input Data to MNEBSA, and Input Data to MNEBTL

SERVICE AREA: USA/ET

OLON = -115.0

NE = 10

BWMN = 0.6

PTER = 0.1

OAER = 2.0

I	ELON(I)	ELAT(I)
1	-69.2	47.0
2	-68.4	47.3
3	-66.9	44.8
4	-69.9	41.5
5	-75.5	35.6
6	-81.8	24.6
7	-85.8	30.2
8	-87.6	38.7
9	-90.5	46.6
10	-88.0	47.5

Output Data from MNEBSA, and
Input Data to MNEBTL

BLON = -82.5

BLAT = 37.2

BWMA = 4.26

BWMI = 2.34

ORAN = 124.0

AREA = 7.828

Output Data from MNEBTL

I	TLRC(I)
1	0.106
2	0.071
3	0.000
4	0.131
5	0.452
6	0.015
7	0.455
8	0.596
9	0.011
10	0.193

Table 2. Example of Minimum Elliptical Beam Calculation
(All angles are in degrees.)

Input Data to MNEBSA, and Input Data to MNEBTL

SERVICE AREA: USA/PT

OLON = -175.0

NE = 10

BWMN = 0.6

PTER = 0.1

OAER = 2.0

I	ELON(I)	ELAT(I)
1	-116.0	49.0
2	-114.6	45.5
3	-115.8	46.2
4	-114.0	42.0
5	-114.8	32.5
6	-124.7	48.4
7	-122.8	49.0
8	-124.2	40.4
9	-120.7	34.6
10	-117.1	32.5

Output Data from MNEBSA, and
Input Data to MNEBTL

BLON = -121.0

BLAT = 40.8

BWMA = 3.79

BWMI = 0.69

ORAN = 133.0

AREA = 2.054

Output Data from MNEBTL

I	TLRC(I)
1	0.004
2	0.000
3	0.022
4	0.004
5	0.011
6	0.005
7	0.070
8	0.111
9	0.055
10	0.036

longitude and latitude of 10 earth stations shown as ELON and ELAT in each table are used as the input data. The calculated tolerance (TLRC) values in both tables indicate that the MNEBSA subroutine yields elliptical beams that are almost ideal for all practical purposes.

On the central computer of the U.S. Department of Commerce Boulder Laboratories, the computation time for each example is approximately 2 s, and the corresponding computation cost is approximately $0.7 in U.S. dollars; the clock cycle time of the central processing unit of the computer is approximately 25 ns, and the instruction timings are 100 ns, 125 ns, and 500 ns for the floating-point addition, multiplication, and division, respectively. The computation times and costs increase approximately linearly as the number of earth stations used for calculation increases. For the number of earth stations in a range between 4 and 20, the computation times range approximately between 1.5 s and 4 s on the same computer with the approximate costs ranging between $0.5 and $1.2. These computation costs of the proposed method can be compared favorably with those of the existing method based on the nonlinear programming technique. Although no published data of computation times or costs are available at this time, an engineer of the company that has developed the existing method has estimated the costs to range approximately between $2 and $3 on a computer of a comparable size. As reported in the CCIR input document, the computation time of the earlier version is about 40 s for the same example as Table 1 of this report with an estimated cost of approximately $10.

4. CONCLUDING REMARKS

We have described a method and its associated computer subroutine package for determining the minimum elliptical beam of a satellite antenna. The method is based on a straightforward and exhaustive search. We have shown that the associated computer subroutine package determines the minimum elliptical beams with reasonable computation times and computation costs.

The method described here also has other advantages. The user of this method (or the associated subroutine package) needs to give only the orbital position of the satellite and the geographical locations of earth stations together with the minimum beamwidth and the pointing and orientation-angle errors. The user is not required to make any initial guesses that may be necessary for some other methods. As another advantage we can point out that the described method will not yield any local optimum. This advantage is obvious since the search is exhaustive over possible ranges of all parameters.

We have shown that our method works better than another method based on the technique of nonlinear programming. This result, however, does not imply that the nonlinear programming technique is ineffective in general. Rather, this result should be interpreted to imply that the nonlinear programming technique does not match our specific problem for which a simpler approach can work better.

5. ACKNOWLEDGMENTS

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APPENDIX A: OFF-AXIS AND ORIENTATION ANGLES OF AN EARTH STATION

Determination of the minimum elliptical beam of a satellite antenna involves the off-axis angle and the orientation angle of the earth station. Calculation of the gain of a satellite antenna in the direction of an earth station also involves the off-axis angle of the earth station and, in the case of a noncircular beam, the orientation angle of the earth station as well. The off-axis angle of an earth station is the angle between the beam axis of the satellite antenna and the vector from the satellite to the earth station. The orientation angle of the earth station is defined as the angle measured counterclockwise, in a plane normal to the beam axis of the satellite antenna, from a line parallel to the equatorial plane to the projection of the vector from the boresight point to the earth station. This appendix describes the procedure for calculating the off-axis and orientation angles of an earth station from given locations of the orbital position of the satellite, the boresight point of the satellite antenna on the surface of the earth, and the earth station.

We assume that the satellite orbital position, the location of the satellite-antenna boresight point, and the location of the earth station are given in terms of their longitude and latitude angles, with plus signs indicating the east longitude and north latitude, respectively, as a general convention. We represent the location of the boresight point of a satellite antenna by the longitude and latitude of the point on the surface of the earth, at which the beam axis of the satellite antenna intersects the surface of the earth.

Throughout this appendix, we take the radius of the earth as the unit of length. We use subscripts b, e, and o to denote the boresight point, the earth station, and the orbital position, respectively.

To calculate the off-axis and orientation angles of an earth station, we introduce two coordinate systems. They are called the earth-center coordinate system and the boresight-point coordinate system.

We first introduce a coordinate system called the earth-center coordinate system. It is a Cartesian system. We call the three axes of the coordinate system the x, y, and z axes. The origin of the coordinate system is the center of the earth. The positive x axis intersects the surface of the earth at 0°E and 0°N. The positive y axis intersects the surface of the earth at 90°E and 0°N. The positive z axis points toward the north pole.

In the earth-center coordinate system, the three coordinates of a point are given by

$$x = r \cos\theta \cos\phi \quad ,$$
$$y = r \cos\theta \sin\phi \quad , \qquad \text{(A-1)}$$
$$z = r \sin\theta \quad ,$$

where r is the distance between the point in question and the center of the earth (i.e., the origin of the coordinate system) measured with the radius of the earth as the unit, and θ and ϕ are the latitude and longitude of the point in question. A point on the surface of the earth is represented by (A-1) with

$$r = 1 \quad , \qquad \text{(A-2)}$$

since the radius of the earth has been taken as the unit of the length. Equation (A-2) applies to all earth stations as well as the boresight point of the satellite antenna on the surface of the earth. The earth-center coordinates of the orbital position, x_o, y_o, and z_o, are represented also by (A-1) with the distance of the orbital position from the center of the earth, r_o, and the latitude and longitude of the orbital position, θ_o and α_o. For the orbital position of a geostationary satellite, we have

$$r_o = 6.6239,$$
$$\theta_o = 0. \qquad \text{(A-3)}$$

Note, however, that all other relations described in this appendix hold without regard to (A-3).

When an orbital position of the satellite and a boresight point on the surface of the earth are given, we define another coordinate system called the boresight-point coordinate system. It is also a Cartesian system. We call the three axes of the coordinate system the x', y', and z' axes. The origin of the coordinate system is the boresight point on the surface of the earth. The positive x' axis points toward the orbital position of the satellite. The y' axis is parallel to the equatorial plane. The positive z' axis is on the north side of the x'-y' plane. Again, the radius of the earth is taken as the unit of length. In this coordinate system, the coordinates of the orbital position are represented by

$$x'_o = \sqrt{(x_o - x_b)^2 + (y_o - y_b)^2 + (z_o - z_b)^2},$$
$$y'_o = 0 \quad , \qquad \text{(A-4)}$$
$$z'_o = 0 \quad ,$$

where x_o, y_o, and z_o are the earth-center coordinates of the orbital position, and

x_b, y_b, and z_b are the same coordinates of the boresight point.

When the boresight-point coordinates of an earth station are given, the off-axis and orientation angles of the earth station can be calculated without difficulty. Let the off-axis and orientation angles of an earth station be denoted by α_e and β_e, respectively, and the boresight-point coordinates of the earth station, by x'_e, y'_e, and z'_e. Then, α_e and β_e are represented by

$$\alpha_e = \tan^{-1} \left(\sqrt{y'^2_e + z'^2_e} \,/\, (x'_o - x'_e)\right) ,$$
$$\beta_e = \tan^{-1} (z'_e \,/\, y'_e) , \tag{A-5}$$

where x'_o is the x' coordinate of the orbital position, given in (A-4).

Conversely, when α_e and β_e of an earth station are given, its boresight-point coordinates, x'_e, y'_e, and z'_e, can also be calculated. Since the distance between a point on the surface of the earth and the center of the earth is unity, the coordinates must satisfy, in addition to (A-5),

$$(x'_e - x'_c)^2 + (y'_e - y'_c)^2 + (z'_e - z'_c)^2 = 1, \tag{A-6}$$

where x'_c, y'_c, and z'_c are the boresight-point coordinates of the center of the earth. Since the origin of the boresight-point coordinate system (i.e., the boresight point) is also on the surface of the earth, we have

$$x'^2_c + y'^2_c + z'^2_c = 1. \tag{A-7}$$

With the help of (A-7), we can solve (A-5) and (A-6) with respect to x'_e, y'_e, and z'_e. The results are

$$\begin{aligned} x'_e &= - B + \sqrt{B^2 - C}, \\ y'_e &= (x'_o - x'_e) \tan \alpha_e \cos \beta_e, \\ z'_e &= (x'_o - x'_e) \tan \alpha_e \sin \beta_e, \\ B &= - x'_o \sin^2 \alpha_e - x'_c \cos^2 \beta_e + E, \\ C &= x'_o (x'_o \sin^2 \alpha_e - 2E), \\ E &= (y'_c \cos \beta_e + z'_c \sin \beta_e) \sin \alpha_e \cos \alpha_e. \end{aligned} \tag{A-8}$$

As the expression for x'_e suggests, x'_e has been determined as a root of a quadratic equation. Out of two roots of the quadratic equation, a root that is closer to zero (or the x' coordinate of the boresight point) has been selected;

the other root has a more negative value and corresponds to a point that is invisible from the satellite.

Since the relation between the two angles and the boresight-point coordinates of an earth station has been established, the remaining problem is the coordinate transformation of the earth station from the earth-center coordinate system to the boresight-point coordinate system and vice versa.

Coordinate transformation from the earth-center coordinate (x, y, z) to the boresight-point coordinate (x', y', z') is represented by

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x - x_b \\ y - y_b \\ z - z_b \end{bmatrix} , \qquad \text{(A-9)}$$

where a_{ij}'s, $i = 1, 2, 3$, $j = 1, 2, 3$, are the coefficients of coordinate transformation, and x_b, y_b, and z_b are the earth-center coordinates of the boresight point on the surface of the earth, calculated by (A-1) and (A-2) with the latitude and longitude of the boresight point, θ_b and ϕ_b. Since both coordinate systems are Cartesian with the same length of unit vectors, the following orthonormal relations among the coefficients a_{ij}'s hold:

$$a_{i1}a_{j1} + a_{i2}a_{j2} + a_{i3}a_{j3} = \delta_{ij} = \begin{cases} 1 \text{ when } i = j , \\ 0 \text{ when } i \neq j , \end{cases}$$

$$a_{1i}a_{1j} + a_{2i}a_{2j} + a_{3i}a_{3j} = \delta_{ij} = \begin{cases} 1 \text{ when } i = j , \\ 0 \text{ when } i \neq j . \end{cases} \qquad \text{(A-10)}$$

The inverse transformation from the boresight-point coordinate to the earth-center coordinate is represented by

$$\begin{bmatrix} x - x_b \\ y - y_b \\ z - z_b \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \qquad \text{(A-11)}$$

Determination of a_{ij}'s will follow.

Since point (x'_o, y'_o, z'_o) represented by (A-4) in the boresight-point coordinate system is represented as point (x_o, y_o, z_o) in the earth-center coordinate system, we obtain, from (A-11),

$$a_{11} = (x_o - x_b)/x'_o \quad ,$$

$$a_{12} = (y_o - y_b)/x'_o \quad , \qquad \text{(A-12)}$$

$$a_{13} = (z_o - z_b)/x'_o \quad .$$

Thus, three coefficients, a_{11}, a_{12}, and a_{13}, have been determined.

Since the y' axis is parallel to the equatorial plane, we have the relation that

$$z = z_b \text{ when } x' = 0 \text{ and } z' = 0 \quad . \qquad \text{(A-13)}$$

From (A-11) and (A-13), we obtain

$$a_{23} = 0 \quad . \qquad \text{(A-14)}$$

Thus, a_{23} has been determined as the fourth coefficient.

Two coefficients, a_{21} and a_{22}, are determined concurrently. From (A-10) and (A-14), we have

$$a_{11}a_{21} + a_{12}a_{22} = 0 \quad .$$

$$a_{21}{}^2 + a_{22}{}^2 = 1 \quad .$$

Solving this set of equations, we obtain

$$a_{21} = \mp\, a_{12}/\sqrt{a_{11}{}^2 + a_{12}{}^2} \quad ,$$

$$a_{22} = \pm\, a_{11}/\sqrt{a_{11}{}^2 + a_{12}{}^2} \quad .$$

Since the positive z' axis is on the north side of the x'-y' plane, the angle between the positive z axis and the positive z' axis is less than 90°, and so is the angle between the x-y plane and the x'-y' plane. Therefore, the signs of x' and y' are the same as what are obtained by rotating the x-y plane. This dictates that a_{11} and a_{22} must have an identical sign. We obtain

$$a_{21} = -\, a_{12}/\sqrt{a_{11}{}^2 + a_{12}{}^2} \quad ,$$

$$\qquad \text{(A-15)}$$

$$a_{22} = a_{11}/\sqrt{a_{11}{}^2 + a_{12}{}^2} \quad .$$

Thus, we have determined a_{21} and a_{22} as the fifth and sixth coefficients.

From (A-10) and (A-14), we obtain

$$a_{33} = \pm\sqrt{1 - a_{13}^2} \quad .$$

Since the z' axis is on the north side of the x'-y' plane, or z and z' must have an identical sign, we obtain

$$a_{33} = \sqrt{1 - a_{13}^2} \quad . \tag{A-16}$$

Thus, a_{33} has been determined as the seventh coefficient.

From (A-10) and (A-14) we have

$$a_{11}a_{13} + a_{31}a_{33} = 0$$

and, therefore, we obtain

$$a_{31} = - a_{11}a_{13}/a_{33} \quad . \tag{A-17}$$

Thus, a_{31} has been determined as the eighth coefficient.

Similarly, from (A-10) and (A-14), we obtain

$$a_{32} = - a_{12}a_{13}/a_{33} \quad . \tag{A-18}$$

Thus, a_{32} has been determined as the ninth and last coefficient.

APPENDIX B: FORTRAN LISTING OF THE MNEBSA SUBROUTINE PACKAGE

The following listing includes, in the first two pages, the listing of a check program, CKMNEB, that illustrates how to call the MNEBSA and MNEBTL subroutines. The CKMNEB program produced, among others, the results shown in Tables 1 and 2 in the text. (The CKMNEB program is also written in ANSI Standard Fortran except the PROGRAM statement. The SECOND function called by this program is the internal clock of the computer of the U.S. Department of Commerce Boulder Laboratories; it is included in neither intrinsic functions nor basic external functions of ANSI Standard Fortran.)

```
      PROGRAM  CKMNEB(OUTPUT,TAPE6=OUTPUT)
C PROGRAM CHECK FOR MNEBSA AND MNEBTL
C DECLARATION STATEMENTS
      DIMENSION   LSA(4)
      DIMENSION   OLON(4),NE(4),ELON(50,4),ELAT(50,4)
      DIMENSION   ELN1(3),ELT1(3)
      EQUIVALENCE (ELN1(1),ELON(1,1)), (ELT1(1),ELAT(1,1))
      DIMENSION   ELN2(6),ELT2(6)
      EQUIVALENCE (ELN2(1),ELON(1,2)), (ELT2(1),ELAT(1,2))
      DIMENSION   ELN3(10),ELT3(10)
      EQUIVALENCE (ELN3(1),ELON(1,3)), (ELT3(1),ELAT(1,3))
      DIMENSION   ELN4(10),ELT4(10)
      EQUIVALENCE (ELN4(1),ELON(1,4)), (ELT4(1),ELAT(1,4))
      DIMENSION   EOFA(50),EORA(50),TLRC(50)
      DATA  ND/4/
      DATA  LSA(1)/6HRRB   /, OLON(1)/-101.6/
      DATA  NE(1)/3/,
     1  ELN1(1), ELN1(2), ELN1(3)/
     2   -59.60,  -59.40,  -59.60/,
     3  ELT1(1), ELT1(2), ELT1(3)/
     4    13.30,   13.20,   13.10/
      DATA  LSA(2)/6HCAN/3 /, OLON(2)/-145.0/
      DATA  NE(2)/6/,
     1  ELN2(1), ELN2(2), ELN2(3), ELN2(4), ELN2(5), ELN2(6)/
     2  -106.00,  -95.00,  -89.00,  -95.20,  -95.20, -106.00/,
     3  ELT2(1), ELT2(2), ELT2(3), ELT2(4), ELT2(5), ELT2(6)/
     4    70.00,   70.00,   56.90,   52.80,   49.00,   49.00/
      DATA  LSA(3)/6HUSA/ET/, OLON(3)/-115.0/
      DATA  NE(3)/10/,
     1  ELN3(1), ELN3(2), ELN3(3), ELN3(4), ELN3(5), ELN3(6),
     2  ELN3(7), ELN3(8), ELN3(9), ELN3(10)/
     3   -69.20,  -68.40,  -66.90,  -69.90,  -75.50,  -81.80,
     4   -85.80,  -87.60,  -90.50,  -88.00/,
     5  ELT3(1), ELT3(2), ELT3(3), ELT3(4), ELT3(5), ELT3(6),
     6  ELT3(7), ELT3(8), ELT3(9), ELT3(10)/
     7    47.00,   47.30,   44.80,   41.50,   35.60,   24.60,
     8    30.20,   38.70,   46.60,   47.50/
```

```
      DATA  LSA(4)/6HUSA/PT/, OLON(4)/-175.0/
      DATA  NE(4)/10/,
     1  ELN4(1), ELN4(2), ELN4(3), ELN4(4), ELN4(5), ELN4(6),
     2  ELN4(7), ELN4(8), ELN4(9), ELN4(10)/
     3  -116.00, -114.60, -115.80, -114.00, -114.80, -124.70,
     4  -122.80, -124.20, -120.70, -117.10/,
     5  ELT4(1), ELT4(2), ELT4(3), ELT4(4), ELT4(5), ELT4(6),
     6  ELT4(7), ELT4(8), ELT4(9), ELT4(10)/
     7    49.00,   45.50,   46.20,   42.00,   32.50,   48.40,
     8    49.00,   40.40,   34.60,   32.50/
      DATA  BWMN,PTER,OAER/0.6,0.1,2.0/
      DATA  NAME/6HCKMNEB/
C CALCULATION
   10 DO 39  JD=1,ND
        OLONJ=OLON(JD)
        NEJ=NE(JD)
   20   TO=SECOND(CP)
        CALL MNEBSA(OLONJ,NEJ,ELON(1,JD),ELAT(1,JD),
     1              BWMN,PTER,OAER,
     2              BLON,BLAT,BWMA,BWMI,ORAN,AREA)
        TM1=SECOND(CP)-TO
        TO=SECOND(CP)
        CALL MNEBTL(OLONJ,NEJ,ELON(1,JD),ELAT(1,JD),
     1              PTER,OAER,BLON,BLAT,BWMA,BWMI,ORAN,
     2              EOFA,EORA,TLRC)
        TM2=SECOND(CP)-TO
   30   WRITE (6,6030)  NAME,LSA(JD),OLONJ,NEJ,BWMN,PTER,OAER,
     1                  BLON,BLAT,BWMA,BWMI,ORAN,AREA
   31   WRITE (6,6031)  PTER,OAER
        WRITE (6,6032)  (J,ELON(J,JD),ELAT(J,JD),
     1                   EOFA(J),EORA(J),TLRC(J),J=1,NEJ)
        WRITE (6,6033)  TM1,TM2
   39 CONTINUE
      STOP
C FORMAT STATEMENTS
 6030 FORMAT(1H1,A6,5X,
     1  15HSERVICE AREA = ,A6,5X,6HOLON =,F8.2,5X,4HNE =,I3////
     2  3X,22HMINIMUM ELLIPSE (BWMN=,F4.2,7H, PTER=,F4.2,
     3  7H, OAER=,F4.2,1H)//
     4  5X,6HBLON =,F8.2/5X,6HBLAT =,F8.2/5X,6HBWMA =,F8.2/
     5  5X,6HBWMI =,F8.2/5X,6HORAN =,F8.2/5X,6HAREA =,F8.3)
 6031 FORMAT(1X///3X,22HTOLERANCE CHECK (PTER=,F4.2,
     1  7H, OAER=,F4.2,1H)//
     2  24X,14HEARTH STATIONS/
     3  11X,40HLONGI-   LATI-    OFF-    ORIEN-   TOLE-/
     4  11X,40HTUDE     TUDE     AXIS    TATION   RANCE/
     5  11X,40H                  ANGLE   ANGLE         /
     6  4X,1HI,6X,40H(ELON)  (ELAT)   (EOFA)   (EORA)  (TLRC)/)
 6032 FORMAT(1X,I4,F12.2,F8.2,F9.3,F9.2,F8.3)
 6033 FORMAT(1X///3X,28HCOMPUTATION TIME OF MNEBSA =,F6.3,2H S//
     1  3X,28HCOMPUTATION TIME OF MNEBTL =,F6.3,2H S)
      END
```

```
      SUBROUTINE  MNEBSA(OLON,NE,ELON,ELAT,BWMN,PTER,OAER,
     1                    BLON,BLAT,BWMA,BWMI,ORAN,AREA)
C THIS SUBROUTINE DETERMINES THE MINIMUM ELLIPTICAL BEAM OF A
C GEOSTATIONARY SATELLITE ANTENNA, GIVEN THE ORBITAL POSITION OF
C THE SATELLITE AND THE LOCATIONS OF THE EARTH STATIONS.
C THE INPUT PARAMETERS ARE
C     OLON = LONGITUDE OF THE ORBITAL POSITION (IN DEGREES),
C     NE   = NUMBER OF EARTH STATIONS (MUST NOT EXCEED 50),
C     ELON = ARRAY OF DIMENSION NE THAT CONTAINS THE LONGI-
C            TUDES OF THE EARTH STATIONS (IN DEGREES),
C     ELAT = ARRAY OF DIMENSION NE THAT CONTAINS THE LATI-
C            TUDES OF THE EARTH STATIONS (IN DEGREES),
C     BWMN = MINIMUM BEAMWIDTH (IN DEGREES),
C     PTER = POINTING ERROR (IN DEGREES),
C     OAER = ORIENTATION-ANGLE ERROR (IN DEGREES).
C WHEN A NEGATIVE VALUE IS GIVEN TO PTER OR OAER, ITS ABSOLUTE
C VALUE WILL BE USED AS PTER OR OAER.
C THE OUTPUT PARAMETERS ARE
C     BLON = LONGITUDE OF THE BORESIGHT POINT ON THE SURFACE
C            OF THE EARTH (IN DEGREES),
C     BLAT = LATITUDE  OF THE BORESIGHT POINT ON THE SURFACE
C            OF THE EARTH (IN DEGREES),
C     BWMA = MAJOR-AXIS BEAMWIDTH (IN DEGREES),
C     BWMI = MINOR-AXIS BEAMWIDTH (IN DEGREES),
C     ORAN = ORIENTATION ANGLE OF THE ELLIPSE, I.E., IN A
C            PLANE NORMAL TO THE BEAM AXIS, THE ANGLE MEAS-
C            URED COUNTERCLOCKWISE FROM A LINE PARALLEL TO
C            THE EQUATORIAL PLANE TO THE MAJOR AXIS OF THE
C            ELLIPSE TO THE NEAREST DEGREE (IN DEGREES),
C     AREA = AREA (OR SOLID ANGLE) OF THE BEAM
C            (IN DEGREES SQUARED).
C THIS SUBROUTINE CALLS THE MNEBBN AND MNEBBR SUBROUTINES.
C THIS SUBROUTINE ASSUMES THAT THE VECTOR FROM THE SATELLITE
C ORBITAL POSITION TO ANY EARTH STATION DOES NOT CROSS THE SEMI-
C PLANE OF THE 90 DEGREES EAST LONGITUDE.
C DECLARATION STATEMENTS
      DIMENSION   ELON(NE),ELAT(NE)
      COMMON/MNE1/XE(50),YE(50),ZE(50)
      DIMENSION   JLNP(10),JLTP(10)
      DATA  JLIN1,NR1/128,6/, JLIN2,NR2/32,5/
      DATA  RO,ZO/6.6239,0.0/
C PI AND RADIAN-TO-DEGREE RATIO
   10 PI=2.0*ATAN2(1.0,0.0)
      RAD=PI/180.0
C EARTH-CENTER COORDINATES OF THE ORBITAL POSITION
   20 XO=COS(RAD*OLON)*RO
      YO=SIN(RAD*OLON)*RO
C EARTH-CENTER COORDINATES OF THE EARTH STATIONS
      DO 21  JE=1,NE
        COSELA=COS(RAD*ELAT(JE))
        XE(JE)=COSELA*COS(RAD*ELON(JE))
        YE(JE)=COSELA*SIN(RAD*ELON(JE))
        ZE(JE)=SIN(RAD*ELAT(JE))
   21 CONTINUE
```

```
C RANGE OF BORESIGHT POINT
   30 BLONMN=ELON(1)
      IF(BLONMN.GT.90.0)       BLONMN=BLONMN-360.0
      BLONMX=BLONMN
      BLATMN=ELAT(1)
      BLATMX=BLATMN
      DO 31  JE=2,NE
        ELONJ=ELON(JE)
        IF(ELONJ.GT.90.0)      ELONJ=ELONJ-360.0
        BLONMN=AMIN1(ELONJ,BLONMN)
        BLONMX=AMAX1(ELONJ,BLONMX)
        ELATJ=ELAT(JE)
        BLATMN=AMIN1(ELATJ,BLATMN)
        BLATMX=AMAX1(ELATJ,BLATMX)
   31 CONTINUE
      OLONI=OLON
      IF(OLONI.GT.90.0)   OLONI=OLON-360.0
      BLONMN=AMIN1((OLONI+BLONMN)/2.0,BLONMN)
      BLONMX=AMAX1((OLONI+BLONMX)/2.0,BLONMX)
      BLATMN=AMIN1(BLATMN/2.0,BLATMN)
      BLATMX=AMAX1(BLATMX/2.0,BLATMX)
C SEARCH FOR MINIMUM ELLIPTICAL BEAM WITHOUT POINTING ERROR,
C ORENTATION-ANGLE ROUND OFF, AND ORIENTATION-ANGLE ERROR
   40 PDTMI=1.0E+6
      DO 69  IR=1,NR1
        IF(IR.NE.1)         GO TO 42
   41   JLNCT=INT((BLONMN+BLONMX)/2.0+360.5)*10
        JLNMN=JLNCT-INT((BLONMX-BLONMN)/25.6)*128
        JLNMX=JLNCT+JLNCT-JLNMN
        JLNIN=JLIN1
        JLTCT=INT((BLATMN+BLATMX)/2.0+180.5)*10
        JLTMN=JLTCT-INT((BLATMX-BLATMN)/25.6)*128
        JLTMX=JLTCT+JLTCT-JLTMN
        JLTIN=JLIN1
        GO TO 50
   42   JLNMN=JLNMI-JLNIN
        JLNMX=JLNMI+JLNIN
        JLNIN=JLNIN/2
        JLTMN=JLTMI-JLTIN
        JLTMX=JLTMI+JLTIN
        JLTIN=JLTIN/2
   50   DO 59  JLN=JLNMN,JLNMX,JLNIN
          IF(IR.EQ.1)          GO TO 52
          JLNFLG=1
          DO 51  JJLN=1,NJLN
            IF(JLN.EQ.JLNP(JJLN))        GO TO 52
   51     CONTINUE
          JLNFLG=0
   52     BLONJ=FLOAT(JLN-3600)/10.0
          DO 58  JLT=JLTMN,JLTMX,JLTIN
            IF(IR.EQ.1)        GO TO 54
            IF(JLNFLG.EQ.0)              GO TO 54
```

```
              DO 53  JJLT=1,NJLT
                IF(JLT.EQ.JLTP(JJLT))        GO TO 58
   53         CONTINUE
   54         BLATJ=FLOAT(JLT-1800)/10.0
              CALL MNEBBN(XO,YO,ZO,NE,XE,YE,ZE,BLONJ,BLATJ,
     1                    OA,AR,PDT)
              IF(PDT.GE.PDTMI)   GO TO 58
              JLNMI=JLN
              JLTMI=JLT
              OAMI=OA
              ARMI=AR
              PDTMI=PDT
   58       CONTINUE
   59     CONTINUE
   60     IF(IR.EQ.NR1)      GO TO 69
          NJLN=0
          DO 61  JLN=JLNMN,JLNMX,JLNIN
            NJLN=NJLN+1
            JLNP(NJLN)=JLN
   61     CONTINUE
          NJLT=0
          DO 62  JLT=JLTMN,JLTMX,JLTIN
            NJLT=NJLT+1
            JLTP(NJLT)=JLT
   62     CONTINUE
   69 CONTINUE
      BLONO=FLOAT(JLNMI-3600)/10.0
      BLATO=FLOAT(JLTMI-1800)/10.0
      ORANO=OAMI
      ARO=ARMI
C FURTHER SEARCH FOR MINIMUM ELLIPTICAL BEAM WITH THE POINTING
C ERROR, ORIENTATION-ANGLE ROUND OFF, AND ORIENTATION-ANGLE
C ERROR
   70 PDTMI=1.0E+6
      JLNMI=BLONO*10.0+3600.5
      JLNIN=JLIN2
      JLTMI=BLATO*10.0+1800.5
      JLTIN=JLIN2
      DO 89  IR=1,NR2
        JLNMN=JLNMI-JLNIN
        JLNMX=JLNMI+JLNIN
        JLNIN=JLNIN/2
        JLTMN=JLTMI-JLTIN
        JLTMX=JLTMI+JLTIN
        JLTIN=JLTIN/2
        DO 79  JLN=JLNMN,JLNMX,JLNIN
          IF(IR.EQ.1)      GO TO 72
          JLNFLG=1
          DO 71  JJLN=1,NJLN
            IF(JLN.EQ.JLNP(JJLN))        GO TO 72
   71     CONTINUE
          JLNFLG=0
```

```
72      BLONJ=FLOAT(JLN-3600)/10.0
        DO 78  JLT=JLTMN,JLTMX,JLTIN
          IF(IR.EQ.1)   GO TO 74
          IF(JLNFLG.EQ.0)               GO TO 74
          DO 73  JJLT=1,NJLT
            IF(JLT.EQ.JLTP(JJLT))       GO TO 78
73        CONTINUE
74        BLATJ=FLOAT(JLT-1800)/10.0
          CALL MNEBBR(XO,YO,ZO,NE,XE,YE,ZE,
  1                   ORANO,ARO,PTER,OAER,BLONJ,BLATJ,
  2                   OA,AR,PDT)
          IF(PDT.GE.PDTMI)              GO TO 78
          JLNMI=JLN
          JLTMI=JLT
          OAMI=OA
          ARMI=AR
          PDTMI=PDT
78      CONTINUE
79    CONTINUE
80    IF(IR.EQ.NR2)     GO TO 89
      NJLN=0
      DO 81  JLN=JLNMN,JLNMX,JLNIN
        NJLN=NJLN+1
        JLNP(NJLN)=JLN
81    CONTINUE
      NJLT=0
      DO 82  JLT=JLTMN,JLTMX,JLTIN
        NJLT=NJLT+1
        JLTP(NJLT)=JLT
82    CONTINUE
89  CONTINUE
    BLON=FLOAT(JLNMI-3600)/10.0
    BLAT=FLOAT(JLTMI-1800)/10.0
    BWMA=2.0*SQRT(PDTMI/ARMI)/RAD
    BWMI=BWMA*ARMI
    ORAN=OAMI
    BWMA=AMAX1(BWMN,BWMA)
    BWMI=AMAX1(BWMN,BWMI)
    IF(BWMA.LE.BWMN.AND.BWMI.LE.BWMN)  ORAN=0.0
    AREA=BWMA*BWMI*PI/4.0
    RETURN
    END
```

```
      SUBROUTINE  MNEBTL(OLON,NE,ELON,ELAT,PTER,OAER,
     1                    BLON,BLAT,BWMA,BWMI,ORAN,
     2                    EOFA,EORA,TLRC)
C THIS SUBROUTINE CALCULATES THE TOLERANCES, AND OTHER RELATED
C VARIABLES, OF EARTH STATIONS, RELATIVE TO AN ELLIPTICAL BEAM
C OF A GEOSTATIONARY SATELLITE ANTENNA.
C THE INPUT PARAMETERS ARE
C      OLON = LONGITUDE OF THE ORBITAL POSITION (IN DEGREES),
C      NE   = NUMBER OF EARTH STATIONS (MUST NOT EXCEED 50),
C      ELON = ARRAY OF DIMENSION NE THAT CONTAINS THE LONGI-
C             TUDES OF THE EARTH STATIONS (IN DEGREES),
C      ELAT = ARRAY OF DIMENSION NE THAT CONTAINS THE LATI-
C             TUDES OF THE EARTH STATIONS (IN DEGREES),
C      PTER = POINTING ERROR (IN DEGREES),
C      OAER = ORIENTATION-ANGLE ERROR (IN DEGREES),
C      BLON = LONGITUDE OF THE BORESIGHT POINT ON THE SURFACE
C             OF THE EARTH (IN DEGREES),
C      BLAT = LATITUDE  OF THE BORESIGHT POINT ON THE SURFACE
C             OF THE EARTH (IN DEGREES),
C      BWMA = MAJOR-AXIS BEAMWIDTH (IN DEGREES),
C      BWMI = MINOR-AXIS BEAMWIDTH (IN DEGREES),
C      ORAN = ORIENTATION ANGLE OF THE ELLIPSE, I.E., IN A
C             PLANE NORMAL TO THE BEAM AXIS, THE ANGLE MEAS-
C             URED COUNTERCLOCKWISE FROM A LINE PARALLEL TO
C             THE EQUATORIAL PLANE TO THE MAJOR AXIS OF THE
C             ELLIPSE (IN DEGREES).
C THE OUTPUT PARAMETERS ARE ALL ARRAYS OF DIMENSION NE WHERE THE
C CALCULATED VALUES OF THE FOLLOWING VARIABLES ARE TO BE STORED.
C      EOFA = OFF-AXIS ANGLE OF THE EARTH STATION,
C      EORA = ORIENTATION ANGLE OF THE EARTH STATION, I.E.,
C             THE ANGLE MEASURED COUNTERCLOCKWISE, IN A PLANE
C             NORMAL TO THE BEAM AXIS, FROM A LINE PARALLEL
C             TO THE EQUATORIAL PLANE TO THE PROJECTION OF
C             THE VECTOR FROM THE BORESIGHT POINT TO THE
C             THE EARTH STATION,
C      TLRC = TOLERANCE OF THE EARTH STATION, I.E., THE
C             DISTANCE BETWEEN THE EARTH STATION AND THE
C             CIRCUMFERENCE OF THE ELLIPSE, EXPRESSED IN
C             THE ANGLE SEEN FROM THE SATELLITE ANTENNA,
C ALL IN DEGREES.
C THIS SUBROUTINE CALLS THE MNEBOA SUBROUTINE.
C DECLARATION STATEMENTS
      DIMENSION   ELON(NE),ELAT(NE),
     1            EOFA(NE),EORA(NE),TLRC(NE)
      COMMON/MNE1/XE(50),YE(50),ZE(50)
      COMMON/MNE2/ALFA(50),BETA(50)
      DATA  RO,ZO/6.6239,0.0/
C RADIAN-TO-DEGREE RATIO
   10 RAD=ATAN2(1.0,0.0)/90.0
C EARTH-CENTER COORDINATES OF THE ORBITAL POSITION
   20 XO=COS(RAD*OLON)*RO
      YO=SIN(RAD*OLON)*RO
```

```
C EARTH-CENTER COORDINATES OF THE BORESIGHT POINT
   30 COSBLA=COS(RAD*BLAT)
      XB=COSBLA*COS(RAD*BLON)
      YB=COSBLA*SIN(RAD*BLON)
      ZB=SIN(RAD*BLAT)
C EARTH-CENTER COORDINATES OF THE EARTH STATIONS
   40 DO 41  JE=1,NE
        COSELA=COS(RAD*ELAT(JE))
        XE(JE)=COSELA*COS(RAD*ELON(JE))
        YE(JE)=COSELA*SIN(RAD*ELON(JE))
        ZE(JE)=SIN(RAD*ELAT(JE))
   41 CONTINUE
C OFF-AXIS AND ORIENTATION ANGLES OF EARTH STATIONS
   50 CALL MNEBOA(XO,YO,ZO,XB,YB,ZB,NE,XE,YE,ZE,ALFA,BETA)
C SEMI-AXIS BEAMWIDTHS
   60 A=RAD*BWMA/2.0
      B=RAD*BWMI/2.0
      ASQ=A*A
      BSQ=B*B
C TOLERANCES OF EARTH STATIONS
   70 DO 79  JE=1,NE
        DMN=1.0E+6
        DO 72  JOE=1,3
          OAJ=RAD*(ORAN+OAER*FLOAT(JOE-2))
          YEDP=ABS(ALFA(JE)*COS(BETA(JE)-OAJ))
          ZEDP=ABS(ALFA(JE)*SIN(BETA(JE)-OAJ))
          Y1DP=AMIN1(YEDP,A)
          Y2DP=A/B*SQRT(AMAX1(0.0,BSQ-ZEDP**2))
          DO 71  JYDP=1,101
            YDP=Y1DP+(Y2DP-Y1DP)*FLOAT(JYDP-1)/100.0
            ZDP=B/A*SQRT(AMAX1(0.0,ASQ-YDP**2))
            DMN=AMIN1(SQRT((YDP-YEDP)**2+(ZDP-ZEDP)**2),DMN)
   71     CONTINUE
   72   CONTINUE
        EOFA(JE)=ALFA(JE)/RAD
        EORA(JE)=BETA(JE)/RAD
        SGN=1.0
        IF(((YEDP/A)**2+(ZEDP/B)**2).GT.1.0)  SGN=-1.0
        TLRC(JE)=SGN*DMN/RAD-ABS(PTER)
   79 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  MNEBBN(XO,YO,ZO,NE,XE,YE,ZE,BLON,BLAT,
     1                     OAM,ARM,PDTM)
C THIS SUBROUTINE CALCULATES, FOR A BORESIGHT POINT, THE MINIMUM
C VALUE (MINIMIZED OVER THE ORIENTATION ANGLE AND AXIAL RATIO
C VALUES) OF THE PRODUCT OF THE SEMIMAJOR AND SEMIMINOR AXES OF
C THE ELLIPTICAL BEAM OF A SATELLITE ANTENNA.  CALCULATION DOES
C NOT INCLUDE THE POINTING ERROR, ORIENTATION-ANGLE ROUND OFF,
C OR ORIENTATION-ANGLE ERROR.
C THE INPUT PARAMETERS ARE
C      XO, YO, ZO
C            = EARTH-CENTER COORDINATES OF THE SATELLITE
C              ORBITAL POSITION,
C      NE    = NUMBER OF EARTH STATIONS (MUST NOT EXCEED 50),
C      XE, YE, ZE
C            = ARRAYS OF DIMENSION NE CONTAINING THE EARTH-
C              CENTER COORDINATES OF THE EARTH STATIONS,
C      BLON = LONGITUDE OF THE BORESIGHT POINT ON THE SURFACE
C              OF THE EARTH (IN DEGREES),
C      BLAT = LATITUDE  OF THE BORESIGHT POINT ON THE SURFACE
C              OF THE EARTH (IN DEGREES).
C THE OUTPUT PARAMETERS ARE
C      OAM   = OPTIMUM VALUE OF THE ORIENTATION ANGLE OF THE
C              ELLIPSE (IN DEGREES),
C      ARM   = OPTIMUM VALUE OF THE AXIAL RATIO,
C      PDTM = MINIMUM VALUE OF THE PRODUCT OF THE SEMIMAJOR
C              AXIS AND SEMIMINOR AXIS OF THE ELLIPTICAL BEAM
C              (IN RADIANS SQUARED).
C THIS SUBROUTINE CALLS THE MNEBOA SUBROUTINE.
C DECLARATION STATEMENTS
      DIMENSION    XE(NE),YE(NE),ZE(NE)
      COMMON/MNE2/ALFA(50),BETA(50)
      COMMON/MNE3/YDPSQ(50),ZDPSQ(50),DMMY(200)
      DATA  NOA,ANOA/1024,1024.0/, JOAINO,NRO/128,8/
      DATA  NAR,ANAR/1024,1024.0/, JARINO,NRA/128,6/
C PI AND THE RADIAN-TO-DEGREE RATIO
   10 PI=2.0*ATAN2(1.0,0.0)
      RAD=PI/180.0
C EARTH-CENTER COORDINATES OF THE BORESIGHT POINT
   20 COSBLA=COS(RAD*BLAT)
      XB=COSBLA*COS(RAD*BLON)
      YB=COSBLA*SIN(RAD*BLON)
      ZB=SIN(RAD*BLAT)
C OFF-AXIS AND ORIENTATION ANGLES OF THE EARTH POINTS
   30 CALL MNEBOA(XO,YO,ZO,XB,YB,ZB,NE,XE,YE,ZE,ALFA,BETA)
C MAIN DO-LOOPS (2) WITH RESPECT TO THE ORIENTATION ANGLE
   40 PDTMI=1.0E+6
      DO 59  IRO=1,NRO
        IF(IRO.NE.1)      GO TO 42
   41   JOAIN=JOAINO
        JOAMN=JOAINO
        JOAMX=NOA
        GO TO 45
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   42   IF(IRO.NE.2)        JOAIN=JOAIN/2
        IF(JOAIN.LE.1)      GO TO 59
        JOAMN=JOAMI-JOAIN/2
        JOAMX=JOAMN+JOAIN
   45   DO 58  JOA=JOAMN,JOAMX,JOAIN
          OAJ=FLOAT(MOD(JOA,NOA))/ANOA*PI
          DO 46  JE=1,NE
            YDPSQ(JE)=(ALFA(JE)*COS(BETA(JE)-OAJ))**2
            ZDPSQ(JE)=ALFA(JE)**2-YDPSQ(JE)
   46     CONTINUE
C INNER DO-LOOPS (2) WITH RESPECT TO THE AXIAL RATIO
   50     DO 56  IRA=1,NRA
            IF(IRA.NE.1)        GO TO 52
            JARMN=JARINO
            JARMX=NAR
            JARIN=JARINO
            GO TO 53
   52       IF(IRA.NE.2)        JARIN=JARIN/2
            JARMN=MAXO(1,JARMI-JARIN/2)
            JARMX=MINO(JARMI+JARIN/2,NAR)
            JARIN=JARMX-JARMN
            IF(JARIN.EQ.0)      GO TO 56
   53       DO 55  JAR=JARMN,JARMX,JARIN
              RAR=ANAR/FLOAT(JAR)
              RARSQ=RAR*RAR
              RSQMX=0.0
              DO 54  JE=1,NE
                RSQMX=AMAX1(RSQMX,YDPSQ(JE)+ZDPSQ(JE)*RARSQ)
   54         CONTINUE
              PDT=RSQMX/RAR
              IF(PDT.GE.PDTMI)        GO TO 55
              JOAMI=JOA
              JARMI=JAR
              PDTMI=PDT
   55       CONTINUE
   56     CONTINUE
   58   CONTINUE
   59 CONTINUE
   60 OAM=FLOAT(MOD(JOAMI,NOA))/ANOA*180.0
      ARM=FLOAT(JARMI)/ANAR
      PDTM=PDTMI
      RETURN
      END
```

```
      SUBROUTINE  MNEBBR(XO,YO,ZO,NE,XE,YE,ZE,
     1                    ORAO,ARO,PTER,OAER,BLON,BLAT,
     2                    OAM,ARM,PDTM)
C THIS SUBROUTINE CALCULATES, FOR A BORESIGHT POINT, THE MINIMUM
C VALUE (MINIMIZED OVER THE ORIENTATION ANGLE AND AXIAL RATIO
C VALUES) OF THE PRODUCT OF THE SEMIMAJOR AND SEMIMINOR AXES OF
C THE ELLIPTICAL BEAM OF A SATELLITE ANTENNA.  CALCULATION DOES
C INCLUDE THE POINTING ERROR, ORIENTATION-ANGLE ROUND OFF, AND
C ORIENTATION-ANGLE ERROR.
C THE INPUT PARAMETERS ARE
C       XO, YO, ZO
C            = EARTH-CENTER COORDINATES OF THE SATELLITE
C              ORBITAL POSITION,
C       NE   = NUMBER OF EARTH STATIONS (MUST NOT EXCEED 50),
C       XE, YE, ZE
C            = ARRAYS OF DIMENSION NE CONTAINING THE EARTH-
C              CENTER COORDINATES OF THE EARTH STATIONS,
C       ORAO = OPTIMUM VALUE OF THE ORIENTATION ANGLE OF THE
C              ELLIPSE WITHOUT POINTING ERROR, ORIENTATION-
C              ANGLE ROUND OFF, AND ORIENTATION-ANGLE ERROR
C              (IN DEGREES),
C       ARO  = OPTIMUM VALUE OF THE AXIAL RATIO OF THE ELLIPSE
C              WITHOUT POINTING ERROR, ORIENTATION-ANGLE ROUND
C              OFF, AND ORIENTATION-ANGLE ERROR,
C       PTER = POINTING ERROR (IN DEGREES),
C       OAER = ORIENTATION-ANGLE ERROR (IN DEGREES),
C       BLON = LONGITUDE OF THE BORESIGHT POINT ON THE SURFACE
C              OF THE EARTH (IN DEGREES),
C       BLAT = LATITUDE  OF THE BORESIGHT POINT ON THE SURFACE
C              OF THE EARTH (IN DEGREES).
C THE OUTPUT PARAMETERS ARE
C       OAM  = OPTIMUM VALUE OF THE ORIENTATION ANGLE OF THE
C              ELLIPSE (IN DEGREES),
C       ARM  = OPTIMUM VALUE OF THE AXIAL RATIO,
C       PDTM = MINIMUM VALUE OF THE PRODUCT OF THE SEMIMAJOR
C              AXIS AND SEMIMINOR AXIS OF THE ELLIPTICAL BEAM
C              (IN RADIANS SQUARED).
C THIS SUBROUTINE CALLS THE MNEBOA SUBROUTINE.
C DECLARATION STATEMENTS
      DIMENSION   XE(NE),YE(NE),ZE(NE)
      COMMON/MNE2/ALFA(50),BETA(50)
      COMMON/MNE3/YDPSQ(50,3),ZDP(50,3)
      DATA  NRO/11/
      DATA  NAR,ANAR/1024,1024.0/, JARINO,NRA/64,7/
C RADIAN-TO-DEGREE RATIO
   10 RAD=ATAN2(1.0,0.0)/90.0
C EARTH-CENTER COORDINATES OF THE BORESIGHT POINT
   20 COSBLA=COS(RAD*BLAT)
      XB=COSBLA*COS(RAD*BLON)
      YB=COSBLA*SIN(RAD*BLON)
      ZB=SIN(RAD*BLAT)
C OFF-AXIS AND ORIENTATION ANGLES OF THE EARTH STATIONS
   30 CALL MNEBOA(XO,YO,ZO,XB,YB,ZB,NE,XE,YE,ZE,ALFA,BETA)
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```
C MAIN DO-LOOP WITH RESPECT TO THE ORIENTATION ANGLE
   40 PDTMI=1.0E+6
      DO 59  IRO=1,NRO
        IF(IRO.LE.2)       JOA=INT(ORAO)+IRO-1
        IF(IRO.GE.3.AND.JOAMI.LE.INT(ORAO))   JOA=JOAMI-1
        IF(IRO.GE.3.AND.JOAMI.GT.INT(ORAO))   JOA=JOAMI+1
        APTER=RAD*ABS(PTER)
        DO 42  JOE=1,3
          OAJ=(FLOAT(JOA)+ABS(OAER)*FLOAT(JOE-2))*RAD
          DO 41  JE=1,NE
            ZDPO=ABS(ALFA(JE)*SIN(BETA(JE)-OAJ))
            YDPSQ(JE,JOE)=ALFA(JE)**2-ZDPO**2
            ZDP(JE,JOE)=ZDPO+APTER
   41     CONTINUE
   42   CONTINUE
C INNER DO-LOOPS (2) WITH RESPECT TO THE AXIAL RATIO
   50   DO 58  IRA=1,NRA
          IF(IRA.NE.1)    GO TO 52
   51     JARO=ARO*ANAR+0.5
          JARMN=MAX0(1,JARO-JARINO)
          JARMX=MIN0(JARO+JARINO,NAR)
          JARIN=(JARMX-JARMN)/2
          GO TO 53
   52     IF(IRA.NE.2)    JARIN=JARIN/2
          JARMN=MAX0(1,JARMI-JARIN/2)
          JARMX=MIN0(JARMI+JARIN/2,NAR)
          JARIN=JARMX-JARMN
          IF(JARIN.EQ.0)  GO TO 58
   53     DO 57  JAR=JARMN,JARMX,JARIN
            RAR=ANAR/FLOAT(JAR)
            RSQMX=0.0
            DO 55  JOE=1,3
              DO 54  JE=1,NE
                ZDPC=ZDP(JE,JOE)*RAR-APTER
                RSQMX=AMAX1(RSQMX,YDPSQ(JE,JOE)+ZDPC**2)
   54         CONTINUE
   55       CONTINUE
            PDT=((SQRT(RSQMX)+APTER)**2)/RAR
            IF(PDT.GE.PDTMI)   GO TO 57
            JOAMI=JOA
            JARMI=JAR
            PDTMI=PDT
   57     CONTINUE
   58   CONTINUE
        IF(IRO.GE.3.AND.JOA.NE.JOAMI)    GO TO 60
   59 CONTINUE
   60 OAM=MOD(JOAMI+180,180)
      ARM=FLOAT(JARMI)/ANAR
      PDTM=PDTMI
      RETURN
      END
```

```
      SUBROUTINE  MNEBOA(XO,YO,ZO,XB,YB,ZB,NE,XE,YE,ZE,
     1                    ALFA,BETA)
C THIS SUBROUTINE CALCULATES THE OFF-AXIS AND ORIENTATION ANGLES
C OF THE EARTH STATIONS.
C THE INPUT PARAMETERS ARE
C       XO, YO, ZO
C             = EARTH-CENTER COORDINATES OF THE SATELLITE
C               ORBITAL POSITION,
C       XB, YB, ZB
C             = EARTH-CENTER COORDINATES OF THE BORESIGHT
C               POINT ON THE SURFACE OF THE EARTH,
C       NE    = NUMBER OF EARTH STATIONS,
C       XE, YE, ZE
C             = ARRAYS OF DIMENSION NE CONTAINING THE EARTH-
C               CENTER COORDINATES OF THE EARTH STATIONS.
C THE OUTPUT PARAMETERS ARE BOTH ARRAYS OF DIMENSION NE, WHERE
C THE CALCULATED VALUES OF THE FOLLOWING VARIABLES FOR THE EARTH
C STATIONS ARE TO BE STORED.
C       ALFA = OFF-AXIS ANGLES (IN RADIANS),
C       BETA = ORIENTATION ANGLES (IN RADIANS).
C DECLARATION STATEMENTS
      DIMENSION   XE(NE),YE(NE),ZE(NE),ALFA(NE),BETA(NE)
C CALCULATION OF THE COEFFICIENTS OF COORDINATE TRANSFORMATION
   10 A11=XO-XB
      A12=YO-YB
      A13=ZO-ZB
      XOP=SQRT(A11*A11+A12*A12+A13*A13)
      A11=A11/XOP
      A12=A12/XOP
      A13=A13/XOP
      AA=SQRT(A11*A11+A12*A12)
      A21=-A12/AA
      A22=A11/AA
      A23=0.0
      A33=SQRT(1.0-A13*A13)
      A31=-A11*A13/A33
      A32=-A12*A13/A33
C COORDINATE TRANSFORMATION AND CALCULATION OF THE ANGLES
   20 DO 21  JE=1,NE
        XEP=A11*(XE(JE)-XB)+A12*(YE(JE)-YB)+A13*(ZE(JE)-ZB)
        YEP=A21*(XE(JE)-XB)+A22*(YE(JE)-YB)+A23*(ZE(JE)-ZB)
        ZEP=A31*(XE(JE)-XB)+A32*(YE(JE)-YB)+A33*(ZE(JE)-ZB)
        ALFA(JE)=ATAN2(SQRT(YEP*YEP+ZEP*ZEP),(XOP-XEP))
        IF(ALFA(JE).EQ.0.0)    BETA(JE)=0.0
        IF(ALFA(JE).NE.0.0)    BETA(JE)=ATAN2(ZEP,YEP)
   21 CONTINUE
      RETURN
      END
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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

The 1977 World Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service decided that elliptical antenna beams should be used for satellite antennas for planning purposes. Since it is desirable to minimize the antenna beam solid angle, we have developed a method for determining the minimum elliptical beam of a satellite antenna. The method is based on a straightforward and exhaustive search for a minimum elliptical beam over all possible ranges of all necessary parameters that specify the elliptical beam. Although it may sound contrary, this method is more efficient than an existing method based on the advanced technique of nonlinear programming. This report describes the method and the computer program that implements the method. It also gives some examples.

16. Key Words *(Alphabetical order, separated by semicolons)*

BSS (broadcasting-satellite service), elliptical antenna beam, minimum elliptical antenna beam, satellite antenna

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