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# Fading Signals in the MF Band

A.D. Spaulding

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Malcolm Baldrige, Secretary

Bernard J. Wunder, Jr., Assistant Secretary
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TABLE OF CONTENTS

## LIST OF FIGURES



## LIST OF TABLES

FADING SIGNALS IN THE MF BAND

A. D. Spaulding*

In the MF band, interfering signals from one or more broadcast stations can be received. During nighttime periods, this interference is caused by fading signals, propagated from distances of more than about 200 miles. It is the purpose of this report to provide the techniques needed to properly determine the degree of interference from one or more distant broadcasting stations. Computer algorithms are developed which determine the actual overall fading distribution of one or a combination of two fading signals, when the short-term fading of each is represented by a Rayleigh distribution and the long-term fading of median values is given by a log-normal distribution. Also, a simple approximate means of determining the distribution of the median value of a signal given by the sum of any number of fading signals is given. This approximate method is accurate only for the smaller percentage points. It is found that the techniques described here lead to somewhat higher values for the sum of the signals than previous methods.

Key Words: MF fading, fading signals, signal interference

## 1. INTRODUCTION

Propagation of medium frequency (MF) radio waves involves both the ground-wave and sky-wave modes. Normal, local broadcasting service is provided by the ground-wave mode. Sky-wave propagation is important during nighttime hours, when the ionospheric absorption is much reduced and signals can be obtained to provide service to distant locations. Propagation of distant signals via the sky-wave mode can also lead to interference to local services.

In recent years, it has become apparent that certain Western Hemisphere nations intend to modify their MF broadcasting operations, primarily by using higher power transmitters and transmitting signals on more frequencies. In order to address this problem and limit potential interference to current and future operations in the Western Hemisphere, the International Telecommunications Union (ITU) scheduled a Regional Adminstrative MF Broadcasting Conference of the Western Hemisphere (Region 2). The first session of the Conference was held in Buenos Aires in March 1980 and established the basis for preparing a frequency assignment plan for the MF broadcasting band in Region 2. This first session considered propagation data, required field strengths, transmitting antenna characteristics, transmitter power, and planning. The second session, convened in November 1981, drew up an agreement and an associated plan of frequency assignments in the MF broadcasting band for

*The author is with the U. S. Department of Commerce, National Telecommunications and Information Administration, Institute for Telecommunication Sciences, Boulder, Colorado, 80303.

Region 2. It is the purpose of this report to provide the techniques needed to properly determine the degree of interference to U. S. operations from one or more distant broadcast stations in Region 2. During nighttime periods, this interference is caused by fading signals propagated from distances of more than about 200 miles.

The Federal Communications Commission (FCC, 1976, §73.182) states: "Objectionable nighttime interference from another broadcast station is the degree of interference produced when, at a specified field intensity contour with respect to the desired station, the field intensity of an undesired station (or the root-sum-square value of field intensities of two or more stations on the same frequency) exceeds for 10 percent or more of the time the values set forth in the standards." In these FCC standards, the interfering field strength value exceeded 10 percent of the time refers to average (or in some cases median) field strength over some short period of time, usually one hour. The problem then is to determine the actual fading distribution for any collection of interfering fading signals, taking into account both the short-term fading within, say, an hour and the long-term fading.

A review of the CCIR (1978a, 1978b, 1978c) literature concerning fading signals and medium frequency sky-wave propagation leads to the following general conclusion:

> For short term fading (e.g., fading within an hour) the field strength observed usually obeys the Rayleigh distribution at the higher frequencies in the MF band.
>
> For long term fading (fading of median field strengths for a given hour from day-to-day) the median field strengths measured at a given time vary randomly from night-to-night with a log-normal distribution. The spread of values, measured over a period of days which is short enough to exclude seasonal and solar-cycle effects, shows no obvious dependence on frequency or path length. On most paths the difference between the field strengths exceeded 10 percent and 50 percent of the nights lies between 3.5 and 8.8 dB, with 5.5 dB being regarded as typical.

The FCC (1976, §73.190, Figure 1), gives average sky-wave field strength for the second hour after sunset as a function of distance. This figure presents the data parametric in the percent exceeded, giving values for 95%, 90%, 70%, 50%, 30%, 10%, and 5%. This FCC figure is reproduced here as Figure 1. Figure 2 of this report presents these data parametric in range. The data are given in terms of field strength in dB versus percent time exceeded and is graphed on normal probability coordinates so that log-normal distributions plot as straight lines. As can be seen from Figure 2, the log-normal distribution represents the data quite accurately. Note also that there is no dependency with distance on the fading range. (The

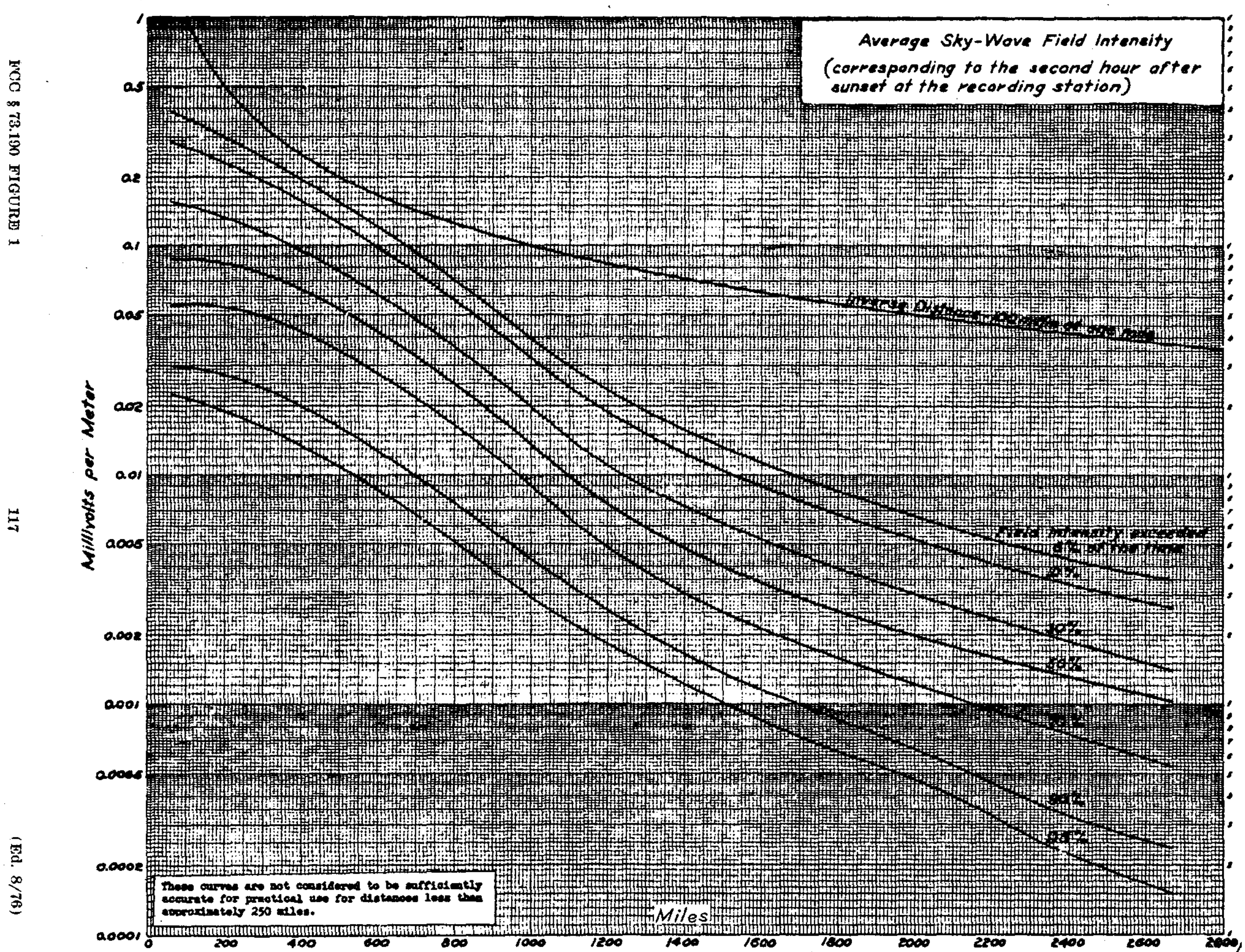


Figure 1. Sky-wave field intensity data from FCC (1976, §73.190), based on a radiated field of 100 mV/m at one mile.

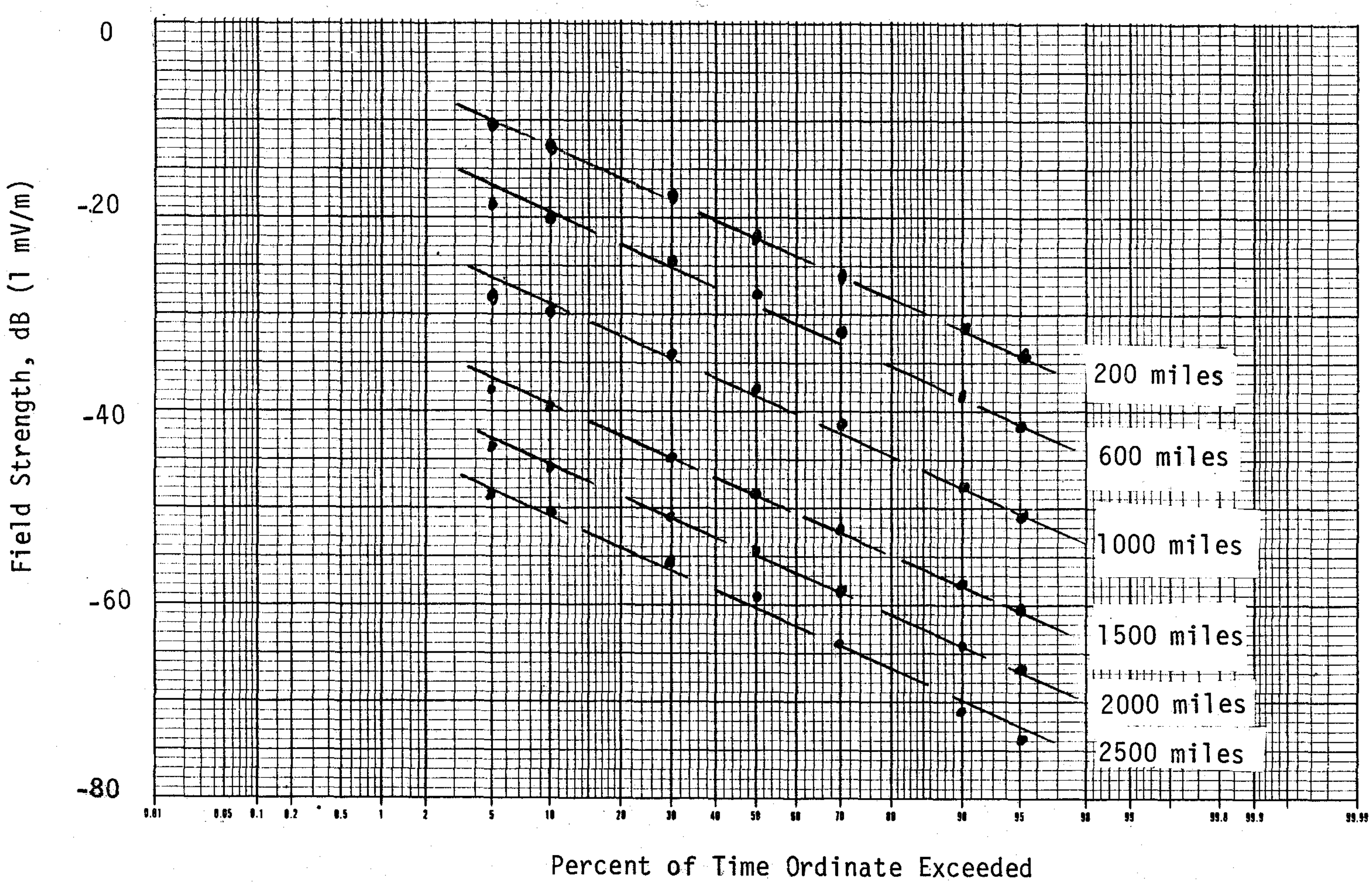


Figure 2. The FCC data of Figure 1 plotted parametric in path length.

fading range is defined as the difference in dB between the upper and lower decile values, that is, the 10% and 90% points.) The fading range, independent of distance, is 19 dB. This is larger than that indicated by the CCIR documents mentioned above (i.e., 7 to 17.6 dB, with 11 dB being "typical")

We have, in general, two fading distributions to consider, the Rayleigh and the log-normal. It should be mentioned, however, that, at low frequency (LF) and, presumably, the lower portion of the MF band, the short term fading distribution can more closely resemble the log-normal than the Rayleigh (CCIR, Report 431-2). Figure 3 shows a cumulative Rayleigh distribution, plotted on "Rayleigh coordinates." The Rayleigh distribution in Figure 3 is plotted relative to its median value, and note that upper decile is 5 dB above the median and that the lower decile is 8 dB below the median. In order to determine the percentage of time for a given hour that a given field strength will be exceeded by one or a combination of interfering signals, we need to obtain the entire fading distribution, considering both short-term and long-term fading. In the past, this has been usually handled by simply approximating the overall fading distribution by a log-normal distribution with a fading range somewhat larger than the fading range of the long-term fading log-normal distribution. For example, CCIR (1978b) states: "It is reasonable to assume that the field strength exceeded for 10% of the total time on a series of nights, during short periods centered on a specific time is 8 dB greater at MF, than the median field strength derived from propagation curves or formulae." This says to use a fading range of 16 dB instead of the "typical" long-term fading range of 11 dB. If we are only interested in the 10% point, and if care is used, the above procedure may be reasonable. However, as we shall see, the overall fading distribution is generally quite different from log-normal. Also, we may be interested in the sum of many interfering signals, each with its own overall fading distribution.

In an effort to obtain data that provide more reliable estimates of MF sky-wave signal strength, the Institute for Telecommunication Sciences (ITS), along with the FCC, has implemented a monitoring program at Cabo Rojo, Puerto Rico. Highly directional Beverage antennas are being used to monitor signals between 2300 and 0600 hours, local time, from discrete locations in South America. In addition, signals from a station in Spain and a station in Senegal are being monitored to provide data from Europe and Africa to locations in Region 2. The data are being recorded continuously using a controlling computer, a programmable receiver, and a programmable digital voltmeter. The system is capable of automatically measuring the received signal level of ten (10) different signals. The signal level on each

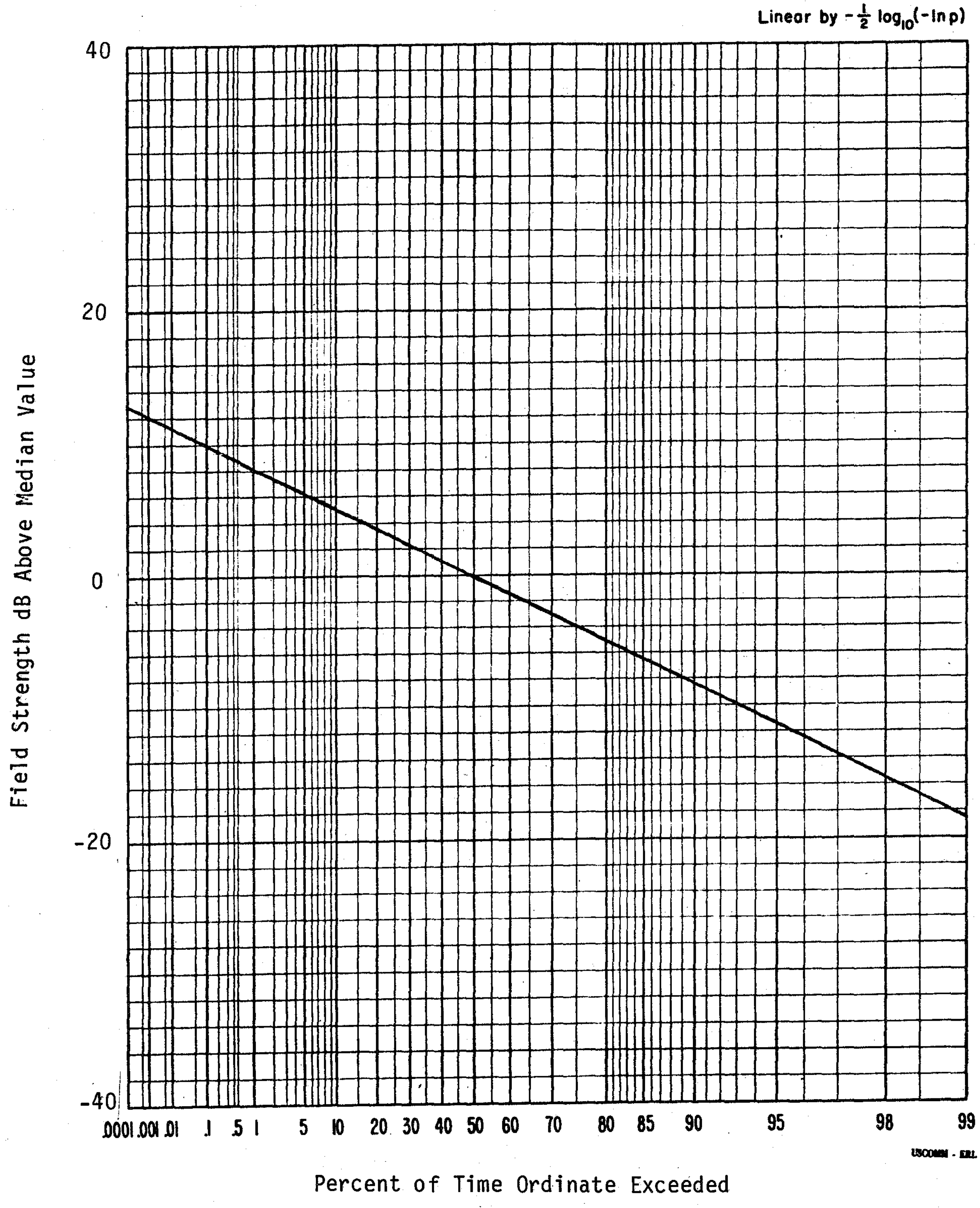


Figure 3. Cumulative Rayleigh distribution, given relative to its median value.

frequency is averaged over a one-minute period. The average signal level and the variance for each of these one-minute periods are then recorded on magnetic tape along with date/time information for later analysis. A preliminary set of data from this monitoring has been given by Washburn et al. (1982). Figures 4-9, from Washburn et al. (1982) show the hourly median values of received signals along with the upper and lower decile values for a 27-day period, 0130-0230 hours local time, from six stations.

If the short-term (within an hour) fading distribution is Rayleigh, then, as we saw from Figure 3, the upper decile is 5 dB above the median and the lower decile is 8 dB below the median. Figures 4-9 show that the hourly variations satisfy this reasonably well for many of the hours, but that there is enough variation from the above to indicate that it may be worthwhile to take a closer look at the short-term fading distribution. In this report we will assume the short-term fading distribution to be Rayleigh.

Figures 10-15 show the hourly medians from Figures 4-9 given as distributions on normal probability coordinates. We see that, in all cases, the log-normal distributions closely approximate the long-term fading of the hourly median values, but the fading range is quite variable and substantially smaller than that given by the FCC data of Figure 2.

We next will proceed generally and develop methods to obtain the entire fading distribution, considering both short-term and long-term fading.

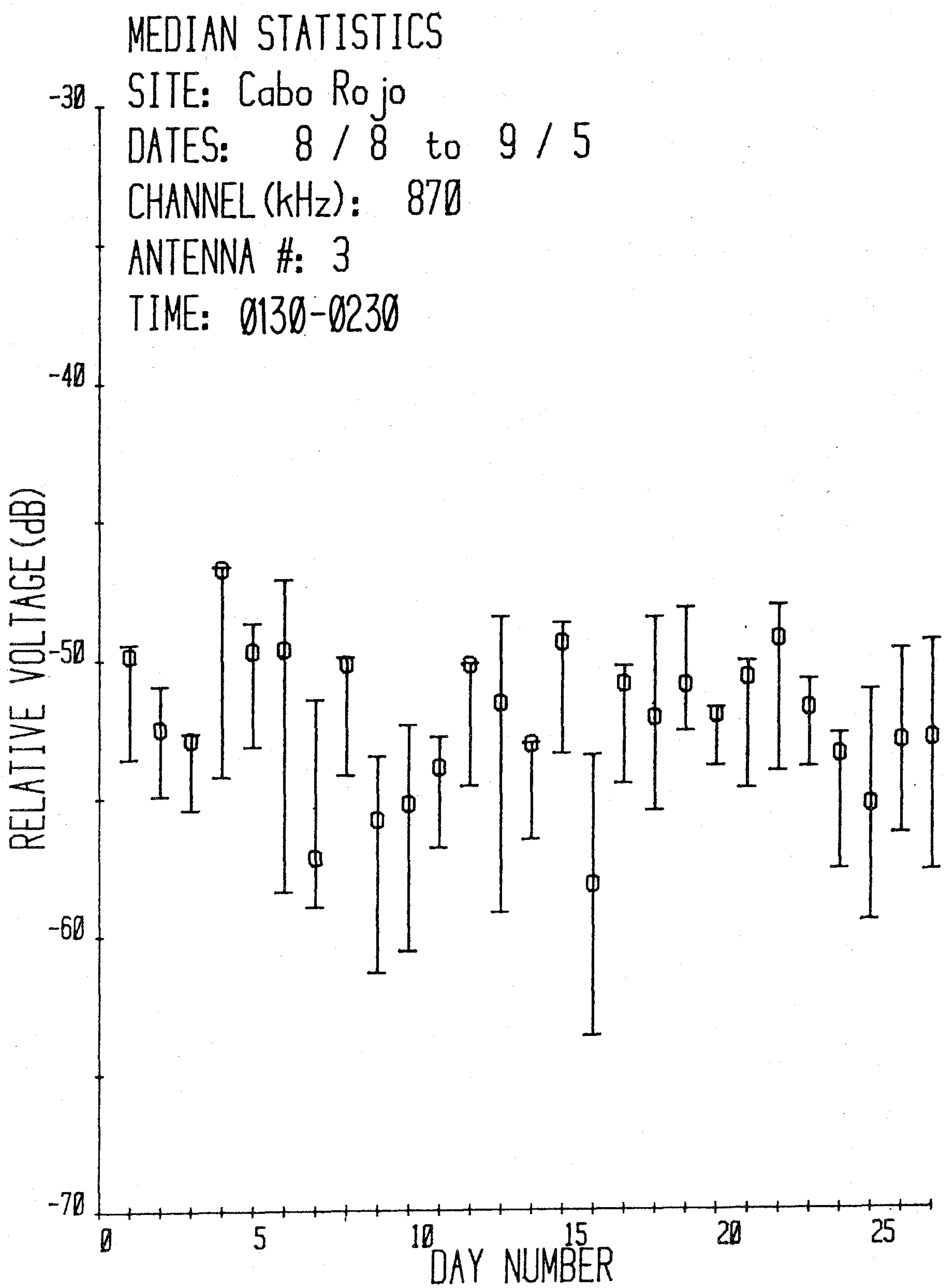


Figure 4. Hourly medians, upper and lower deciles: LRA 100 kW Buenos Aires, Argentina; Beverage antenna #3, 165°T.

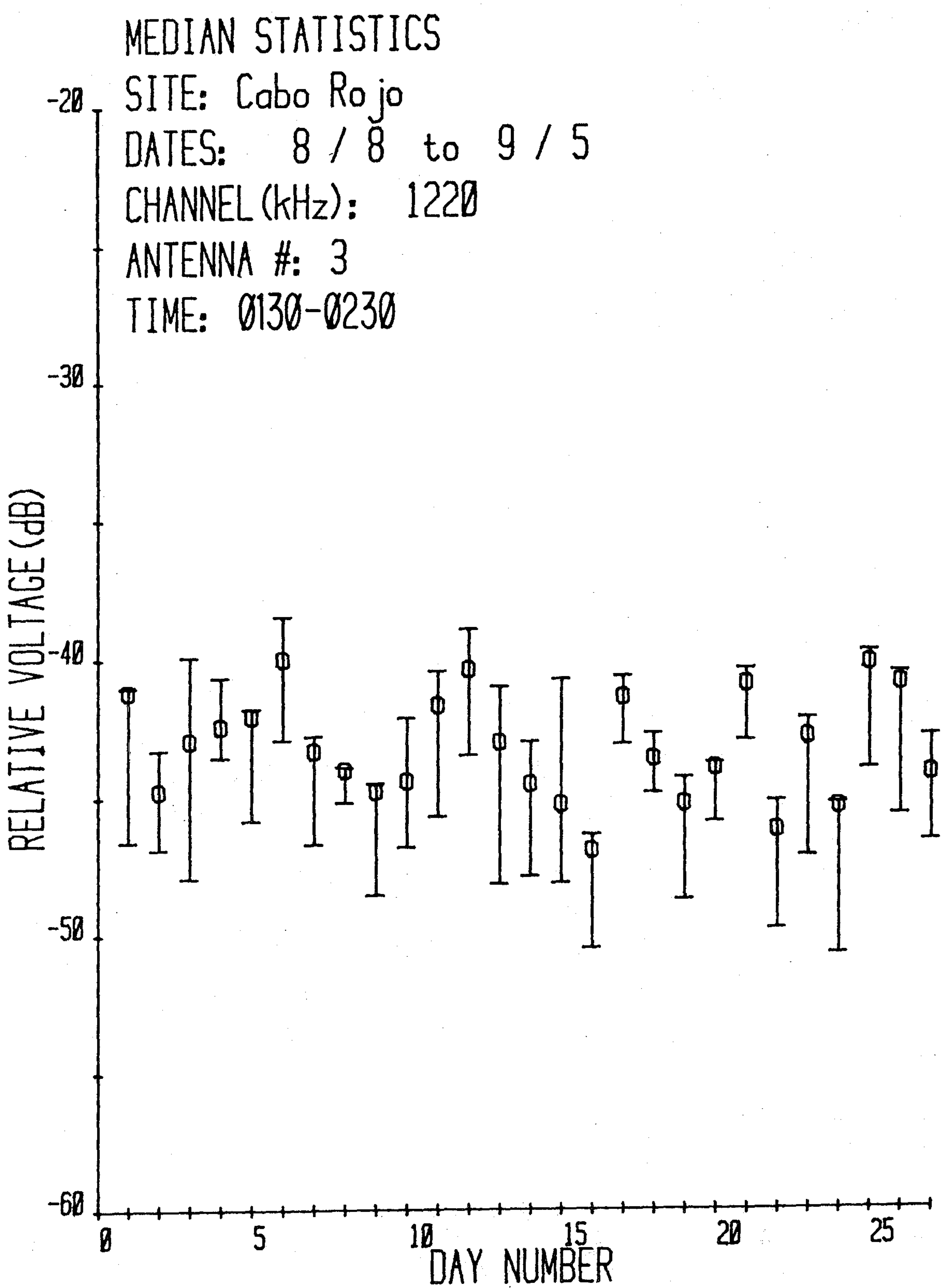


Figure 5. Hourly medians, upper and lower deciles: ZYJ458 150 kW Rio de Janeiro, Brazil; Beverage antenna #3, 165°T.

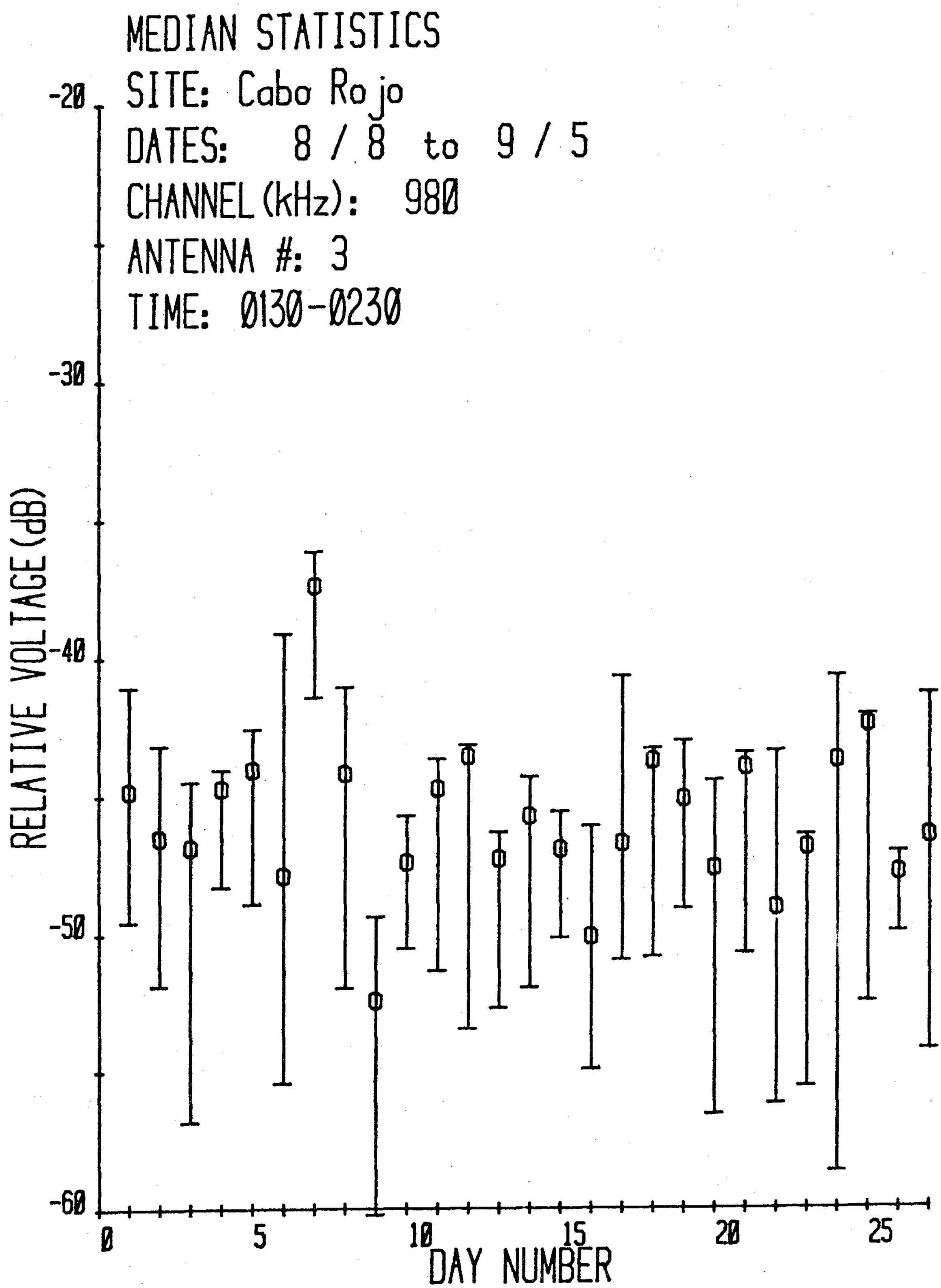


Figure 6. Hourly medians, upper and lower deciles: ZYH707 600 kW Brasilia, Brazil; Beverage antenna #3, 165°T.

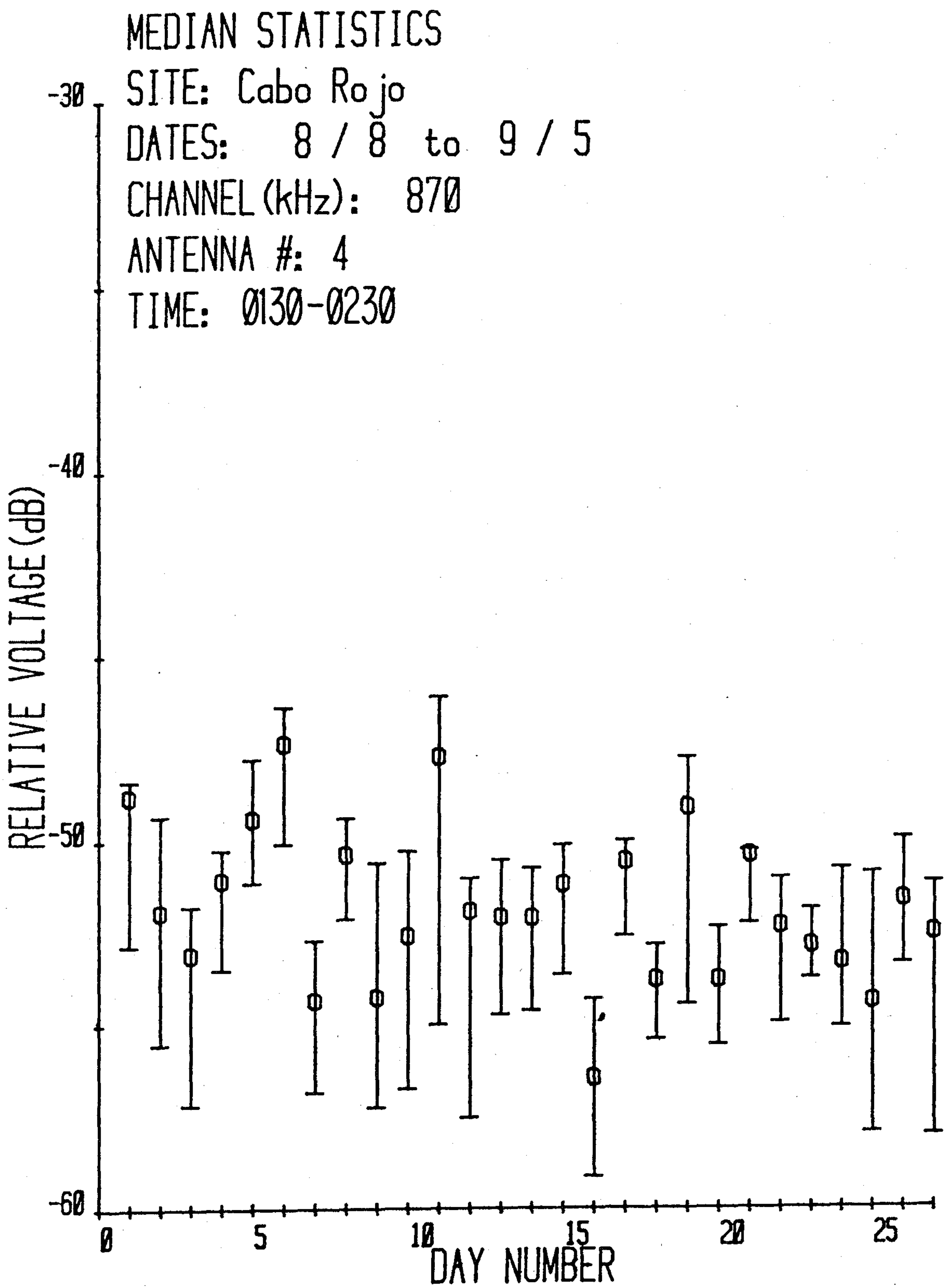


Figure 7. Hourly medians, upper and lower deciles: Radio Cristal 20 kW Guayaquil, Ecuador; Beverage antenna #4, 180°T.

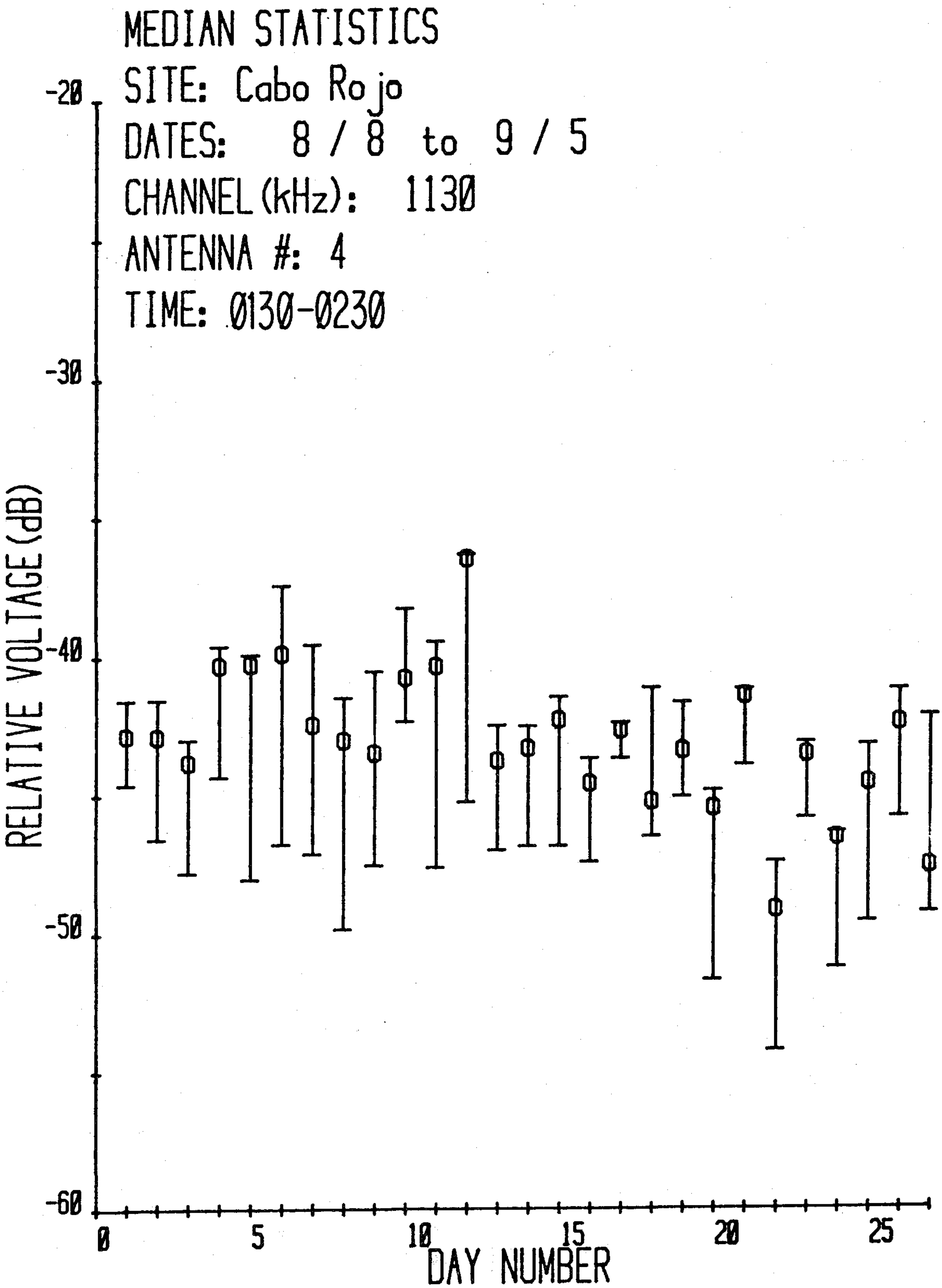


Figure 8. Hourly medians, upper and lower deciles: 20 kW Bogota, Colombia; Beverage antenna #4, 180°T.

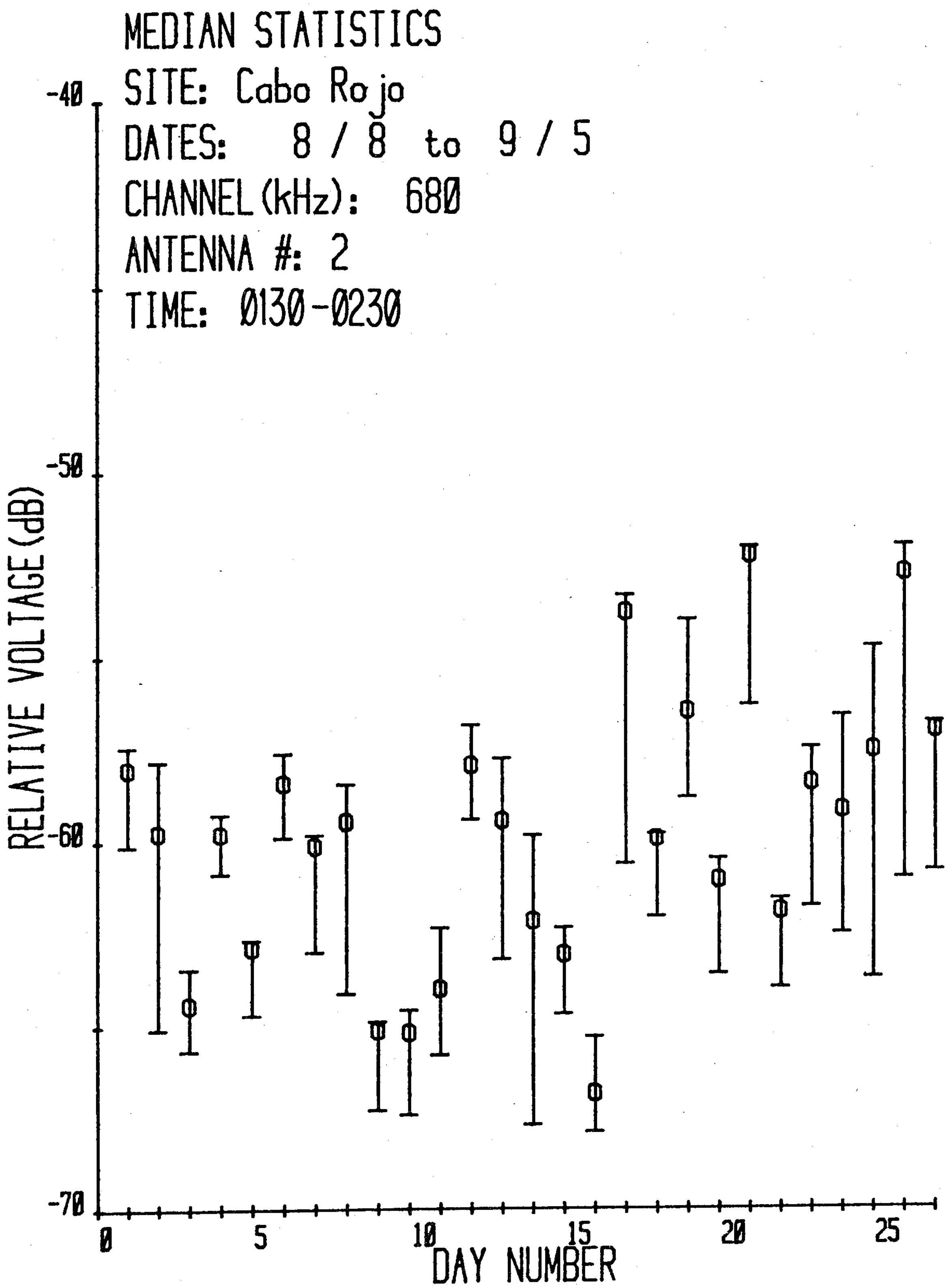


Figure 9. Hourly medians, upper and lower deciles: HJBO 100 kW Zambrano, Colombia; Beverage antenna #2, 140°T.

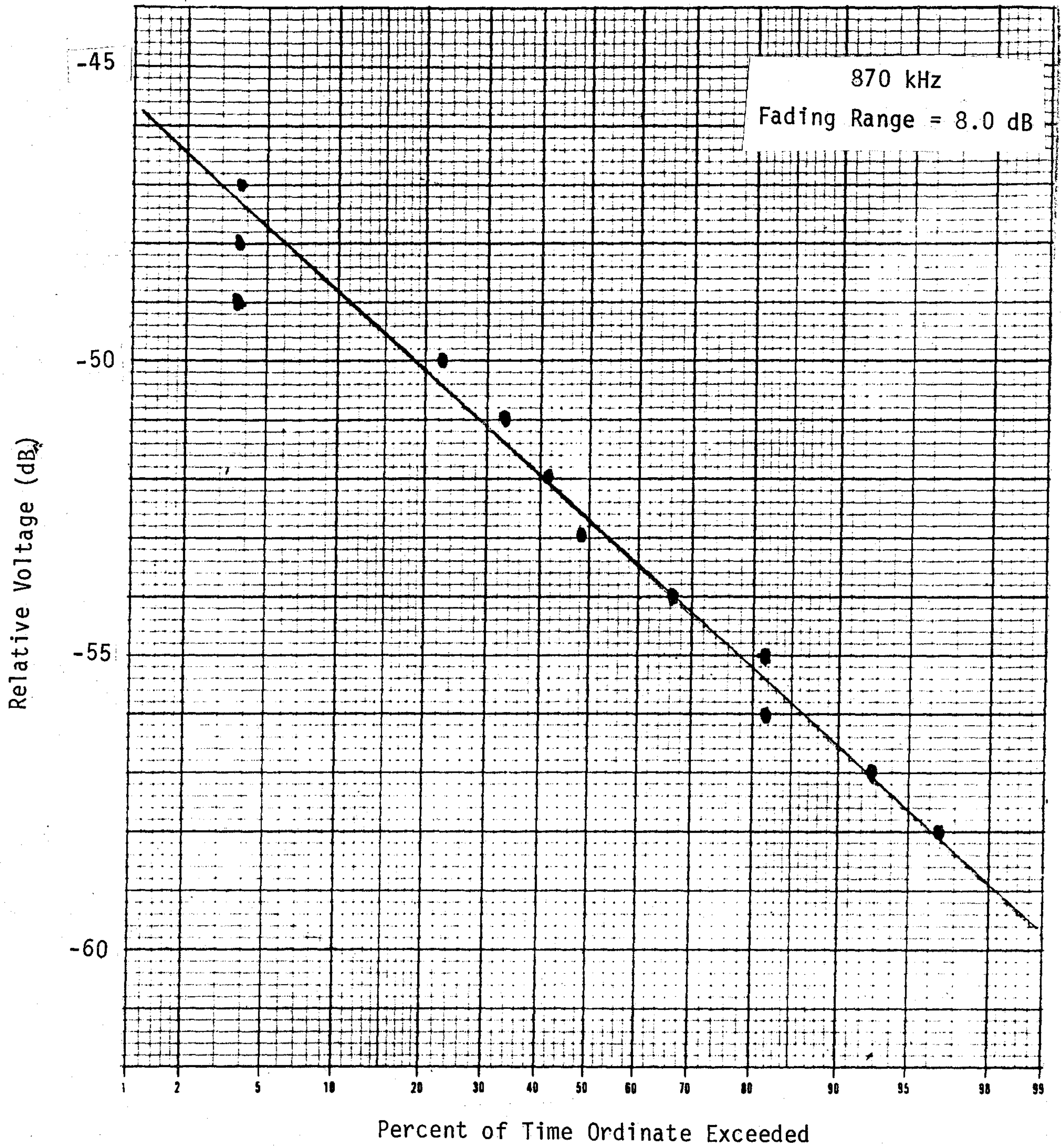


Figure 10. Cumulative distribution of the median data of Figure 4.

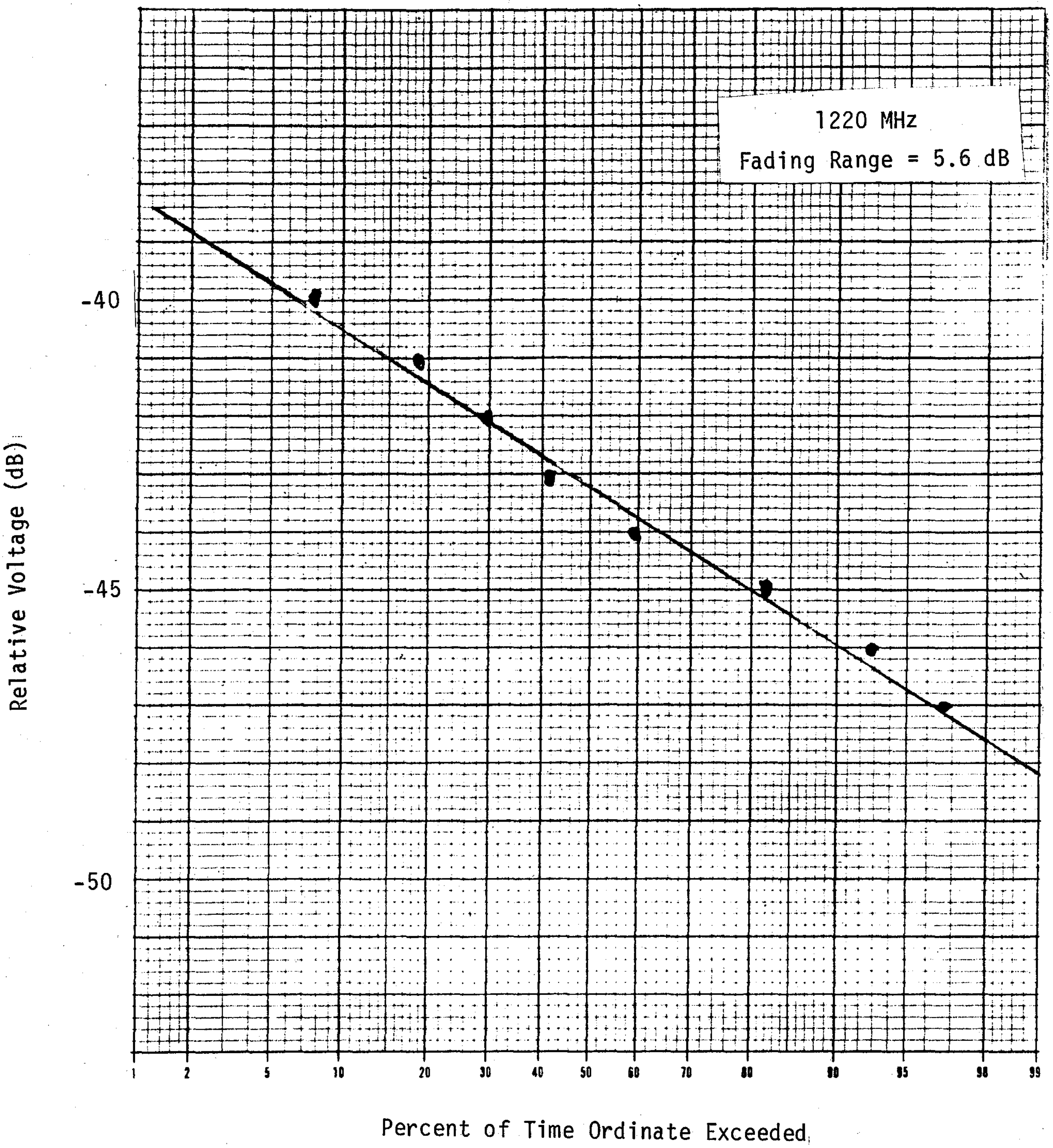


Figure 11. Cumulative distribution of the median data of Figure 5.

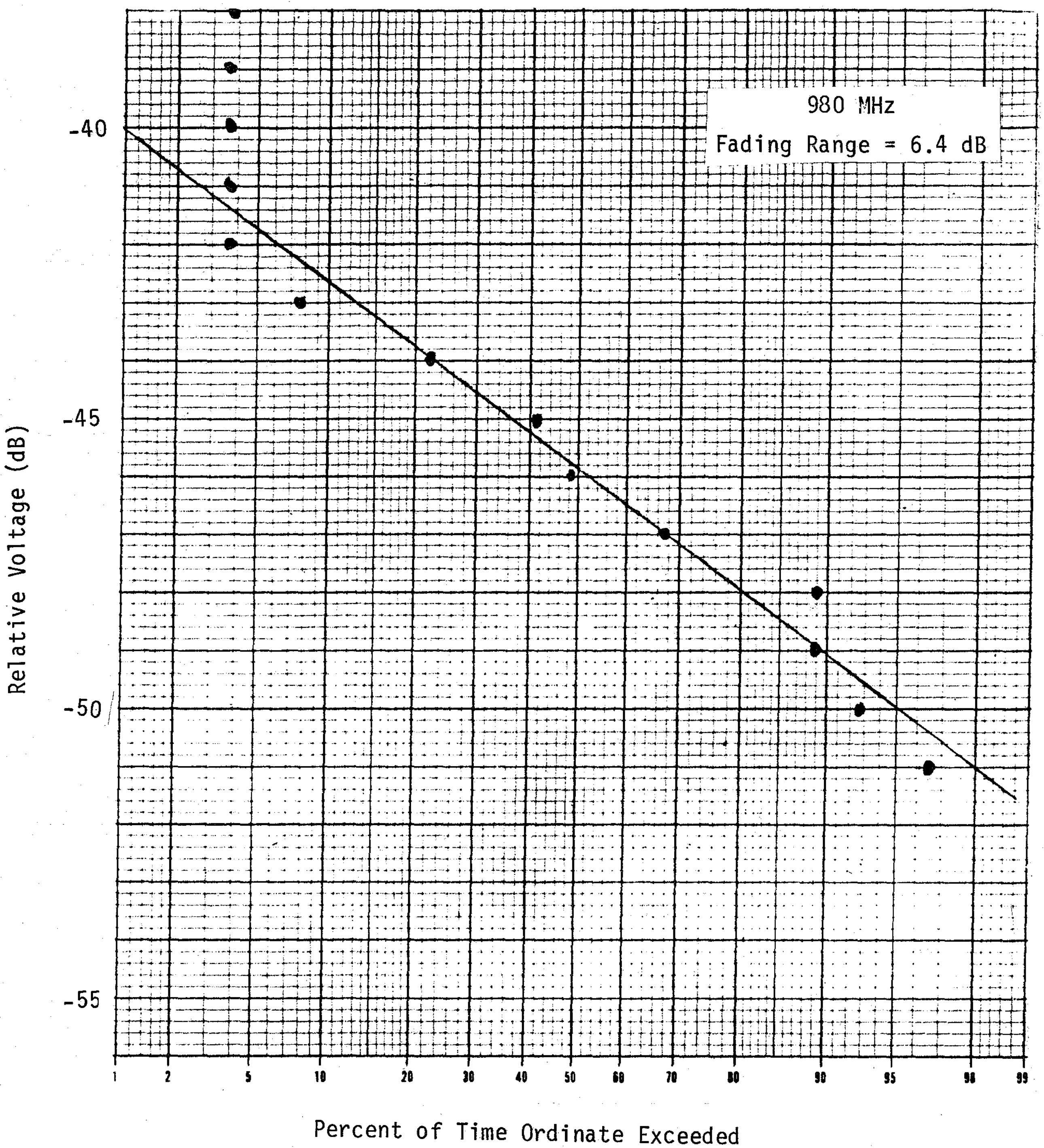


Figure 12. Cumulative distribution of the median data of Figure 6.

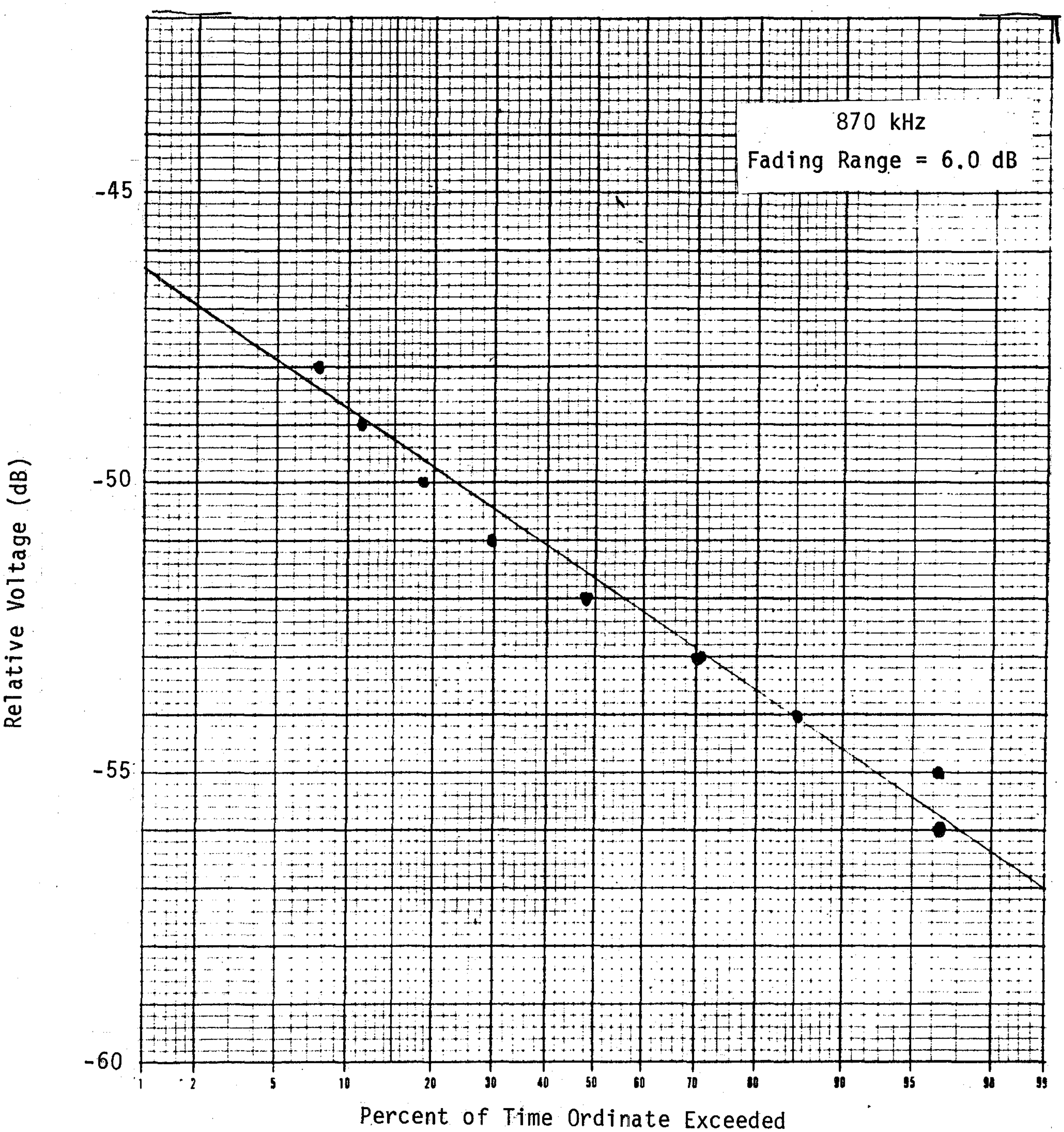


Figure 13. Cumulative distribution of the median data of Figure 7.

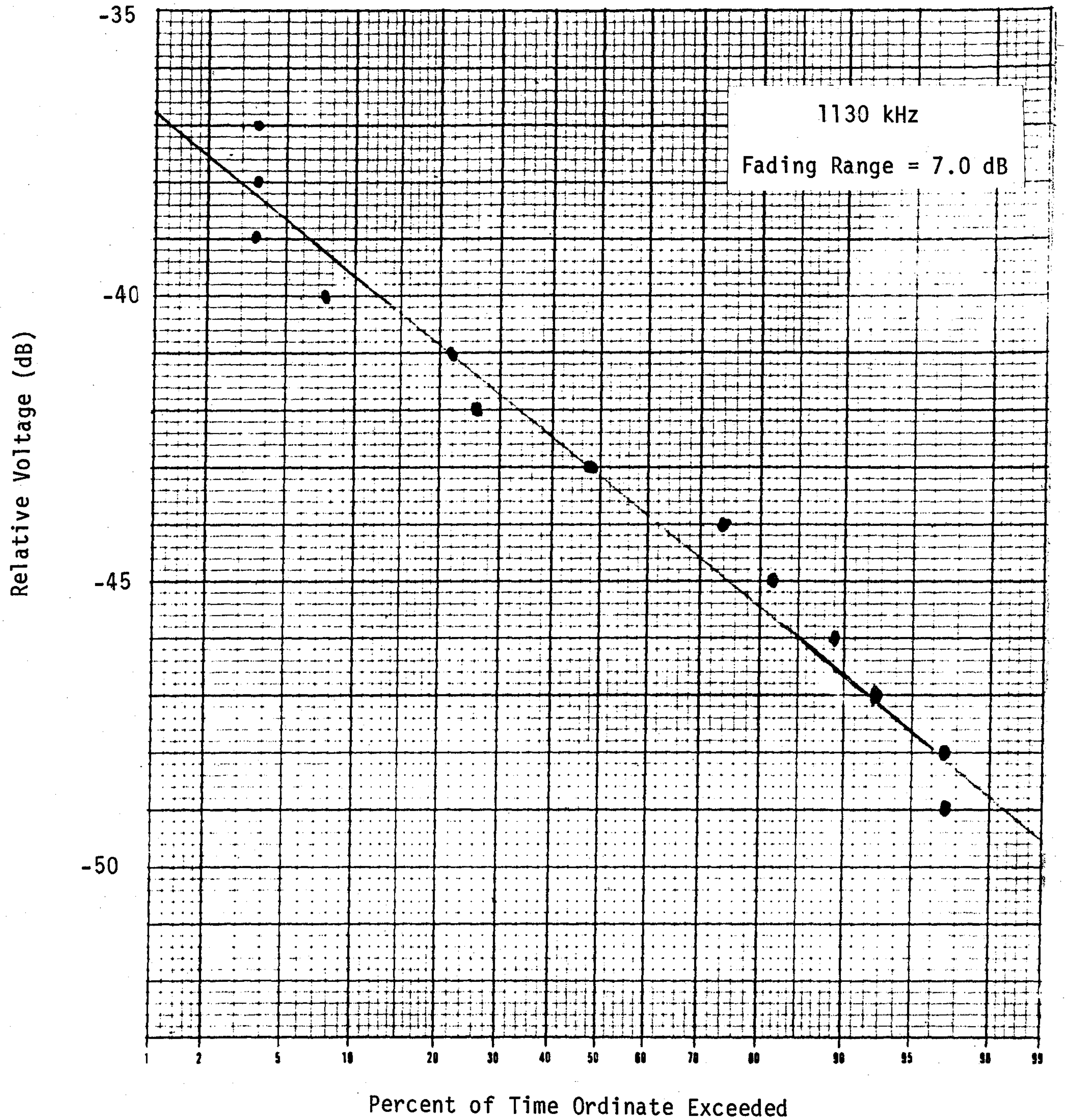


Figure 14. Cumulative distribution of the median data of Figure 8.

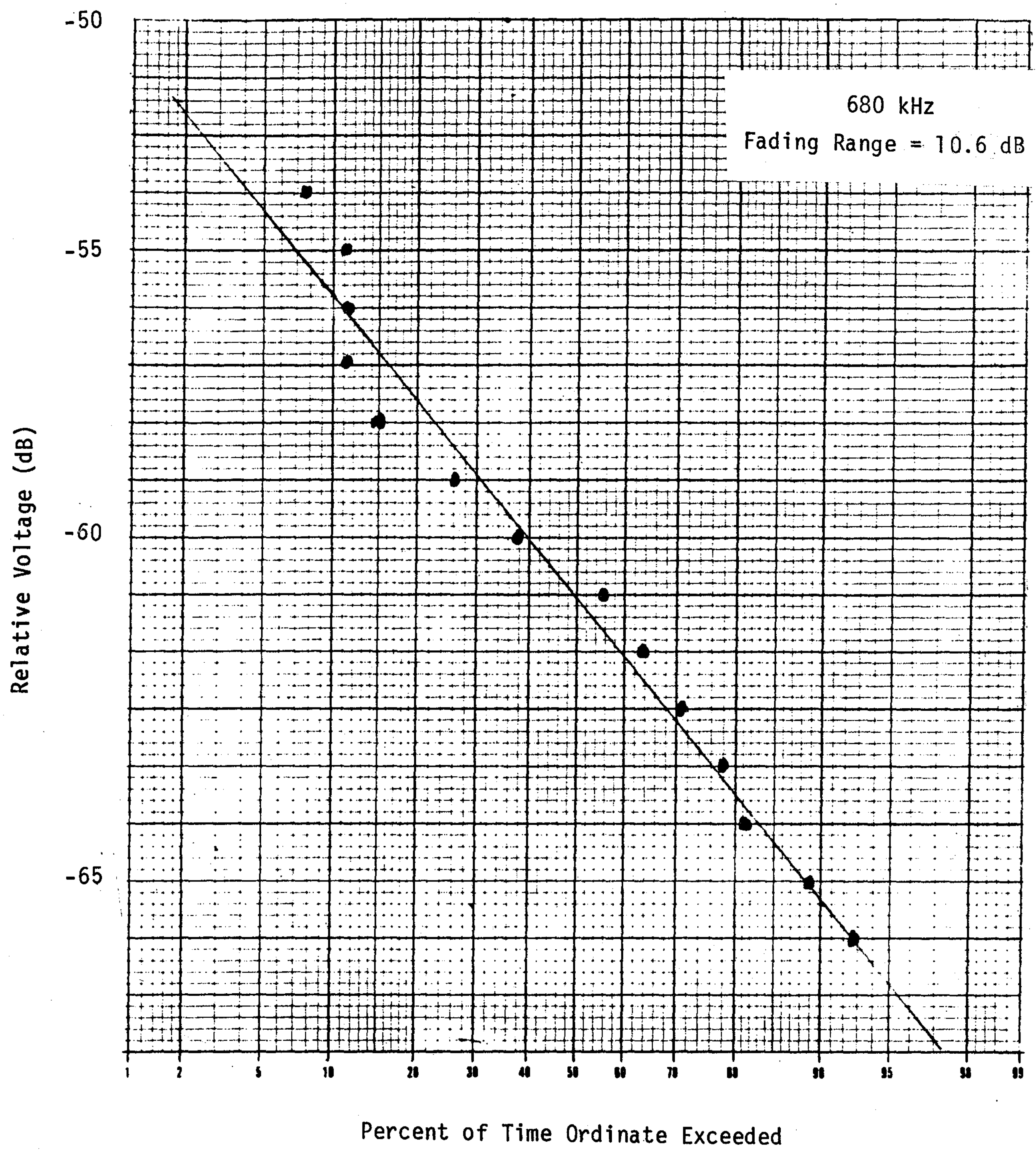


Figure 15. Cumulative distribution of median data of Figure 9.

## 2. THE COMPLETE FADING DISTRIBUTION

We want to obtain the complete fading distribution, considering both short-term and long-term fading, so that we can determine the interfering signal level that will be exceeded for any percentage of time. This is the information we need to properly determine the degree of objectionable interference to any desired broadcast signal.

For the short-term, within an hour, fading, the signal is represented by the Rayleigh distribution. The Rayleigh probability density function, pdf, for a signal S, is

$$p_S(x) = \frac{x}{\alpha} e^{-x^2/2\alpha} , \quad o \leq x < \infty , \tag{1}$$

and has only one parameter, $\alpha$. The exceedance probability is given by

$$\text{Prob}[S \geq S_o] = e^{-S_o^2/2\alpha} \quad . \tag{2}$$

In terms of the mean value $E[x] = \overline{x}$;

$$\alpha = \frac{2\overline{x}^2}{\pi} \quad , \tag{3}$$

and the median value, m, is given by

$$m = 0.9394 \ \overline{x} \ , \tag{4}$$

so that

$$\alpha = 0.7213 \ m^2 \ . \tag{5}$$

We can consider the short-term fading as a conditional probability, conditional on its median value m (and therefore $\alpha$). This is denoted $p_S(x|m)$.

Next, this median value is log-normally distributed. In terms of m, if $y = 20 \log m = 8.686 \ \ell n \ m$,

$$p_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} \quad e^{-1/2\left(\frac{y - \mu}{\sigma}\right)^2}, \quad -\infty < y < \infty \tag{6}$$

which is the normal (Gaussian) distribution with mean $\mu$ and standard deviation $\sigma$. The distribution of m is then given by

$$p_M(m) = \frac{8.686}{m\sqrt{2\pi\sigma^2}} \; e^{-1/2\left(\frac{20 \log m - \mu}{\sigma}\right)^2}, \; o < m < \infty \quad . \tag{7}$$

Therefore, the pdf of the "complete" fading distribution for our signal S is given by

$$p_S(x) = \int_0^\infty p_S(x|m)p_M(m)dm \quad . \tag{8}$$

Rather than the pdf given by (8), we are usually interested in the exceedance probability, or

$$\mathrm{Prob}[S > S_o] = \int_{S_o}^\infty \int_0^\infty p_S(x|m)p_M(m)dm \tag{9}$$

Using (7) and (2), (9) gives us, finally,

$$\mathrm{Prob}[S > S_o] = \int_0^\infty \exp\left(\frac{-S_o^2}{1.4426\, m^2}\right) \frac{8.686}{m\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{20 \log m - \mu}{\sigma}\right)^2\right\} dm \quad . \tag{10}$$

The above is a complicated integral requiring numerical integration. In (10), or (7), $\mu$ is the mean value (in dB) of the signal, and is the value usually obtained from propagation curves or models, and $\sigma$ is the standard deviation (dB). In terms of the fading range

$$\text{Fading range} = 2.54\ \sigma \ \text{dB} \quad . \tag{11}$$

In the above, note that $S_o$ and m are field strengths (voltage) and $\mu$ and $\sigma$ are dB quantities. For the log-normal distribution, by using the moment generating function for the normal distribution, it is easily shown that

$$E[m] = \exp\ [\mu/c + 1/2\ \sigma^2/c^2]\ , \tag{12}$$

and

$$E[m^2] = \exp\ [2\mu/c + 2\sigma^2/c^2]\ , \tag{13}$$

where $c = 8.686.$

E[m] is the overall average field strength [for the log-normal distribution (7)].

In (10), if we let $\mu$ = 20 log $\gamma$ and make the change of variable $z = m/\gamma$, we obtain

$$\text{Prob}[S > S_o] = \int_0^\infty \exp\left(\frac{-S_o^2/\gamma^2}{1.4426z^2}\right)\frac{8.686}{z\sigma\sqrt{2\pi}} \quad \exp\left\{-1/2\left(\frac{20 \log z}{\sigma}\right)\right\} dz \;. \qquad (14)$$

This shows that the only parameter which determines the shape of the resulting distribution is the fading range (which determines $\sigma$), and that the results can be given relative to the median value $\mu$ (or the corresponding voltage $\gamma$ ). In the Appendix, a listing of the computer algorithm FADE1 is given. This program was developed to compute the overall fading exceedance distribution given by (10) or (14). For versatility, the parameter $\mu$ has been maintained in the program. Figure 16 shows some sample results, for fading ranges of 6 and 16 dB. Also shown as dashed lines on Figure 16 are log-normal distributions with fading ranges of 6 and 16 dB. As can be seen from Figure 16, the overall distribution is not close to log-normal, especially for "smaller" fading ranges.

The above results are for a single interfering signal. Next, suppose we have two signals, each with short-term Rayleigh fading and long-term log-normal fading of the median values. We require the distribution of S, where $S = S_1 + S_2$. In general, the distribution of a random variable which is the sum of two other independent random variables is given by the convolution of the pdf's of the two random variables. We cannot, however, for our case, obtain the required pdf of S by convolving two pdf's given by (8) since the two signals arrive with arbitrary phases so that their magnitudes cannot be added. We can add the signal powers, but this is a difficult approach. What we need is the distribution of the phasor sum of $S_1$ and $S_2$. Both $S_1$ and $S_2$ have conditional Rayleigh pdf's and uniformly distributed phases. This means that the quadrature components are zero mean Gaussian processes, and the convolution of two Gaussian pdf's is again Gaussian. So, the pdf of the phasor sum of our two signals is again given by a Rayleigh pdf.

The above results in the "complete" fading exceedance probability distribution for our interfering signal, composed of two signals, being given by

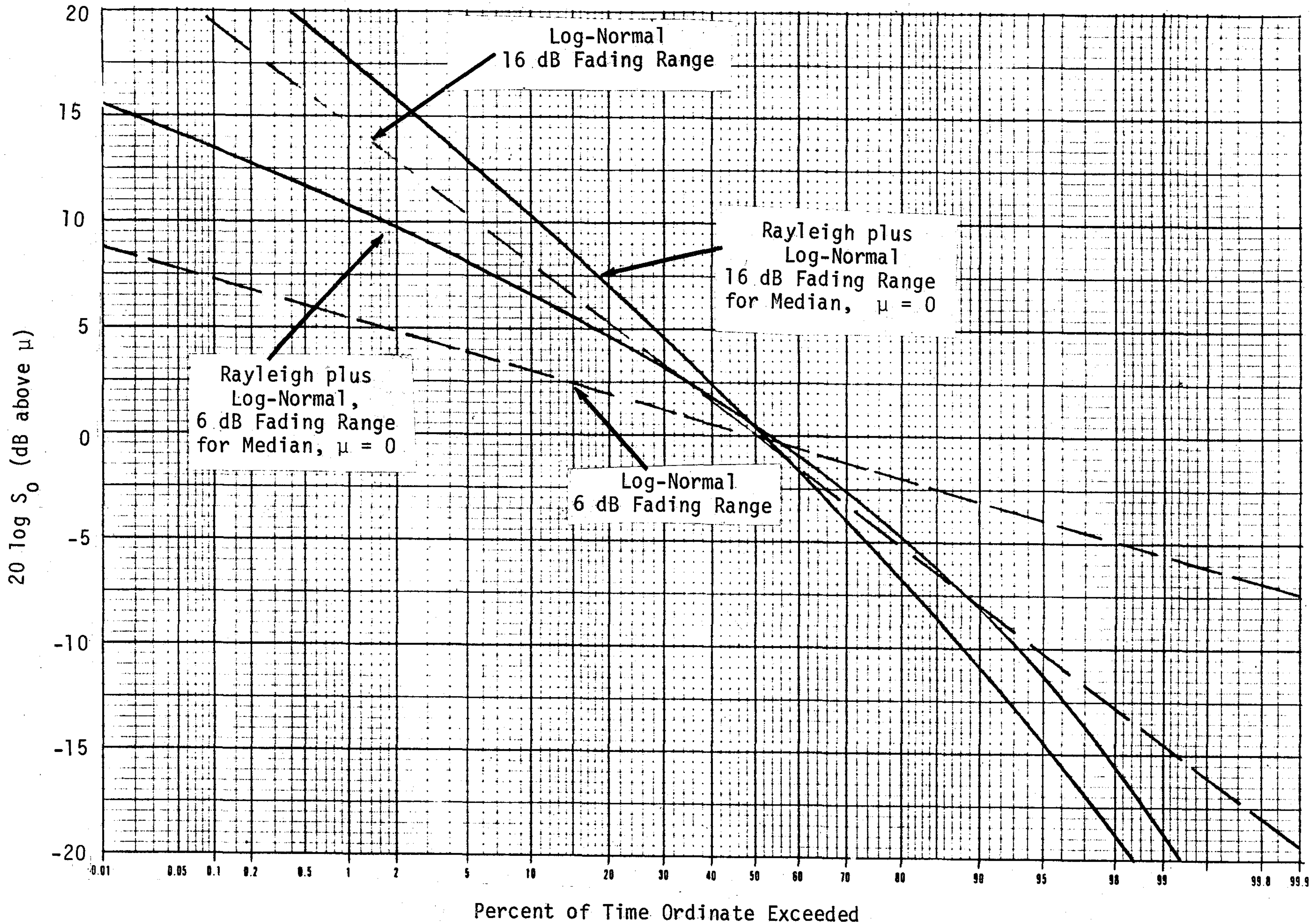


Figure 16. Cumulative distribution of one fading signal, considering both short-term and long-term fading, from Equation 10.

$$\text{Prob}[S > S_0] = \int_0^\infty \int_0^\infty \exp\{-S_0^2/1.4426(m_1^2 + m_2^2)\}$$

$$\times \frac{8.686}{m_1\sigma_1\sqrt{2\pi}} \exp\left\{-1/2\left(\frac{20 \log m_1 - \mu_1}{\sigma_1}\right)^2\right\} \frac{8.686}{m_2\sigma_2\sqrt{2\pi}} \exp\left\{-1/2\left(\frac{20 \log m_2 - \mu_2}{\sigma_2}\right)^2\right\} dm_1 dm_2. \quad (15)$$

where $m_1$ and $m_2$ are the fading median values (voltage) and $\mu_1$, $\mu_2$, $\sigma_1$, and $\sigma_2$ are the corresponding mean values and standard deviations (dB). In the Appendix, a listing of the computer algorithm FADE2 is given. This program was developed to compute the overall fading exceedance distribution given by (15). Figure 17 shows some example results. On Figure 17, the distribution resulting from two signals, both with the same mean and both with a 6 dB fading range, is given. The distribution for one signal with the same mean and fading is also given for comparison. Also given is the distribution resulting from two signals, one with a mean 10 dB below the other and with the larger signal having a 6 dB fading range and the smaller signal a 12 dB fading range. The program FADE2 can be used, of course, for any combination of two independent signals.

While the above procedure can be continued for three or more signals, the high dimensionability of the resulting integrals would result in problems both with computation time and accuracy.

Using the results developed above for one or two interfering signals, the actual field strength exceeded for any chosen percentage of time can be determined and, therefore, the interfering effects of these signals can be properly determined.

While the above takes into account both the short-term and long-term fading distributions, many "standards" are given in terms only of median values; that is, the percentage of hours (usually chosen as 10%) for which a given hourly median will be exceeded is required. This says that only the long-term fading distributions are considered. The next section presents an approximate method which can be used to obtain the hourly median value which will be exceeded for 10% (or any other small percentage) of the hours for any combination of any number of interfering signals.

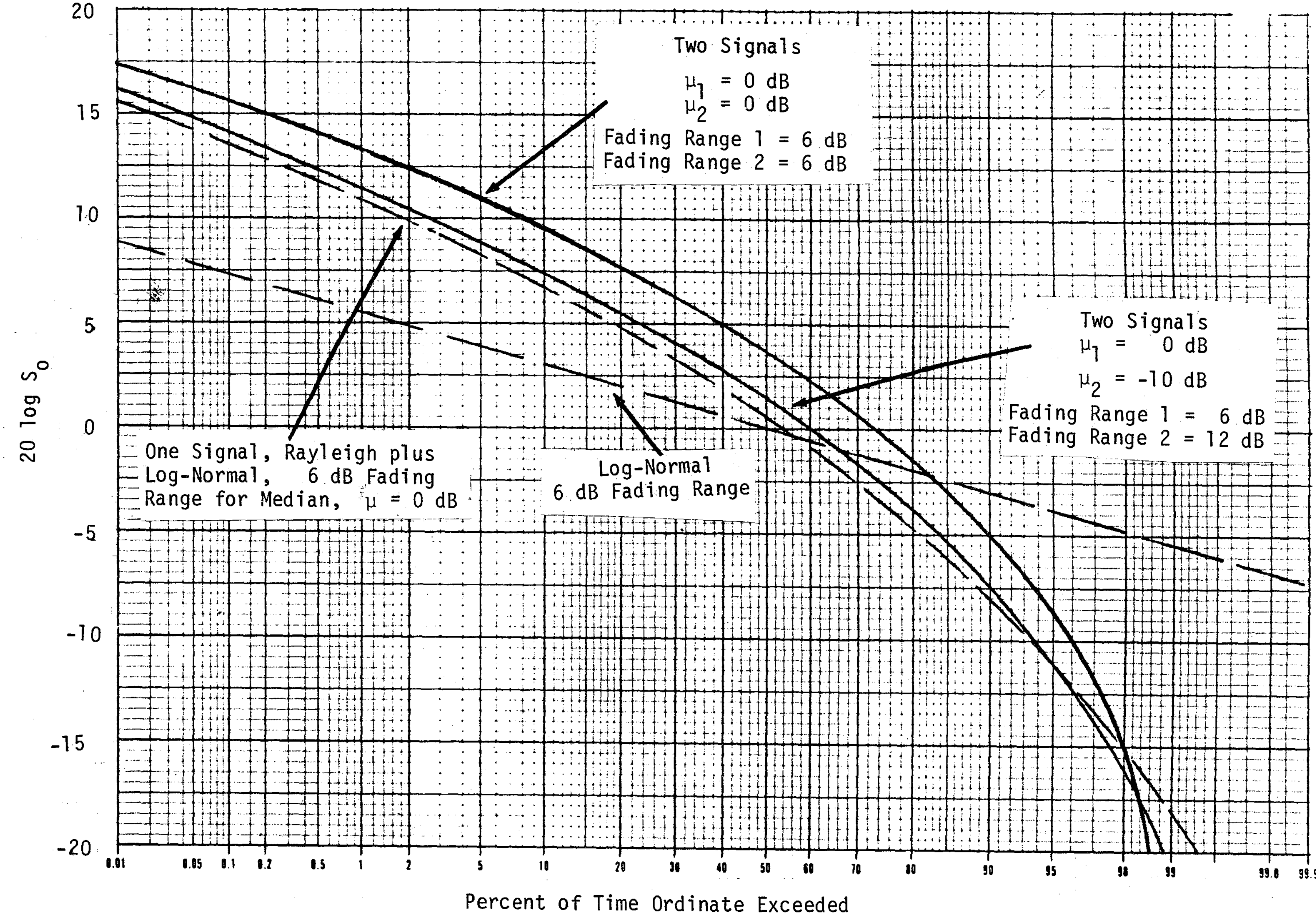


Figure 17. Cumulative distribution of the sum of two fading signals, from Equation 15.

## 3. APPROXIMATE LONG-TERM FADING DISTRIBUTION FOR N SIGNALS

Suppose we have n signals, each with their hourly median values log-normally distributed. We now require the distribution of the median of the sum of these n signals. It turns out that if n is large, the distribution of the sum is closely represented by another log-normal distribution. Also, if one of the n signals dominates the others, the resulting distribution cannot be far from log-normal. However, we are generally interested in a relatively small n and, quite often, with signals of similar size. The resulting distribution for the sum is now, definitely not log-normal and can be determined by convolution. Such convolutions of log-normal pdf's have been performed in the past with, for our purposes, some quite interesting results. It turns out that the method presented below will give quite accurate results for the larger signal strengths (i.e., the small percentage points of the resultant distribution) and appears to be most accurate around the 10% point. [For an indication of the truth of this conjecture, see Norton et al. (1952) and Appendix A of Gierhart et al. (1970).] In developing this approach, the FCC data given earlier in Figures 1 and 2 will be used, but, of course, any log-normal distributions could be used (e.g., Figures 10-15). Here we are considering the median of the sum of n signals, $S_T = S_1 + S_2 + \ldots + S_n$. Since the distributions involved are distributions of median values expressed in dB, it is proper to directly add the "signals." This corresponds to adding signal powers (since $20 \log S = 10 \log S^2$), whereas previously we were concerned with the actual resultant field strength and required the phasor sum of the component signals. The approximate approach simply assumes that the resultant distribution of the sum of the log-normally distributed variables is again log-normal.

The pdf of our i-th signal, $S_i$, is

$$p_{S_i}(x_i) = \frac{8.686}{x_i\sqrt{2\pi\sigma_i^{\,2}}}\; e^{-1/2\left(\frac{20\log x_i - \mu_i}{\sigma_i}\right)^2} . \tag{16}$$

As shown earlier,

$$E[x_i] = \exp\left[\mu_i/c + 1/2\ \sigma_i^{\,2}/c^2\right] \tag{17}$$

and

$$E[x_i^{\,2}] = \exp\left[2\mu_i/c + 2\sigma_i^{\,2}/c^2\right] , \tag{18}$$

where c = 8.686.

(Remember that the variable $y_i = 20 \log x_i = 8.686 \ln x_i$ is normally distributed with mean $\mu_i$ and standard deviation $\sigma_i$.) Letting $\alpha_i$ denote the mean of $x_i$ and $\beta_i$, the variance of $x_i$, then

$$\alpha_i = E[x_i] \quad \text{and}$$

$$\beta_i = E[x^2{}_i] - E^2[x_i] \quad \text{or}$$

$$\beta_i = \alpha^2{}_i[\exp(\sigma^2{}_i/c^2) - 1] \quad . \tag{19}$$

The above (17), (18), and (19) can be solved to give

$$\sigma_i{}^2 = c^2 \ln\,(1 + \beta_i/\alpha_i{}^2) \quad \text{and}$$

$$\mu_i = c(\ln\alpha_i - 1/2\ \alpha_i{}^2/c^2). \tag{20}$$

Since our n received log-normally fading signals are independent, the mean, $\alpha_T$, and variance, $\beta_T$, of the sum are given by:

$$\alpha_T = \sum_{i=1}^{n} \alpha_i , \beta_T = \sum_{i=1}^{n} \beta_i \quad . \tag{21}$$

Then these $\alpha_T$ and $\beta_T$'s can be used in (20) to obtain $\sigma_T$ and $\mu_T$, with the resulting pdf then given by

$$\hat{p}_{S_T}(y) = \frac{1}{\sqrt{2\pi\sigma_T{}^2}}\ e^{-1/2\left(\frac{y - \mu_T}{\sigma_T}\right)^2} , \tag{22}$$

where $S_T$ is the sum (in dB) of our n received signals. The pdf in (22) has a hat on it to indicate that this is only an approximation which is valid, in general, only for the small percentage points and, in particular, quite good around 10%.

Table 1 gives the parameters for each of the distributions in Figure 2. Note that the fading range is given by 2.54 $\sigma$.

Table 1. The Log-normal Distribution Parameters for the FCC Data.

| Distance Miles | $\alpha$ | $\beta$ | $\mu$(dB) | $\sigma$(dB) |
|---|---|---|---|---|
| 200 | 0.1151 | 0.0145 | -22.0 | 7.48 |
| 600 | 0.0514 | 0.00291 | -29.0 | 7.48 |
| 1000 | 0.0172 | 0.000326 | -38.5 | 7.48 |
| 1500 | $5.44\times10^{-3}$ | $1.55\times10^{-5}$ | -48.5 | 7.48 |
| 2000 | $2.73\times10^{-3}$ | $8.19\times10^{-6}$ | -54.5 | 7.48 |
| 2500 | $1.45\times10^{-3}$ | $2.31\times10^{-6}$ | -60.0 | 7.48 |

We will now consider two examples to illustrate use of the above. First, suppose we have three signals, all from 600 miles, then

$$\alpha_T = 3(0.0514) = 0.1542,$$

and

$$\beta_T = 3(0.00291) = 0.00873.$$

Then, using (20),

$$\sigma_T{}^2 = (8.686)^2 \ell n \left[1 + \frac{0.00873}{(0.1542)^2}\right] = 23.6,$$

$$\sigma_T = 4.86 \text{ dB},$$

and

$$\mu_T = 8.686 \; [\ell n(0.1542) - \tfrac{1}{2} \; (23.6)/(8.686)^2]$$

$$\mu_T = -17.6 \text{ dB}.$$

Figure 18 shows the results for this example, and we see that, for the interfering signal composed of the sum of our three signals, the median value exceeding 10% is -11.5 dB or 0.266 mV/m.

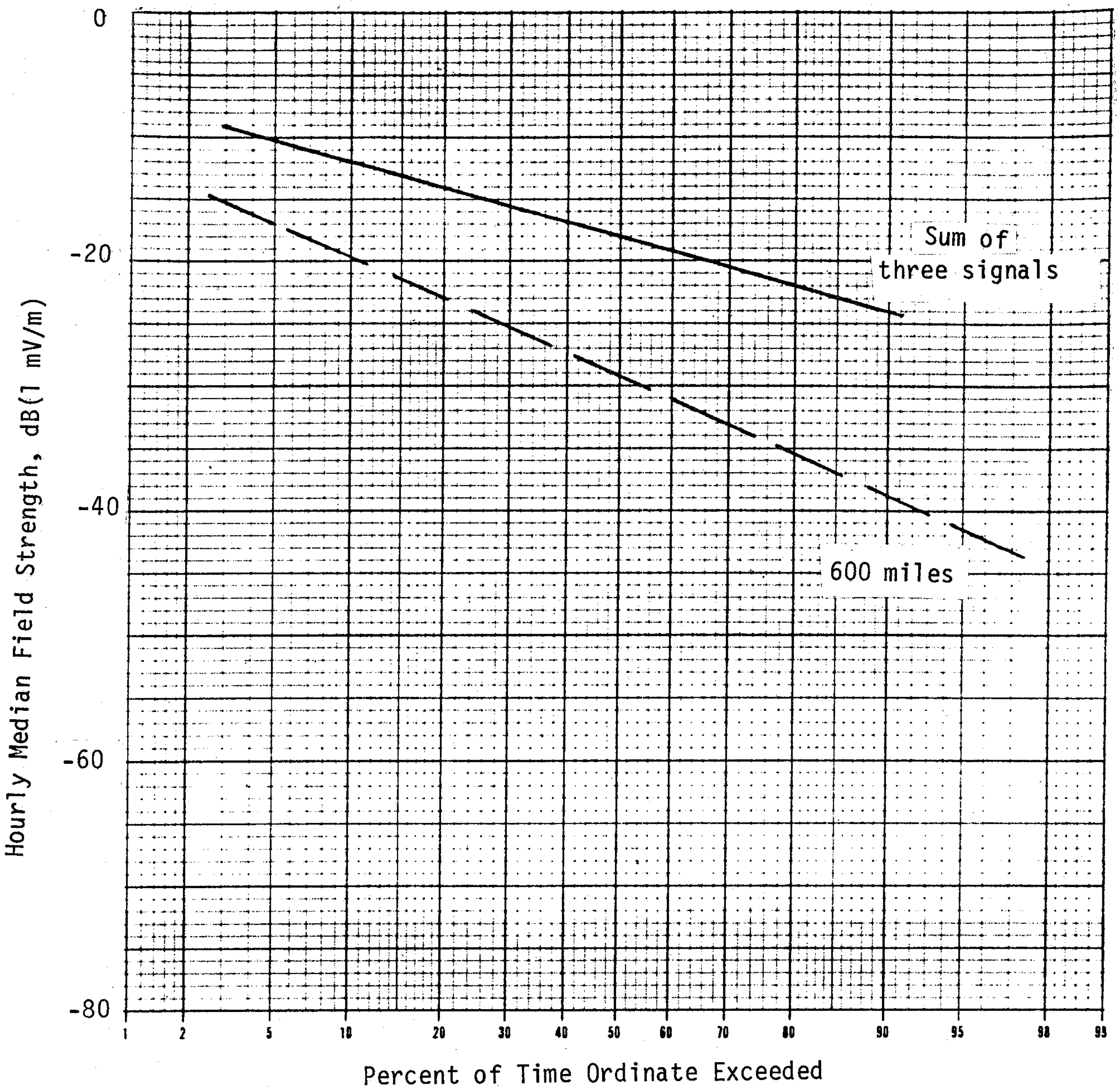


Figure 18. The distribution of the median of the sum of three signals, each from a distance of 600 miles (Figure 2).

Next, suppose we have five signals from 1500 miles, two signals from 1000 miles, and one signal from 600 miles. Then we obtain

$$
\begin{aligned}
\alpha_T &= 0.1130 \\
\beta_T &= 0.00364 \\
\sigma_T &= 4.35 \quad \text{dB} \\
\mu_T &= -20.03 \quad \text{dB.}
\end{aligned}
$$

Figure 19 shows the results for this example, and we see that, for this case, the value exceeded 10% of the time is -14.5 dB or 0.188 mV/m.

The FCC (1976, §73.182) procedure is to take the root-sum-square value of the 10% values of the individual signals composing the sum and to exclude small signals that are less than 50% of the root-sum-square value of the higher signals already included. For our two examples, the FCC procedure gives the 10% value for the sum to be -14.2 dB (.194 mV/m) compared to -11.5 dB (.266 mV/m) calculated above. For the second example, the FCC method gives -15.7 dB (.146 mV/m) as compared to -14.5 dB (.188 mV/m) calculated above.

It would appear from the above that the "FCC method" gives results comparable to the log-normal approach. However, this is not always the case. Consider the example where we have ten signals from 600 miles. Using Figure 1, the FCC approach gives

$$\text{10\% value} = \sqrt{10(0.098)^2} = .309 \ \frac{\text{mV}}{\text{m}} \quad .$$

Using the log-normal approach, which now, since n is relatively large, is certainly quite accurate at the 10% point, we have:

$$
\begin{aligned}
\alpha_T &= 0.514 \\
\beta_T &= 0.291 \\
\sigma_T &= 2.8 \quad \text{dB} \\
\text{fading range} &= 7.11 \quad \text{dB} \\
\mu_T &= -6.2 \quad \text{dB}
\end{aligned}
$$

$$\textbf{10\% value} = \mu_T + \frac{\text{fading range}}{2} = 2.65 \text{ dB}$$

$$\text{10\% value} = .737 \text{ mV/m.}$$

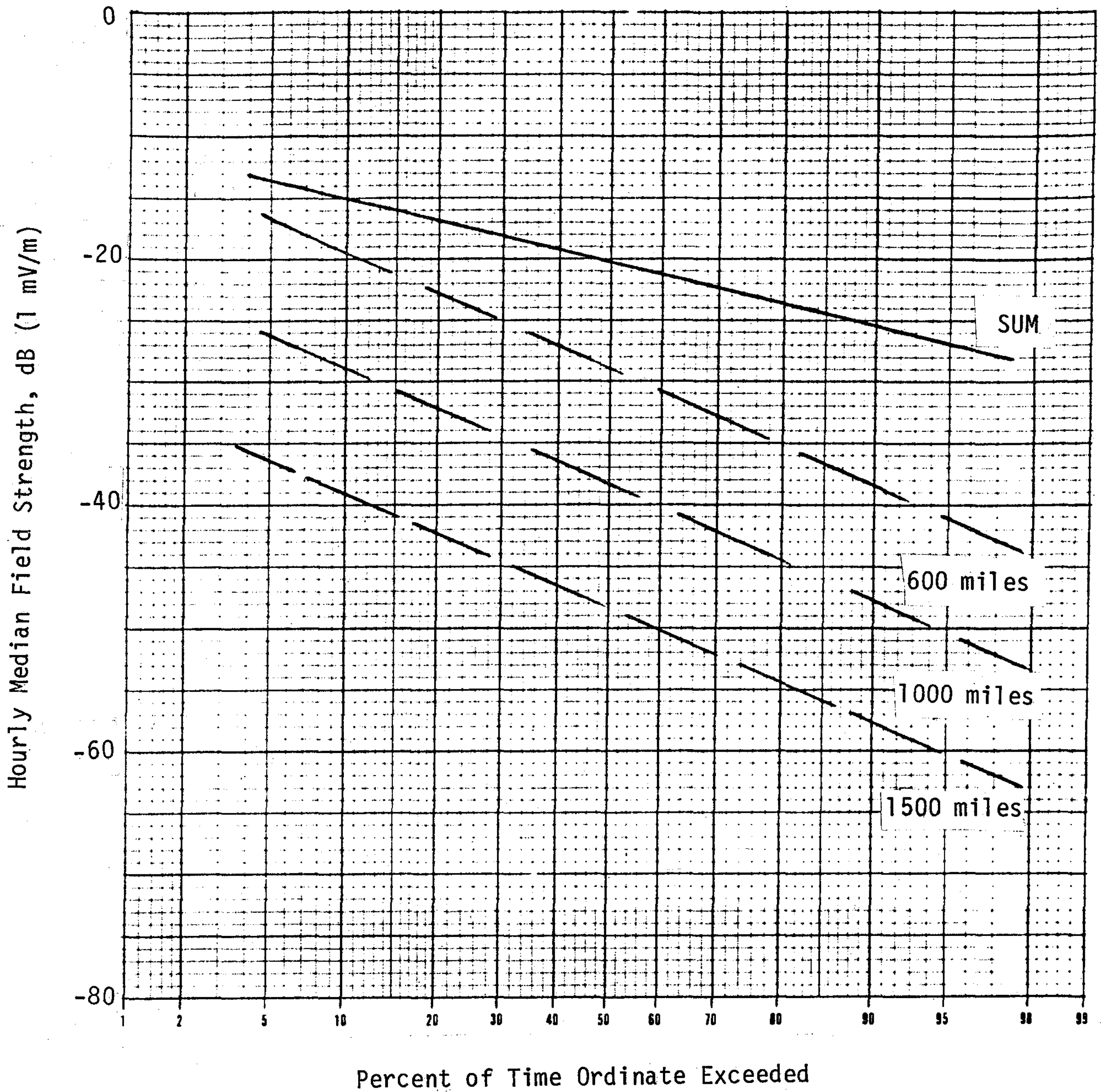


Figure 19. The distribution of the median of the sum of 8 signals, 5 from a distance of 1500 miles, 2 from 1000 miles, and 1 from 600 miles (Figure 2).

If one is concerned only with hourly median values, the above "log-normal method" gives a simple procedure which can be used to determine the small (10% or less) percentage points of the distributions for the median of the sum of n interfering signals. This method is substantially more accurate than the FCC "root-sum-square method," and predicts values higher than the FCC method. It should be noted, however, that, as was shown in Section 2, the short-term fading can have a significant effect on the distribution of the actual interfering field strength with, of course, a correspondingly significant effort on the interference effects of the undesired signals.

## 4. CONCLUSIONS AND DISCUSSION

It is common practice to attempt to determine the interfering effects of undesired signals in the MF band by considering only the distribution of the hourly median field strength values over some period of time (e.g., a season). In this report, we have presented a method of approximately determining the small percentage large field-strength portion of the distribution of the hourly median values for a fading signal that is composed of any number, n, of component fading signals. This method, the "log-normal" method, is significantly more accurate than the "root-sum-square of 10% values" method.

It is more useful, however, in determining the interfering effects of the undesired signals to know the overall distribution of the actual received field strength taking into account the variations within the hour as well as the variations from day-to-day for that hour over, for example, a season. In this report we have developed computer algorithms that compute the "true" distribution of the received field strength where the short-term, within an hour, fading distribution is given by a Rayleigh distribution, and the long-term fading of the hourly median dB value is given by a log-normal distribution. Algorithms are given for one and for the sum of two such fading signals. As can be seen from Figures 16 and 17, the short-term fading can have a significant effect on the resulting overall fading distribution, especially for the smaller log-normal fading ranges.

In this report, we followed the standard assumption that the short-term fading distribution is Rayleigh. The recent measurements at Cabo Rojo, Puerto Rico, for the Institute for Telecommunication Sciences and the FCC (Figures 4-9) would seem to indicate that the within-the-hour variation can depart, sometimes significantly, from that given by the Rayleigh distribution. This means that it may be useful to take a closer look at the short-term fading characteristics.

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## APPENDIX: PROGRAM LISTINGS

In this Appendix we simply list the computer programs (in FORTRAN4) used for the sample calculations given in the report and required for similar calculations. The programs are essentially self-explanatory via the comment statements, but some further explanation may be helpful.

The first program given is FADE1, which calculates the overall fading distribution (exceedance percentages) for one signal using Equation 10. The mean value ($\mu$ of Equation 10) is termed AMED (the mean and median values are the same), and must be supplied along with the fading rate (or rates) for the log-normal distribution of the median values. The shape of the distribution does not depend on AMED, and the results can be given relative to the mean value. The mean is left in the program as a parameter, so that the actual mean (from some propagation model, say) can be used in FADE1 and the results applied directly, for example, to some plotting routine.

The second program given is FADE2, which calculates the overall fading distribution for a signal which is the sum of two fading signals using Equation 15. The two mean values (termed AMED1 and AMED2) must be supplied along with the two fading ranges (FADE1 and FADE2).

In these programs, IRAY and SYSTEMC are used. This is to suppress an exponent underflow error message for the particular computer used (CYBER 170/750) and is not, in general, required.

```
      PROGRAM FADE1(INPUT,OUTPUT)
C     THIS PROGRAM COMPUTES THE EXCEEDANCE PROBABILITY FOR
C     A SIGNAL WHICH IS RAYLEIGH FADING WITHIN AN HOUR AND
C     WHOSE HOURLY MEDIAN VALUE IS LOG-NORMALLY FADING
C     WITH MEAN AMED AND FADING RANGE 2.54*SIGMA.
C     PROGRAM USES WEDDLE INTEGRATION OVER LOGARITHMIC
C     INTERVALS.
      DIMENSION IRAY(6),FADR(9),Y(7)
      DATA IRAY/-1,-1,-1,0,-1,-1/
      DATA FADR/4.,6.,8.,10.,12.,14.,16.,18.,20./
      CALL SYSTEMC(115,IRAY)
      PRINT 6
    6 FORMAT(1H1)
C     SPECIFY THE MEAN VALUE,MU,EQUATION 10
      AMED=0.0
      DO 70 J=1,9
C     SPECIFY THE FADING RANGE OF MEDIANS
      SIGMA=FADR(J)/2.54
      PRINT 7, AMED, FADR(J)
      DO 60 K=1,31
      SDB=AMED-30.+2.0*(K-1)
      S=10.**(SDB/20.)
      SUM=0.
      DO 50 L=1,40
      BDB=-160.+5.*(L-1)
      CDB=-160.+5.*L
      B=10.**(BDB/20.)
      C=10.**(CDB/20.)
      DX=(C-B)/6.
      DO 40 M=1,7
      X=B+(M-1)*DX
      E=EXP(-S*S/(1.4426*X*X))
      F=(20.*ALOG10(X)-AMED)/SIGMA
      EE=EXP(-0.5*F*F)
      Y(M)=3.465213*E*EE/(X*SIGMA)
   40 CONTINUE
      SS=0.3*DX*(Y(1)+5.*Y(2)+Y(3)+6.*Y(4)+Y(5)+5.*Y(6)+Y(7))
      SUM=SUM+SS
C     CHANGE EXCEEDANCE PROBABILITIES TO PERCENTAGES
      SUMP=SUM*100.
   50 CONTINUE
      PRINT 8,SDB,SUMP
   60 CONTINUE
      PRINT 6
   70 CONTINUE
    7 FORMAT(2X,2(F5.1))
    8 FORMAT(5X,F5.1,2X,1PE9.3)
      END
```

```
      PROGRAM FADE2(INPUT,OUTPUT)
C     THIS PROGRAM COMPUTES THE EXCEEDANCE PROBABILITY FOR
C     THE PHASOR SUM OF TWO FADING SIGNALS WHICH FADE
C     AS IN PROGRAM FADE1.
C     PROGRAM USES MULTIPLE HARDY INTEGRATION OVER LOGARITHMIC
C     INTERVALS FOR THE DOUBLE INTEGRAL.
      DIMENSION IRAY(6),MM(4),Z(4),DSUM(4),W(5),FY(22,4)
      DATA IRAY/-1,-1,-1,0,-1,-1/
      DATA W/0.093333,0.54,0.733333,0.54,0.093333/
      DATA MM/1,3,5,6/
      CALL SYSTEMC(115,IRAY)
      PRINT 6
    6 FORMAT(1H1)
C     SPECIFY THE TWO MEAN VALUES,MU1 AND MU2,EQUATION 15
      AMED1=0.0
      AMED2=0.0
      A1=10.**(AMED1/10.)
      A2=10.**(AMED2/10.)
C     SPECIFY THE TWO FADING RANGES OF MEDIANS
      FADR1=6.0
      FADR2=6.0
      SIGMA1=FADR1/2.54
      SIGMA2=FADR2/2.54
      PRINT 7, AMED1,AMED2,FADR1,FADR2
      DO 60 K=1,13
      SDB=AMED1-30.+5.0*(K-1)
      S=10.**(SDB/20.)
      SUMM=0. $ ZZEND=0.0
      DO 50 L=1,20
      BDB=-160.+10.*(L-1)
      CDB=BDB+10.
      B=10.**(BDB/20.)
      C=10.**(CDB/20.)
      DX=(C-B)/6.
      DSUM(1)=DSUM(2)=DSUM(3)=DSUM(4)=0.0
      DO 40 M=1,4
      X=B+MM(M)*DX
      XP=20.*ALOG10(X)/SIGMA2
      FX=(3.465213/(X*SIGMA2))*EXP(-0.5*XP*XP)
      SUM=0.0 $ ZEND=0.0
      DO 30 L1=1,20
      EDB=-160.+10.*(L1-1)
      FDB=EDB+10.
      E=10.**(EDB/20.)
      F=10.**(FDB/20.)
      DY=(F-E)/6.
      DO 20 M1=1,4
      Y=E+MM(M1)*DY
      IF(M.NE.1.OR.L.NE.1) GO TO 15
      YP=20.*ALOG10(Y)/SIGMA1
      FY(L1,M1)=(3.465213/(Y*SIGMA1))*EXP(-0.5*YP*YP)
   15 FF=EXP(-S*S/(1.4426*(A1*Y*Y+A2*X*X)))
      Z(M1)=FF*FY(L1,M1)*FX
   20 CONTINUE
```

```
      SS=3.*DY*(W(1)*ZEND+W(2)*Z(1)+W(3)*Z(2)+W(4)*Z(3)+W(5)*Z(4))
      ZEND=Z(4)
      SUM=SUM+SS
   30 CONTINUE
      DSUM(M)=SUM
   40 CONTINUE
      SSS=3.*DX*(W(1)*ZZEND+W(2)*DSUM(1)+W(3)*DSUM(2)+
     1W(4)*DSUM(3)+W(5)*DSUM(4))
      ZZEND=DSUM(4)
      SUMM=SUMM+SSS
   50 CONTINUE
C     CHANGE EXCEEDANCE PROBABILITIES TO PERCENTAGES
      SUMMP=SUMM*100.
      PRINT 8, SDB,SUMMP
   60 CONTINUE
    7 FORMAT(2X,4(F5.1),/)
    8 FORMAT(5X,F6.1,2X,1PE9.3)
      END
```

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In the MF band, interfering signals from one or more broadcast stations can be received. During nighttime periods, this interference is caused by fading signals, propagated from distances of more than about 200 miles. It is the purpose of this report to provide the techniques needed to properly determine the degree of interference from one or more distant broadcasting stations. Computer algorithms are developed which determine the actual overall fading distribution of one or a combination of two fading signals, when the short-term fading of each is represented by a Rayleigh distribution and the long-term fading of median values is given by a log-normal distribution. Also, a simple approximate means of determining the distribution of the median value of a signal given by the sum of any number of fading signals is given. This approximate method is accurate only for the smaller percentage points. It is found that the techniques described here lead to somewhat higher values for the sum of the signals than previous methods.

16. Key Words *(Alphabetical order, separated by semicolons)*
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