NTIA Report 83-139

The Effect of Bandwidth and Interference Rejection on the Spectrum Efficiency of Land Mobile Radio Systems

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for Communications and Information

December 1983

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

THE EFFECT OF BANDWIDTH AND INTERFERENCE REJECTION ON THE SPECTRUM EFFICIENCY OF LAND MOBILE RADIO SYSTEMS

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Analytical and empirical results concerning the relative spectrum efficiency of two generic (wideband and narrowband) land mobile radio systems are presented. Graph theoretic frequency assignment techniques are used to relate spectrum usage to transmitter bandwidth, interference rejection characteristics, transmitter locations, frequency assignment techniques, and other system and deployment characteristics. A curve-fit equation is presented for estimating the amount of spectrum needed to assign frequencies to transmitters of both types when they are randomly located in a square geographical area. Spectrum usage when transmitters are located at preferred sites or clustered near city centers is also discussed. In most cases considered the narrowband systems use less spectrum than the wideband systems even though the narrowband systems require greater protection from cochannel interference.

Key words: spectrum management; frequency assignment; graph theory; spectrum efficiency

1. INTRODUCTION

The total number of land mobile radio systems that can operate successfully in a given geographical area and frequency band is a complicated function of the system characteristics, the deployment plan, and the operational procedures. In this report, we study the effect of channel bandwidth, interference resistance, and base station location on the total number of systems that can be assigned in an area.

Our approach is to compare the total bandwidth required by a fixed number of two generic systems - a "wideband" system and a "narrowband" system - under different deployments. The interference rejection characteristics of the systems are represented by frequency-distance separation rules, and the systems are assigned channels with a computer program that minimizes the total bandwidth required to assign all systems.

It is assumed that inter-system interference is prevented by imposing frequency-distance rules on the base stations. In practice, these rules are

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*The authors are with the Institute for Telecommunication Sciences, National Telecommunications and Information Administration, U.S. Department of Commerce, Boulder, CO 80303 determined by relating propagation loss, required signal-to-interference ratio, and the receiver filter characteristics. Table 1 shows the frequency-distance rules for the two generic systems used in this study. The narrowband system is assumed to require greater protection from cochannel interference (less processing gain available) so its cochannel separation distance is greater than that of the wideband system.

TABLE 1. FREQUENCY-DISTANCE SEPARATION RULES FOR TWO GENERIC LAND MOBILE RADIO SYSTEMS

W: Wideband Systems - 12.5 kHz Channel Spacing
N: Narrowband Systems - 5 kHz Channel Spacing

Interaction		Frequency Separation (kHz)	Channel Separation	Distance Separation (mi)
1	W-W	0	0	110
2	W-W	12.5	1	35
3	W-W	25	2	1
4	N-N	0	0	155
5	N-N	5	1	8
6	N-N	10	2	4
7	N-N	15	3	2
8	N-N	20	4	1
9	N-N	25	5	1
10	W-N, N-W	0	0	130
11	W-N, N-W	12.5	1	8
12	W-N, N-W	25	2	1

The channels for the wideband system are nominally 25 kHz, but assignments are allowed every 12.5 kHz, so the wideband systems need greater adjacent channel protection than the narrowband systems. However, the narrowband systems need to be protected for more channels on each side of the assigned channel.

Cross system frequency-distance rules (Table 1, 10-12) are defined for scenarios in which both systems are deployed in the same band and area.

Both analytical and empirical approaches have been used. Most of the data presented and discussed in this report were generated for this study. Relevant data from previous studies are also presented. To be consistent with other studies and published reports we use the term "transmitter" when referring to a land mobile system. For this study a land mobile system consists of a base station and mobile radios, all of which use the same channel for both transmitting and receiving. For purposes of applying the frequency-distance rules, the location of a system is considered to be the location of the base station.

The empirical results come from using a frequency assignment computer program. Hale (1981) describes early versions of the frequency assignment algorithms implemented in the computer program. The current frequency assignment algorithms are implemented in a computer program described by Cronin and Berry (A User's Guide to the Frequency Assignement Computer Program ASIGN, to be published as an NTIA Technical Report in 1984). The program uses several modified graph coloring algorithms to assign frequencies to transmitters given the frequency constraints between transmitters. The constraints consist of forbidden channel separations restricting the frequencies that can be assigned to transmitters in order to prevent interference. For studies such as this the program also generates transmitter locations and applies frequency-distance rules to produce the frequency constraints between transmitters.

The effects on spectrum usage of various distributions of the transmitters are discussed. Results are presented for transmitters uniformly distributed in a square, exponentially distributed about several city centers, and transmitters colocated at preferred locations. The dependence of spectrum usage on frequency-distance rules, the sizes of geographical areas, and frequency assignment algorithms in these cases is also discussed.

Frequency assignment problems typically involve a frequency band of finite width. The difficulty of measuring the saturation point of a frequency band is discussed. The size of the largest clique formed by transmitters is presented as a useful indication of saturation for this study.

A method of interleaving wideband and narrowband transmitters in one frequency band is investigated. This restricts the frequencies that can be assigned to transmitters. These additional restrictions may cause the spectrum to be used with less efficiency than possible with separate, adjacent frequency bands for each type of transmitter. The effects of interleaving transmitters are presented and discussed.

2. BASE STATIONS UNIFORMLY DISTRIBUTED IN A SQUARE

2.1 The Calculation Procedure

The effects of different system characteristics on spectrum usage are determined by repeated calculations of the required bandwidth for 10 transmitter sets for each set of characteristics. The transmitters in a transmitter set are located in a square in the plane. The x and y coordinates of each transmitter location are generated by a random number generator. The system characteristics include the frequency-distance rules, the number of transmitters, and various parameters concerning the distribution of the transmitters. For the data discussed in this section the x and y coordinates of the transmitter locations were generated by a random number generator using a uniform probability distribution. Each frequency assignment problem consists of one set of characteristics and one transmitter set.

For each problem 14 suboptimal frequency assignment algorithms were used to assign frequencies. Thirteen of the 14 algorithms are generalizations of heuristic graph coloring algorithms. They assign frequencies using a small amount of spectrum, perhaps the least possible amount of spectrum. These algorithms choose the order in which transmitters are assigned frequencies and the frequencies assigned. One of the algorithms simulates a typical method of assigning frequencies and is used for comparison with the graph theoretic methods. This is called the random order algorithm. Transmitters are assigned frequencies in an arbitrary order. This simulates the typical order of assignments used by administrations, i.e., frequencies are assigned when applicants apply for them.

The amount of spectrum used to assign frequencies to all transmitters of a frequency assignment problem is called the span of the assignment. The 14 frequency assignment algorithms produce 14 frequency assignments. One or more of the assignments use the least amount of span and this is called the minimum span. The minimum span is also called the required span or the required bandwidth. Each data value compiled for this study is the average minimum span for all 10 transmitter sets for each set of system characteristics.

2.2 The Effect of Adjacent Channel Separations

The adjacent channel separation distances are necessary because the off-frequency rejection of the receivers is not perfect, nor is the frequency stability of the transmitters. Berry and Cronin (1983) have studied the effects of such required separations on the span of frequency assignments. Figure 1 shows the percentage increase in frequency span required for frequency assignments to transmitters that are randomly located in a square when the x and y coordinates of the locations are uniformly distributed. The increase is given as a function of D(1)/D(0), the ratio of the first adjacent channel separation distance to the cochannel separation distance. Figure 1 shows that very little additional span is needed if D(1)/D(0) is less than 0.4. The first adjacent channel separation distances in Table 1 are smaller than this, so they add very little to the required span when the transmitter locations are uniformly distributed.

Figure 2 shows the additional span needed when there is a second adjacent channel distance separation in addition to a first adjacent channel distance separation. In this figure D(2)/D(0) is the ratio of the second adjacent channel distance to the cochannel distance. Provided that D(2)/D(0) is less than 0.4, little additional span is required. So the second adjacent channel separation distances in Table 1 do not change the required span very much, provided that the transmitter locations are random and uniformly distributed. Similar results can be expected for the third, fourth, and fifth adjacent channel distances of the narrowband transmitter frequency-distance rules. The effect of these rules when the transmitters are grouped at favored locations is considered in Section 3.

2.3 Spectrum Use Computed for Various Cases

Table 2 shows the average span required for uniformly distributed and random transmitter locations within a square for a number of different cases. Each entry in the table is the average minimum frequency span (in number of channels) over 10 transmitter sets. The average size of the largest initial clique is also listed. These data are tabulated as a function of two variables: the number of transmitters (N), and the ratio of the cochannel distance to the length of the side of the square in which the transmitters are located (R).

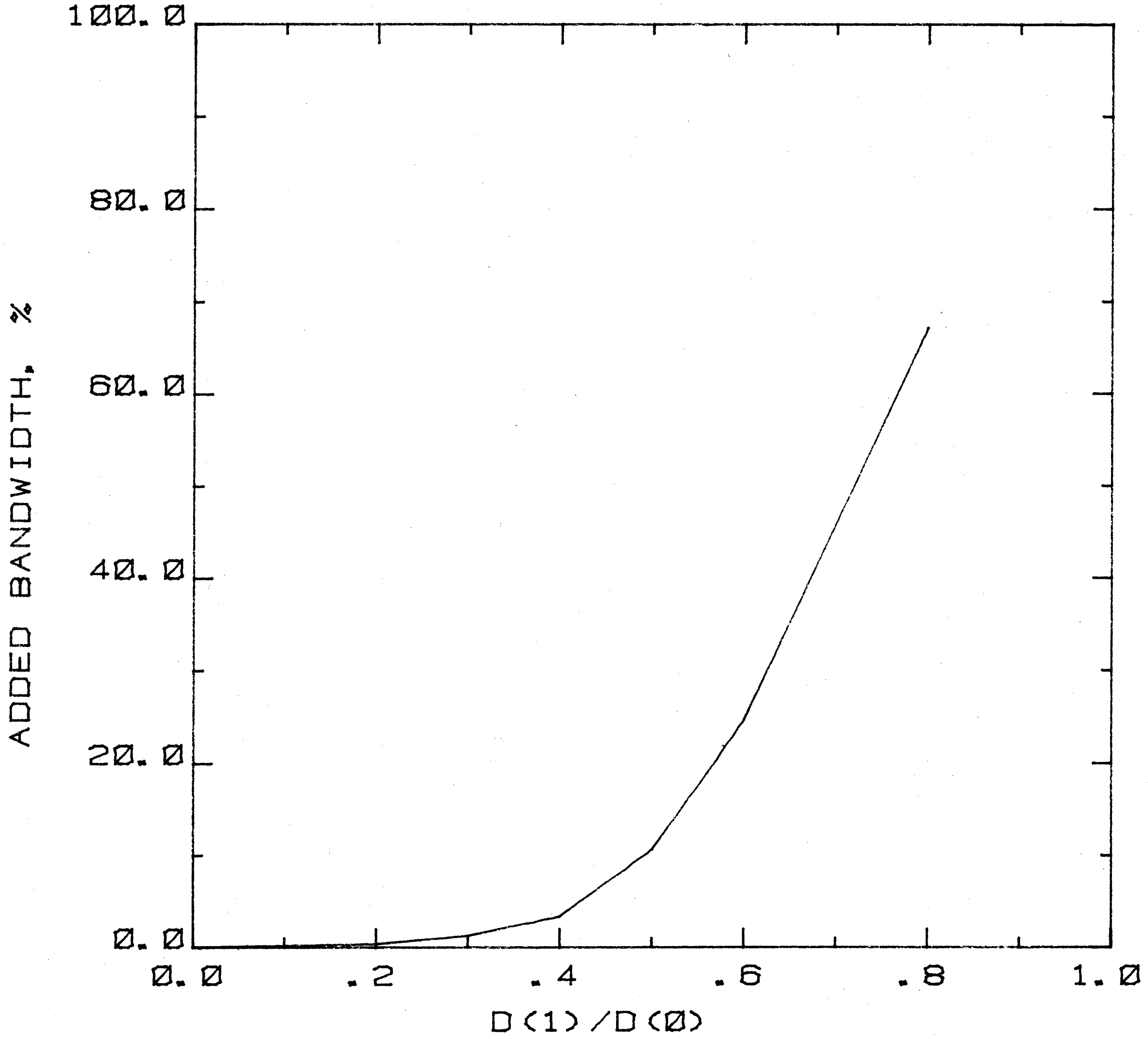

Figure 1. Percentage increase in total bandwidth needed to assign frequencies to all transmitters when an adjacent channel distance separation is required. The horizontal axis is the ratio of the required adjacent channel separation distance to the cochannel separation distance.

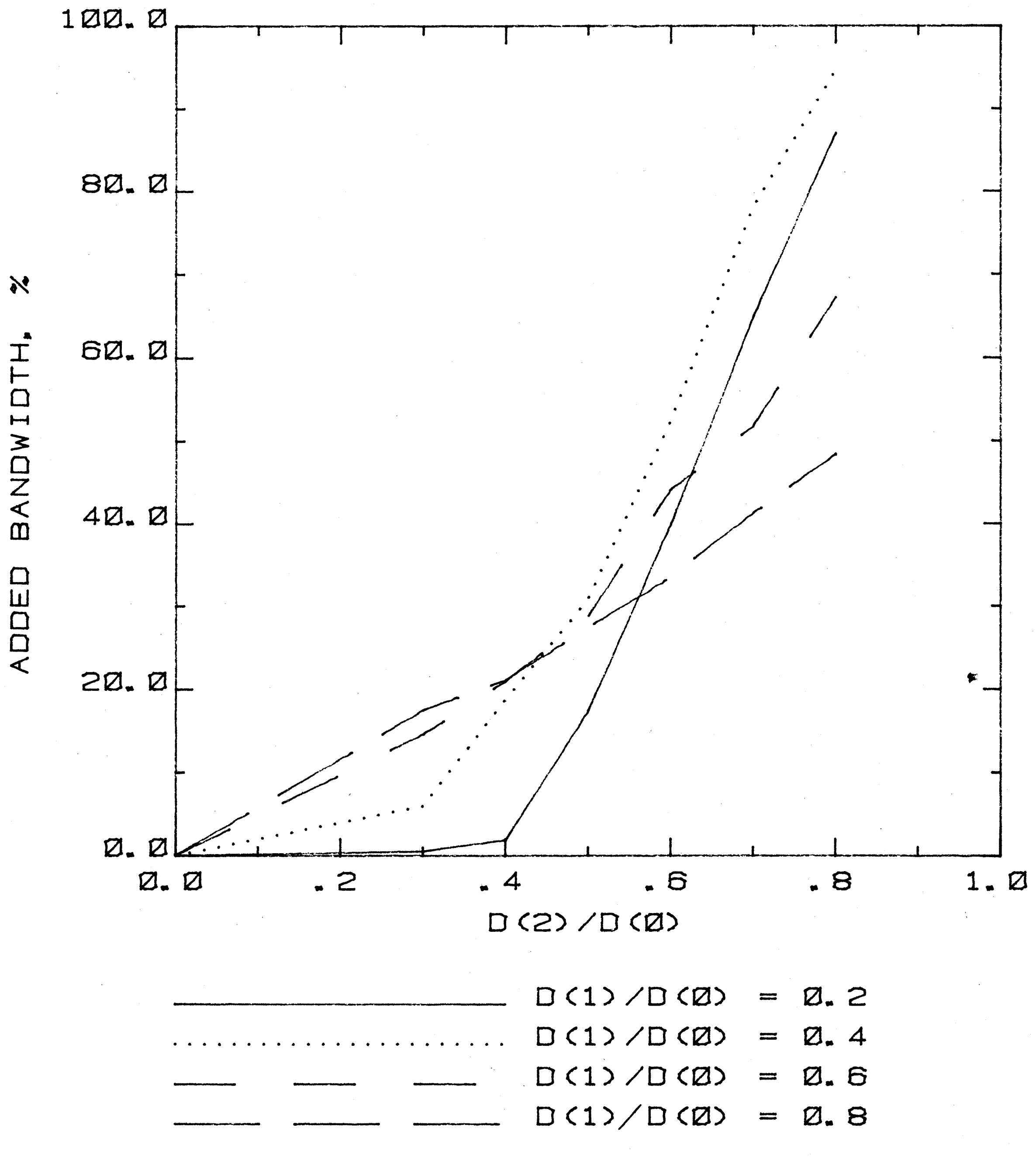

Figure 2. Percentage increase in total bandwidth needed to assign frequencies to all transmitters when a second adjacent channel distance separation is required. The increase is relative to the bandwidth needed when the adjacent channel separation distance is that shown in the legend.

TABLE 2. REQUIRED SPAN TO ASSIGN N BASE STATIONS IN A SQUARE WHEN

R = (cochannel separation distance)/(square side)
S = Average minimum span (channels)
C = Average size of largest initial clique (stations)

	N									
	50		100		200		300		400	
R	S	C	S	C	S	C	S	C	S	C
.0800	3.0	2.8	4.9	4.8	6.6	6.5	8.2	8.1	9.6	9.6
.1375									16.2	15.6
.1692	5.2	5.1	8.4	8.3	13.0	12.5	17.4	17.2	24.4	20.2
.2500									36.4	35.6
.2750									40.5	39.7
.3333	12.0	11.8	17.8	17.5	31.0	30.1	42.9	41.4	54.3	51.7
.3875									69.0	63.4
.4151									75.6	71.0
.5000	17.3	17.2	30.2	30.0	55.1	53.9	81.8	77.7	103.0	97.9
.5500									123.4	115.5
.7143	28.1	27.8	51.8	50.2	97.9	96.0	143.4	138.8		

Nine of the table entries were taken from a previous study in which the frequency distance rules consisted of only a cochannel distance. The remaining 21 entries were generated by using the frequency-distance rules described for the wideband transmitters.

A clique is a set of transmitters each of which imposes frequency constraints on all other transmitters in the clique. The size of a clique is the number of transmitters belonging to it. The constraints may be only cochannel constraints or they may be cochannel and adjacent channel constraints. An optimal frequency assignment for the transmitters in a clique assigns a different channel to each transmitter. The spectrum occupied by each transmitter in a clique must not be occupied by any other transmitter in the clique because of the cochannel constraints between transmitters. So a lower bound on the span (in number of channels) needed for an optimal frequency assignment for transmitters in a clique is the number of transmitters in the clique.

Each algorithm chooses the order in which transmitters are assigned frequencies. The first several transmitters assigned frequencies form an easily recognizable clique. This is called the initial clique. The size of the largest initial clique found by one of the 14 algorithms provides a lower bound on the frequency span. For frequency assignment problems involving only cochannel constraints, this lower bound has been found to be within 3.3% of the span, on average. When the span is equal to the largest initial clique, the frequency assignment is optimum. The data in Table 2 shows that the average size of the largest initial clique is very close to the average minimum span for the table entries obtained by using the wideband frequency-distance rules. This information together with the discussion in Section 2.2 indicates that the data from the previous study can be used for this study because the average minimum span values would not change significantly if it were obtained using the wideband frequency-distance rules. For the same reasons this data can be used for studying the spectrum usage of narrowband transmitters. The significant distance separation is the cochannel distance separation and the significant parameter is the ratio of the cochannel distance to the length of the side of the square in which the transmitters are located.

The data in Table 2 can be used to answer spectrum management questions.

Example 1:

Find the average minimum number of channels needed to assign frequencies to 400 wideband transmitters uniformly distributed in a square 400 mi X 400 mi. The ratio of the cochannel distance to the length of the side of the square is R = 110/400 = .275. The table entry under the 400 transmitter column for R = .275 shows 40.5 channels. Each wideband channel occupies 12.5 kHz, for a total of 512.5 kHz for 41 channels.

Example 2:

Find the average minimum number of channels needed to assign frequencies to 400 narrowband transmitters uniformly distributed in a square 400 mi X 400 mi. For this case, R = 155/400 = .3875. The table entry for N = 400 and R = .3875 shows 69.0 channels. Each narrowband channel occupies 5 kHz, for a total of 345 kHz.

2.4 Empirical Formula for the Required Span

The data in Table 2 were curve-fit using a polynomial least squares curve-fitting routine. The formula (to three significant digits) for the number of channels S is:

$$S = 4.19 + N(.140\, R + .722\, R^2).$$

Table 3 shows the absolute difference between the curve-fit value and the value shown in Table 2 and the difference as a percentage of the Table 2 value for all N and R values. The average absolute difference is .95 channels; the average percent difference is 6.9%. The five entries for which R = .08 account for much of the average percentage error although the absolute errors are small. For this value of R and a cochannel distance of 110 mi the square size is very large, 1375 mi X 1375 mi. For a cochannel distance of 155 mi the square size is 1937 mi X 1937 mi. For the other 25 entries, the average percentage error is 3.7%. The curve-fit equation can be used reliably for interpolating between the data values in Table 2 and for some limited extrapolation from those values.

TABLE 3. DIFFERENCE BETWEEN EMPIRICAL FORMULA FOR REQUIRED SPAN AND COMPUTED RESULTS IN TABLE 2

D = Absolute difference (channels)
% = Percent difference

	N									
	50		100		200		300		400	
R	D	%	D	%	D	%	D	%	D	%
.0800	2.0	66.1	.88	17.9	.76	11.5	.74	9.0	.92	9.6
.1375									1.2	7.1
.1692	1.2	23.3	.23	2.7	.07	.5	.10	.6	2.5	10.1
.2500									.15	.4
.2750									.94	2.3
.3333	1.5	12.2	.92	5.2	1.4	4.6	.64	1.5	.65	1.2
.3875									.26	.4
.4151									1.6	2.1
.5000	.59	3.4	.96	3.2	.81	1.5	2.5	3.0	1.4	1.4
.5500									1.0	.8
.7143	.49	1.7	.77	1.5	.03	0.	1.3	.9		

Example 3:

Find the number of wideband transmitters uniformly and randomly distributed in a square 330 mi X 330 mi that can be assigned frequencies in a frequency band of 500 kHz. Wideband channels are 12.5 kHz wide, so 500 kHz is 40 channels. Solve the curve-fit equation for N, $N = (S-4.19)/(.140\,R + .722\,R^2)$. Then R = 110/330 = .333, and S = 40. So the approximate number of transmitters is 283.

The amount of spectrum, in kHz, needed for a frequency assignment for N narrowband transmitters in a square whose side is M mi is:

$$5\ (\text{number of channels}) = 21.0 + N(108.58/M + 86730.38/M^2)\ \text{kHz}.$$

Similarly, the amount of spectrum needed for N wideband transmitters is:

$$12.5\ (\text{number of channels}) = 52.4 + N(192.5/M + 109202.5/M^2)\ \text{kHz}.$$

The wideband transmitters always use more spectrum because the bandwidth is more than twice that of narrowband transmitters. The wideband cochannel distance is smaller and, although the first adjacent channel distance is much larger, adjacent channel constraints have little effect on the required frequency span.

2.5 Qualifications and Limitations

The data and the curve-fit discussed above must be qualified in several ways. The transmitter locations are uniformly (but randomly) distributed for all problems. The average minimum frequency span would be different if this distribution were different. This is particularly true if many transmitters are located in preferred locations (see Section 3). Since the adjacent channel distances are relatively small for both wideband and narrowband transmitter frequency-distance rules and the transmitters are uniformly distributed, there is not a significant number of adjacent channel constraints between transmitters. Thus, the frequency span is determined by the cochannel constraints. The adjacent channel constraints could easily be the determining characteristic of the required frequency spans for transmitters located at preferred sites. Similar comments apply to the narrowband transmitter locations.

The adjacent channel constraints also determine the frequency span when the transmitter locations are uniformly distributed in a square that is so small that there are many adjacent channel constraints between transmitters. When R is greater than $\sqrt{2}$, all transmitters impose cochannel constraints on all other transmitters. So the frequency span, in channels, is at least n, the number of transmitters. Reducing the square size even more can not increase the number of cochannel constraints but the number of adjacent channel constraints will increase. All wideband transmitters in a 25 mi X 25 mi square are within the cochannel and first adjacent channel distance of each other. So at least 2n-1 channels are needed for a frequency assignment for n transmitters.

For larger squares there may be few adjacent channel constraints but the large numbers of cochannel constraints still dictate a large frequency span. All wideband transmitters in a 50 mi X 50 mi square are within the cochannel distance of each other. So at least n channels are needed for a frequency assignment for n transmitters. All narrowband transmitters in squares whose sides are less than 100 mi are within the cochannel distance of each other. At least n 5 kHz channels are needed for a frequency assignment for n transmitters in these sizes of squares.

The curve-fit of the data in Table 2 shows the average minimum span to be a linear function of N. With enough first adjacent channel constraints the slope of this linear dependence on N would double. If there are significant numbers of other adjacent channel constraints the average minimum span would increase at a greater rate as a function of N.

The frequency span is very dependent on the algorithm used to assign frequencies. For this study the minimum span for any one problem is the minimum achieved by one or more of 14 algorithms. Each algorithm chooses the order in which the transmitters are assigned frequencies and the frequencies assigned. The random order algorithm assigns frequencies to transmitters in the order in which the transmitters were generated by the random number generator. The frequency assigned to each transmitter is the smallest acceptable frequency. The first adjacent channel distance for the wideband transmitter frequency-distance rules is a large enough fraction of the cochannel distance to be significant for the random order algorithm. The random order algorithm produces frequency assignments that use approximately 17% more spectrum than the best of the other algorithms. With relatively small adjacent channel distances, the percentage increase in spectrum usage when using the random order algorithm is approximately 11% for narrowband transmitters.

The average size of the largest initial clique is very close to the average minimum span for these problems. So the best (smallest span) frequency assignments are optimal or very close to optimal. If frequencies are assigned using methods that are not so close to optimal, the data in Table 2 must be modified to show more required channels.

2.6 Saturation of Limited Allocation Bands

A useful statistic for comparing the wideband and narrowband radio systems is the number of transmitters of each type that can be assigned frequencies in a frequency band of fixed size. This "saturation point" of a frequency band is difficult to define and difficult to measure. Very broadly, a band is considered saturated when it becomes impossible to find acceptable frequencies for applicants who wish to transmit. Clearly, the size of the frequency band is not the only factor that determines saturation. The transmitter characteristics, the transmitter locations, and other factors are also important. In this section the difficulty of measuring saturation is demonstrated by discussing the merits of some calculable and related statistics.

The program counts the number of transmitters that cannot be assigned frequencies within a given frequency band for a particular frequency assignment problem. This is not a reliable measure of saturation since, as discussed above, the number of transmitters that can be assigned frequencies within a frequency band depends very much on the distribution of the transmitters in the geographical area and on the algorithm used to assign frequencies. The random order algorithm produces frequency assignments that use the spectrum less efficiently than assignments produced by other algorithms. So the random order algorithm would reject more transmitters than other algorithms and the first transmitter rejected would be earlier in the assignment order than the first rejection of more optimal algorithms.

Further, the rejection of n transmitters does not necessarily imply that these transmitters cannot be assigned frequencies within the allowed frequency band without violating the frequency-distance rules. There may be one transmitter that imposes many frequency constraints on these n transmitters. By removing this one transmitter it may be possible to assign frequencies to the n transmitters within the allowed frequency band since there would then be fewer frequency constraints imposed upon them. The count of the number of transmitters that cannot be assigned frequencies within a frequency band should be considered an error indicator when it is not zero. It indicates only that the allowed frequency band is too small and does not necessarily indicate the best action to be taken to correct the error.

Zoellner (1973) has also investigated this question. In his study the transmitters are randomly located in a 100 mi X 100 mi square using a uniform probability distribution. Frequency constraints are derived from frequency-distance rules. The cochannel distance is 30 mi, the first adjacent channel distance is 10 mi, and the second adjacent channel distance is 5 mi. These distances for this size of a square are similar enough to the wideband frequency-distance rules to allow comparison of Zoellner's data to the data of this study. The transmitters are assigned channels in random order and there are four channel selection methods: the smallest acceptable (SA), the least heavily occupied, if an occupied channel is acceptable (LHO), the random selection of any of the acceptable channels (RA), and the Max-F*D method which attempts to choose a channel that is "as far away as possible" from previously assigned transmitters. Zoellner used a fixed band of 20 channels. For each channel selection method he counted the number of transmitters that are assigned channels by the time one transmitter is found that cannot be assigned an acceptable channel. The average number of transmitters assigned before the first rejection, for each method is:

SA	103.3
LHO	81.9
RA	81.6
Max-F*D	94.4.

Using the curve-fit equation with R = 30/100 = .3 and the number of channels, S = 20, the best of the algorithms of this study will accommodate an average of 148 transmitters in a band of 20 channels with no rejected transmitters. Thus, the range of the frequency spans indicates that the "first rejection" method is not useful for measuring saturation.

Zoellner measures saturation by counting the number of transmitters that can be assigned channels after 100 transmitters in a row are rejected in a band of 20 channels. For all channel selection methods this is approximately 187 transmitters. However, the number of transmitter locations generated to reach 100 successive rejections ranges from 767 to 856 for the four channel selection methods. Significant variations in the number of transmitters assigned can be expected when much smaller numbers of successive rejections are used as a measure of the saturation point. It seems that an "nth rejection" method is not a useful measure of saturation.

Ideally, saturation is a characteristic of the generalized graph of a frequency assignment problem. The question of being able to add transmitters without requiring more frequency span can be answered by examining the graph structure if an exact frequency assignment algorithm is used. Some analysis of graph structure follows below. Inexact algorithms, however, may use more frequency span than is required by the structure of the graph. Realistically, the method of assigning frequencies is a factor to be considered in any discussion of saturation.

Table 2 shows the average sizes of the largest initial cliques for each set of assignment problems. For the problems of this study the number of transmitters in the largest initial clique is a lower bound for and a good estimate of the minimum span. The size of this clique is determined by the structure of the generalized graph and is independent of the algorithm used to assign frequencies. Since it is a good estimate of the minimum span, the relationships between the size of this clique and R and N are similar to those of the average minimum span to R and N. Also, since the transmitters are distributed uniformly over the square, it is probable that there are other cliques in the graph of the same size or nearly the same size. For R and N values Table 2 shows the average minimum span needed for a frequency assignment. If R or N is increased, the size of the largest initial clique also increases and, for this reason alone, the span must increase. In this sense the average size of the largest initial clique provides some measure of saturation.

3. NON-UNIFORM DISTRIBUTION OF BASE STATION LOCATIONS

3.1 Preferred Locations

A preferred location is a mountain top or tall building. Many base station antennas are located at preferred locations. Applying the wideband frequency-distance rules, all wideband transmitters in a preferred location are within 1 mi of each other. There are cochannel, first adjacent, and second adjacent channel constraints between each pair of transmitters. An optimum frequency assignment for all transmitters in one preferred location would assign a different channel to each transmitter and these channels would be separated by two channels. The lower bound on the frequency span for any problem in which wideband transmitters are located in preferred locations is 3n-2, where n is the number of transmitters at the preferred location having the largest number of transmitters. This lower bound does not change even if the preferred locations are widely distributed over a large area.

A characteristic of transmitters distributed in preferred locations is that only a few transmitters require a large span for frequency assignments. If 400 transmitters are equally distributed among 5 preferred locations, there are 80 transmitters at each location. Each preferred location of 80 transmitters needs at least 3(80)-2 = 238 channels for a channel assignment that observes the wideband frequency-distance rules. If 400 transmitters are equally distributed among 25 preferred locations, there are 16 transmitters at each location and 1 such location needs a span of 3(16)-2 = 46 channels. For comparison, the data in Table 2 show that 400 transmitters uniformly distributed in a square 400 mi X 400 mi can be assigned channels within approximately the same span needed for only 16 transmitters in a preferred location, 40.5 channels.

If two preferred locations are within the cochannel distance of each other, every transmitter in both locations must be assigned a different channel. Transmitters at each location also have first and second adjacent channel constraints with every other transmitter at that location. The channels assigned transmitters at each location must be separated by at least two channels as discussed above. If few preferred locations are within the cochannel distance of each other, the channels assigned can be staggered so that the span required is not much more than that required for the transmitters at one location. This may also be possible if some preferred locations are within the first adjacent channel distance of each other. If preferred locations are not sparsely distributed over a large area there would be many frequency constraints between transmitters located in different preferred locations. In such situations a small change in the size of the square, the cochannel distance, the number of preferred locations, or the number of transmitters in a preferred location can cause a large change in the required frequency span. By contrast, if transmitters are distributed uniformly, small changes in these parameters produce small changes in the required frequency span.

Similar comments apply to narrowband transmitters located in preferred locations. Every pair of narrowband transmitters in a preferred location has cochannel, first, second, third, fourth, and fifth adjacent channel constraints between them. So if n narrowband transmitters are located in one preferred location, a frequency span of at least 6n-5 channels is needed for a frequency assignment that does not violate the frequency-distance rules. So the number of channels needed for a frequency assignment to n narrowband transmitters in a preferred location is approximately twice the number of channels needed for n wideband transmitters.

Since the wideband transmitter channel width is more than twice the channel width of narrowband transmitters, the wideband transmitters use more spectrum when considering only one preferred location. Due to the greater cochannel distance for narrowband transmitters, it is more likely that preferred locations of narrowband transmitters will be within a cochannel distance of each other. However, it is also more likely that preferred locations of wideband transmitters will be within a first adjacent channel distance of each other because of the greater first adjacent channel distance. Generalizations are difficult to make about the comparative spectrum usage of the two types of transmitters in preferred locations. The spectrum usage is very dependent on the distribution of the preferred locations.

3.2 Base Stations Clustered Near City Centers

Many transmitter sets were generated in which several city centers are uniformly distributed in a 400 mi X 400 mi square and 400 transmitters are distributed about the city centers with an exponential probability distribution. The number of transmitters distributed about each city center decreases exponentially as the distance from the city center increases. The numbers of city centers are 10 and 25 with 40 and 16 transmitters, respectively, distributed about each center. The mean of the exponential distribution is also varied. This is the mean of the distance from the city center and approximately 63% of the transmitters distributed about each city center are within this distance of the city center. The mean is either 50, 25, or 10 miles. For each combination of the number of city centers and the mean of the exponential distribution 10 transmitter sets were generated. The wideband frequency-distance rules were applied and frequencies assigned to all transmitter sets. The narrowband frequency-distance rules were applied to all transmitter sets having 10 city centers and frequencies assigned. The computation time for these problems is relatively large and restricts the range of the parameters for which data can be generated. The average minimum frequency span and the average size of the largest initial clique for each combination of parameters is displayed in Table 4.

TABLE 4. REQUIRED SPAN FOR N BASE STATIONS CLUSTERED NEAR CITY CENTERS

S = Average minimum span (channels)
C = Average size of largest initial clique (stations)

Mean About City Centers	Number of City Centers					
	Wideband Systems				Narrowband Systems	
	10		25		10	
	S	C	S	C	S	C
50	89.0	87.4	63.5	62.9	112.7	112.5
25	96.3	94.4	80.6	79.3	132.5	131.8
10	117.0	112.0	84.2	83.7	152.1	152.1

The results in Table 4 show that the frequency span increases as the number of city centers decreases, the mean of the exponential distribution decreases, and the cochannel distance increases. When the bandwidths of the wideband and narrowband transmitters are used in computing spectrum consumed, the results show the narrowband transmitters using less spectrum for the cases that are directly comparable (Table 5).

TABLE 5. SPECTRUM CONSUMED BY BASE STATIONS CLUSTERED NEAR TEN CITY CENTERS

Mean About City Centers	Average Minimum Span (kHz)	
	Wideband Systems	Narrowband Systems
50	1112.5	563.5
25	1203.75	662.5
10	1462.5	760.5

For all combinations of the parameters the average minimum frequency span is very close to the average size of the largest initial clique. The span is close to the span needed if all adjacent channel distances of the frequency-distance rules were 0 mi. The adjacent channel distances of both the wideband and narrowband frequency-distance rules have little effect on the required frequency span.

Over the 10 transmitter sets used to obtain the data values in Table 2 the range of the frequency span is approximately 12% of the average minimum span. When the transmitters are clustered near city centers this percentage over 10 transmitter sets can be 4 or 5 times as much. The small numbers of city centers located in a large square and the small numbers of transmitters clustered about the city centers account for this wide range. Ten city clustered about the city centers account for this wide range. Ten city centers can be located in a 400 mi X 400 mi square so that none are within a cochannel distance of each other, using the wideband frequency-distance rules. In this case the frequency span is largely dependent on the span needed for the frequency assignment to the transmitters of the one city where the transmitters impose the most constraints upon each other. If some city centers are within a cochannel distance of each other, the required frequency span increases, perhaps by a large amount. When the mean of the exponential distribution is small, this situation is similar to that of preferred locations and the comments of the previous section apply.

4. INTERLEAVED SERVICES WITH ALTERNATE CHANNEL ASSIGNMENTS AND A FIXED CHANNEL WIDTH

Consider a service in which frequency assignments are restricted so that wideband transmitters and narrowband transmitters are assigned alternate 12.5 kHz channels. That is, assume that wideband transmitters are assigned only on even increments of 12.5 kHz and narrowband transmitters are assigned only on odd increments of 12.5 kHz. By this arrangement alone many of the potential frequency constraints between transmitters are observed. Since wideband transmitters will not be assigned the same channel as narrowband transmitters and since each type of transmitter must have transmitters of the same type assigned to its second adjacent channel, cochannel and second adjacent channel constraints between wideband and narrowband transmitters are observed. Since each wideband transmitter must have narrowband transmitters assigned to its first adjacent channels, first adjacent channel constraints between wideband transmitters are observed. Similarly, and also since narrowband transmitters occupy only 5 kHz, first through third adjacent 5 kHz channel constraints between narrowband transmitters are observed.

The only significant distance between wideband and narrowband transmitters is the first adjacent channel distance, 8 mi (Table 1, 10-12). This is the only distance that can produce frequency constraints between wideband and narrowband transmitters that are not already observed. Between wideband and wideband transmitters the significant distances are the cochannel distance, 110 mi, and the second adjacent channel distance, 1 mi. Between narrowband and narrowband transmitters the significant distances are the cochannel distance, 155 mi, and the fourth and fifth adjacent channel distances, 1 mi.

If transmitter locations are randomly distributed using a uniform probability distribution, the adjacent channel distances discussed above will typically have little effect on the required frequency span because they are so small (see Section 2.2). There are very few transmitters within 8 mi of each other except in the most dense environments. The cochannel constraints between transmitters are the determining factor for any frequency assignment. So the interactions between wideband and narrowband transmitters can be ignored for estimating the required frequency span for a frequency assignment. As a result, the questions concerning the wideband transmitter frequency assignments can be discussed without regard for the narrowband transmitters. The alternating channel assignment restriction eliminates significant interactions between wideband and narrowband transmitters. Similarly, the questions concerning narrowband transmitter frequency assignments can be discussed without regard for the wideband transmitters.

If there is a fixed band of 150 kHz and each wideband and narrowband transmitter occupies 12.5 kHz, then there are 12 channels, 6 for each type of transmitter. Table 2 and the curve-fit equation in Section 2.4 can be used to determine the number of transmitters in a particular geographical area that can be assigned channels within a band of 6 channels. From Table 2 it is evident that a band of 6 channels allows only sparsely distributed transmitters with cochannel distances of 110 mi and 155 mi. The R = .08 and N = 200 entry shows that 6.6 channels are needed to assign 200 wideband transmitters randomly distributed in a square 1375 mi X 1375 mi and 200 narrowband transmitters in a square 1938 mi X 1938 mi.

Now consider interleaved services in preferred locations. If any wideband transmitters are located in preferred locations there are cochannel, first adjacent, and second adjacent channel constraints between any two transmitters at that location. An optimum channel assignment would have every third channel assigned to one of these transmitters. The required span for any one location is 3n-2 channels, where n is the number of transmitters at that location. However, the third channel from any channel assigned to a wideband transmitter is reserved for narrowband transmitters. So channels assigned to colocated wideband transmitters must be separated by at least 4 channels. With a frequency band of 12 channels this restricts preferred locations to include no more than 3 wideband transmitters.

Similarly, there are cochannel and first through fifth adjacent channel constraints between each pair of colocated narrowband transmitters. Assume that each narrowband channel occupies 5 kHz and, also, restrict narrowband transmitters to be assigned center frequencies that are multiples of 12.5 kHz. Then the fifth adjacent 5 kHz channel is within the second adjacent 12.5 kHz channel. So colocated narrowband transmitters must be assigned channels that are separated by at least 3 channels. Each third channel from every channel assigned to a narrowband transmitter is reserved for a wideband transmitter. So colocated narrowband transmitters must also be assigned channels that are separated by at least 4 channels and preferred locations of narrowband transmitters must contain no more than 3 transmitters if the frequency band is 150 kHz wide.

Wideband and narrowband transmitters in preferred locations have cochannel, first adjacent, and second adjacent channel constraints between each pair of a wideband transmitter and a narrowband transmitter. So channels assigned must be separated by at least three channels. Since the third adjacent channel of one type of transmitter is reserved for a transmitter of the other type, this separation need not be increased.

In general, transmitters in preferred locations need a large span for only a few transmitters. For interleaved transmitters the span required may be larger than necessitated by the frequency-distance rules because of the restrictions limiting wideband and narrowband transmitters to be assigned alternating channels.

5. SUMMARY AND CONCLUSIONS

Analytical and empirical results concerning the spectrum efficiency of the wideband and narrowband transmitters described in Table 1 are presented and discussed. The required frequency span for various numbers of transmitters, sizes of squares in which the transmitters are located, distributions of the transmitters, and the two frequency-distance rules were obtained using the frequency assignment computer program. The data for uniformly distributed transmitters were curve-fit and the curve-fit equation shows how problem parameters are related when base station locations have a random, uniform distribution.

The curve-fit equation indicates that of the two postulated land mobile systems, the narrowband systems that are uniformly distributed in large enough squares always require less spectrum than uniformly distributed wideband systems. Over the range of parameters tested and described in Section 3.2, narrowband systems also use less spectrum than wideband systems when base stations are exponentially distributed about city centers.

The required frequency span of a frequency assignment depends on the frequency assignment algorithm and the distribution of the transmitters in addition to the number of transmitters, the frequency-distance rules, and the size of the geographical area in which the transmitters are located. When the transmitters are uniformly distributed in large enough squares or exponentially distributed about city centers as described in Section 3.2, the adjacent channel distances of both the wideband and narrowband frequency-distance rules have little effect on the required frequency span when graph theoretic frequency assignment techniques are used. If transmitters are assigned frequencies using a random order algorithm, more spectrum will be used than is required. This increase in spectrum usage is larger for wideband transmitters because there are more first adjacent channel constraints between wideband transmitters than there are between narrowband transmitters due to the larger first adjacent channel distance of the wideband frequency-distance rules. The order in which transmitters are assigned frequencies becomes more significant as the frequency constraints between transmitters become more varied and complex.

The frequency span required for transmitters located in preferred locations is much greater that that required for uniformly distributed transmitters and transmitters exponentially distributed about city centers because of the effects of even very small adjacent channel distances. Narrowband transmitters located at one preferred location use less spectrum than the same number of wideband transmitters at one preferred location because the channel width of narrowband transmitters is less than half the channel width of wideband transmitters. For several preferred locations the required frequency span depends on the distribution of those locations. Preferred locations of narrowband transmitters are more likely to be within a cochannel distance of each other but are less likely to be within an adjacent channel distance of each other when compared to wideband transmitters. As the number of frequency constraints increases, the required frequency span increases.

Interleaving narrowband and wideband transmitters as described in Section 4 is not more spectrum efficient than segregating the transmitters into adjacent frequency bands if the transmitters are distributed in such a way that there are few adjacent channel constraints between narrowband transmitters and wideband transmitters. The restriction of assigning narrowband and wideband transmitters to alternate channels and disallowing the sharing of channels ensures that any cochannel constraints between narrowband transmitters and wideband transmitters are observed. First adjacent channel constraints between transmitters of the same type are also observed by this restriction. If there are few adjacent channel constraints, the cochannel constraints between transmitters of the same type determine the required frequency span.

The problem of judging when a frequency band is saturated was discussed. Further analysis of generalized graphs and frequency assignment techniques is needed to be able to adequately define and measure the saturation point of a band. Arguments were presented to support the use of the size of the largest initial clique as useful data for indicating saturation for this study.

6. ACKNOWLEDGMENT

We thank Gerald Hurt of NTIA's Office of Spectrum Management for suggesting this investigation and helping define the bandwidths and frequency-distance rules for the wideband and narrowband systems.

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FORM **NTIA-29**
(4-80)

U.S. DEPARTMENT OF COMMERCE
NAT'L. TELECOMMUNICATIONS AND INFORMATION ADMINISTRATION

BIBLIOGRAPHIC DATA SHEET

1. PUBLICATION NO.
NTIA Report 83-139

2. Gov't Accession No.

3. Recipient's Accession No.

4. TITLE AND SUBTITLE
The Effect of Bandwidth and Interference Rejection on the Spectrum Efficiency of Land Mobile Radio Systems

5. Publication Date
December 1983

6. Performing Organization Code
NTIA/ITS.S4

7. AUTHOR(S)
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8. PERFORMING ORGANIZATION NAME AND ADDRESS
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325 Broadway
Boulder, CO 80303

9. Project/Task/Work Unit No.
9101123

10. Contract/Grant No.

11. Sponsoring Organization Name and Address
NTIA
Herbert C. Hoover Bldg.
14th & Constitution Ave., NW
Washington, DC, 20230

12. Type of Report and Period Covered

13.

14. SUPPLEMENTARY NOTES

15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Analytical and empirical results concerning the relative spectrum efficiency of two generic (wideband and narrowband) land mobile radio systems are presented. Graph theoretic frequency assignment techniques are used to relate spectrum usage to transmitter bandwidth, interference rejection characteristics, transmitter locations, frequency assignment techniques, and other system and deployment characteristics. A curve-fit equation is presented for estimating the amount of spectrum needed to assign frequencies to transmitters of both types when they are randomly located in a square geographical area. Spectrum usage when transmitters are located at preferred sites or clustered near city centers is also discussed. In most cases considered the narrowband systems use less spectrum than the wideband systems even though the narrowband systems require greater protection from cochannel interference.

16. Key Words *(Alphabetical order, separated by semicolons)*

spectrum management; frequency assignment; graph theory; spectrum efficiency

17. AVAILABILITY STATEMENT

☒ UNLIMITED.

☐ FOR OFFICIAL DISTRIBUTION.

18. Security Class. *(This report)*
"U"

19. Security Class. *(This page)*
"U"

20. Number of pages
23

21. Price: