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Polarization Angles of Linearly Polarized Antennas and Radio Waves in Satellite Communications

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POLARIZATION ANGLES OF LINEARLY POLARIZED ANTENNAS AND RADIO WAVES IN SATELLITE COMMUNICATIONS

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Calculating the polarization angles of antennas and radio waves is necessary for analyzing the mutual interference when a linearly polarized emission is used in satellite communications. A method for calculating the polarization angle has been developed. It is based on several coordinate systems appropriate for representing the polarization angle and on a series of coordinate transformations and of projections of vectors in coordinate planes. It is applicable to a variety of cases of practical importance. Comparisons are made with some existing methods applicable to special cases, and some numerical examples are given. A computer subroutine package that implements the developed method is also presented.

Key words: antenna, linear polarization, polarization angle, radio wave, satellite communication

1. INTRODUCTION

When a linearly polarized emission is used in satellite communications, the polarization angle of a radio wave at an earth point (a point on the surface of the earth) must be calculated for estimating the rain attenuation and cross-polarization discrimination (CCIR, 1982a, Report 564-2). A satellite transmitting antenna that produces horizontal polarization at an earth point does not necessarily produce the same polarization at another earth point (Shkarofsky and Moody, 1976; Dougherty, 1980; CCIR, 1982a, Report 814-1). Shkarofsky and Moody (1976) give an equation for approximately calculating, at an arbitrary earth point, the polarization angle of an incident radio wave from a satellite antenna that produces a horizontally polarized radio wave at the aim point (or boresight point) of the satellite antenna. Annex I to CCIR Report 814-1 (CCIR, 1982a) gives an equation for calculating the polarization angle when the meridian of the aim point is the same as that of the satellite. Dougherty (1980) gives a set of equations for calculating the polarization angle of a radio wave from a satellite antenna in a more general case.

Similarly, an earth transmitting antenna that produces a horizontally polarized radio wave in the direction of the propagation path (i.e., in a

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plane normal to the propagation path) to the desired satellite does not necessarily produce the same polarization in the direction to another satellite. When the earth transmitting antenna is not located at the aim point of the desired satellite, calculation of the polarization angle of a radio wave in the direction of the propagation path to an arbitrary satellite is more involved than in the downlink case described in the preceding paragraph.

When a desired radio wave is interfered with by an interfering (or undesired) radio wave at a receiving antenna and both the desired and interfering waves are linearly polarized, the difference between the polarization angle of the interfering wave and the polarization angle of the receiving antenna must be calculated in the direction of the propagation path of the interfering radio wave. This calculation is required for evaluating the combined effective gain of the transmitting and receiving antennas for the interfering wave (CCIR, 1982b, Sec. 6.2.10). It involves calculation of the polarization angle of a receiving antenna in the direction of the propagation path of the interfering radio wave. Calculation of the polarization angle of a receiving antenna is rather simple for an uplink case but is quite involved for a downlink case. It has turned out that such calculations for an uplink and a downlink case are mathematically equivalent to the calculations of the polarization angle of a radio wave for a downlink and an uplink case, respectively.

This report presents a method for calculating the polarization angles. The method is based on several coordinate systems appropriate for representing the polarization angles. It performs a series of coordinate transformations among the coordinate systems and of projections of vectors in coordinate planes.

The method presented in this report is applicable to either a downlink or an uplink case. It includes polarization angles relative to both the equatorial plane and the local horizontal plane. It is applicable even when the latitude of the satellite is not zero.

Basic concepts regarding the polarization angles are described in Section 2. A method for calculating the polarization angle is presented in detail in Section 3. Comparisons are made with some existing methods applicable to special cases in Section 4, and some numerical examples are given in Section 5. Cartesian coordinate systems useful for the polarization angle calculations are described in Appendix A. A Fortran subroutine package that implements the method is described in Appendix B.

2. BASIC CONCEPTS

Basic concepts regarding polarization vectors and polarization angles are established, and three-dimensional Cartesian coordinate systems convenient for describing polarization angles are outlined.

2.1. Polarization Vectors

A polarization vector is a vector parallel to the electrical vector of an antenna or a radio wave.

The polarization vector of an antenna is normal (perpendicular) to the direction of the main beam of the antenna.

The polarization vector of a radio wave at a receiving point is normal to the propagation path from the transmitting point to the receiving point. It is coplanar with the polarization vector of the transmitting antenna and the propagation path. It is therefore parallel to the projection of the polarization vector of the transmitting antenna in the direction of the propagation path (i.e., on a plane normal to the propagation path). It is parallel to the polarization vector of the transmitting antenna if, and only if, the propagation path is normal to the polarization vector of the transmitting antenna. It is defined without reference to a receiving antenna; it is not necessarily the same as the polarization vector of a receiving antenna when the antenna exists at the receiving point.

2.2. Polarization Angles

A polarization angle is the angle of the projection of a polarization vector in a specified plane, measured from a reference line in the plane. A line parallel to the equatorial plane or a line parallel to the local horizontal plane in the specified plane is generally used as the reference line. A convention is used here that the polarization angle is measured counterclockwise in the plane when the plane is looked at in the direction from the transmitting point to the receiving point. (More specifically, the polarization angle is measured counterclockwise in a plane on the opposite side to the receiving point when the plane is considered at the transmitting point, and on the same side as the transmitting point when the plane is considered at the receiving point.) Selection of the sense (or the positive direction) of the reference line is immaterial since the polarization angle is significant only in the principal value, i.e., the angle modulus 180°.

The polarization angle of an antenna is the angle of the projection of the polarization vector of the antenna in a specified plane.

The polarization angle of a radio wave at a receiving point is the angle of the polarization vector of the radio wave in the plane normal to the propagation path. It is equal to the polarization angle of the projection, in the direction of the propagation path, of the polarization vector of the transmitting antenna. The polarization angle is defined without reference to a receiving antenna; it is not necessarily equal to the polarization angle of a receiving antenna when the antenna exists at the receiving point.

2.3. Coordinate Systems

In all cases of practical importance in satellite communications, the plane of projection in which the polarization angle is measured is always normal to a path between an earth point and a satellite point. It is, therefore, convenient to consider a three-dimensional Cartesian coordinate system associated with the path between the earth and satellite points in such a way that the origin of the coordinate is the earth point, the first coordinate axis is parallel to the reference line, and the third coordinate axis points toward the satellite.

In this coordinate system, projection of a vector on the plane of projection is effected by simply equating the third component of the vector to zero. Mathematically, it is represented by multiplication, on the left side of a three-element column vector or on the right side of a row vector, of a 3 X 3 matrix called the projection matrix, (P), in which the diagonal elements of the first and second rows are equal to unity and all other elements are equal to zero.

In this coordinate system, the third component of a projected vector is zero. The polarization angle of a polarization vector is represented by the arctangent of the ratio of the second component to the first component of the polarization vector.

Two such coodinate systems useful for the polarization angle calculations are described in Appendix A. Coordinate transformation from a reference system called the earth-center coordinate system, to these two systems is also described in the appendix.

3. CALCULATION OF POLARIZATION ANGLES

In this section, the problems are defined and a method for solving the problems is presented.

3.1. Definition of the Problems

Consider a configuration shown in Figure 1. It consists of three earth points (E, A, and AE), and two satellite points (S and SE). There is an antenna at each of the five points. The path between E and S is the propagation path of our concern; the desired polarization angle of the antenna and radio wave are measured in the direction of this path, i.e., in a plane normal to this path.

Satellite point SE is the position of the satellite that serves the earth point E. The antenna at E is aligned to the antenna at SE; i.e., the main beam of the antenna at E points toward SE, and the polarization vector of the antenna at E is parallel to the projection of the polarization vector of the antenna at SE in the direction of the path between E and SE.

Earth point A is the aim point of the satellite antenna at S. The antennas at A and S are mutually aligned; i.e., the main beam of the antenna at A points toward S, the main beam of the antenna at S points toward A, and the polarization vectors of the antennas at A and S are parallel to each other. The polarization angle of the antenna at S is specified in terms of the polarization angle of the antenna at A.

Earth point AE is the aim point of the satellite antenna at SE. The antennas at AE and SE are mutually aligned. The polarization angle of the antenna at SE is specified in terms of the polarization angle of the antenna at AE.

There are two problems of our concern for the downlink case. The first problem is to calculate the polarization angle of the radio wave from S to E for a specified value of the polarization angle of the antenna at S. The second problem is to calculate the polarization angle of the antenna at E in

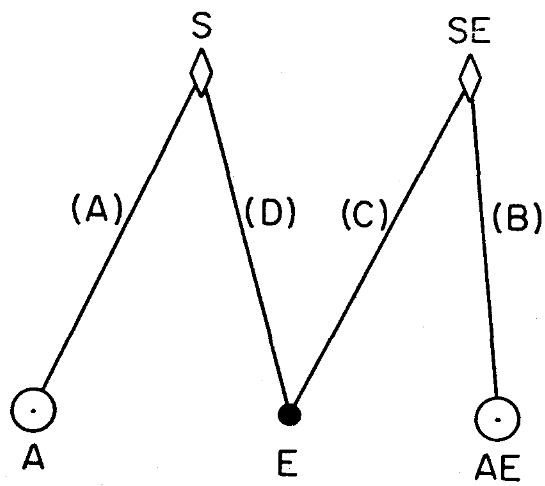

Figure 1. A basic configuration of earth and satellite points. (The path between earth point E and satellite point S is the propagation path of our concern. Point SE is the position of the satellite that serves the earth point at E. Earth points A and AE are the aim points of the satellite antennas at S and SE, respectively. The characters with parentheses denote the matrices for the coordinate transformations from a reference coordinate system to the coordinate systems associated with the paths.)

the direction of the path between S and E for a specified value of the polarization angle of the antenna at SE.

There are also two problems of our concern for the uplink case. The first problem is to calculate the polarization angle of the radio wave from E to S for a specified value of the polarization angle of the antenna at SE. The second problem is to calculate the polarization angle of the antenna at S in the direction of the path between E and S for a specified value of the polarization angle of the antenna at S.

3.2. Polarization Angle of a Downlink Radio Wave

In this subsection, we solve the first problem for the downlink case. The problem here is to calculate the polarization angle of the radio wave from S to E when the polarization angle of the satellite antenna at S is specified in terms of the polarization angle of the earth antenna at A. It is equivalent to projecting a vector, which is originally specified in a plane normal to the path between A and S, on another plane normal to the path between E and S.

We consider two Cartesian coordinate systems associated with the path between A and S and with the path between E and S, as discussed in Subsection 2.3. We denote the 3 X 3 matrices of coordinate transformation from a reference coordinate system to the first and second coordinate systems by (A) and (D), respectively. We also denote the three-element column vectors representing the polarization vector of the satellite antenna at S in the first coordinate system and the polarization vector of the radio wave from S to E in the second coordinate system by (U) and (X), respectively. Since the polarization vectors are orthogonal to the third coordinate axes of their respective coordinate systems, the third elements of (U) and (X) are zero. The first and second elements of (U) are equal to the cosine and sine of the specified polarization angle value of the satellite antenna. The desired polarization angle value of the radio wave is equal to the arctangent of the ratio of the second element to the first element of (X). Our problem is now to determine (X) when (U) is given.

An expression for the polarization vector of the satellite antenna in the second coordinate system is obtained by a coordinate transformation of (U) from the first to the second coordinate system, and the coordinate transformation is effected by multiplying $(D)(A)^{-1}$ on the left side of (U), where $(A)^{-1}$ is an inverse matrix of (A). Since the polarization vector of the radio wave from S to E is parallel to the projection of the polarization vector of the satellite antenna, its expression is obtained by multiplying the projection matrix, (P), defined in Subsection 2.3 on the left side of the expression for the polarization vector of the satellite antenna. Thus, we have, as the expression for the polarization vector of the radio wave from S to E, a column vector.

$$(X) = (E)(U), \tag{1}$$

where

$$(E) = (P)(D)(A)^{-1}. \tag{2}$$

The desired polarization angle can be calculated as the arctangent of the ratio of the second element to the first element of (X).

In practice, calculation of only four elements in the first two rows and first two columns of (E) is required. Since the third element of (U) is zero, the three elements in the third column are not used. Since the three elements in the third row of $(D)(A)^{-1}$ will be equated to zero by (P), they do not have to be calculated. Therefore, the 2 X 2 matrix consisting of the first two rows and first two columns of (E) is considered to act as an operator essentially on a two-element column vector consisting of the first two elements of (U). Note, however, that since the 2 X 2 matrix is not generally orthogonal, it should not be considered to represent a simple rotation in a plane; the polarization angle of (X) depends also on the polarization angle of (U).

3.3. Polarization Angle of a Downlink Receiving Antenna

In this subsection, we solve the second problem for the downlink case. The problem here is to calculate the polarization angle of the earth receiving antenna at E in the direction of the path between S and E when the polarization angle of the satellite antenna at SE is specified in terms of the polarization angle of the earth antenna at AE. Since the earth receiving antenna at E is aligned to the satellite transmitting antenna at SE, the desired angle is the polarization angle of the projection, in the direction of the path between S and E, of the polarization vector of the earth antenna at E, which is parallel to the projection, in the direction of the path between SE and E, of the polarization vector of the satellite antenna at SE. The problem here, therefore, can be considered in two steps of projection: first projecting a vector, originally specified in a plane normal to the path between AE and SE, on a second plane normal to the path between E and SE; and next projecting the vector thus projected on a third plane normal to the path between E and S.

We consider three Cartesian coordinate systems associated with the path between AE and SE, with the path between E and SE, and with the path between E and S, as discussed in Subsection 2.3. We denote the 3 X 3 matrices of coordinate transformation from a reference coordinate system to the first, second, and third coordinate systems by (B), (C), and (D), respectively. We also denote the three-element column vectors representing the polarization vector of the satellite antenna of SE in the first coordinate system, the polarization vector of the earth antenna at E in the second system, and the projection of the polarization vector of the earth antenna at E in the third system by (V), (W), and (Y), respectively. The first and second elements of (V) are equal to the cosine and sine of the specified polarization angle of the satellite antenna, and the third element is zero. The desired polarization angle of the earth receiving antenna at E in the direction of the path between E and S is equal to the arctangent of the ratio of the second element to the first element of (Y). Our problem is now to determine (Y) when (V) is given.

The polarization vector of the radio wave from SE to E is determined in the same manner as in Subsection 3.2. We have

$$(W) = (F)(V), \tag{3}$$

where

$$(F) = (P)(C)(B)^{-1}. \tag{4}$$

Since the polarization vector of the earth receiving antenna at E in the third coordinate system is parallel to the projection of the polarization vector in the second coordinate system, the former is related to the latter by

$$(Y) = (G)(W), \tag{5}$$

where

$$(G) = (P)(D)(C)^{-1}. \tag{6}$$

Thus, we can determine (Y) from (V) with (3) through (6). The desired polarization angle can be calculated as the arctangent of the ratio of the second element to the first element of (Y).

Similar comments as given in the last paragraph of Subsection 3.2 apply also to this subsection.

3.4. Polarization Angle of an Uplink Radio Wave

In this subsection, we solve the first problem for the uplink case. The problem here is to calculate the polarization angle of the radio wave from E to S when the polarization angle of the satellite antenna at SE is specified in terms of the polarization angle of the earth antenna at AE. Since the earth transmitting antenna at E is aligned to the satellite receiving antenna at SE, the desired angle is the polarization angle of the projection, in the direction of the path between S and E, of the polarization vector of the earth antenna at E, which is parallel to the projection, in the direction of the path between E and SE, of the polarization vector of the satellite antenna at SE. The problem here, therefore, can be considered in two steps of projection: first projecting a vector, originally specified in a plane normal to the path between AE and SE, on a second plane normal to the path between E and SE; and next projecting the vector thus projected on a third plane normal to the path between E and S. Comparison of these steps of projection with those described in the first paragraph of Subsection 3.3 indicates that the problem here is mathematically equivalent to the problem of Subsection 3.3.

We consider the three Cartesian coordinate systems and three 3 X 3 matrices of coordinate transformation, (B), (C), and (D), considered in Subsection 3.3. We also consider three column vectors, (V), (W), and (Y), used in that subsection. The first and second elements of (V) are equal to the cosine and sine of minus the specified polarization angle value of the satellite antenna, and the third element is zero. Since the problem here is equivalent to the problem of Subsection 3.3, (Y) can be determined from (V) with the use of (3) through (6). The desired polarization angle value can be determined as minus the arctangent of the ratio of the second element to the first element of (Y).

Similar comments as given in the last paragraph of Subsection 3.2 apply also to this subsection.

3.5. Polarization Angle of an Uplink Receiving Antenna

In this subsection, we solve the second problem for the uplink case. The problem here is to calculate the polarization angle of the satellite receiving antenna at S in the direction of the path between E and S when the polarization angle of the satellite antenna is specified in terms of the polarization angle of the earth antenna at A. It is equivalent to projecting a vector, which is originally specified in a plane normal to the path between A and S, on another plane normal to the path between E and S. Comparison of this step of projection with that described in the first paragraph of Subsection 3.2 indicates that the problem of this subsection is mathematically equivalent to the problem of Subsection 3.2.

We consider two Cartesian coordinate systems and two 3 X 3 matrices of coordinate transformation, (A) and (D), considered in Subsection 3.2. We also consider two column vectors, (U) and (X), used in that subsection. The first and second elements of (U) are equal to the cosine and sine of minus the specified polarization angle of the satellite antenna, and the third element is zero. Since the problem here is equivalent to the problem of Subsection 2.2, (X) can be determined from (U) with the use of (1) and (2). The desired polarization angle can be calculated as minus the arctangent of the ratio of the second element to the first element of (X).

Similar comments as given in the last paragraph of Subsection 3.2 apply also to this subsection.

4. SPECIAL CASES

We consider the polarization angle of a downlink radio wave, i.e., the first problem for a downlink case discussed in Subsection 3.2. For the purpose of comparison with some existing formulas, we place some simplifying assumptions and convert the representation of the polarization angle of the radio wave given in (1) and (2) in the subsection to a closed-form representation. The assumptions are (i) that the radio waves are specified or measured relative to the local horizontals, (ii) that the radio wave is horizontally polarized at the aim point, and (iii) that the satellite is above the equator of the earth. Because of the first assumption, we use the local-horizontal coordinate systems throughout this section.

We use the same symbols and notations as used in Subsections 3.1 and 3.2. In addition, we denote the polarization angle by β, the latitude by θ, the longitude by ϕ, and the normalized distance from the center of the earth (normalized with the radius of the earth as the unit) by r. In this section, we use a subscript a for the aim point, A, subscript e for the earth point in question, E, and subscript s for the satellite point, S. We also use the convention of denoting an element of a matrix or a vector by a lower-case character corresponding to the matrix or vector name with one or two subscripts, with the first subscript being the row number, and the second, the column number.

Since the second assumption dictates that

$$\beta_a = 0, \tag{7}$$

we have

$$\begin{aligned} u_1 &= \cos \beta_a = 1, \\ u_2 &= \sin \beta_a = 0, \\ u_3 &= 0. \end{aligned} \tag{8}$$

As the expression for the polarization angle of the radio wave from S to E, therefore, we have

$$\tan \beta_e = e_{21}/e_{11}. \tag{9}$$

This relation indicates that we must calculate only e_{21} and e_{11}. This simplifies the calculation.

The third assumption dictates that

$$\phi_s = 0. \tag{10}$$

This relation reduces complexity of calculating the element of the matrices.

As the expression of the desired polarization angle value under the three assumptions, we obtain

$$\tan \beta_e = g \cdot h,$$

$$g = \frac{r_s - \cos\theta_e \cos(\phi_e - \phi_s)}{(r_s^2 - 2r_s \cos\theta_e \cos(\phi_e - \phi_s) + 1)^{1/2}} \tag{11}$$

$$h = \frac{\sin\theta_a \cos\theta_e \sin(\phi_e - \phi_s) - \cos\theta_a \sin\theta_e \sin(\phi_a - \phi_s)}{\sin\theta_a \sin\theta_e + \cos\theta_a \cos\theta_e \sin(\phi_a - \phi_s) \sin(\phi_e - \phi_s)} .$$

The first factor, g, is a function of r_s, but not of θ_a nor ϕ_a. The second factor, h, is a function of θ_a and ϕ_a but not of r_s.

If we denote by δ the angle between the satellite, S, and the earth point in question, E, seen from the center of the earth, we have the relation

$$\cos\delta = \cos\theta_e \cos(\phi_e - \phi_s). \tag{12}$$

With the use of δ, we have

$$g = \{1 + [\sin\delta/(r_s - \cos\delta)]^2\}^{-1/2} \tag{13}$$

as an alternative expression of g.

A Satellite of Infinite Distance

In addition to the three assumptions listed in the first paragraph of this section, we further assume, as the fourth assumption, that the distance of the satellite from the earth is so large that it can be considered infinite, i.e.,

$$r_s = \infty. \tag{14}$$

When r_s increases beyond a limit, the g factor in (11) approaches unity. In the limiting case of (14), β_e is expressed by

$$\tan\beta_e = h$$

$$= \frac{\sin\theta_a \cos\theta_e \sin(\phi_e - \phi_s) - \cos\theta_a \sin\theta_e \sin(\phi_a - \phi_s)}{\sin\theta_a \sin\theta_e + \cos\theta_a \cos\theta_e \sin(\phi_a - \phi_s) \sin(\phi_e - \phi_s)} . \quad (15)$$

This expression agrees with the formula given by Shkarofsky and Moody (1976), as corrected by Shkarofsky (1977).

Aim Point on the Same Meridian as the Satellite

In addition to the three assumptions listed in the first paragraph of this section, we further assume, as another fourth assumption, that the aim point is on the same meridian as the satellite, i.e.,

$$\phi_a = \phi_s . \quad (16)$$

In this case, the expression for the h factor in (11) reduces to a simple expression

$$h = \sin(\phi_e - \phi_s)/\tan\theta_e .$$

The expression for β_e reduces to

$$\tan\beta_e = [\sin(\phi_e - \phi_s)/\tan\theta_e] \{1 + [\sin\delta/(r_s - \cos\delta)]^2\}^{-1/2} . \quad (17)$$

Note that the result is independent of θ_a, which is the latitude of the aim point.

For this case, the CCIR (1982a, Report 814-1, Annex I) gives a similar expression which differs from (17) in that the power in the expression is +1/2 instead of -1/2. It can be shown that the expression given by CCIR would be obtained if the polarization vector of the radio wave were assumed to be parallel to the equatorial plane at any point on the surface of the earth. Obviously, however, such an assumption is not justified; it holds only if the earth point is on the equator or on the same meridian as the satellite.

5. AN EXAMPLE

As an example, we consider a configuration consisting of 15 earth points and 3 geostationary satellites. The geographical data are given in Table 1 and Figure 2. In this table, the IE, IS, and ISE numbers are the earth point number, the satellite number, and the satellite number of the satellite that serves the earth point; ELON, ELAT, SLON, SLAT, ALON, and ALAT are the longitude and latitude of the earth point, the satellite, and the aim point of the satellite antenna, all in degrees; EELV and AELV are the elevation of the earth point and the aim point, both in meters; and PAS is the polarization angle of the satellite antenna. The first five earth points represent a pentagon that roughly covers the combined area of the U.S. Pacific Time Zone and U.S. Mountain Time Zone, designated as USA/PM. The next five earth points are for the U.S. Central Time Zone, designated as USA/C, and the last five earth points are for Mexico, designated as MEX. The satellite that serves the USA/PM area is located at -115° longitude, due south of the center of the area. The satellite for USA/C is located at -85°, due south of the eastern edge of the area. The satellite for MEX is located at -130°, west of the service area, to take care of the so-called eclipse protection. The aim point of each satellite antenna is selected as the intersection between the surface of the earth (of zero elevation) and the beam center of the minimum elliptical beam that covers the area; it is determined by the method described by Akima, (1981).

As described in Subsection 3.1, each satellite antenna and the earth antenna at its aim point are mutually aligned, and earth antennas in each service area are aligned to the satellite antenna that serves the area. Assuming that the polarization angles of the satellite antennas are specified at their respective aim points, we calculate the polarization angle of the radio wave and the difference in polarization angles between the radio wave and the receiving antenna for each of the paths between the satellites and earth points.

In regard to the polarization angles of the satellite antennas, we consider two cases. In the first case, we take a line parallel to the local horizontal plane at each aim point as the reference line and set the polarization angles of the satellite antennas to 0° (horizontal), 90° (vertical), and 0° (horizontal) for the USA/PM, USA/C, and MEX service areas, respectively. In the second case, we take a line parallel to the equatorial

Table 1. Input Data for the Example

EARTH POINT DATA

IE	ISE	ELON	ELAT	EELV
1	1	-125.0	49.0	0.0
2	1	-125.0	35.0	0.0
3	1	-117.0	32.0	0.0
4	1	-105.0	32.0	2500.0
5	1	-105.0	49.0	1000.0
6	2	-105.0	49.0	1000.0
7	2	-105.0	32.0	2500.0
8	2	-98.0	26.0	30.0
9	2	-85.0	30.0	10.0
10	2	-85.0	49.0	300.0
11	3	-120.0	32.0	0.0
12	3	-105.0	18.0	0.0
13	3	-92.0	14.0	0.0
14	3	-86.0	22.0	0.0
15	3	-105.0	32.0	2500.0

SATELLITE DATA

IS	SLON	SLAT	ALON	ALAT	AELV
1	-115.0	0.0	-114.0	39.3	0.0
2	-85.0	0.0	-94.1	37.2	0.0
3	-130.0	0.0	-102.2	23.5	0.0

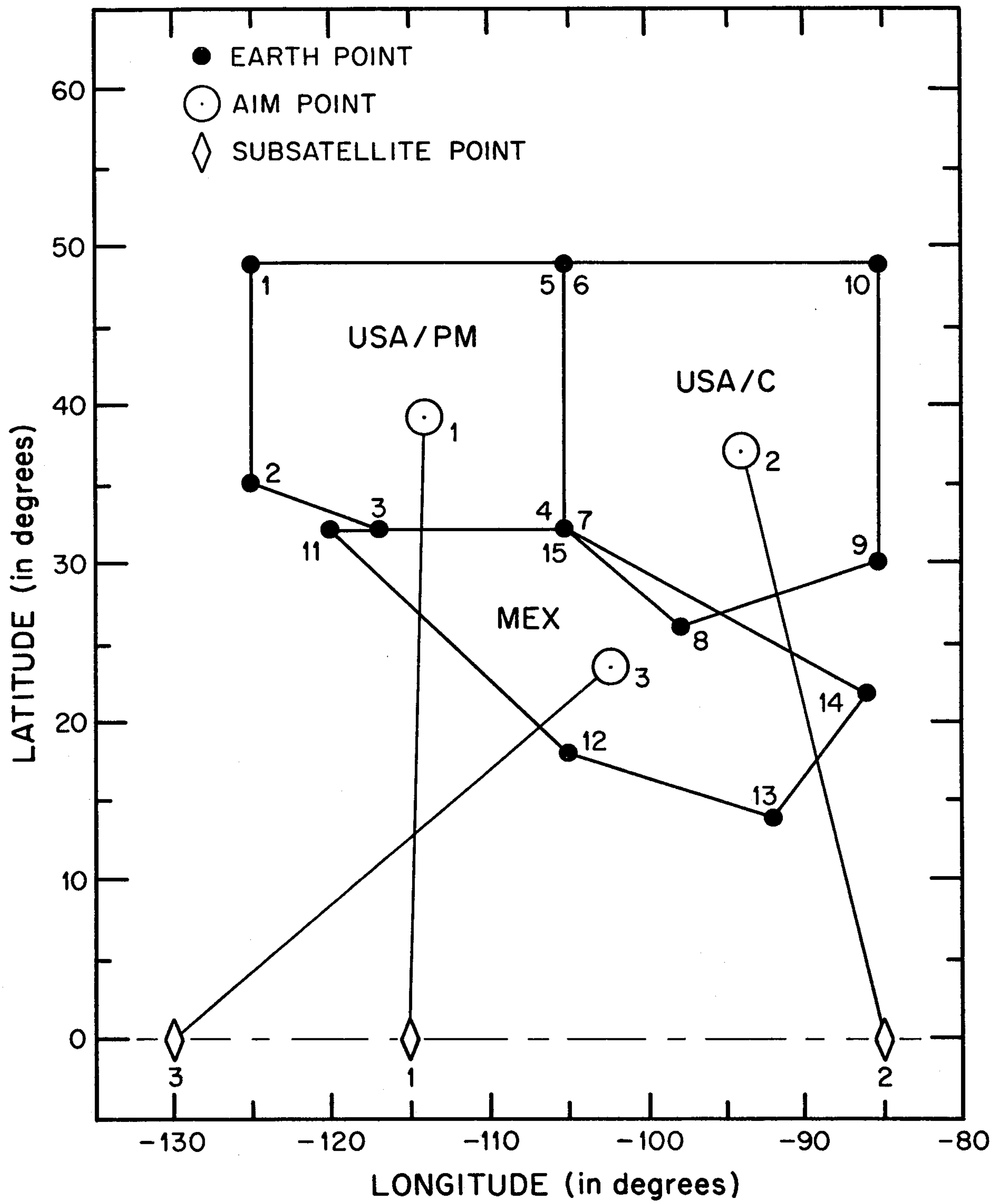

Figure 2. A geographical representation of the earth points, subsatellite points, and aim points used in the example.

plane at each aim point as the reference line and set the polarization angles of the satellite antennas to 0°, 90°, and 0° (i.e., the same angle values).

We use a line parallel to the local horizontal plane at each earth point as the reference line for the polarization angle of the radio wave. For the calculation of the polarization angle difference between the radio wave and the receiving antenna, the selection of the reference line does not matter as long as the same reference line is used for the two angles.

The calculated results for the first and second cases are shown in Tables 2 and 3, respectively. In each table, the upper half lists the polarization angles of the radio wave, and the lower half, the polarization angle differences. The IE, IS, and ISE numbers are the earth point number, the satellite number, and the satellite number of the satellite that serves the earth point. The polarization angles and the differences that correspond to IS=ISE are for desired paths, while all others are for interfering paths.

The polarization angles of the radio waves for IS=ISE for the uplink and downlink case for an identical path are essentially equal to each other. The only difference is in their signs; the opposite signs result from the difference in the definition of the polarization angle.

Table 2 indicates that the polarization angles of the radio waves for IS=ISE can differ considerably from the polarization angles of the radio wave set at the aim points (i.e., 0°, 90°, and 0° for ISE=1, 2, and 3, respectively) within the service areas.

In both Tables 2 and 3, the polarization angle differences for IS=ISE are all zero, as expected. The angle differences for the uplink and downlink cases for an identical path, are equal to each other, also as expected.

Table 2 indicates that, in the first case, the orthogonality is pretty well maintained between service areas 1 and 2 (i.e., service areas corresponding to ISE=1 and 2), but not between service areas 2 and 3. The parallel relation is not maintained between the service areas 1 and 3 either. These results are explainable by the fact that the direction of the path from the third satellite (IS=3) to its aim point is very different from the paths for the other satellites.

In contrast to the results in Table 2, Table 3 indicates that the parallel or orthogonal relations are well maintained among all the three service areas in the second case. This suggests that the use of the equatorial plane is better than the use of the local horizontal plane as the

Table 2. Calculated Results for the Example--Case 1
(Satellite antenna polarizations specified in reference to the local horizontal plane)

ISE	IE	POLARIZATION ANGLE OF RADIO WAVE					
		UPLINK			DOWNLINK		
		TO			FROM		
		IS=1	IS=2	IS=3	IS=1	IS=2	IS=3
	1	9.73	26.20	-1.09	-9.73	72.67	-42.45
	2	15.08	40.47	-4.27	-15.08	59.41	-39.77
1	3	4.40	38.47	-17.03	-4.40	61.64	-27.10
	4	-14.25	26.91	-31.23	14.25	73.17	-12.86
	5	-7.30	13.62	-16.81	7.30	85.24	-26.63
	6	75.24	-85.24	67.34	7.30	85.24	-26.63
	7	67.38	-73.17	52.27	14.25	73.17	-12.86
2	8	51.59	-77.08	38.43	29.62	77.08	.37
	9	42.00	78.29	35.39	39.49	-78.29	3.73
	10	60.40	78.29	55.92	22.09	-78.29	-15.28
	11	54.64	-82.41	31.36	-9.12	59.41	-31.36
	12	19.35	-75.91	-5.42	26.86	55.49	5.42
3	13	-9.73	84.23	-20.83	56.15	75.78	20.83
	14	-2.87	59.16	-12.71	48.83	-80.75	12.71
	15	31.21	83.87	12.86	14.25	73.17	-12.86

ISE	IE	POLARIZATION ANGLE DIFFERENCE					
		UPLINK			DOWNLINK		
		TO			FROM		
		IS=1	IS=2	IS=3	IS=1	IS=2	IS=3
	1	0.00	-81.14	-43.54	0.00	-81.14	-43.54
	2	0.00	-80.13	-44.04	0.00	-80.13	-44.04
1	3	0.00	-79.88	-44.12	0.00	-79.88	-44.12
	4	0.00	-79.91	-44.09	0.00	-79.91	-44.09
	5	0.00	-81.14	-43.45	0.00	-81.14	-43.45
	6	82.54	0.00	40.71	82.54	0.00	40.71
	7	81.63	0.00	39.41	81.63	0.00	39.41
2	8	81.21	0.00	38.79	81.21	0.00	38.79
	9	81.49	0.00	39.12	81.49	0.00	39.12
	10	82.48	-.00	40.64	82.48	0.00	40.64
	11	45.52	-23.00	0.00	45.52	-23.00	0.00
	12	46.20	-20.43	0.00	46.20	-20.43	0.00
3	13	46.42	-19.99	0.00	46.42	-19.99	0.00
	14	45.95	-21.59	0.00	45.95	-21.59	0.00
	15	45.46	-22.96	0.00	45.46	-22.96	0.00

Table 3. Calculated Results for the Example--Case 2
(Satellite antenna polarizations specified in reference to the equatorial plane)

ISE	IE	POLARIZATION ANGLE OF RADIO WAVE: UPLINK TO IS=1	UPLINK TO IS=2	UPLINK TO IS=3	DOWNLINK FROM IS=1	DOWNLINK FROM IS=2	DOWNLINK FROM IS=3
	1	8.50	24.74	-2.38	-8.50	60.84	4.83
	2	13.85	38.99	-5.55	-13.85	47.60	7.48
1	3	3.17	36.99	-18.31	-3.17	49.83	20.10
	4	-15.48	25.43	-32.52	15.48	61.34	34.29
	5	-8.53	12.16	-18.10	8.53	73.40	20.53
	6	85.19	-73.40	75.08	8.53	73.40	20.53
	7	77.20	-61.34	59.73	15.48	61.34	34.29
2	8	61.37	-65.24	45.83	30.85	65.24	47.51
	9	51.83	-89.87	42.93	40.73	89.87	50.88
	10	70.36	-89.87	63.72	23.32	89.87	31.83
	11	6.09	36.06	-15.85	-7.89	47.60	15.85
	12	-29.15	42.56	-52.56	28.08	43.65	52.56
3	13	-58.21	23.04	-68.01	57.38	63.92	68.01
	14	-51.32	-2.03	-59.89	50.06	87.41	59.89
	15	-17.27	22.25	-34.29	15.48	61.34	34.29

ISE	IE	POLARIZATION ANGLE DIFFERENCE: UPLINK TO IS=1	UPLINK TO IS=2	UPLINK TO IS=3	DOWNLINK FROM IS=1	DOWNLINK FROM IS=2	DOWNLINK FROM IS=3
	1	0.00	85.58	2.45	0.00	85.58	2.45
	2	0.00	86.59	1.92	0.00	86.59	1.92
1	3	0.00	86.82	1.79	0.00	86.82	1.79
	4	0.00	86.77	1.78	0.00	86.77	1.78
	5	0.00	85.56	2.44	0.00	85.56	2.44
	6	-86.27	0.00	-84.39	-86.27	0.00	-84.39
	7	-87.33	0.00	-85.98	-87.33	0.00	-85.98
2	8	-87.77	0.00	-86.66	-87.77	0.00	-86.66
	9	-87.45	0.00	-86.20	-87.45	0.00	-86.20
	10	-86.32	0.00	-84.45	-86.32	0.00	-84.45
	11	-1.81	83.66	0.00	-1.81	83.66	0.00
	12	-1.07	86.21	0.00	-1.07	86.21	0.00
3	13	-.83	86.97	0.00	-.83	86.97	.00
	14	-1.26	85.38	0.00	-1.26	85.38	0.00
	15	-1.79	83.59	0.00	-1.79	83.59	0.00

reference in specifying the satellite antenna polarizations for efficiently utilizing the geostationary satellite orbit.

6. CONCLUDING REMARKS

Basic concepts of the polarization vectors and polarization angles have been summarized, and basic problems of practical importance concerning the polarization angles have been identified. A method for calculating the polarization angles of linearly polarized radio waves and antennas has been presented. Comparisons of the method with some existing methods (of limited capability) have been made with generally good coincidence. A numerical example with a rather artificial configuration is given.

The method is based on several coordinate systems appropriate for representing the polarization angles and consists of a series of coordinate transformations among the coordinate systems and of projections of vectors in coordinate planes. Since the method consists of a series of simple and mutually independent steps, development and/or understanding of the method does not require a complicated three-dimensional picture.

The method presented in this report covers both the uplink and downlink cases. In applying the method, the polarization vector of the antenna can be specified, and the polarization angle of the antenna or the radio wave can be measured, in relation to a line parallel to either the equatorial plane of the earth or a local horizontal plane. Application of the method is not limited to a geostationary satellite; the latitude of the satellite need not be zero.

The method is straightforward and can easily be implemented in computer programs. A Fortran subroutine package that implements the method is described in Appendix B to this report.

7. ACKNOWLEDGMENTS

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APPENDIX A. COORDINATE SYSTEMS FOR THE ANTENNA AND PROPAGATION CALCULATIONS

Three Cartesian coordinate systems are described here. They are the earth-center, equatorial-plane, and local-horizontal coordinate systems. The first one is a reference coordinate system. Angles used in antenna and propagation calculations, such as the off-axis angle, orientation angle, polarization angle, etc., are specified or measured in either the second or the third coordinate system. The first one is used for effecting coordinate transformation between a pair of coordinate systems of the second or third type.

Any length can be used as the unit of length as long as it is used consistently in all the three coordinate systems. Perhaps the most convenient way in satellite communications is to use the radius of the earth as the unit.

Earth-center Coordinate System

The earth-center coordinate system is a Cartesian coordinate system. The origin is the center of the earth. The positive x, y, and z axes intersect the surface of the earth at 0° east and 0° north, at 90° east and 0° north, and at 90° north (i.e., the north pole), respectively.

In this coordinate system, the three coordinates of a point, x, y, and z, are represented by

$$
\begin{aligned}
x &= r \cos\theta \cos\phi, \\
y &= r \cos\theta \sin\phi, \qquad \text{(A-1)} \\
z &= r \sin\theta,
\end{aligned}
$$

where r is the distance between the point in question and the center of the earth, and θ and ϕ are the latitude and longitude of the point. Conversely, r, θ, and ϕ of a point are calculated by

$$
\begin{aligned}
r &= (x^2 + y^2 + z^2)^{1/2}, \\
\theta &= \tan^{-1}(z/(x^2 + y^2)^{1/2}), \qquad \text{(A-2)} \\
\phi &= \tan^{-1}(y/x),
\end{aligned}
$$

respectively, when x, y, and z are given for the point.

Equatorial-Plane Coordinate System

The equatorial-plane coordinate system is a Cartesian coordinate system, characterized by two points, a specified earth point (i.e., a point on the surface of the earth) and a satellite point, and by the equatorial plane of the earth. The origin is the earth point. The positive z' axis points toward the satellite point. The x' axis is parallel to the equatorial plane of the earth. As a convention, the sense of the x' axis is taken in such a way that the positive y' axis is on the north side of the z'-x' plane.

In this coordinate system having a specified earth point as the origin, the angle between the vector from the satellite to an arbitrary earth point and the vector from the satellite to the specified earth point is represented by

$$\alpha_e = \tan^{-1}((x_e'^2 + y_e'^2)^{1/2}/(z_s' - z_e')), \qquad \text{(A-3)}$$

where x_e', y_e', and z_e' are the coordinates of the arbitrary earth point, and z_s' is the z' coordinate of the satellite. The orientation angle of an arbitrary earth point relative to the x' axis is represented by

$$\beta_e = \tan^{-1}(y_e'/x_e'). \qquad \text{(A-4)}$$

These angles are useful in calculating the gain pattern of a satellite antenna.

Conversely, when α_e and β_e of an earth point are given, its coordinates, x_e', y_e', and z_e', can also be calculated. If we denote the distance between an earth point (a point on the surface of the earth) and the center of the earth by r_e, the coordinates must satisfy, in addition to (A-3) and (A-4),

$$(x_e' - x_c')^2 + (y_e' - y_c')^2 + (z_e' - z_c')^2 = r_e^2, \qquad \text{(A-5)}$$

where x_c', y_c', and z_c' are the coordinates of the center of the earth. Since the origin of the coordinate system is also on the surface of the earth, we have

$$x_c'^2 + y_c'^2 + z_c'^2 = r_e^2. \qquad \text{(A-6)}$$

With the help of (A-6), we can solve (A-3), (A-4), and (A-5) with respect to x_e', y_e', and z_e'. The results are

$$\begin{aligned} x_e' &= R \sin\alpha_e \cos\beta_e, \\ y_e' &= R \sin\alpha_e \sin\beta_e, \\ z_e' &= z_s' - R \cos\alpha_e, \end{aligned} \qquad \text{(A-7)}$$

where

$$\begin{aligned} R &= B - (B^2 - C)^{1/2}, \\ B &= (x_c' \cos\beta_e + y_c' \sin\beta_e)\sin\alpha_e + (z_s' - z_c')\cos\alpha_e, \\ C &= (z_s' - z_c')^2 - z_c'^2. \end{aligned} \qquad \text{(A-8)}$$

As the expression for R in (A-8) suggests, R has been determined as a root of a quadratic equation. Out of two roots of the quadratic equation, a root that leads to an x_e' value closer to zero has been selected; the other root leads to a more negative x_e' value and corresponds to an earth point that is invisible from the satellite.

The polarization angle of a radio wave is equivalent to the orientation angle mathematically. It is represented by

$$\beta_p = \tan^{-1}(y_p'/x_p'), \qquad \text{(A-9)}$$

where x_p' and y_p' are the x' and y' components of the polarization vector of the radio wave.

This coordinate system is essentially the same as the one called the boresight-point coordinate system and used for calculating the orientation angle of a minimum elliptical beam of a satellite antenna by Akima (1981).

Local-Horizontal Coordinate System

The local-horizontal coordinate system is a Cartesian coordinate system, characterized by two points, a specified earth point (i.e., a point on the surface of the earth) and a satellite point, and by the local horizontal plane at the earth point. The origin is the earth point. The positive z' axis points toward the satellite point. The x' axis is parallel to the local horizontal plane at the earth point. As a convention, the sense of the x' axis is taken in such a way that the y' coordinate of the earth center is negative. (When the earth center, the specified earth point, and the satellite point are collinear, the local horizontal line is normal to the z' axis and the x' axis is not uniquely determined. In this case, the x' axis is taken, as a convention in the same way as in the equatorial-plane coordinate system.)

This coordinate system is the same as the equatorial-plane coordinate system except that the x' and y' axes are taken differently. Equations (A-3) through (A-9) hold also for this coordinate system.

Coordinate Transformation

We denote the earth-center coordinates of a point by x, y, and z, and denote the equatorial-plane or local-horizontal coordinates by x', y', and z'. We use subscripts o and s for the specified earth point and the satellite point, respectively. Note that the specified earth point is the origin of the equatorial-plane or local-horizontal coordinate system and that the satellite is on the positive z' axis.

Coordinate transformation from the earth-center coordinates to the equatorial-plane or local-horizontal coordinates is represented by a matrix equation

$$X' = A\,(X - X_o) = AX - AX_o, \qquad \text{(A-10)}$$

where

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \text{(A-11)}$$

is a 3 X 1 matrix or a three-element column vector containing the earth-center coordinates of a point, where

$$X' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \qquad \text{(A-12)}$$

is a 3 X 1 matrix or a three-element column vector containing the new coordinates of the point, and where

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad \text{(A-13)}$$

is the 3 X 3 matrix of coordinate transformation. Conversely, transformation from the equatorial-plane or local-horizontal coordinates to the earth-center coordinates is represented by

$$X - X_o = A^{-1}X', \quad \text{(A-14)}$$

where A^{-1} is the inverse matrix of A.

Since all coordinate systems involved are Cartesian with the same length of unit vectors, the A matrix is orthogonal the inverse of A is equal to the transpose of A. Therefore, a useful relation

$$A^{-1} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{22} & a_{33} \end{pmatrix} \quad \text{(A-15)}$$

is obtained.

Note that each column of A^{-1} or each row of A is the x, y, and z components of the unit vector in the positive direction of the x', y', or z' axis. Also note that each column of A or each row of A^{-1} is the x', y', or z' components of the unit vector in the positive direction of the x, y, or z axis.

<u>Transformation from the Earth-Center Coordinates to the Equatorial-Plane Coordinates</u>

Three elements, a_{31}, a_{32}, and a_{33}, are the x, y, and z components of the unit vector in the positive direction of the z' axis. Since the satellite point is represented by (x_s, y_s, z_s) and by (0, 0, z_s') in the earth-center and the new coordinate systems, respectively, we obtain, from (A-14),

$$a_{31} = (x_s - x_o)/z_s' ,$$
$$a_{32} = (y_s - y_o)/z_s' , \quad \text{(A-16)}$$
$$a_{33} = (z_s - z_o)/z_s' .$$

The z' coordinate of the satellite, z_s', is equal to the distance between the satellite and the origin of the new coordinate system and is represented by

$$z_s' = ((x_s - x_o)^2 + (y_s - y_o)^2 + (z_s - z_o)^2)^{1/2} . \qquad \text{(A-17)}$$

The x' axis of the equatorial-plane coordinate system is parallel to the equatorial plane. This means that

$$z = z_o \text{ when } y' = 0 \text{ and } z' = 0.$$

From (A-14) together with (A-11), (A-12), and (A-15), we have

$$z - z_o = a_{13}x' + a_{23}y' + a_{33}z' .$$

Combining these two relations, we obtain

$$a_{13} = 0 . \qquad \text{(A-18)}$$

Three elements, a_{11}, a_{12}, and a_{13}, constitute a unit vector in the positive direction of the x' axis, which is orthogonal to the z' axis. Noting (A-18), therefore, we have

$$a_{31}a_{11} + a_{32}a_{12} = 0 ,$$

$$a_{11}^2 + a_{12}^2 = 1 .$$

Solving this set of equations with respect to a_{11} and a_{12}, we have

$$a_{11} = \mp a_{32}/(a_{31}^2 + a_{32}^2)^{1/2} ,$$

$$a_{12} = \pm a_{31}/(a_{31}^2 + a_{32}^2)^{1/2} .$$

Since the positive y' axis is taken on the north side of the z'-x' plane, the a_{23} element, which is the z component of the unit vector in the positive direction of the y' axis, must be positive. Since this unit vector is equal to the vector product of the two unit vectors in the positive directions of the z' and x' axes, we have

$$a_{23} = a_{31}a_{12} - a_{32}a_{11} = \pm (a_{31}^2 + a_{32}^2)^{1/2} \quad .$$

Therefore, taking the upper signs in the above expressions for a_{11} and a_{12}, we obtain

$$a_{11} = - a_{32}/(a_{31}^2 + a_{32}^2)^{1/2} \;,$$
$$a_{12} = a_{31}/(a_{31}^2 + a_{32}^2)^{1/2} \quad . \tag{A-19}$$

Finally, since the new coordinate system is a Cartesian system, and since the remaining three elements, a_{21}, a_{22}, and a_{23}, constitute a unit vector in the positive direction of the y' axis, which is the vector product of the two vectors in the positive directions of the z' and x' axes, we have

$$a_{21} = a_{32}a_{13} - a_{33}a_{12} \;,$$
$$a_{22} = a_{33}a_{11} - a_{31}a_{13} \;, \tag{A-20}$$
$$a_{23} = a_{31}a_{12} - a_{32}a_{11} \;.$$

Thus, all nine elements of the A matrix have been determined.

Transformation from the Earth-Center Coordinates to the Local-Horizontal Coordinates

The origin and the z' axis in the local-horizontal coordinate system are the same as those in the equatorial-plane coordinate system. Therefore, three elements a_{31}, a_{32}, and a_{33} are determined also by (A-16) and (A-17).

To determine three elements, a_{11}, a_{12}, and a_{13}, we consider three vectors. The first one is the vector from the earth center to the origin of the coordinate system, the second is from the origin of the coordinate system to the satellite, and the third is from the earth center to the satellite. Obviously, the third vector is the sum of the first and the second vectors. Since the x' axis is parallel to the local horizontal plane in the local-horizontal coordinate system, it is orthogonal to the first vector. Since it is also orthogonal to the z' axis, which is in the same direction as the second vector, it is also orthogonal to the second vector. It is, therefore, also orthogonal to the third vector. When the earth center, the origin of the coordinate system, and the satellite are not collinear, the vector product of the first and the third factors is a nonzero vector, and the x' axis is parallel to the vector product. Therefore, three elements, a_{11}, a_{12}, and a_{13}, that constitute a unit vector in the positive direction of the x' axis, are represented by

$$\begin{aligned} a_{11} &= (y_o z_s - z_o y_s)/c , \\ a_{12} &= (z_o x_s - x_o z_s)/c , \\ a_{13} &= (x_o y_s - y_o x_s)/c , \end{aligned} \qquad \text{(A-21)}$$

where c is a constant. Since the three elements constitute a unit vector, c is determined as

$$c = ((y_o z_s - z_o y_s)^2 + (z_o x_s - x_o z_s)^2 + (x_o y_s - y_o x_s)^2)^{1/2} . \qquad \text{(A-22)}$$

Since the positive direction of the x' axis is also parallel to the vector product of two unit vectors in the positive directions of the y' and z' axes, both the first vector and the positive y' axis point to the same side of the z'-x' plane. Therefore, the y' coordinate of the earth center is negative. (This reasoning can be verified more directly by calculating the y' coordinate of the earth center after determining the remaining three elements.)

When the earth center, the origin of the coordinate system, and the satellite are collinear, the x' axis cannot be determined by the above procedure. In this case, we take the x' axis in the same way as in the equatorial-plane coordinate system. Therefore, we use (A-18) and (A-19) for the three elements, a_{11}, a_{12}, and a_{13}.

Like the equatorial-plane coordinate system, the local-horizontal coordinate system is also a Cartesian system. Therefore, three elements a_{21}, a_{22}, and a_{23} are determined also by (A-20).

Reference to Appendix A

Akima, H. (1981), A method for determining the minimum elliptical beam of a satellite antenna, NTIA Report 81-88, October. (NTIS Order No. PB82-153966)

APPENDIX B. A FORTRAN SUBROUTINE PACKAGE FOR CALCULATING POLARIZATION ANGLES

The Fortran subroutine package described in this appendix implements the method, developed in the text, for calculating polarization angles of radio waves and antennas in satellite communications. The package consists of two subroutines, i.e., PLANGA and CTEC2P. The PLANGA subroutine interfaces with the user; it calculates the polarization angles. The CTEC2P subroutine is a supporting subroutine and calculates the matrix elements for the coordinate transformations for PLANGA.

This package is written in ANSI-77 Standard Fortran (Publication X3.9-1978, ANSI, 345 E. 47th St., New York, NY 10017). Conversion of this package to ANSI-66 Fortran (Publication X3.9-1966, ANSI, New York, NY) requires very minor efforts.

A Fortran listing of the package follows. The user information of each subroutine including the descriptions of the input and output parameters is given in the beginning of each subroutine.

```
      SUBROUTINE  PLANGA(IRLE,ILNK,XE,YE,ZE,
     1                   IPS,PAS,XS,YS,ZS,XA,YA,ZA,
     2                   IPSE,PASE,XSE,YSE,ZSE,XAE,YAE,ZAE,
     3                   PARW,DPAA)
C THIS SUBROUTINE CALCULATES, FOR AN UPLINK OR A DOWNLINK PATH,
C THE POLARIZATION ANGLE OF THE RADIO WAVE (OR THE PROJECTION OF
C THE TRANSMITTING ANTENNA) AND THE DIFFERENCE IN THE POLARIZA-
C TION ANGLE BETWEEN THE RADIO WAVE AND THE PROJECTION OF THE
C RECEIVING ANTENNA IN A PLANE NORMAL TO THE PATH BETWEEN THE
C TRANSMITTING AND RECEIVING ANTENNAS.
C THIS SUBROUTINE INVOLVES TWO SATELLITES: ONE IS THE SATELLITE
C IN QUESTION AND THE OTHER IS THE SATELLITE THAT SERVES THE
C EARTH POINT.
C WHEN ONE ANTENNA IS CIRCULARLY POLARIZED AND THE OTHER ANTENNA
C IS LINEARLY POLARIZED FOR THE PATH, THIS SUBROUTINE RETURNS A
C VALUE OF 45 DEGREES AS THE DIFFERENCE IN THE POLARIZATION
C ANGLE BETWEEN THE TWO ANTENNAS.
C THIS SUBROUTINE CALLS THE CTEC2P SUBROUTINE.
C THE INPUT ARGUMENTS ARE
C   IRLE    = INDEX FOR THE REFERENCE LINE, RELATIVE TO WHICH
C             THE POLARIZATION ANGLE OF THE RADIO WAVE IS
C             MEASURED AT THE EARTH POINT
C           = 1  FOR A LINE PARALLEL TO THE EQUATORIAL PLANE
C           = 2  FOR A LINE PARALLEL TO THE LOCAL HORIZONTAL
C                PLANE (DEFAULT VALUE),
C   ILNK    = INDEX FOR UPLINK OR DOWNLINK CASE
C           = 1  FOR UPLINK
C           = 2  FOR DOWNLINK,
C   XE, YE, ZE
C           = EARTH-CENTER COORDINATES OF THE EARTH POINT,
C   IPS     = INDEX OF THE POLARIZATION TYPE OF THE ANTENNA
C             OF THE SATELLITE IN QUESTION
C           = 0 FOR CIRCULAR POLARIZATION
C           = 1 FOR LINEAR POLARIZATION WHEN THE ANGLE IS
C               SPECIFIED RELATIVE TO A LINE PARALLEL TO THE
C               EQUATORIAL PLANE AT THE AIM POINT
C           = 2 FOR LINEAR POLARIZATION WHEN THE ANGLE IS
C               SPECIFIED RELATIVE TO A LINE PARALLEL TO THE
C               LOCAL HORIZONTAL PLANE AT THE AIM POINT,
C   PAS     = POLARIZATION ANGLE (IN DEGREES) OF THE ANTENNA
C             OF THE SATELLITE IN QUESTION SPECIFIED AT ITS
C             AIM POINT
C             (MUST BE +45.0 OR -45.0 WHEN IPS.EQ.0),
C   XS, YS, ZS
C           = EARTH-CENTER COORDINATES OF THE SATELLITE IN
C             QUESTION,
C   XA, YA, ZA
C           = EARTH-CENTER COORDINATES OF THE AIM POINT (ON
C             THE SURFACE OF THE EARTH) OF THE ANTENNA OF THE
C             SATELLITE IN QUESTION,
C   IPSE    = INDEX OF THE POLARIZATION TYPE OF THE ANTENNA
C             OF THE SATELLITE THAT SERVES THE EARTH POINT
C           = 0 FOR CIRCULAR POLARIZATION
```

```
C           = 1 FOR LINEAR POLARIZATION WHEN THE ANGLE IS
C               SPECIFIED RELATIVE TO A LINE PARALLEL TO THE
C               EQUATORIAL PLANE AT THE AIM POINT
C           = 2 FOR LINEAR POLARIZATION WHEN THE ANGLE IS
C               SPECIFIED RELATIVE TO A LINE PARALLEL TO THE
C               LOCAL HORIZONTAL PLANE AT THE AIM POINT,
C   PASE    = POLARIZATION ANGLE (IN DEGREES) OF THE ANTENNA
C             OF THE SATELLITE THAT SERVES THE EARTH POINT
C             SPECIFIED AT ITS AIM POINT
C             (MUST BE +45.0 OR -45.0 WHEN IPSE.EQ.0),
C   XSE, YSE, ZSE
C           = EARTH-CENTER COORDINATES OF THE SATELLITE THAT
C             SERVES THE EARTH POINT,
C   XAE, YAE, ZAE
C           = EARTH-CENTER COORDINATES OF THE AIM POINT (ON
C             THE SURFACE OF THE EARTH) OF THE ANTENNA OF THE
C             SATELLITE THAT SERVE THE EARTH POINT.
C THE OUTPUT ARGUMENTS ARE
C   PARW    = POLARIZATION ANGLE (IN DEGREES) OF THE RADIO
C             WAVE,
C   DPAA    = DIFFERENCE IN THE POLARIZATION ANGLE (IN DE-
C             GREES) BETWEEN THE RADIO WAVE AND THE PROJEC-
C             TION OF THE RECEIVING ANTENNA,
C BOTH MEASURED IN A PLANE NORMAL TO THE PATH BETWEEN THE TRANS-
C MITTING AND RECEIVING ANTENNAS.
C SPECIFICATION STATEMENTS
      EQUIVALENCE (A31,B31,C31,D31),(A32,B32,C32,D32),
     1            (A33,B33,C33,D33)
      SAVE  INIT,CFDTR,CFRTD
      DATA  INIT/0/
C CALCULATION
C UNIVERSAL CONSTANTS
   10 IF(INIT.LE.0)  THEN
        INIT=1
        CFDTR=ATAN2(1.0,0.0)/90.0
        CFRTD=1.0/CFDTR
      END IF
C POLARIZATION ANGLES OF THE PROJECTIONS OF THE ANTENNAS
C - CIRCULARLY POLARIZED ANTENNAS
C - - SATELLITE ANTENNA
   20 IF((ILNK.EQ.1.AND.IPS.LE.0.AND.IPSE.LE.0).OR.
     1   (ILNK.EQ.2.AND.IPS.LE.0))                    THEN
        PPSA=PAS
        SPDT=(XA-XS)*(XE-XS)+(YA-YS)*(YE-YS)+(ZA-ZS)*(ZE-ZS)
        IF(SPDT.LT.0.0)    PPSA=-PPSA
      END IF
C - - EARTH ANTENNA
   30 IF((ILNK.EQ.1.AND.IPSE.LE.0).OR.
     1   (ILNK.EQ.2.AND.IPS.LE.0.AND.IPSE.LE.0))  THEN
        PPEA=PASE
        SPDT=(XS-XE)*(XSE-XE)+(YS-YE)*(YSE-YE)+(ZS-ZE)*(ZSE-ZE)
        IF(SPDT.LT.0.0)    PPEA=-PPEA
      END IF
```

```
C - LINEARLY POLARIZED ANTENNAS
C - - COMMON CALCULATION FOR BOTH ANTENNAS
   40 IF((ILNK.EQ.1.AND.IPSE.GE.1).OR.
     1    (ILNK.EQ.2.AND.IPS.GE.1))    THEN
        IF(IRLE.EQ.1)  THEN
          INCS=1
        ELSE
          INCS=2
        END IF
        CALL CTEC2P(INCS,XE,YE,ZE,XS,YS,ZS,
     1                D11,D12,D13,D21,D22,D23,D31,D32,D33)
      END IF
C - - SATELLITE ANTENNA
   50 IF((ILNK.EQ.1.AND.IPS.GE.1.AND.IPSE.GE.1).OR.
     1    (ILNK.EQ.2.AND.IPS.GE.1))                    THEN
        BTU=PAS*CFDTR
        IF(ILNK.EQ.1)      BTU=-BTU
        UU1=COS(BTU)
        UU2=SIN(BTU)
        CALL CTEC2P(IPS,XA,YA,ZA,XS,YS,ZS,
     1                A11,A12,A13,A21,A22,A23,A31,A32,A33)
        E11=D11*A11+D12*A12+D13*A13
        E12=D11*A21+D12*A22+D13*A23
        E21=D21*A11+D22*A12+D23*A13
        E22=D21*A21+D22*A22+D23*A23
        XX1=E11*UU1+E12*UU2
        XX2=E21*UU1+E22*UU2
        BTX=ATAN2(XX2,XX1)
        PPSA=BTX*CFRTD
        IF(ILNK.EQ.1)      PPSA=-PPSA
        PPSA=90.0-MOD(-PPSA+450.0,180.0)
      END IF
C - - EARTH ANTENNA
   60 IF((ILNK.EQ.1.AND.IPSE.GE.1).OR.
     1    (ILNK.EQ.2.AND.IPS.GE.1.AND.IPSE.GE.1))  THEN
        BTV=PASE*CFDTR
        IF(ILNK.EQ.1)      BTV=-BTV
        VV1=COS(BTV)
        VV2=SIN(BTV)
        CALL CTEC2P(IPSE,XAE,YAE,ZAE,XSE,YSE,ZSE,
     1                B11,B12,B13,B21,B22,B23,B31,B32,B33)
        CALL CTEC2P(IPSE,XE,YE,ZE,XSE,YSE,ZSE,
     1                C11,C12,C13,C21,C22,C23,C31,C32,C33)
        F11=C11*B11+C12*B12+C13*B13
        F12=C11*B21+C12*B22+C13*B23
        F21=C21*B11+C22*B12+C23*B13
        F22=C21*B21+C22*B22+C23*B23
        WW1=F11*VV1+F12*VV2
        WW2=F21*VV1+F22*VV2
        G11=D11*C11+D12*C12+D13*C13
        G12=D11*C21+D12*C22+D13*C23
        G21=D21*C11+D22*C12+D23*C13
        G22=D21*C21+D22*C22+D23*C23
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```
         YY1=G11*WW1+G12*WW2
         YY2=G21*WW1+G22*WW2
         BTY=ATAN2(YY2,YY1)
         PPEA=BTY*CFRTD
         IF(ILNK.EQ.1)      PPEA=-PPEA
         PPEA=90.0-MOD(-PPEA+450.0,180.0)
       END IF
C POLARIZATION ANGLE OF THE RADIO WAVE
   70  IF(ILNK.EQ.1)        PARW=PPEA
       IF(ILNK.EQ.2)        PARW=PPSA
C
C POLARIZATION ANGLE DIFFERENCE
   80  IF((IPS.LE.0.AND.IPSE.LE.0).OR.
     1    (IPS.GE.1.AND.IPSE.GE.1))  THEN
         IF(ILNK.EQ.1)      PPRA=PPSA
         IF(ILNK.EQ.2)      PPRA=PPEA
         DPAA=PARW-PPRA
         DPAA=90.0-MOD(-DPAA+450.0,180.0)
       ELSE
         DPAA=45.0
       END IF
       RETURN
       END
```

```
      SUBROUTINE  CTEC2P(INCS,XO,YO,ZO,XS,YS,ZS,
     1                    A11,A12,A13,A21,A22,A23,A31,A32,A33)
C THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE 3*3 MATRIX FOR
C COORDINATE TRANSFORMATION FROM THE EARTH-CENTER COORDINATE
C SYSTEM TO THE EQUATORIAL-PLANE COORDINATE SYSTEM OR THE LOCAL-
C HORIZONTAL COORDINATE SYSTEM.
C THE EARTH-CENTER COORDINATE SYSTEM IS A CARTESIAN SYSTEM.  THE
C ORIGIN IS THE CENTER OF THE EARTH.  THE POSITIVE X, Y, AND Z
C AXES INTERSECT THE SURFACE OF THE EARTH AT 0 DEGREES EAST AND
C 0 DEGREES NORTH, AT 90 DEGREES EAST AND 0 DEGREES NORTH, AND
C AT 90 DEGREES NORTH (THE NORTH POLE), RESPECTIVELY.
C THE EQUATORIAL-PLANE COORDINATE SYSTEM IS A CARTESIAN SYSTEM,
C CHARACTERIZED BY TWO POINTS, AN EARTH POINT AND A SATELLITE
C POINT, AND BY THE EQUATORIAL PLANE OF THE EARTH.  THE ORIGIN
C IS THE EARTH POINT.  THE POSITIVE Z' AXIS POINTS TOWARD THE
C SATELLITE POINT.  THE X' AXIS IS PARALLEL TO THE EQUATORIAL
C PLANE OF THE EARTH.  THE SENSE OF THE X' AXIS IS TAKEN IN SUCH
C A WAY THAT THE POSITIVE Y' AXIS IS ON THE NORTH SIDE OF THE
C Z'-X' PLANE.
C THE LOCAL-HORIZONTAL COORDINATE SYSTEM IS A CARTESIAN SYSTEM,
C CHARACTERIZED BY TWO POINTS, AN EARTH POINT AND A SATELLITE
C POINT, AND BY THE LOCAL HORIZONTAL PLANE AT THE EARTH POINT.
C THE ORIGIN IS THE EARTH POINT.  THE POSITIVE Z' AXIS POINTS
C TOWARD THE SATELLITE POINT.  THE X' AXIS IS PARALLEL TO THE
C LOCAL HORIZONTAL LINE AT THE EARTH POINT.  THE SENSE OF THE
C X' AXIS IS TAKEN IN SUCH A WAY THAT THE Y' COORDINATE OF THE
C EARTH CENTER IS NEGATIVE.  (WHEN THE EARTH POINT, THE SATEL-
C LITE POINT, AND THE EARTH CENTER ARE COLLINEAR, THE LOCAL
C HORIZONTAL LINE IS UNDEFINED.  IN THIS CASE, THE X' AXIS IS
C TAKEN, AS A CONVENTION, IN THE SAME WAY AS IN THE EQUATORIAL-
C PLANE COORDINATE SYSTEM.)
C THE INPUT ARGUMENTS ARE
C   INCS    = INDEX SPECIFYING THE NEW COORDINATE SYSTEM
C           = 1  FOR THE EQUATORIAL-PLANE COORDINATE SYSTEM
C           = 2  FOR THE LOCAL-HORIZONTAL COORDINATE SYSTEM,
C   XO, YO, ZO
C           = EARTH-CENTER COORDINATES OF THE EARTH POINT,
C   XS, YS, ZS
C           = EARTH-CENTER COORDINATES OF THE SATELLITE
C             POINT.
C THE OUTPUT ARGUMENTS ARE THE ELEMENTS OF THE MATRIX THAT
C TRANSFORMS THE EARTH-CENTER X-Y-Z COORDINATE SYSTEM TO THE
C EQUATORIAL-PLANE OR THE LOCAL-HORIZONTAL X'-Y'-Z' COORDINATE
C SYSTEM IN THE FOLLOWING MATRIX EQUATION
C      (X')   (A11 A12 A13) (X-XO)
C      (Y') = (A21 A22 A23) (Y-YO)
C      (Z')   (A31 A32 A33) (Z-ZO)
C SPECIFICATION STATEMENT
      DATA  EPSLN/1.0E-12/
C CALCULATION
C UNIT VECTOR  --  Z' AXIS
   10 A1=XS-XO
      A2=YS-YO
      A3=ZS-ZO
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```
      ASQ=A1*A1+A2*A2+A3*A3
      IF(ASQ.LE.0.0)      GO TO 90
      A=SQRT(ASQ)
      A31=A1/A
      A32=A2/A
      A33=A3/A
C UNIT VECTOR  --  X' AXIS
   20 IF(INCS.LE.1)       GO TO 25
C - EQUATORIAL-PLANE COORDINATE SYSTEM
      A1=Y0*ZS-Z0*YS
      A2=Z0*XS-X0*ZS
      A3=X0*YS-Y0*XS
      ASQ=A1*A1+A2*A2+A3*A3
      ASQMN=(X0*X0+Y0*Y0+Z0*Z0)*(XS*XS+YS*YS+ZS*ZS)*EPSLN
      IF(ASQ.LE.ASQMN)    GO TO 25
      A=SQRT(ASQ)
      A11=A1/A
      A12=A2/A
      A13=A3/A
      GO TO 30
C - LOCAL-HORIZONTAL COORDINATE SYSTEM
   25 A1=-A32
      A2= A31
      ASQ=A1*A1+A2*A2
      IF(ASQ.LE.0.0)      GO TO 91
      A=SQRT(ASQ)
      A11=A1/A
      A12=A2/A
      A13=0.0
C UNIT VECTOR  --  Y' AXIS
   30 A21=A32*A13-A33*A12
      A22=A33*A11-A31*A13
      A23=A31*A12-A32*A11
      RETURN
C ERROR EXIT
   90 PRINT 99090, X0,Y0,Z0
      RETURN
   91 PRINT 99091, X0,Y0
      RETURN
C FORMAT STATEMENT
99090 FORMAT(1X/
     1  1X,'***   IDENTICAL INPUT POINTS.'/
     2  3X,'X0=XS=',E11.3,3X,'Y0=YS=',E11.3,3X,'Z0=ZS=',E11.3/
     3  1X,'ERROR DETECTED IN ROUTINE   CTEC2P'/)
99091 FORMAT(1X/
     1  1X,'***   IDENTICAL INPUT X AND Y VALUES.'/
     2  3X,'X0=XS=',E11.3,3X,'Y0=YS=',E11.3/
     3  1X,'ERROR DETECTED IN ROUTINE   CTEC2P'/)
      END
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Calculating the polarization angles of antennas and radio waves is necessary for analyzing the mutual interference when a linearly polarized emission is used in satellite communications. A method for calculating the polarization angle has been developed. It is based on several coordinate systems appropriate for representing the polarization angle and on a series of coordinate transformations and of projections of vectors in coordinate planes. It is applicable to a variety of cases of practical importance. Comparisons are made with some existing methods applicable to special cases, and some numerical examples are given. A computer subroutine package that implements the developed method is also presented.

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