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An Extended Single-Error-State Model for Bit Error Statistics

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AN EXTENDED SINGLE-ERROR-STATE MODEL FOR BIT ERROR STATISTICS

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Fritchman's single-error-state (SES) model for describing the error statistics of digital communication channels is modified to allow the prediction of error statistics as a function of the bit error rate (BER). From the data samples of a measurement program, cumulative distributions (CD's) of the statistics are obtained, and simple analytic relationships are derived between error statistics and the BER at any CD level. A computer program based on the extended SES model has been written that evaluates the block error rate, burst error rate, error gap distribution, and counting distribution for any desired BER. Comparisons are shown of model predictions and measured data taken over a switched telecommunications network.

Key words: bit error statistics; error statistics modeling; digital communication channels

1. INTRODUCTION

1.1 Background

During the development of digital communication systems in the last 25 years, there have been a number of error statistics models suggested by various workers. A survey paper by Kanal and Sastry (1978) provides a comprehensive overview of the subject and describes the theory and methods of many of the proposed models. Most of the individual papers presenting the models have included data comparisons and show good agreement between model and measurements.

The question naturally arises: why, after 25 years, hasn't one or more of these models been adopted by system designers or performance evaluators to predict system error statistics of interest, such as block error rate or error burst rate? The answer lies in the fact that a designer needs to know what the effects on error statistics are when the bit error rate (BER) is varied over a range of values. For instance, how does the block error rate change as BER goes from, say, 10^{-3} to 10^{-5}? Few of the models can answer this question because their predictions are valid only for a particular BER--the BER that is associated with the particular experiment or collection of measurements they

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are describing at the time. The limitations of this "characteristic" or "long-term" BER has been recognized by Johannes (1984), who discusses the need for a more practically oriented approach to error statistics modeling.

An exception to the above limitation is provided by the two-state Markov model (Crow, 1978), of which the binary symmetric channel (BSC) is a special case. The two-state Markov gives analytic relationships between the error statistics and BER and, consequently, is sometimes used under appropriate circumstances to predict performance. Unfortunately, the two-state Markov model does not, in general, characterize real channels very accurately.

1.2 Data Acquisition

In order to obtain bit error statistics over digital communication links, measurements are often made of bit error rates (BER's) and block error rates (BLER's). An interval of time is chosen, which determines the number of bits N_t transmitted during the interval. The BER is then the number of bits in error during the interval divided by N_t. For the BLER the interval is divided into blocks of N bits and a count is made of the number of blocks in which one or more bit errors occur. The BLER is then the ratio of this number to the total number of blocks in the interval. Ideally, measurements should be made for a wide range of N values.

As a practical matter when many samples are taken, a particular BER will be associated with many different values of BLER, depending on the bit error distribution within the sample interval. A plot of BLER (for a given N) versus BER often results in a scatter diagram with little apparent relationship between the two quantities. However, if cumulative distributions (CD's) of BER and BLER are obtained from a large number of samples, one-to-one relationships can be determined by reading values at the same CD level. This process, of course, can also be used to relate BER to other measurable quantities such as received signal level, signal-to-noise ratio, burst error rates, etc.

The above procedure was followed in some switched network measurements made by Bell Labs and presented in a paper by Balkovic et al. (1971). Figure 1 is the reproduction of a figure from that paper showing the cumulative distributions (for a 1200 b/s data rate) of the burst error rate, the bit error rate, and block error rates for block sizes N = 100, 500, 1000, 5000, and 10000. The ordinate, labeled "percent of calls", gives the CD levels at which the various measured quantities were determined.

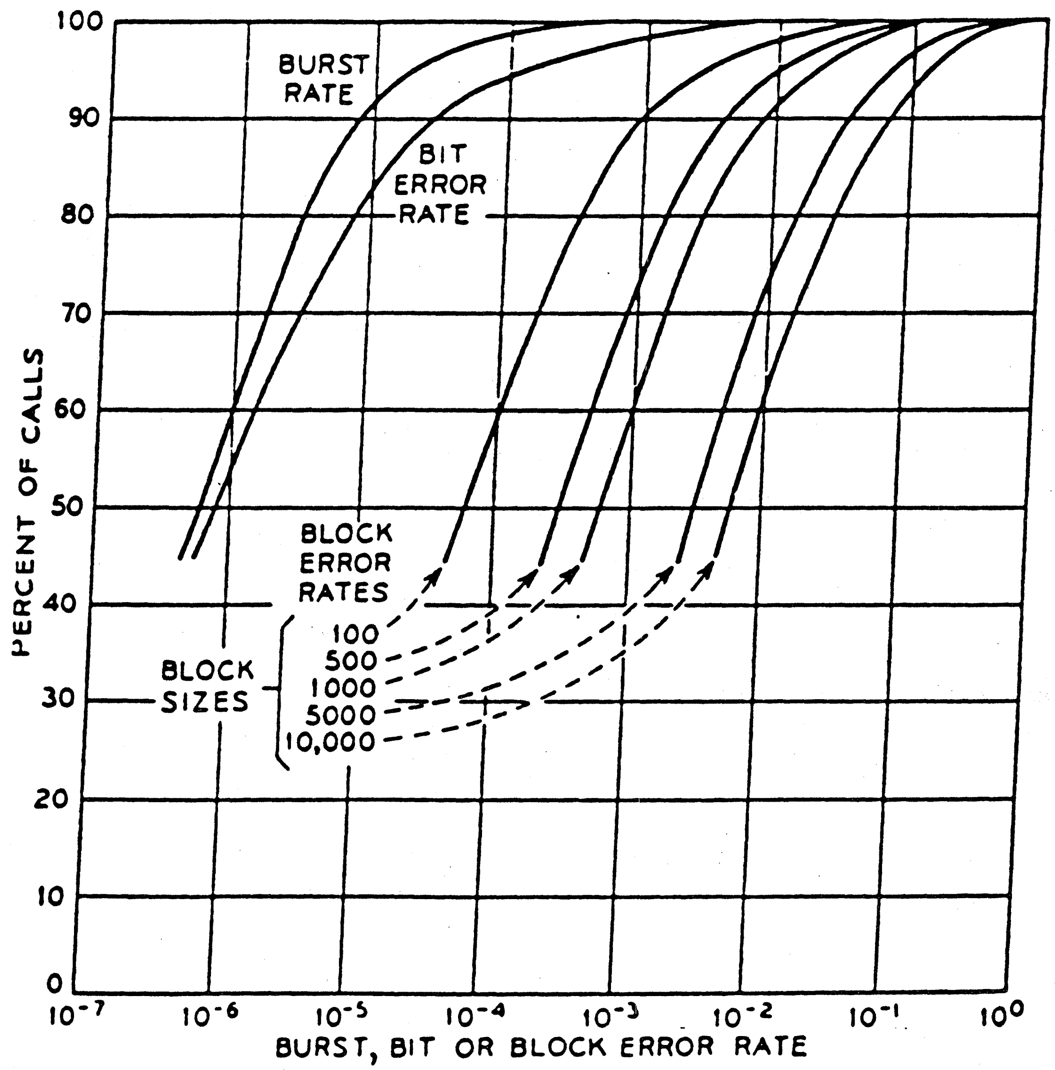

Figure 1. Reproduction of Figure 5 by Balkovic et al. (1971, p. 1362) showing measured cumulative distributions of burst, bit, and block error rates for a data rate of 1200 b/s.

1.3 Model Limitations

The samples (or calls) measured during the course of the experiment show a wide range of BER's, and this is typical of measurements over any type of data channel: switched network, HF, troposcatter, etc. Most error statistics models describe methods of obtaining the statistics of interest for a given experiment over a channel but determine only one overall, "limiting" BER; the effects of other values of BER on the statistics of the channel are difficult, if not impossible, to evaluate. In other words, no practical analytic relationship between BER and the other statistics is available.

The one exception to this appears to be the two-state Markov model and, especially, the binary symmetric channel (BSC) model, which is a special case of the two-state Markov. For instance, in a BSC the relationship between the block error probability, BLER(N), and p=BER is simply

$$BLER(N) = 1 - (1-p)^N, \qquad \text{(BSC)}. \tag{1}$$

Given any value of BER, one can easily evaluate BLER(N) for possible use in error control requirements or performance evaluations. Any finite sample yields only an estimate of a model parameter, such as p in (1), so that an exact fit cannot be expected from finite data even if the model is exactly correct. However, the parameters, such as p, and their sample estimates are not distinguished by notation in the following.

Unfortunately, the BSC (and even the two-state Markov) model is not satisfactory as a descriptor of real channels. On the other hand, more sophisticated models, while better characterizing actual conditions during a given experiment, have the disadvantage of the limiting BER problem mentioned above. In previous work (Vogler, 1986), it has been suggested that Fritchman's single-error-state (SES) model can provide a reasonable compromise between the analytic intractability of sophistication and the nonrealism of two-state Markov. Comparison of Fritchman's model with some of the measurements presented in Balkovic et al. (1971) shows reasonable agreement in such statistics as the block error rate and the cumulative counting distribution. However, the model is applicable only at the overall BER--in this case, at the 80% level in Figure 1 where BER = 7.6×10^{-6}. What to expect from the statistics of the channel at any other value of BER cannot be determined from the model.

The restriction on Fritchman's model (and, in fact, most other models except the two-state Markov) arises from the manner in which the model parameters are found from the data. No process is provided for going from one BER to another. Thus, the measured BLER at, say, the 90% level in Figure 1 cannot be estimated unless one were suddenly to assume a BSC for convenience. But a quick calculation from (1) shows that the channel is not characterized by complete independence of the bit errors.

In the next section, the SES model will be extended to allow the prediction of error statistics for any BER. The extension is based on Fritchman's model and retains the shortcomings inherent in that model, i.e., the inability to account for conditional error gap dependence. However, it has the advantage of relatively simple analytic form and is easily programmed for a computer. The validity of the model can be determined only by comparison with actual measurements and, because of the variety of error statistics it is capable of evaluating, many experiments do not collect enough data (or the right kind) to provide the necessary frequency counts for testing. However, the switched network measurement program conducted in 1969-70 by Bell Laboratories included estimates of a number of statistical quantities of interest here. These data, presented in Balkovic et al. (1971), have been used extensively to compare with the extended model. Besides the measurements shown in Figure 1, other (fixed-BER) statistics from the paper are shown in a later section.

2. Formulation of Extended SES Model

The analytic form of BER (=p) in a single-error-state model is (see Appendix A)

$$p = \left[1 + \sum_{i=1}^{NS-1} A_i \rho_i / (1 - \rho_i)\right]^{-1} \tag{2}$$

where NS denotes the number of Markov states and the A_i, ρ_i may be determined by fitting to measured data. It can be noticed that a sequence of BER values can be formed by successively eliminating the (end) terms of the summation; and, furthermore, the sequence will be monotonically increasing if we stipulate that $\rho_i < \rho_{i+1}$ and $A_i > 0$. Thus, we have the sequence

$$p_S = [1 + \sum_{i=1}^{S} A_i\rho_i/(1 - \rho_i)]^{-1},\ S = 1,2,\ \ldots,\ NS - 1 \tag{3}$$

$$\rho_i < \rho_{i+1};\ p_S > p_{S+1}.$$

The variable p can now be made to take on continuous values from 0 to 1 by assuming certain conditions on the A_i and ρ_i, conditions that allow a smooth variation from one fitted p value to the next. One of the simplest conditions is to use the following form for the continuous variable p:

$$p = [1 + SP + \hat{A}\,\hat{\rho}/(1 - \hat{\rho})]^{-1}, \tag{4}$$

$$SP \equiv \sum_{i=1}^{S-1} A_i\rho_i/(1 - \rho_i),$$

where $\hat{A}$ is defined as

$$\hat{A} = cA_S(\hat{\rho} - \rho_{S-1})/\hat{\rho}(\rho_S - \rho_{S-1}),$$

$$c = \ln\{p/p_{S-1}\}/\ln\{p_S/p_{S-1}\}. \tag{5}$$

It can then be shown that, for a given value of p such that $p_{S-1} \geq p \geq p_S$,

$$\hat{\rho} = (\rho_{S-1} + C_1)/(1 + C_1), \tag{6}$$

where

$$C_1 = \{(1 - p)/p - SP\}(\rho_S - \rho_{S-1})/cA_S. \tag{7}$$

To extend the range of p to zero ($p < p_{NS-1}$), we define ρ_{NS}, A_{NS}, and c as unity in the equations for $\hat{A}$ and $\hat{\rho}$.

Equation (4), along with (5) and (6), provides relationships between a variable p and the fitting parameters A, ρ used in the SES model. Furthermore, the p function has the required values at the interval endpoints; i.e., at $p = p_{S-1}$: $\hat{A}\,\hat{\rho}/(1 - \hat{\rho}) = 0$,

at $p = p_S$: $\hat{A}\,\hat{\rho}/(1 - \hat{\rho}) = A_S \rho_S/(1 - \rho_S)$.

In Fritchman's model the A_i, ρ_i are calculated from measured error gap distribution (EGD) data, and the parameters in turn then determine a fixed value of BER. If predictions of error statistics at some other BER are desired, another set of measured samples must be acquired with the hope that the measured BER will be somewhere near the desired BER.

In the present approach, a more convenient statistic for fitting is the block error rate, BLER(N), at a particular N. For instance if CD's of measured BLER(N) and BER are known as in Figure 1, the parameters A and ρ can be found in the following way.

The expression for the block error rate (BLER=B) in the SES model can be given as [see Appendix A, equations (A2) and (A3)]

$$\{1 - B(N)\}/p = SB + A_x \, \rho_x^N/(1 - \rho_x), \tag{8}$$

$$SB \equiv \sum_{i=1}^{S-1} A_i \, \rho_i^N/(1 - \rho_i),$$

where S = 1, 2,..., NS-1, and A_x, ρ_x denote values to be determined for a given p (see below). It is to be understood that SB and SP are identically zero when S = 1.

Since BLER(1) = BER = p, (8) provides two relationships between BLER and p, i.e., N = 1 and the value of N selected for the fitting. Choosing NS-1 pairs of values, B(N) and p (at a fixed N), A_x and ρ_x in (8) are calculated at each pair by

$$\rho_x = \exp[(\ln R)/(N - 1)], \tag{9}$$

$$A_x = \{(1 - p)/p - SP\}(1 - \rho_x)/\rho_x \tag{10}$$

where

$$R = \rho_x^{N-1} = \{(1 - B)/p - SB\}/\{(1 - p)/p - SP\}. \qquad (11)$$

The order of evaluation is to start with the largest valued (B, p) pair (designated in the computer program of Appendix B as B_1, p_1). The A's and ρ's are then determined successively from each data pair as shown in (8) - (11).

The above procedure, along with (4), allows the numerical evaluation of A_i, ρ_i for any desired p. With one additional condition [from A(14)],

$$EGD(1) = \sum_i A_i \rho_i \leq 1, \qquad (12)$$

we can now computerize and predict single-error-state error statistics as functions of the bit error rate for any practical measurement system.

3. Comparisons of Model and Measurements

An example using the extended SES model is shown in Figure 2 for the Balkovic et al. (1971) data from Figure 1. The A_i, ρ_i were determined from the BLER(1000) measurements and result in the solid curve of Figure 2. These same A_i, ρ_i are then used to generate the curves for BLER(10^4) and BLER(100). It can be seen that the model predicts fairly good agreement with the N = 100 and 10^4 data.

Data points at four values of BER were used in the fitting process: BER = 10^{-6}, 10^{-5}, 10^{-4} and 10^{-3}. Since the distributions were not sufficiently detailed to give accurate readings at larger BER, two pairs of A_i, ρ_i (obtained from measured EGD data) were assumed in this region: (A_1, ρ_1) = (0.3087, 0.6521) and (A_2, ρ_2) = (0.16, 0.990278); thus, altogether, six pairs of A,ρ were determined. The computer algorithm that determines the fitting parameters allows the user to adjust arbitrary values--those that cannot be obtained directly from the measured distributions--in accordance with the criterion of (12). Ideally, the data distributions should show enough detail over the whole range of BER so that assumed values are unnecessary. The data point at BER = 0.003 was not used in the fitting process because of the uncertainty involved in reading Figure 1 at this BER value.

The question arises as to whether the choice of an N other than 1000 will affect the prediction capability. This is partially answered in the next two figures where the A_i, ρ_i are determined from BLER(100) data in Figure 3 and from BLER(10^4) data in Figure 4. Predictions of the remaining two block error rates in each case again agree fairly well with the measured data. Further

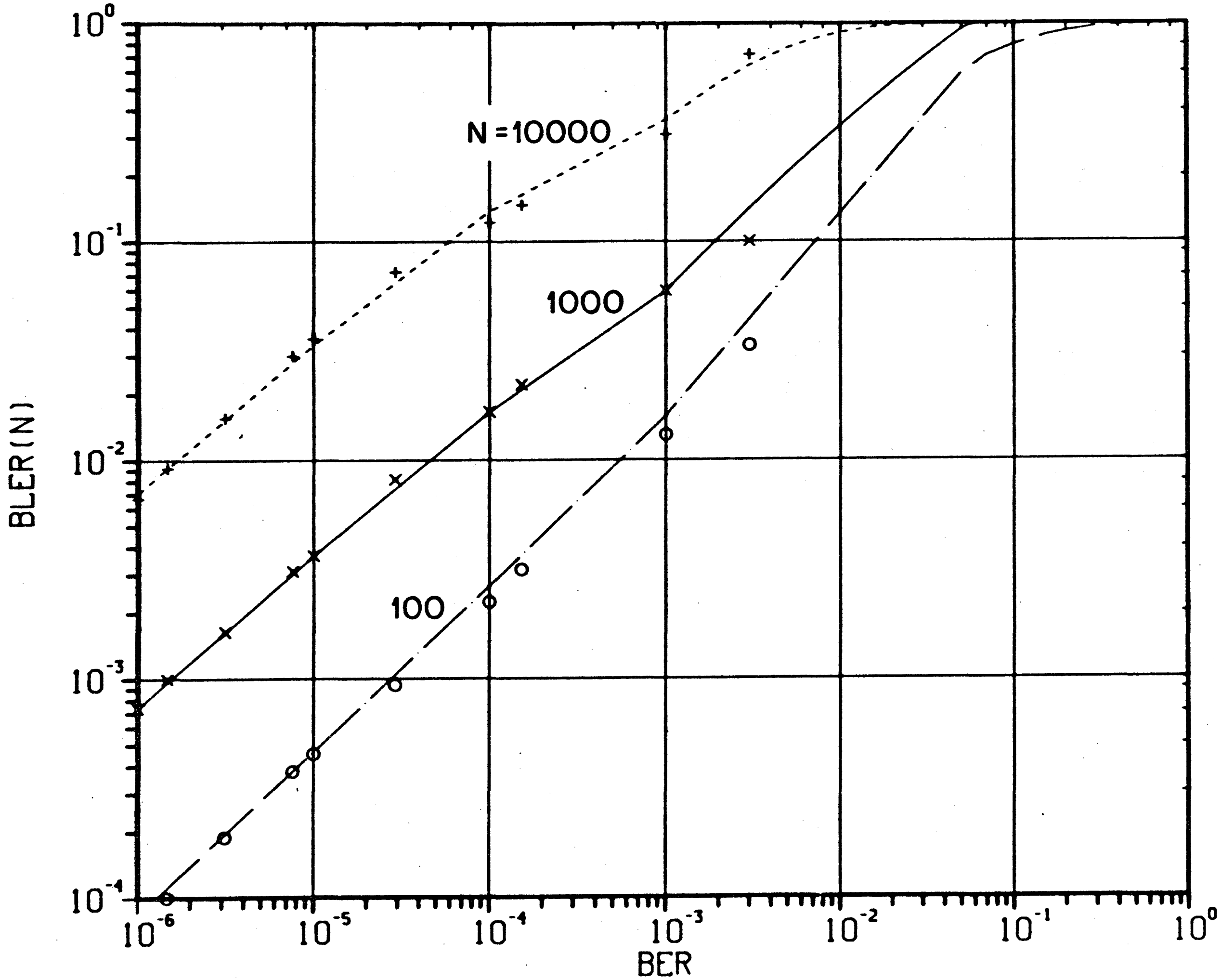

Figure 2. Prediction curves and measured points of block error rates, BLER(N), versus BER for block sizes N = 100, 1000, 10000. Model parameters were obtained from BLER(1000) data (Balkovic et al., 1971; p. 1362).

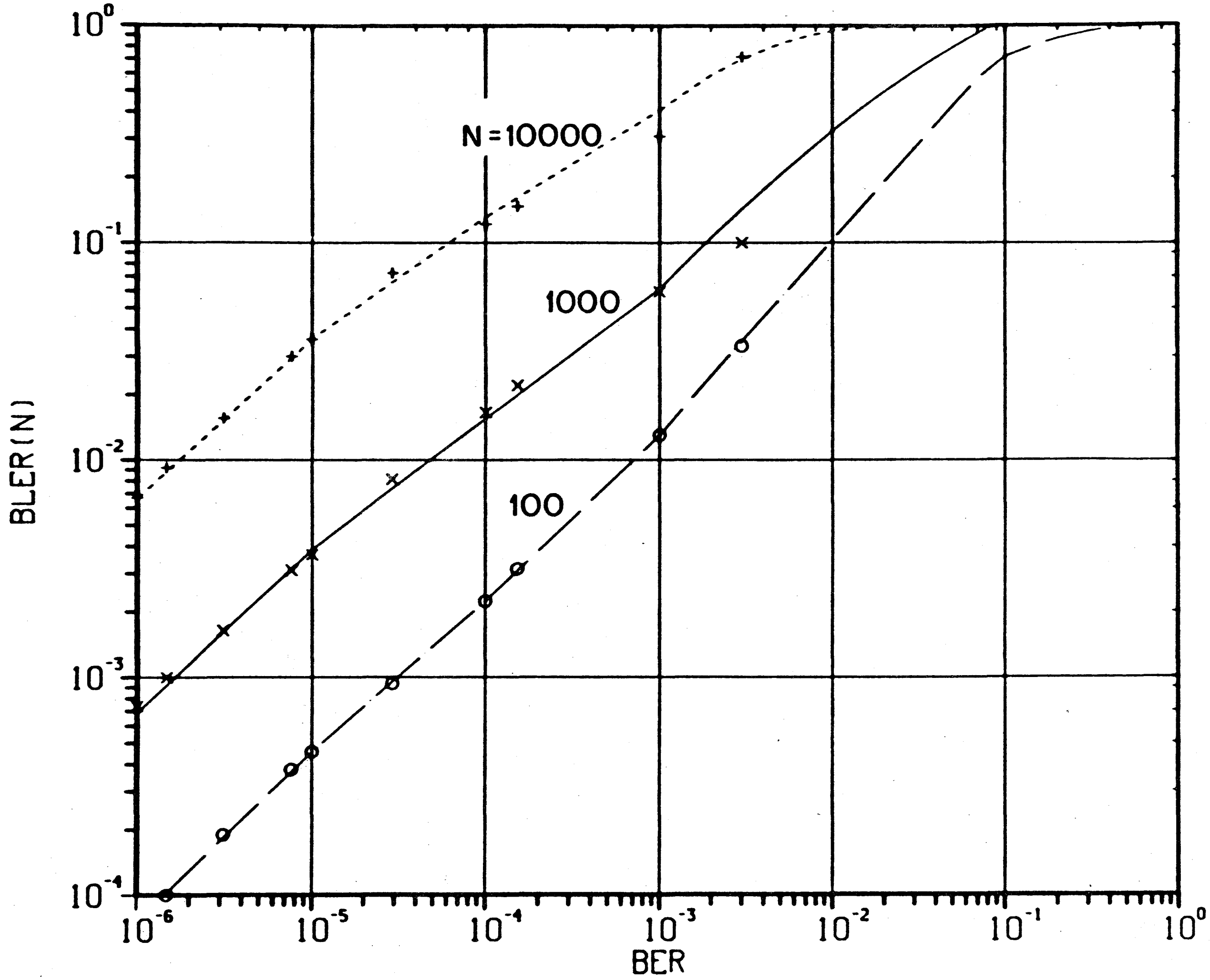

Figure 3. Prediction curves and measured points of block error rates, BLER(N), versus BER for block sizes N = 100, 1000, 10000. Model parameters were obtained from BLER(100) data (Balkovic et al., 1971; p. 1362).

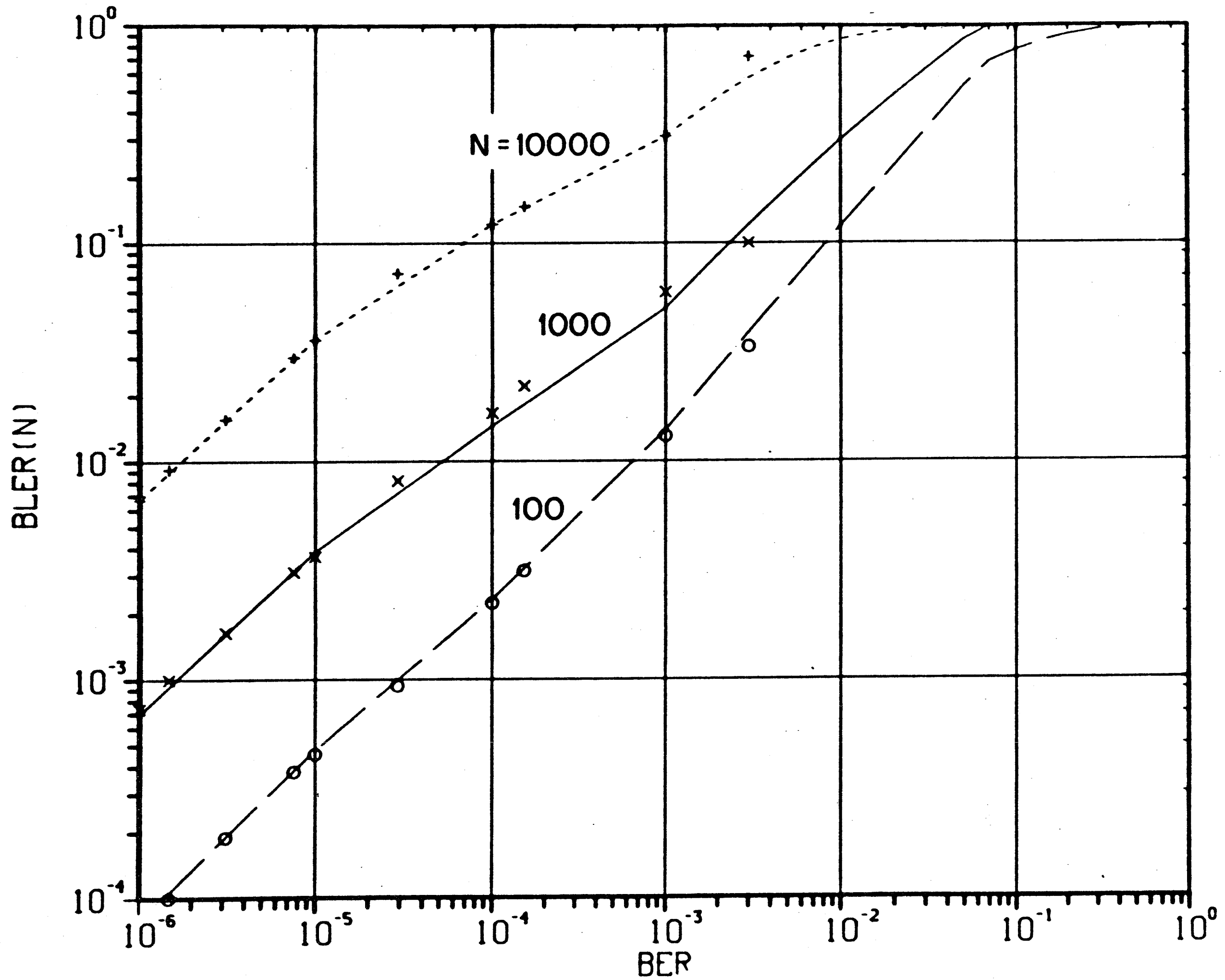

Figure 4. Prediction curves and measured points of block error rates, BLER(N), versus BER for block sizes N = 100, 1000, and 10000. Model parameters were obtained from BLER(10000) data (Balkovic et al., 1971; p. 1362).

study would be needed to determine if there is an optimum block length for the initial fit. It is likely that smaller block lengths would be better simply because of practical considerations in taking measurement samples.

Model validation of other statistics also is possible using the Balkovic data. For instance, an error burst is defined to be a succession of one or more bits beginning and ending with an error and separated from neighboring bursts by K or more error free bits (Balkovic et al., 1971, p. 1361). Burst rate, PB(K), is the number of bursts in a sample divided by the number of bits in the sample.

With the observation that the beginning of a burst must follow gaps with lengths $\geq$ K, then it follows that the total number of bursts is equal to the total number of gaps with lengths $\geq$ K. Thus,

$$\begin{aligned} PB(K) &= \frac{\text{number of bursts}}{\text{number of bits}} = \frac{\text{number of bursts}}{N_e/p} \\ &= p\ EGD(K), \end{aligned} \tag{13}$$

where $p = BER = N_e/N_t$, and N_e, N_t are the total number of bit errors and total number of bits, respectively. EGD(K) is the usual error gap distribution (see Appendix A).

The SES model burst rate for K = 50 (the value used in the Balkovic data) is shown in Figure 5 and compared with the measured data from Figure 1. The A_i, ρ_i are those used in Figure 2--the N = 1000 fit. Again, there is fairly good agreement between model and measurement, especially at smaller BER. If the A_i, ρ_i from Figures 3 or 4 are used, the agreement is similar to that shown in Figure 5.

The block error probability at BER = 10^{-6}, 10^{-5}, 10^{-4}, and 10^{-3} using the A_i, ρ_i of Figure 2 is shown in Figure 6 together with data points at block lengths of 100, 500, 1000, 5000, and 10^4. The agreement shown could have been expected from a study of the results in Figure 2, but the added data points at N = 500 and 5000 help to validate the model. Comparisons at other values of BER have been made and, in all cases tested, the agreement for BLER(500) and BLER(5000) is at least as good as that indicated in Figure 6. Let it again be emphasized that, as far as user input to the computer program is concerned, the comparisons at fixed BER involve only the single set of fitting parameters used

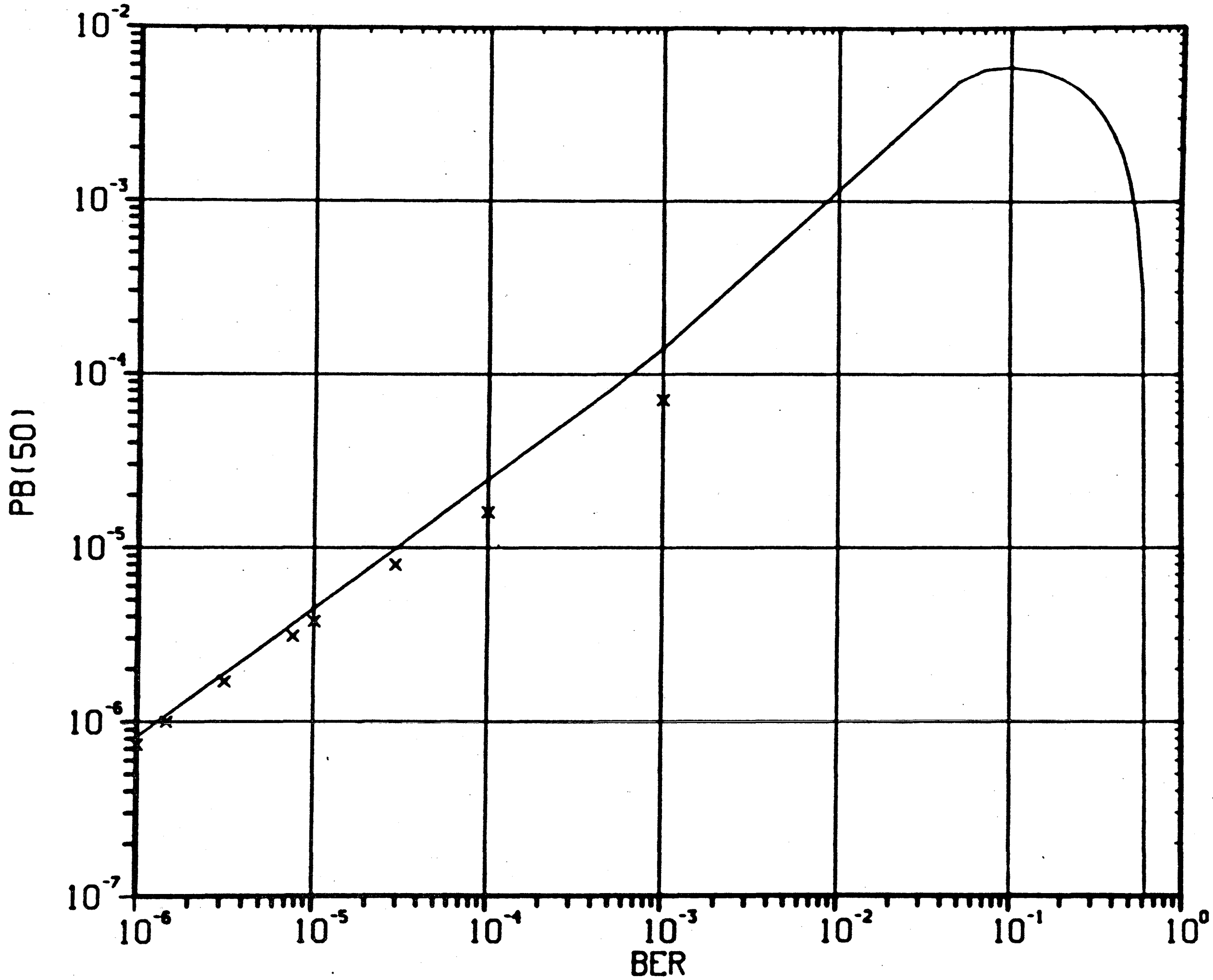

Figure 5. Prediction curve and measured points of the error burst rate, PB(K), versus BER for the burst parameter K = 50. Model parameters are the same as in Figure 2, and measured data is from Balkovic et al. (1971, p. 1362).

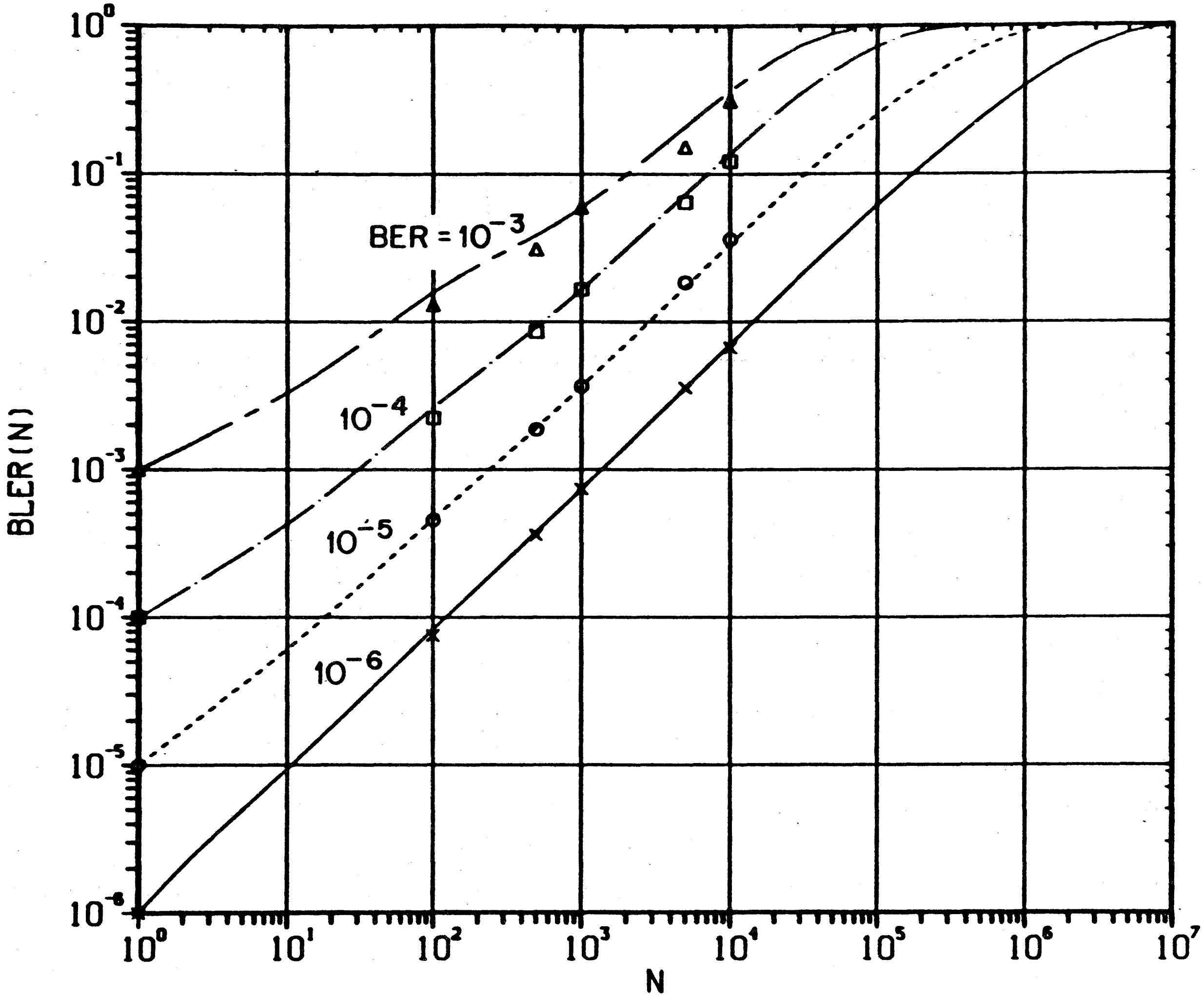

Figure 6. Block error rates versus block size N for BER = 10^{-3}, 10^{-4}, 10^{-5}, and 10^{-6}. Model parameters are the same as in Figure 2, and measured data is from Balkovic et al. (1971, p. 1362).

in Figure 2; i.e., the six pairs of A_i, ρ_i that were determined from the BLER(1000) fit.

Another statistic for which measured data are available is the error gap distribution (EGD). The Balkovic EGD data (Figure 17 of that paper using the curve labeled "Pooled") appears to be characterized by a BER of about 7.6×10^{-6}, i.e., the BER at the 80% level of Figure 1. The comparison of data and model (using the A_i, ρ_i of Figure 2) is shown in Figure 7. It is seen that predicted probabilities of gap lengths greater than about 1000 are noticeably optimistic, and agreement is not as close as some of the other statistics. Whether this is consistently so at other BER cannot be answered because the data are not available.

Yet another statistic for which comparisons can be made is the cumulative counting distribution PC(M,N): the probability of M or more errors in a block of N bits. Measured data are presented in Balkovic et al. (1971, Figure 14, p. 1371) for BER = 7.6×10^{-6}, and some comparisons are shown in Figure 8. The model is consistently high in the middle portions of the curves for N = 200, 500, and 1000. Perhaps this is caused by some conditional error-free-gap dependency at larger gap lengths that cannot be accounted for in a single error state model. The data points at M = 10 are not predicted very well by the model nor are the points at M = 1. In the latter case one would expect better agreement with the measurements because PC(1, N) = BLER(N); but the Balkovic 1200 b/s data shows two different values in this case. Whether different CD levels have been used for the two quantities cannot be determined from the text.

Finally, in Figure 9, comparisons are made of the cumulative burst length distribution PBC(K,L) for a gap characteristic of K = 50 and BER = 7.6×10^{-6}. The measured data are taken from the 1200 b/s curve in Figure 12, p. 1369 of the Balkovic paper, and the solid curve shows the prediction of the extended SES model. It is apparent that a first order Markov process does not accurately portray the distribution, although the values are within a factor of two for $L < 40$. It should also be recognized that a valid frequency count of long burst lengths requires a very large number of samples, and the data values near 100 are less reliable. To indicate the superiority of an SES model over the two-state Markov, the latter prediction is shown as the dashed curve in Figure 9. A much improved prediction is possible if we are given the added

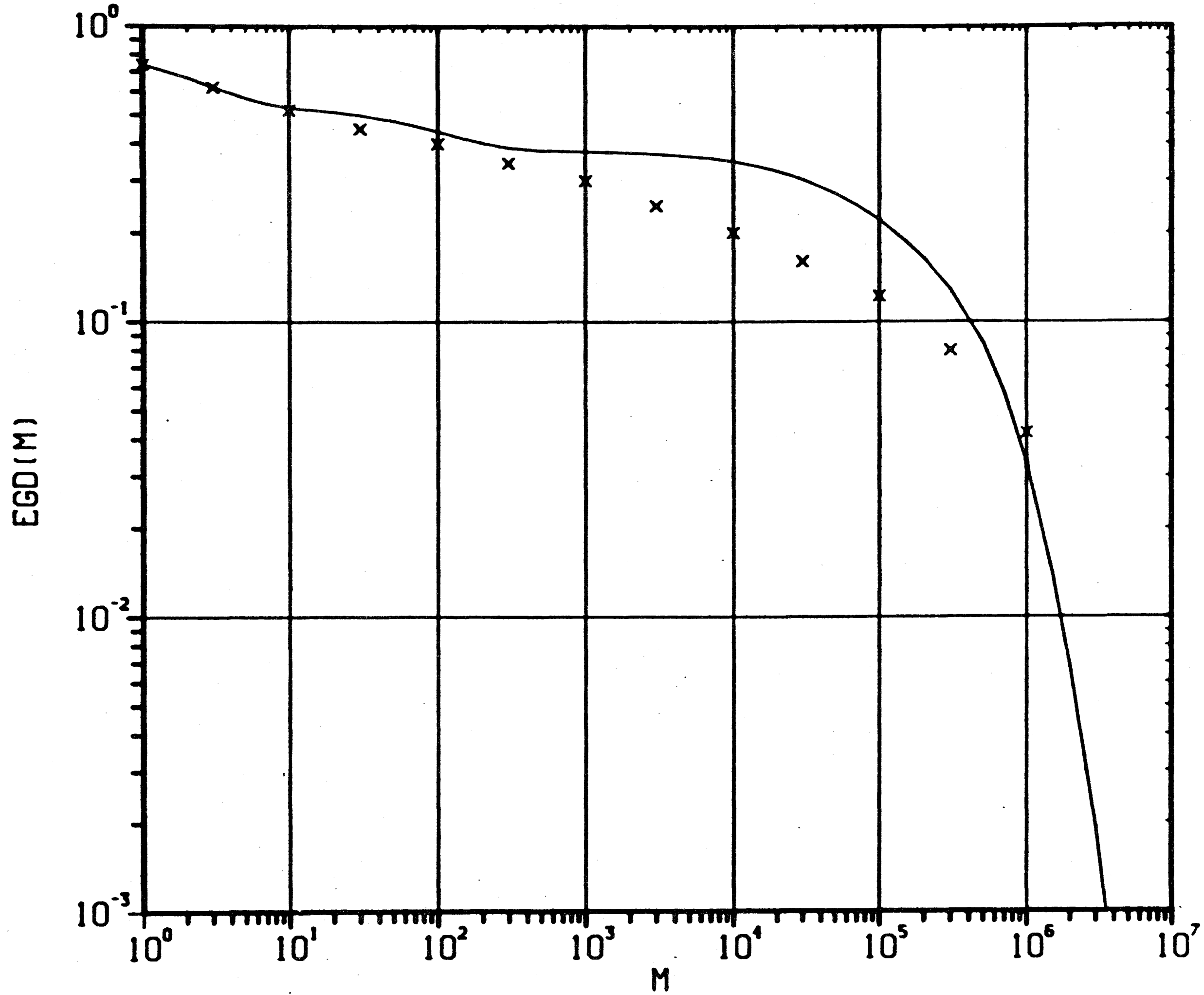

Figure 7. Prediction curve and measured points of the error gap distribution EGD(M) for BER = 7.6×10^{-6}. Model parameters are the same as in Figure 2, and measured data is from Balkovic et al. (1971, p. 1373).

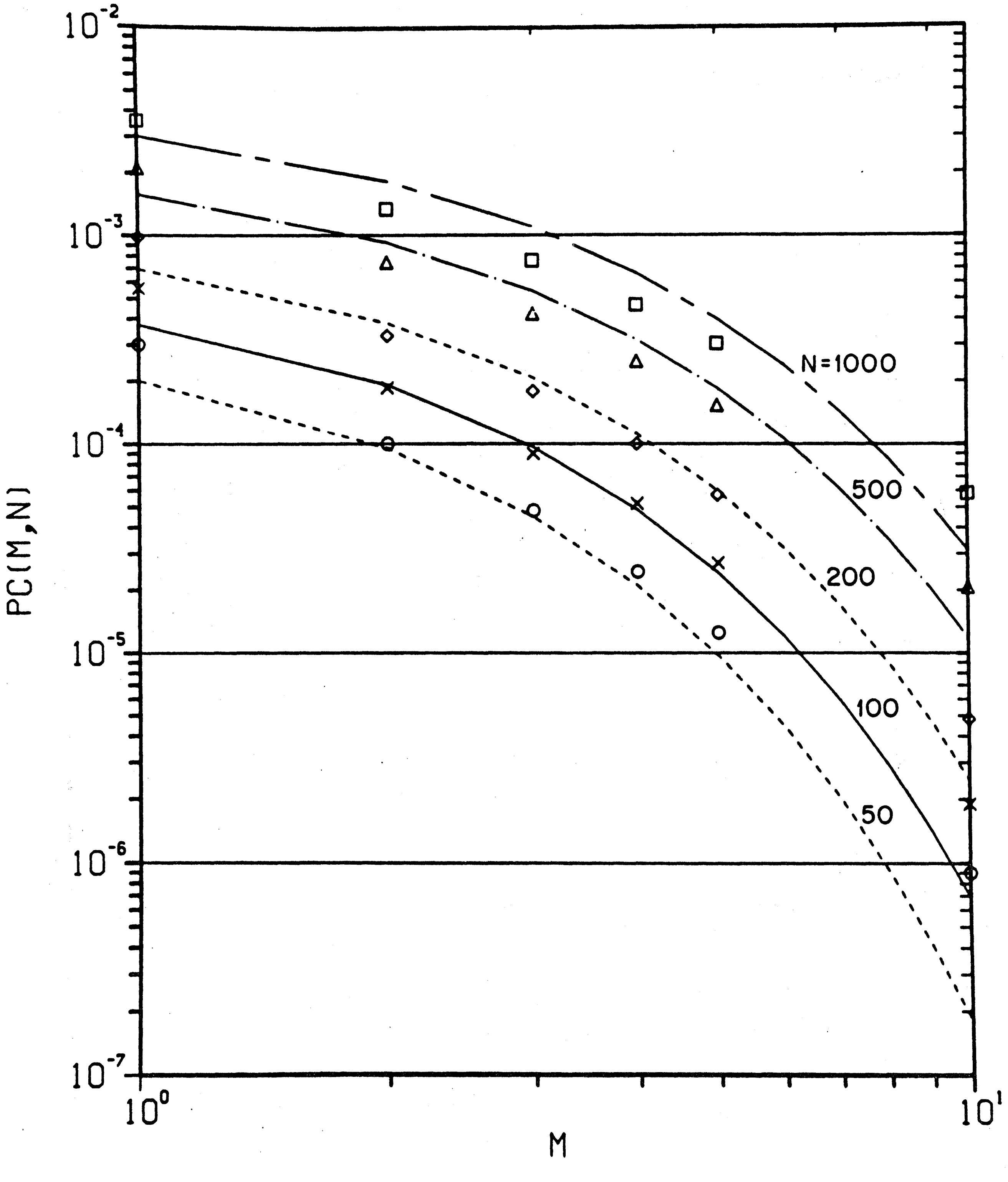

Figure 8. Prediction curves and measured points of the cumulative counting distribution PC(M,N) for BER = 7.6×10^{-6}. Model parameters are the same as in Figure 2, and measured data is from Balkovic et al. (1971, p. 1371).

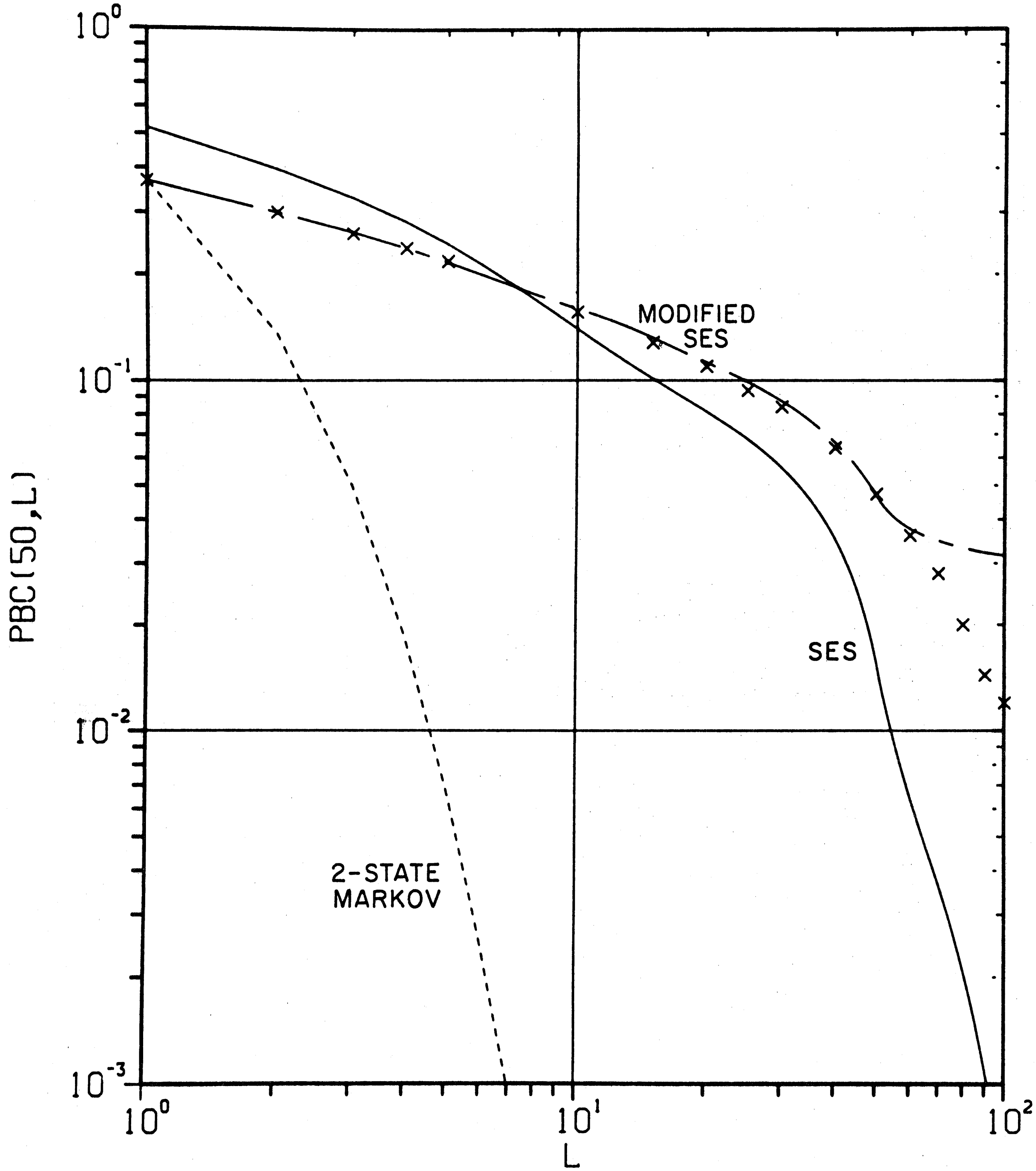

Figure 9. Prediction curves and measured points of the burst length distribution PBC(50,L) for BER = 7.6×10^{-6}. The solid curve is the extended SES model (same parameters as in Figure 2) and measured data is from Balkovic et al. (1971, p. 1369).

information of the two measured values, PBC(K,1) and PBC(K,2). Denoting these by P_{m1} and P_{m2}, we form the factors $\alpha_0=(1-P_{m1})/(1-P_{s1})$ and $\alpha_1=(P_{m1}-P_{m2})/(P_{s1}-P_{s2})$, where P_{s1} and P_{s2} denote the corresponding SES values. Using these factors to multiply appropriate terms of the equation for PBC (see (A17), Section A.4) results in the curve labeled "Modified SES" in Figure 9. This curve, of course, cannot be derived strictly from first order Markov theory, but is simply a semi-empirical approximation to the burst length distribution.

4. CONCLUSION

4.1 Summary

The single-error-state model presented here evaluates error statistics as functions of the bit error rate. Previous models develop procedures for calculating the statistics, but the results are applicable only to a characteristic BER that is determined from some particular sample collection. An exception, of course, is the two-state Markov model, which provides analytic relationships between the error statistics and BER. This model (and particularly the BSC) is often used for design predictions simply because it is one of the few models available that allows the engineer to estimate the statistical effects of varying the BER. However, the two-state Markov model is, in general, not representative of real channels.

In this paper an extended form of the SES model has been developed and compared with some measured switched network error statistics. Reasonable agreement is found for the block error rate and the burst error rate as a function of BER. Somewhat poorer agreement is found in the comparisons with the error gap distribution and the cumulative counting distribution at a particular BER.

The model has been implemented in a computer program that calculates the block error rate, burst error rate, error gap distribution, counting distribution, and cumulative counting distribution. Also included in the program is the procedure for evaluating the fitting parameters, which are obtained simply by entering pairs of BLER versus BER measured data.

Because data sampling on a real channel is essentially a statistical process, the entity being sampled will have a range of values, with little apparent relationship to any other statistic during any one particular sample. The most reasonable way to organize the data is to form cumulative distributions of the statistics of interest (BER, BLER, etc.) from the collection of sample measurements; this then will provide one-to-one correspondences among the statistics at any particular CD level. The organization of the data in the above manner is implicit in the extended SES model presented here.

4.2 Future Studies

Although the comparisons of model and switched network measured data in the preceding sections serve as an initial evaluation of the model, it is highly desirable to compare data from other types of communication channels. Measurement over HF, VHF, troposcatter, and satellite channels should be undertaken and efforts made to acquire a comprehensive set of statistical data. The quantities of interest should include not only those concerned with link performance objectives, but also those useful in the evaluation of efficient coding schemes.

Future studies might also wish to investigate whether characteristic values of the fitting parameters can be used to describe different types of communication channels. The establishment of a criterion such as this would greatly aid system designers in developing their required performance objectives. In network design, the overall evaluation of various combinations of individual link dependence could be investigated in considerable detail since link statistics are easily calculated once a set of fitting parameters are given.

A related problem for future consideration is to determine if there is a distribution or family of distributions that can describe BER. If, for different channel types under prescribed conditions, "characteristic" BER distributions can be found, then a complete and fully analytic description of statistical information is available. Furthermore, the statistics can be related directly to transmission parameters such as received signal levels and signal-to-noise ratios. Notice that this is exactly the procedure followed in most texts on digital communication; however, the discussions are, by necessity, limited to very simple channels such as the BSC. The development of analytic relationships for more realistic channels will broaden the application of theory to actual systems.

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APPENDIX A: EQUATIONS OF THE SES MODEL

A.1 The Block Error Rate, BLER(N)

The single-error-state (SES) model developed by Fritchman (1967) can be described as a multistate Markov chain, one state of which is denoted the error state. The state transition probability matrix and state diagram are shown in Figure A-1, which indicates some of the notation used in later equations and also shows the relationships between the transitions and the parameters A, ρ introduced by Fritchman. The number of states is denoted by the symbol NS, and this symbol when used as a subscript also identifies parameters associated with the error state. Note that transitions between nonerror states (1, 2, ..., NS-1) are not allowed.

Since the model is based on a Markov process, one may calculate the state equilibrium probabilities π_i (Cox and Miller, 1965; pp. 101ff), which in terms of A and ρ ($A > 0$, $0 < \rho < 1$) become

$$1/\pi_{NS} = 1 + \sum_{i=1}^{NS-1} A_i\rho_i/(1 - \rho_i), \quad (\pi_{NS} = \text{BER} = p), \tag{A1a}$$

$$\pi_i = \pi_{NS} A_i\rho_i/(1 - \rho_i), \quad i = 1, \ldots, NS-1. \tag{A1b}$$

Using these results along with the transition probabilities of Figure A-1, one may derive a simple expression for the block error rate BLER(N), the probability of one or more errors in a block of N bits. If we denote the probability of no errors in a block of N bits by P(0,N), then

$$P(0,N) = \sum_{i=1}^{NS-1} \pi_i\rho_i^{N-1} = p \sum_{i=1}^{NS-1} A_i\rho_i^N/(1 - \rho_i), \quad N \geq 1, \tag{A2}$$

and

$$\text{BLER}(N) = 1 - P(0,N). \tag{A3}$$

Equation (A2) can also be obtained from the formula for P(0,N) given by Kanal and Sastry (1978). The usual notational convention is assumed wherein a "0" denotes a correct bit, a "1" denotes an error, and a positive integer exponent indicates the number of consecutive identical bits. Thus, a string of

$$
P = \begin{bmatrix} P_{11} & 0 & \cdots & P_{1,NS} \\ 0 & P_{22} & \cdots & P_{2,NS} \\ \vdots & \vdots & & \vdots \\ P_{NS,1} & P_{NS,2} & \cdots & P_{NS,NS} \end{bmatrix} = \begin{bmatrix} \rho_1 & 0 & \cdots & (1-\rho_1) \\ 0 & \rho_2 & \cdots & (1-\rho_2) \\ \vdots & \vdots & & \vdots \\ A_1\rho_1 & A_2\rho_2 & \cdots & \rho_{NS} \end{bmatrix}
$$

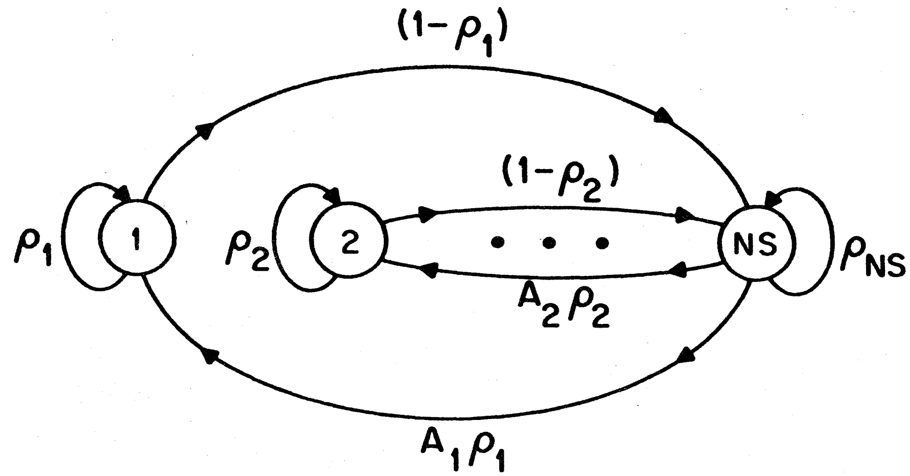

Figure A-1. The transition probability matrix P and state diagram for Fritchman's single-error-state model.

k correct bits between two incorrect bits would be written as 10^k1. From Kanal and Sastry (1978),

$$P(0,N) = 1 - p\,[N - \sum_{j=1}^{N-1} (N-j)P(0^{j-1}1/1)],\ N \geq 1, \tag{A4}$$

where $P(0^{j-1}1/1)$ is the conditional probability of the bit sequence $0^{j-1}1$ given an error. As stated above, (A4) may be algebraically reduced to (A2) in the case of the SES model.

A.2 Counting Distributions, P(M,N) and PC(M,N)

The counting distribution P(M,N) is defined as the probability of exactly M errors in a block of N bits. Elliott (1965) has shown that, in terms of the error statistic probabilities, P(M,N) is given by

$$P(M,N) = p \sum_{k=0}^{N-M} P(0^k/1)R(M,N-k),\ 1 \leq M \leq N, \tag{A5}$$

where

$$R(M,i) = \sum_{k=0}^{i-M} P(0^k1/1)R(M-1,\ i-k-1),\ 2 \leq M \leq i, \tag{A6a}$$

$$R(1,i) = P(0^{i-1}/1),\ i \geq 1. \tag{A6b}$$

Simpler expressions, comparable to (A2), apparently are not available generally for P(M,N), although three special cases are derivable for the SES model:

$$P(N,N) = p\,\rho_{NS}^{N-1},\ N \geq 1; \tag{A7}$$

$$P(N-1,N) = p\,\rho_{NS}^{N-2}\,[2(1 - \rho_{NS}) + (\frac{N-2}{\rho_{NS}}) \sum_{i=1}^{NS-1} A_i\rho_i(1-\rho_i)],\ N \geq 2; \tag{A8}$$

$$P(1,N) = p[\sum_{i=1}^{NS-1} A_i \rho_i^{N-1} \{(N-2)A_i + 2\}$$

$$+ 2 \sum_{j \neq k} (A_j \rho_j)(A_k \rho_k)\{(\rho_j^{N-2} - \rho_k^{N-2})/(\rho_j - \rho_k)\}], \quad \text{(A9)}$$

$$j, k = 1, \ldots, NS - 1; \; N \geq 2.$$

The algorithm that calculates P(M,N) in the computer program is based only on (A5) and, because of the recurrence relations in (A6), requires more computer time as N becomes large.

The cumulative counting distribution PC(M,N) is defined as the probability of M or more errors in a block of N bits and is computed from P(M,N) by

$$PC(M,N) = \sum_{k=M}^{N} P(k,N), \; 0 \leq M \leq N, \; N \geq 1. \quad \text{(A10)}$$

From the preceding definitions, it is apparent that the following relationships hold:

$$PC(0,N) = 1, \quad \text{(A11a)}$$

$$PC(1,N) = 1 - P(0,N) = BLER(N), \quad \text{(A11b)}$$

$$PC(1,1) = P(1,1) = BLER(1) = BER. \quad \text{(A11c)}$$

A.3 The Error Gap Distribution, EGD(M)

The bit sequence $10^{\ell-1}1$ delineates what is defined as an error gap (or error-free gap) that is characterized by a gap length, GL = ℓ (ℓ = 1, 2, 3, ...). The error gap distribution (EGD) may be written as

$$EGD(\ell-1) = Pr\{GL \geq \ell\} = \sum_{k=\ell}^{\infty} P(0^{k-1}1/1)$$

$$= P(0^{\ell-1}/1), \quad \ell \geq 1. \tag{A12}$$

From Figure A-1 and the relationships of (A12), it follows that, in the SES model,

$$EGD(M) = \sum_{i=1}^{NS-1} A_i \rho_i^M, \quad M \geq 1, \tag{A13}$$

$$EGD(1) = \sum_{i=1}^{NS-1} A_i \rho_i = 1 - \rho_{NS}. \tag{A14}$$

Equation (A14) is important in that it serves as a condition that must be satisfied when determining the fitting parameters A_i, ρ_i [see (12), Section 2]. Furthermore, a measured value of $\rho_{NS} = P(1/1)$ at a particular BER can be useful for estimating A_1, ρ_1 in those situations where the CD's of BLER(N) and BER show insufficient detail at larger BER.

A.4 Error Burst Statistics

An error burst of gap and length characteristics K and L, respectively, is defined to be the pattern, $1\beta 1$, where β is of length (L-1) bits and may contain any succession of 0's or 1's except $0^i (i \geq K)$; the burst pattern is separated from neighboring bursts by K or more error-free bits. Using this definition, we can express the conditional probability of the burst pattern, $P(\beta 1/1) = BC(K,L)$, as

$$BC(K,L) = \begin{cases} \sum_{\ell=0}^{L-1} P(0^{L-1-\ell}1/1)BC(K,\ell), & L \leq K \\ \sum_{\ell=L-K}^{L-1} P(0^{L-1-\ell}1/1)BC(k,\ell), & L > K \end{cases} \tag{A15}$$

where $BC(K,0) = 1$ and $K \geq 1$. The function $BC(L,L)$ is identical to the autocorrelation function defined by Elliott (1965).

The burst error rate $PB(K,L)$ is defined as the probability of occurrence of a burst and, since a burst is always preceded and followed by the patterns 10^i and $0^i1 (i \geq K)$, it follows that

$$PB(K,L) = pP(0^K/1)P(\beta 1/1)P(0^K/1)$$

$$= p[EGD(K)]^2 BC(K,L), \quad (K,L \geq 0) \qquad (A16)$$

where $EGD(0)$ and $BC(0,0)$ are defined as unity.

The cumulative distribution over L of error bursts for a constant K, $PBC(K,L)$, may be obtained from

$$PBC(K,L) = \{1/pEGD(K)\} \sum_{\ell=L}^{\infty} PB(K,L)$$

$$= EGD(K) \sum_{\ell=L}^{\infty} BC(K,\ell), \qquad (A17)$$

where $PBC(K,0) = 1$. Note that the summation of (A16) over $0 \leq L \leq \infty$ is simply $pEGD(K) = PB(K)$, which is the same expression given by (13) in Section 3. This follows by expanding the summation and rearranging the terms.

A.5 The Two-State Markov and BSC Models

It is often interesting to compare the two-state Markov model with measurements, and, for convenience, this section lists formulas of some of the more common statistics. We assume the independent variables to be p = BER and $\rho_2 = P(1/1)$; a BSC is the special case, $\rho_2 = p$. Then

$$A_1 = (1 - \rho_2)/\rho_1, \ \rho_1 = (1 - 2p + p\rho_2)/(1 - p), \quad \text{(2-state)}$$

$$A_1 = 1, \ \rho_1 = 1 - p, \quad \text{(BSC)} \qquad (A18)$$

and

$$\mathrm{BLER}(N) = \begin{cases} 1 - (1 - p)\rho_1^{N-1}, & \text{(2-state)} \\ 1 - (1 - p)^N, & \text{(BSC)} \end{cases} \tag{A19}$$

$$\mathrm{EGD}(M) = \begin{cases} (1 - \rho_2)\rho_1^{M-1}, & \text{(2-state)} \\ (1 - p)^M, & \text{(BSC)} \end{cases} \tag{A20}$$

$$\mathrm{PB}(K) = \begin{cases} p(1 - \rho_2)\rho_1^{K-1}, & \text{(2-state)} \\ p(1 - p)^K, & \text{(BSC)} \end{cases} \tag{A21}$$

The counting distribution for the two-state Markov model is given by (A5) and (A6), and PB(K,L) is given by (A16) and (A15) where

$$P(0^k/1) = \begin{cases} (1 - \rho_2)\rho_1^{k-1}, & k > 0 \\ 1, & k = 0 \end{cases} \tag{A22}$$

$$P(0^k 1/1) = \begin{cases} (1-\rho_2)(1-\rho_1)\rho_1^{k-1}, & k > 0 \\ \rho_2, & k = 0 \end{cases} \tag{A23}$$

For a BSC,

$$P(M,N) = \binom{N}{M} p^M (1 - p)^{N-M}, \qquad \text{(BSC)} \tag{A24}$$

where $\binom{N}{M}$ denotes the binomial coefficient.

APPENDIX B: THE COMPUTER PROGRAM

This appendix contains a Fortran listing of the computer program PRBTAB that calculates and tabulates bit error statistics based on the extended SES model. The fitting parameters A_i, ρ_i can be either read in directly or evaluated by reading in pairs of BLER(N) versus BER measured data. The program is interactive, with data and responses being entered from a terminal keyboard. After choosing a desired statistic (BLER, EGD, etc.), the user enters a data point (usually either a block size N or BER) and the value of the statistic is displayed. This value, together with the data point value, is also stored in a file that can be saved and used later for plotting purposes.

The program is written in FORTRAN 77 and contains 446 lines of code plus informative comments. Statistics that can be calculated include BLER(N), PB(K), PB(K,L) and PBC(K,L), EGD(M), P(M,N), and PC(M,N). A listing is given in the following pages.

```
      PROGRAM PRBTAB
C
C     CALCULATES AND TABULATES(IN FILE PRBANS) THE STATISTICS,
C      1.BLER(N) OR PB(K) VS BER, FOR FIXED N OR K.
C      2.FOR FIXED BER:
C        EGD(M) VS M;  P(M,N) VS M, FOR FIXED N;
C        PC(M,N) VS M, FOR FIXED N;  BLER(N) VS N;
C        PB(K,L) VS L, FOR FIXED K.
C     THE A(I),RHO(I) MAY BE READ IN EITHER AS GIVENS
C     OR DETERMINED FROM BLER VS BER DATA.
C     NOTES:
C      1.THE NOTATION, P=BER AND B(N)=BLER(N), IS OFTEN USED.
C      2.FOR 2-STATE MARKOV OR BSC, ENTER: NS=2,A(1),RHO(1)=0.
C      3.BLOCK LENGTH N IS USUALLY A POSITIVE INTEGER,
C        BUT FOR B(N) VS P, MAY BE NON-INTEGER AND/OR NEGATIVE.
C      4.REFERENCES: (1)COMPARISONS OF THE TWO-STATE...
C                    (2)EXTENDED SES MODEL...
C
      COMMON/PARAM/AH(19),ROH(19),T(19),NSH,CALCP
      DIMENSION A(0:20),RO(0:20),BDAT(2001),PDAT(2001),B(19),P(19)
      OPEN(1,FILE='PRBANS')
      EPSA=1.E-10
      EPSR=2.E6
      GDNS=0.
      SPNS=0.
      A(0)=1.
      RO(0)=0.
      PRINT*,'IF B VS P DATA ARE GIVEN, ENTER 1; OTHERWISE 0'
      READ*,ITYPE
      IF(ITYPE.EQ.1) GO TO 100
      PRINT*,'ENTER NUMBER OF STATES,NS'
      READ*,NS
      RO(NS)=1.
      A(NS)=1.
      NS1=NS-1
      PRINT*,'ENTER A(I),RHO(I) FOR I=1,NS-1 (RHO INCREASING)'
      READ*,(A(J),J=1,NS1),(RO(K),K=1,NS1)
      DO 995 I=1,NS-1
      ARO=A(I)*RO(I)
      GDNS=GDNS+ARO
      SPNS=SPNS+ARO/(1.-RO(I))
995   CONTINUE
      PRINT*,'AT P(NS-1), EGD(1)=',GDNS
      PRINT*,' '
1000  PRINT*,'ENTER 1 FOR STATS VS P; OR 0 FOR STATS AT FIXED P'
      READ*,ITYP
      IF(ITYP.NE.1) GO TO 1010
      IMAX=0
      PRINT*,'ENTER 1 FOR BLER(N) VS P; OR 0 FOR PB(K) VS P'
      READ*,ITYB
      IF(ITYB.EQ.0) THEN
        PRINT*,'ENTER BURST PARAMETER,K'
        READ*,KK
        GO TO 1010
      END IF
      PPINT*,'ENTER BLOCK LENGTH,N'
      READ*,BN
1010  PRINT*,'ENTER P; OR -1 TO TABULATE'
      READ*,PX
      IF(INT(PX).EQ.-1) GO TO 2000
C
C     DETERMINE A(I),RHO(I) FOR THE GIVEN P
C
      CT=(1.-PX)/PX
```

```
      C=CT
      PF=CT
      CLC=1.
      DO 10 I=1,NS
      II=I
      IF(II.EQ.NS) GO TO 20
      CT=CT-A(II)*RO(II)/(1.-RO(II))
      IF(CT.LT.0.) GO TO 15
      C=CT
10    CONTINUE
15    IF(II.EQ.1) GO TO 20
      PS=1./(PF-CT+1.)
      PL=1./(PF-C+1.)
      CLC=ALOG(PX/PL)/ALOG(PS/PL)
      CLC=AMAX1(CLC,EPSA)
20    C=C*(RO(II)-RO(II-1))/(CLC*A(II))
      ROHAT=(RO(II-1)+C)/(1.+C)
      AHAT=CLC*A(II)*(ROHAT-RO(II-1))/(ROHAT*(RO(II)-RO(II-1)))
      AHAT=AMAX1(AHAT,EPSA)
      NSH=II+1
      ROH(NSH-1)=ROHAT
      ROHF=1./(1.-ROHAT)
      AH(NSH-1)=AHAT
      IF((NSH-NS).GT.0) THEN
        GDT=1.-GDNS
        IF((GDT.LE.0.).OR.(NS.EQ.2)) THEN
          NSH=NS
          AH(NSH-1)=A(NSH-1)
          SPT=SPNS-A(NSH-1)*RO(NSH-1)/(1.-RO(NSH-1))
        ELSE
          AH(NSH-1)=AMIN1(AHAT,GDT)
          SPT=SPNS
        END IF
        RAT=(PF-SPT)/AH(NSH-1)
        ROH(NSH-1)=RAT/(1.+RAT)
        ROHF=1.+RAT
      END IF
      DO 30 K=1,NSH-2
      ROH(K)=RO(K)
      AH(K)=A(K)
30    CONTINUE
      T(NSH-1)=AH(NSH-1)*ROH(NSH-1)
      SUMP=T(NSH-1)*ROHF
      DO 35 I=1,NSH-2
      T(I)=AH(I)*ROH(I)
      SUMP=SUMP+T(I)/(1.-ROH(I))
35    CONTINUE
      CALCP=1./(SUMP+1.)
      IF(ITYP.NE.1) THEN
        IF(AH(NSH-1).LE.EPSA) NSH=NSH-1
        DO 40 I=1,NSH-1
        PRINT*,'AH(',I,')=',AH(I),' RHOH(',I,')=',ROH(I)
40      CONTINUE
        CALL PRB
        GO TO 1000
      END IF
C
C     COMPUTE PB(K) FOR THE GIVEN P
C
      IF(ITYB.EQ.0) THEN
        SUMG=0.
        DO 41 J=1,NSH-1
        F=0.
        IF(REAL(KK)*ALOG10(ROH(J)).GT.-100.) F=ROH(J)**KK
```

```
        SUMG=SUMG+AH(J)*F
41      CONTINUE
        SUMG=CALCP*SUMG
        PRINT*,'CALC P=',CALCP,' PB(',KK,')=',SUMG
        PRINT*,' '
        IMAX=IMAX+1
        BDAT(IMAX)=SUMG
        PDAT(IMAX)=CALCP
        GO TO 1010
      END IF
C
C     COMPUTE BLER(N) FOR THE GIVEN P
C
      IF(ROHF.GT.EPSR) THEN
        X=-(BN/ROHF)*(1.+.5/ROHF)
        GO TO 42
      END IF
      X=BN*ALOG(ROH(NSH-1))
42    IF(X.LT.-674.) X=-674.
      SUM=AH(NSH-1)*EXP(X)*ROHF
      DO 50 I=1,NSH-2
      X=BN*ALOG(ROH(I))
      IF(X.LT.-674.) X=-674.
      SUM=SUM+AH(I)*EXP(X)/(1.-ROH(I))
50    CONTINUE
      ANS=CALCP*SUM
      PRINT*,'CALC P=',CALCP,' BLER(',BN,')=',1.-ANS
      PRINT*,' '
      IMAX=IMAX+1
      BDAT(IMAX)=1.-ANS
      PDAT(IMAX)=CALCP
      GO TO 1010
C
C     DETERMINE A(I),RHO(I) FROM B(N) VS P DATA
C
100   PRINT*,'ENTER NO. OF STATES,NS; AND BLOCK LENGTH,N (N.GT.1)'
      READ*,NS,N
      RO(NS)=1.
      A(NS)=1.
      SB=0.
      SP=0.
      PRINT*,'ENTER (NS-1) VALUES OF BLER AND P (DECREASING)'
      READ*,(B(J),J=1,NS-1),(P(K),K=1,NS-1)
102   DO 110 I=1,NS-1
      IF(B(I).EQ.1.) THEN
        PRINT*,'ENTER A(',I,'),RO(',I,')'
        READ*,AT,ROT
        A(I)=AT
        RO(I)=ROT
        GO TO 105
      END IF
      PF=(1.-P(I))/P(I)
      BF=(1.-B(I))/P(I)
      RX=(BF-SB)/(PF-SP)
      RO(I)=EXP(ALOG(RX)/(N-1))
      IF(RO(I).GE.1.) THEN
        PRINT*,'RHO(',I,')=',RO(I),'  IS NOT VALID.'
        PRINT*,'ENTER SMALLER VALUE OF A(',I-1,')'
        SB=0.
        SP=0.
        GDNS=0.
        GO TO 102
      END IF
      A(I)=(PF-SP)*(1.-RO(I))/RO(I)
```

```
105   X=N*ALOG(RO(I))
      IF(X.LT.-674.) X=-674.
      SB=SB+A(I)*EXP(X)/(1.-RO(I))
      ARO=A(I)*RO(I)
      GDNS=GDNS+ARO
      SP=SP+ARO/(1.-RO(I))
      SPNS=SP
110   CONTINUE
      DO 120 I=1,NS-1
      PRINT*,'A(',I,')=',A(I),' RHO(',I,')=',RO(I)
120   CONTINUE
      PRINT*,'AT P(NS-1), EGD(1)=',GDNS
      PRINT*,' '
      PRINT*,'ENTER 1 TO CALCULATE THE STATISTICS;'
      PRINT*,'OR 0 TO REDO THE A(I),RHO(I); OR -99 TO STOP'
      READ*,IDEC
      IF(IDEC.EQ.-99) GO TO 99
      IF(IDEC.EQ.0) THEN
        SB=0.
        SP=0.
        GDNS=0.
        GO TO 102
      END IF
      GO TO 1000
C
C     TABULATE IN FILE PRBANS
C
2000  IF(ITYB.EQ.0) THEN
        WRITE(1,'(1H1,8HHEAD,PB(,I5,14H) VS BER,CALC.)')KK
        GO TO 2010
      END IF
      WRITE(1,'(1H1,10HHEAD,BLER(,F10.4,14H) VS BER,CALC.)')BN
2010  WRITE(1,'(1X,14HNO. OF POINTS=,I4,)')IMAX
      WRITE(1,'(1X)')
      JMIN=1
      JMAX=MIN0(JMIN+7,IMAX)
2030  WRITE(1,'(1X,8E9.4)')(BDAT(J),J=JMIN,JMAX)
      WRITE(1,'(1X,8E9.4)')(PDAT(J),J=JMIN,JMAX)
      WRITE(1,'(1X)')
      JMIN=JMAX+1
      JMAX=MIN0(JMIN+7,IMAX)
      IF(JMIN.GT.IMAX) THEN
        PRINT*,'ENTER 1 TO CONTINUE; OR -99 TO STOP'
        READ*,ICONT
        IF(ICONT.EQ.-99) GO TO 99
        GO TO 1000
      END IF
      GO TO 2030
99    STOP
      END
```

```
      SUBROUTINE PRB
C
C     CALCULATES P(M,N), THE PROBABILITY OF M ERRORS
C     IN A BLOCK OF N BITS (BY ELLIOTT'S FORMULA);
C     OR BLER(N)=1-P(0,N), THE BLOCK ERROR RATE
C     (THE PROBABILITY OF 1 OR MORE ERRORS IN A BLOCK
C     OF N BITS-- BY THE IMPROVED FORMULA; INPUT: N);
C     OR EGD(M), THE ERROR GAP DISTRIBUTION (THE PROB-
C     ABILITY OF M CORRECT BITS FOLLOWING AN ERROR);
C     OR PC(M,N), THE PROBABILITY OF M OR MORE ERRORS
C     IN A BLOCK OF N BITS-- (PC(1,N)=BLER(N));
C     OR PB(K,L), THE BURST ERROR RATE WITH
C     GAP AND LENGTH CHARACTERISTICS, K AND L;
C     OR PBC(K,L), THE PROBABILITY OF ERROR BURSTS
C     WITH FIXED K AND LENGTHS GE L; PBC(K,0)=1.
C       INPUT: NS,A(K),RHO(K), 2 LE NS LE 20;
C              M,N, 0 LE M LE N, 1 LE N LE 131000;
C              1 LE K LE 1000, 0 LE L LE 1000.
C              (FOR BLER(N), N MAY EXCEED 131000.)
C
      COMMON/PARAM/AH(19),ROH(19),T(19),NSH,CALCP
      DIMENSION PD(30000),PP(30000),R(30000)
      DIMENSION TP(19),YDAT(2001),XDAT(2001)
      NS=NSH
      NS1=NS-1
      PNS=1.
      S1=0.
      DO 5 I=1,NS1
      S1=S1+T(I)
      PNS=PNS-T(I)
5     CONTINUE
      BER=CALCP
      PRINT*,'BER=',BER,' RHONS=',PNS
      PRINT*,' '
7     PRINT*,'ENTER ITYP FOR DESIRED STATISTIC, WHERE ITYP IS:'
      PRINT*,'1 FOR EGD(M); OR 2 FOR P(M,N);'
      PRINT*,'OR 3 FOR PC(M,N); OR 4 FOR BLER(N);'
      PRINT*,'OR 5 FOR PB(K,L); OR 6 FOR PBC(K,L)'
      READ*,ITYP
      IMAX=0
      IF(ITYP-2) 200,10,300
10    PRINT*,'ENTER M,N FOR P(M,N); OR 0,0 TO TABULATE'
      READ*,M,N
      IF(N.EQ.0) GO TO 190
C
C     COMPUTE P(M,N)
C
11    DO 12 I=1,NS1
      T(I)=AH(I)*ROH(I)
12    CONTINUE
      NM=N+1-M
      PD(1)=1.
      PD(2)=S1
      PP(1)=PNS
      PP(2)=0.
      DO 13 I=1,NS1
      TP(I)=T(I)*(1.-ROH(I))
      PP(2)=PP(2)+TP(I)
13    CONTINUE
      IF(M.EQ.0) GO TO 405
      R(NM)=1.
      IF(NM.EQ.1) GO TO 15
      R(NM-1)=S1
15    IF(NM.LT.3) GO TO 30
```

```
      M1=NM-1
      M2=NM-2
      DO 20 I=1,M2
      PD(I+2)=0.
      PP(I+2)=0.
      DO 17 J=1,NS1
      T(J)=T(J)*ROH(J)
      TP(J)=TP(J)*ROH(J)
      PD(I+2)=PD(I+2)+T(J)
      PP(I+2)=PP(I+2)+TP(J)
17    CONTINUE
      R(M1-I)=PD(I+2)
20    CONTINUE
30    IF(M.EQ.1) GO TO 65
      ITOT=1
40    IMIN=1
50    IP=0
      RT=0.
      DO 60 I=IMIN,NM
      IP=IP+1
      RT=RT+PP(IP)*R(I)
60    CONTINUE
      R(IMIN)=RT
      IMIN=IMIN+1
      IF(IMIN.LE.NM) GO TO 50
      ITOT=ITOT+1
      IF(ITOT.LT.M) GO TO 40
65    IP=0
      RT=0
      DO 70 I=1,NM
      IP=IP+1
      RT=RT+PD(IP)*R(I)
70    CONTINUE
      ANS=BER*RT
71    IF(ITYP.EQ.3) GO TO 330
      IF(ITYP.EQ.2) GO TO 72
      PRINT*,'BLER(',N,')=',BLER
      IMAX=IMAX+1
      YDAT(IMAX)=BLER
      XDAT(IMAX)=N
      GO TO 400
72    PRINT*,'P(',M,',',N,')=',ANS
      IMAX=IMAX+1
      YDAT(IMAX)=ANS
      XDAT(IMAX)=M
      GO TO 10
190   WRITE(1,'(1H1,9HHEAD,BER=,E9.4,5H,P(M,,I5,7H),CALC.)')CALCP,N
      GO TO 1000
C
C     COMPUTE EGD(M)
C
200   PRINT*,'ENTER M FOR EGD(M); OR -1 TO TABULATE'
      READ*,M
      IF(M.EQ.-1) GO TO 290
205   SUMG=0.
      DO 210 J=1,NS1
      F=0.
      IF(REAL(M)*ALOG10(ROH(J)).GT.-100.) F=ROH(J)**M
      SUMG=SUMG+AH(J)*F
210   CONTINUE
      IF(ITYP.EQ.5) GO TO 510
      IF(ITYP.EQ.6) GO TO 610
      PRINT*,'EGD(',M,')=',SUMG
      IMAX=IMAX+1
```

```
      YDAT(IMAX)=SUMG
      XDAT(IMAX)=M
      GO TO 200
290   WRITE(1,'(1H1,9HHEAD,BER=,E9.4,13H,EGD(M),CALC.)')CALCP
      GO TO 1000
C
C     COMPUTE PC(M,N)
C
300   IF(ITYP.GT.3) GO TO 400
      PRINT*,'ENTER (MAXIMUM)M,N FOR PC(M,N)'
      READ*,MMAX,N
      SUMC=1.
      M=-1
320   IF(M.EQ.MMAX-1) GO TO 390
      M=M+1
      IF(M.EQ.0) GO TO 405
      GO TO 11
330   SUMC=SUMC-ANS
      PRINT*,'PC(',M+1,',',N,')=',SUMC
      IMAX=IMAX+1
      YDAT(IMAX)=SUMC
      XDAT(IMAX)=M+1
      GO TO 320
390   WRITE(1,'(1H1,9HHEAD,BER=,E9.4,6H,PC(M,,I5,7H),CALC.)')CALCP,N
      GO TO 1000
C
C     COMPUTE BLER(N) OR P(0,N)
C
400   IF(ITYP.GT.4) GO TO 500
      PRINT*,'ENTER N FOR BLER(N); OR -1 TO TABULATE'
      READ*,N
      IF(N.EQ.-1) GO TO 490
405   SUM=0.
      DO 410 I=1,NS1
      X=N*ALOG(ROH(I))
      IF(X.LT.-674.) X=-674.
      SUM=SUM+AH(I)*EXP(X)/(1.-ROH(I))
410   CONTINUE
      ANS=CALCP*SUM
      BLER=1.-ANS
      GO TO 71
490   WRITE(1,'(1H1,9HHEAD,BER=,E9.4,14H,BLER(N),CALC.)')CALCP
      GO TO 1000
C
C     COMPUTE PB(K,L)
C
500   IF(ITYP.GT.5) GO TO 600
      PRINT*,'ENTER K,L FOR PB(K,L); OR 0,0 TO TABULATE'
      READ*,KK,LL
      IF(KK.EQ.0) GO TO 590
      M=KK
      GO TO 205
510   PB=CALCP*SUMG*SUMG
      BCKL=1.
      IF(LL.NE.0) CALL COREL(KK,LL,BCKL)
      PB=PB*BCKL
      PRINT*,'PB(',KK,',',LL,')=',PB
      IMAX=IMAX+1
      YDAT(IMAX)=PB
      XDAT(IMAX)=LL
      GO TO 500
590   WRITE(1,'(1H1,9HHEAD,BER=,E9.4,4H,PB(,I5,1H,,I5,7H),CALC.)')
     XCALCP,KK,LL
      GO TO 1000
```

```
C
C     COMPUTE PBC(K,L), L GE 1
C
600   PRINT*,'ENTER K,(MAXIMUM)L FOR PBC(K,L)'
      READ*,KK,LMAX
      M=KK
      GO TO 205
610   SUMB=0.
      LL=-1
620   IF(LL.EQ.LMAX-1) GO TO 690
      LL=LL+1
      BCKL=1.
      IF(LL.NE.0) CALL COREL(KK,LL,BCKL)
      SUMB=SUMB+BCKL
      PBC=1.-SUMG*SUMB
      PRINT*,'PBC(',KK,',',LL+1,')=',PBC
      IMAX=IMAX+1
      YDAT(IMAX)=PBC
      XDAT(IMAX)=LL+1
      GO TO 620
690   WRITE(1,'(1H1,9HHEAD,BER=,E9.4,5H,PBC(,I5,9H,L),CALC.)')
     XCALCP,KK
      GO TO 1000
C
C     TABULATE IN FILE PRBANS
C
1000  WRITE(1,'(1X,14HNO. OF POINTS=,I4,)')IMAX
      WRITE(1,'(1X)')
      JMIN=1
      JMAX=MIN0(JMIN+7,IMAX)
1010  WRITE(1,'(1X,8E9.4)')(YDAT(J),J=JMIN,JMAX)
      WRITE(1,'(1X,8F9.0)')(XDAT(J),J=JMIN,JMAX)
      WRITE(1,'(1X)')
      JMIN=JMAX+1
      JMAX=MIN0(JMIN+7,IMAX)
      IF(JMIN.GT.IMAX) GO TO 1020
      GO TO 1010
1020  PRINT*,'ENTER 1 FOR MORE STATS AT THE GIVEN BER;'
      PRINT*,'OR 0 FOR A NEW BER; OR -99 TO STOP'
      READ*,ICONT
      IF(ICONT.EQ.-99) GO TO 99
      IF(ICONT.EQ.0) RETURN
      GO TO 7
99    STOP
      END
```

```
      SUBROUTINE COREL(K,L,BCKL)
C
C     CALCULATES BC(K,L), THE BURST CORRELATION FUNCTION;
C     1 LE L,K LE 1000.
C     NOTE: BC(L,L) IS THE AUTOCORRELATION FUNCTION
C     FROM ELLIOTT(1965),EQ.(3),P.105.
C
      COMMON/PARAM/AH(19),ROH(19),T(19),NSH,CALCP
      DIMENSION DEG(1000),BC(0:1000),V(19)
      BC(0)=1.
C
C     COMPUTE EXPONENT TEST VALUES
C
      DO 10 I=1,NSH-1
      V(I)=-100./ALOG10(ROH(I))
10    CONTINUE
C
C     COMPUTE EGD DIFFERENTIALS (ARRAY DEG)
C     AND BURST CORRELATION FUNCTIONS (ARRAY BC)
C
      LUP=MINO(K,L)
      DO 40 J=1,LUP
      SUMD=0.
      DO 20 I=1,NSH-1
      RC=1.-ROH(I)
      IF(J.EQ.1) RC=ROH(I)
      F1=AH(I)*RC
      F=0.
      IF(REAL(J).LT.V(I)) F=ROH(I)**(J-1)
      SUMD=SUMD+F1*F
20    CONTINUE
      DEG(J)=SUMD
      IF(J.EQ.1) DEG(J)=1.-DEG(J)
      SUMC=0.
      DO 30 JJ=1,J
      SUMC=SUMC+DEG(JJ)*BC(J-JJ)
30    CONTINUE
      BC(J)=SUMC
40    CONTINUE
      BCKL=SUMC
      IF(L.LE.K) RETURN
      JUP=L-K
      DO 60 J=1,JUP
      SUMC=0.
      DO 50 JJ=1,K
      SUMC=SUMC+DEG(JJ)*BC(K+J-JJ)
50    CONTINUE
      BC(K+J)=SUMC
60    CONTINUE
      BCKL=SUMC
      RETURN
      END
```

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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

Fritchman's single-error-state (SES) model for describing the error statistics of digital communication channels is modified to allow the prediction of error statistics as a function of the bit error rate (BER). From the data samples of a measurement program, cumulative distributions (CD's) of the statistics are obtained, and simple analytic relationships are derived between error statistics and the BER at any CD level. A computer program based on the extended SES model has been written that evaluates the block error rate, burst error rate, error gap distribution, and counting distribution for any desired BER. Comparisons are shown of model predictions and measured data taken over a switched telecommunications network.

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