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A Model of a Shaped-Beam Emission Pattern of a Satellite Antenna for Interference Analysis

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A MODEL OF A SHAPED-BEAM EMISSION PATTERN OF A SATELLITE ANTENNA FOR INTERFERENCE ANALYSIS

Hiroshi Akima*

For efficient use of the geostationary satellite orbit, mutual interference among satellite systems must be analyzed in the planning stage of the systems. To conserve the transmitter power, many satellite antennas in the FSS (fixed-satellite service) use the so-called shaped-beam emission patterns that cover their service areas. A computer model of a shaped-beam pattern is needed in the analysis of mutual interference. We present a simple model for calculating the antenna gain in the direction of an earth point from several contour lines given on the map of the Earth, each corresponding to an antenna gain value.

Key words: antenna emission pattern; FSS (fixed-satellite service); satellite antenna; satellite communication; shaped-beam antenna

1. INTRODUCTION

For efficient use of the geostationary satellite orbit by communication satellites, mutual interference among satellite systems must be analyzed in the planning stage of the systems. The analysis of mutual interference involves calculations of satellite antenna gains in the directions of wanted and unwanted earth points, earth-station antenna gains in the directions of wanted and unwanted satellites, and propagation factors along the wanted and unwanted paths. Because of the high degree of complexity of the calculations, use of a computer is necessary for the analysis. It is, therefore, essential for the analysis to develop computer models that allow the user to calculate the satellite and earth-station antenna gains as well as the propagation factors or to model the satellite and earth-station antenna emission patterns as well as the propagation phenomena. Some existing models for the antenna patterns and propagation phenomena are described by Akima (1985).

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To conserve the transmitter power and suppress the potential interference, many antennas of satellites in the FSS (fixed-satellite service) use so-called shaped-beam emission patterns, each approximately covering the wanted service area. A computer model of a shaped-beam emission pattern of a satellite antenna that calculates the antenna gain in the direction of an earth point is needed for the analysis of mutual interference among the FSS systems.

Typically, a shaped-beam emission pattern of a satellite antenna is given graphically on a map of the Earth with several contour lines, each corresponding to a gain value. (As described later, some patterns are more complicated; a pattern may have two contour lines or more for a gain value, and it may have two maximum gain points or more for a pattern.) A possible way of using such contour lines for the computer model of the shaped-beam satellite antenna pattern is to approximate each contour line with a polygon and store the coordinates of the polygon points in the data base. It is desirable that the model is based on such polygon-point data.

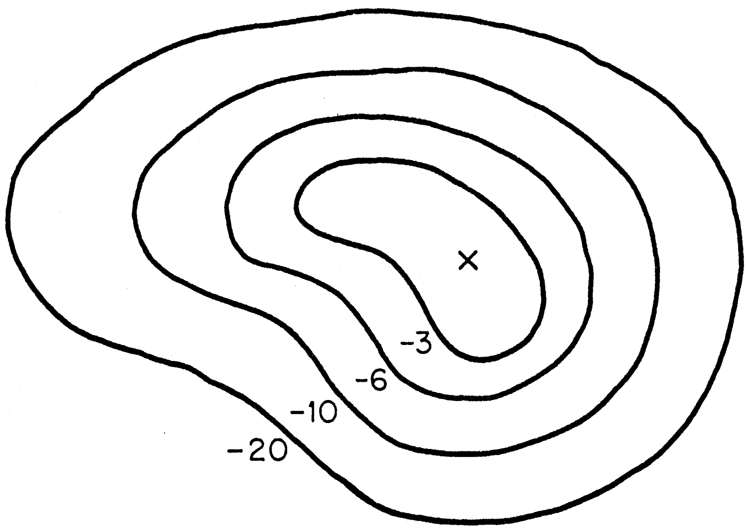
We have developed a new model for a shaped-beam emission pattern of a satellite antenna. The model calculates the antenna gain in the direction of an earth point by linearly interpolating two gain values corresponding to the two contours between which the earth point in question lies with respect to the distances from the earth point to the two contour polygons. The antenna gain values resulting from the model are continuous but, in general, are not smooth on the contour polygons and elsewhere. The model does not produce excessive undulations (or wiggles). This model can easily be implemented in a computer program.

The model has been outlined by Akima (1985). This report describes the model in detail. Section 2 of this report describes the shaped-beam emission pattern. Section 3 lists some characteristics of the model that are necessary or desirable. Section 4 reviews several candidate models, establishes guidelines for developing the model, and develops the model. Section 5 describes the model in detail. Section 6 describes a special case of the model. Some examples are given in Section 7. The method for representing the location of an earth point relative to the location of a satellite used in the model is described in Appendix A. A Fortran subroutine package that implements the model is described in Appendix B.

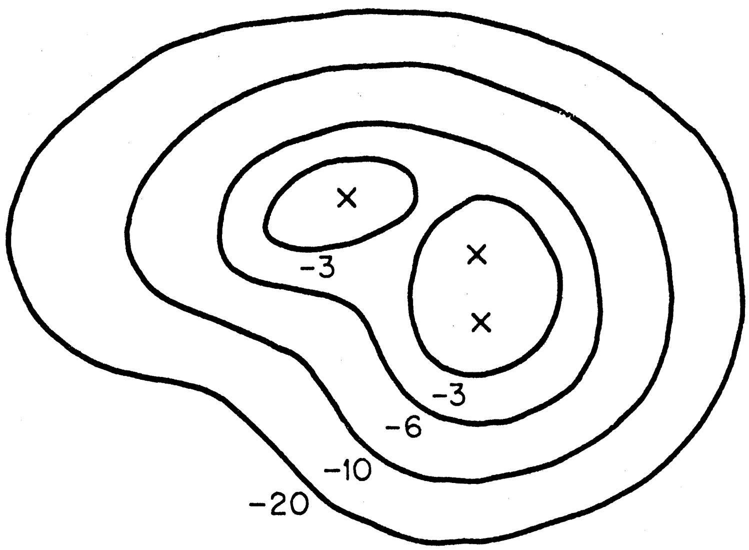
2. SHAPED-BEAM EMISSION PATTERN OF SATELLITE ANTENNA

Typically, as mentioned earlier, a shaped-beam emission pattern of a satellite antenna is given graphically on a map of the Earth with several contour lines, each corresponding to a gain value, with a maximum gain point, which is the point on the surface of the Earth and in the direction of which the antenna gain takes a maximum value. Some patterns are rather simple; only one contour line corresponds to an antenna gain and there is only one maximum gain point, as illustrated in Figure 1(a). Some patterns are more complicated; there may be two contour lines or more for a gain value, and there may be two maximum gain points or more for a pattern, as illustrated in Figure 1(b). In our model, we approximate each contour line, drawn on the surface of the Earth, with a polygon, also on the surface of the Earth, and store the coordinates of the polygon points in the data base for use by the model as the input data to the model.

Since each polygon point is an earth point (that is a point on the surface of the Earth), we consider how to represent the location of an earth point. The location of an earth point can be represented in terms of its longitude and latitude or in terms of three coordinates in the earth-center coordinate system, that is a Cartesian coordinate system having the center of the Earth as its origin. When the location of a satellite is given, the location of an earth point can also be represented in relation to the satellite location in terms of angles seen from the satellite. Since the location of the satellite is always given for the shaped-beam emission pattern, we use the angle representation of a polygon point. We use the so-called "pitch" and "roll" angles of each polygon point seen from the satellite. The "pitch" and "roll" angles of an earth point seen from a satellite are defined as the angles of the earth point seen from the satellite and measured in the east and north directions from the line connecting the satellite to its subsatellite point. (The subsatellite point of a satellite is the point on the surface of the Earth having the same longitude and latitude as the satellite; it is the earth point closest to the satellite.) The "pitch" and "roll" angles are described in more detail in Appendix A. Mathematical relations between these angles and the locations of a satellite and an earth point as well as some computer subroutines for these relations are also given in Appendix A.



(a) A simple pattern



(b) A more complicated pattern with multiple contours for a gain value and with multiple maximum gain points

Figure 1. Schematic representation of shaped-beam patterns. (The "x" mark is a maximum gain point. The gain value for each contour is in decibels relative to the maximum gain of the antenna.)

3. NECESSARY OR DESIRABLE CHARACTERISTICS FOR THE MODEL

Some characteristics of the model are considered necessary or desirable for the model of a shaped-beam emission pattern of a satellite antenna of the FSS system. In this section, we examine several characteristics of the model with the hope that we can establish guidelines for developing the model.

Capability of Handling Complicated Emission Patterns

In some emission patterns, there are two contour lines or more corresponding to an antenna gain value, and there are two maximum gain points or more for a pattern, as illustrated earlier in Figure 1(b). The model must be able to handle, without difficulty, a complicated emission pattern that has multiple contour lines for an antenna gain value and multiple maximum gain points for a pattern. This means that the underlying algorithm (or the set of mathematical procedures) of the model must consist of a unique and tractable sequence of calculation steps even for a complicated emission pattern.

Computer Implementation

The model must be such that one can easily implement it as a computer subroutine package and use it without difficulty as a part of an analysis program of mutual interferences. Since the program is a large program and the implemented model is used very many times, the implemented model must not require an excessive storage or computation time.

Continuity

The antenna gain values resulting from the model must be continuous with respect to the location of the earth point in the direction of which calculation of the antenna gain is desired. In other words, the antenna gain value must not jump as the earth point in question moves on the surface of the Earth. Importance of this requirement is obvious if one considers the physical nature of an emission pattern of a satellite antenna.

Freedom from Excessive Undulations

The antenna gain values resulting from the model must not exhibit excessive undulations. Although it is difficult to define excessive undulation rigorously, we consider it desirable if the antenna gain value in the direction of an earth point that lies between two contour lines remains between the two antenna gain values that correspond to the two contour lines. Importance of this requirement is also obvious if one considers the nature of the model for an emission pattern of a satellite antenna. The requirement for freedom from excessive undulations is one of the most difficult requirements to satisfy in developing an interpolation method in general.

Smoothness

It is desirable for the model that the antenna gain values resulting from the model are smooth with respect to the location of the earth point in the direction of which calculation of the antenna gain is desired. (Smoothness of the antenna gain values means that the first derivatives of the antenna gain values with respect to the location of the earth point are continuous.) Although smoothness is considered desirable, it is not considered an absolute necessity insofar as the model is used in an analysis program of mutual interferences. As a matter of fact, many reference antenna patterns recommended by the CCIR (International Radio Consultative Committee) or adopted by the ITU (International Telecommunication Union) are not necessarily smooth (CCIR, 1982; ITU, 1982). If the requirement for smoothness conflicts with other requirements, therefore, we must be prepared to abandon the smoothness requirement.

Summary

All characteristics listed here except smoothness are considered necessary for the model. Smoothness is considered desirable if the requirement for it does not conflict with other requirements. If we try to obtain smooth gain values, however, the algorithm of the model will become complicated and almost intractable. As will be described later, therefore, we abandon the smoothness requirement. The decision to abandon this requirement is perhaps the biggest step in our effort for developing the model, although it is a rather painful decision. The decision is the key to the success for developing our model.

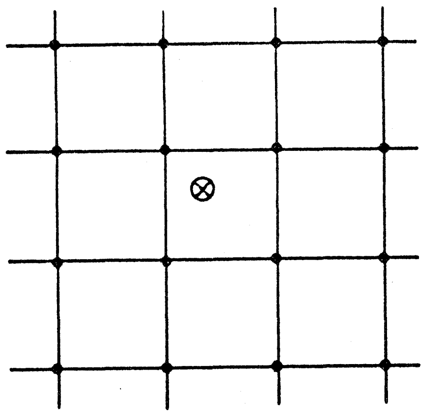
4. DEVELOPMENT OF THE MODEL

Several models were considered earlier as candidates for a shaped-beam emission pattern of a satellite antenna (Akima et al., 1981). One of the models considered there assumes that the antenna gain values are given at rectangular grid points, as shown in Figure 2(a). It is based on bivariate interpolation for regular-grid data points such as the one developed by Akima (1974). Obviously, this model is not applicable when input data are given in the form of gain contour lines.

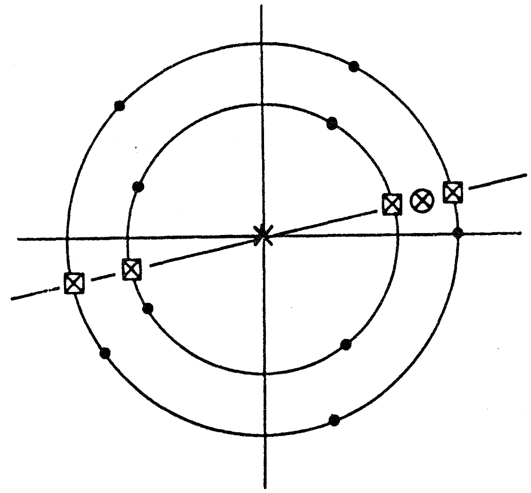
Another model considered there assumes that the antenna gain values are given on concentric circles in a polar coordinate system, as shown in Figure 2(b). Obviously, this model is not applicable either when input data are given in the form of gain contour lines.

Another model considered there assumes that the gain values are given at irregularly distributed points, as shown in Figure 2(c). It is based on bivariate interpolation for irregularly distributed data points such as the one developed by Akima (1978). Although this model is applicable in principle to any case, it usually requires a large storage and long calculation time. This model sometimes produces excessive undulations of the result. We can eliminate these excessive undulations at the sacrifice of smoothness of the results with the use of linear interpolation locally (i.e., by using a piece of plane instead of a piece of curved surface), but we cannot significantly improve the storage and calculation time requirements. The basic defect of this model is that the model does not take full advantage of the fact that the data points are grouped by gain values as the contour polygon points. This model is the last resort for the case where the input data points are irregularly scattered.

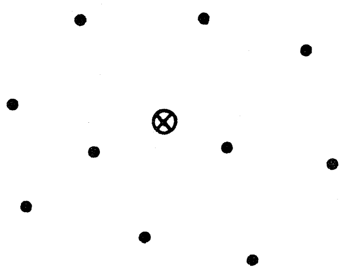
Another model considered there assumes that several gain contours and a maximum gain point are given, as illustrated in Figure 2(d). It is based on univariate interpolation, such as the one developed by Akima (1970), on the straight line passing through the maximum gain point (marked by the "x" mark) and the point for which calculation of the antenna gain is desired (i.e., the point marked by the encircled "x" mark). The algorithm of this model consists of two steps. In the first step, the model determines the points where the straight line intersects the contour lines (i.e., the points marked by the "x" marks enclosed by squares) by applying the univariate interpolation to the radius (or the distance from the maximum gain point) of the polygon points as a



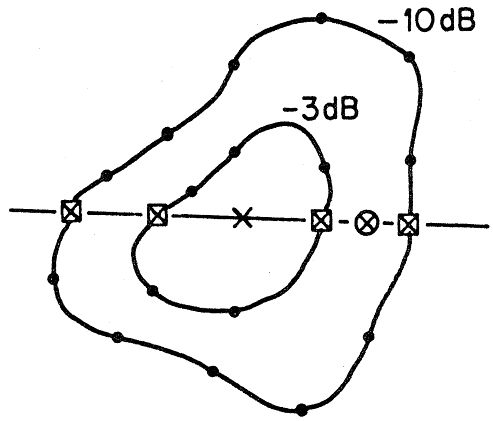
(a) Rectangular grid



(b) Concentric circles



(c) Irregularly distributed points



(d) Gain contours with a maximum gain point

Figure 2. Various models considered in a previous study. (The circular dot is a given data point; the "x" mark is a maximum gain point; the encircled "x" mark is the point for which gain calculation is desired; and the "x" mark enclosed by a square is the point necessary during calculation. The gain values for the contours are relative to the maximum gain of the antenna.)

function of the orientation angle of the point. In the second step, the model calculates the gain value for the desired point with univariate interpolation on the straight line. This model takes full advantage of the grouped data points. It produces smooth results but may sometimes produce excessive undulations. Again, the excessive undulations can be eliminated at the sacrifice of smoothness with the use of linear interpolation locally. This model does not require excessive storage area or long calculation time. It looks promising.

This model, however, sometimes yields an unreasonable gain value. In the illustrative example in Figure 3, the relative gain value at the point in question (marked by the encircled "x" mark) is supposed to be close to the value for the outer contour, which is -10 dB, but the value resulting from the model is likely to be close to the middle of the two values for the two contours, which is -7 dB. This example indicates that the distances from the point in question to the contours are more important than those on the straight line.

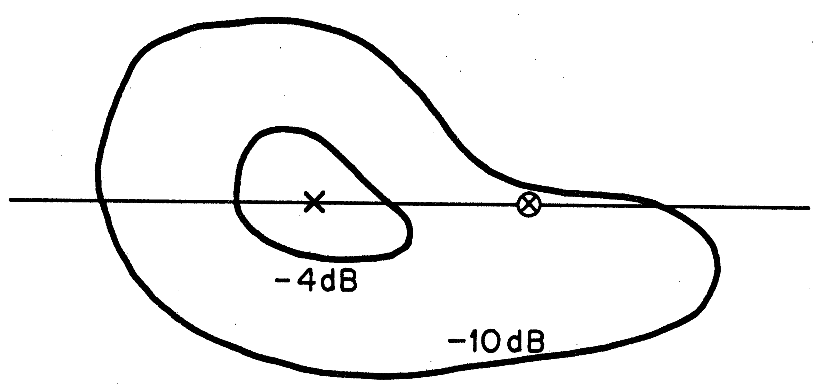


Figure 3. Another example of a contour model. (The gain value for each contour is relative to the maximum gain.)

This model has another disadvantage. If the model is used for a complicated pattern with two maximum gain points or more, such as the one shown earlier in Figure 1(b), one must select one maximum gain point out of the two points or more. We were not successful, however, in establishing an algorithm for such a selection while satisfying the requirements for other characteristics including continuity of the antenna gain values.

One of the lessons we have learned from the examination of the above model is that local use of linear interpolation is effective in suppressing excessive undulation although we have to sacrifice smoothness of the results.

Another lesson we have learned is that, if we base our model on global procedures and use all maximum gain points and contour lines for calculating the antenna gain in the direction of an earth point, the algorithm for the model may become almost intractable. We base our model on local procedures. We restrict the number of contour lines to be used at a time. We use the maximum gain points only when they are needed.

Still another lesson we have learned is the advisability of the use of distances from the earth point to the contour lines. The use of the distances will satisfy the requirement for continuity of the results that dictates that, when the earth point approaches a contour line, calculated (interpolated) gain value must approach the gain value for the contour line.

When the earth point in question lies between two contour lines, we use only the two contour lines and calculate the antenna gain in the direction of the earth point by locally interpolating two gain values that correspond to the two contour lines. By doing so, we can avoid all complications resulting from the multiple contour lines for a gain value and multiple maximum gain points. The resulting gain values are not generally smooth when the earth point moves over a contour line, but we can maintain continuity and freedom from excessive undulations, depending on the interpolation method to be used.

When the earth point in question lies between two contour lines, we use linear interpolation of antenna gain values with respect to the distances from the earth point to the contours. Linear interpolation is not only simple but also has the advantage that it never produces undulations.

We use the maximum gain point only when the earth point in question lies inside the innermost contour. In this case, we use quadratic interpolation of the gain values for the maximum gain point and for the contour line with respect to the distances from the earth point to the maximum gain point and to the contour line. Use of quadratic interpolation is based on the reasoning that, in many cases, a surface can be closely approximated in the neighborhood of the maximum point with a paraboloid.

In lieu of the distance from an earth point to a contour line, we use the distance from the earth point to a polygon that approximates the contour line. We could have included a procedure for supplementing more points in the interval between each pair of polygon points, such as the one developed by Akima (1970, Appendix B), but we have decided not to include such a procedure in the model. This decision is purely for simplicity. The precision that the model can achieve, therefore, depends on how many polygon points per contour line the user supplies as the input data to the model.

5. THE MODEL

The model we have developed is based on the distance between the earth point in the direction of which we wish to calculate the antenna gain and the polygon that approximates the contour. When the earth point in question lies between two polygons, the model linearly interpolates the antenna gain with respect to the distances from the point to the polygons. When the earth point in question lies inside the innermost polygon, the model uses quadratic interpolation with the distances from the point to the maximum gain point and the polygon. When the point in question lies outside the outermost polygon, the model uses linear extrapolation with the two outermost polygons. Descriptions of specific items follow.

As mentioned earlier, the model uses a two-dimensional Cartesian coordinate system with the "pitch" and "roll" angles as the abscissa and ordinate, respectively. (See Appendix A for more details on these angles.) The model calculates the distance d between two earth points (x_1, y_1) and (x_2, y_2) simply as the square root of the sum of squares of the differences in the abscissa and ordinate in this coordinate system, i.e.,

$$d = [(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}. \qquad (1)$$

The resultant distance is a close approximation of the angle between the two earth points seen from the satellite. The distance coincides with the angle when the two earth points lie on either one of the coordinate axes.

The model uses the distance between an earth point and a polygon in the same coordinate system. The model calculates the distance as the minimum, taken over all sides of the polygon, of the distances between the earth point and the sides of the polygon. The distance between an earth point and a side of the polygon is the distance between the earth point and a straight line that contains the side of the polygon when the earth point lies inside the belt area which is bounded by two straight lines that are perpendicular to the side and pass through the end points of the side, as shown in Figure 4(a). The distance between an earth point and a side is the minimum of the two distances between the earth point and the two end points of the side when the earth point lies outside the belt area, as shown in Figure 4(b).

For Figure 4(a), the distance d between the earth point P(x,y) and the side between $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is calculated as the vector product of the vector from P_1 to P times the vector from P_1 to P_2 divided by the distance between P_1 and P_2, i.e.,

$$d = [(x - x_1)(y_2 - y_1) - (y - y_1)(x_2 - x_1)]/[(x_2 - x_1)^2 + (y_2 - y_1)^2]^{1/2}. \tag{2}$$

For Figure 4(b), the distance is calculated as the minimum of the distances from P to P_1 and P_2, i.e.,

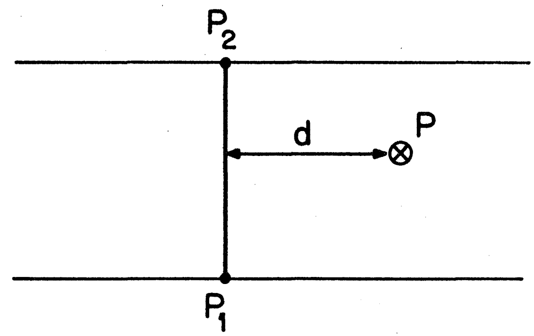
$$d = \min\{[(x - x_i)^2 + (y - y_i)^2]^{1/2}, i = 1, 2\}. \tag{3}$$

The decision as to whether or not P lies in the belt area in Figure 4 is made by calculating the scaler product (inner product) s_i (i = 1, 2) of the vector from P_i to P times the vector from P_1 to P_2, i.e.,

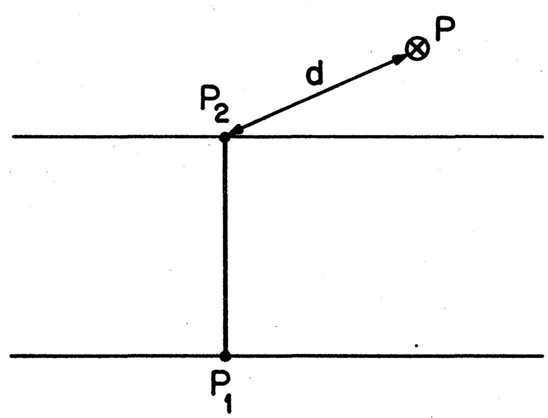
$$s_i = (x - x_i)(x_2 - x_1) + (y - y_i)(y_2 - y_1). \tag{4}$$

We say that P lies in the belt area if and only if s_1 is positive and s_2 is negative.

The model includes a procedure for determining whether the earth point in question lies inside or outside the polygon. To simplify this procedure, we have placed a restriction on the data structure of the emission pattern that polygon points be given counterclockwise for each polygon. Then, when the



(a) When the point lies inside the belt area



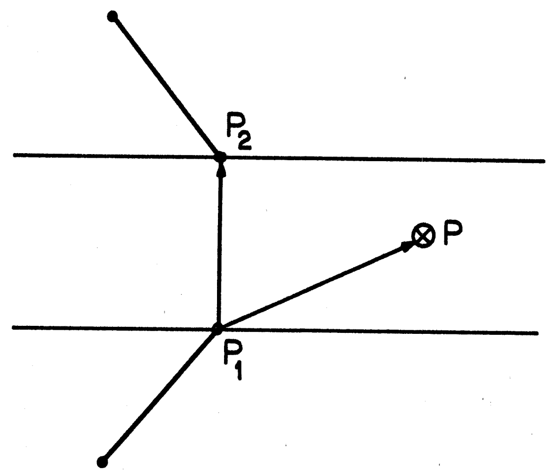
(b) When the point lies outside the belt area

Figure 4. Distance between a point and a side of a polygon.

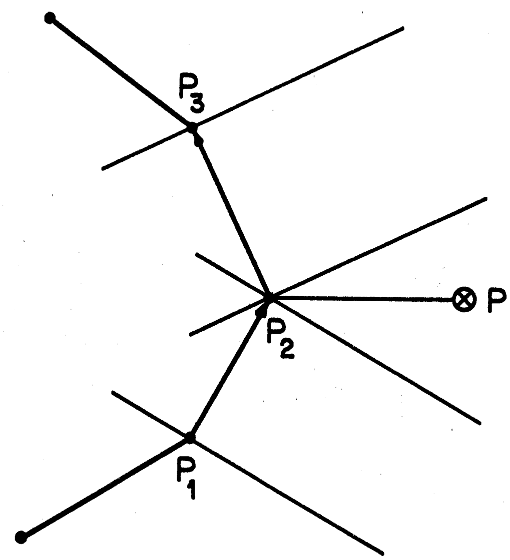
earth point P lies in the belt area perpendicular to the closest side of the polygon between P_1 and P_2, as shown in Figure 5(a), we say that the earth point P lies outside the polygon if the vector product of the vector from P_1 to P times the vector from P_1 to P_2 is positive. When the earth point P lies in a triangular area with the closest polygon point P_2 as the vertex, as shown in Figure 5(b), we say that the earth point P lies outside the polygon if the vector product of the vector from P_1 to P_2 times the vector from P_2 to P_3 is positive.

The procedure just described assumes that a contour line is given as a closed line or as a polygon. When only a portion of a contour line is given as an open line, "outside" of a polygon should be interpreted as the "right side" of the line. When a contour line is given as an open line, the model extends the contour line by linearly extending the first and the last sides. Depending on the relative locations and the directions of these two sides, there are several cases as shown in Figure 6. The extended lines may open wider and wider as shown in Figure 6(a). The lines may be parallel or near parallel as shown in Figure 6(b). The lines may cross each other on the extended portions of both lines as shown in Figure 6(c). The lines may even cross each other on the extended portion of one side but on the other side or on the extension in the opposite direction of the other side as shown in Figure 6(d). For Figures 6(a) and 6(b), the model assumes that the first and last sides are extended indefinitely. For Figure 6(c), the model calculates the location of the point at which the two lines cross each other, calculates the center of gravity of the triangle composed of the crossing point and the first and last contour points, and assumes that the line is a closed line with the center of gravity added as a virtual point. For Figure 6(d), which is an unlikely case, the model assumes that the line is a closed line with a virtual side connecting the last point and the first point. The distinction between Figures (6b) and (6c) is rather arbitrary, and the angle between the two lines of 10° is arbitrarily set as the boundary of the two cases. The use of the center of gravity is also arbitrary.

When the earth point in question lies inside the innermost contour line, as shown in Figure 7(a), the model calculates the antenna gain by quadratic interpolation with respect to the distances from the point in question to the maximum gain point and to the contour line. If we denote the antenna gains corresponding to the maximum gain point and to the innermost contour line by G_m

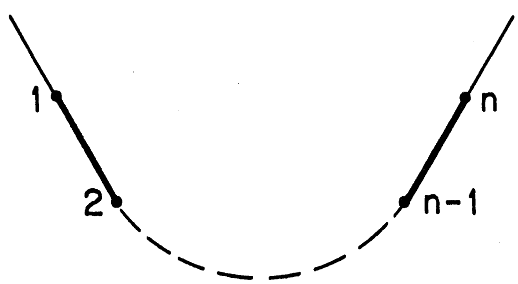


(a) When the point lies inside the belt area

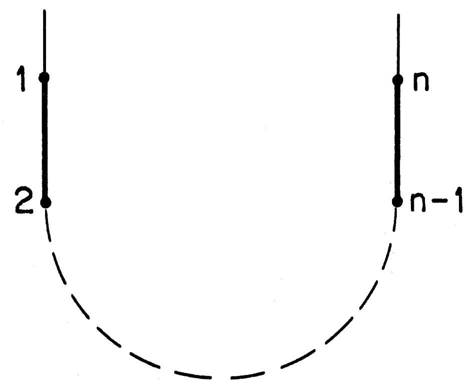


(b) When the point lies inside the triangular area

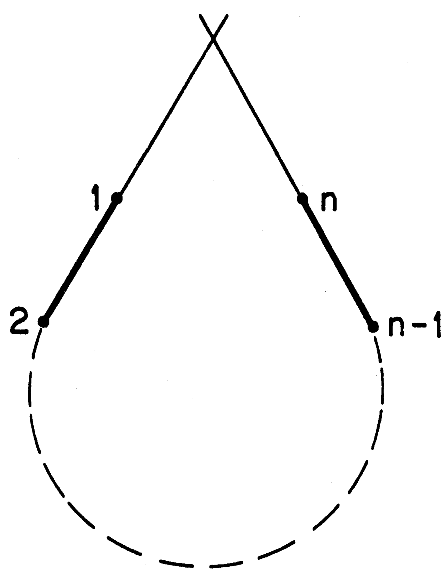
Figure 5. Two cases to be considered in determining whether or not a point lies inside a polygon.



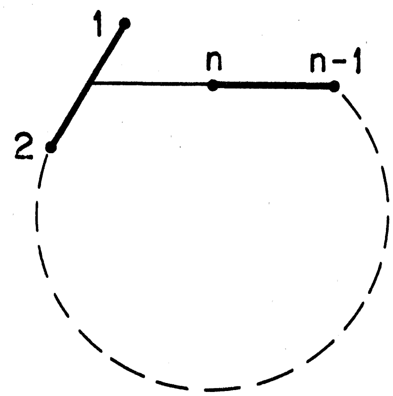
(a) Two extended lines open wider



(b) Two extended lines are parallel

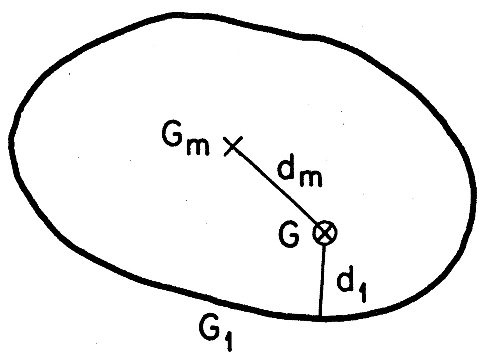


(c) Two extended lines cross each other on the extended portions

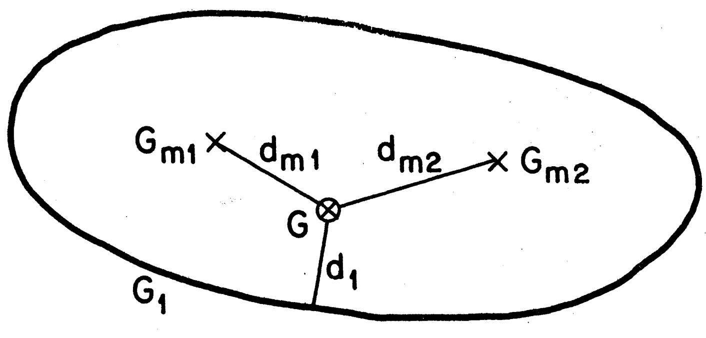


(d) One extended line crosses the other side of the polygon

Figure 6. Various configurations of the first and last sides of open contour lines.



(a) A pattern with a single maximum gain point



(b) A pattern with multiple maximum gain points

Figure 7. An earth point inside the innermost contour line.

and G_1, respectively, and the distances from the point in question to the maximum gain point and to the polygon by d_m and d_1, the antenna gain in the direction of the earth point in question, G, is represented by

$$G = G_m + (G_1 - G_m)[d_m/(d_m + d_1)]^2. \quad (5)$$

For an emission pattern having multiple maximum gain points, such as the one shown in Figure 7(b), the model calculates the G value with each maximum gain point independently and select the maximum value out of the calculated G values, i.e.,

$$G = \max_i \{G_{mi} + (G_1 - G_{m1})[d_{mi}/(d_{mi} + d_{1i})]^2\}. \quad (6)$$

Thus, no complications will be caused by multiple maximum gain points.

When the earth point in question lies between two contours, as shown in Figure 8(a), the model calculates the antenna gain by linear interpolation with respect to the distances from the point to the contours. If we denote the antenna gains that correspond to the two contours by G_1 and G_2 and the distances to the two contours by d_1 and d_2, the antenna gain in the direction of the earth point in question, G, is represented by

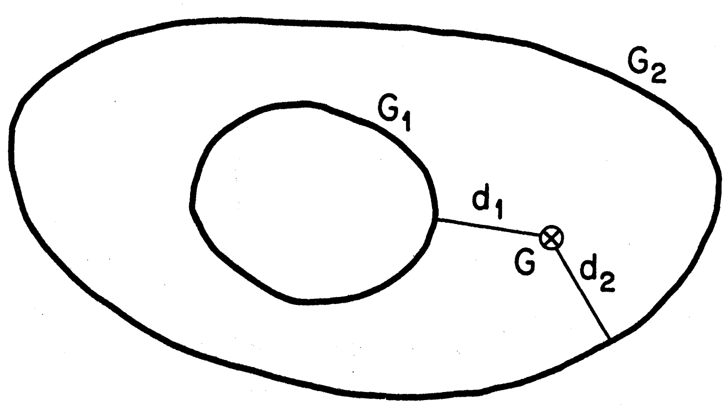
$$G = G_1 + (G_2 - G_1)[d_1/(d_1 + d_2)]. \quad (7)$$

When there are two inner contours or more inside the outer contour, as shown in Figure 8(b), the model uses the minimum of the distances to the inner contours as d_1, i.e.,

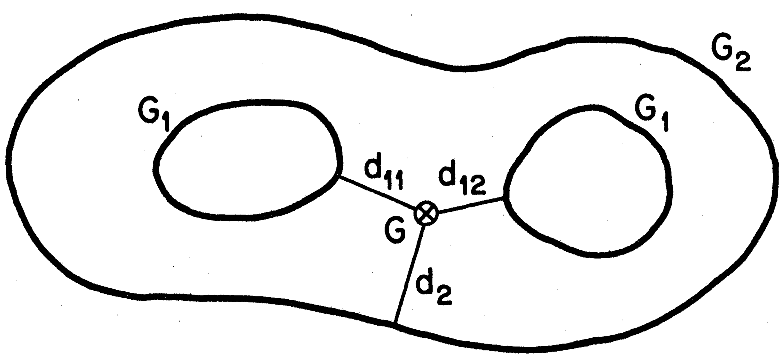
$$d_1 = \min_i\{d_{1i}\}. \quad (8)$$

We thus realize that the model can easily handle an emission pattern with multiple contour lines for an antenna gain value.

When the earth point in question lies outside the outermost contour line, as shown in Figure 9, the model calculates the antenna gain by linear extrapolation with respect to the distances from the point in question to the two outermost contour lines. If we denote the antenna gains that correspond to the two contours by G_1 and G_2 and the distances to the two contours by d_1 and



(a) A case with a single inner contour



(b) A case with multiple inner contours

Figure 8. An earth point between two contour lines.

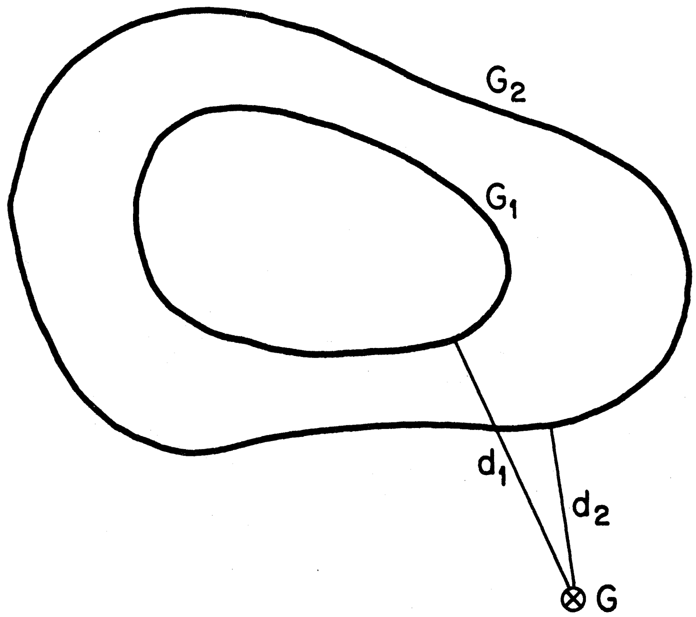


Figure 9. An earth point outside the outermost contour line.

d_2, the antenna gain in the direction of the earth point in question, G, is represented by

$$G = G_1 + (G_2 - G_1)[d_1/(d_1 - d_2)]. \tag{9}$$

When the antenna gain thus calculated falls below the residual gain value (i.e., the gain value for large off-axis angles), the latter value will be used as the antenna gain for the earth point.

6. POLYGON PATTERNS (SPECIAL-CASE PATTERNS)

The developed model includes, as special-case patterns, two emission patterns that are specified with a polygon (or polygons) corresponding to a single antenna gain value. One of these patterns assumes maximum gain points inside each polygon, while the other does not. Each of these special-case emission patterns is called the single-polygon pattern, or in short, the polygon pattern. The latter name may sound inappropriate, but this pattern is the first product in which we have ever used the polygon data successfully.

In the polygon patterns, the gain is calculated for an earth point with the distance between the earth point and the polygon and with an assumed gain fall-off curve. We assume that the distance between the earth point and the polygon closely approximates the difference in the off-axis angle between the earth point and the point on the fall-off curve that corresponds to the gain value of the polygon. The same procedures as described in the preceding section for the shaped-beam pattern (general case) are used in these polygon patterns (special case) for calculating the distance between an earth point and a polygon and for determining whether an earth point is inside or outside a polygon.

A gain fall-off curve is usually given as a function of the normalized off-axis angle, normalized with a reference beamwidth as a unit. Some curves use two normalized off-axis angles, normalized with two reference beamwidths. A reference beamwidth can be the beamwidth of the beamlet (or a small beam) which is a function of, and inversely proportional to, the frequency and the diameter of the antenna reflector (usually a paraboloid) of the satellite. The normalized off-axis angle with this reference beamwidth generally applies to a fast roll-off pattern or the fast roll-off portion of a pattern. Another reference beamwidth can be the effective beamwidth of the antenna that is inversely proportional to the maximum gain value. The normalized off-axis angle with this reference beamwidth generally applies to an antenna without fast roll-off design or the skirt portion of a fast roll-off pattern.

The polygon patterns currently included in the model are based on the roll-off curve of the fast roll-off pattern of the BSS (broadcasting-satellite service) satellite antenna, adopted by the 1983 RARC-BS-R2 (Regional Administrative Radio Conference for the Planning of the Broadcasting-Satellite Service in Region 2) (ITU, 1983). In the polygon pattern that assumes a maximum gain point (or maximum gain points) inside each polygon, the roll-off curve is used without modifications. Use of this curve leads to calculation of the gain for an earth point inside the polygon in the same manner as described in the preceding section for the general shaped-beam pattern, i.e., calculation with (5) or (6). (See Figure 7.) In the other polygon pattern that assumes no maximum gain points, the roll-off curve inside the polygon is slightly modified to allow calculation of the antenna gain without maximum gain points.

The roll-off curve uses the two reference beamwidths described in the preceding paragraph. The first reference beamwidth, ϕ_r, is the 3-dB beamwidth of the beamlet and is one of the input arguments to the model. We denote by r' the normalized off-axis angle normalized with ϕ_r and we assume that r' is calculated by

$$r' = (-G_1/12)^{1/2} \pm d_1/\phi_r, \qquad (10)$$

where G_1 is the relative antenna gain value of the polygon in decibels and is one of the input arguments to the model, and where d_1 is the distance between the earth point and the polygon. The double sign is negative for an earth point inside the polygon and positive otherwise. In deriving (10), we have assumed that the antenna gain (in decibels) decreases near the maximum gain point as the square of the angle from the maximum gain point.

The second reference beamwidth, ϕ_0, is the effective 3-dB beamwidth of the antenna and is estimated from the relation

$$G_m = 44.447 - 20 \log (\phi_0), \qquad (11)$$

where G_m is the maximum gain in decibels and is one of the input arguments to the model. We denote by r the normalized off-axis angle normalized by ϕ_0, and we assume that r is calculated from the relation

$$r = 0.5 + (r' - 0.5)\ (\phi_r/\phi_0). \qquad (12)$$

This relation means that both r and r' take an identical value of 0.5 for the relative antenna gain value of -3dB.

The antenna gain in the polygon pattern that assumes a maximum gain point is represented by

$$
\begin{aligned}
G &= G_1\,[d_m/(d_m + d_1)]^2 && \text{for } r' < r_1', \\
&= 12\, r'^2 && \text{for } r_1' < r' < 1.4499, \\
&= -25.227 && \text{for } 1.4499 < r',\ r < 1.4499, \\
&= -22 - 20 \log (r) && \text{for } 1.4499 < r.
\end{aligned}
\tag{13}
$$

In this equation, d_m and d_1 are the distances from the earth point to the maximum gain point and to the polygon, respectively, and r_1' is the value of r' that corresponds to the polygon for G_1 and is calculated by substituting $d_1 = 0$ in (10).

The equation for the polygon pattern that does not assume a maximum gain point is the same as (13) except that the first two segments are replaced by

$$
\begin{aligned}
G &= 0 && \text{for } r' < 0, \\
&= -12\, r'^2 && \text{for } 0 < r' < 1.4499.
\end{aligned}
\tag{14}
$$

Since (14) does not include d_m, this pattern does not require any knowledge about the location of maximum gain points.

When the antenna gain thus calculated by either polygon pattern falls below the residual gain value (i.e., the gain value for large off-axis angles, the latter value will be used as the antenna gain for the earth point. Note that there is a plateau with the gain value of -25.227 dB in this roll-off curve.

7. EXAMPLES

To illustrate how the model works, we present some examples in this section. In the first example, we gave the model the input data of three maximum gain points and seven contour lines for the relative gain values of -2, -2, -4, -6, -10, -20, -30 dB with a maximum of 30 contour points per contour line. The contour maps for the relative antenna gain calculated by the model are shown in Figures 10 and 11. Figure 10 depicts contours for smaller values of antenna gain in a wide area, while Figure 11 depicts contours for larger gain values in a narrower area. The results depicted in both figures look reasonable.

With partial data of the same pattern, we also calculated the antenna gain of polygon patterns. Figures 12 and 13 depict the contour map resulting from the two -2 dB contours with the three maximum gain points. In this example, the 3-dB beamwidth of the beamlet is assumed to be 2.0°. The maximum gain is assumed to be 32 dBi, and the effective 3-dB beamwidth of the antenna estimated from the maximum gain value is approximately 4.2°. Figure 12 demonstrates the general nature of the fast roll-off emission pattern adopted by the 1983 RARC-BS-R2 (ITU, 1983). The contours for the values from -2 dB through -25 dB are rather closely packed, consistent with a fast roll-off pattern. The contours for -25 dB and -26 dB are widely separated, consistent with the fact that there is a plateau between these two values. The gain falls off gradually outside the plateau. Comparison of Figure 13 with Figure 11 indicates that the contours for -1 dB in both figures are identical as expected.

Figures 14 and 15 depict the contour maps resulting from the -4 dB contour without the maximum gain points. In this example, the 3-dB beamwidth of the beamlet is assumed to be 1.8°. The maximum gain is assumed to be 32 dBi as in Figures 12 and 13 and, therefore, the effective 3-dB beamwidth of the antenna is approximately 4.2°. Figure 14 demonstrates the general nature of the fast roll-off pattern adopted by the 1983 RARC-BS-R2, i.e., fast fall off of the gain near the beam center, existence of a plateau, and gradual fall off of the gain in the skirt area. Comparison of Figure 15 with Figure 13 indicates that the contours inside the given polygon in the pattern without maximum gain points are more closely packed than in the pattern with maximum gain points.

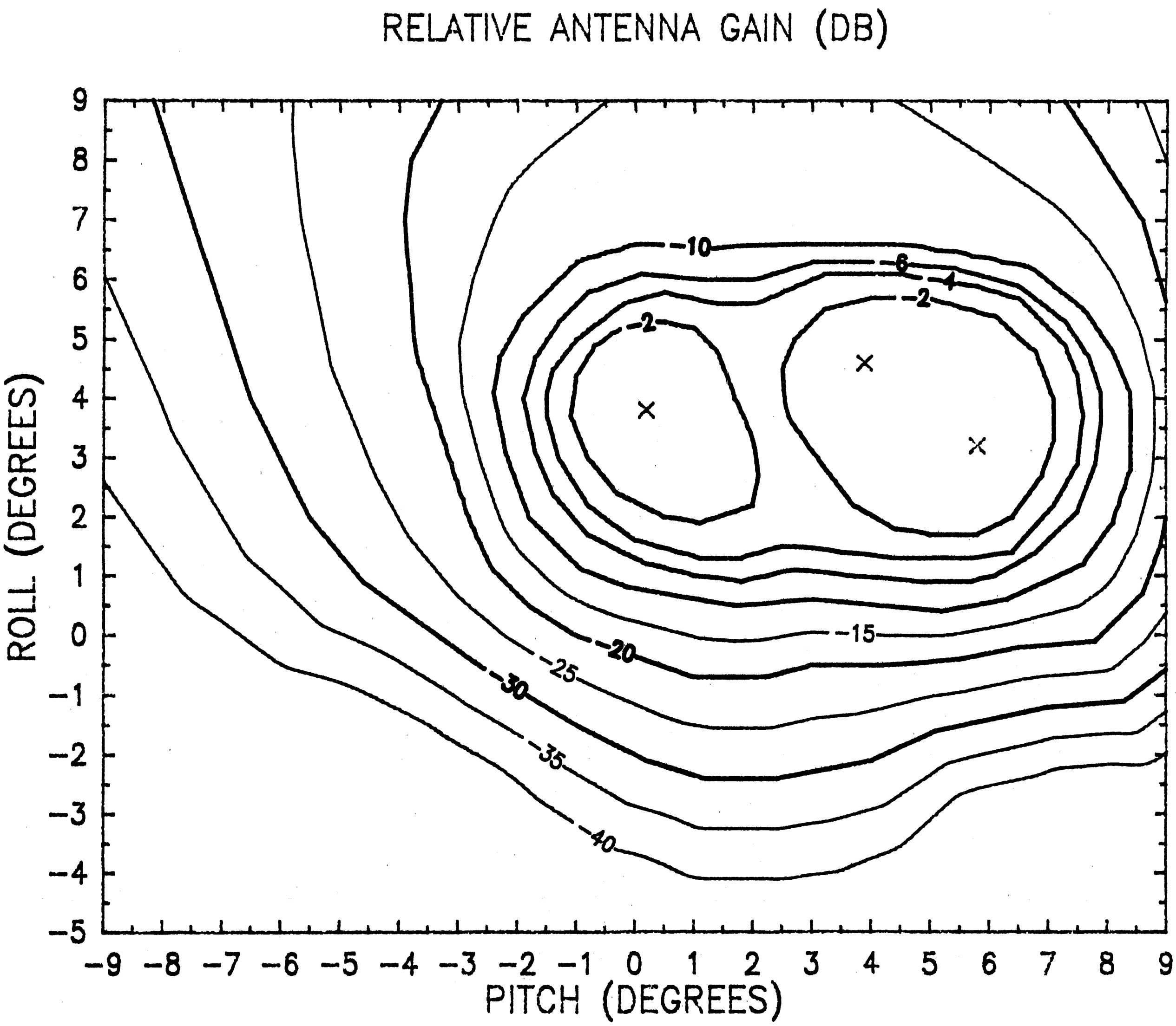


Figure 10. An example of the shaped-beam pattern. (The "x" marks are given maximum gain points. The heavier lines are given contours, while the lighter lines are the contours calculated by the model.)

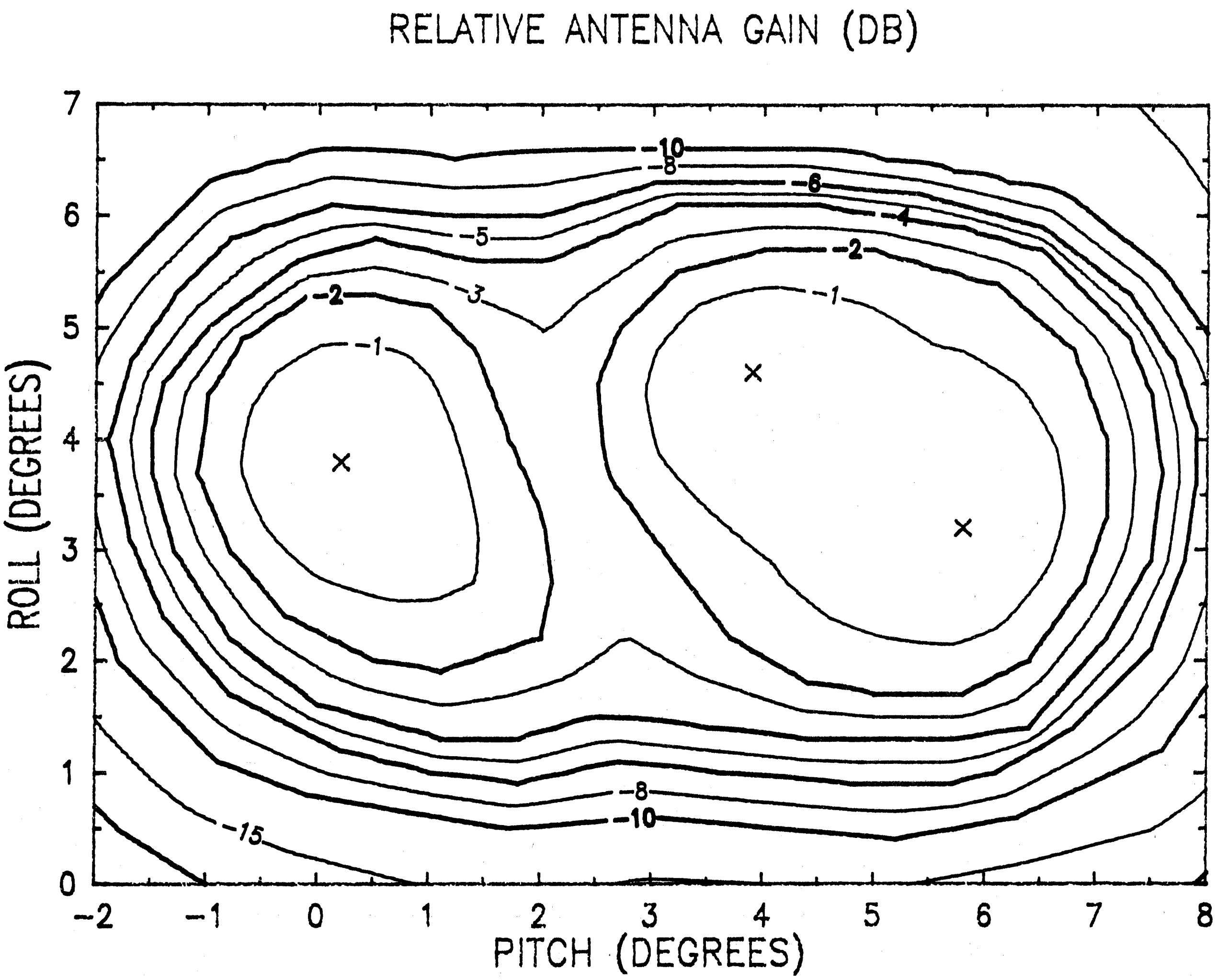


Figure 11. An example of the shaped-beam pattern. (A partial enlarged plotting of Figure 10 with additional contours.)

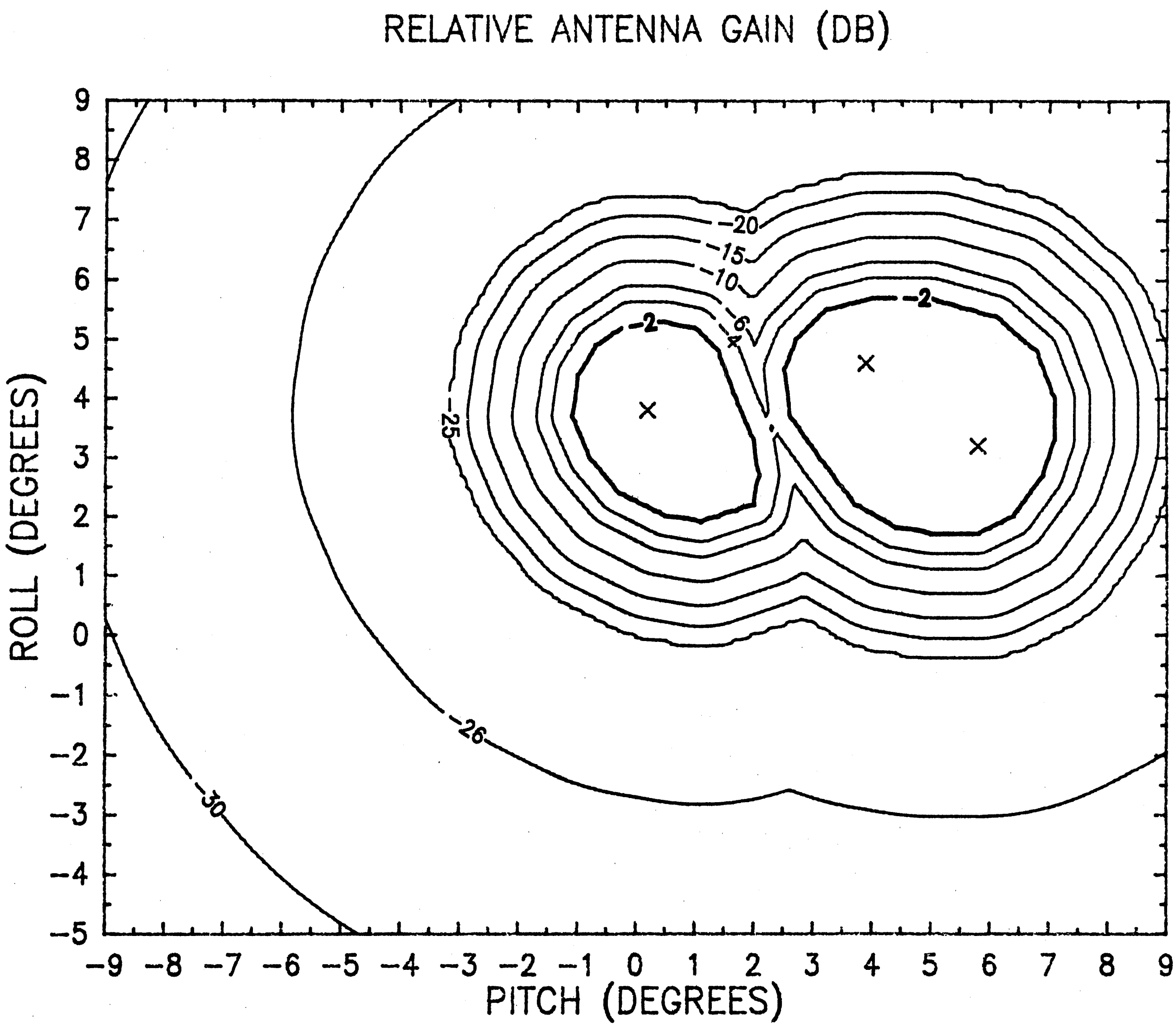


Figure 12. An example of the polygon pattern with maximum gain points. (The "x" marks are given maximum gain points. The heavier lines are given contours, while the lighter lines are the contours calculated by the model.)

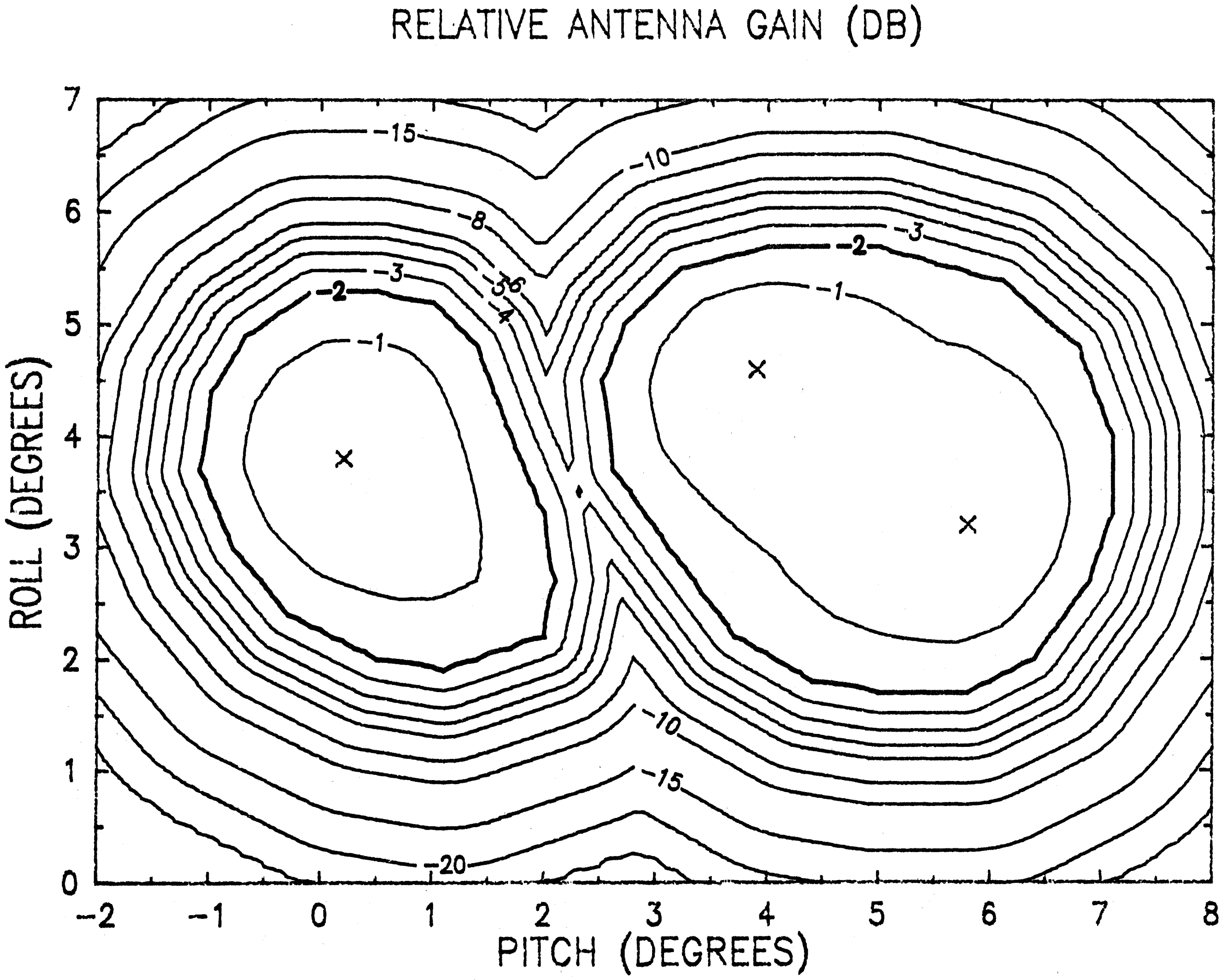


Figure 13. An example of the polygon pattern with maximum gain points. (A partial enlarged plotting of Figure 12 with additional contours.)

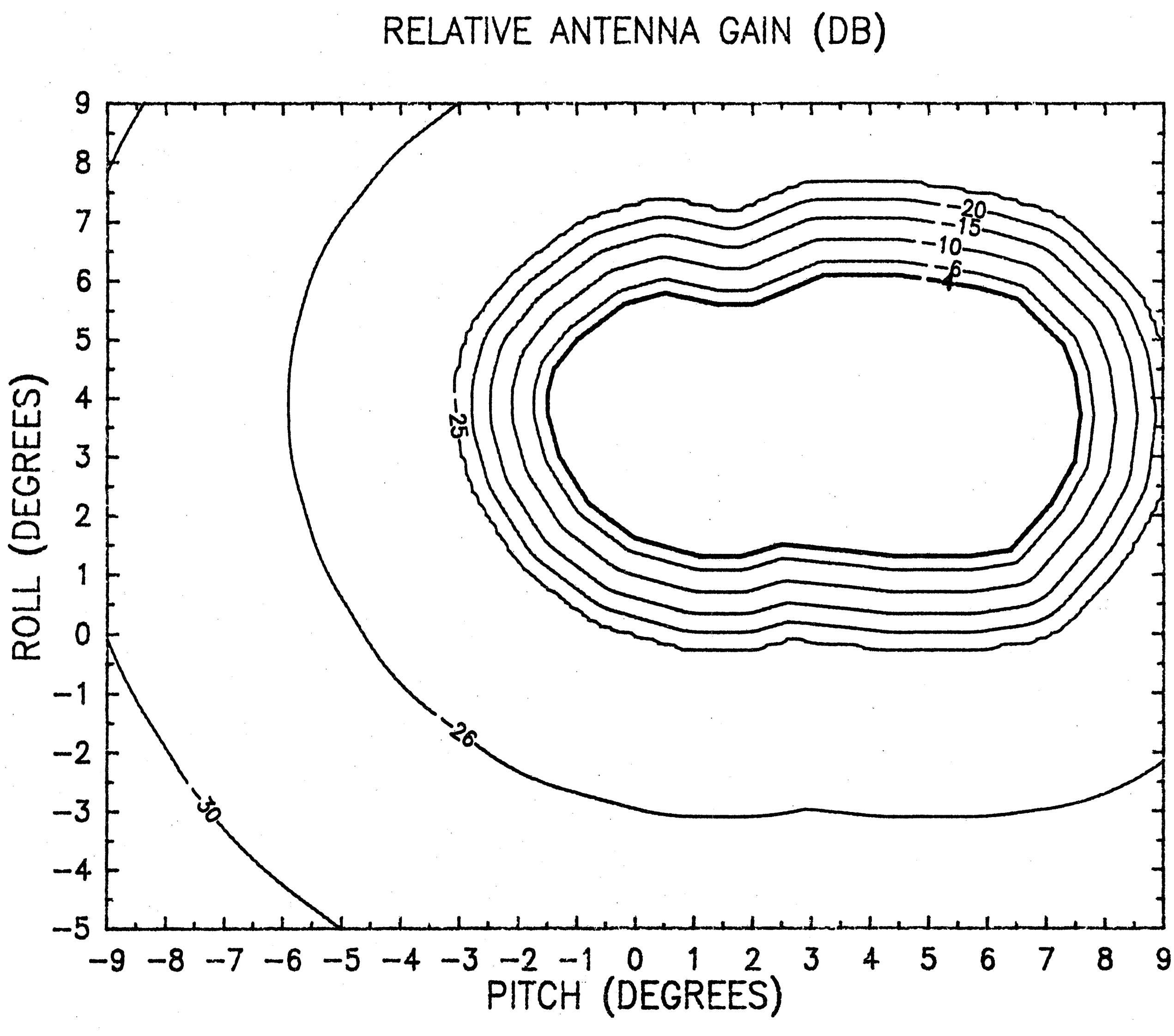


Figure 14. An example of the polygon pattern without maximum gain points. (The heavier lines are given contours, while the lighter lines are the contours calculated by the model.)

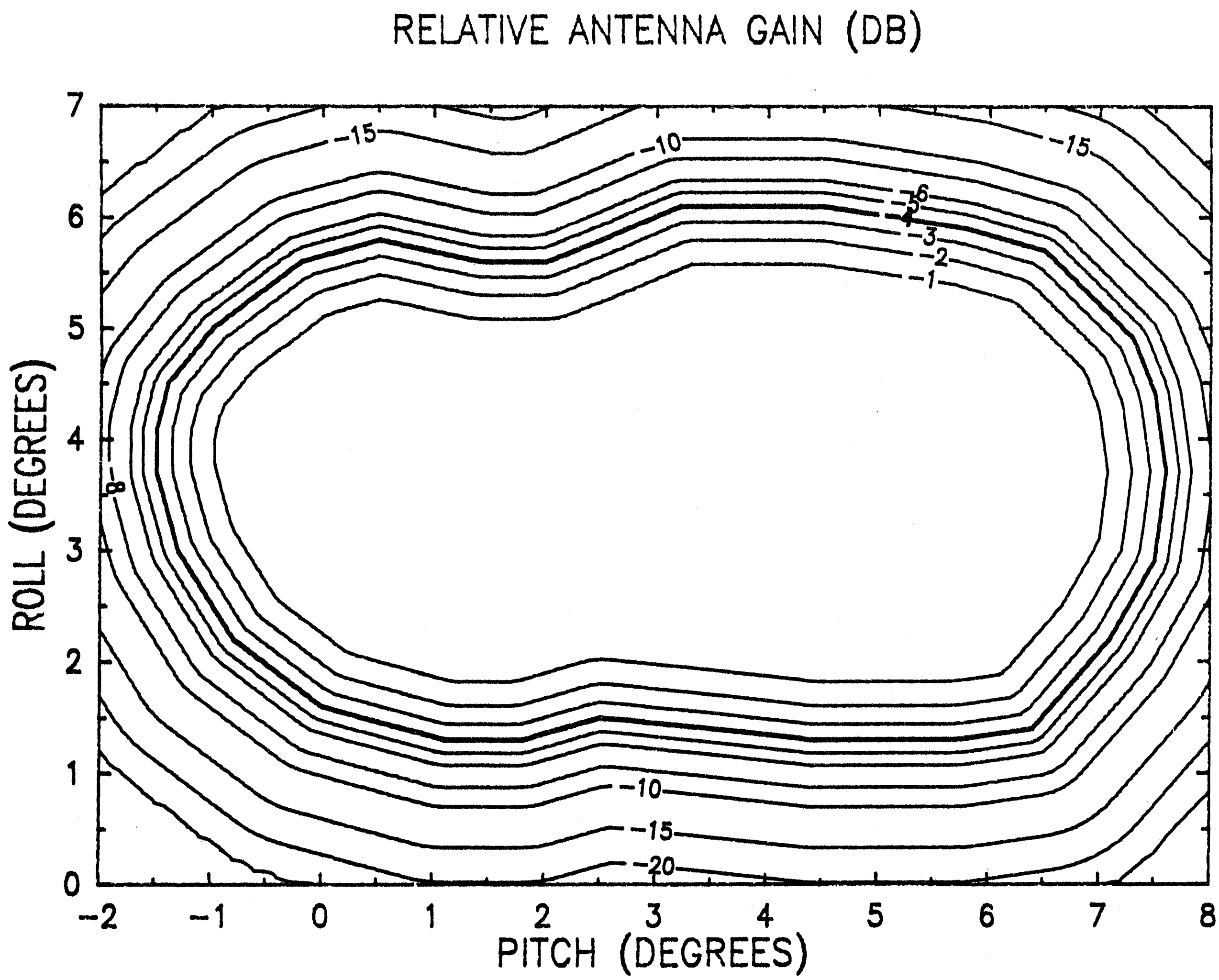


Figure 15. An example of the polygon pattern with maximum gain points. (A partial enlarged plotting of Figure 14 with additional contours.)

8. CONCLUSIONS

A model of the shaped-beam emission pattern takes the data of the emission pattern and calculates the antenna gain value in the direction of an earth point. Having a good model of shaped-beam emission pattern of a satellite antenna is essential for the analysis of mutual interferences among the FSS (fixed-satellite service) systems.

We have reviewed some models of shaped-beam emission pattern of a satellite antenna considered in a previous study, established the guidelines for developing a model based on the analysis of necessary or desirable characteristics of the model, and developed a new model. We have explained how we have developed a new model and described the developed model in detail.

The antenna gain values resulting from the model are continuous and free from undulations. The model can handle, without difficulty, a complicated pattern that has multiple maximum gain points and multiple contours for a single gain value. The model does not require a large memory area in the computer or a long computation time. Perhaps the only disadvantage of the model is that the resulting gain values are not smooth, but this is not considered serious in many applications as long as the model is used as a part of an interference analysis program.

The developed model also includes, as a special case, the so-called polygon pattern, which allows the user to calculate the antenna gain from a given gain contour (or contours) corresponding to only one gain value. This pattern is also described in this report.

The model has been implemented in a computer subprogram package. The package is presented in detail in Appendix B to this report. The appendix includes a complete Fortran listing of the package.

To represent the location of an earth point relative to the location of a satellite, the model uses the so-called "pitch" and "roll" angles, which are the angles of the line connecting the earth point to the satellite measured, in the east and north directions, respectively, from the line connecting the subsatellite point to the satellite. Mathematical relations concerning these angles are described in Appendix A, together with computer subprograms that implement the relations.

The model has been used in the U.S. preparatory effort for the 1985/88 WARC (World Administrative Radio Conference) on Space Services, often referred to as the 1985/88 WARC-ORB, sponsored by the ITU (International Telecommunication Union). At their request, the model has been submitted to the IFRB (International Frequency Registration Board) of the ITU for their possible use for the WARC-ORB.

9. ACKNOWLEDGMENTS

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APPENDIX A. PITCH AND ROLL ANGLES OF AN EARTH POINT

The location of an earth point (i.e., a point on the surface of the Earth) relative to the location of a satellite can be represented in various ways. In our model, we use the so-called "pitch" and "roll" angles to represent the location of an earth point. These angles are defined as the angles of the earth point seen from the satellite and measured in the east and north directions from the line connecting the satellite to its subsatellite point. (The subsatellite point of a satellite is the point on the surface of the Earth having the same longitude and latitude as the satellite; it is the earth point closest to the satellite.) The terms "pitch" and "roll" may sound strange at first, but the use of these terms can be recognized if we consider a "ship" located in parallel to the equatorial plane of the Earth at the location of the satellite with the "mast" pointing toward the Earth. If the "ocean" is calm and the ship does not pitch nor roll, the mast points toward the subsatellite point. If the ocean is not calm and the ship pitches and rolls, the earth point that the mast points toward moves on the surface of the Earth. At an instant a set of pitch and roll angles corresponds to the direction of an earth point. Since this correspondence is unique with a fixed satellite point, we can use the pitch and roll angles to represent the location of an earth point relative to the satellite location.

In a two-dimensional Cartesian coordinate with the pitch and roll angles as the abscissa and ordinate, respectively, the subsatellite point of the satellite is projected at the origin of the coordinate. When the satellite is positioned above the Equator of the Earth, the Equator is projected on the abscissa, and the meridian that passes through the subsatellite point is projected on the ordinate. When the satellite is a geostationary satellite, the whole Earth is projected in a circle of the radius of 8.7° with its center at the origin of the coordinate.

In this appendix, we present mathematical relations that relate the pitch and roll angles to more conventional representation of the location of an earth point. For this purpose, we introduce the following two Cartesian coordinate systems, i.e., the earth-center coordinate system and the subsatellite-point coordinate system.

Earth-Center Coordinate System. The origin of this coordinate system is the center of the Earth. The positive x, y, and z axes intersect the surface of the Earth at 0° east and 0° north, at 90° east and 0° north, and at 90° north (i.e., the north pole), respectively.

Subsatellite-Point Coordinate System. This coordinate system is characterized by two points, a satellite point and its subsatellite point. The origin is the subsatellite point. The positive z' axis points toward the satellite point. The x' axis is parallel to the equatorial plane of the Earth. As a convention, the sense of the x' axis is taken in such a way that the positive y' axis is on the north side of the z'-x' plane. This coordinate system is a special case of the equatorial-plane coordinate system, described in detail in "Technical basis for the Geostationary Satellite Orbit Analysis Program (GSOAP) Version 2," by Akima, NTIA Report 85-183, November 1985, NTIS Order No. PB86-151750.

To represent the pitch and roll angles of an earth point mathematically, we also introduce the following symbols:

π = pitch angle of the earth point,

ρ = roll angle of the earth point,

r_s = radius of the geostationary satellite orbit,

r_e = radius of the Earth,

ϕ = longitude of a point,

θ = latitude of a point,

x, y, z = earth-center coordinates of a point,

x', y', z' = subsatellite-point coordinates of a point.

We use subscripts, s and e, with some of the above symbols to denote the satellite and earth point, respectively.

With these symbols, we have familiar expressions of the earth-center coordinates of an earth point; i.e.,

$$x_e = r_e \cos\theta_e \cos\phi_e,$$

$$y_e = r_e \cos\theta_e \sin\phi_e, \qquad \text{(A-1)}$$

$$z_e = r_e \sin\theta_e.$$

From these earth-center coordinates of the earth point, we can calculate the subsatellite-point coordinates of the earth point by effecting a modified (simplified) version of coordinate transformation from the earth-center coordinate system to the equatorial-plane coordinate system, also described by Akima. Then, with the subsatellite-point coordinates of the earth point, the pitch and roll angles of the earth point can be calculated by

$$\pi = \tan^{-1}[x_e'/(r_s - r_e - z_e')],$$
$$\rho = \tan^{-1}[y_e'/(r_s - r_e - z_e')]. \qquad \text{(A-2)}$$

When the earth-center coordinates of every point are already calculated, this procedure is perhaps the simplest way of calculating the pitch and roll angles of an earth point.

We can represent the pitch and roll angles of an earth point in terms of polar coordinates (i.e., longitude, latitude, and radius) of the satellite and earth point. Representing the elements of the coordinate transformation matrix in terms of polar coordinates of the satellite, and calculating and representing the subsatellite-point coordinates of the earth point in terms of polar coordinates of the satellite and the earth point, we obtain the following relations:

$$\pi = \tan^{-1}\{[\cos\theta_e \sin(\phi_e - \phi_s)] / [r_s/r_e - \cos\theta_s \cos\theta_e \cos(\phi_e - \phi_s) - \sin\theta_s \sin\theta_e]\},$$
$$\rho = \tan^{-1}\{[\cos\theta_s \sin\theta_e - \sin\theta_s \cos\theta_e \cos(\phi_e - \phi_s)] / [r_s/r_e - \cos\theta_s \cos\theta_e \cos(\phi_e - \phi_s) - \sin\theta_s \sin\theta_e]\}. \qquad \text{(A-3)}$$

When the satellite is on the Equator of the Earth, the above expression of the pitch and roll angles can be simplified as follows:

$$\pi = \tan^{-1}\{[r_e \cos\theta_e \sin(\phi_e - \phi_s)]/[r_s - r_e \cos\theta_e \cos(\phi_e - \phi_s)]\},$$
$$\rho = \tan^{-1}\{[r_e \sin\theta_e]/[r_s - r_e \cos\theta_e \cos(\phi_e - \phi_s)]\}. \qquad \text{(A-4)}$$

These last expressions can also be obtained intuitively from the geometry of the satellite and the earth point without going through the rather complicated procedure of coordinate transformation.

Conversely, when the pitch and roll angles of an earth point are given, the subsatellite-point coordinates of the earth point can be calculated by

$$x_e' = (r_s - z'') \tan\pi,$$

$$y_e' = (r_s - z'') \tan\rho, \qquad \text{(A-5)}$$

$$z_e' = z'' - r_e,$$

where

$$z'' = \{t^2 r_s + [r_e{}^2 - t^2 (r_s{}^2 - r_e{}^2)]^{1/2}\}/(1 + t^2),$$

$$\text{(A-6)}$$

$$t^2 = \tan^2\pi + \tan^2\rho.$$

To derive these equations, we have used the fact that the distance between the earth point and the center of the Earth is equal to r_e. From the subsatellite-point coordinates of the earth point thus calculated, we can calculate the earth-center coordinates of the earth point, x_e, y_e, and z_e, by effecting the coordinate transformation which is the inverse transformation of the one described earlier. From x_e, y_e, and z_e, we can calculate the longitude and latitude of the earth point by

$$\theta_e = \sin^{-1}(z_e/r_e),$$

$$\text{(A-7)}$$

$$\phi_e = \tan^{-1}(y_e/x_e).$$

Equations in (A-7) are the inverse relations of (A-1).

We can also obtain expressions of longitude and latitude of the earth point by solving (A-6). If we let

$$R = r_e [\cos\theta_s \cos\theta_e \cos(\phi_e - \phi_s) + \sin\theta_s \sin\theta_e], \qquad \text{(A-8)}$$

we have, from (A-6),

$$(r_s - R)\tan\pi = \cos\theta_e \sin(\phi_e - \phi_s),$$

$$(r_s - R)\tan\rho = \cos\theta_s \sin\theta_e - \sin\theta_s \cos\theta_e \cos(\phi_e - \phi_s). \quad \text{(A-9)}$$

Solving (A-9) with respect to R, we obtain

$$R = \{t^2 r_s + [r_e^2 - t^2 (r_s^2 - r_e^2)]^{1/2}\}/(1 + t^2), \quad \text{(A-10)}$$

where t^2 is the same as in (A-6). Note that R in (A-10) is equal to z" in (A-6). Once R is calculated, we can calculate ϕ_e and θ_e from (A-8) and (A-9) as

$$\phi_e = \phi_s + \tan^{-1}\{[(r_s - R)\tan\pi \cos\theta_s]/(R - r_e \sin\theta_s \sin\theta_e)\},$$

$$\theta_e = \sin^{-1}\{[(r_s - R)\tan\rho \cos\theta_s + R \sin\theta_s]/r_e\}. \quad \text{(A-11)}$$

When the satellite is on the Equator of the Earth, we can simplify (A-11) to

$$\phi_e = \phi_s + \tan^{-1}\{[(r_s - R)\tan\pi]/R\},$$

$$\theta_e = \sin^{-1}\{[(r_s - R)\tan\rho]/r_e\}, \quad \text{(A-12)}$$

by setting $\theta_s = 0$.

We have implemented the above algorithms in four Fortran subroutine subprograms, i.e., EPACEC, EPACLL, EPECAC, and EPLLAC. The first two subroutines calculate the pitch and roll angles of earth points relative to the location of a satellite from the locations of the satellite and the earth points. The EPACEC subroutine calculates the pitch and roll angles from the earth-center coordinates of the satellite and the earth points. The EPACLL subroutine calculates the same from the longitudes and latitudes of the satellite and the earth points. The remaining two subroutines calculate the locations of earth points from the locations of the satellite and the pitch and roll angles of earth points relative to the location of the satellite. The EPECAC subroutine calculates the earth-center coordinates of earth points from the earth-center coordinates of the satellite and the pitch and roll angles of the earth points.

The EPLLAC subroutine calculates the longitudes and latitudes of earth points from the longitude and latitude of the satellite and the pitch and roll angles of the earth points.

Fortran listings of the four subroutines follow. These subroutines are written in ANSI (American National Standards Institute) Standard Fortran (Publication X3.9-1978, ANSI, 345 East 47th Street, New York, NY 10017). The user information of each subroutine including the description of the input and output arguments is given in the beginning of each subroutine.

```
      SUBROUTINE  EPACEC(XS,YS,ZS,NE,XE,YE,ZE, EPAE,EPAN)
C THIS SUBROUTINE CALCULATES THE ANGLE COORDINATES OF EARTH
C POINTS RELATIVE TO THE LOCATION OF A SATELLITE FROM THE EARTH-
C CENTER COORDINATES OF THE SATELLITE AND EARTH POINTS.
C THE ANGLE COORDINATES OF AN EARTH POINT RELATIVE TO THE LOCA-
C TION OF A SATELLITE ARE THE ANGLES OF A LINE CONNECTING THE
C SATELLITE AND THE EARTH POINT MEASURED IN THE EAST AND NORTH
C DIRECTIONS FROM THE LINE CONNECTING THE SATELLITE AND ITS
C SUBSATELLITE POINT.  THESE ANGLE COORDINATES ARE ALSO CALLED
C THE 'PITCH' AND 'ROLL' ANGLES OF THE EARTH POINT.
C THIS SUBROUTINE IS BASED ON THE COORDINATE TRANSFORMATION FROM
C THE EARTH-CENTER COORDINATE SYSTEM TO THE SUBSATELLITE-POINT
C COORDINATE SYSTEM, WHICH IS A SPECIAL CASE OF THE EQUATORIAL-
C PLANE COORDINATE SYSTEM.
C THE EARTH-CENTER COORDINATE SYSTEM IS A CARTESIAN SYSTEM.  THE
C ORIGIN IS THE CENTER OF THE EARTH.  THE POSITIVE X, Y, AND Z
C AXES INTERSECT THE SURFACE OF THE EARTH AT 0 DEGREES EAST AND
C 0 DEGREES NORTH, AT 90 DEGREES EAST AND 0 DEGREES NORTH, AND
C AT 90 DEGREES NORTH (THE NORTH POLE), RESPECTIVELY.
C THE EQUATORIAL-PLANE COORDINATE SYSTEM IS A CARTESIAN SYSTEM,
C CHARACTERIZED BY TWO POINTS, AN EARTH POINT AND A SATELLITE
C POINT, AND BY THE EQUATORIAL PLANE OF THE EARTH.  THE ORIGIN
C IS THE EARTH POINT.  THE POSITIVE Z' AXIS POINTS TOWARD THE
C SATELLITE POINT.  THE X' AXIS IS PARALLEL TO THE EQUATORIAL
C PLANE OF THE EARTH.  THE SENSE OF THE X' AXIS IS TAKEN IN SUCH
C A WAY THAT THE POSITIVE Y' AXIS IS ON THE NORTH SIDE OF THE
C Z'-X' PLANE.
C THE SUBSATELLITE-POINT COORDINATE SYSTEM, WHICH IS A SPECIAL
C CASE OF THE EQUATORIAL-PLANE COORDINATE SYSTEM, HAS ITS ORIGIN
C AT THE SUBSATELLITE POINT ON THE SATELLITE.  IN THIS COORDI-
C NATE SYSTEM, THE CENTER OF THE EARTH IS ON THE NEGATIVE Z'
C AXIS.
C THE INPUT ARGUMENTS ARE
C   XS, YS, ZS
C          = EARTH-CENTER COORDINATES OF THE SATELLITE
C            (IN KM),
C   NE     = NUMBER OF EARTH POINTS,
C   XE, YE, ZE
C          = ARRAYS OF DIMENSION NE CONTAINING THE EARTH-
C            CENTER COORDINATES OF THE EARTH POINTS (IN KM).
C THE OUTPUT ARGUMENTS ARE
C   EPAE, EPAN
C          = ARRAYS OF DIMENSION NE WHERE THE ANGLE COORDI-
C            NATES OF THE EARTH POINTS (IN DEGREES) IN THE
C            EAST AND NORTH DIRECTIONS ARE TO BE STORED.
C SPECIFICATION STATEMENT
      DIMENSION   XE(*),YE(*),ZE(*),
     1            EPAE(*),EPAN(*)
      PARAMETER   (RE0=6378.2,RG0=42164.0)
      SAVE  INIT,CFRTD,RGRE
      DATA  INIT/0/
C CALCULATION
C INITIALIZATION
   10 IF(INIT.LE.0)  THEN
```

```
          INIT=1
          CFRTD=90.0/ATAN2(1.0,0.0)
          RGRE=RGO/REO
        END IF
C MAIN CALCULATION
C UNIT VECTOR  --  Z' AXIS
   20 A31=XS/RGO
      A32=YS/RGO
      A33=ZS/RGO
C UNIT VECTOR  --  X' AXIS
   30 A1=-A32
      A2= A31
      A=SQRT(A1*A1+A2*A2)
      A11=A1/A
      A12=A2/A
C     A13=0.0
C UNIT VECTOR  --  Y' AXIS
   40 A21=         -A33*A12
C  40 A21=A32*A13-A33*A12
      A22=A33*A11
C     A22=A33*A11-A31*A13
      A23=A31*A12-A32*A11
C DO-LOOP WITH RESPECT TO THE EARTH-POINT NUMBER
   50 DO 59  IE=1,NE
C TRANSFORMATION TO THE SUBSATELLITE-POINT COORDINATES
        DXE=XE(IE)-XS/RGRE
        DYE=YE(IE)-YS/RGRE
        DZE=ZE(IE)-ZS/RGRE
        XEP=A11*DXE+A12*DYE
C       XEP=A11*DXE+A12*DYE+A13*DZE
        YEP=A21*DXE+A22*DYE+A23*DZE
        ZEP=A31*DXE+A32*DYE+A33*DZE
C CALCULATION OF ANGLE COORDINATES
        DZP=RGO-REO-ZEP
        EPAE(IE)=CFRTD*ATAN2(XEP,DZP)
        EPAN(IE)=CFRTD*ATAN2(YEP,DZP)
   59 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  EPACLL(SLON,SLAT,NE,ELON,ELAT, EPAE,EPAN)
C THIS SUBROUTINE CALCULATES THE ANGLE COORDINATES OF EARTH
C POINTS RELATIVE TO THE LOCATION OF A SATELLITE FROM THE LONGI-
C TUDES AND LATITUDES OF THE SATELLITE AND EARTH POINTS.
C THE ANGLE COORDINATES OF AN EARTH POINT RELATIVE TO THE LOCA-
C TION OF A SATELLITE ARE THE ANGLES OF A LINE CONNECTING THE
C SATELLITE AND THE EARTH POINT MEASURED IN THE EAST AND NORTH
C DIRECTIONS FROM THE LINE CONNECTING THE SATELLITE AND ITS
C SUBSATELLITE POINT.  THESE ANGLE COORDINATES ARE ALSO CALLED
C THE 'PITCH' AND 'ROLL' ANGLES OF THE EARTH POINT.
C THE INPUT ARGUMENTS ARE
C   SLON, SLAT
C          = LONGITUDE AND LATITUDE OF THE SATELLITE
C            (IN DEGREES),
C   NE     = NUMBER OF EARTH POINTS,
C   ELON, ELAT
C          = ARRAYS OF DIMENSION NE CONTAINING THE LONGI-
C            TUDES AND LATITUDES OF THE EARTH POINTS
C            (IN DEGREES).
C THE OUTPUT ARGUMENTS ARE
C   EPAE, EPAN
C          = ARRAYS OF DIMENSION NE WHERE THE ANGLE COORDI-
C            NATES OF THE EARTH POINTS (IN DEGREES) IN THE
C            EAST AND NORTH DIRECTIONS ARE TO BE STORED.
C SPECIFICATION STATEMENT
      DIMENSION   ELON(*),ELAT(*),
     1            EPAE(*),EPAN(*)
      PARAMETER   (RE0=6378.2,RG0=42164.0)
      SAVE  INIT,CFDTR,CFRTD,RGRE
      DATA  INIT/0/
C CALCULATION
C INITIALIZATION
   10 IF(INIT.LE.0)  THEN
        INIT=1
        CFDTR=ATAN2(1.0,0.0)/90.0
        CFRTD=1.0/CFDTR
        RGRE=RG0/RE0
      END IF
C MAIN CALCULATION
C SINE AND COSINE OF THE SATELLITE LATITUDE
   20 IF(SLAT.NE.0.0)  THEN
        THS=CFDTR*SLAT
        SINTHS=SIN(THS)
        COSTHS=COS(THS)
      ELSE
        SINTHS=0.0
        COSTHS=1.0
      END IF
C DO-LOOP WITH RESPECT TO THE EARTH-POINT NUMBER
C CALCULATION OF ANGLE COORDINATES
   50 DO 59  IE=1,NE
        THE=CFDTR*ELAT(IE)
        SINTHE=SIN(THE)
        COSTHE=COS(THE)
```

```
        DPH=CFDTR*(ELON(IE)-SLON)
        SINDPH=SIN(DPH)
        COSDPH=COS(DPH)
        DENOM=RGRE-COSTHS*COSTHE*COSDPH-SINTHS*SINTHE
        EPAE(IE)=CFRTD*ATAN2(COSTHE*SINDPH,DENOM)
        EPAN(IE)
     1     =CFRTD*ATAN2(COSTHS*SINTHE-SINTHS*COSTHE*COSDPH,DENOM)
   59 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  EPECAC(XS,YS,ZS,NE,EPAE,EPAN, XE,YE,ZE)
C THIS SUBROUTINE CALCULATES THE EARTH-CENTER COORDINATES OF
C EARTH POINTS FROM THE ANGLE COORDINATES OF THE EARTH POINTS
C RELATIVE TO THE LOCATION OF A SATELLITE AND THE EARTH-CENTER
C COORDINATES OF THE SATELLITE.
C THE ANGLE COORDINATES OF AN EARTH POINT RELATIVE TO THE LOCA-
C TION OF A SATELLITE ARE THE ANGLES OF A LINE CONNECTING THE
C SATELLITE AND THE EARTH POINT MEASURED IN THE EAST AND NORTH
C DIRECTIONS FROM THE LINE CONNECTING THE SATELLITE AND ITS
C SUBSATELLITE POINT.  THESE ANGLE COORDINATES ARE ALSO CALLED
C THE 'PITCH' AND 'ROLL' ANGLES OF THE EARTH POINT.
C THIS SUBROUTINE IS BASED ON THE COORDINATE TRANSFORMATION FROM
C THE SUBSATELLITE-POINT COORDINATE SYSTEM, WHICH IS A SPECIAL
C CASE OF THE EQUATORIAL-PLANE COORDINATE SYSTEM, TO THE EARTH-
C CENTER COORDINATE SYSTEM.
C THE EARTH-CENTER COORDINATE SYSTEM IS A CARTESIAN SYSTEM.  THE
C ORIGIN IS THE CENTER OF THE EARTH.  THE POSITIVE X, Y, AND Z
C AXES INTERSECT THE SURFACE OF THE EARTH AT 0 DEGREES EAST AND
C 0 DEGREES NORTH, AT 90 DEGREES EAST AND 0 DEGREES NORTH, AND
C AT 90 DEGREES NORTH (THE NORTH POLE), RESPECTIVELY.
C THE EQUATORIAL-PLANE COORDINATE SYSTEM IS A CARTESIAN SYSTEM,
C CHARACTERIZED BY TWO POINTS, AN EARTH POINT AND A SATELLITE
C POINT, AND BY THE EQUATORIAL PLANE OF THE EARTH.  THE ORIGIN
C IS THE EARTH POINT.  THE POSITIVE Z' AXIS POINTS TOWARD THE
C SATELLITE POINT.  THE X' AXIS IS PARALLEL TO THE EQUATORIAL
C PLANE OF THE EARTH.  THE SENSE OF THE X' AXIS IS TAKEN IN SUCH
C A WAY THAT THE POSITIVE Y' AXIS IS ON THE NORTH SIDE OF THE
C Z'-X' PLANE.
C THE SUBSATELLITE-POINT COORDINATE SYSTEM, WHICH IS A SPECIAL
C CASE OF THE EQUATORIAL-PLANE COORDINATE SYSTEM, HAS ITS ORIGIN
C AT THE SUBSATELLITE POINT ON THE SATELLITE.  IN THIS COORDI-
C NATE SYSTEM, THE CENTER OF THE EARTH IS ON THE NEGATIVE Z'
C AXIS.
C THE INPUT ARGUMENTS ARE
C   XS, YS, ZS
C             = EARTH-CENTER COORDINATES OF THE SATELLITE
C               (IN KM),
C   NE        = NUMBER OF EARTH POINTS,
C   EPAE, EPAN
C             = ARRAYS OF DIMENSION NE CONTAINING THE ANGLE
C               COORDINATES OF THE EARTH POINTS (IN DEGREES)
C               IN THE EAST AND NORTH DIRECTIONS.
C THE OUTPUT ARGUMENTS ARE
C   XE, YE, ZE
C             = ARRAYS OF DIMENSION NE WHERE THE EARTH-CENTER
C               COORDINATES OF THE EARTH POINTS (IN KM) ARE
C               TO BE STORED.
C SPECIFICATION STATEMENT
      DIMENSION   EPAE(*),EPAN(*),
     1            XE(*),YE(*),ZE(*)
      PARAMETER   (RE0=6378.2,RG0=42164.0)
      SAVE  INIT,CFDTR,RGRE,RE0SQ,RG0SQ
      DATA  INIT/0/
C CALCULATION
```

```
C INITIALIZATION
   10 IF(INIT.LE.0)  THEN
        INIT=1
        CFDTR=ATAN2(1.0,0.0)/90.0
        RGRE=RGO/REO
        REOSQ=REO**2
        RGOSQ=RGO**2
      END IF
C MAIN CALCULATION
C UNIT VECTOR  --  Z' AXIS
   20 A31=XS/RGO
      A32=YS/RGO
      A33=ZS/RGO
C UNIT VECTOR  --  X' AXIS
   30 A1=-A32
      A2= A31
      A=SQRT(A1*A1+A2*A2)
      A11=A1/A
      A12=A2/A
C     A13=0.0
C UNIT VECTOR  --  Y' AXIS
   40 A21=        -A33*A12
C  40 A21=A32*A13-A33*A12
      A22=A33*A11
C     A22=A33*A11-A31*A13
      A23=A31*A12-A32*A11
C DO-LOOP WITH RESPECT TO THE EARTH-POINT NUMBER
   50 DO 59  IE=1,NE
C CALCULATION OF THE SUBSATELLITE-POINT COORDINATES
        TANAE=TAN(CFDTR*EPAE(IE))
        TANAN=TAN(CFDTR*EPAN(IE))
        TS=TANAE*TANAE+TANAN*TANAN
        D=REOSQ-TS*(RGOSQ-REOSQ)
        IF(D.LT.0.0)      GO TO 90
        ZDP=(TS*RGO+SQRT(D))/(1.0+TS)
        XEP=(RGO-ZDP)*TANAE
        YEP=(RGO-ZDP)*TANAN
        ZEP=ZDP-REO
C TRANSFORMATION TO THE EARTH-CENTER COORDINATES
        XE(IE)=A11*XEP+A21*YEP+A31*ZEP+XS/RGRE
        YE(IE)=A12*XEP+A22*YEP+A32*ZEP+YS/RGRE
        ZE(IE)=        A23*YEP+A33*ZEP+ZS/RGRE
C       ZE(IE)=A13*XEP+A23*YEP+A33*ZEP+ZS/RGRE
   59 CONTINUE
      RETURN
C ERROR STOP
   90 PRINT 99090, IE,EPAE(IE),EPAN(IE)
      STOP
C FORMAT STATEMENTS
99090 FORMAT(1X/' ***   EARTH POINT DOES NOT EXIST.'/
     1  9X,'IE =',I4,5X,'EPAE =',F7.3,5X,'EPAN =',F7.3/
     2  7X,'ERROR DETECTED IN ROUTINE   EPECAC'/1H1)
      END
```

```
      SUBROUTINE  EPLLAC(SLON,SLAT,NE,EPAE,EPAN, ELON,ELAT)
C THIS SUBROUTINE CALCULATES THE LONGITUDE AND LATITUDE OF EARTH
C POINTS FROM THE ANGLE COORDINATES OF THE EARTH POINTS RELATIVE
C TO THE LOCATION OF A SATELLITE AND THE LONGITUDE AND LATITUDE
C OF THE SATELLITE.
C THE ANGLE COORDINATES OF AN EARTH POINT RELATIVE TO THE LOCA-
C TION OF A SATELLITE ARE THE ANGLES OF A LINE CONNECTING THE
C SATELLITE AND THE EARTH POINT MEASURED IN THE EAST AND NORTH
C DIRECTIONS FROM THE LINE CONNECTING THE SATELLITE AND ITS
C SUBSATELLITE POINT.  THESE ANGLE COORDINATES ARE ALSO CALLED
C THE 'PITCH' AND 'ROLL' ANGLES OF THE EARTH POINT.
C THE INPUT ARGUMENTS ARE
C   SLON, SLAT
C          = LONGITUDE AND LATITUDE OF THE SATELLITE
C            (IN DEGREES),
C   NE     = NUMBER OF EARTH POINTS,
C   EPAE, EPAN
C          = ARRAYS OF DIMENSION NE CONTAINING THE ANGLE
C            COORDINATES OF THE EARTH POINTS (IN DEGREES)
C            IN THE EAST AND NORTH DIRECTIONS.
C THE OUTPUT ARGUMENTS ARE
C   ELON, ELAT
C          = ARRAYS OF DIMENSION NE WHERE THE LONGITUDES AND
C            LATITUDES OF THE EARTH POINTS (IN DEGREES) ARE
C            TO BE STORED.
C SPECIFICATION STATEMENT
      DIMENSION   EPAE(*),EPAN(*),
     1            ELON(*),ELAT(*)
      PARAMETER   (REO=6378.2,RGO=42164.0)
      SAVE  INIT,CFDTR,CFRTD,REOSQ,RGOSQ
      DATA  INIT/0/
C CALCULATION
C INITIALIZATION
   10 IF(INIT.LE.0)  THEN
        INIT=1
        CFDTR=ATAN2(1.0,0.0)/90.0
        CFRTD=1.0/CFDTR
        REOSQ=REO**2
        RGOSQ=RGO**2
      END IF
C MAIN CALCULATION
C DO-LOOP WITH RESPECT TO THE EARTH-POINT NUMBER
   50 DO 59  IE=1,NE
        TANAE=TAN(CFDTR*EPAE(IE))
        TANAN=TAN(CFDTR*EPAN(IE))
        TS=TANAE*TANAE+TANAN*TANAN
        D=REOSQ-TS*(RGOSQ-REOSQ)
        IF(D.LT.0.0)      GO TO 90
        RR=(TS*RGO+SQRT(D))/(1.0+TS)
        DPH=ATAN2((RGO-RR)*TANAE,RR)
        THE=ASIN((RGO-RR)*TANAN/REO)
        ELON(IE)=MOD(CFRTD*DPH+SLON+540.0,360.0)-180.0
        ELAT(IE)=CFRTD*THE
   59 CONTINUE
```

```
      RETURN
C ERROR STOP
   90 PRINT 99090, IE,EPAE(IE),EPAN(IE)
      STOP
C FORMAT STATEMENTS
99090 FORMAT(1X/' ***   EARTH POINT DOES NOT EXIST.'/
     1  9X,'IE =',I4,5X,'EPAE =',F7.3,5X,'EPAN =',F7.3/
     2  7X,'ERROR DETECTED IN ROUTINE   EPLLAC'/1H1)
      END
```

APPENDIX B. THE ANGSSB SUBPROGRAM PACKAGE

The Fortran subprogram package described in this appendix calculates the gain of a satellite antenna of a shaped-beam or polygon emission pattern in the direction of an earth point, which is a point on the surface of the Earth. The package consists of two subroutine subprograms, i.e., ANGSSB and DSPTLN. The ANGSSB subroutine interfaces with the user; it takes the data of the emission pattern and calculates the antenna gain. The DSPTLN subroutine is a supporting subroutine called by ANGSSB; it calculates the distance between a point and a polygon (or an open line) in a plane determines whether inside or outside of the polygon (or the left side or the right side of the open line) the point lies.

This package is written in ANSI (American National Standards Institute) Standard Fortran (Publication X3.9-1978, ANSI, 345 East 47th Street, New York, NY 10017).

A Fortran listing of the package follows. The user information of each subroutine including the description of the input and output arguments is given in the beginning of each subroutine.

```
      SUBROUTINE  ANGSSB(SAPT,NMGP,GMX,PMGAE,PMGAN,GRSI,BWREF,
     1                   NCPMX,NCL,GCCL,KCL,NCP,CPAE,CPAN,
     2                   EPAE,EPAN,
     3                   GACP,GAXP,GAMX,OFAA)
C THIS SUBROUTINE CALCULATES THE COPOLAR AND CROSSPOLAR ANTENNA
C GAINS (RELATIVE TO THE COPOLAR ON-AXIS GAIN) OF A SATELLITE
C ANTENNA OF A SHAPED-BEAM OR POLYGON PATTERN FOR THE DIRECTION
C OF AN EARTH POINT.
C THE ANTENNA PATTERNS COVERED BY THIS SUBROUTINE ARE
C   SGSB01 - GENERAL SATELLITE ANTENNA SHAPED-BEAM PATTERN,
C            SPECIFIED WITH SEVERAL GAIN CONTOURS (DEFAULT),
C   SGPP83 - GENERAL SATELLITE ANTENNA POLYGON PATTERN,
C            BASED ON THE FAST ROLL-OFF GAIN CURVES OF THE
C            SBFR83 PATTERN,
C   SGPPM1 - GENERAL SATELLITE ANTENNA POLYGON PATTERN,
C            BASED ON THE FAST ROLL-OFF GAIN CURVES OF THE
C            SBFRM1 PATTERN.
C THIS SUBROUTINE CALLS THE DSPTLN SUBROUTINE.
C RESTRICTIONS  --
C   (1) FOR A SHAPED-BEAM PATTERN, CONTOUR DATA MUST BE GIVEN
C       TO THIS SUBROUTINE FOR TWO GAIN VALUES OF MORE.
C   (2) MULTIPLE CONTOURS MUST NOT BE GIVEN FOR A GAIN VALUE
C       EXCEPT FOR ONE OF THE THREE HIGHEST GAIN VALUES.
C THE INPUT ARGUMENTS ARE
C   SAPT   = CHARACTER VARIABLE OF LENGTH SIX (6) FOR THE
C            TYPE OF THE SATELLITE ANTENNA PATTERN,
C   NMGP   = NUMBER OF MAXIMUM GAIN POINTS,
C   GMX    = ARRAY OF DIMENSION NMGP CONTAINING, AS THE ITH
C            ELEMENT, THE ABSOLUTE GAIN AT THE ITH MAXIMUM
C            GAIN POINT (IN DBI)
C            (MUST BE GIVEN IN A NON-INCREASING ORDER),
C   PMGAE, PMGAN
C          = ARRAYS OF DIMENSION NMGP CONTAINING, AS THE ITH
C            ELEMENTS, THE COORDINATES OF THE ITH MAXIMUM
C            GAIN POINT, MEASURED AT THE SATELLITE IN ANGLES
C            (IN DEGREES) IN THE EAST AND NORTH DIRECTIONS
C            FROM THE LINE CONNECTING THE SATELLITE AND ITS
C            SUBSATELLITE POINT,
C   GRSI   = RESIDUAL GAIN (I.E., THE GAIN FOR A LARGE OFF-
C            AXIS ANGLE) (IN DBI),
C   BWREF  = REFERENCE BEAMWIDTH (I.E., THE BEAMWIDTH OF THE
C            BEAMLET) (IN DEGREES) FOR THE SGPP83 AND SGPPM1
C            PATTERNS
C            (IDLE FOR THE SGSB01 PATTERN),
C   NCPMX  = MAXIMUM NUMBER OF CONTOUR POINTS IN A CONTOUR
C            LINE,
C   NCL    = NUMBER OF CONTOUR LINES,
C   GCCL   = ARRAY OF DIMENSION NCL CONTAINING, AS THE LTH
C            ELEMENT, THE COPOLAR GAIN (RELATIVE TO THE
C            MAXIMUM COPOLAR GAIN) OF THE LTH CONTOUR LINE
C            (IN DB)
C            (THE VALUES MUST BE GIVEN IN A NON-INCREASING
C            ORDER.),
C   KCL    = INTEGER ARRAY OF DIMENSION NCL CONTAINING, AS
```

```
C             THE LTH ELEMENT, THE KEY TO THE TYPE OF THE
C             LTH CONTOUR LINE
C           = 0  FOR AN OPEN LINE (DEFAULT)
C           = 1  FOR A CLOSED LINE,
C   NCP     = INTEGER ARRAY OF DIMENSION NCL CONTAINING, AS
C             THE LTH ELEMENT, THE NUMBER OF CONTOUR POINTS
C             IN THE LTH CONTOUR LINE,
C   CPAE, CPAN
C           = DOUBLY DIMENSIONED ARRAYS OF DIMENSION
C             (NCPMX,NCL) CONTAINING, IN THE KTH ROWS AND LTH
C             COLUMNS, THE COORDINATES OF THE KTH POINT OF
C             THE LTH CONTOUR, MEASURED AT THE SATELLITE IN
C             ANGLES (IN DEGREES) IN THE EAST AND NORTH
C             DIRECTIONS FROM THE LINE CONNECTING THE
C             SATELLITE AND ITS SUBSATELLITE POINT
C             (THE CONTOUR POINTS FOR EACH CONTOUR LINE MUST
C             BE GIVEN COUNTERCLOCKWISE.),
C   EPAE, EPAN
C           = COORDINATES OF THE EARTH POINT, MEASURED AT THE
C             SATELLITE IN ANGLES (IN DEGREES) IN THE EAST
C             AND NORTH DIRECTIONS FROM THE LINE CONNECTING
C             THE SATELLITE AND ITS SUBSATELLITE POINT,
C WHERE I = 1, 2, ..., NMGP, K = 1, 2, ..., NCP, AND L = 1, 2,
C ..., NCL.
C THE OUTPUT ARGUMENTS ARE
C   GACP, GAXP
C           = COPOLAR AND CROSSPOLAR ANTENNA GAINS (IN DB
C             RELATIVE TO THE MAXIMUM COPOLAR GAIN), RESPEC-
C             TIVELY, OF THE SATELLITE ANTENNA IN THE DIREC-
C             TION OF THE EARTH POINT,
C   GAMX    = MAXIMUM COPOLAR GAIN (IN DBI),
C   OFAA    = OFF-AXIS ANGLE OF THE EARTH POINT.
C SPECIFICATION STATEMENTS
      CHARACTER   SAPT*6
      DIMENSION   GMX(*),PMGAE(*),PMGAN(*),
     1            GCCL(*),KCL(*),NCP(*),
     2            CPAE(NCPMX,*),CPAN(NCPMX,*)
      DIMENSION   NMCL(3)
      SAVE  INIT,CFGTBW
      DATA  INIT/0/
C CALCULATION
C INITIALIZATION
   10 IF(INIT.LE.0)  THEN
        INIT=1
        CFGTBW=LOG(10.0)/20.0
      END IF
C BRANCHING FOR POLYGON PATTERNS
   50 IF(SAPT.EQ.'SGPP83'.OR.SAPT.EQ.'SGPPM1')     GO TO 300
C THE SGSB01 PATTERN (A SHAPED-BEAM PATTERN)  (DEFAULT)
C NUMBER OF CONTOUR GAIN VALUES AND NUMBER OF MULTIPLE CONTOUR
C LINES FOR EACH OF THE FIRST THREE GAIN VALUES
  100 NMCL(1)=1
      NMCL(2)=0
      NMCL(3)=0
```

```
      ICGV=1
      DO 101  ICL=2,NCL
        IF(GCCL(ICL).NE.GCCL(ICL-1))     ICGV=ICGV+1
        IF(ICGV.LE.3)     NMCL(ICGV)=NMCL(ICGV)+1
  101 CONTINUE
      NCGV=ICGV
      NCGVM5=MIN(NCGV,5)
      GO TO (900,110,120,120,140) NCGVM5
C CALCULATION FOR THE FIRST GAIN VALUE CONTOUR
  110 ICLMN=1
      ICLMX=NMCL(1)
      ASSIGN 500 TO LBLO
      IF(NCGV.EQ.2)  THEN
        ASSIGN 120 TO LBL1
      ELSE
        ASSIGN 520 TO LBL1
      END IF
      GO TO 700
C CALCULATION FOR THE SECOND GAIN VALUE CONTOUR
  120 ICLMN=NMCL(1)+1
      ICLMX=NMCL(1)+NMCL(2)
      IF(NCGV.EQ.2)  THEN
        ASSIGN 520 TO LBLO
        ASSIGN 520 TO LBL1
      ELSE
        ASSIGN 110 TO LBLO
        ASSIGN 130 TO LBL1
      END IF
      GO TO 700
C CALCULATION FOR THE THIRD GAIN VALUE CONTOUR
  130 ICLMN=NMCL(1)+NMCL(2)+1
      ICLMX=NMCL(1)+NMCL(2)+NMCL(3)
      ASSIGN 520 TO LBLO
      IF(NCGV.EQ.4)  THEN
        ASSIGN 170 TO LBL1
      ELSE
        ASSIGN 520 TO LBL1
      END IF
      GO TO 700
C CALCULATION FOR THE FOURTH GAIN VALUE CONTOUR
  140 ICLMN=NMCL(1)+NMCL(2)+NMCL(3)+1
      ICLMX=ICLMN
      ASSIGN 120 TO LBLO
      IF(NCGV.EQ.5)  THEN
        ASSIGN 170 TO LBL1
      ELSE
        ASSIGN 150 TO LBL1
      END IF
      GO TO 700
C CALCULATION FOR THE SECOND LAST GAIN VALUE CONTOUR
  150 ICLMN=NCL-1
      ICLMX=ICLMN
      ASSIGN 160 TO LBLO
      ASSIGN 170 TO LBL1
```

```
      GO TO 700
C BINARY SEARCH FOR THE LARGEST CONTOUR NUMBER THAT DOES NOT
C ENCLOSE THE EARTH POINT IN IT
  160 ICL=(ICL1+ICL2)/2
      IF(ICL.LE.ICL1)     GO TO 520
      ICLMN=ICL
      ICLMX=ICLMN
      ASSIGN 160 TO LBL0
      ASSIGN 160 TO LBL1
      GO TO 700
C CALCULATION FOR THE LAST GAIN VALUE CONTOUR
  170 ICLMN=NCL
      ICLMX=ICLMN
      ASSIGN 520 TO LBL0
      ASSIGN 520 TO LBL1
      GO TO 700
C THE SGPP83 OR SGPPM1 PATTERN (A POLYGON PATTERN)
C CALCULATION FOR THE GAIN VALUE CONTOURS
  300 NMCL(1)=NCL
      ICLMN=1
      ICLMX=NCL
      IF(SAPT.EQ.'SGPP83')  THEN
        ASSIGN 500 TO LBL0
      ELSE
        ASSIGN 540 TO LBL0
      END IF
      ASSIGN 560 TO LBL1
      GO TO 700
C INTERNAL ROUTINES FOR INTERPOLATION AND EXTRAPOLATION
C INTERPOLATION WHEN THE EARTH POINT IS INSIDE ONE OF THE FIRST
C GAIN VALUE CONTOURS (FOR THE SGSB01 AND SGPP83 PATTERNS)
  500 D1=SQRT(D1SQ)
      GC=GCCL(1)
      DO 501  IMGP=1,NMGP
        IF(NMCL(1).GT.1)  THEN
          CALL DSPTLN(KCL(ICL1),NCP(ICL1),
     1                CPAE(1,ICL1),CPAN(1,ICL1),
     2                PMGAE(IMGP),PMGAN(IMGP), DSQ,ISOR)
          IF(ISOR.GT.0)   GO TO 501
        END IF
        DM=SQRT((PMGAE(IMGP)-EPAE)**2+(PMGAN(IMGP)-EPAN)**2)
        DGMX=GMX(IMGP)-GMX(1)
        GCI=DGMX+(GCCL(1)-DGMX)*((DM/(DM+D1))**2)
        GC=MAX(GC,GCI)
  501 CONTINUE
      GACP=GC
      GAXP=-30.0
      GAMX=GMX(1)
      IF(SAPT.EQ.'SGPP83')  THEN
        OFAA=SQRT(-GACP/12.0)*BWREF
      ELSE
        OFAA=0.0
      END IF
      RETURN
```

```
C INTERPOLATION WHEN THE EARTH POINT IS OUTSIDE THE FIRST GAIN
C VALUE CONTOURS (FOR THE SGSB01 PATTERN)
  520 D1=SQRT(D1SQ)
      D2=SQRT(D2SQ)
      IF(ISOR2.GT.0)      D2=-D2
      GC=GCCL(ICL1)+(GCCL(ICL2)-GCCL(ICL1))*(D1/(D1+D2))
      GACP=MAX(GC,MIN(GRSI-GMX(1),GCCL(NCL)))
      GAXP=MIN(-30.0,GACP)
      GAMX=GMX(1)
      OFAA=0.0
      RETURN
C INTERPOLATION WHEN THE EARTH POINT IS INSIDE ONE OF THE FIRST
C GAIN VALUE CONTOURS (FOR THE SGPPM1 PATTERN)
  540 RAP=MAX(0.0,SQRT(-GCCL(1)/12.0)-SQRT(D1SQ)/BWREF)
      GACP=-12.0*RAP*RAP
      GAXP=-30.0
      GAMX=GMX(1)
      OFAA=RAP*BWREF
      RETURN
C EXTRAPOLATION WHEN THE EARTH POINT IS OUTSIDE THE GAIN VALUE
C CONTOURS (FOR THE SGPP83 AND SGPPM1 PATTERNS)
  560 RAP=SQRT(-GCCL(1)/12.0)+SQRT(D2SQ)/BWREF
      IF(RAP.LE.1.4499)  THEN
        GC=-12.0*RAP*RAP
      ELSE
        PHI0=EXP((44.447-GMX(1))*CFGTBW)
        RA=(RAP-0.5)*(BWREF/PHI0)+0.5
        IF(RA.LE.1.4499)  THEN
          GC=-25.227
        ELSE
          GC=MAX(-22.0-20.0*LOG10(RA),GRSI-GMX(1))
        END IF
      END IF
      GACP=GC
      GAXP=MIN(-30.0,GACP)
      GAMX=GMX(1)
      OFAA=RAP*BWREF
      RETURN
C INTERNAL ROUTINE FOR CALCULATING THE DISTANCE FROM AN EARTH
C POINT TO A SET OF CONTOURS AND FOR DETERMINING THE SIDE
  700 DO 701  ICL=ICLMN,ICLMX
        CALL DSPTLN(KCL(ICL),NCP(ICL),CPAE(1,ICL),CPAN(1,ICL),
     1                  EPAE,EPAN, DSQ,ISOR)
        IF(ISOR.LE.0)  THEN
          IF(ICLMN.EQ.1)  THEN
            ICL1=ICL
            D1SQ=DSQ
          ELSE
            ICL2=ICL
            D2SQ=DSQ
            ISOR2=ISOR
          END IF
          GO TO LBLO
        ELSE
```

```
          IF(ICL.EQ.ICLMN)  THEN
            ICLI=ICL
            DSQI=DSQ
          END IF
          IF(DSQ.LT.DSQI)  THEN
            ICLI=ICL
            DSQI=DSQ
          END IF
        END IF
  701 CONTINUE
      IF(ICLMX.LT.NCL)  THEN
        ICL1=ICLI
        D1SQ=DSQI
      ELSE
        ICL2=ICLI
        D2SQ=DSQI
        ISOR2=ISOR
      END IF
      GO TO LBL1
C ERROR STOP
  900 PRINT 99900, NCL,(GCCL(ICL),ICL=1,NCL)
      PRINT 99901
      STOP
C FORMAT STATEMENTS
99900 FORMAT(1X/' ***   ONLY ONE GAIN CONTOUR VALUE'/
     1  9X,'NCL =',I3/
     2  9X,'GCCL ='/
     3  (11X,10F8.3))
99901 FORMAT(1X/' ERROR DETECTED IN ROUTINE   ANGSSB')
      END
```

```
      SUBROUTINE  DSPTLN(KCLL,NLP,XLP,YLP,XQ,YQ, DSQ,ISOR)
C THIS SUBROUTINE CALCULATES THE DISTANCE FROM A POINT TO A LINE
C AND DETERMINES WHICH SIDE OF THE LINE (INSIDE OR OUTSIDE FOR A
C CLOSED LOOP, AND LEFT SIDE OR RIGHT SIDE FOR AN OPEN LINE) THE
C POINT LIES.
C THE INPUT ARGUMENTS ARE
C   KCLL    = KEY TO THE TYPE OF THE LINE
C           = 0  FOR AN OPEN LINE (DEFAULT)
C           = 1  FOR A CLOSED LINE,
C   NLP     = NUMBER OF LINE POINTS THAT APPROXIMATE THE
C             LINE,
C   XLP, YLP
C           = ARRAYS OF DIMENSION NLP CONTAINING, AS THE ITH
C             ELEMENTS, THE X AND Y COORDINATES OF THE ITH
C             LINE POINT
C             (THE LINE POINTS MUST BE GIVEN COUNTERCLOCKWISE
C             WHEN THE LINE IS A CLOSED LOOP.),
C   XQ, YQ = X AND Y COORDINATES OF THE POINT IN QUESTION.
C THE OUTPUT ARGUMENTS ARE
C   DSQ     = SQUARE OF THE DISTANCE FROM THE POINT TO THE
C             LINE,
C   ISOR    = INDEX FOR THE SIDE OF THE LINE
C           = 1  WHEN THE POINT IS OUTSIDE A CLOSED LINE OR
C                ON THE RIGHT SIDE OF AN OPEN LINE
C           = 0  OTHERWISE.
C SPECIFICATION STATEMENTS
      DIMENSION   XLP(*),YLP(*)
      PARAMETER   (TAN10=0.1763,EPSLN=1.0E-4)
C CALCULATION
C SETS LOCAL VARIABLES FOR THE KEY TO THE TYPE OF THE LINE AND
C THE NUMBER OF LINE POINT.  RESETS THEM AND CREATES A VIRTUAL
C LINE POINT WHEN NECESSARY.
   10 KCLL0=KCLL
      IF(KCLL0.NE.1)      KCLL0=0
      NLP0=NLP
      NLP1=NLP0
      IF(KCLL0.EQ.1)      GO TO 20
      IF(NLP0.LE.3)       GO TO 20
      X1=XLP(1)
      Y1=YLP(1)
      X2=XLP(2)
      Y2=YLP(2)
      X3=XLP(NLP0-1)
      Y3=YLP(NLP0-1)
      X4=XLP(NLP0)
      Y4=YLP(NLP0)
      DX12=X2-X1
      DY12=Y2-Y1
      DX34=X4-X3
      DY34=Y4-Y3
      SP=DX12*DX34+DY12*DY34
      VP=DX12*DY34-DY12*DX34
      IF(VP.GE.(SP*TAN10))     GO TO 20
      IF(VP.GT.(-SP*TAN10))  THEN
```

```
          DX23=X3-X2
          DY23=Y3-Y2
          IF((DX12*DX23+DY12*DY23).GT.0.0)      GO TO 20
          IF((DX23*DX34+DY23*DY34).GT.0.0)      GO TO 20
        END IF
        KCLLO=1
        DX14=X4-X1
        DY14=Y4-Y1
        XVLP=(DX12*DY34*X4-DY12*DX34*X1-DX12*DX34*DY14)/VP
        YVLP=(DY12*DX34*Y4-DX12*DY34*Y1-DY12*DY34*DX14)/(-VP)
        IF(((XVLP-X1)*DX12+(YVLP-Y1)*DY12).GE.0.0)   GO TO 20
        IF(((XVLP-X4)*DX34+(YVLP-Y4)*DY34).LE.0.0)   GO TO 20
        NLP1=NLP1+1
        XVLP=(X1+X4+XVLP)/3.0
        YVLP=(Y1+Y4+YVLP)/3.0
C CALCULATES THE DISTANCE AND DETERMINES THE CLOSEST LINE POINT
C THAT REPRESENTS THE DISTANCE.  (WHEN THE POINT IN QUESTION IS
C CLOSER TO A SIDE THAN TO ANY LINE POINT, THE CLOSEST LINE
C POINT NUMBER WILL BE SET TO ZERO.)
     20 DO 29  ILP1=1,NLP1
          IF(KCLLO.EQ.0.AND.ILP1.EQ.NLP1)       GO TO 29
          IF(ILP1.EQ.1)  THEN
            X1=XLP(1)
            Y1=YLP(1)
            DX10=XQ-X1
            DY10=YQ-Y1
          ELSE
            X1=X2
            Y1=Y2
            DX10=DX20
            DY10=DY20
          END IF
          ILP2=MOD(ILP1,NLP1)+1
          IF(ILP2.LE.NLP0)  THEN
            X2=XLP(ILP2)
            Y2=YLP(ILP2)
          ELSE
            X2=XVLP
            Y2=YVLP
          END IF
          DX20=XQ-X2
          DY20=YQ-Y2
          DX12=X2-X1
          DY12=Y2-Y1
C CHECKS IF THE POINT IN QUESTION LIES INSIDE THE BELT AREA.
C CALCULATES THE DISTANCE AND REGISTER THE CLOSEST LINE POINT
C NUMBER WHEN THE POINT IN QUESTION LIES OUTSIDE.
          VPI=0.0
          ILPI=0
          SP1=DX10*DX12+DY10*DY12
          SP2=DX20*DX12+DY20*DY12
          IF(SP1.LE.0.0)  THEN
            IF(KCLLO.EQ.1.OR.ILP1.NE.1)  THEN
              DSI=DX10**2+DY10**2
```

```
      ILPI=ILP1
     END IF
    ELSE IF(SP2.GE.0.0)  THEN
     IF(KCLL0.EQ.1.OR.ILP2.NE.NLP1)  THEN
      DSI=DX20**2+DY20**2
      ILPI=ILP2
     END IF
    END IF
C CALCULATES THE VECTOR PRODUCT AND THE DISTANCE WHEN THE POINT
C IN QUESTION LIES INSIDE THE BELT AREA.
    IF(ILPI.EQ.0)  THEN
     VPI=DX10*DY12-DY10*DX12
     DSI=(VPI**2)/(DX12**2+DY12**2)
    END IF
C REGISTERS AND UPDATES THE MINIMUM DISTANCE.
    IF(ILP1.EQ.1)  THEN
     DSM=DSI
     VPM=VPI
     ILPM=ILPI
    END IF
    IF(DSI.LT.DSM)  THEN
     DSM=DSI
     VPM=VPI
     ILPM=ILPI
    END IF
 29 CONTINUE
   DSQ=DSM
C CALCULATES THE VECTOR PRODUCT OF THE TWO SIDES WHEN THE POINT
C IN QUESTION IS CLOSER TO A LINE POINT THAN ANY SIDE.
 30 IF(ILPM.NE.0)  THEN
    ILP1=MOD(ILPM+NLP1-2,NLP1)+1
    IF(ILP1.LE.NLP0)  THEN
     X1=XLP(ILP1)
     Y1=YLP(ILP1)
    ELSE
     X1=XVLP
     Y1=YVLP
    END IF
    ILP2=ILPM
    IF(ILP2.LE.NLP0)  THEN
     X2=XLP(ILP2)
     Y2=YLP(ILP2)
    ELSE
     X2=XVLP
     Y2=YVLP
    END IF
    ILP3=MOD(ILPM,NLP1)+1
    IF(ILP3.LE.NLP0)  THEN
     X3=XLP(ILP3)
     Y3=YLP(ILP3)
    ELSE
     X3=XVLP
     Y3=YVLP
    END IF
```

```
    DX12=X2-X1
    DY12=Y2-Y1
    DX23=X3-X2
    DY23=Y3-Y2
    VPM=DX12*DY23-DY12*DX23
    SPM=DX12*DX23+DY12*DY23
    IF(ABS(VPM).LT.ABS(SPM)*EPSLN)  THEN
      DX10=XQ-X1
      DY10=YQ-Y1
      VPM=DX10*DY12-DY10*DX12
    END IF
   END IF
C DETERMINES WHICH SIDE OF THE LINE THE POINT LIES.
40 IF(VPM.GE.0.0)  THEN
    ISOR=1
   ELSE
    ISOR=0
   END IF
   RETURN
   END
```

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For efficient use of the geostationary satellite orbit, mutual interference among satellite systems must be analyzed in the planning stage of the systems. To conserve the transmitter power, many satellite antennas in the FSS (fixed-satellite service) use the so-called shaped-beam emission patterns that cover their service areas. A computer model of a shaped-beam pattern is needed in the analysis of mutual interference. We present a simple model for calculating the antenna gain in the direction of an earth point from several contour lines given on the map of the Earth, each corresponding to an antenna gain value.

16. Key Words *(Alphabetical order, separated by semicolons)*

Antenna emission pattern; FSS (fixed-satellite service); satellite antenna; satellite communication; shaped-beam antenna

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