ESSA RESEARCH LABORATORIES
Institute for Telecommunication Sciences
Boulder, Colorado
July 1969

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Final Report Phase C Part 2
In Support of Hard Rock Silo Development
Program 125B
Contract F04701-68 F-0072
Task 2. 1b(7)

Technical Memorandum ERLTM-ITS 191

U.S. DEPARTMENT OF COMMERCE / ENVIRONMENTAL SCIENCE SERVICES ADMINISTRATION

ESSA RESEARCH LABORATORIES

INSTITUTE FOR TELECOMMUNICATION SCIENCES

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UHF PROPAGATION FROM BURIED ANTENNAS

George A. Hufford

This paper poses, and attempts to answer, some of the questions involved when a UHF antenna is buried in the ground. A description of the basic theory is given, together with illustrative calculations for many situations of interest. Graphs are presented for the patterns and related parameters of a variety of buried antennas, including electric and magnetic doublets, half-wave dipoles, and the annular slot antenna. It is hoped that these graphs may be of direct help in designing systems which must use such buried antennas. Of all the antennas investigated, for example, it is the annular slot which seems to be the most efficient radiator.

Key Words: Antenna theory, buried antennas, radio propagation, radiation gain, UHF antennas.

1. INTRODUCTION

An antenna buried in the ground has a somewhat different environment from the free space normally assumed in theoretical analyses of antennas. Many of the consequences of this different environment have been discussed by Baños (1966), deBettencourt (1966), King and Iizuka (1963), Wait (1961), Williams (1963), and others. These discussions, however, have been concerned largely with low frequencies. At higher frequencies, in particular at UHF, there are other considerations. Basic theory, it is true, remains the same, but the emphasis is changed. Skin depths are smaller, and so there is a tendency to use what is normally called a "poor" ground, such as impervious granite or basalt or even especially poured, low-loss concrete. This, coupled with the natural effects of the higher frequency, means that, while at low frequencies the loss tangent of the ground is normally very large, at high frequencies it is very small.

Numerical results are therefore somewhat different, and when approximations are desired, an entirely new set must be used.

At high frequencies, too, a wider variety of antenna is available. Consequently, emphasis is often shifted to a discussion of antenna details that are otherwise unimportant. One result is that a slightly more refined theory is needed.

It is the purpose of this paper to pose, and to attempt to answer, some of the questions involved. We begin with a development of the basic theory that we believe to be a bit more general than has been given heretofore. We are thus able to treat arbitrary distributions of both electric and magnetic currents. In further sections we give brief discussions of the current distribution along a linear antenna and of the power budget for arbitrary antennas. Finally, we give illustrative calculations for several situations of interest. It is hoped that these calculations can be directly used in the design of systems with buried antennas.

The basic problem concerns a homogeneous ground with (complex) propagation constant $k_1 = \omega\sqrt{\epsilon_1\mu_1}$ and intrinsic impedance $Z_1 = \sqrt{\mu_1/\epsilon_1}$. Above it, and separated from it by a plane interface, is the air with (real) propagation constant $k_o = \omega\sqrt{\epsilon_o\mu_o}$ and intrinsic impedance $Z_o = \sqrt{\mu_o/\epsilon_o}$. An antenna, of arbitrary configuration, is entirely underground. We suppose that it can be represented as a distribution of electric current $\overline{J}$ and magnetic current $\overline{M}$.

As in figure 1, we set up a coordinate system whose z - axis is vertical and whose origin is at the "center" of the antenna. The air-ground interface is defined by z = h, so that h is the nominal depth of the antenna below ground. We freely use, whenever convenient, either rectangular (x, y, z), cylindrical (ρ, ϕ, z), or spherical (r, θ, ϕ) coordinates, together with the corresponding unit vectors

$\bar{i}_x, \bar{i}_y, \bar{i}_z, \bar{i}_\rho, \bar{i}_\phi, \bar{i}_r, \bar{i}_\theta$. The radial vector, in particular, may be expressed as

$$\bar{r} = x\,\bar{i}_x + y\,\bar{i}_y + z\,\bar{i}_z = \rho\,\bar{i}_\rho + z\,\bar{i}_z = r\,\bar{i}_r\,. \qquad (1.1)$$

SI (MKSA) units are used throughout, and fields and currents are expressed as complex phasors with the factor $\sqrt{2}\,e^{-i\omega t}$ supressed. Most of the notation and much of the basic theory are taken from Stratton (1941) and Harrington (1961).

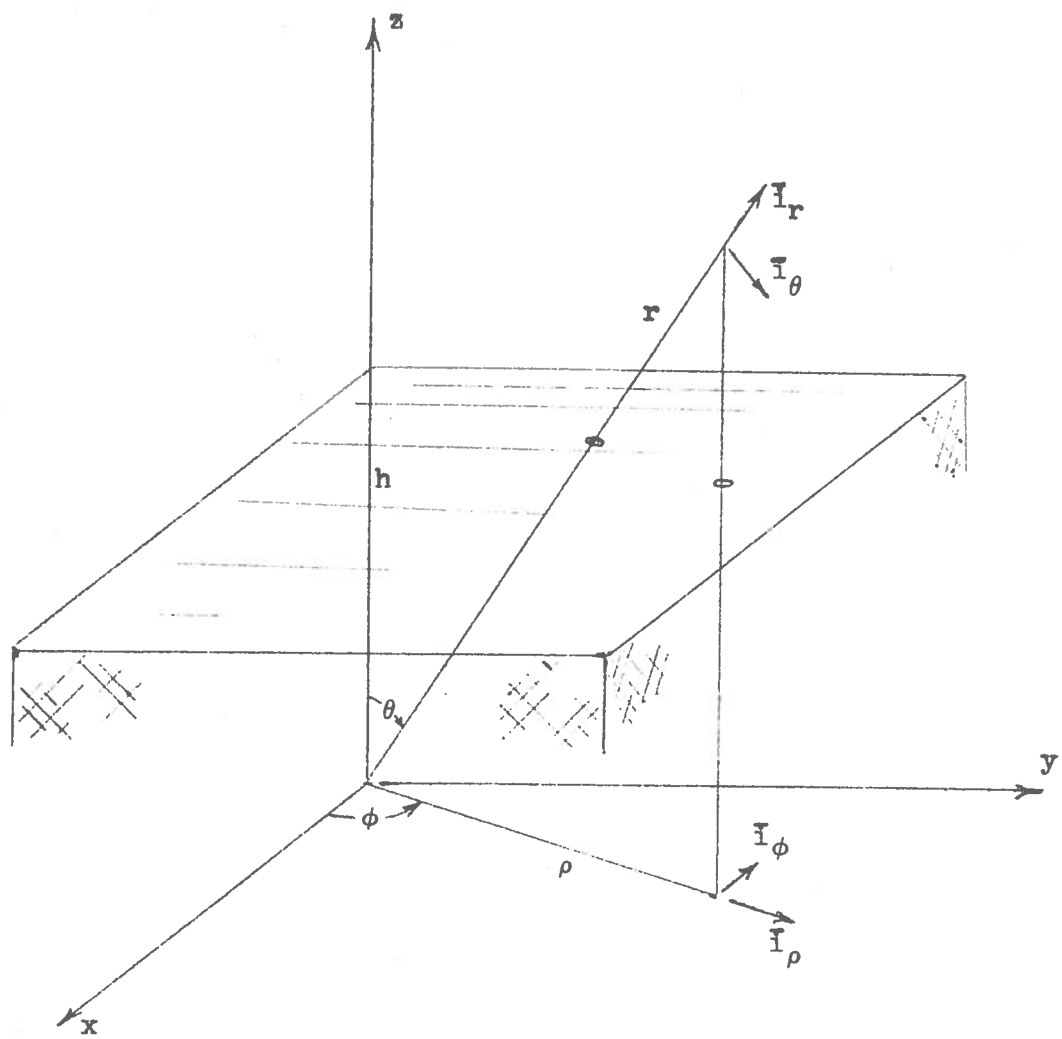

Figure 1. Path geometry.

2. ELEMENTARY DOUBLETS - THE SPACE WAVE

Let an elementary electric doublet with polarization vector $\overline{p}$ be located at the origin. If ground filled all of space, then the fields would be given by

$$\overline{E} = ik_1 Z_1 \overline{p}\psi + \frac{iZ_1}{k_1} (\overline{p} \cdot \nabla) \nabla\psi, \tag{2.1}$$

$$\overline{H} = - \overline{p} \times \nabla\psi ,$$

where

$$\psi = \frac{e^{ik_1 r}}{4\pi r} = \frac{ik_1}{8\pi^2} \int_0^{2\pi} d\phi_1' \int_0^{\chi - i\infty} \sin\theta_1' \, d\theta_1' \, e^{ik_1 \overline{i}_{r1}' \cdot \overline{r}}, \tag{2.2}$$

and

$$\overline{i}_{r1}' = \sin\theta_1' \, \overline{i}_{\rho 1}' + \cos\theta_1' \, \overline{i}_z , \tag{2.3}$$

$$\overline{i}_{\rho 1}' = \cos\phi_1' \, \overline{i}_x + \sin\phi_1' \, \overline{i}_y .$$

If $\chi = \pi/2 - \arg k_1$, then the integral in (2.2) converges for all values of $\overline{r}$ satisfying $z \geq 0$ or, what is the same thing, $\theta \leq \pi/2$. This integral expresses ψ (and hence $\overline{E}$, $\overline{H}$) as a superposition of (inhomogeneous) plane waves, each of which has the direction vector $\overline{i}_{r1}'$. Note that $\overline{i}_{r1}' \cdot \overline{i}_{r1}' = 1$, although, since θ_1' can be complex, $\overline{i}_{r1}'$ is not, strictly speaking, a unit vector.

To solve our problem, we may use this integral together with the theory of the reflection and refraction of plane waves by a plane interface. The field given by (2.1) and (2.2) is the incident field. It will give rise to a reflected field, which is a superposition of reflected plane waves, and to a transmitted field, which is a superposition of transmitted plane waves.

If a plane wave, incident from below the interface, has the direction vector $\overline{i}_{r1}$, then the transmitted plane wave will have the direction vector $\overline{i}_r$ given by Snell's law,

$$\overline{i}_r = \sin\theta\,\overline{i}_\rho + \cos\theta\,\overline{i}_z, \tag{2.4}$$

$$\overline{i}_\rho = \overline{i}_{\rho 1},$$

$$k_o \sin\theta = k_1 \sin\theta_1.$$

Since we are interested only in the transmitted wave, it will serve us best to change the variables of integration in (2.2) to those corresponding to the transmitted direction vector. We find

$$k_1 \int_o^{2\pi} d\phi_1' \int_o^{\chi - i\infty} d\theta_1' \sin\theta_1' \; F(\theta_1', \phi_1') \tag{2.5}$$

$$= k_o \int_o^{2\pi} d\phi' \int_o^{\frac{\pi}{2} - i\infty} \sin\theta' d\theta' \frac{k_o \cos\theta'}{k_1 \cos\theta_1'} \; F(\theta_1', \phi'),$$

where F is any integrable function and θ_1' may be determined in terms of θ' by means of (2.4).

Setting

$$\overline{E}_e = \frac{-k_1^2 Z_1}{8\pi^2} \int_o^{2\pi} d\phi_1' \int_o^{\chi - i\infty} \sin\theta_1' d\theta_1' \, \overline{p} \cdot \overline{i}_{\phi 1}' \, \overline{i}_{\phi 1}' \, e^{ik_1 \overline{i}_{r1}' \cdot \overline{r}}, \tag{2.6}$$

$$\overline{H}_m = \frac{-k_1^2}{8\pi^2} \int_o^{2\pi} d\phi_1' \int_o^{\chi - i\infty} \sin\theta_1' d\theta_1' \, \overline{p} \cdot \overline{i}_{\phi 1}' \times \overline{i}_{r1}' \, \overline{i}_{\phi 1}' \, e^{ik_1 \overline{i}_{r1}' \cdot \overline{r}},$$

and

$$\overline{H}_e = \frac{-i}{k_1 Z_1} \nabla \times \overline{E}_e, \quad \overline{E}_m = \frac{iZ_1}{k_1} \nabla \times \overline{H}_m, \tag{2.7}$$

it may be seen that (2.1) and (2.2) give

$$\overline{E} = \overline{E}_e + \overline{E}_m, \quad \overline{H} = \overline{H}_e + \overline{H}_m. \tag{2.8}$$

This decomposition of the incident field is a convenient one, since the field $\overline{E}_e$, $\overline{H}_e$ is a superposition of transverse electric plane waves, while $\overline{E}_m$, $\overline{H}_m$ is a superposition of transverse magnetic plane waves. We might call $\overline{E}_e$, $\overline{H}_e$ the TE (or horizontally polarized) component and $\overline{E}_m$, $\overline{H}_m$ the TM (or vertically polarized) component.

It follows that the transmitted field is given by (2.8), wherein the integrals (2.6) are changed by introducing the Fresnel coefficients and by suitably changing the phase factor.

To compute the transmitted phase factor, it is most convenient to require the position vector to be measured from a point on the interface. If we set $\overline{r} = \overline{r}_o + h\overline{i}_z$, then $\overline{r}_o$ would be such a position vector. The phase $k_1\overline{i}'_{r1} \cdot \overline{r} = k_1\overline{i}'_{r1} \cdot \overline{r}_o + k_1 h \cos\theta'_1$ of an incident wave would become, in the transmitted wave,

$$k_o\overline{i}'_r \cdot \overline{r}_o + k_1 h \cos\theta'_1 = k_o\overline{i}'_r \cdot \overline{r} + h(k_1\cos\theta'_1 - k_o\cos\theta'). \quad (2.9)$$

Assembling all of these results, we find that <u>the field in air</u> is given by (2.8), where now

$$\overline{E}_e = \frac{-k_o^2 Z_o}{8\pi^2}\int_o^{2\pi} d\phi' \int_o^{\frac{\pi}{2}-i\infty} \sin\theta'\, d\theta' \frac{2Z_1\cos\theta'}{Z_o\cos\theta'_1 + Z_1\cos\theta'}\, \overline{p}\cdot\overline{i}'_\phi\overline{i}'_\phi$$

$$\cdot\; e^{ih(k_1\cos\theta'_1 - k_o\cos\theta')}\, e^{ik_o\overline{i}'_r\cdot\overline{r}}, \quad (2.10)$$

$$\overline{H}_m = \frac{-k_o^2}{8\pi^2}\int_o^{2\pi} d\phi' \int_o^{\frac{\pi}{2}-i\infty} \sin\theta'\, d\theta' \frac{2Z_1\cos\theta'}{Z_1\cos\theta'_1 + Z_o\cos\theta'}\, \overline{p}\cdot\overline{i}'_\phi \times \overline{i}'_{r1}\,\overline{i}'_\phi$$

$$\cdot\; e^{ih(k_1\cos\theta'_1 - k_o\cos\theta')}\, e^{ik_o\overline{i}'_r\cdot\overline{r}},$$

and

$$\overline{H}_e = \frac{-i}{k_o Z_o}\nabla\times\overline{E}_e, \quad \overline{E}_m = \frac{iZ_o}{k_o}\nabla\times\overline{H}_m. \quad (2.11)$$

These equations give a full-wave theoretic solution to our problem. From them we can pick out quantities of more immediate interest. The radiation field, for example, is the first term of the asymptotic expression as $r \to \infty$. Since the phase $k_o r \overline{i}'_r \cdot \overline{i}_r$ is stationary when $\overline{i}'_r = \overline{i}_r$, we quickly see that

$$\overline{E}_e \sim ik_o Z_o \frac{2Z_1 \cos\theta}{Z_o \cos\theta_1 + Z_1 \cos\theta} \overline{p} \cdot \overline{i}_\phi \overline{i}_\phi$$

$$e^{ih(k_1 \cos\theta_1 - k_o \cos\theta)} \frac{e^{ik_o r}}{4\pi r}, \tag{2.12}$$

$$\overline{H}_m \sim ik_o \frac{2Z_1 \cos\theta}{Z_1 \cos\theta_1 + Z_o \cos\theta} \cdot \overline{p} \cdot \overline{i}_\phi \times \overline{i}_{r1} \overline{i}_\phi$$

$$e^{ih(k_1 \cos\theta_1 - k_o \cos\theta)} \frac{e^{ik_o r}}{4\pi r},$$

and

$$\overline{H}_e \sim \frac{1}{Z_o} \overline{i}_r \times \overline{E}_e, \quad \overline{E}_m \sim -Z_o \overline{i}_r \times \overline{H}_m. \tag{2.13}$$

This is the complete space wave excited by an electric doublet. The solution for a magnetic doublet can be found in exactly the same way. A simpler approach, however, is to use the solutions we already have together with the principle of duality. Thus, if $\overline{q}$ represents a magnetic doublet located at the origin, then the radiation field in air is given immediately by (2.8) and (2.13), where now

$$\overline{E}_e \sim -ik_o \frac{2Z_o \cos\theta}{Z_o \cos\theta_1 + Z_1 \cos\theta} \overline{q} \cdot \overline{i}_\phi \times \overline{i}_{r1} \overline{i}_\phi$$

$$e^{ih(k_1 \cos\theta_1 - k_o \cos\theta)} \frac{e^{ik_o r}}{4\pi r}, \tag{2.14}$$

$$\overline{H}_m \sim \frac{ik_o}{Z_o} \frac{2Z_o \cos\theta}{Z_1 \cos\theta_1 + Z_o \cos\theta} \overline{q} \cdot \overline{i}_\phi \overline{i}_\phi$$

$$e^{ih(k_1 \cos\theta_1 - k_o \cos\theta)} \frac{e^{ik_o r}}{4\pi r}.$$

3. ELEMENTARY DOUBLETS — THE GROUND WAVE

In discussions of the ground wave, two parameters continually appear. The first is the critical angle θ_c. This is the value θ_1 attains when $\theta = \pi/2$. It is given by

$$\sin\theta_c = k_o/k_1, \quad \cos\theta_c = \sqrt{k_1^2 - k_o^2}/k_1 . \tag{3.1}$$

The second parameter might be called the surface transfer impedance δ. It is given by

$$\delta = \begin{cases} \delta_e = \dfrac{Z_o}{Z_1}\cos\theta_c & \text{for TE waves} \\[2ex] \delta_m = \dfrac{Z_1}{Z_o}\cos\theta_c & \text{for TM waves.} \end{cases} \tag{3.2}$$

We meet these parameters first in the classical problem of an isotropic radiator. For definiteness, suppose that the radiator is located on the z-axis at a height z_1 (either positive or negative) above the interface, and consider the field distant ρ at a height z_2 above the interface. According to the Sommerfeld theory of propagation over a plane, if z_1 and z_2 are not too large, then the gain relative to free space is given quite accurately by

$$y = F\left(\frac{ik_o\rho}{2}\,\delta^2\right) U(z_1)\; U(z_2) \tag{3.3}$$

$$\sim \frac{2i}{k_o\rho\,\delta^2}\; U(z_1)\; U(z_2),$$

where

$$F(p) = 2 + 2i\sqrt{\pi p}\;\operatorname{werf}\sqrt{p} \sim -1/p, \tag{3.4}$$

$$U(z) = \begin{cases} 1 - ik_o\delta z & z \geq 0 \\[1ex] e^{-ik_1\cos\theta_c z} & z \leq 0. \end{cases}$$

The function werf z is the "complex error function" (see Abramowitz and Stegun, 1964) defined by

$$\operatorname{werf} z = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2}}{z - t}\, dt, \tag{3.5}$$

the contour of integration passing below the pole at $t = z$. At UHF the "numerical distance" $ik_o\rho\delta^2/2$ is normally so large that the asymptotic results quoted above are entirely adequate.

Let us, however, return to the vector wave problem in which we are interested. Although we could probably use the scalar results quoted above, it is more certain and more instructive to make our derivations directly from (2.10). For this purpose, we set $\overline{r} = \rho\overline{i}_\rho + (h + z)\overline{i}_z$ and ask for asymptotic results as $\rho \to \infty$. The quantity $z \geq 0$ is not the coordinate value here; it is the height of the observation point above the interface.

Since

$$\overline{i}'_r \cdot \overline{r} = \rho \sin\theta' \cos(\phi' - \phi) + (h + z)\cos\theta', \tag{3.6}$$

we find, for example,

$$\overline{E}_e = \frac{-k_o^2 Z_o}{8\pi^2}\int_0^{2\pi} d\phi' \int_0^{\frac{\pi}{2} - i\infty} \sin\theta' d\theta' \,\overline{F}\cos\theta'\, e^{i(k_1 h\cos\theta'_1 + k_o z\cos\theta')} \cdot e^{ik_o\rho\sin\theta'\cos(\phi' - \phi)}, \tag{3.7}$$

where

$$\overline{F} = \frac{2Z_1}{Z_o\cos\theta'_1 + Z_1\cos\theta'}\; \overline{p} \cdot \overline{i}'_\phi\, \overline{i}'_\phi\,. \tag{3.8}$$

The phase $k_o\rho \sin\theta'\cos(\phi' - \phi)$ is stationary when $\phi' = \phi$, $\theta' = \pi/2$. At this point, however, the integrand vanishes because of the factor $\cos\theta'$, and it is therefore necessary to treat higher order terms. Setting $\hat{\theta} = \theta' - \pi/2$, $\hat{\phi} = \phi - \phi'$, we first note that

$$k_o\rho\sin\theta'\cos(\phi' - \phi) = k_o\rho\left(1 - (\hat{\theta}^2 + \hat{\phi}^2)/2 + O\left((\hat{\theta}^2 + \hat{\phi}^2)^2\right)\right). \tag{3.9}$$

With this in mind, we see that if F is any smooth function, we may write down the general asymptotic result

$$\int d\phi' \int d\theta' \cos\theta' \, F(\theta', \phi') \, e^{ik_o\rho \sin\theta' \cos(\phi' - \phi)} \tag{3.10}$$

$$= e^{ik_o\rho} \int d\phi' \int d\theta' (a\hat{\theta} + b\hat{\theta}^2 + c\hat{\theta}\hat{\phi} + O((\hat{\theta} + \hat{\phi})\,\hat{\theta}^2))$$

$$\cdot \; e^{-ik_o\rho(\hat{\theta}^2 + \hat{\phi}^2)/2} \sim \frac{2\pi b}{(ik_o\rho)^2} e^{ik_o\rho} .$$

As for the number b, we have

$$b = \frac{1}{2}\frac{\partial^2}{\partial\theta'^2}\left[\cos\theta' F(\theta', \phi') \, e^{ik_o\rho(\sin\theta'\cos\hat{\phi} - 1 + (\hat{\theta}^2 + \hat{\phi}^2)/2)}\right]_{\substack{\theta' = \pi/2 \\ \phi' = \phi}}$$

$$= -\frac{\partial F}{\partial\theta'}\left(\frac{\pi}{2}, \phi\right) . \tag{3.11}$$

Before applying these results to (2.10), we first note the behavior of θ_1' near the stationary point. From (2.4) it follows, indeed, that θ_1' is also stationary. Its rate of change therefore plays no role in the derivative (3.11), and we may replace it throughout by the constant θ_c. In particular, the vector $\overline{i}'_{r1}$ may be replaced by

$$\overline{i}_{rc} = \sin\theta_c \, \overline{i}_\rho + \cos\theta_c \, \overline{i}_z . \tag{3.12}$$

Assembling these results together, we find that the field in the air but near the ground is given by (2.8), where now

$$\overline{E}_e \sim -\frac{2Z_1}{\cos\theta_c}\left(\frac{Z_1}{Z_o\cos\theta_c} - ik_o z\right) e^{ik_1 h\cos\theta_c}\,\overline{p}\cdot\overline{i}_\phi\,\overline{i}_\phi\,\frac{e^{ik_o\rho}}{4\pi\rho^2} \qquad (3.13)$$

$$= ik_o Z_o \frac{2i}{k_o\rho\delta_e^2}\left(1 - ik_o\delta_e z\right) e^{ik_1 h\cos\theta_c}\,\overline{p}\cdot\overline{i}_\phi\,\overline{i}_\phi\,\frac{e^{ik_o\rho}}{4\pi\rho},$$

$$\overline{H}_m \sim -\frac{2}{\cos\theta_c}\left(\frac{Z_o}{Z_1\cos\theta_c} - ik_o z\right) e^{ik_1 h\cos\theta_c}\,\overline{p}\cdot\overline{i}_\phi\times\overline{i}_{rc}\,\overline{i}_\phi\,\frac{e^{ik_o\rho}}{4\pi\rho^2}$$

$$= ik_o\frac{Z_1}{Z_o}\frac{2i}{k_o\rho\delta_m^2}\left(1 - ik_o\delta_m z\right) e^{ik_1 h\cos\theta_c}\,\overline{p}\cdot\overline{i}_\phi\times\overline{i}_{rc}\,\overline{i}_\phi\,\frac{e^{ik_o\rho}}{4\pi\rho}.$$

One may note the similarity between these results and (3.3). The other components of the field are given by (2.11), which may be written (to the same degree of approximation) in the form

$$\overline{E}_m \sim -Z_o\left(\overline{i}_\rho\times\overline{H}_m - \delta_m\overline{i}_z\times\overline{H}_m\big|_{z=0}\right), \qquad (3.14)$$

$$\overline{H}_e \sim \frac{1}{Z_o}\left(\overline{i}_\rho\times\overline{E}_e - \delta_e\overline{i}_z\times\overline{E}_e\big|_{z=0}\right).$$

The last terms in these expressions are the terms which provide the familiar wave tilt.

Finally, we note that if the field is excited by the magnetic doublet $\overline{q}$, then the principle of duality would say that the field satisfies (2.8) and (3.14), where

$$\overline{E}_e \sim -ik_o\frac{Z_o}{Z_1}\frac{2i}{k_o\rho\delta_e^2}\left(1 - ik_o\delta_e z\right) e^{ik_1 h\cos\theta_c}\,\overline{q}\cdot\overline{i}_\phi\times\overline{i}_{rc}\,\overline{i}_\phi\,\frac{e^{ik_o\rho}}{4\pi\rho},$$

$$\overline{H}_m \sim \frac{ik_o}{Z_o}\frac{2i}{k_o\rho\delta_m^2}\left(1 - ik_o\delta_m z\right) e^{ik_1 h\cos\theta_c}\,\overline{q}\cdot\overline{i}_\phi\,\overline{i}_\phi\,\frac{e^{ik_o\rho}}{4\pi\rho}. \qquad (3.15)$$

4. REAL ANTENNAS

Suppose an antenna can be represented by the electric current density $\overline{J}(\overline{r})$. (We assume that $\overline{J}$ vanishes everywhere except inside a small volume completely under ground and centered around our origin.) Then the fields in air are given by a superposition of the fields of (2.10). We merely replace $\overline{p}$ by $\overline{J}$ and integrate over the volume of the antenna.

But we may take advantage of the fact that both expressions (2.10) are linear in $\overline{p}$, and we may carry out this proposed integration first. Remembering that the radial vector $\overline{r}$ and the depth h must be altered so as to correspond to each element of current, we see that the field excited by this antenna will still be given by (2.10), provided only that we replace $\overline{p}$ by $\overline{P}(\theta', \phi')$, where

$$\overline{P}(\theta, \phi) = \int \overline{J}(\overline{r}')\, e^{-iz'(k_1 \cos\theta_1 - k_o \cos\theta)}\, e^{-ik_o \overline{i}_r \cdot \overline{r}'}\, dv(\overline{r}') \qquad (4.1)$$

$$= \int \overline{J}(r')\, e^{-ik_o \rho' \sin\theta \cos(\phi' - \phi) - ik_1 z' \cos\theta_1}\, dv(\overline{r}').$$

Repeating our arguments concerning asymptotic expansions, we see that the radiation field and the ground wave are given precisely by (2.12) and (3.13) with this same replacement of $\overline{P}$ for $\overline{p}$. (In the case of the ground wave one should, in accordance with (3.11), include also a term $\partial\overline{P}(\pi/2, \phi)/\partial\theta$. But it follows immediately from (4.1) that this derivative will always vanish.)

We might note that, since $k_o \sin\theta = k_1 \sin\theta_1$, the phase of (4.1) can be written in the simpler form

$$k_1 \left(\rho' \sin\theta_1 \cos(\phi' - \phi) + z' \cos\theta_1\right) = k_1 \overline{i}_{r1} \cdot \overline{r}'. \qquad (4.2)$$

Note also that if the antenna is located all at one depth (so that we may set $z' = 0$ in (4.1)), then $\overline{P}$ is the same as though the antenna were in air. This remark assumes, of course, that the current distribution remains fixed in the two environments.

If, furthermore, the antenna is represented by the magnetic current density $\overline{M}(\overline{r})$, then the same arguments will be valid. Consequently, to determine the resulting fields we need merely to replace $\overline{q}$ in (2.14) by the vector

$$\overline{Q}(\theta, \phi) = \int \overline{M}(\overline{r}')\, e^{-ik_1 \overline{i}_{r1} \cdot \overline{r}'}\, dv(\overline{r}'). \tag{4.3}$$

To complete this section, we give two simple examples.

4.1 The Thin Dipole

We may assume, to a good degree of approximation, that the current on a thin linear antenna is sinusoidal with some calculable propagation constant k_a. (If the antenna is bare, then $k_a = k_1$. We treat sheathed antennas in sec. 5.)

Consider, then, a center fed dipole of length 2ℓ, oriented along an arbitrary unit vector $\overline{i}_a$. The current is given by

$$\overline{I} = I_o \sin k_a(\ell - |s|)\, \overline{i}_a \quad \text{at } \overline{r} = s\, \overline{i}_a, \tag{4.4}$$

and, substituting this into (4.1), we find

$$\overline{P} = I_o \overline{i}_a \int_{-\ell}^{\ell} \sin k_a (\ell - |s|)\, e^{-ik_1 s \overline{i}_{r1} \cdot \overline{i}_a}\, ds \tag{4.5}$$

$$= 2I_o k_a \frac{\cos\left(k_1 \ell \overline{i}_{r1} \cdot \overline{i}_a\right) - \cos k_a \ell}{k_a^2 - k_1^2\left(\overline{i}_{r1} \cdot \overline{i}_a\right)^2}\, \overline{i}_a .$$

This formula should be used with some caution since k_1 and k_a are probably complex. The number I_o is, of course, the "anti-node" current.

Note the particular case of a vertical dipole when $k_a = k_1$. Then $\overline{i}_{r1} \cdot \overline{i}_a = \cos \theta_1$, and

$$\overline{P} = 2I_o k_1 \frac{\cos(k_1 \ell \cos\theta_1) - \cos k_1 \ell}{k_1^2 \sin^2\theta_1} \overline{i}_z \tag{4.6}$$

$$\approx 2I_o \ell \sin k_1 \ell \, \overline{i}_z .$$

The latter approximation assumes that $\theta_1 \approx 0$, as it is in most applications. We therefore conclude that no matter what its length, a bare vertical dipole or monopole will behave very much as though it were a simple doublet.

4.2 An Annular Slot

A thin slot antenna may be represented (see Booker, 1946) by a magnetic current $\overline{V}$ flowing along the slot. Suppose, then, that we have a horizontal annular slot of radius a, uniforrnly illuminated. We may set

$$\overline{V} = V \overline{i}_\phi \quad \text{at} \quad \overline{r} = a \, \overline{i}_\rho , \tag{4.7}$$

and

$$\overline{Q} = Va \int_o^{2\pi} \overline{i}_\phi{}' \, e^{-ik_o a \overline{i}_r \cdot \overline{i}_\rho{}'} \, d\phi ' \tag{4.8}$$

$$= -2\pi i \; V a \, J_1(k_o a \sin\theta) \; \overline{i}_\phi .$$

5. SHEATHED ANTENNAS

The current on a linear antenna acts, to a first order approximation, as though the antenna were a transmission line propagating in its principal, or TM, mode. If the antenna is perfectly conducting and bare, then this principal mode has the propagation constant of the surrounding medium. But if the antenna is buried, then it is often desirable, for either mechanical or electrical reasons, to sheath it with a dielectric coating. One must then ask how much such a sheath will affect the propagation constant.

At low frequencies the ground acts like a fairly good conductor, and it is common to think of a sheathed antenna as behaving like a coaxial line. At higher frequencies, however, it is probably more reasonable to think of it as a surface wave- or G-line such as was first described by Gobau (1950). Actually, the analysis is identical for the two situations.

Let us suppose, then, that our antenna consists of a perfect conductor, circular in cross section, and of radius a. Surrounding it is the sheath of radius b, propagation constant k_2, and intrinsic impedance Z_2. This structure is then imbedded in the ground with constants k_1, Z_1. Following Gobau we find that the propagation constant k_a must satisfy (approximately) the characteristic equation

$$\frac{\varkappa_1^2}{\varkappa_2^2} \log\left(\frac{Cb\varkappa_1}{2i}\right) = \eta \log \frac{b}{a}, \tag{5.1}$$

where $\varkappa_1^2 = k_1^2 - k_a^2$, $\varkappa_2^2 = k_2^2 - k_a^2$, $\eta = k_1Z_2/k_2Z_1$, and $C = e^{\gamma} = 1.78107$, γ being Euler's constant. If we set

$$k_a^2 = k_1^2 + \left(\frac{2}{Cb}\right)^2 \left(\frac{b}{a}\right)^{2\eta} \zeta , \qquad (5.2)$$

then (5.1) may be rewritten in the form

$$\zeta \log \frac{1}{\zeta} = \frac{C^2 b^2}{2} \left(\frac{a}{b}\right)^{2\eta} \left(k_2^2 - k_1^2\right) \log \left(\frac{b}{a}\right)^{\eta} . \qquad (5.3)$$

This equation is easily solved by recursion. Examples are given later in figure 25. Note, however, that ζ will be complex, and that this means we must carefully choose the proper branch for log $(1/\zeta)$. The approximations involved in deriving (5.1) require ζ to be small, and it will follow that therefore the argument of ζ must be approximately equal to that of the difference $k_2^2 - k_1^2$. In the case of interest to us, k_1 and k_2 are nearly real and $k_1 > k_2$. This means that the difference $k_2^2 - k_1^2$ is near the negative real axis, and we must choose arg ζ near $+\pi$ or near $-\pi$. As it turns out, neither choice is very plausible. They are both associated with leaky, backfire modes.

In the case of the G-line (where $k_1 \leq k_2$) or the leaky coaxial line (where k_1^2 is large and nearly pure imaginary), the choice of arg ζ is fairly obvious. In the present case, however, the situation is not at all clear. It deserves more study than we are able to give here.

6. POWER CONSIDERATIONS

An investigation of the power budget of a buried antenna presents several difficulties. Let us begin with an illustration.

Consider an electrical doublet $\bar{p}$ in a situation where the ground occupies all of space. The radial component of the Poynting vector is given by

$$\bar{i}_r \cdot \bar{S} = \bar{i}_r \times \bar{E} \cdot \bar{H}^* \tag{6.1}$$

$$= \frac{\|\bar{p} \times \bar{i}_r\|^2}{16\pi^2 r^2} Z_1 |k_1|^2 \left(1 - \frac{1}{ik_1 r} - \frac{1}{k_1^2 r^2}\right)$$

$$\cdot \left(1 + \frac{1}{ik_1^* r}\right) e^{-2\mathrm{Im}k_1 r},$$

where we have used the notation $\|\bar{v}\|^2 = \bar{v} \cdot \bar{v}^*$ to denote the Hermitian norm of a (possibly complex) vector. Integrating this expression over a sphere of radius r, we find that the power transported across that sphere can be expressed in the form

$$W(r) = W_r e^{-2\mathrm{Im}k_1 r} + W_i, \tag{6.2}$$

where

$$W_r = \frac{\|\bar{p}\|^2}{6\pi} |k_1|^2 \operatorname{Re} Z_1, \tag{6.3}$$

$$W_i(r) = \frac{\|\bar{p}\|^2}{6\pi} \operatorname{Re} Z_1 \left(\frac{2\,\mathrm{Im}\,k_1}{r} + \frac{2i\mathrm{Im}\,k_1}{k_1 r^2} + \frac{i}{k_1 r^3}\right) e^{-2\mathrm{Im}\,k_1 r}.$$

The first term, $W_r \exp(-2\mathrm{Im}\,k_1 r)$, we recognize immediately as the power lost to the radiated field. The exponential factor is readily understandable as the absorption of the field by the medium. We may term W_r the total radiated power.

If k_1, Z_1 are real, then the second term W_i vanishes. Otherwise, however, it is positive, and may be attributed to heat losses in the induction field. As $r \to 0$, W_i becomes infinitely large. This is not unreasonable, since the antenna, being infinitesimal, requires infinitely large fields in its immediate vicinity. But any real antenna must occupy some finite space, and then these losses will be finite. An illustration of their magnitude is given later in figure 11.

When the situation involves an arbitrary antenna, we may expect somewhat the same phenomena to appear. The power budget might take the form

$$W_{in} = W_a + W_i + W_r, \tag{6.4}$$

where W_{in} is the antenna input power, W_a the heat losses in the antenna itself, W_i the heat losses in the induction field, and W_r the radiated power.

In a theoretical analysis, the field of an antenna will usually be derived on the assumption of some representative current present somewhere on the antenna. If the ground occupies all of space, then the resulting W_r can be computed from the radiation field (in ground) by integrating the Poynting vector over a large sphere and dropping the factor $\exp(-2\mathrm{Im}\, k_1\, r)$.

On the other hand, W_a and W_i depend so much on the details of the antenna structure that only gross estimates are possible. Even their definition will often be a matter of subjective choice, since in many situations the antenna will be inseparable from the medium. Indeed, one might lump together W_a and W_i, attributing both to "antenna losses," and thus taking the stand that the immediate vicinity is an integral part of the antenna.

With these observations in mind, it seems entirely reasonable to define a radiation gain relative to free space as the field excited by an antenna relative to that excited by an isotropic radiator (in free space) whose radiated power is also W_r.

In the case of a buried antenna there is one further consideration. The field will be reflected at the interface and return to the antenna, producing an induced emf. This reaction upsets the voltage-current relationship so that the radiated power is not the quantity found so simply above. Another way to say this is to note that if one integrates the Poynting vector of the direct, reflected, and transmitted waves (taking due account of the absorption factor), then one will not obtain the same number as if there were only the direct wave.

In one description of this problem, one writes

$$W_r = W_{ro} \frac{W_r}{W_{ro}}, \tag{6.5}$$

where W_r is the true radiated power, and W_{ro} is the radiated power in the absence of the interface. Since the radiated powers are proportional to the radiation resistances (relative to the reference current) we can rewrite this in the form

$$W_r = W_{ro} \frac{R_r}{R_{ro}}, \tag{6.6}$$

where R_r, R_{ro} are the corresponding resistance. The ratio R_r/R_{ro} may be included in the power budget as yet another gain. Studies of this ratio have been made (in other situations) by King (1956), Vogler and Noble (1964), Sinha and Bhattacharyya (1966), and others. One concludes that if the antenna is more than a few wavelengths, or more than a few skin depths, from the interface, then the resultant gain is negligible.

For purposes of this paper, we shall ignore the effects of such a gain and assume that W_r and W_{ro} are identical.

7. CALCULATIONS

It is convenient to introduce the complex refractive index n of the ground, setting

$$n^2 = \epsilon_1/\epsilon_o = \epsilon_r + i\sigma/\epsilon_o\omega, \tag{7.1}$$

where ϵ_r is the dielectric constant of the ground relative to air, and σ is the conductivity. Assuming, as is usually the case, that $\mu_1 = \mu_o$, we have

$$k_1 = nk_o, \qquad Z_1 = Z_o/n, \tag{7.2}$$

$$\sin\theta_1 = \sin\theta/n, \qquad \cos\theta_1 = \sqrt{n^2 - \sin^2\theta}/n,$$

$$\sin\theta_c = 1/n, \qquad \cos\theta_c = \sqrt{n^2 - 1}/n.$$

Turning now to the representation of the field components, if an isotropic radiator in free space radiates the power W_r, then the fields are given by

$$E_f = \sqrt{\frac{W_r Z_o}{4\pi}}\ \frac{e^{ik_o r}}{r}, \quad H_f = \sqrt{\frac{W_r}{4\pi Z_o}}\ \frac{e^{ik_o r}}{r}. \tag{7.3}$$

We may then normalize the fields from a real antenna by setting

$$\overline{E} = \overline{E}_o E_f, \quad \overline{H} = \overline{H}_o H_f. \tag{7.4}$$

The factors $\overline{E}_o$, $\overline{H}_o$ might be called the <u>vector gain relative to free space</u>.

Using this notation, we may write (2.12) in the form

$$\overline{E}_{eo} \sim L(n)\, C_e(\theta)\, D(h)\, A_e(\theta, \phi)\, \overline{i}_\phi, \tag{7.5}$$

$$\overline{H}_{mo} \sim L(n)\, C_m(\theta)\, D(h)\, A_m(\theta, \phi)\, \overline{i}_\phi,$$

where

$$L = \frac{2}{\sqrt{\mathrm{Re}\, n}\,(n+1)}, \tag{7.6}$$

$$C_e = \frac{(n+1)\cos\theta}{n\cos\theta_1 + \cos\theta} \qquad C_m = \frac{(n+1)\cos\theta}{\cos\theta_1 + n\cos\theta},$$

$$D(h) = e^{ik_o h(n\cos\theta_1 - \cos\theta)},$$

$$A_e = ik_o \left(\frac{Z_o \mathrm{Re}\, n}{4\pi W_r}\right)^{1/2} \overline{P}\cdot\overline{i}_\phi,$$

$$A_m = ik_o \left(\frac{Z_o \mathrm{Re}\, n}{4\pi W_r}\right)^{1/2} \overline{P}\cdot\overline{i}_\phi \times \overline{i}_{r1}.$$

In these expressions we have separated the fields into factors that seem useful to us. These factors depend principally on the indicated parameters, although the other parameters may also play a role. We shall call L the <u>interface loss</u>, C_e, C_m the <u>electric</u> and <u>magnetic cutback factors</u>, D the <u>depth attenuation</u>, and A_e, A_m the <u>antenna patterns</u>.

Note that $\overline{E}_{mo} = \overline{i}_r \times \overline{H}_{mo}$, and, since $\overline{i}_r \times \overline{i}_\phi = -\overline{i}_\theta$, it will follow that $\overline{E}_{mo}$ is precisely $\overline{H}_{mo}$ with the unit vector $\overline{i}_\phi$ replaced by $\overline{i}_\theta$. It follows that E_{eo} is the ϕ- component of $\overline{E}$ and H_{mo} the θ- component. The situation in the case of the ground wave is, however, more complicated, and we prefer to retain the present E - H notation.

For the ground wave we may rewrite (3.13) in the form

$$\overline{E}_{eo} \sim \frac{i}{k_o\rho} \; S_e(n)\, U_e(z)\, D\,(h)\, A_e\left(\frac{\pi}{2}, \phi\right) \overline{i}_\phi, \tag{7.7}$$

$$\overline{H}_{mo} \sim \frac{i}{k_o\rho} \; S_m(n)\, U_m(z)\, D\,(h)\, A_m\left(\frac{\pi}{2}, \phi\right) \overline{i}_\phi,$$

where

$$S_e = \frac{2}{\sqrt{\operatorname{Re} n}\,(n^2 - 1)} = \frac{2}{\sqrt{\operatorname{Re} n}\,\delta_e^2} = \frac{1}{n-1}\, L, \tag{7.8}$$

$$S_m = \frac{2n^3}{\sqrt{\operatorname{Re} n}\,(n^2 - 1)} = \frac{2}{n\sqrt{\operatorname{Re} n}\,\delta_m^2} = \frac{n^3}{n-1}\, L.$$

The functions U_e, U_m are both equal to the function U defined in (3.4), with δ replaced by δ_e, δ_m, respectively. Furthermore, the function D(h) is, in this situation, equal to U (-h). The functions S_e, S_m are the only new factors (other than the important $i/k_o\rho$) introduced here. We shall call them the <u>electric</u> and <u>magnetic surface wave coupling</u> factors.

If the antenna is represented by magnetic currents, then we obtain very similar results. From (2.14) and (3.15) we find that the space wave is given by (7.5), and that the ground wave is given by (7.7), except that the antenna patterns A_e, A_m must be replaced by

$$A_e' = -ik_o \left(\frac{\operatorname{Re} n}{4\pi Z_o W_r}\right)^{1/2} n\,\overline{Q} \cdot \overline{i}_\phi \times \overline{i}_{r1}, \tag{7.9}$$

$$A_m' = ik_o \left(\frac{\operatorname{Re} n}{4\pi Z_o W_r}\right)^{1/2} n\,\overline{Q} \cdot \overline{i}_\phi.$$

All of the above factors, except for the antenna patterns themselves, are independent of the antenna, and therefore they have a universal applicability. In figures 2 - 10 we have constructed several graphs of them.

Figure 2 is a plot of $|L|$ vs. ϵ_r. So long as the loss tangent of the ground is small we have $n \approx \sqrt{\epsilon_r}$ to a first order approximation. It follows that n, and consequently such functions as L, S_e, S_m, are insensitive to the conductivity σ and the frequency f. We may, for example, use without much harm the approximation

$$L \approx \frac{2}{\epsilon_r^{1/4}(1 + \epsilon_r^{1/2})} \approx 2\,\epsilon_r^{-3/4}. \tag{7.10}$$

The second approximation is valid when $\epsilon_r >> 1$.

Figures 3 and 4 show $|C_e|$, $|C_m|$ plotted vs. elevation angle, $90^{\circ} - \theta$. If, again, $|n| >> 1$, then $\cos\theta_1 \approx 1$, and

$$C_e \approx \frac{(n + 1)\cos\theta}{n + \cos\theta} \approx \cos\theta, \tag{7.11}$$

$$C_m \approx \frac{(n + 1)\cos\theta}{1 + n\cos\theta}.$$

These approximations show what is probably the greatest difference between the TE and TM components of the field. For at small elevation angles $\cos\theta \approx 0$, and C_m is nearly $(n + 1)$ times as large as C_e. This fact is clearly brought out in figure 4.

Setting

$$\Delta = \mathrm{Im}\, k_o n \cos\theta_1, \tag{7.12}$$

we find

$$|D(h)| = e^{-\Delta h}, \tag{7.13}$$

so that there is an attenuation with depth of Δ nepers/m. Graphs of Δ are plotted in figures 5, 6, and 7. For small loss tangents,

$$n \approx \sqrt{\varepsilon_r} + i \frac{\sigma}{2\sqrt{\varepsilon_r}\,\varepsilon_o\,\omega} \tag{7.14}$$

so that

$$\Delta \approx \frac{\sigma k_o}{2\sqrt{\varepsilon_r}\,\varepsilon_o\,\omega} = \frac{\sigma}{2\sqrt{\varepsilon_r}} Z_o. \tag{7.15}$$

It follows that Δ is insensitive to changes in θ or in frequency. This fact is demonstrated again in figures 6 and 7.

Turning to the ground wave, $|S_e|$ and $|S_m|$ are plotted vs. ε_r in figure 8, and $|U_e|$, $|U_m|$ vs. height in figure 9. Here again is demonstrated the fact that the TM (or vertically polarized) component is much easier to excite than the TE component. Note also that, if $n \approx 1$, then the asymptotic expansions we have used require much larger ρ. If desired, the function F of (3.4) may be introduced instead.

The term "antenna pattern" is justified here only by the expressions we shall soon obtain for particular antennas. But in any case, the remaining factors ($D(h)$ and A_e or A_m) in (7.5) and (7.7) make up a kind of "basic attenuation" which is independent of the antenna. To show the transition between the ground wave and the space wave, we have plotted, in figure 10, the height-gain curves of $|LC_m|$ and $|S_m U_m / k_o \rho|$ that would be found at varying distances. Labeling these two expressions as waves is, of course, somewhat anomalous. They are really two different approximations of (2.10) which should be accurate in two different, overlapping regions. As figure 10 suggests, the common region where both are accurate is quite extensive. Incidentally, it would be entirely possible to develop a uniform approximation which would encompass both ground wave and space wave in one expression.

One might also note that the curves of figure 10 are of even greater universality than we have indicated. This is because the antenna pattern is almost constant near the horizon. As we saw in section 4, the derivative of $\overline{P}$ with respect to θ vanishes on the horizon. Consequently, the derivatives of A_e and A_m also vanish on the horizon. In short, if we are interested only in low angles of elevation, it is a good approximation to say that the antenna patterns are constant with respect to elevation, and to set them equal to the values they have along the horizon.

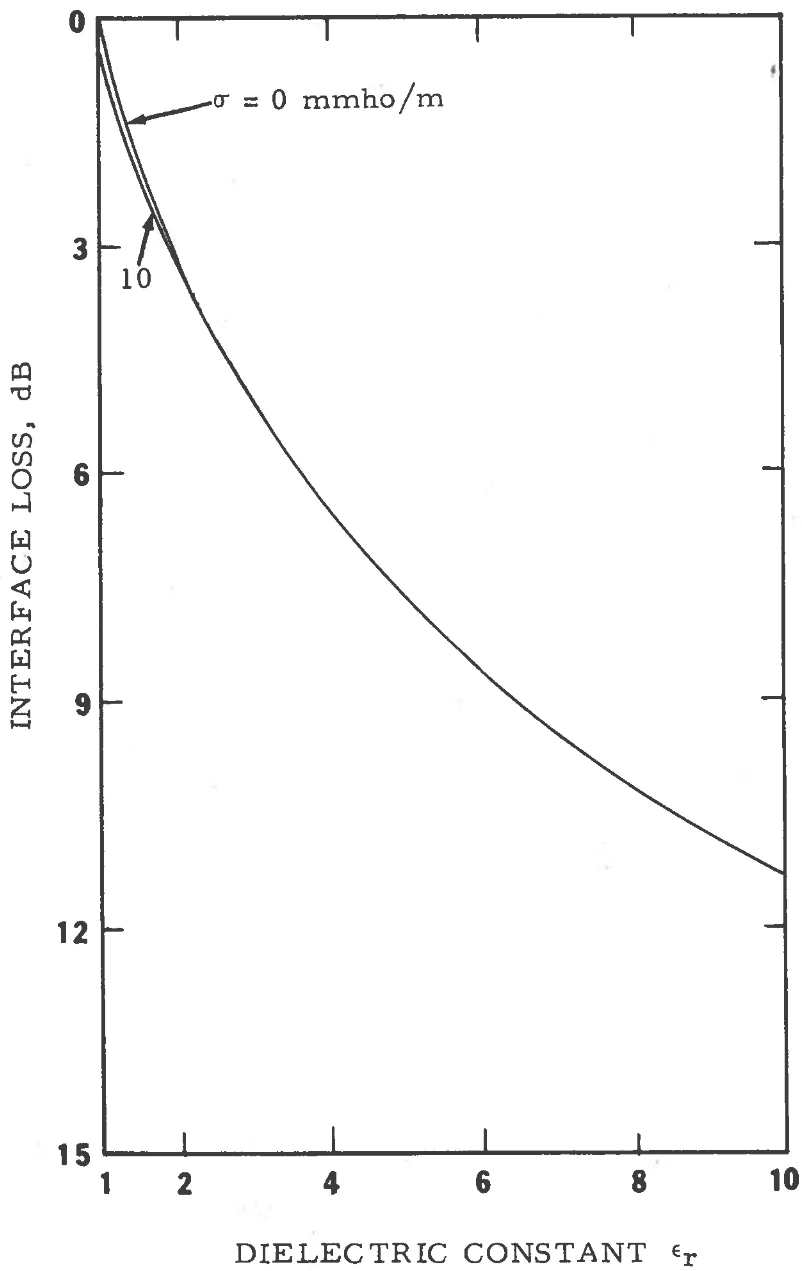

Figure 2. Interface loss.

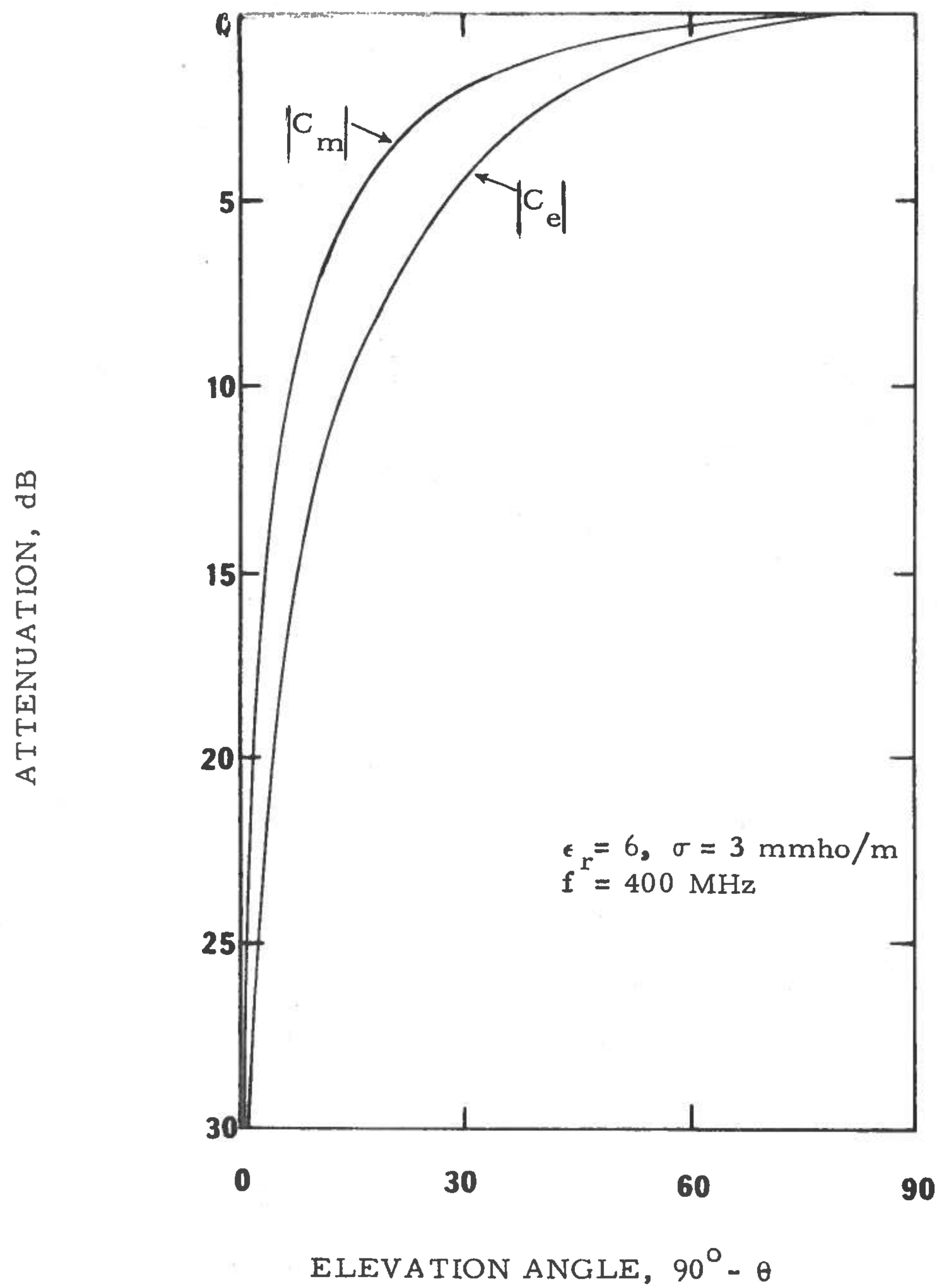

Figure 3. Cutback factors.

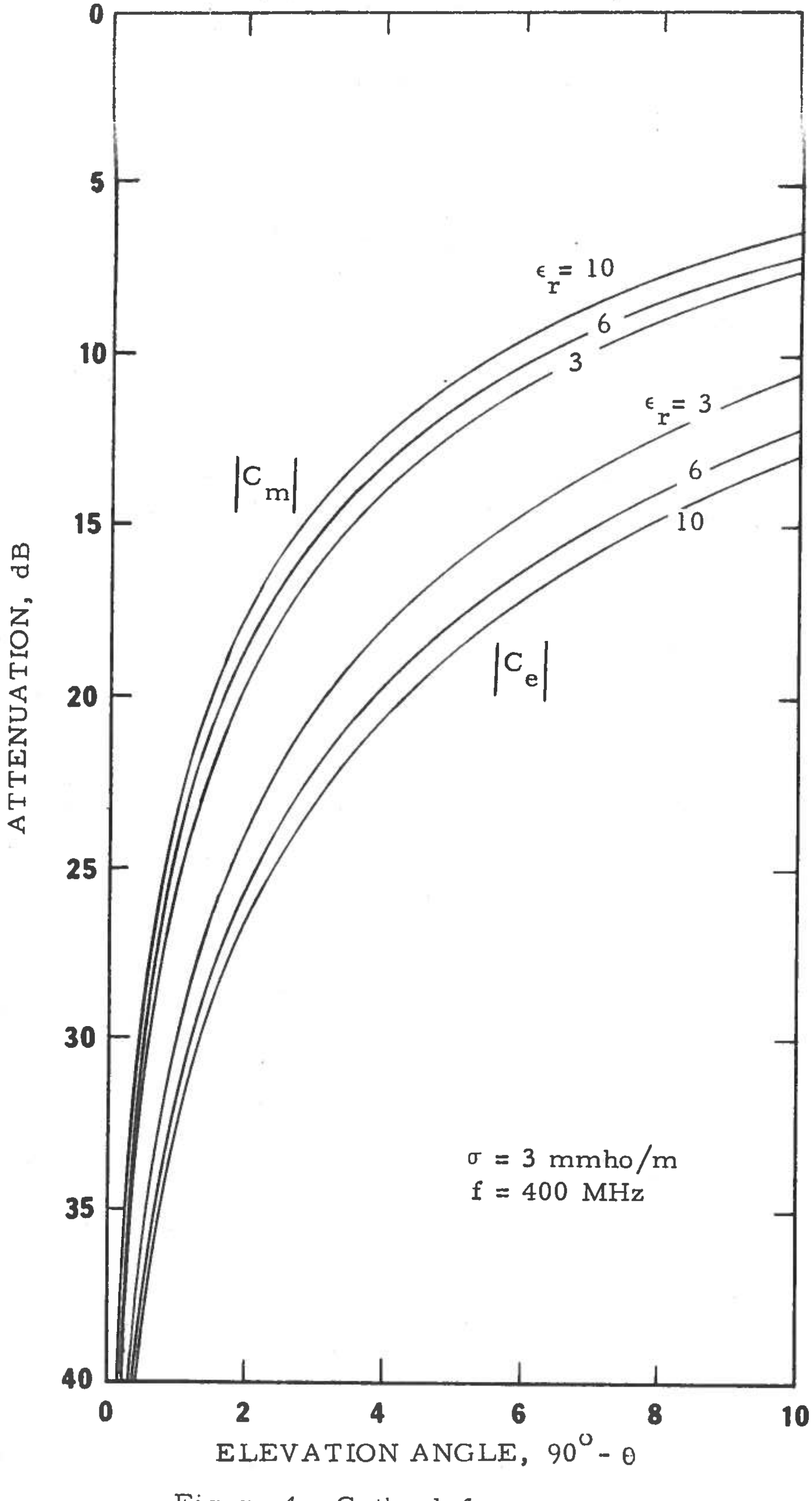

Figure 4. Cutback factors, detail.

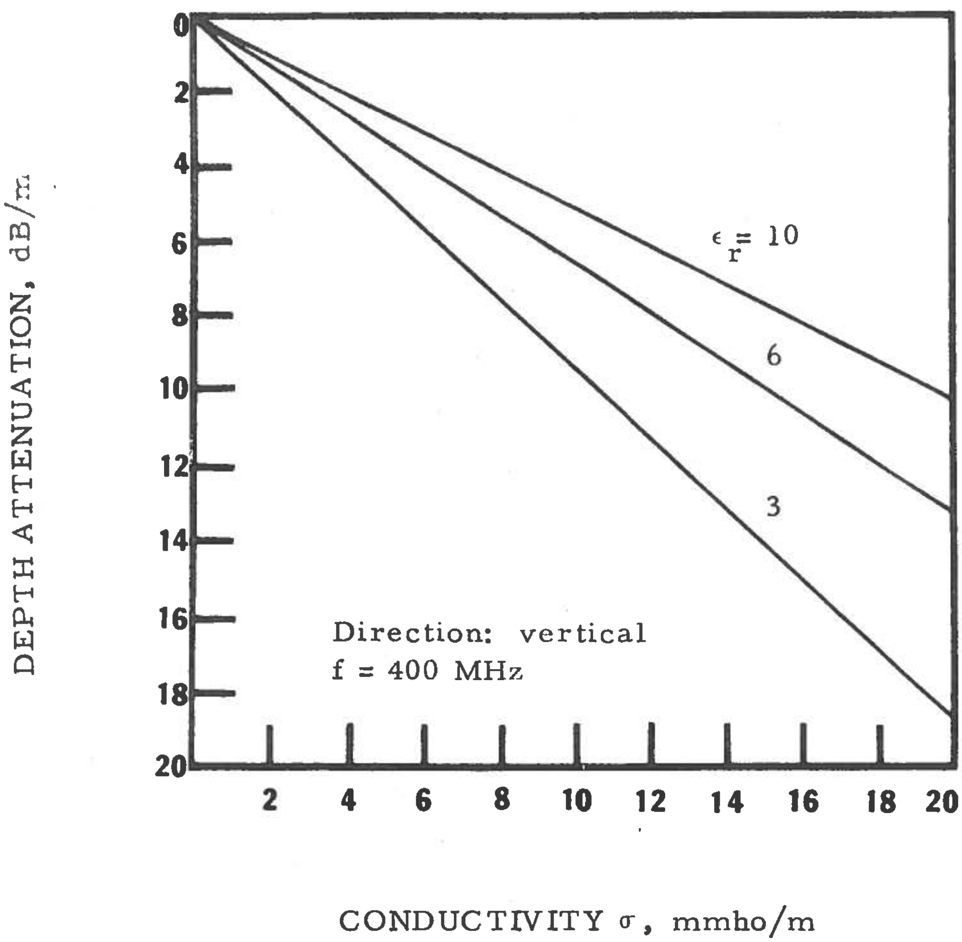

Figure 5. Depth attenuation.

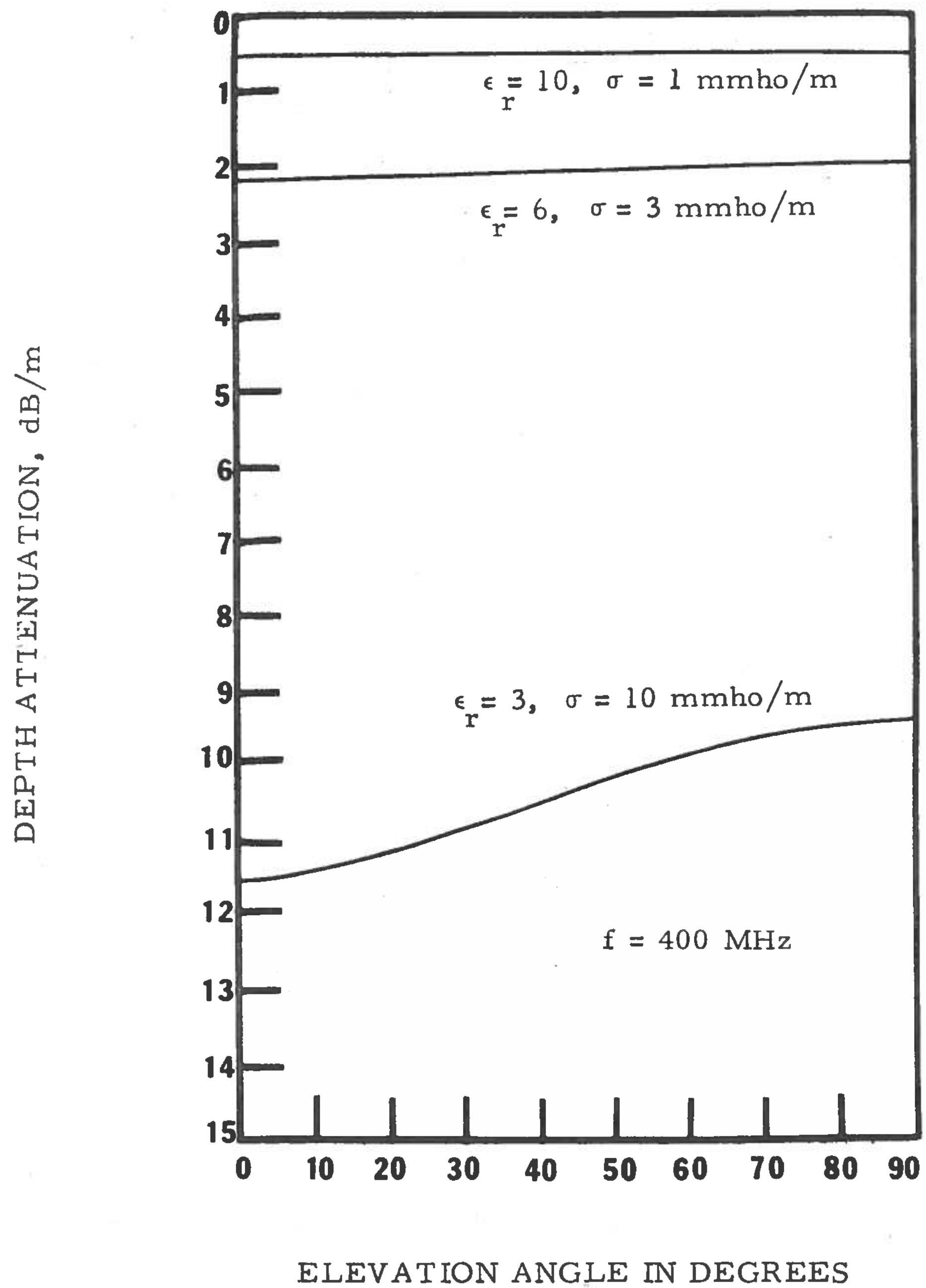

Figure 6. Depth attenuation, variation with elevation.

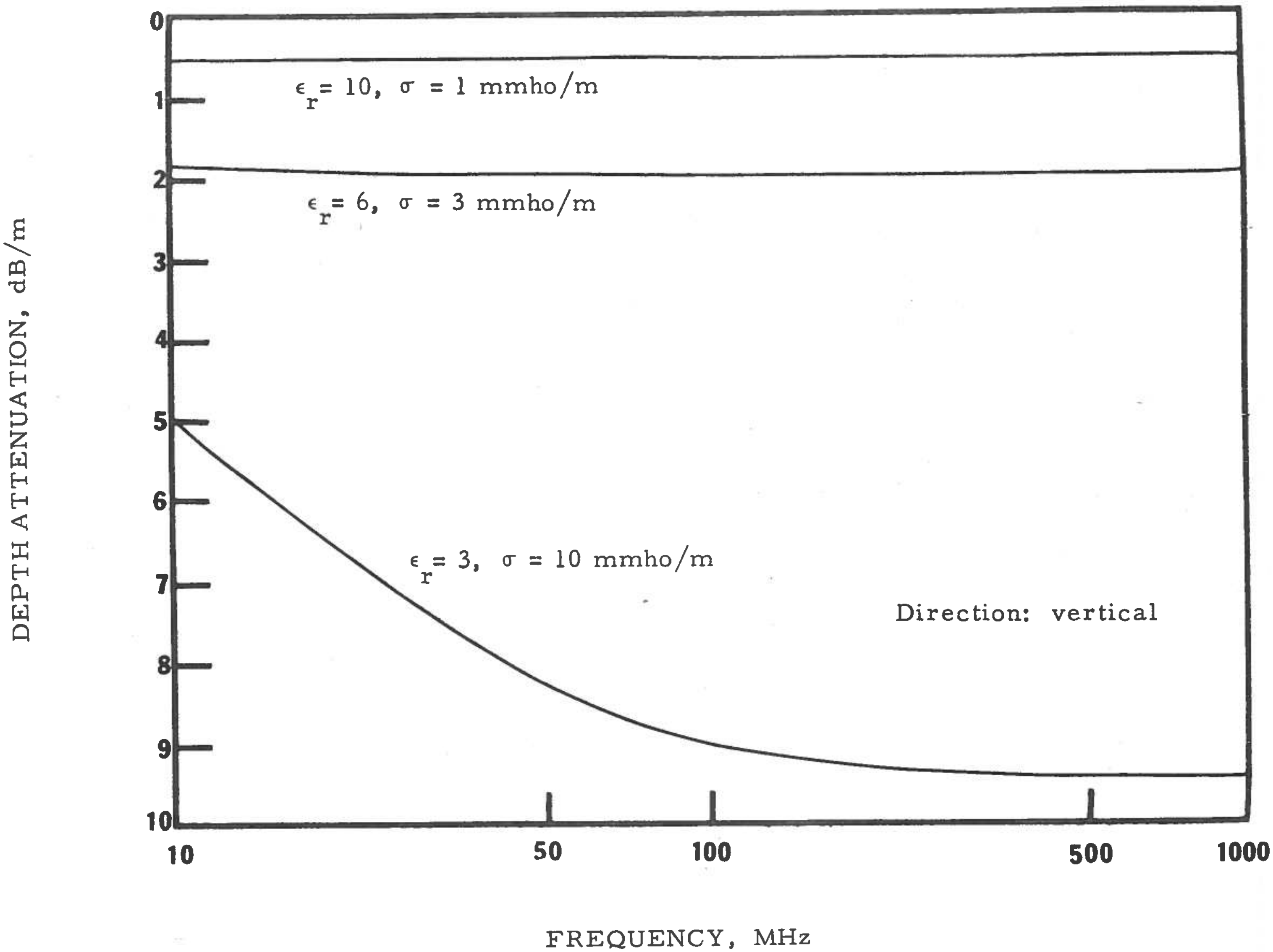

Figure 7. Depth attenuation, variation with frequency .

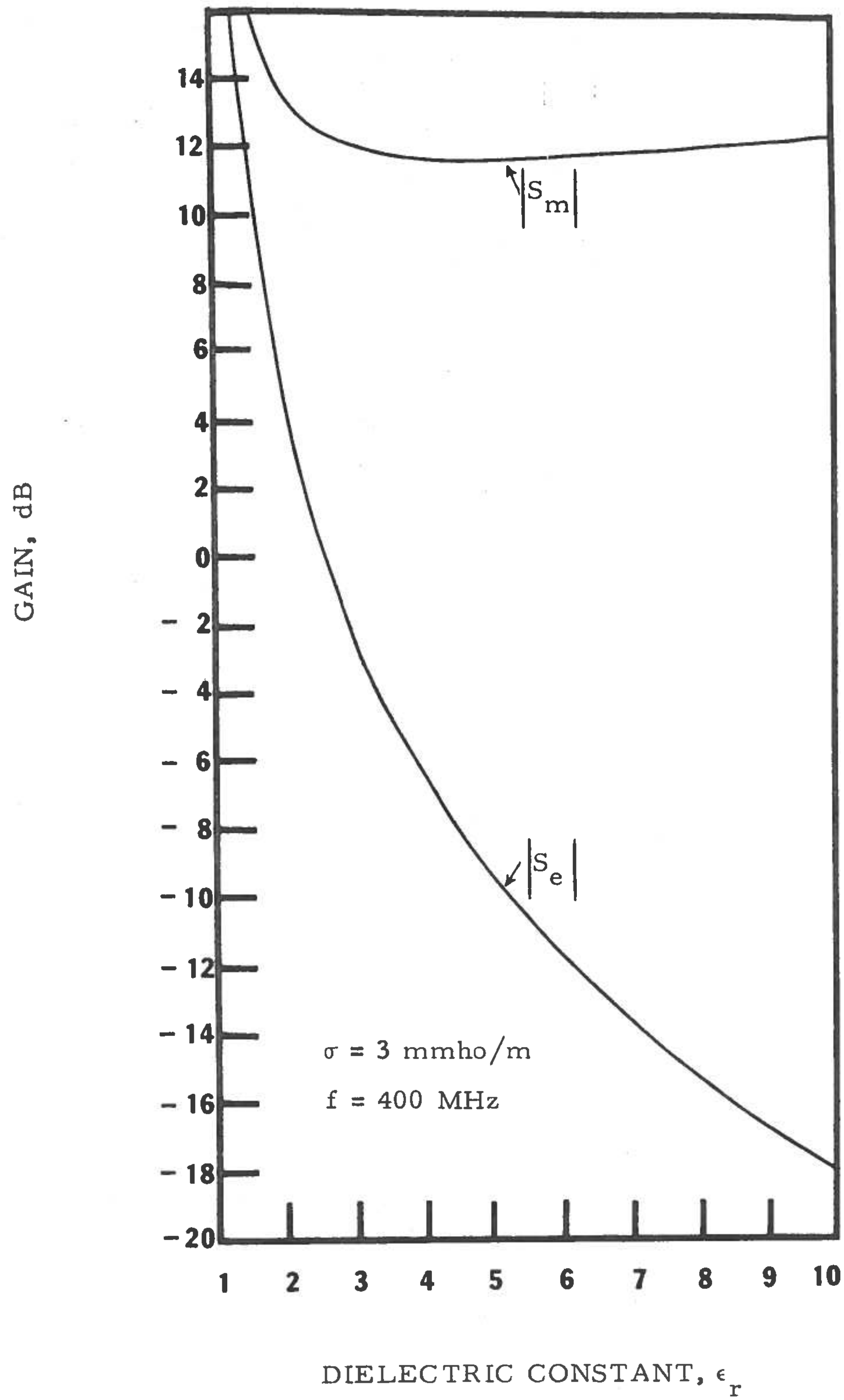

Figure 8. Surface wave-coupling factors.

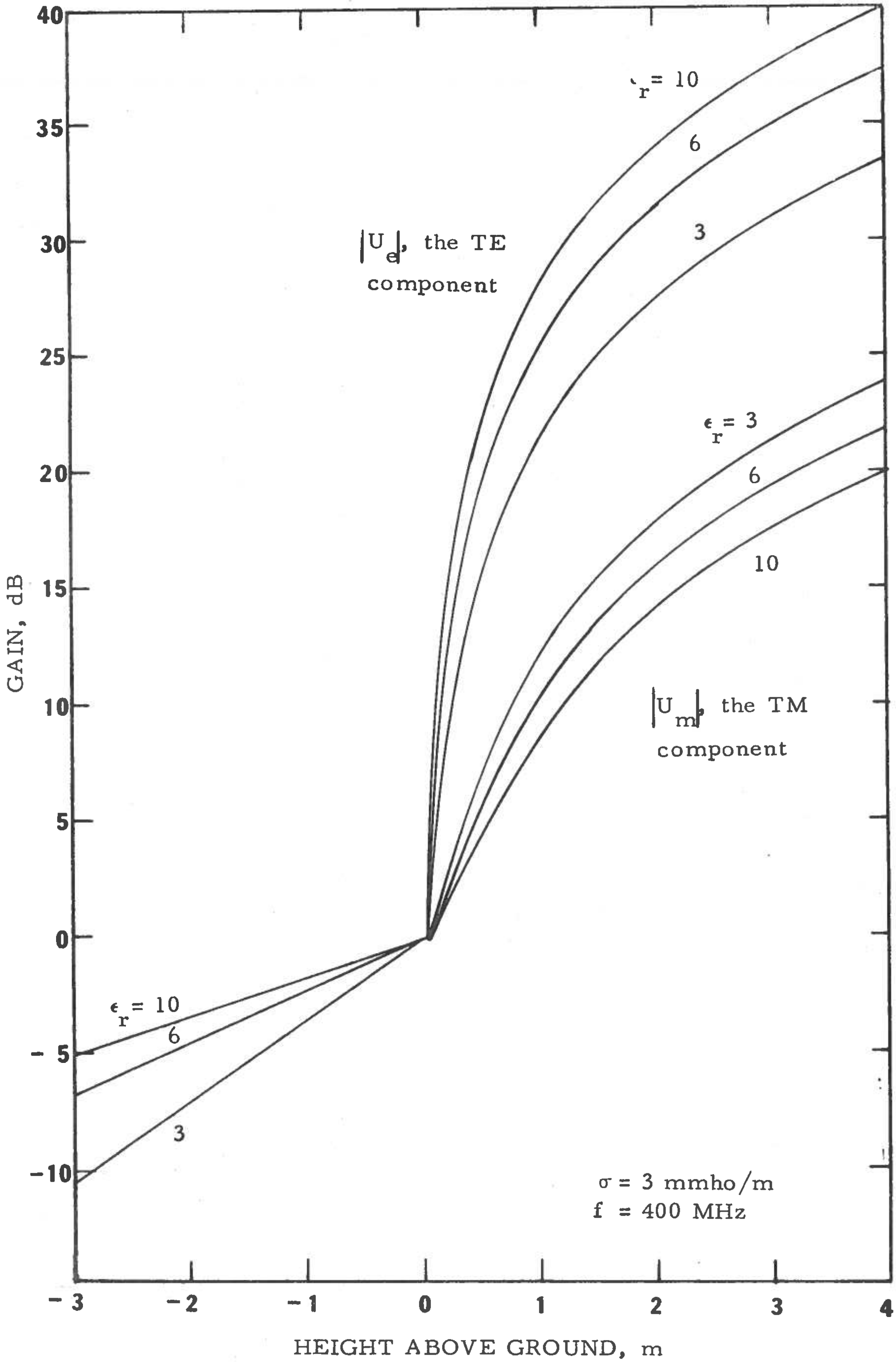

Figure 9. Height gain of the ground wave.

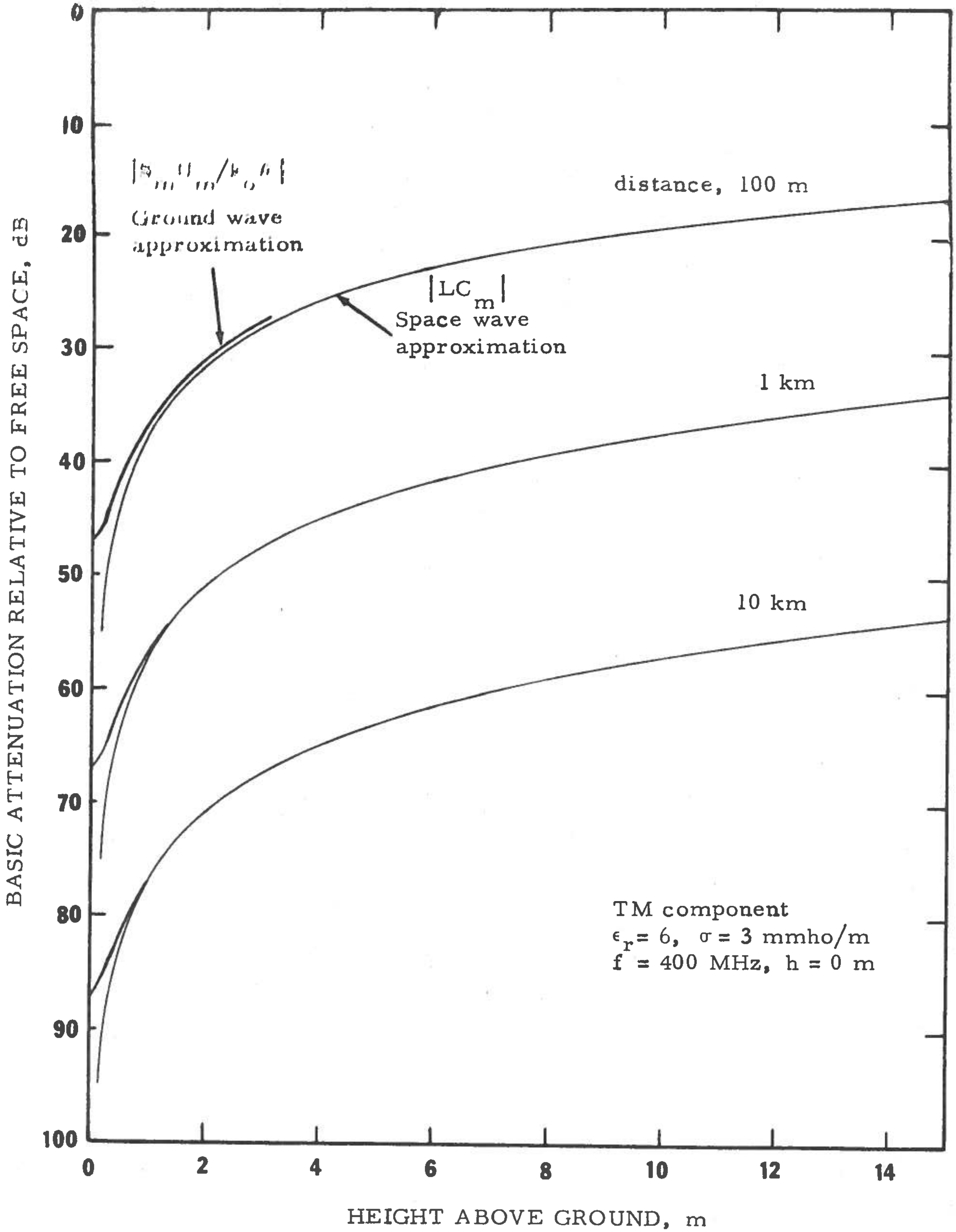

Figure 10. Basic attenuation, a comparison between the space wave and the ground wave approximations.

7.1 The Vertical Electric Doublet (VED)

Let $\bar{p}$ represent any electric doublet. In the notation of section 6 we have

$$W_r = \frac{k_o^2 Z_o \,||\bar{p}||^2}{6\pi} \operatorname{Re} n, \tag{7.16}$$

$$W_i(r) = \frac{k_o^2 Z_o \,||\bar{p}||^2}{6\pi} \operatorname{Re} \frac{1}{n} \left(\frac{2 \operatorname{Im} n}{k_o r} + \frac{2i \operatorname{Im} n}{n k_o^2 r^2} + \frac{i}{n k_o^3 r^3} \right) \cdot e^{-2k_o r \operatorname{Im} n} .$$

As suggested in section 6, it is of some interest to consider the case of a lossless antenna whose pattern is that of a doublet, but which occupies a sphere of radius r. Despite the fact that the antenna is lossless, it still is not a perfect radiator, since the induction field has heat losses equal to $W_i(r)$. It is reasonable, then, to compute an efficiency

$$\frac{W_r}{W_r + W_i(r)}, \tag{7.17}$$

and this we have done in figure 11. As one can see, the problem, while not negligible, is not severe.

In the case of a VED, we have $\bar{p} = p\bar{i}_z$ and

$$A_e = 0, \tag{7.18}$$

$$A_m = -i\sqrt{3/2}\, \sin\theta_1 = -i\sqrt{3/2}\, \sin\theta/n.$$

In figures 12 and 13 we have plotted the resulting E_θ field. Figure 12 is a polar plot of the (normalized) space wave pattern defined by (7.5), while figure 13 is a rectangular plot from which may be read the product of all terms entering into (7.5). As mentioned before, this product comprises the radiation gain of the buried antenna (relative to an isotropic radiator in free space).

Since $|A_m(\pi/2)| = \sqrt{3/2}\ /n$, the low elevation gain is rather small. As we shall see, a VED is a relatively inefficient device with which to couple energy into the field in air.

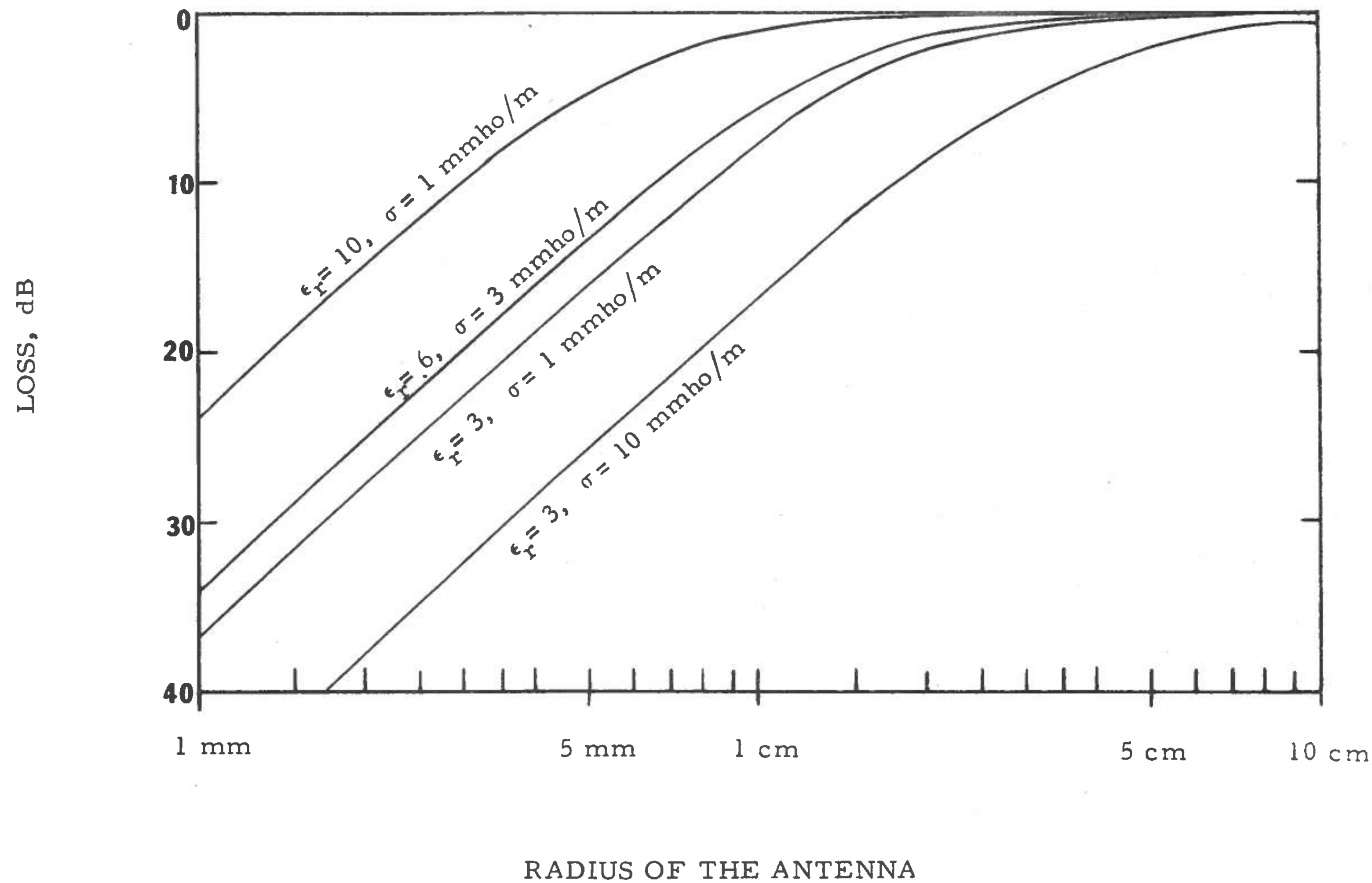

Figure 11. Induction field heat losses for a spherical antenna with a doublet-like field.

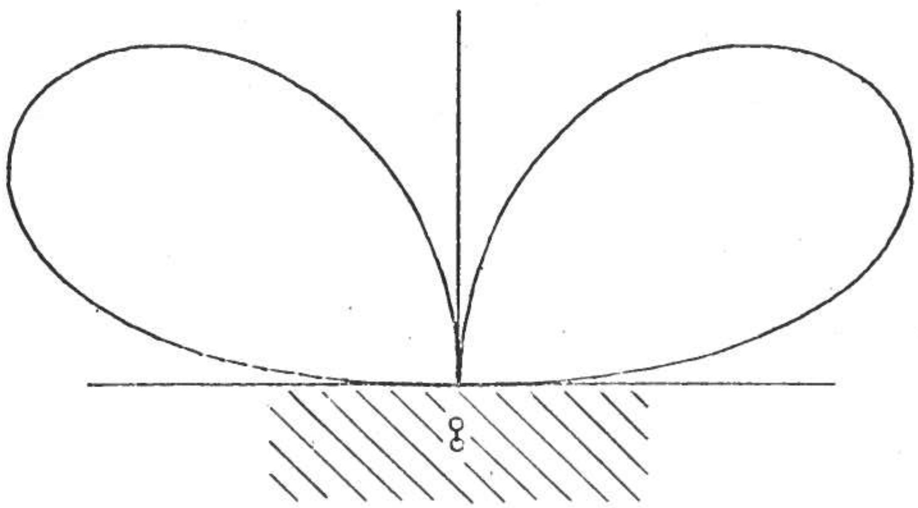

Figure 12. Vertical pattern of a buried VED; $\epsilon_r = 6$, $\sigma = 3$ mmho/m, f = 400 MHz, h = 1 m.

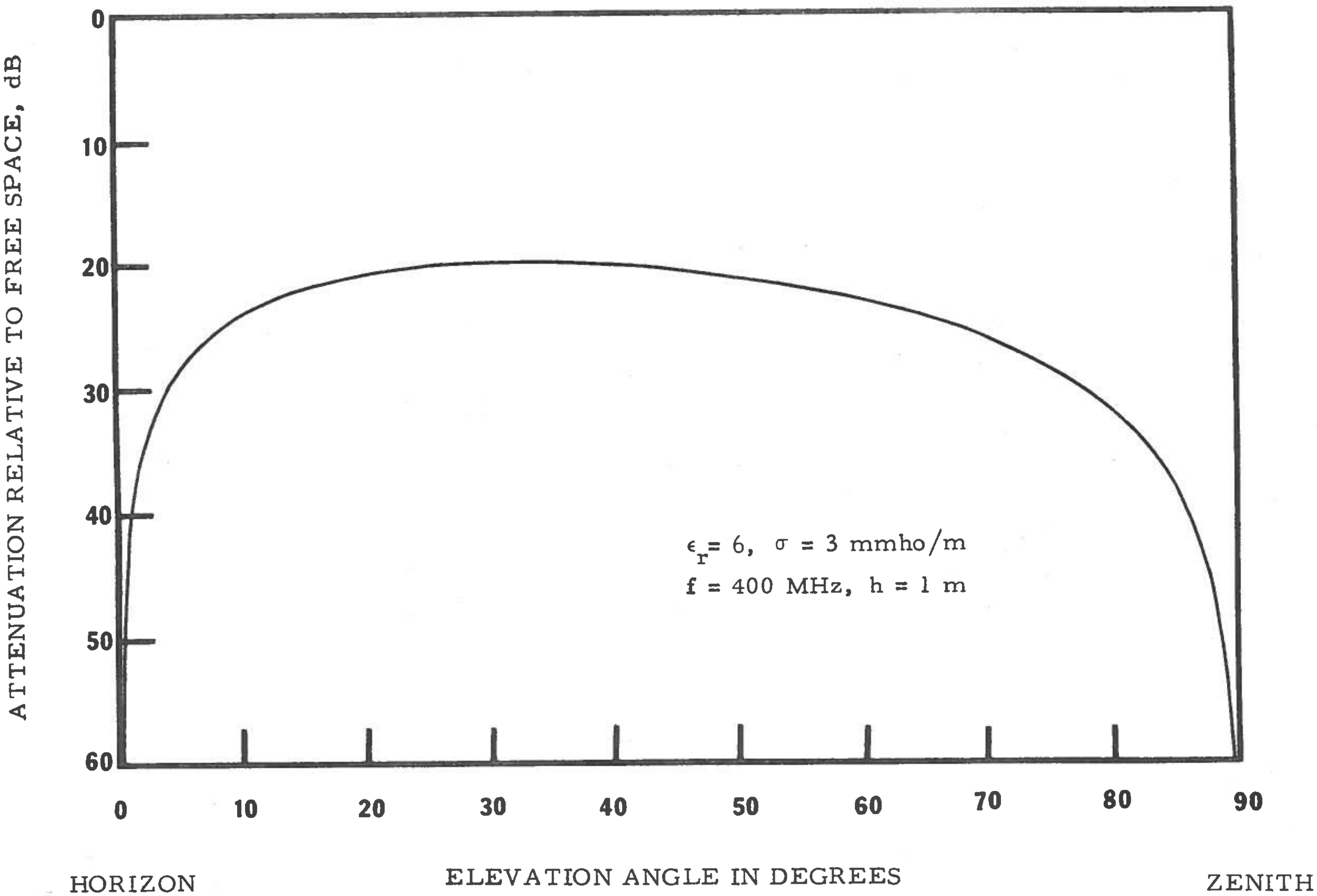

Figure 13. Radiation gain from a buried VED.

7.2 The Horizontal Electric Doublet (HED)

We set $\bar{p} = p\bar{i}_y$ so that the x-axis is broadside to the doublet. Then

$$A_e = i\sqrt{3/2}\ \cos\phi, \tag{7.19}$$

$$A_m = i\sqrt{3/2}\ \cos\theta_1\ \sin\phi.$$

Thus both the TE (E_ϕ) and the TM (E_θ) components are excited. If we may assume $\cos\theta_1 \approx 1$ then the excitation of both fields is about the same, and the fact that the cutback factors emphasize the TM component at low angles means that in reality this component is likely to be the stronger. Note that the "wrong" (TM) component is actually better excited by such an HED than it is by the more natural VED.

Vertical patterns of the E_ϕ (broadside) field and the E_θ (endfire) field are shown in figure 14. Azimuthal patterns of both, together with the total flux, are shown in figure15. Figure 16 is a rectangular plot of the two vertical patterns, while figure 17 is a rectangular plot of the azimuthal ground wave patterns at zero height.

Because there are two components of the field, both of which have small imaginary parts, the resultant field will have a polarization which is slightly elliptical. How slight this ellipticity is, is shown in figure 18, where the axial ratio is plotted.

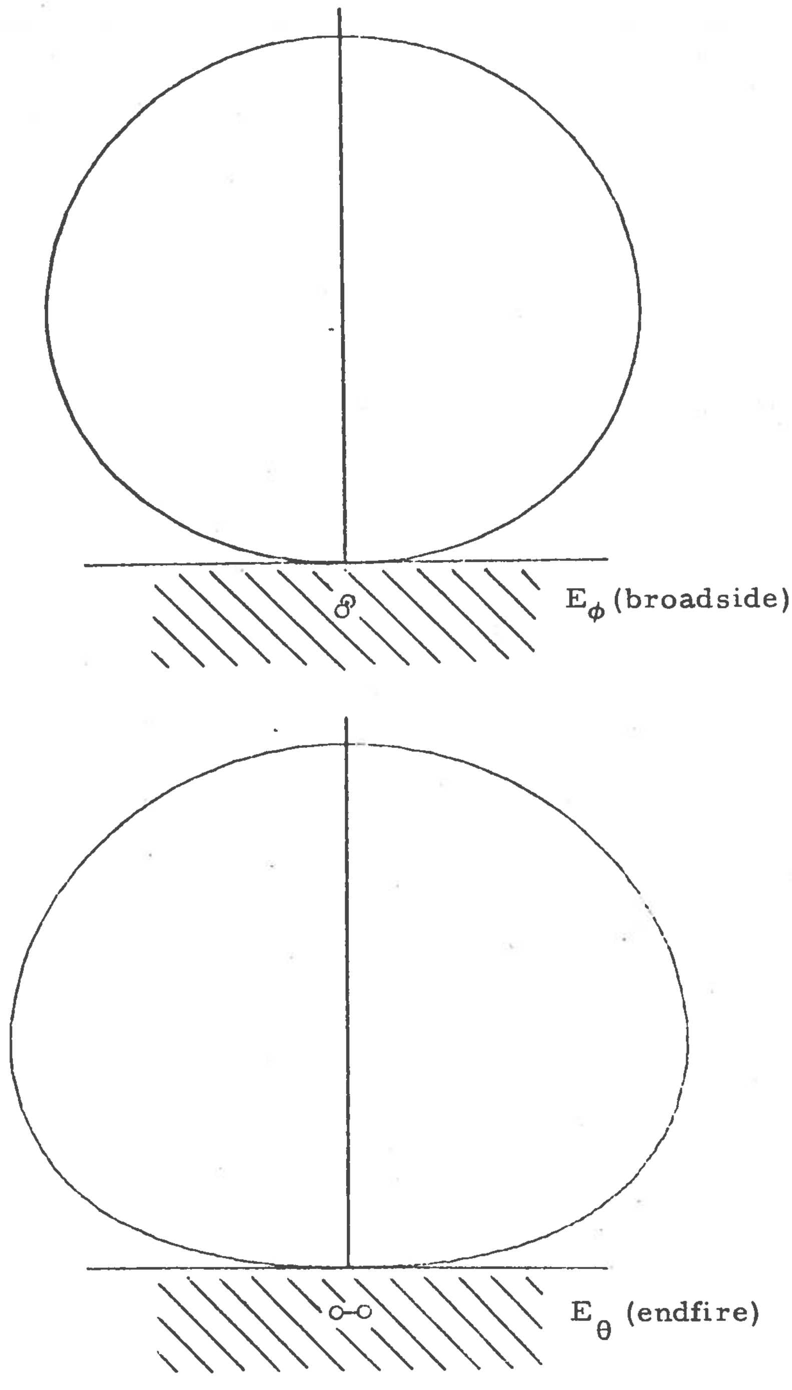

Figure 14. Vertical patterns of a buried HED; $\epsilon_r = 6$, $\sigma = 3$ mmho/m, f = 400 MHz, h = 1 m.

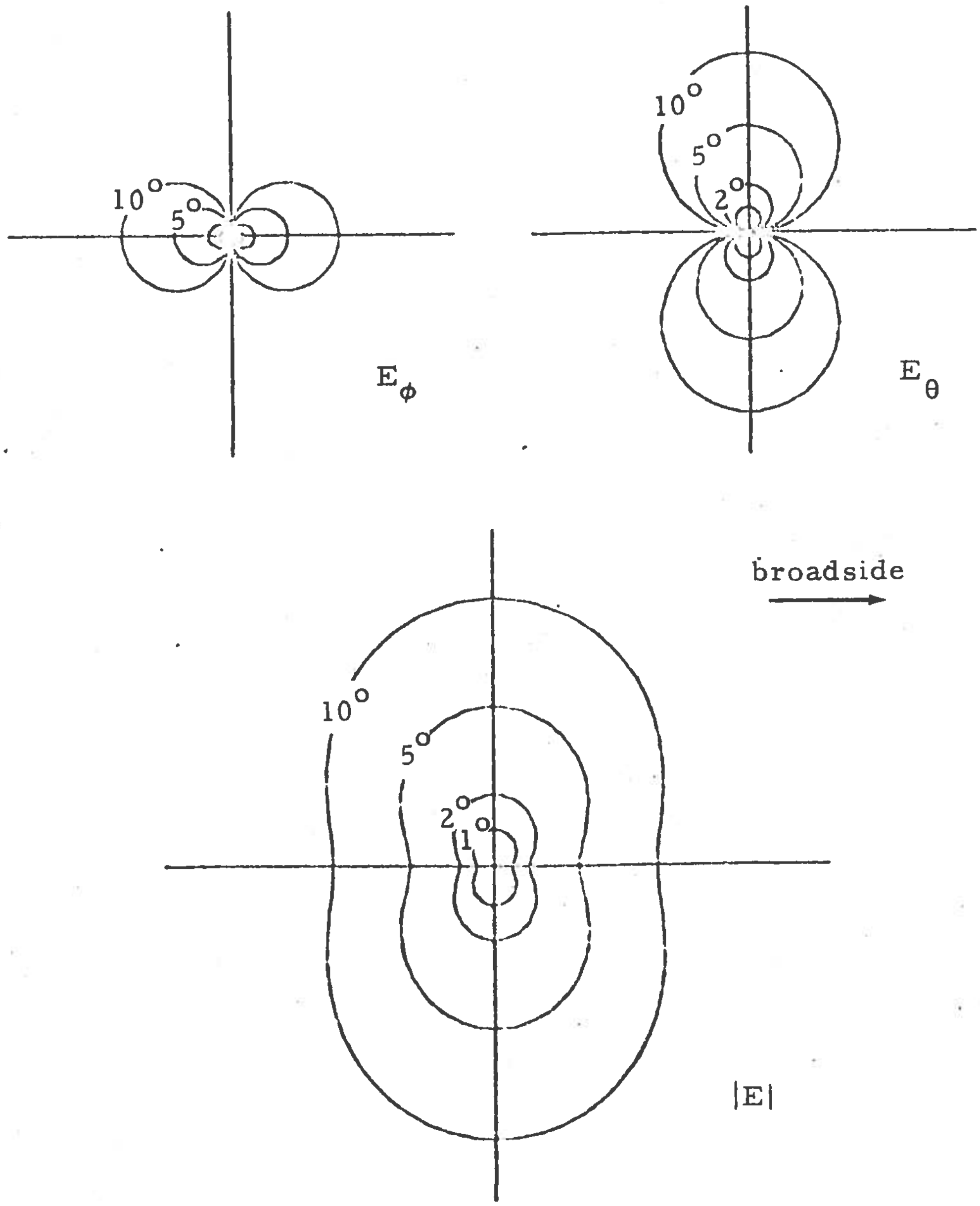

Figure 15. Azimuthal patterns of a buried HED; $\epsilon_r = 6$, $\sigma = 3$ mmho/m, f = 400 MHz, h = 1 m.
The patterns are drawn at various elevation angles.

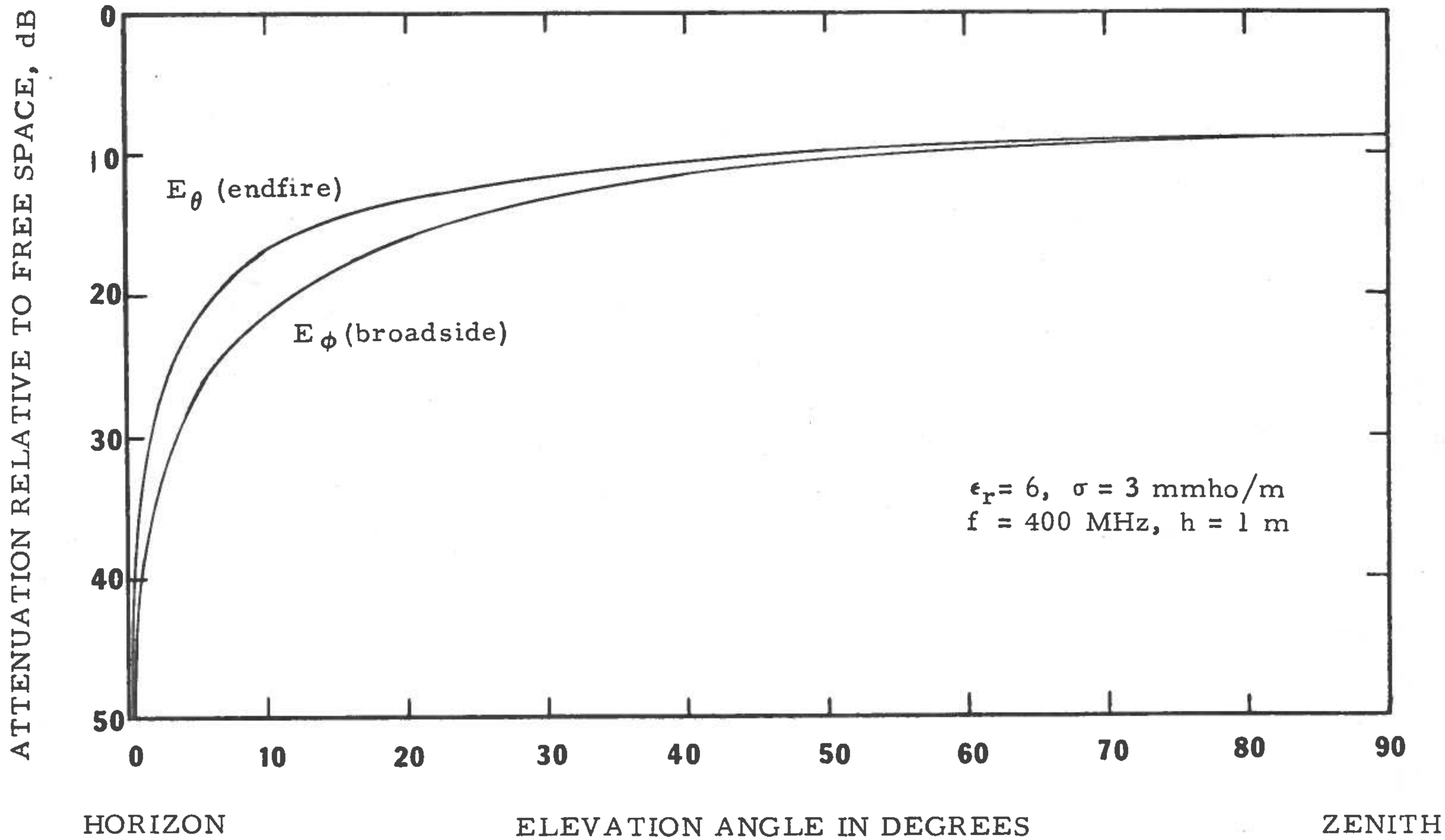

Figure 16. Radiation gain of a buried HED.

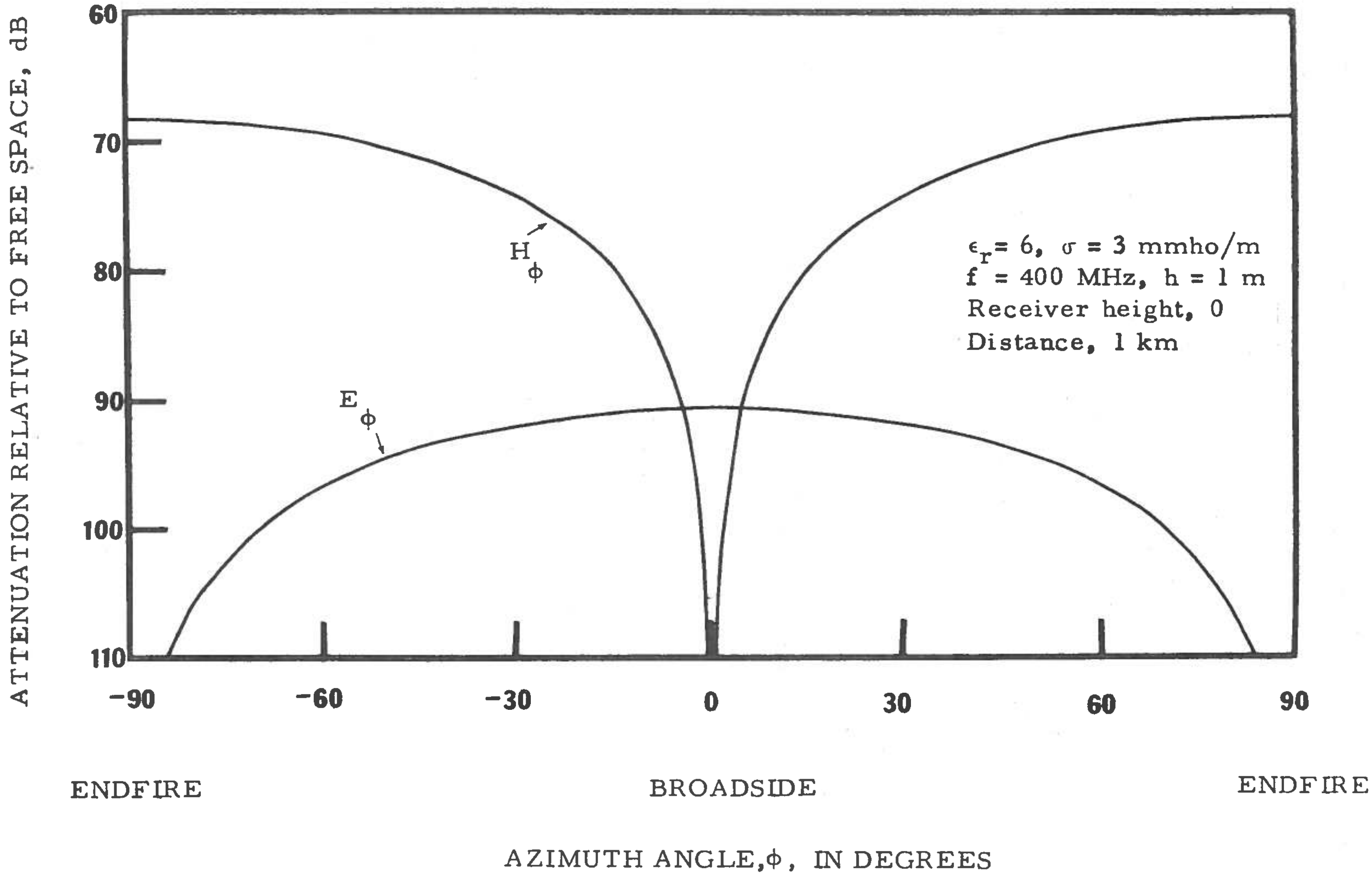

Figure 17. The ground wave of a buried HED.

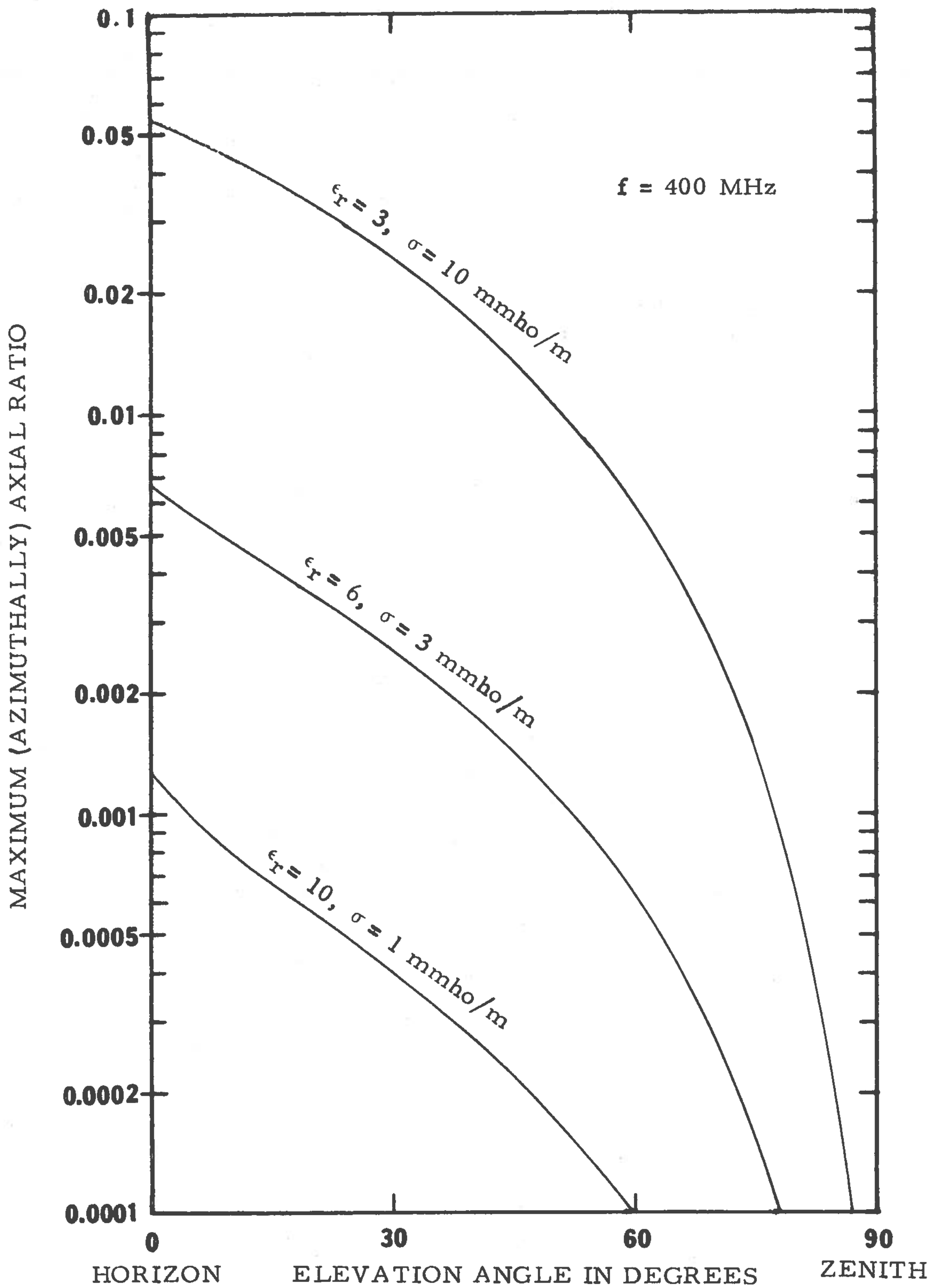

Figure 18. Space wave ellipticity for a buried HED.

7.3 Tipped Doublets

Setting $\overline{p} = p(\cos\alpha\,\overline{i}_y - \sin\alpha\,\overline{i}_z)$, we find

$$A_e = i\sqrt{\frac{3}{2}}\cos\alpha\cos\phi, \tag{7.20}$$

$$A_m = i\sqrt{\frac{3}{2}}(\sin\alpha\sin\theta_1 + \cos\alpha\cos\theta_1\sin\phi).$$

This corresponds to an electric doublet lying in the yz - plane and making a (real) angle α with the y - axis. It is tipped away from the horizontal.

An interesting problem is how to tip the doublet so as to maximize the field at zero elevation in, say, the y - direction. If we set $\theta = \pi/2$, $\phi = \pi/2$, (7.20) becomes

$$A_e = 0, \tag{7.21}$$

$$A_m = i\sqrt{3/2}\cos(\theta_c - \alpha).$$

A plot of this value of $|A_m|$ versus α is given in figure 19. The maximum value of $\sqrt{3/2}\cosh\mathrm{Im}\,\theta_c$ is attained when $\alpha = \mathrm{Re}\,\theta_c$. If the conductivity is high, this might be an important consideration, but otherwise, as one can see, there is little advantage to be gained.

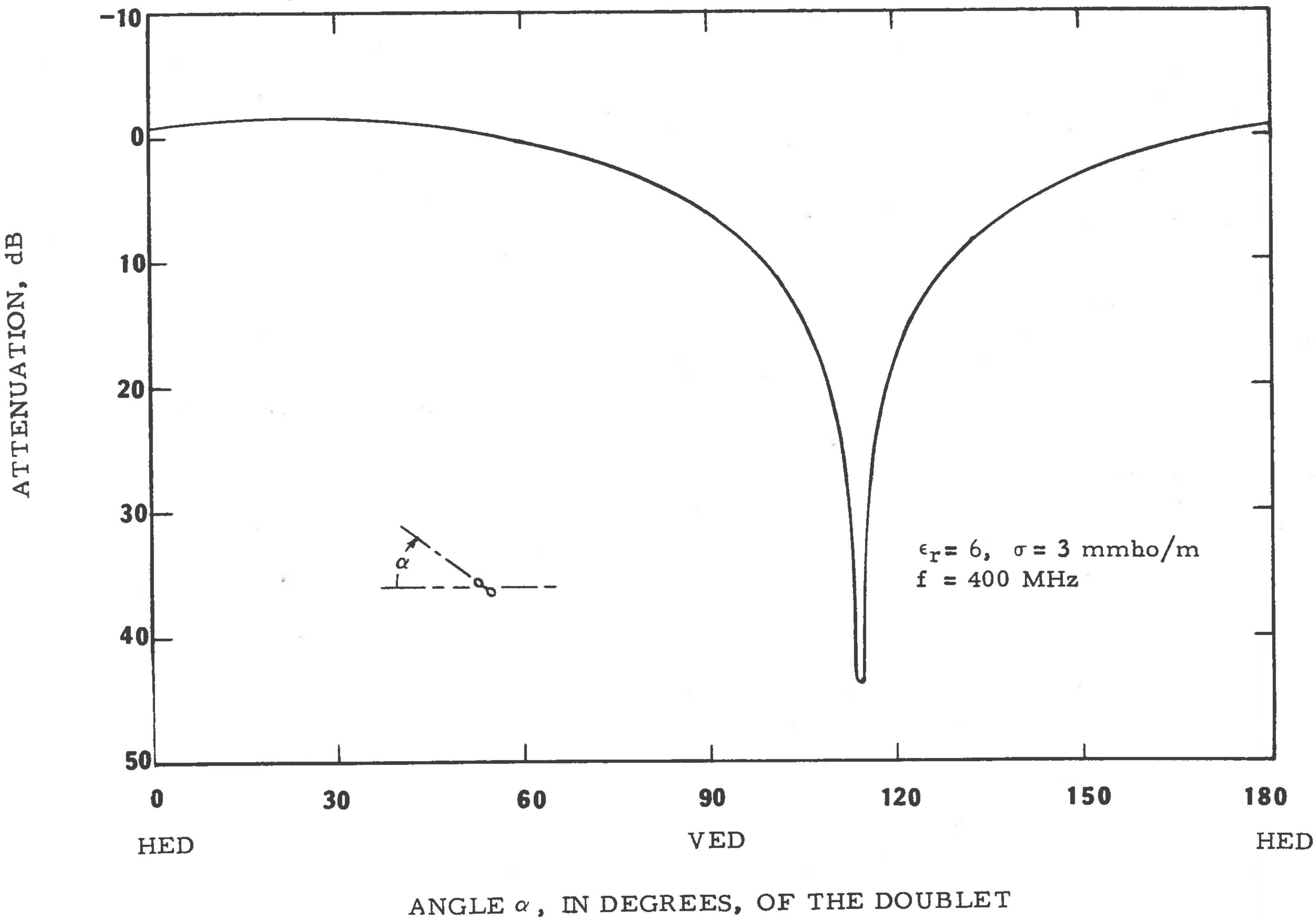

Figure 19. Endfire horizontal gain of a tipped electric doublet.

7.4 Magnetic Doublets

For a magnetic doublet $\bar{q}$, we find

$$W_r = \frac{k_o^2 \, || \bar{q} ||^2}{6\pi Z_o} \, |n|^2 \, \mathrm{Re}\, n. \tag{7.22}$$

Inserting this into (7.9) we find immediately that in the case of a VMD

$$\bar{q} = q \, \bar{i}_z , \tag{7.23}$$

$$A_e' = i \frac{n}{|n|} \sqrt{3/2} \sin \theta_1 = i \sqrt{3/2} \sin \theta / |n|,$$

$$A_m' = 0,$$

and in the case of an HMD

$$\bar{q} = q \, \bar{i}_y, \tag{7.24}$$

$$A_e' = -i \frac{n}{|n|} \sqrt{3/2} \cos \theta_1 \sin \phi,$$

$$A_m' = i \frac{n}{|n|} \sqrt{3/2} \cos \phi.$$

These results are slightly different from those for the corresponding electric doublets. The radiation gains have been plotted in figures 20 and 21. One notes that the VMD is very inefficient, but that the HMD is slightly more efficient than the HED.

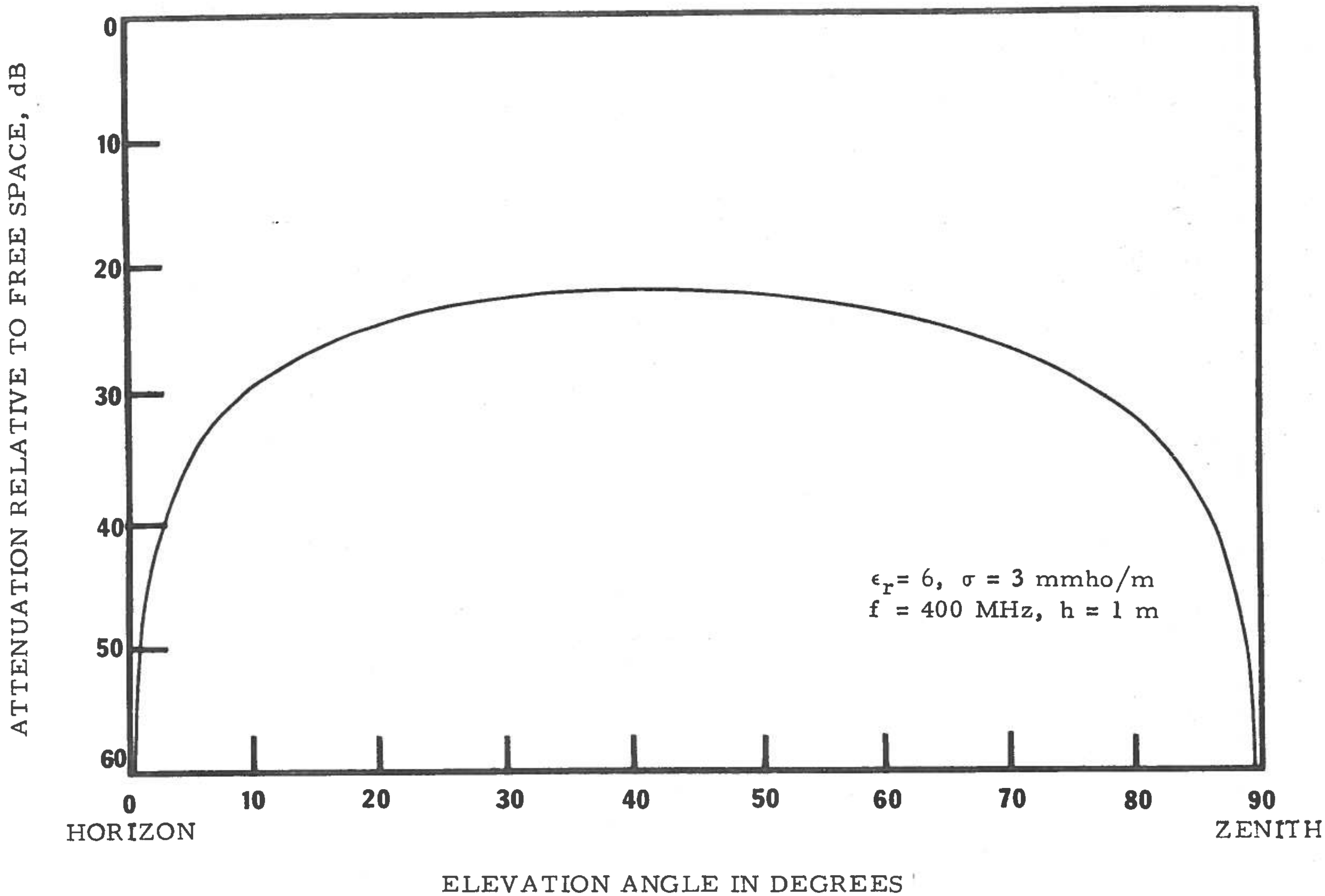

Figure 20. Radiation gain of a buried VMD.

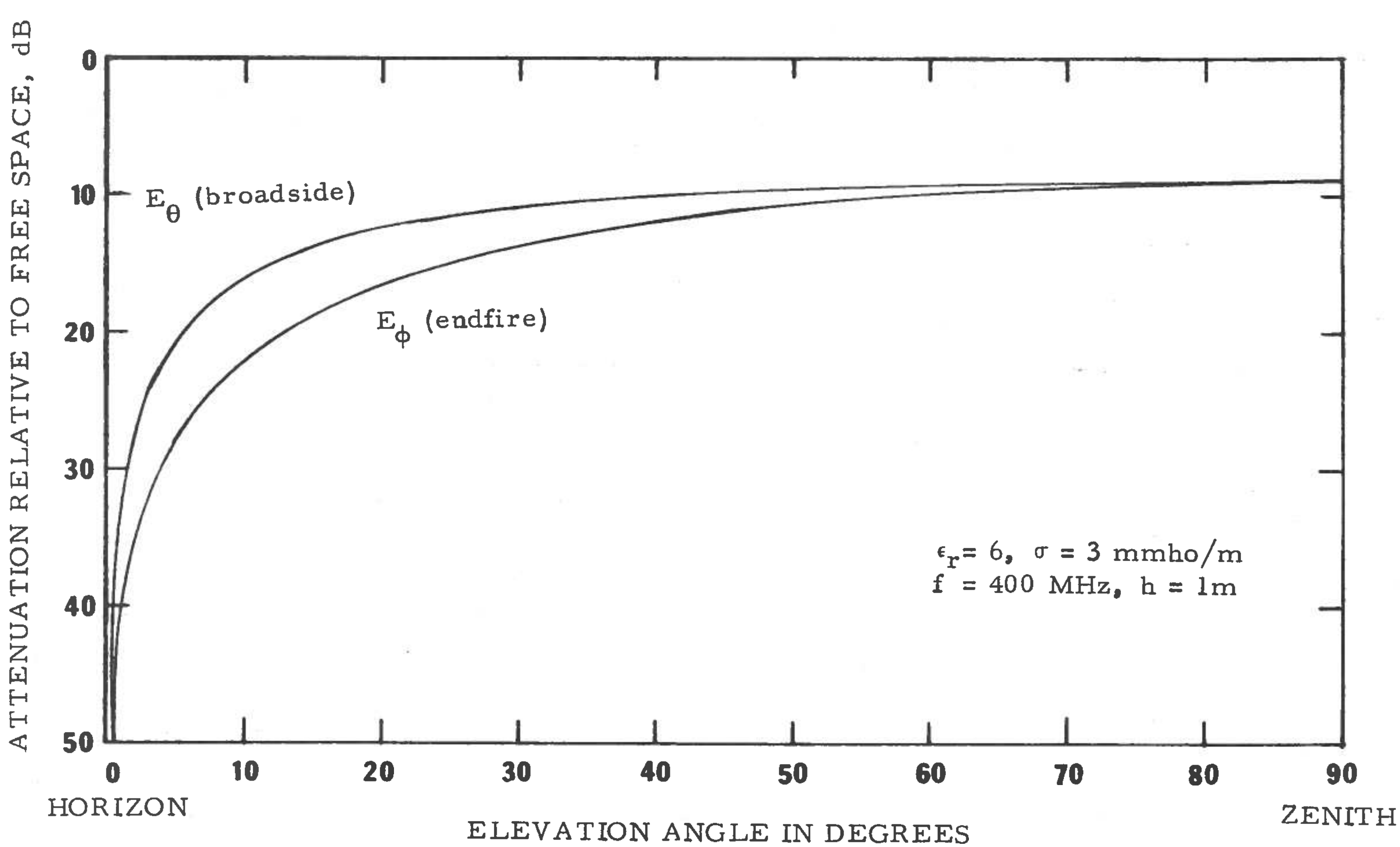

Figure 21. Radiation gain of a buried HMD.

7.5 The Horizontal Bare Dipole

Let an arbitrary antenna be represented by the current density $\overline{J}$. If we set

$$\overline{P}_1(\theta, \phi) = \int \overline{J}(\overline{r}\,') e^{-ik_1 \overline{i}_r \cdot \overline{r}\,'} dv(\overline{r}\,'), \tag{7.25}$$

then we quickly find

$$W_r = \frac{k_o^2 Z_o \operatorname{Re} n}{16\pi^2} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \, ||\overline{P}_1 \times \overline{i}_r||^2. \tag{7.26}$$

If the dipole of section 4.1 is bare (so that $k_a = k_1 = k_o n$), then

$$\overline{P} = \frac{2I_o}{k_o n} \frac{\cos(k_o \ell n \overline{i}_a \cdot \overline{i}_{r1}) - \cos(k_o \ell n)}{1 - (\overline{i}_a \cdot \overline{i}_{r1})^2} \overline{i}_a, \tag{7.27}$$

and

$$||\overline{P}_1 \times \overline{i}_r||^2 = \left(\frac{2I_o}{k_o |n|}\right)^2 \frac{|\cos(k_o \ell n \cos\theta) - \cos(k_o \ell n)|^2}{\sin^2\theta}. \tag{7.28}$$

Clearly, the fields excited by such a dipole will be the same as those excited by a similarly oriented doublet, except that there will be an additional scalar factor (essentially the same scalar factor which appears in (7.27)), which must be taken into account.

As suggested in section 4.1, if the dipole is vertically oriented, the vector $\overline{P}$ is almost a constant, and the corresponding pattern of radiation will therefore be very nearly that of the VED. Only the efficiencies can differ.

As a matter of fact, the vector $\overline{P}$ will be nearly constant in almost any case. This is because the vector $\overline{i}_{r1}$, itself, does not change very radically, even though the point of observation varies over the entire upper hemisphere.

As an illustration, let the dipole be horizontal with $\bar{i}_a = \bar{i}_y$. Then we find

$$\bar{i}_a \cdot \bar{i}_{r1} = \sin\theta_1 \sin\phi = \sin\theta \sin\phi/n. \qquad (7.29)$$

Inserting this into (7.27) and thence into (7.6) we may compute the resulting patterns and gains. Such computations are presented in figures 22, 23, and 24. For the length of the dipole, we elected to set $2\ell = \lambda_1/2$, where $\lambda_1 = \lambda_o/\mathrm{Re}\, n$ is the wave length in ground. Note that the fields of such a dipole are almost indistinguishable from those of a simple HED.

The integral (7.26) is necessary only in the computations for the radiation gain in figure 24. It was computed by numerical quadratures.

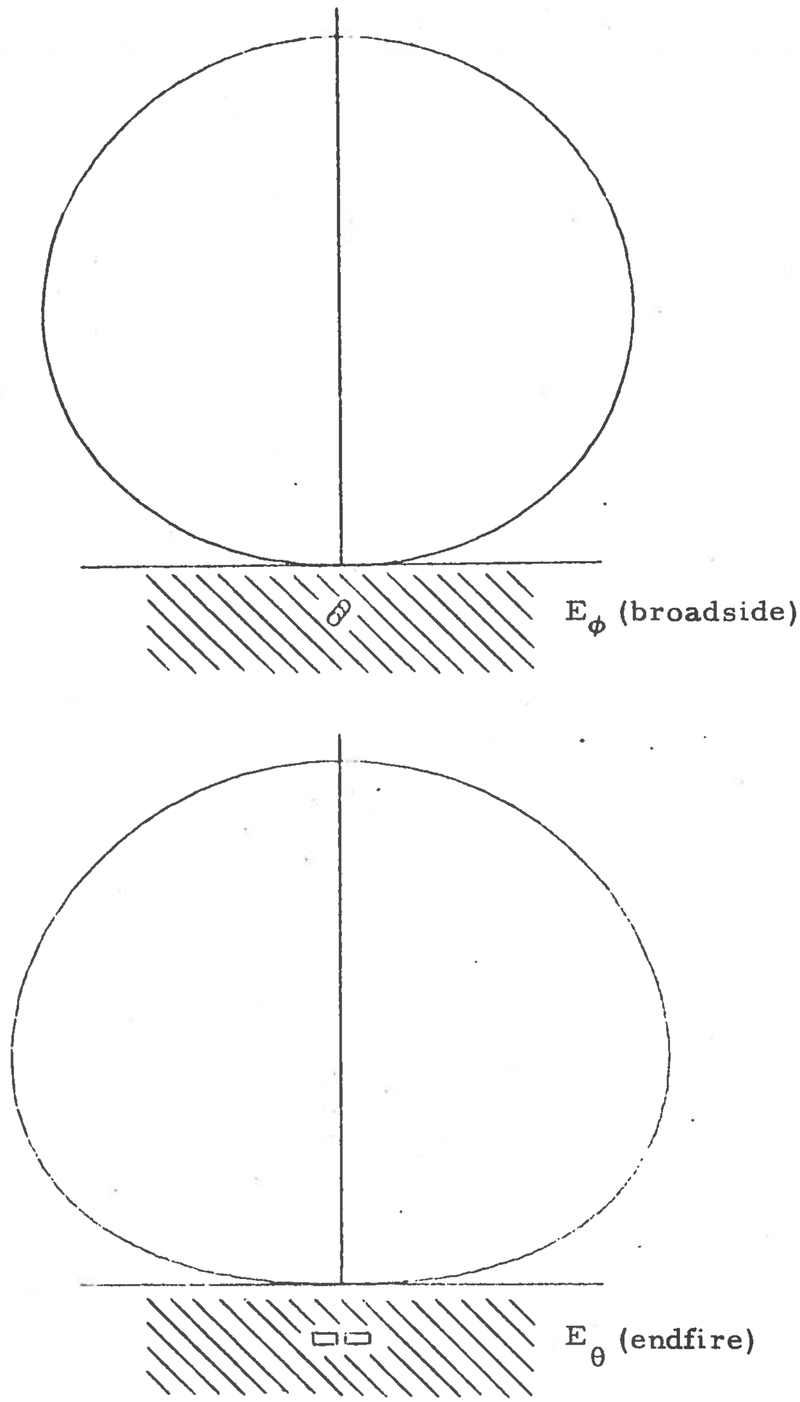

Figure 22. Vertical patterns of a buried horizontal bare dipole. Length, $\lambda_1/2 = 15.3$ cm, $\epsilon_r = 6$, $\sigma = 3$ mmho/m, $f = 400$ MHz, $h = 1$ m.

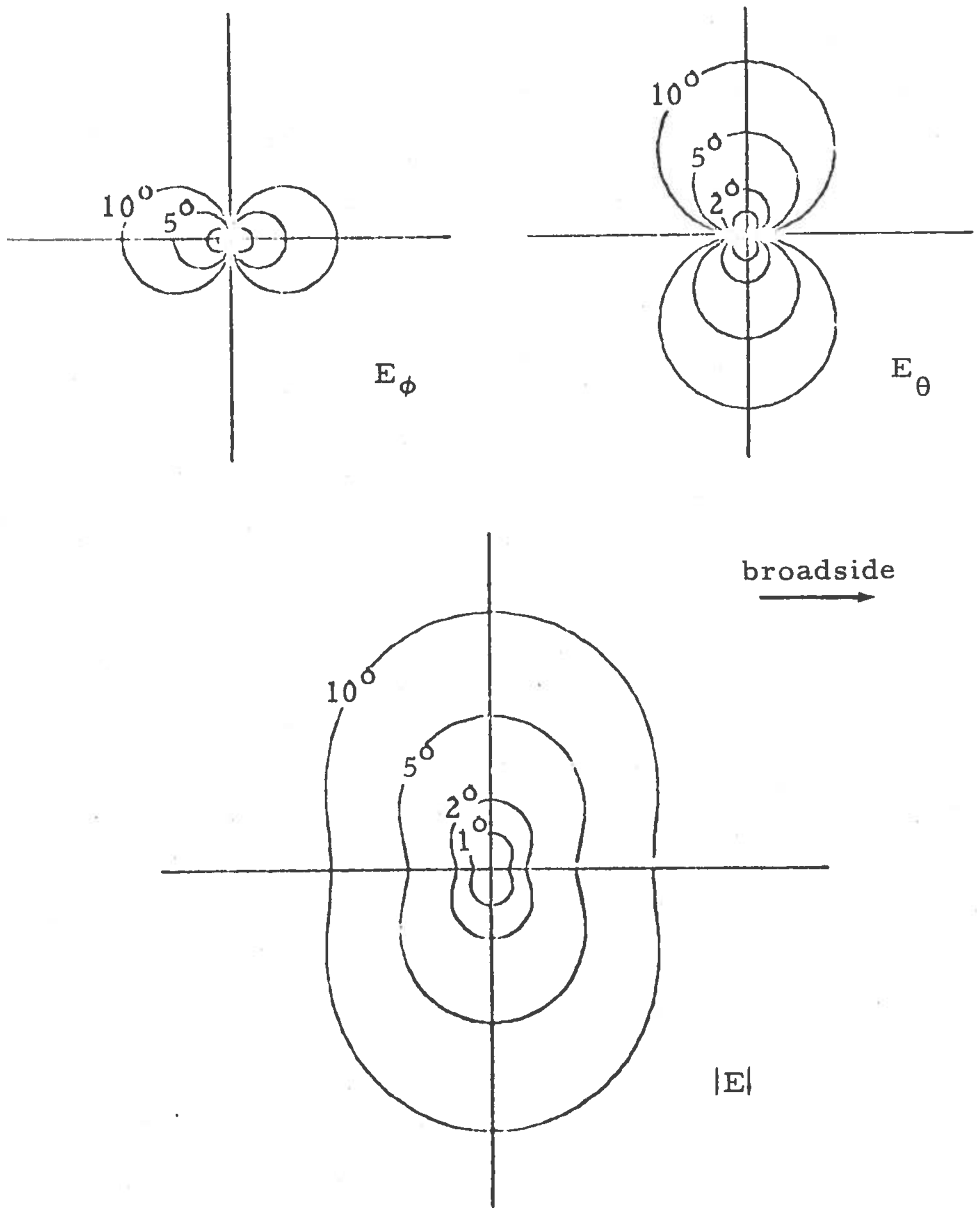

Figure 23. Azimuthal patterns of a buried horizontal bare dipole. Length, $\lambda_1/2 = 15.3$ cm, $\epsilon_r = 6$, $\sigma = 3$ mmho/m, f = 400 MHz, h = 1 m.
The patterns are drawn at various elevation angles

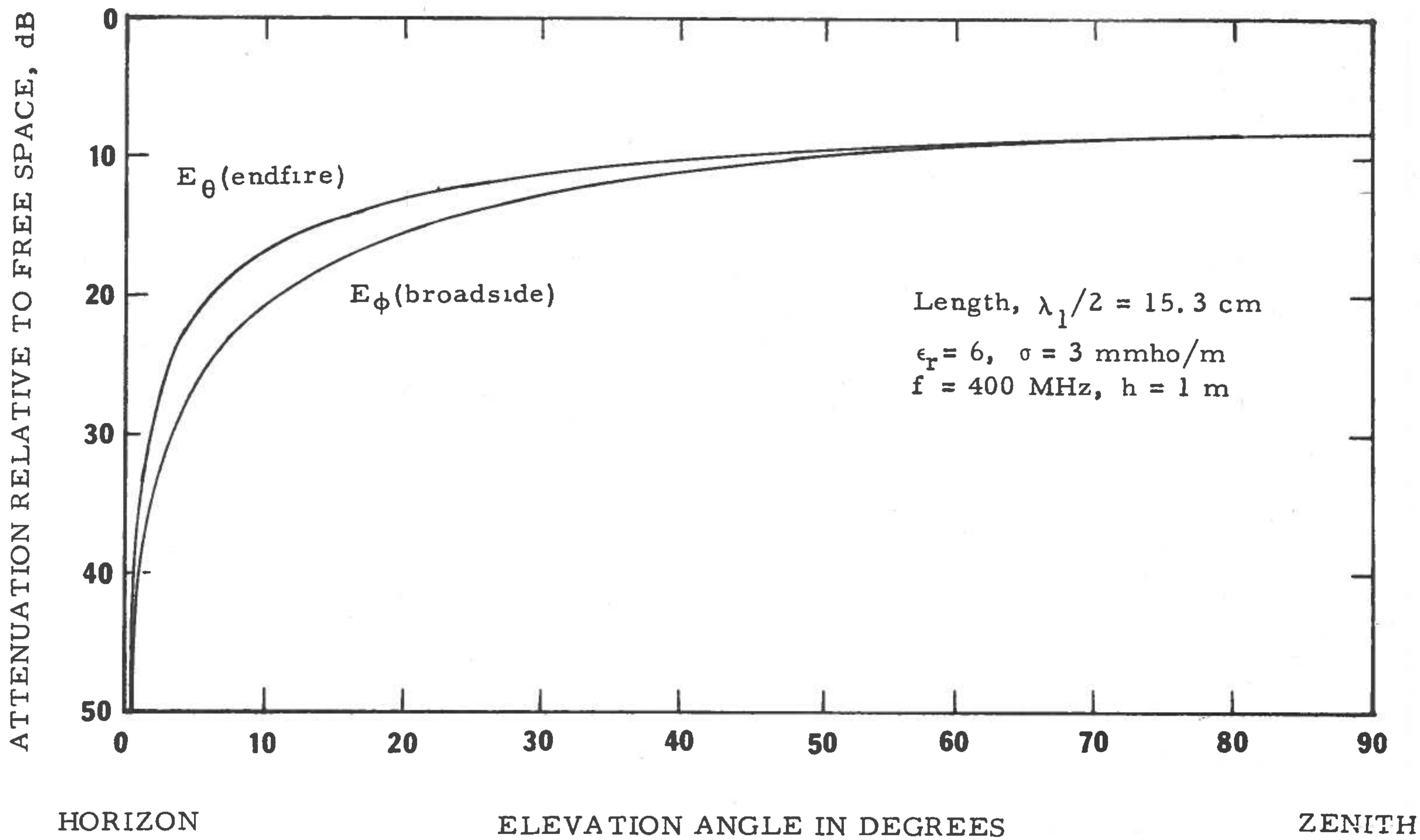

Figure 24. Radiation gain of a buried horizontal bare dipole.

7.6 The Horizontal Sheathed Dipole

As suggested in section 5, putting a sheath around the antenna may considerably alter its properties. If we suppose that the sheath has permeability μ_o and (complex) permittivity ϵ_2, then the η of section 5 is equal to ϵ_1/ϵ_2. Dividing (5.2) by k_o^2, we may define an effective refractive index n_a for the antenna,

$$n_a^2 = n^2 + \left(\frac{2}{Ck_o b}\right)^2 \left(\frac{b}{a}\right)^2 \epsilon_1/\epsilon_2 \, \zeta, \qquad (7.30)$$

where ζ is the solution to (5.3). A graph of n_a^2 vs. b/a is given in figure 25. For this illustration, we have supposed that the sheath is composed of lucite, and that the dipole is a rather fat one with $a = 16$ mm.

The values in figure 25 seem implausible. The imaginary part, in particular, is so large that the resulting current will be excessively attenuated. Nevertheless, these values are associated with modes that seem to us to be the least implausible of all those arising from solutions to (5.3). Still, we have no guarantee that this mode is really the one excited by the feed.

Under the assumption that our procedure has provided the proper antenna mode, we have plotted the radiation gain of a horizontal sheathed dipole for one of the more extreme cases in figure 26. Note that the antenna pattern is still almost indistinguishable from that of an HED. This is because, as in the case of the bare dipole, the vector $\bar{i}_{r1}$ remains relatively constant.

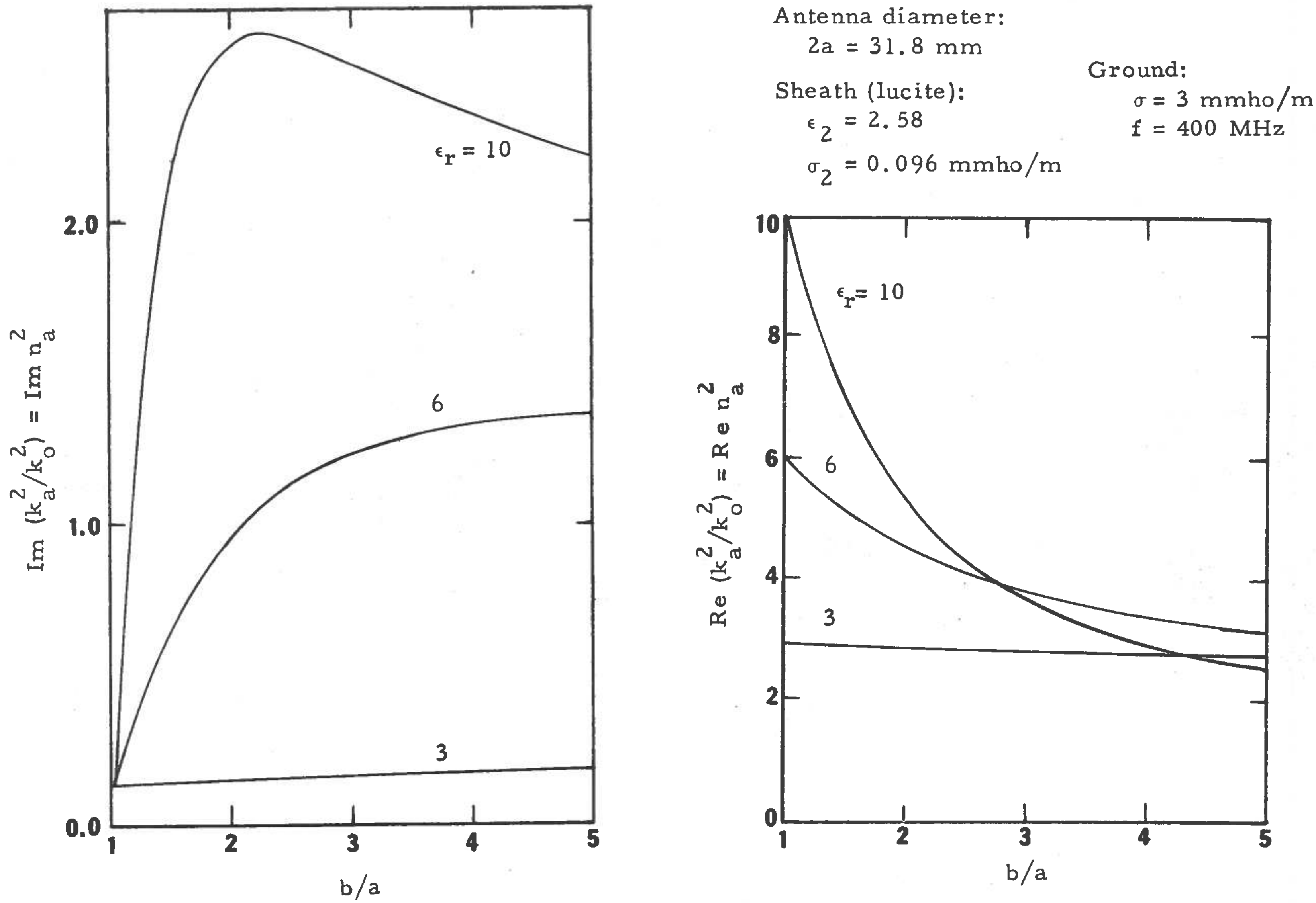

Figure 25. Propagation constant of a sheathed line.

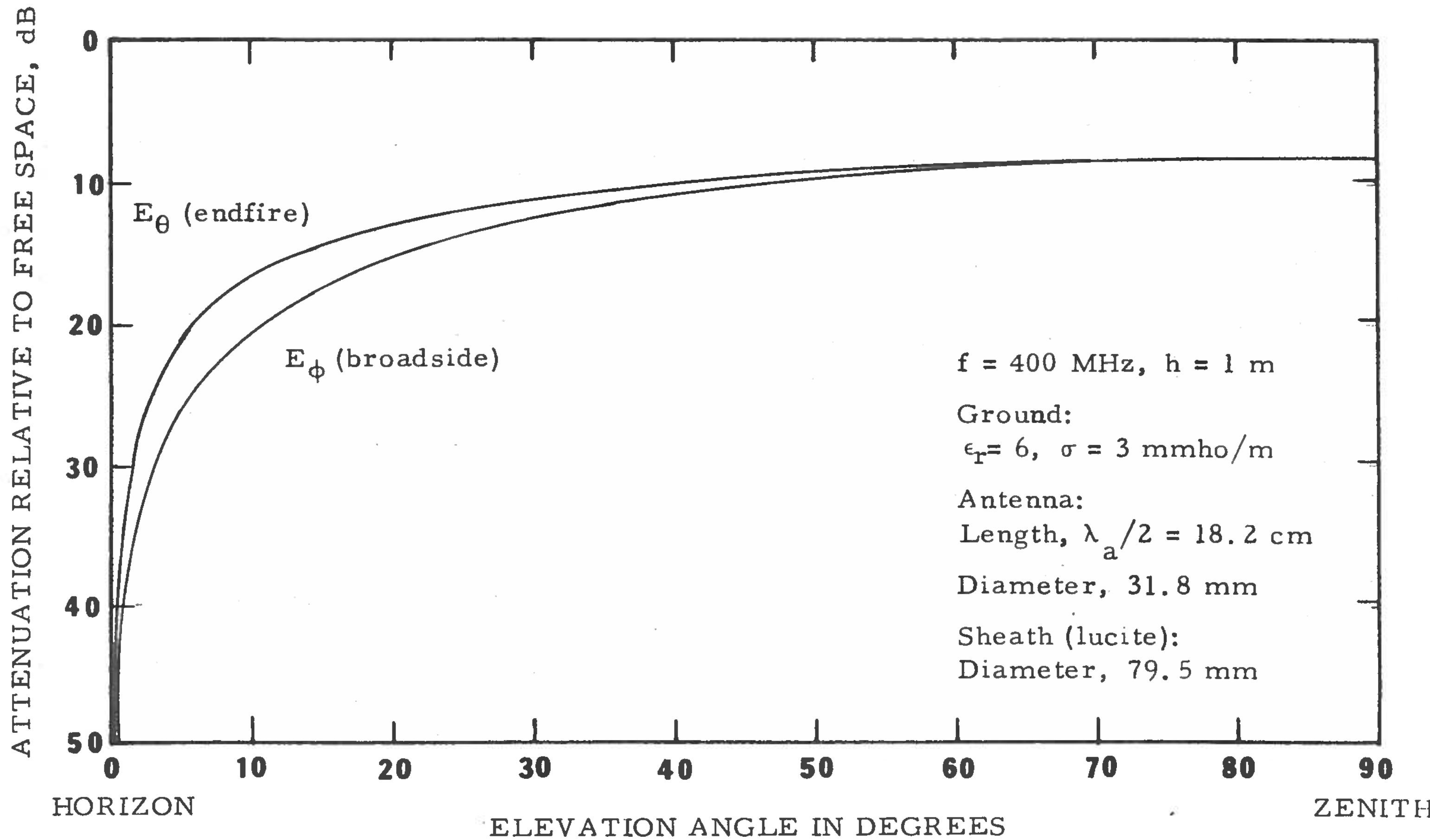

Figure 26. Radiation gain of a buried horizontal sheathed dipole.

7.7 The Annular Slot

When the antenna is represented by a magnetic current density $\overline{M}$, we set

$$\overline{Q}_1 = \int \overline{M}(\overline{r}\,') e^{-ik_1 \overline{i}_r \cdot \overline{r}\,'} dv(\overline{r}\,'), \tag{7.31}$$

whence

$$W_r = \frac{k_o^2}{16\pi^2 Z_o} |n|^2 \text{Re}\, n \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \, ||\overline{Q}_1 \times \overline{i}_r||^2. \tag{7.32}$$

For the annular slot of section 4.2 we find

$$||\overline{Q}_1 \times \overline{i}_r||^2 = (2\pi V a)^2 \, |J_1(k_o an \sin\theta)|^2, \tag{7.33}$$

and

$$A_e' = 0, \tag{7.34}$$

$$A_m' = k_o \left(\frac{\text{Re}\, n}{4\pi Z_o W_r}\right)^{1/2} n\, 2\pi V a\, J_1(k_o a \sin\theta),$$

where W_r can be found from (7.32) and (7.33) by numerical quadratures.

The fields, of course, vary with the radius a. This variation, however, is not immediately obvious, since W_r is also a function of a. In figure 27 we have plotted the horizontal gain $A_m'(\pi/2)$ as a function of the radius. The maximum of this function corresponds to an optimal value of the radius when one is interested only in low angle fields. In figure 28 we have plotted the first maximum as a function of ϵ_r.

Finally, figures 29 and 30 give vertical patterns of annular slots of selected radii. Two of these radii correspond to maxima of the graph in figure 27.

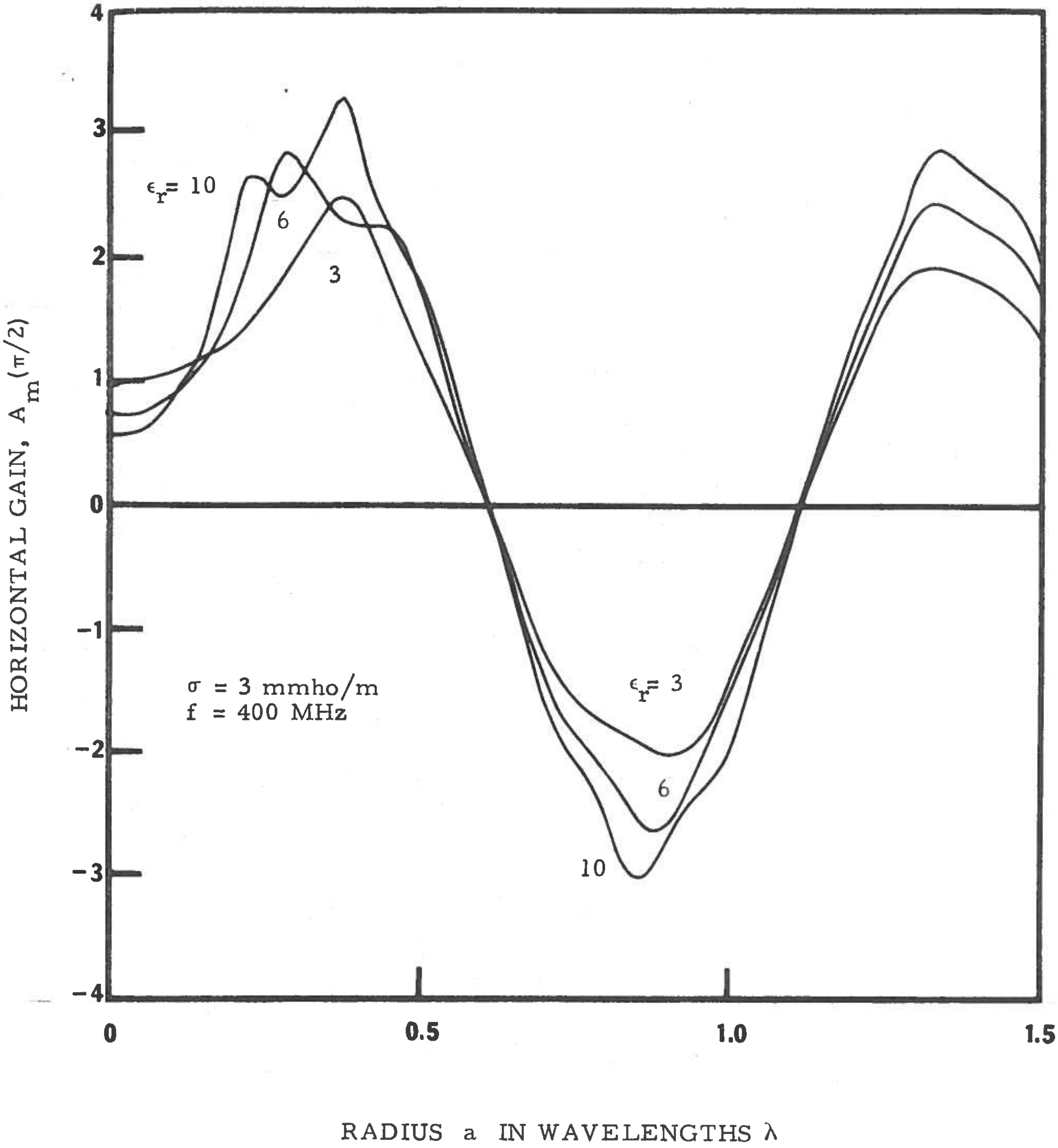

Figure 27. Horizontal gain of buried annular slots.

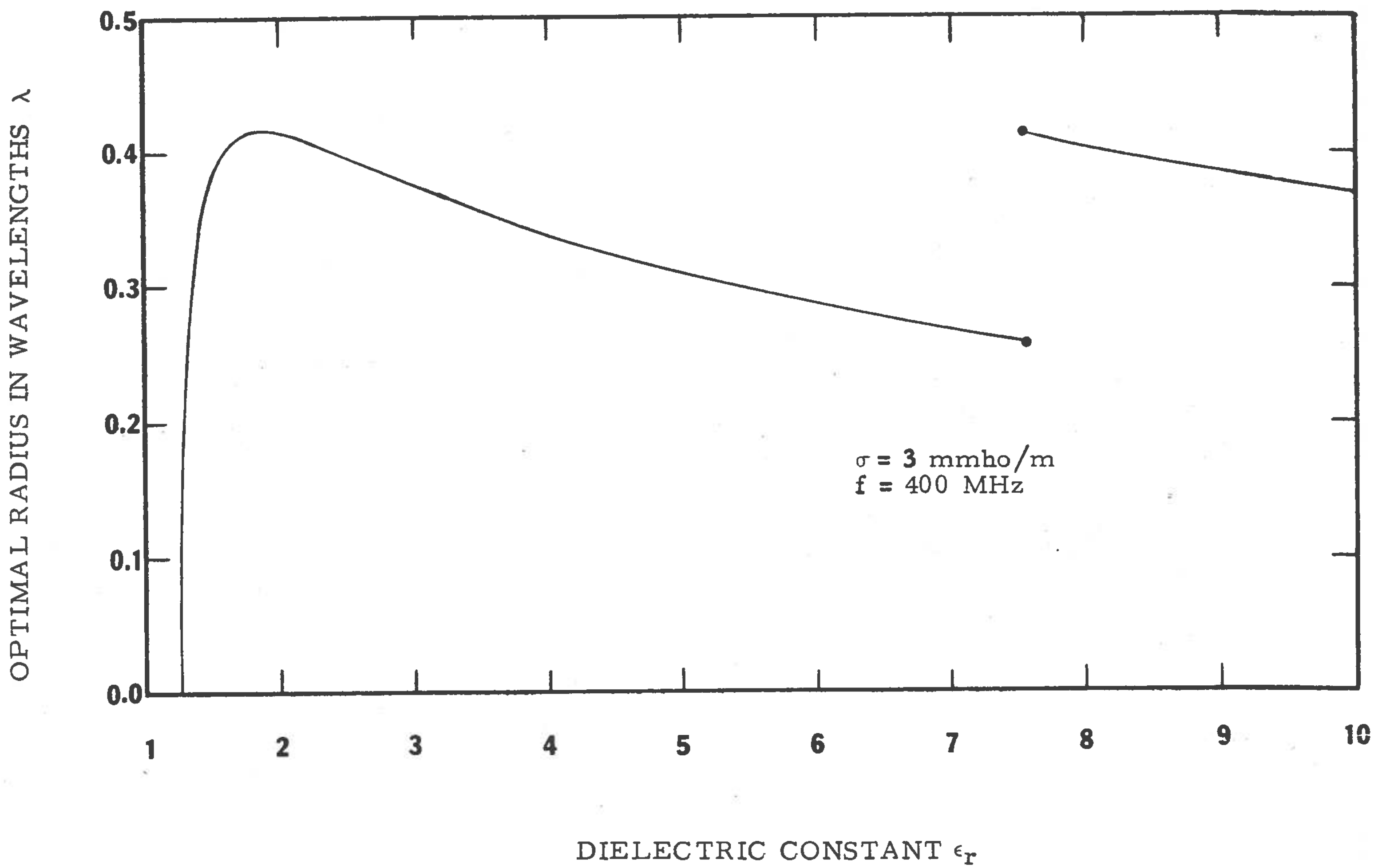

Figure 28. Optimal radius of a buried annular slot.

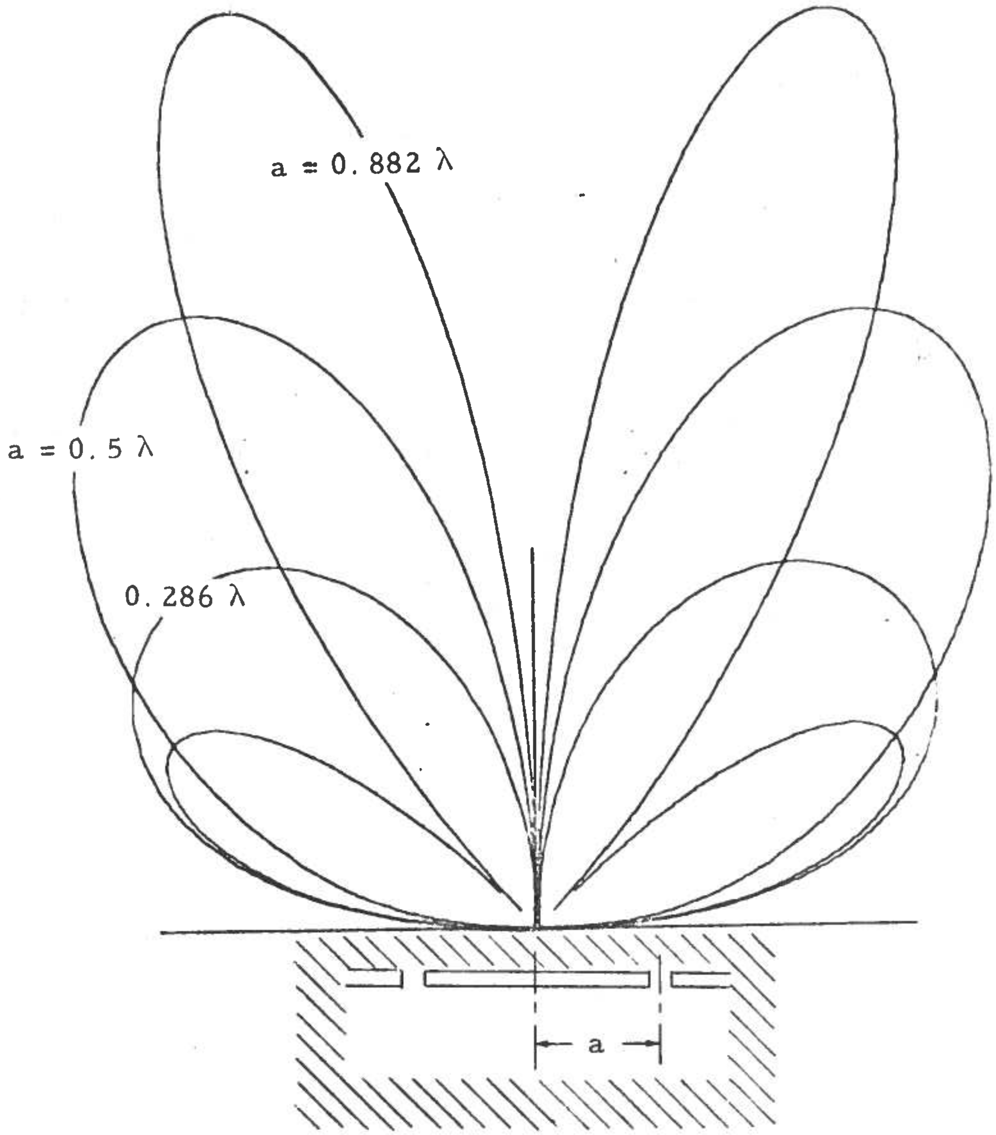

Figure 29. Vertical patterns of buried annular slots; $\epsilon_r = 6$, $\sigma = 3$ mmho/m, f = 400 MHz, h = 1 m

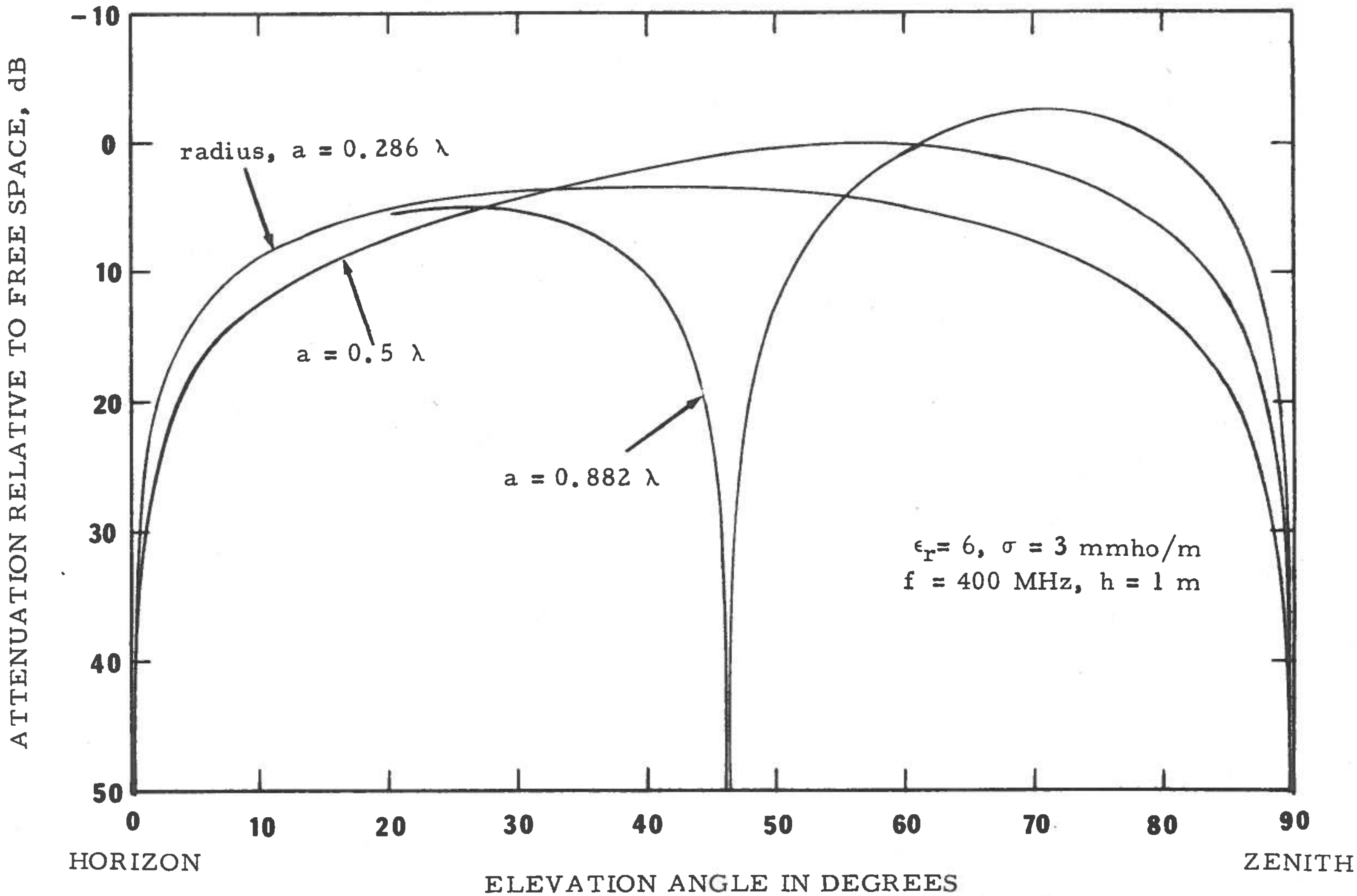

Figure 30. Radiation gain of buried annular slots.

8. CONCLUSIONS

Although burying an antenna will reduce considerably its radiated field, the numbers cited in this paper show that it will not render communication impossible. The many graphs presented here may be of some use in the design of UHF radio systems which must include such buried antennas.

In using these graphs, however, caution should be exercised. On the theoretical side, for example, the curves of radiation gain have made no allowance for induction field heat losses, nor have they included a possible interaction between the interface and the radiation resistance. On the practical side, it should be pointed out that our theory has assumed a homogeneous ground and a plane interface. If the ground is alluvium containing an occasional large boulder, or if it has been made deliberately nonhomogeneous by encasing the antenna in a special material, then the radiation patterns may be altered considerably. And if there exist above ground objects which can effectively scatter the field, then one can expect a marked interference pattern.

As for the actual design of the antenna, we have, for example, reconfirmed the fact that it is the TM (or vertically polarized) component of the field that should, for best efficiency, be excited. Somewhat surprisingly, the vertical doublet is not an effective radiator for this purpose. For comparison, we have plotted in figure 31 the horizontal gain of various antennas vs. the dielectric constant of the ground. (In the case of those antennas with a directivity in the azimuth, it is the maximum horizontal gain that is shown.) One sees immediately that the annular slot is by far the best of these antennas. If a simpler structure is desired, one might choose the horizontal dipole, or perhaps a vertically oriented loop antenna. On the other hand, more complicated antennas which produce higher horizontal gains, are, of course, possible. Arrays of slots or stacked arrays of dipoles come to mind first.

We have also suggested that the reduction of induction field heat losses may be an important consideration. To accomplish this, current densities should be kept small and sharp corners should be avoided. In this regard, dielectric sheathing seems desirable. Unfortunately, what effect such a sheathing will have on the current distribution of the antenna, is not entirely resolved.

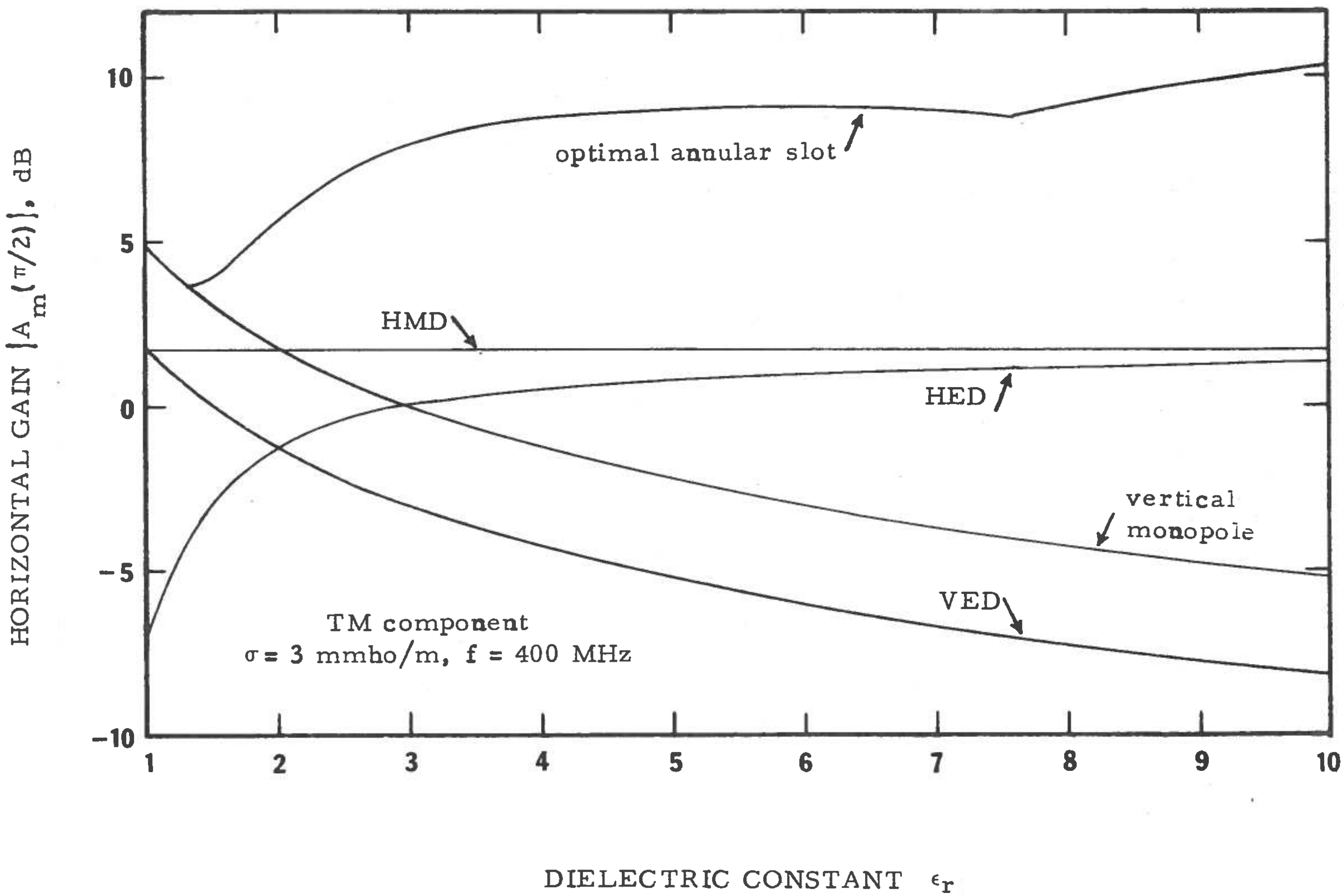

Figure 31. Horizontal gain of various antennas

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