A MODEL FOR THE COMPLEX PERMITTIVITY OF ICE AT FREQUENCIES BELOW 1 THz

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Using empirical fits to published data, formulas are suggested for the complex permittivity of ice. The radio frequency may range from 0 to 1000 GHz, and the temperature from 0° to −40° C. It then becomes apparent where additional data would be welcome.

Keywords: complex permittivity; ice; millimeter waves; radiowave propagation; semi-empirical models

1. INTRODUCTION

The interaction of radio waves with naturally occuring ice is of importance for several aspects of remote sensing. It may also be of some importance for communication links, particularly when falling snow or ice clouds are involved. An analysis of these effects depends first of all on the electromagnetic properties of ice and it is these properties that we want to study here. Using data available in the literature, we hope to develop a model that describes the complex permittivity as a function of radio frequency and temperature and that will prove useful for subsequent studies. Our goal is to cover the frequencies from 0 to 1000 GHz and the temperatures from 0° to, say, −40°C, these ranges being chosen so as to imitate Liebe's MPM (the Millimeterwave Propagation Model) [1].

Surveys of current knowledge concerning the permittivity of ice over the entire electromagnetic spectrum are given in [2] and [3], and those more restricted to radio frequencies in [4]–[6]. These all affirm that the general properties of ice are well understood but that there are important gaps of detail where existing data are both sparse and inconsistent.

In the general picture there is a dipole relaxation, which is apparent at low frequencies, and a set of absorption lines in the infrared spectrum. In between there are no new phenomena. The striking fact about the dipole process is that the relaxation frequency is very low: its maximum value (at 0°C) is 7.3 kHz. And

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on the other hand, the lowest absorption line is at about 4.8 THz. Thus it would seem that between 1 MHz and 1 THz, ice is almost a perfect dielectric. It exhibits absorption properties only to the extent that the processes on either side have far wings that are still active.

The Debye formula for the dipole relaxation process describes the complex permittivity as

$$\epsilon(f) = \epsilon' + i\epsilon'' = \frac{\epsilon_0 - \epsilon_1}{1 - if/g_D} + \epsilon_1 \tag{1}$$

where f is the radio frequency and the parameters are real and positive. The parameter ϵ_0 is the static dielectric constant, ϵ_1 the asymptotic dielectric constant, and g_D the relaxation frequency. In (1) we have also displayed the usual convention that the permittivity ϵ may be separated into its real part ϵ' (often called the *dielectric constant*) and its imaginary part ϵ'' (the *loss factor*). For large f (*i.e.*, for $f \gg g_D$) we have the approximation

$$\epsilon(f) = \epsilon_1 + i(\epsilon_0 - \epsilon_1)g_D/f \tag{2}$$

so that the real part is nearly constant and the imaginary part is small and inversely proportional to frequency.

On the other side of the spectrum, the Lorentz formula to describe how an absorption line contributes to the permittivity may be written as

$$\epsilon_a(f) = \frac{m_r f}{f_r - ig_r - f} \tag{3}$$

where f_r is the resonant frequency, g_r the halfwidth of the line, and m_r determines its strength. When f is small (*i.e.*, when $f \ll f_r$) we may ignore its contribution to the denominator so that (3) becomes a simple linear function in f. If there are many such absorption lines their contributions will all add together, but in the low frequency approximation the result will still be linear.

These remarks are not new (see, *e.g.*, [7]). There seem, however, to be enough new data and new theory to provide new and, we hope, more accurate formulas.

2. THE MODEL

From the foregoing comments there results the proposed model

$$\begin{aligned} \epsilon' &= \epsilon_1 \\ \epsilon'' &= \alpha(t)/f + \beta(t)f \end{aligned} \tag{4}$$

where the coefficients α and β are to be functions of the temperature t (°C). Of course, as with ϵ'', there should be extra terms in the formula for ϵ'. But we assume these are all small with respect to ϵ_1 and may be ignored. We would then expect (4) to be valid from 1 MHz to 1 THz. If it is necessary to continue the model to lower frequencies one need only replace (2) by (1). If higher frequencies are required, the separate infrared absorption lines must be included.

Of the many credible studies at the low frequency end, the one by Wörz and Cole [8] seems to be the most recent and the most reliable. It covers a set of careful

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measurements which seem to fit a fairly satisfactory theory. From these results we can find formulas for the parameters in (1) and hence for the coefficient α.

First, as in the MPM, we introduce a "relative inverse temperature" parameter

$$\theta = \frac{300}{273.15 + t} - 1$$

and then

$$\begin{aligned} g_D &= 64.1 e^{-22.1\,\theta} \qquad \text{kHz} \\ \epsilon_0 &= 81.8 + 96\,\theta \\ \epsilon_1 &= 3.15 \end{aligned} \tag{6}$$

whence

$$\alpha = (50.4 + 62\,\theta) \times 10^{-4}\, e^{-22.1\,\theta} \qquad \text{GHz}. \tag{7}$$

The equation here for ϵ_0 is actually a quite accurate interpolation of a slightly more complicated one by Wörz and Cole, and the final digit in ϵ_1 comes from Cumming [9]. We should also point out that Wörz and Cole express doubts about the formula for g_D at temperatures below $-45°$, and that there have been indications that ϵ_1 does vary with temperature. If it does, the variation is only in the second decimal place and we have ignored it here.

The high frequency end of (4) is more difficult and subject to more ambiguity. One imagines that one needs only define β so that (4) fits some set of data showing loss factor versus temperature at a single frequency near 10 GHz. Now, there are three sets [9]–[11] of such data that are usually quoted in the survey papers and one newer set [12] of recently measured data. Unfortunately, these sets are quite inconsistent. They almost seem to describe different phenomena, although they all appear to contain careful measurements using good equipment.

There is, in Mishima *et al.* [13], another set of good-looking data. Using infrared spectroscopy, the measurements cover four temperatures and a frequency range from 240 GHz to 750 GHz. Unhappily, the temperatures are all very low, the highest being at $-71°$. But the data are accompanied with a theory that seems to fit well and that might be used to extrapolate in both temperature and frequency. According to the theory we may write

$$\epsilon'' = 6.14 \times 10^{-5} F(1.118\,(\theta + 1))\, f \tag{8}$$

where f is in GHz, θ is the parameter in (5), and

$$F(x) = x \frac{e^x}{(e^x - 1)^2}. \tag{9}$$

The extrapolation, however, seems only to contribute another inconsistent result. Now, one aspect all the microwave sets do share is that they show an accelerated increase in the loss factor as the temperature approaches 0°. This does not happen with (8), and in fact Mishima *et al.* have written that their formula could suffer at higher temperatures—they point in particular to thermal expansion as causing difficulties.

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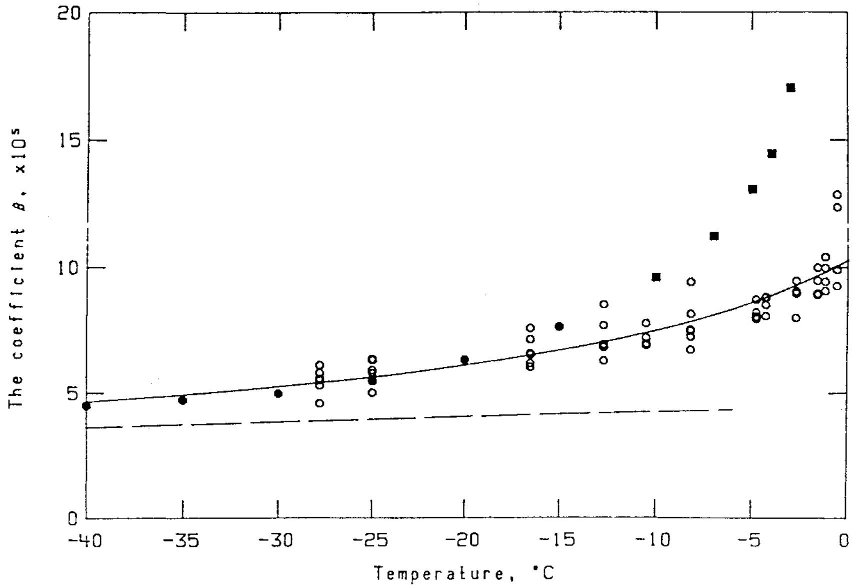

Figure 1. The coefficient β as a function of temperature. Open circles represent the measurements of Wegmüller [12], closed circles and closed squares the measurements of Lamb [10]. The solid line is a least squares fit to both the open and closed circles. The dashed line is the extrapolated formula of Mishima et al.

On the other hand, there are two sets of data that do seem to fit in with the reults of Mishima *et al.*, especially at the lower temperatures. These are the new data of Wegmüller [12] and the older data of Lamb [10]. Our notion, therefore, is to define the coefficient β as given by (8) and then to add to it a correction so as to fit these two sets.

The result we have derived is

$$\beta(t) = (0.445 + 0.00211t) \times 10^{-4} + 0.585 \times 10^{-4}/(1 - t/29.1)^2 \quad \text{GHz}^{-1}. \qquad (10)$$

The two terms linear in temperature are an accurate approximation to (8) and the last term is the correction which provides the higher losses observed at higher temperatures. In terms of the parameter θ this may be rewritten

$$\beta = \left(\frac{0.502 - 0.131\theta}{1+\theta}\right) \times 10^{-4} + 0.542 \times 10^{-6} \left(\frac{1+\theta}{\theta + 0.0073}\right)^2 \quad \text{GHz}^{-1}. \qquad (11)$$

Our proposed model, then, is given by (4) together with (7) and (10) or (11). In Figure 1 we show how this model fits the data of Wegmüller and of Lamb. The plotted points come from measured loss factors by solving (4) for the coefficient β. The values in (10) are derived from a least squares fit to the Wegmüller data in Figure 1 and to the lower temperature Lamb data.

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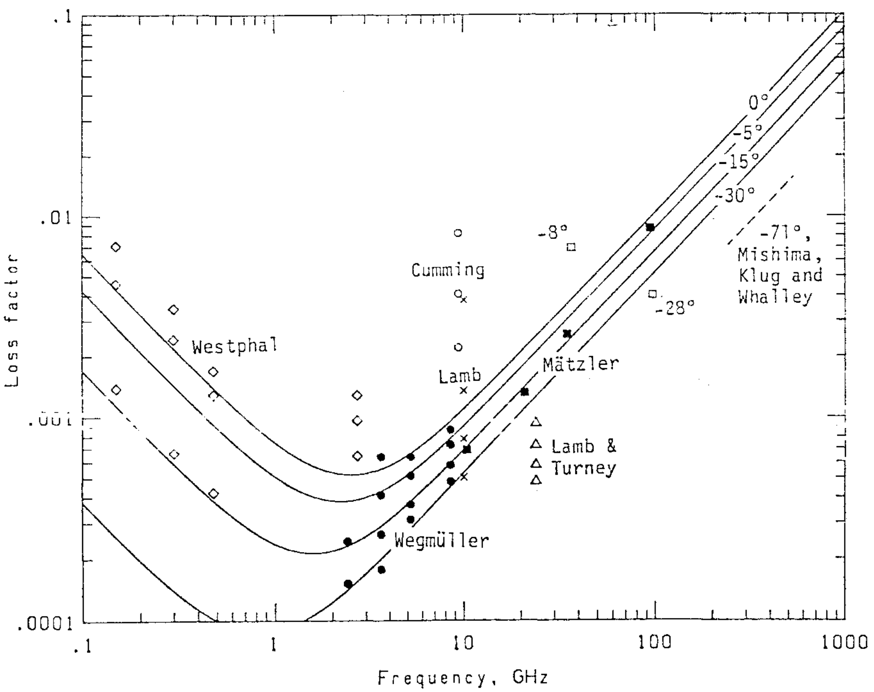

Figure 2. The loss factor ϵ'' of ice as a function of frequency. The solid curves display the proposed model. Measured data: open diamonds, Westphal [3] at −1, −5, and −30°; closed circles, Wegmüller [12]; crosses, Lamb [10]; open triangles, Lamb and Turney [11], all at 0, −5, −15, and −30°; open circles, Cumming [9] at 0, −5, −10°; closed squares, Mätzler [12]; dashed curve, Mishima et al. [13] at −71°; and open squares are isolated measurements [14], [15] at the indicated temperatures.

In Figure 2 we have plotted model predictions against frequency and have included most of the available data that are pertinent. In addition to the microwave data and the data from Mishima *et al.*, there are the UHF data of Westphal (using annealed ice in Antarctica; see [3]), the millimeter-wave data from Mätzler [12], and isolated measurements of Perry and Straiton [14] at −28°, and Glushnev *et al.* [15] at −8°.

3. CONCLUDING REMARKS

We think the proposed model is a good compromise between theory and the several sets of available data. An examination of Figure 2 quickly shows what new measurements would provide critical tests. UHF data at low temperatures and millimeter-wave data at higher temperatures would be particularly useful. Also helpful would be high temperature data in the far infrared.

The question of how impurities affect the permittivity of ice is an important question and one that is difficult to answer. In [12] there is a good discussion of

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saline impurities. But older data [5] are inconsistent and it seems difficult to draw any general conclusions.

As one application of our results, consider the case where the atmosphere contains a low density of ice particles, as in an ice cloud or a snowfall. If conditions allow the use of an equivalent permittivity we would have

$$\epsilon_s = 1 + 3v_s \frac{\epsilon - 1}{\epsilon + 2} \tag{12}$$

where ϵ_s is the permittivity of the air/ice mixture, ϵ is that of ice as given above, and v_s is the specific volume of the ice. When v_s is small this formula is a good approximation to almost any of the standard approaches [6]. If ϵ'' is much smaller than ϵ' (as it is above) then we find

$$\epsilon_s'' = \frac{9}{(\epsilon' + 2)^2} v_s \epsilon'' = 0.339 v_s \epsilon''. \tag{13}$$

Since v_s will be on the order of 10^{-6} this value might be on the order of 10^{-9}. At 30 GHz this would correspond to a specific attenuation of about 0.003 dB/km.

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