An Improved-Accuracy Discrete Sampling Criterion for the Estimation of the Local Mean Voltage of a Mobile Radio Channel

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***Abstract*—This paper presents new recommendations for the discrete sampling of a mobile radio with non-line-of-sight Rayleigh fading and a Jakes power spectrum. The results are derived from a new analytical result for the variance of the local-mean voltage using discrete, spatial sampling with uniform spacing. The variance presented here accounts for the correlation between the samples and significantly changes the number of samples that are required to estimate the local mean voltage with a 1 dB spreading factor.**

Keywords—correlation; discrete sampling; Jakes channel; local mean; mobile channel; Rayleigh; spreading factor; variance

I. INTRODUCTION

Mobile channel measurements can be carried out using a continuous-wave (CW) channel sounding system [1]. This system transmits a CW signal from a fixed location to a receiver located in a van moving at a constant speed shown in Fig. 1. The van is driven over prescribed routes and records the received signal. The signal is received by a vector signal analyzer (VSA), which down-converts the signal into a baseband IQ data stream. Data can be collected on either a continuous-time basis or sampled at uniformly spaced, discrete intervals.

The local mean voltage is obtained by averaging the IQ voltage envelope over the spatial interval $2L$, shown in Fig.1 (a linear receiver is assumed). We present analytical results for the mean and variance of the local mean voltage for both continuous and discrete sampling [2]. The analytical result for the discrete variance yields the recommendation in Section III for both the required number of samples and spacing to achieve a spreading factor of 1 dB (±1/2 dB). This result differs significantly from a procedure given in Parsons [3]. The differences are attributed to the correlations between samples that are not accounted for in Parsons' analysis.

II. CONTINUOUS AND DISCRETE WINDOW AVERAGING

The received complex baseband signal is given by

$$s(t) = r(t)e^{i\varphi(t)}, \tag{1}$$

where $r(t)$ and $\varphi(t)$ are the envelope and phase of the signal. An estimate of the local mean $\widehat{m}$ at a location x can be obtained by averaging the IQ envelope $r(y)$ over a distance $2L$ from $x\text{-}L$ to $x\text{+}L$

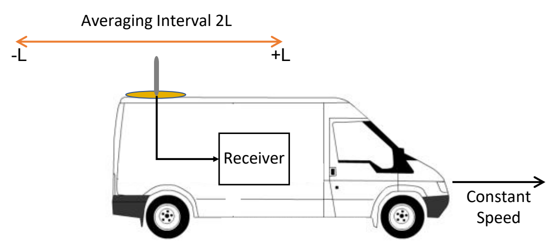

Figure 1. A van collecting mobile channel data from a CW transmitter. The averaging interval is centered about the receiving antenna.

$$\widehat{m}(x) = \frac{1}{2L}\int_{x-L}^{x+L} r(y)dy. \tag{2}$$

For a non-line-of sight (NLOS) channel with a uniform distribution of scatterers around a receiving with a Jakes power spectrum [4], the resulting probability density function (PDF) of the received voltage envelope $r(t)$ is a Rayleigh distributed random variable given by

$$p(r) = \frac{r}{b^2}\exp\left(-\frac{r^2}{2b^2}\right), \tag{3}$$

where $r \geq 0$ and b is the Rayleigh scaling parameter.

The estimated local mean for discrete sampling is

$$\widehat{m} = \frac{1}{N}\sum_{i=1}^{N} r_i\ , \tag{5}$$

where $r_i \equiv r(x_i)$ are samples of the signal envelope.

From [2], The variance of the continuous average (2) is

$$\sigma_{\widehat{m}}^2 = \frac{2b^2}{\frac{L}{\lambda}}\left(1-\frac{\pi}{4}\right)\int_0^{\frac{2L}{\lambda}}\left(1-\frac{x}{\frac{2L}{\lambda}}\right)J_0^2(2\pi x)dx \tag{7}$$

where J_o is a zeroth order Bessel function of the first kind, b is the scaling factor, L is the averaging interval, and λ is the wavelength of the CW signal. Eq. (7) corrects the variance

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originally reported by Lee [5]. For discrete sampling with N uniformly spaced samples, the variance is [2]

$$\sigma_{\hat{m}}^2 = \frac{\sigma_r^2}{N}\left[1 + \frac{2}{N}\sum_{n=1}^{N-1}(N-n)J_0^2\left(\frac{2\pi nd}{\lambda}\right)\right]$$
$$= \sigma_{uncorr}^2 + \sigma_{corr}^2\,, \quad (8)$$

where σ_r^2 is the variance of the Rayleigh PDF (3), and d is distance between samples. Eq. (8) is analogous to the continuous-average result (7). The first term on the right-hand side of (8) is the usual expression for the variance σ_{uncorr}^2 of the mean obtained by averaging N uncorrelated samples. The sum of Bessel functions σ_{corr}^2 accounts for correlations among the samples.

III. Recommendations for Discrete Channel Sampling

In order to quantify variations (in dB) about a local mean at a location x, Lee [4] defines the "2 $\sigma_{\hat{m}}$ spread" (spreading factor) in decibels as

$$2\,\sigma_{\hat{m}}\,spread = 20\,log_{10}\frac{m_r + \sigma_{\hat{m}}}{m_r - \sigma_{\hat{m}}} \quad (dB), \quad (9)$$

where m_r is the mean of the Rayleigh PDF (3), and $\sigma_{\hat{m}}$ is the standard deviation derived from either (7) or (8). For the continuous case, Eq. (7) and Eq. (9) yield a spreading factor of 1 dB (±1/2 dB) for an averaging interval $2L$=60λ. This corrects the recommendation of a 40λ averaging interval originally proposed by Lee [5].

Fig. 2 shows linear plots of the standard deviations obtained from (7) for the continuous case, and discrete results obtained from (8) for the range of $0.25\lambda \le d \le 2.0\lambda$. The specified averaging interval is $2L$=60λ and b=1. The red trace is the continuous result obtained from (7) and (9). As the sample spacing decreases, the standard deviation decreases in a staircase-like fashion, with rapid transitions occurring at half wavelength intervals. This is caused by the oscillitory behavior of the Bessel functions in (8). Close agreement is seen between discrete and continous averging for $d \le 0.45\lambda$, with an absolute difference of less than 0.0002. A spreading factor of 1 dB is obtained over this range when (9) is invoked.

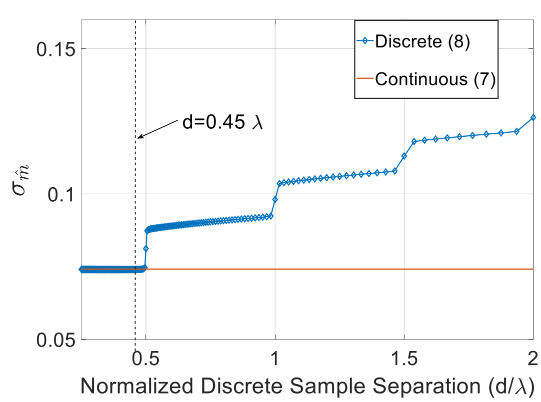

Figure 2. Linear plots of the standard deviations for discrete window averaging from (8) (blue trace) and continuous averagingfrom (7) (red trace) for an averaging interval $2L$=60λ. The Rayleigh scaling factor is b=*1*.

Parsons [3] provides a discrete channel sampling criterion for an NLOS Rayleigh channel with a Jakes power spectrum. Parsons makes two primary assumptions. First, the envelope autocovariance between two adjacent samples is given by Bessel function expression $J_0^2\left(\frac{2\pi d}{\lambda}\right)$. Second, it is assumed that all adjacent samples are uncorrelated, yielding a spacing of d=0.38λ, corresponding to the first zero of the Bessel function. The variance is then computed from the σ_{uncorr}^2 term in (8), which implies that all samples are uncorrelated.

Table 1 shows the required sampling parameters needed to achieve a spreading factor of 1 dB. using both the analysis presented here and that of Parsons. Eq. (9) is used to compute the spreading factor for both cases.

A self-consistent sampling criterion is now proposed, based on (8) and (9). Selecting an averaging interval of $2L$ = 60λ, a sampling spacing of d=0.45λ, and noting that d=2L/(N-1), the resulting number of samples is N = 135. This accounts for the correlations among the channel voltage envelope samples.

Parsons' method yields significantly different results yielding a shorter averaging interval of $2L$=31λ, a sample spacing of d=0.38λ, and N=83 samples to obtain a spreading factor of 1 dB. This significantly underestimates both the required number of samples and the averaging interval. This difference is directly attributable to the assumptions of uncorrelated samples and using the continuous autocovariance to set the sample spacing for uncorrelated samples.

Table 1. Sampling configurations to obtain a 1 dB spreading factor using our proposed method and that of Parsons [3].

Method	Averaging Interval 2L	Sample Spacing d	Number of Required Samples N
Johnk, Lemmon, Dalke	60λ	0.45λ	135
Parsons	31λ	0.38λ	83

IV. Conclusions

A new and self-consistent approach for the discrete spatial sampling of an NLOS radio channel with Rayleigh fading and a Jakes power spectrum has been presented. This approach results in more accurate discrete channel sampling parameters.

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