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U.S. DEPARTMENT OF COMMERCE
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A Wave Hop Propagation Program for an Anisotropic Ionosphere

L. A. BERRY

J. E. HERMAN

BOULDER,
COLORADO
APRIL 1971

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A Wave Hop Propagation Program for an Anisotropic Ionosphere

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INSTITUTE FOR TELECOMMUNICATION SCIENCES

BOULDER, COLORADO 80302

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Errata for OT/ITS RR 11, "A wave hop propagation
program for an anisotropic ionosphere,"
by L. A. Berry and J. E. Herman

- p. 1, line 2 of introduction should read, "Culation of LF and VLF radio propagation under an anisotropic ionosphere"
- p. 19, section 4.4 should read, "See FORTRAN listing of PROGRAM ANIHOP beginning on page 35."
- p. 28, caption should read, "Printed output from ANIHOP sample case, page 5."
- p. 29, caption should read, "Printed output from ANIHOP sample case, page 6."
- p. 31, caption should read, "Printed output from ANIHOP sample case, page 8."
- p. 32, caption should read, "Printed output from ANIHOP sample case, page 9."
- p. 37, insert as statement 30+3 (just before "NPIF..")

```
IF(TPHA(1,NT-1) - (TPHA(1,NT)-3.141592653)*CPHI(NT-1)/CPHI(NT)
1 .GT. 6.283185307 ) TPHA(1,NT-1) = TPHA(1,NT-1) - 6.283185307
```

- p. 43, change statement 950 - 1 from:
 $AJMK = (J+2-M-K) * (M-K) / (K * (K-1)) * AJMK$ to:
 $AJMK = (J+2-M-K) * (M-K) * AJMK / (K * (K-1)).$

Although mathematically equivalent, these two statements are computationally different because they contain mixed mode arithmetic.

- p. 46, change statement 40+9 from:
 $SUM = (KI * (N+1) - KKT) / (KKT * KF) * BJM(LP(L-KI, N), K, NP(N)) * E(KI+2, 1, IQ) + SUM$ to
 $SUM = (KI * (N+1) - KKT) * BJM(LP(L-KI, N), K, NP(N)) * E(KI+2, 1, IQ) / (KKT * KF) + SUM$

- p. 55, replace SUBROUTINE GUDFAZ with the one listed on next page.

SUBROUTINE GUDFAZ(K,C,FAZ,N)

C
C
C
C
C
C
C
C

SMOOTHS PHASE OF IONO REFLECTION COEFFICIENTS FOR ANIHOP
K=1,2,3,4 FOR TEE,TEM,TME,TMM, RESPECTIVELY
C=COS(PHI), ARRANGED IN ORDER OF INCREASING PHI
(=ANGLE OF INCIDENCE)
FAZ=PHASE OF REFLECTION COEFFICIENT, ARRANGED IN
SAME ORDER AS C.
N=NUMBER OF DATA POINTS

DIMENSION C(20),FAZ(4,20)

DATA (PI=3.141592653)

DO 10 I=2,N

J=N+2-I

8 TRY = FAZ(K,J-1) -FAZ(K,J)

IF(ABS(TRY) .LT. PI) GO TO 10

FAZ(K,J-1) = FAZ(K,J-1) - SIGNF(2.*PI, TRY)

GO TO 8

10 CONTINUE

RETURN

END

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FOREWORD

This work was performed for the Defense Atomic Support Agency, under the direction of Mr. Dow E. Evelyn, HQ, DASA.

A WAVE HOP PROPAGATION PROGRAM FOR AN ANISOTROPIC IONOSPHERE

Leslie A. Berry and J. E. Herman

This report documents a digital computer program for computing the propagation of LF and VLF radio waves using the wave hop theory. It supersedes NBS Report 8889. The documentation includes a description of the physical model, detailed mathematical formulas, a main function flow chart, a listing of the FORTRAN source deck, and the input and output of a sample case.

1. INTRODUCTION

This report documents a computer program for the full wave calculation of LF and VLF radio propagation under an anisotropic ionosphere using the wave hop theory. It supersedes NBS Report 8889, "FORTRAN Programs for Full-Wave Calculation of LF and VLF Radio Propagation"

(Berry and Chrisman, 1965a). The revision is necessary because the original program was based on a formulation that is incorrect for an anisotropic ionosphere (see Berry, et al. (1969) for details). The error was significant if the magnitudes of the ionospheric polarization coupling (conversion) coefficients were comparable to those of the reflection coefficients. The program and all its subroutines have been completely rewritten in a more modern version of FORTRAN (CDC FORTRAN 63) and a number of improvements suggested by users have been incorporated.

The next section explains the concepts and model used, so that you will know when it is appropriate to use the program, how to determine the model, and what input information you must provide. Section 3 contains detailed mathematical formulas used, and section 4 describes the computer implementation. This last section includes a test case and a listing of the FORTRAN source deck.

An ionospheric reflection coefficient program which can be used to generate input data for the propagation program is described in the appendix.

2. THE PHYSICAL PROBLEM AND THE MATHEMATICAL MODEL

The problem solved by the computer program can be stated as follows:

Given: A ground-based source of vertically polarized radio waves¹ of known frequency in the LF-VLF band, the electrical constants of the ground and the reflection characteristics of the ionosphere along a great circle path.

Calculate: The amplitude and phase of the propagated vertical electric field on the earth's surface as a function of distance along the path.

The geometry of the assumed model is shown in figure 1. The earth is a smooth sphere with radius a and electrical constants σ (conductivity) and ϵ (dielectric constant). The ionosphere is concentric with known reflection properties characterized by the ionospheric reflection coefficient matrix $\underline{I}(\varphi)$, where φ is the angle of incidence of a wave on the ionosphere. The height of the ionosphere above the earth, h , can be thought of as the virtual, or phase, height. Distance, d , is measured along the ground; we also use the angular distance $\theta = d/a$.

We compute the vertical (radial) electric field at O due to a vertically polarized source at S . The field is the vector sum of several components which travel to O via different paths:

$$E(d) = \sum_{j=0}^J E_j(d), \quad (1)$$

where the subscript j counts the number of times the energy has been reflected from the ionosphere. Thus, E_0 is the ground wave that travels directly along the surface of the earth from S to O . E_1 is the "first hop" (shown as the solid line in figure 1) which has reflected once from the ionosphere; E_2 is the "second hop" (shown as the dashed line in

¹Lewis (1970) derives wave hop formulas for an elevated, arbitrarily-oriented dipole.

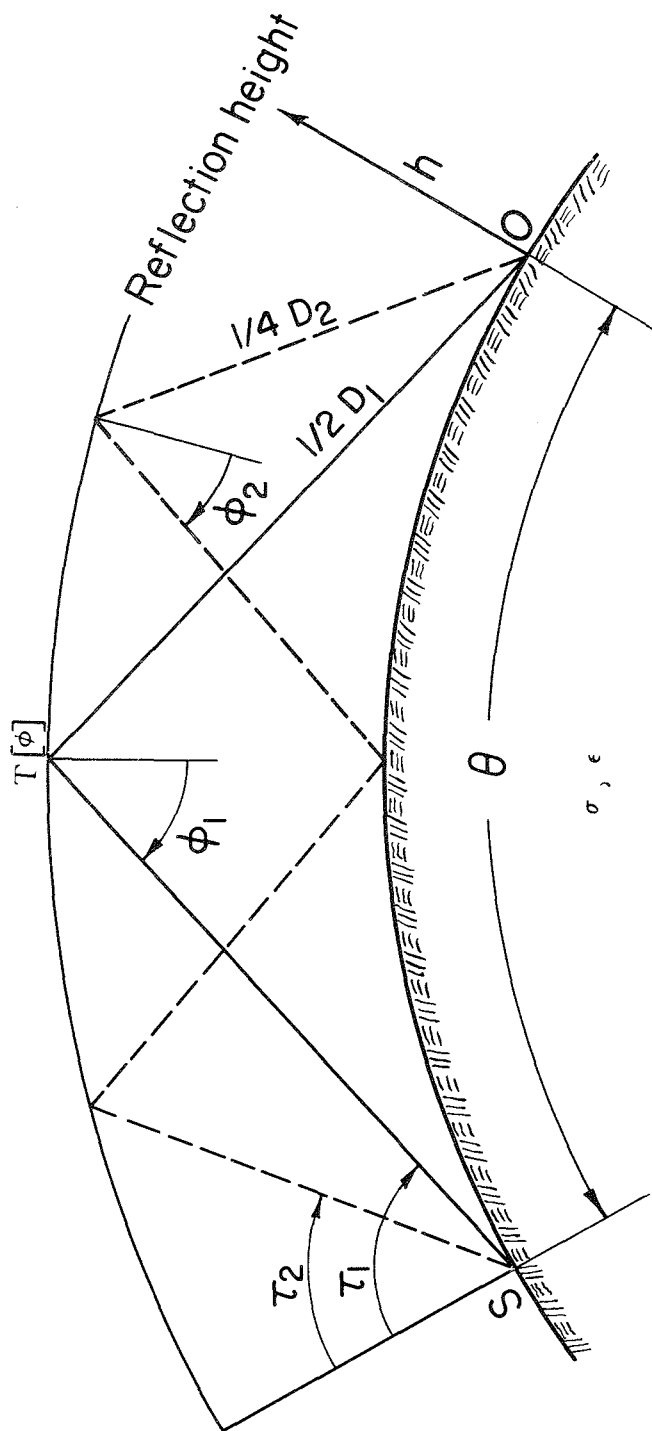


Figure 1. Geometry for wave hop propagation model.

figure 1); and so forth. The angle of incidence of the path of the j -th hop on the ground is denoted τ_j ; the angle of incidence on the ionosphere is denoted φ_j .

The theory is developed by Wait (1961), Berry (1964), and Berry et al. (1969), and the propagation characteristics of the wave hops are discussed by Berry (1964, 1967) and Johler (1970). Wave hop theory is most appropriate at LF (30-300 kHz) for distances less than, say, 6000 km. At lower frequencies a VLF waveguide mode theory (Wait, 1968 and references cited therein; Pappert, 1968) program is usually more appropriate, especially for long paths, although the wave hop program will compute fields correctly down to a few kilohertz and out to distances beyond 10,000 km. This conclusion is illustrated by Morfitt and Halley (1970) who compare fields calculated using this wave hop theory with fields calculated using the NEL mode theory program (Sheddy et al., 1968).

To use the program, you must specify:

- (a) The transmitter's operating frequency and effective radiated power.
- (b) The ground conductivity and relative dielectric constant. These are assumed to be constant along the path. If the conductivity varies significantly along the path, the geometric mean of the path conductivities is a useful approximation. Morgan (1968) provides a worldwide VLF effective-conductivity map which is also useful at low LF. At high LF and above, conductivity maps prepared for MF frequencies, such as that prepared by Barghausen, et al. (1966) may be more appropriate.
- (c) Ionospheric reflection coefficients as a function of angle of incidence. The coefficients should be given for the

entire range of angles of incidence that will be encountered by the program during execution. Belrose (1968), Watt (1967), and Berry and Chrisman (1965b) show graphs of the angle of incidence for various distances and reflection heights. The appendix of this report contains a reflection coefficient program that can be used to compute reflection coefficients for any given ionospheric profile, or reflection coefficients from different sources can be used. Belrose (1968) shows empirically derived reflection coefficients for frequencies in the upper LF and MF bands.

- (d) The distances of interest. These are given by a minimum distance, a distance increment, and a maximum distance.
- (e) The number of hops to be computed and summed. The number may be different for each of five (or fewer) distance ranges, because fewer hops are necessary at short ranges. The number of hops necessary for the series in (1) to converge depends on the amplitude of the ionospheric reflection coefficients, the ground conductivity, and the frequency. The amplitude of the individual terms in (1) are printed out, so it is possible to determine, after the fact, if enough terms were used, but advance determination of the minimum number of hops necessary still requires judgement. For daytime ionospheric reflection coefficients a crude rule-of-thumb is: number of hops = (distance in Mm) + 1. Fewer hops are needed at high LF, more are needed at night.

3. DETAILS OF THE MATHEMATICAL SOLUTION

3.1 Notation

- f = frequency, Hz
 P_r = effective radiated power, watts
 d = distance along the great circle path, m
 a_1 = earth's radius $\approx 6.36739 (10^6)$, m
 h = phase reference height of ionosphere (loosely, "reflection height")
 c = speed of light $\approx 2.997925 (10^8)$ m/s
 μ_0 = $4\pi (10^{-7})$
 σ = ground conductivity, mho/m
 ϵ = relative dielectric constant of ground
 $T_{ee}, T_{em}, T_{me}, T_{mm}$ are ionospheric reflection and conversion coefficients (Johler and Harper, 1962). The subscripts indicate which vector (electric or magnetic) is in the plane of incidence; the first subscript refers to the incident wave and the second to the reflected wave. The equivalence to a more common notation is: $T_{ee} \equiv R_{||}$, $T_{em} \equiv R_{\perp}$, $T_{me} \equiv R_{\perp}$, and $T_{mm} \equiv R_{||}$.

$$\varphi = \text{the angle of incidence on the ionosphere} \quad (2)$$

$$\omega = 2\pi f, \quad k = \omega/c \quad (3)$$

$$k_2 = \sqrt{k^2 \epsilon - i\mu_0 \omega \sigma} = k\sqrt{\epsilon - i \frac{\mu_0 c^2 \sigma}{\omega}} \quad (4)$$

$$a_2 = a_1 + h \quad (5)$$

$$v = (ka_1/2)^{\frac{1}{3}} \quad (6)$$

$$x = v\theta, \quad y = (kh/v) \quad (7)$$

$$z = 1.25/v^2$$

$$q_e = -iv \frac{k}{k_2} \sqrt{1 - \left(\frac{k}{k_2}\right)^2} \quad (8)$$

$$q_m = -iv \frac{k_2}{k} \sqrt{1 - \left(\frac{k}{k_2}\right)^2}$$

$$\theta = d/a_1, \quad x = v\theta \quad (9)$$

$$F = 30 \sqrt{\frac{\pi P_r}{120}}, \quad G = \frac{e^{-ikd+i\pi/4}}{d} \sqrt{\frac{\theta x}{\pi \sin \theta}} \cdot \frac{F}{2}. \quad (10)$$

3.2 The Ground-Wave

The first term of (1) is the ground wave, E_0 . It is given by (compare Wait (1962), Fock (1964))

$$E_0(d) = -4\pi G \sum_s (1+zt) \left(1 + \frac{3+i \cot \theta}{8ka_1 + vt_s}\right) \frac{e^{-i x t_s}}{t_s - q_e} \quad (11)$$

The t_s satisfy

$$W_1'(t_s) - q_e W_1(t_s) = 0, \quad (12)$$

where $W_1(t)$ is the Airy function of the first kind defined by Wait (1962), and the t_s are numbered in order of increasing magnitude of the imaginary part. The first two factors in the s -th term differ from unity by a few percent, at most, and are frequently neglected in ground wave calculations.

3.3 The Wave Hops

For $j \geq 1$ in (1), the wave hops are (Berry, et al., 1969)

$$E_j = I_{j1} T_{ee}^j + \sum_{M=2}^j I_{jM} C_{jM}, \quad (13)$$

where

$$C_{jM} = \sum_{k=1}^{M-1} a_{jMk} T_{ee}^{j+1-M-k} (T_{em} T_{me})^k T_{mm}^{M-1-k}. \quad (14)$$

Then

$$a_{jM1} = j - M + 1, \quad (15)$$

and

$$a_{jMk} = \frac{(j+2-M-k)(M-k)}{k(k-1)} a_{jM(k-1)} \text{ for } k \geq 2.$$

Both I_{jM} and the T_{ik} are implicit functions of d . The functions are made explicit below, beginning with the path integrals, I_{jM} .

The basic formula is (Berry, et al., 1969)

$$I_{jM} = G \int_{\Gamma} (1+zt) e^{-ixt} W_1(t) W_2(t) (1+R_e)^2 p^j R_e^{j-M} R_m^{M-1} dt, \quad (16)$$

where the ground reflection coefficients are

$$R_i = - \frac{W_2'(t)/W_2(t) - q_i}{W_1'(t)/W_1(t) - q_i}, \quad i = e \text{ or } m, \quad (17)$$

and

$$p = \frac{W_2(t) W_1(t-y)}{W_1(t) W_2(t-y)}. \quad (18)$$

Γ runs from ∞ to 0 and down into the third quadrant to $\infty e^{-i2\pi/3}$. However, (16) is an approximation to an integral involving Hankel functions of complex order (Berry, 1964; Berry and Chrisman, 1965c), and the approximation is valid only if $\cos \tau_j$ is small (refer to figure 1).

3.3.1 Geometrical Optics

For short distances, $\cos \tau_j$ is not small so we use the geometrical optics formula:

$$I_{JM} \approx -i F \frac{e^{-ikD_j}}{D_j} B_j \sin^2 \tau_j (1 + \hat{R}_e)^2 \hat{R}_e^{j-M} \hat{R}_m^{M-1}, \quad (19)$$

where

$$D_j = 2j \sqrt{2a_1 a_2 (1 - \cos \frac{\theta}{2j}) + h^2}, \quad (20)$$

$$\sin \tau_j = \frac{2j}{D_j} a_2 \sin \frac{\theta}{2j}, \quad (21)$$

$$\cos \varphi_j = \frac{D_j + 2ja_1 \cos \tau_j}{2ja_2}, \quad (22)$$

and the convergence-divergence coefficient is

$$B_j = \frac{a_2}{a_1} \sqrt{\frac{D_j \sin \tau_j \cos \varphi_j}{a_2 \sin \theta \cos \tau_j}}. \quad (23)$$

The Fresnel ground reflection coefficients are

$$\hat{R}_1 = \frac{s + q_1}{s - q_1}, \text{ where } s = iv \cos \tau_j \quad (24)$$

3.3.2 Saddle Point Approximation.

The relationship between (19) and (16) was shown by Wait (1961). Following his derivation, change the contour Γ so that we integrate from $-\infty$ to ∞ . Since we cross no singularities with this move, it only changes the sign of (16). Then, if $(-t) \gg 1$,

$$W_k(t) \approx (-t)^{\frac{1}{4}} \exp \left[(-1)^k i \left(\frac{2}{3} (-t)^{\frac{3}{2}} + \frac{\pi}{4} \right) \right]$$

and

$$W'_k \approx (-1)^{k-1} i (-t)^{\frac{1}{2}} W_k(t).$$

Substitution into (16) yields

$$I_{JM} \approx -G \int_{-\infty}^{\infty} e^{-i\Omega(t)} (1+zt) (-t)^{\frac{1}{2}} (1+R_e)^2 R_e^{j-M} R_e^{M-1} dt, \quad (26)$$

where

$$\Omega(t) = xt - j \frac{4}{3} (-t)^{\frac{3}{2}} + \frac{4}{3} j(y-t)^{\frac{3}{2}}, \quad (27)$$

and now

$$R_1(t) \approx \frac{(-t)^{\frac{1}{2}} - iq_1}{(-t)^{\frac{1}{2}} + iq_1}.$$

The saddle point approximation to (26) is

$$I_{JM} \approx -i F \frac{e^{-ikd}}{d} \sqrt{\frac{\theta}{\sin \theta}} (1 - z\alpha^2) \left(1 + \frac{x}{2j\alpha}\right) (1+R_e)^2 R_e^{j-M} R_e^{M-1}, \quad (28)$$

where

$$\alpha = (-t_0)^{\frac{1}{2}} = \frac{4j y - x^2}{4j x}. \quad (29)$$

It can be shown (Wait, 1961) that, for $\cos \tau_j \ll 1$,

$$(-t_0)^{\frac{1}{2}} \approx v \cos \tau_j,$$

$$\Omega(t_0) \approx k(D_j - d),$$

$$\left(1 + \frac{x}{2j\alpha}\right)^{\frac{1}{2}} \approx B_j,$$

$$(1 - z\alpha^2) \approx \sin^2 \tau_j,$$

and

$$d \frac{\sin \theta}{\theta} \approx D_j,$$

so (19) and (28), and hence (16), are approximately equal for the conditions $(-t_0) \gg 1$ and $\cos \tau_j \ll 1$. This small region is large enough to overlap (16) and (19).

3.3.3 Numerical Integration.

When τ_j approaches $\pi/2$, (16) must be integrated numerically. We use the Wronskian (Wait, 1962)

$$W_1'(t) W_2(t) - W_2'(t) W_1(t) = 2i$$

and (17) to derive

$$(1 + R_e)^2 = \frac{-4}{W_2^2(t) (W_1'(t) - q_e W_1(t))^2}, \quad (30)$$

and hence

$$I_{jM} = G \int_{\Gamma} (4)(-1)^j (1+zt) e^{-ixt} \left(\frac{W_1(t-y)}{W_2(t-y)} \right)^j \frac{(E_2(t, q_e))^{j-M}}{(E_1(t, q_e))^{j-M+2}} \left(\frac{E_2(t, q_m)}{E_1(t, q_m)} \right)^{M-1} dt, \quad (31)$$

where

$$E_k(t, q_i) = W_k'(t) - q_i W_k(t). \quad (32)$$

3.3.4 Residue Series.

The integrand in (31) has poles of order $j-M+2$ wherever

$$E_1(t, q_e) = 0,$$

and poles of order $M-1$ wherever

$$E_1(t, q_m) = 0.$$

Zeros for both functions exist inside the contour Γ ; they are the well-known solutions used for calculation of ground wave propagation over a spherical earth or diffraction by a finitely conducting sphere (Wait, 1962; Fock, 1965). The zeros of $W_2(t-y)$ are all in the upper half plane outside the contour Γ . Thus, I_{jM} can be evaluated by summing a residue series:

$$I_{JM} = 2\pi i G \sum_s \text{Res}_{JM}(t_s) . \quad (33)$$

We will first discuss methods for calculating residues of high order poles; then return to the problem of finding the pole locations t_s .

Assume that we have an integral

$$I = \oint \frac{A(t)}{B(t)} dt , \quad (34)$$

where $A(t)$ is analytic on and inside the contour and $B(t)$ has a zero of order N at $t = t_0$ inside the contour. Expand $A(t)$, $B(t)$, and their ratio in Laurent series around t_0 and equate coefficients of like powers to obtain (see for example Kaplan (1952), pp. 564-565)

$$\begin{array}{ll} b_0 \hat{a}_0 & = a_0 \\ b_1 \hat{a}_0 + b_0 \hat{a}_1 & = a_1 \\ b_2 \hat{a}_0 + b_1 \hat{a}_1 + b_0 \hat{a}_2 & = a_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ b_{N-1} \hat{a}_0 + b_{N-2} \hat{a}_1 + \cdot & = a_{N-1} \end{array} \quad (35)$$

where

$$a_i = \frac{\left. \frac{d^i}{dt^i} (A(t)) \right|_{t=t_0}}{i!} , \quad (36)$$

and

$$b_i = \frac{\left. \frac{d^{i+N}}{dt^{i+N}} (B(t)) \right|_{t=t_0}}{(i+N)!} ,$$

are the coefficients of the Laurent series for $A(t)$ and $B(t)$ respectively, and the $\hat{a}$ are the coefficients of the series for A/B . By definition

$$\text{Residue}(t_0) = \hat{a}_{N-1} . \quad (37)$$

The set of equations (35) can be solved recursively:

$$\hat{a}_0 = a_0/b_0; \text{ and } \hat{a}_i = (a_i - \sum_{\ell=0}^{i-1} b_{i-\ell} \hat{a}_\ell) / b_0. \quad (38)$$

Comparing (34) and (31), we see that if

$$E_1(t, q_e) = 0 \text{ ("} q_e \text{ poles") ,}$$

$$A(t) = 4(-1)^j (1+zt) e^{-ixt} \left(\frac{W_1(t-y)}{W_2(t-y)} \right)^j \frac{E_2^{j-M}(t, q_e) E_2^{M-1}(t, q_m)}{E_1^{M-1}(t, q_m)}, \quad (39)$$

and

$$B(t) = (E_1(t, q_e))^{j-M+2}.$$

If $E_1(t, q_m) = 0$, ("q_m poles"), then $N = M-1$ (there are no q_m poles for $M = 1$),

$$A(t) = 4(-1)^j (1+zt) e^{-ixt} \left(\frac{W_1(t-y)}{W_2(t-y)} \right)^j \frac{E_2^{j-M}(t, q_e) E_2^{M-1}(t, q_m)}{E_1^{j-M+2}(t, q_e)}, \quad (40)$$

and

$$B(t) = (E_1(t, q_m))^{M-1}.$$

Taking high order derivatives of products of several functions (such as $A(t)$) analytically is very tedious, but such derivatives can easily be evaluated numerically on a computer by repeated application of Liebnitz's rule:

$$(gh)^{(k)} = \sum_{m=0}^k \frac{k!}{m! (k-m)!} g^{(m)} h^{(k-m)}, \quad (41)$$

where g and h are functions of t and

$$g^{(m)} = \frac{d^m}{dt^m} g.$$

In order to apply (41) to (39) and (40), we must be able to compute all derivatives of $(E_1(t, q))^{-1}$. In (41), let $h = g^{-1}$ so that $gh = 1$, and $(gh)^{(k)} = 0$, if $k > 0$. Then (41) becomes

$$0 = g^{(0)} h^{(k)} + \sum_{m=1}^k \frac{k!}{m! (k-m)!} g^{(m)} h^{(k-m)}.$$

Notice that $g^{(0)} = g = 1/h$. Solve for $h^{(k)}$:

$$h^{(k)} = (g^{-1})^{(k)} = -h \sum_{m=1}^k \frac{k!}{m! (k-m)!} g^{(m)} h^{(k-m)}, \quad (42)$$

which is a recursive formula for derivatives of g^{-1} in terms of derivatives of g .

Similarly, recursive formulas for the b_i for a pole of order N are:

$$b_0 = (E_1^{(1)}(t_0, q_p))^N,$$

where $p = e$ or m , as appropriate, and

$$b_i = (E_1^{(1)}(t_0, q_p))^{-1} \sum_{k=1}^i \frac{k(N+1) - i}{i(k+1)!} b_{i-k} E_1^{(k+1)}(t_0, q_p). \quad (43)$$

Finally, all derivatives of the functions involved in (39) and (40) can be found recursively. Assume that $W_k(t)$ and $W'_k(t)$ are given. Then the differential equation for W_k [$W_k^{(2)}(t) = t W_k(t)$] and straightforward mathematical induction show that

$$W_k^{(n)}(t) = t W_k^{(n-2)} + (n-2) W_k^{(n-3)}(t), \text{ for } n \geq 2. \quad (44)$$

Using (32),

$$E_k^{(n)}(t, q) = W_k^{(n+1)}(t) - q W_k^{(n)}(t). \quad (45)$$

Of course,

$$(e^{-ixt})^{(n)} = (-ix)^n e^{-ixt}. \quad (46)$$

We now return to the problem of finding the location of the poles, that is, the zeroes of $E_1(t, q)$. We use Newton's iteration:

$$t \approx \hat{t} - \frac{E_1(\hat{t}, q)}{E_1'(\hat{t}, q)} = \hat{t} - \frac{W_1'(\hat{t}) - q W_1(\hat{t})}{\hat{t} W_1'(\hat{t}) - q W_1(\hat{t})}, \quad (47)$$

where $\hat{t}$ is an approximation to t . For first approximations, we use the known zeroes of $W_1'(t)$, if q is small, and the zeroes of $W_1(t)$ if q is large. The first ten such zeroes are listed in Subroutine TW, which also contains approximate formulas for all the other zeroes.

3.4 Ionospheric Reflection Coefficients

The ionospheric reflection coefficients, T , are functions of the angle of incidence, φ , which depends on distance and the hop number. When the geometrical-optics formula (19) is used, φ is computed with (22). When (28) or (31) is used, φ is given by (29), $\cos \tau_j = (-t_0)^{\frac{1}{2}}/v$, and $\sin \varphi = a_1/a_2 \sin \tau_j$. When I_{JM} is evaluated with the residue series (33), we use these same formulas except now t_0 is the first q_e pole, and hence is complex. Continuation of the ionospheric reflection coefficients into the complex φ -plane is accomplished by approximating the input values along the real axis with analytic functions and continuing these functions analytically.

Following Wait (1962), we approximate T_{ee} and T_{mm} by an exponential

$$T \approx -\exp(A \cos \varphi), \quad (48)$$

where A is a complex number determined from the input data:

$$\begin{aligned} \text{Real}(A) &= \log(|T|)/\cos \varphi, \\ \text{Imag}(A) &= (\text{Phase}(T) - \pi)/\cos \varphi. \end{aligned} \quad (49)$$

The polarization coupling coefficients T_{em} and T_{me} are approximated by fitting their amplitude and phase with linear functions.

4. NOTES ON COMPUTER IMPLEMENTATION

4.1 Program Organization and Flow

Figure 2 shows the large-block program organization and flow. The numbers in the blocks are the statement numbers where the block function begins. Details of program organization and flow are best determined from the statements and comments in the program listing in section 4.4.

4.1.1 Numerical Integration Branch.

We use 48-point Gaussian quadrature to integrate (31) on a finite portion of the contour Γ . First, for $t \gg 1$,

$$W_k(t) \approx t^{-\frac{1}{4}} \exp\left(\frac{2}{3} t^{\frac{3}{2}}\right), \quad (50)$$

and

$$W'_k(t) \approx t^{\frac{1}{2}} W_k(t),$$

so the magnitude of the integrand in (31) is about $4t^{\frac{1}{2}} \exp(-\frac{4}{3} t^{\frac{3}{2}})$. We consider the contribution beyond $t = 4$ to be negligible, so the first portion of the contour of integration runs from 4 to 0 along the real axis. The second portion of the contour is a straight line from 0 to $K(-4-i)$, where K is chosen so that the real part of $-ixt$ is -9 for the shortest distance for which numerical integration would be used. Specifically, $K = -9/(2\sqrt{4+y} - 4)$. The slope of this contour was chosen empirically to be optimum for convergence of the integrand. The rest of the infinite contour Γ contributes little to the integral, so we ignore it.

4.1.2 Residue Series and the Subroutine LPAINR.

The residues are calculated in the subroutine LPAINR. Since only the factor e^{-ixt} in $A(t)$ (see (39) and (40)) depends on distance, all necessary derivatives of $A(t)/e^{-ixt}$ are computed and stored on the

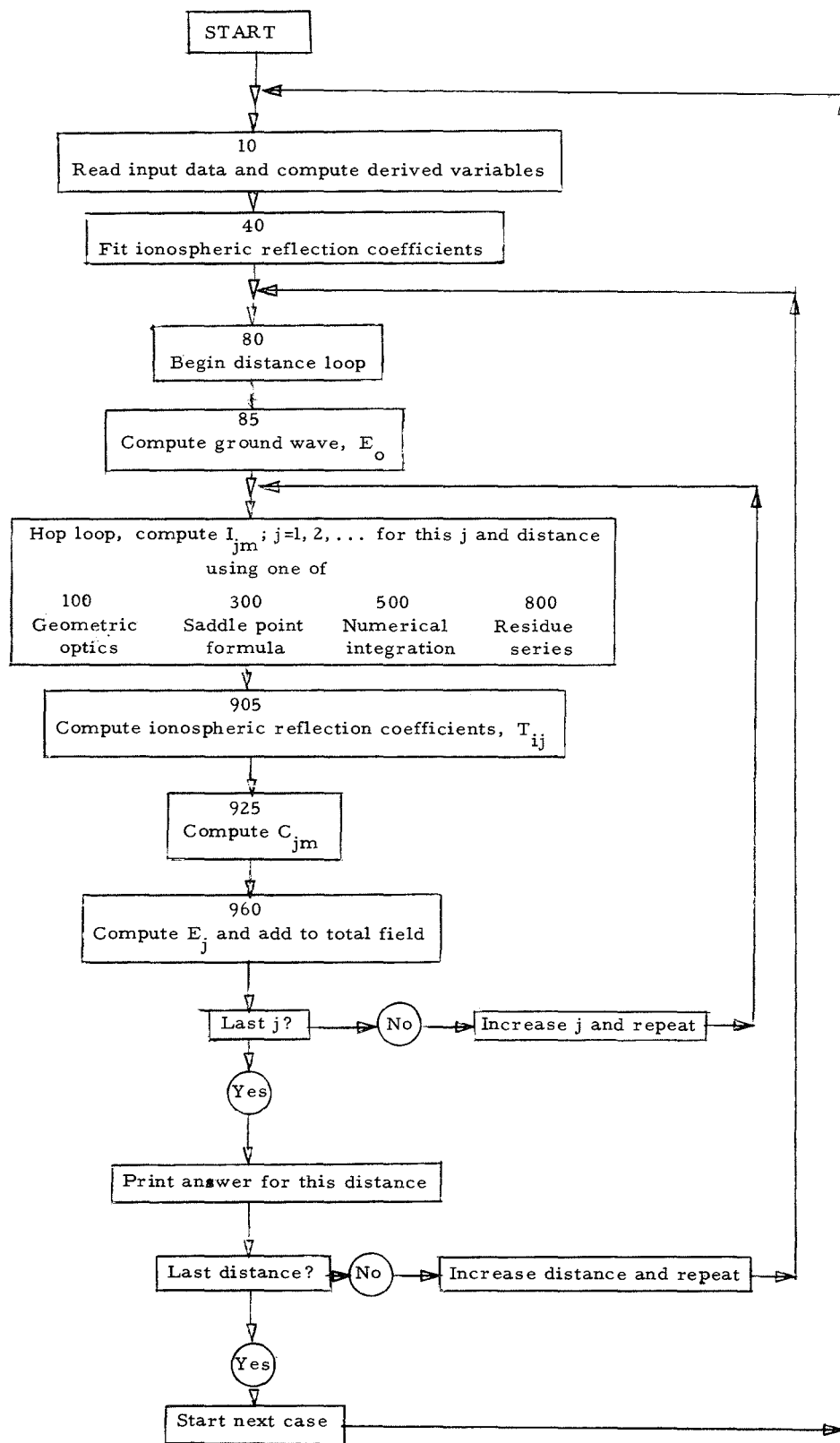


Figure 2. Flow chart for Program ANIHOP.

first entry to LPAINR. The b_i (43) are also independent of distance and are computed and stored on this first entry. Then, on every entry, e^{-ixt} and its derivatives are computed and the derivatives of the product $e^{-ixt} (A(t)/e^{-ixt}) = A(t)$ are computed using (41). The residues are then computed with (38).

The storage in LPAINR is set up for at most 5 hops since the sixth hop would not normally use LPAINR except for paths longer than 10,000 km. Because quantities (such as the b_i) need to be stored only for $M \leq j$ (and for $M > 1$ for the q_m poles), we save some storage by overlaying the arrays for the q_m poles on those for the q_e poles. If necessary, the details can be determined by examining the FORTRAN listing.

4.2 Input Data and a Test Case

Table 1 lists the input data necessary for each case, and figure 3 shows the arrangement of an input data deck. You can stack as many cases in sequence as you want. Execution terminates when an end-of-file card is read by statement 10.

Input data for a test case are listed in figure 4. The reflection coefficients used in this test case were computed with the program ANIREF, which is described in the appendix.

The printout produced by the test case is listed in figures 5-13. The "PHI C" printed out is the "phase lag" related to the phase of the field by

$$\varphi_c = -[\text{phase}(E) + kd + \pi/2],$$

where phase (E) is the phase of the total field or any of its components, as appropriate.

4.3 Important Compiler Characteristics

In our FORTRAN system, the variable index of a DO loop is compared with the maximum value before the DO loop is executed, and if

the index exceeds the maximum value, the loop is skipped. This corresponds with the usual mathematical convention that a sum or product is empty if the lower limit of the index exceeds the upper limit, and we have found it convenient to use this feature in the program and its subroutines. If your compiler tests the index of the DO loop after execution, you will need to modify the program accordingly.

Our computer stores two-dimensional arrays columnwise. That is, if $A(i,j)$ is a two-dimensional array, all of the elements of the column $j=1$ are stored in sequence, followed by all the elements of the column $j=2$, etc. A three-dimensional array can be thought of as an ordered sequence of two-dimensional arrays, with the third subscript being the number of the two-dimensional array. Therefore, a three-dimensional array can be used in place of a two-dimensional array by fixing the value of the third subscript. Similarly, a two-dimensional array can be treated as a one-dimensional array by fixing the value of the second subscript. We have used this feature extensively in subroutine LPAINR.

4.4 FORTRAN Listing of PROGRAM ANIHOP and Its Subroutines

See FORTRAN listing of ANIHOP beginning on page 33.

5. ACKNOWLEDGMENTS

Revision of the propagation program was sponsored by the Defense Atomic Support Agency and monitored by Mr. Dow Evelyn. Mrs. Mary Chrisman wrote the early versions of the program and several of its subroutines. Dr. George Hufford originated the AIRY function subroutine. The reflection coefficient program in the appendix incorporates many improvements developed by Dr. Bernard Wieder.

We would especially like to thank Mr. David Morfitt of NWC, Corona; Dr. Gary Price of SRI; Mr. Burt Gambill of GE Tempo; and Dr. G. Gonzales of University of Miami for many suggestions and discussions of the earlier version of the program.

Table 1. Input Data for the Program ANIHOP

Variable	Physical Quantity	Units	Comments
CNDK		mho/m	Ground conductivity of path.
EPS2			Ground dielectric constant relative to free space.
DMIN	Distance	km	Minimum distance at which field is calculated.
DELTA	Distance	km	Distance increment.
DMAX	Distance	km	Largest distance for which NHOP hops will be used.
NHOP	Number of hops		See preceding comment.
NT	Number		Reflection coefficients are read in for this many angles of incidence.
POWER	Radiated power	watts	CCIR definition.
KASE			Controls punching of output cards; 1 means punch cards; 0 means no cards output.
ICOND	Number		Number of distance blocks to follow. May be 0.
XD(I)	Distance	km	} { } { NH(I) hops will be calculated for XD(I-1) to XD(I), where NH(0) = NHOP, and XD(0) = DMAX.
NH(I)	Number of hops		
ID(I), ITIM, IPHI, PHIA	Various (see comments)		Identification of ionospheric reflection coefficients; such as time, magnetic field, azimuth, etc.
FREQ	Radio frequency	kHz	

Table 1. Input Data for the Program ANIHOP (continued)

Variable	Physical Quantity	Units	Comments
PHI(N)		Degrees	Angle of incidence on ionosphere
TAMP(L, N)	T	radians	Ionospheric reflection coefficient for PHI(N): L = 1 for T_{ee} , L = 2 for T_{em} , L = 3 for T_{me} , L = 4 for T_{mm} .
TPHA(L, N)	Phase of T		
HBOT	Height	km	The input phase(T) is referenced to this height.
HREF	Height	km	Effective height of reflection--phase(T) will re-referenced to this height in program.

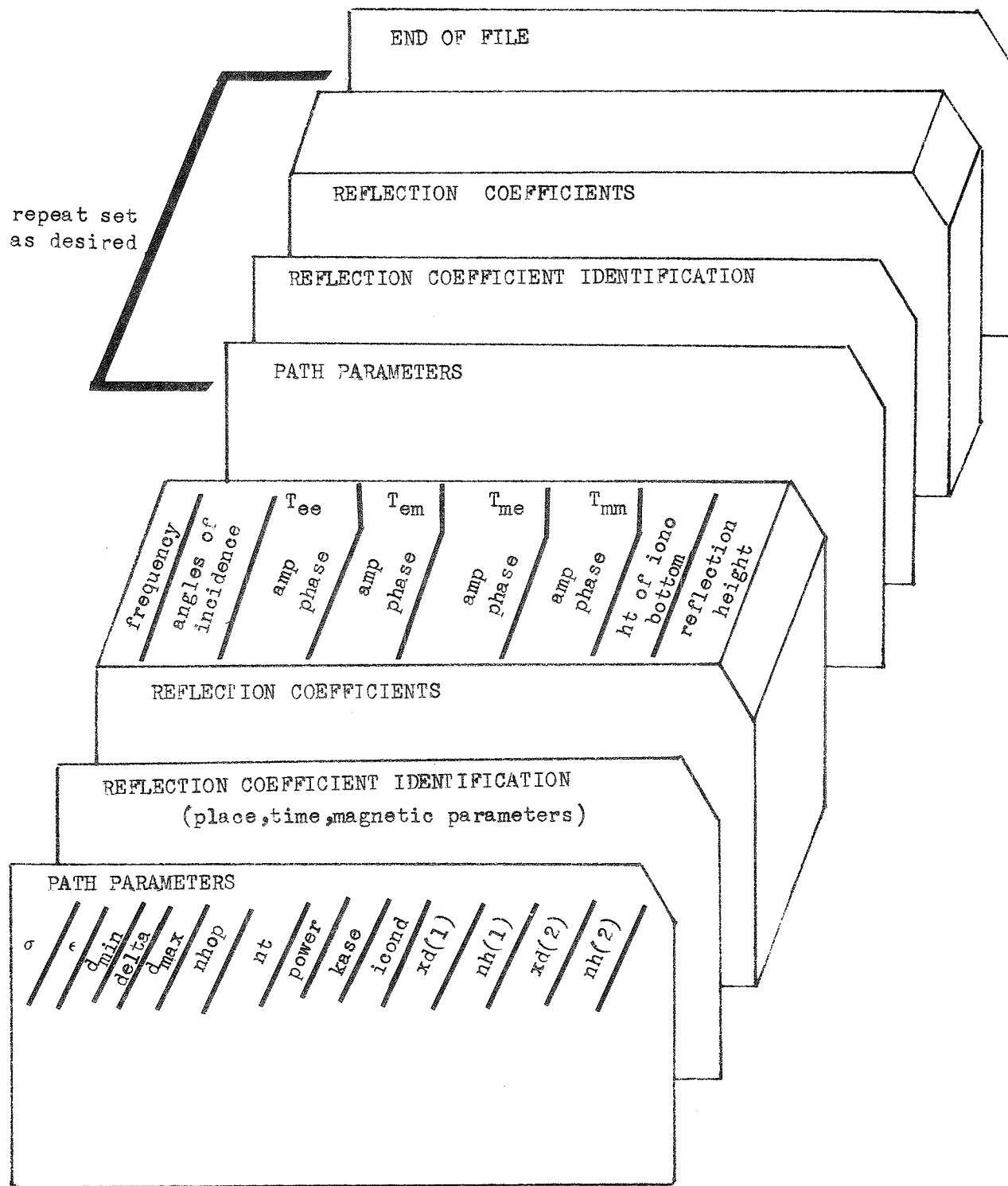


Figure 3. Data deck set-up for program Anihop. The top line on the dummy card is a descriptive label; it should not be punched on the card.

ANIHOP INPUT

σ	ϵ	d_{min}	δ	d_{max}	n_{hop}	n_t	power	kase	icond	$x_d(1)$	$n_h(1)$	$x_d(2)$	$n_h(2)$	$x_d(3)$	$n_h(3)$
4.	80.	600.	200.	1000.	2	6	1000.	0	3	6000.	5	8000.	7	10000.	9
SAN FRAN SEAPATH NIGHT PHIA= 213.5 DIP= 6.5 HM= 0.310 GAUSS identification															
30.065.02.386-001	1.9271.830-001	1.0187.001-002	1.5413.332-001	1.049	74.0	76.3									
30.073.01.627-001	2.8252.093-001	0.9357.233-002	1.8514.838-001	0.565	74.0	74.9									
30.078.01.527-001	2.5762.187-001	1.8929.881-002	0.7885.981-001	1.402	74.0	76.2									
30.080.02.031-001	2.4752.135-001	2.2311.028-001	0.4436.503-001	1.713	74.0	77.1									
30.081.02.410-001	2.4752.081-001	2.3921.027-001	0.2816.781-001	1.865	74.0	77.4									
30.082.02.866-001	2.5042.005-001	2.5481.010-001	0.1257.073-001	2.014	74.0	77.6									
4.	80.	600.	200.	1000.	2	6	1000.	0	3	6000.	5	8000.	7	10000.	9
SAN FRAN SEAPATH NIGHT PHIA= 258.1 DIP= 39.0 HM= 0.370 GAUSS															
30.065.03.816-001	1.1412.662-001	1.1152.343-001	0.8282.081-001	0.630	74.0	77.8									
30.073.02.596-001	2.6713.786-001	0.7353.404-001	0.7832.047-001	1.153	74.0	75.3									
30.078.01.394-001	2.9984.035-001	1.6713.596-001	1.6513.211-001	1.513	74.0	74.6									
30.080.01.270-001	2.6723.977-001	2.0173.533-001	1.9793.909-001	1.730	74.0	76.2									
30.081.01.464-001	2.4953.902-001	2.1853.462-001	2.1404.303-001	1.849	74.0	77.3									
30.082.01.827-001	2.4003.791-001	2.3513.360-001	2.2994.729-001	1.974	74.0	78.2									
4.	80.	600.	200.	1000.	2	6	1000.	0	3	6000.	5	8000.	7	10000.	9
SAN FRAN SEAPATH NIGHT PHIA= 299.2 DIP= 66.9 HM= 0.510 GAUSS															
30.065.03.090-001	0.4292.224-001	1.1422.389-001	1.4034.374-001	1.632	74.0	79.1									
30.073.03.213-001	1.4283.384-001	0.7743.682-001	0.7523.215-001	2.103	74.0	78.7									
30.078.03.986-001	1.8263.487-001	1.7753.877-001	1.8133.480-001	0.338	74.0	79.0									
30.080.04.569-001	1.9983.357-001	2.1563.755-001	2.2094.015-001	0.321	74.0	79.2									
30.081.04.922-001	2.0913.247-001	2.3453.642-001	2.4044.371-001	0.633	74.0	79.3									
30.082.05.316-001	2.1893.103-001	2.5323.489-001	2.5974.782-001	0.933	74.0	79.4									
END OF FILE															

frequency	angle of incidence	amp	phase	amp	phase	amp	phase	amp	phase	ht of iono bottom	reference height
T_{ee}				T_{em}		T_{me}		T_{mm}			

Figure 4. List of input data deck for ANIHOP sample case.

WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR 1000 WATTS RADIATED POWER
 FREQUENCY = 30.00KHZ, EARTH CONDUCTIVITY = 4.0000 MHO/M, RELATIVE PERMITIVITY = 80.00
 IONOSPHERIC MODEL

SAN FRAN SEAPATH NIGHT PHIA= 213.5 DIP= 6.5 HM= 0.310 GAUSS REFERENCE HEIGHT = 77.600

PHI	COS(PHI)	T E E		T E M		T M E		T M M		HBOT	HREF
		AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE		
65.00	0.4226	0.23860	3.840	0.18300	0.895	0.07001	-2.829	0.33320	0.864	74.0	76.3
73.00	0.2924	0.16270	4.149	0.20930	2.259	0.07233	-0.527	0.48380	1.889	74.0	74.9
78.00	0.2079	0.15270	3.517	0.21870	2.833	0.09881	0.153	0.59810	2.343	74.0	76.2
80.00	0.1736	0.20310	3.261	0.21350	3.017	0.10280	0.343	0.65030	2.499	74.0	77.1
81.00	0.1564	0.24100	3.183	0.20810	3.100	0.10270	0.427	0.67810	2.573	74.0	77.4
82.00	0.1392	0.28660	3.134	0.20050	3.178	0.10100	0.505	0.70730	2.644	74.0	77.6

DISTANCE		TOTAL	GR WAVE	HOP 1	HOP 2	HOP 3	HOP 4	HOP 5	HOP 6	HOP 7	HOP 8	HOP 9
600	AMP	2.75-004	3.44-004	1.58-004	8.31-005							
	PHI C	-0.36	0.40	-1.93	-1.61							
800	AMP	1.99-004	2.08-004	1.30-004	3.06-005							
	PHI C	1.09	0.59	2.58	-0.28							
1000	AMP	2.40-004	1.32-004	1.27-004	1.18-005							
	PHI C	1.08	0.78	1.48	-0.75							
1200	AMP	2.07-004	8.50-005	1.28-004	5.27-006	6.56-006	5.85-006	6.72-139				
	PHI C	1.02	0.98	0.93	3.09	2.42	-3.00	-0.03				
1400	AMP	1.88-004	5.57-005	1.24-004	1.17-005	3.45-006	2.96-006	3.35-006				
	PHI C	0.84	1.19	0.62	1.40	0.66	2.87	0.83				
1600	AMP	1.45-004	3.68-005	1.13-004	1.48-005	2.51-006	1.93-006	1.32-006				
	PHI C	0.57	1.39	0.48	-0.60	0.41	-1.21	-1.25				
1800	AMP	9.50-005	2.45-005	9.88-005	1.68-005	3.63-006	1.49-006	8.53-007				
	PHI C	0.60	1.59	0.44	-2.04	2.17	3.01	-0.91				
2000	AMP	7.35-005	1.65-005	8.35-005	1.86-005	4.36-006	1.40-006	6.60-007				
	PHI C	0.76	1.80	0.47	-3.06	-0.98	2.55	1.48				
2200	AMP	7.12-005	1.11-005	7.06-005	1.95-005	4.56-006	1.47-006	5.94-007				
	PHI C	1.16	2.00	0.70	2.49	-2.88	-2.82	-0.71				
2400	AMP	7.66-005	7.51-006	5.76-005	1.97-005	4.52-006	1.51-006	5.86-007				
	PHI C	1.20	2.21	0.87	1.95	1.33	-0.55	-1.47				
2600	AMP	6.54-005	5.11-006	4.56-005	1.94-005	4.37-006	1.45-006	5.97-007				
	PHI C	1.18	2.41	1.01	1.53	-0.30	2.42	-1.08				

Figure 5. Printed output from ANIHOP sample case, page 1.

2800	AMP	5.27-005	3.48-006	3.57-005	1.90-005	4.21-006	1.39-006	5.96-007		
	PHI C	1.18	2.62	1.16	1.21	-1.52	0.42	0.35		
3000	AMP	4.09-005	2.38-006	2.76-005	1.86-005	4.05-006	1.30-006	5.72-007		
	PHI C	1.31	2.82	1.32	0.97	-2.48	-2.06	2.64		
3200	AMP	3.65-005	1.63-006	2.11-005	1.82-005	3.95-006	1.22-006	5.54-007		
	PHI C	1.34	3.03	1.49	0.81	3.03	2.12	-0.82		
3400	AMP	3.19-005	1.12-006	1.60-005	1.76-005	3.89-006	1.18-006	5.50-007		
	PHI C	1.31	-3.05	1.67	0.72	2.41	0.33	2.45		
3600	AMP	2.65-005	7.70-007	1.21-005	1.68-005	3.92-006	1.11-006	5.71-007		
	PHI C	1.25	-2.85	1.85	0.69	1.91	-1.16	0.78		
3800	AMP	2.18-005	5.30-007	9.08-006	1.55-005	4.10-006	1.03-006	5.89-007		
	PHI C	1.26	-2.64	2.04	0.70	1.50	-2.41	-1.60		
4000	AMP	2.07-005	3.66-007	6.78-006	1.43-005	4.40-006	9.78-007	6.12-007		
	PHI C	1.28	-2.44	2.23	0.77	1.19	2.80	2.63		
4200	AMP	2.06-005	2.53-007	5.04-006	1.34-005	4.77-006	9.44-007	6.37-007		
	PHI C	1.25	-2.23	2.42	0.98	0.95	1.90	0.85		
4400	AMP	1.82-005	1.75-007	3.73-006	1.23-005	5.13-006	9.31-007	6.57-007		
	PHI C	1.27	-2.03	2.61	1.22	0.80	1.14	-0.70		
4600	AMP	1.54-005	1.21-007	2.76-006	1.10-005	5.42-006	9.34-007	6.64-007		
	PHI C	1.36	-1.82	2.81	1.47	0.70	0.53	-2.02		
4800	AMP	1.30-005	8.42-008	2.03-006	9.18-006	5.59-006	9.36-007	6.73-007		
	PHI C	1.39	-1.62	3.00	1.61	0.66	0.05	3.12		
5000	AMP	1.12-005	5.84-008	1.49-006	7.59-006	5.63-006	9.27-007	6.86-007		
	PHI C	1.39	-1.41	-3.08	1.77	0.65	-0.31	2.12		
5200	AMP	9.32-006	4.06-008	1.10-006	6.20-006	5.52-006	9.44-007	7.01-007		
	PHI C	1.35	-1.21	-2.88	1.93	0.68	-0.58	1.25		
5400	AMP	7.42-006	2.83-008	8.02-007	5.02-006	5.28-006	1.01-006	7.17-007		
	PHI C	1.29	-1.00	-2.69	2.10	0.73	-0.74	0.50		
5600	AMP	5.66-006	1.97-008	5.86-007	4.02-006	4.95-006	1.12-006	7.33-007		
	PHI C	1.23	-0.80	-2.49	2.28	0.80	-0.79	-0.14		
5800	AMP	4.19-006	1.37-008	4.28-007	3.20-006	4.53-006	1.27-006	7.55-007		
	PHI C	1.18	-0.59	-2.29	2.46	0.89	-0.75	-0.69		
6000	AMP	3.16-006	9.57-009	3.12-007	2.53-006	4.16-006	1.45-006	7.80-007		
	PHI C	1.13	-0.39	-2.09	2.64	1.01	-0.65	-1.15		
6200	AMP	2.90-006	6.68-009	2.27-007	1.98-006	3.92-006	1.63-006	7.98-007	3.70-007	1.85-007
	PHI C	1.10	-0.18	-1.89	2.82	1.20	-0.52	-1.53	0.74	-0.78
6400	AMP	2.18-006	4.67-009	1.65-007	1.55-006	3.64-006	1.79-006	8.11-007	3.78-007	1.88-007
	PHI C	1.22	0.02	-1.69	3.01	1.42	-0.37	-1.83	-0.02	-2.08

Figure 6. Printed output from ANIHOP sample case, page 2.

6600	AMP	1.80-006	3.26-009	1.20-007	1.20-006	3.33-006	1.92-006	8.28-007	3.87-007	1.91-007	
	PHI C	1.51	0.22	-1.49	-3.09	1.66	-0.20	-2.09	-0.69	3.03	
6800	AMP	1.64-006	2.28-009	8.74-008	9.31-007	3.00-006	2.00-006	8.61-007	3.96-007	1.95-007	
	PHI C	1.82	0.43	-1.28	-2.90	1.91	-0.04	-2.28	-1.28	1.96	
7000	AMP	1.40-006	1.60-009	6.34-008	7.17-007	2.58-006	2.02-006	9.08-007	4.07-007	1.99-007	
	PHI C	2.01	0.63	-1.08	-2.71	2.11	0.13	-2.39	-1.80	0.99	
7200	AMP	1.16-006	1.12-009	4.60-008	5.50-007	2.17-006	1.99-006	9.55-007	4.23-007	2.04-007	
	PHI C	2.22	0.84	-0.88	-2.51	2.29	0.30	-2.41	-2.24	0.12	
7400	AMP	1.07-006	7.84-010	3.33-008	4.21-007	1.81-006	1.91-006	9.87-007	4.41-007	2.08-007	
	PHI C	2.52	1.04	-0.68	-2.32	2.46	0.46	-2.36	-2.61	-0.66	
7600	AMP	1.10-006	5.50-010	2.42-008	3.21-007	1.50-006	1.79-006	9.99-007	4.58-007	2.13-007	
	PHI C	2.77	1.25	-0.48	-2.13	2.64	0.63	-2.23	-2.91	-1.36	
7800	AMP	1.14-006	3.86-010	1.75-008	2.44-007	1.23-006	1.65-006	9.89-007	4.72-007	2.20-007	
	PHI C	2.90	1.45	-0.27	-1.93	2.82	0.80	-2.05	3.13	-1.99	
8000	AMP	1.10-006	2.71-010	1.27-008	1.85-007	1.00-006	1.52-006	9.66-007	4.90-007	2.27-007	
	PHI C	2.90	1.66	-0.07	-1.73	3.00	0.98	-1.83	2.92	-2.54	
8200	AMP	9.70-007	1.90-010	9.17-009	1.40-007	8.10-007	1.41-006	9.35-007	5.17-007	2.33-007	1.18-007
	PHI C	2.86	1.86	0.13	-1.54	-3.10	1.20	-1.57	2.76	-3.03	-0.48
8400	AMP	1.02-006	1.34-010	6.63-009	1.06-007	6.52-007	1.30-006	9.04-007	5.56-007	2.41-007	1.22-007
	PHI C	2.73	2.07	0.33	-1.34	-2.92	1.44	-1.29	2.66	2.82	-1.27
8600	AMP	1.06-006	9.39-011	4.80-009	7.99-008	5.23-007	1.18-006	8.73-007	6.02-007	2.50-007	1.25-007
	PHI C	2.58	2.27	0.54	-1.14	-2.73	1.69	-1.01	2.62	2.45	-1.99
8800	AMP	1.07-006	6.61-011	3.47-009	6.01-008	4.17-007	1.07-006	8.42-007	6.44-007	2.62-007	1.29-007
	PHI C	2.49	2.48	0.74	-0.94	-2.54	1.95	-0.72	2.66	2.14	-2.65
9000	AMP	1.06-006	4.65-011	2.51-009	4.52-008	3.31-007	9.56-007	8.10-007	6.74-007	2.72-007	1.33-007
	PHI C	2.47	2.68	0.94	-0.75	-2.35	2.22	-0.45	2.75	1.90	3.04
9200	AMP	1.00-006	3.27-011	1.81-009	3.39-008	2.62-007	8.40-007	7.72-007	6.86-007	2.80-007	1.38-007
	PHI C	2.49	2.89	1.14	-0.55	-2.16	2.48	-0.19	2.89	1.70	2.51
9400	AMP	9.18-007	2.31-011	1.31-009	2.54-008	2.07-007	7.15-007	7.28-007	6.78-007	2.89-007	1.43-007
	PHI C	2.47	3.09	1.35	-0.35	-1.97	2.67	0.07	3.07	1.53	2.04
9600	AMP	8.11-007	1.63-011	9.46-010	1.90-008	1.62-007	6.03-007	6.79-007	6.52-007	3.03-007	1.48-007
	PHI C	2.43	-2.99	1.55	-0.15	-1.78	2.87	0.31	-3.01	1.39	1.63
9800	AMP	6.76-007	1.15-011	6.84-010	1.42-008	1.27-007	5.06-007	6.24-007	6.14-007	3.26-007	1.53-007
	PHI C	2.34	-2.78	1.75	0.05	-1.58	3.06	0.54	-2.77	1.31	1.26
10000	AMP	5.40-007	8.09-012	4.94-010	1.06-008	9.92-008	4.21-007	5.76-007	5.67-007	3.54-007	1.59-007
	PHI C	2.16	-2.58	1.96	0.25	-1.39	-3.03	0.77	-2.51	1.29	0.93

Figure 7. Printed output from ANIHOP sample case, page 3.

WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR 1000 WATTS RADIATED POWER
 FREQUENCY = 30.00KHZ, EARTH CONDUCTIVITY = 4.0000 MHO/M, RELATIVE PERMITIVITY = 80.00

IONOSPHERIC MODEL

SAN FRAN SEAPATH NIGHT PHIA= 258.1 DIP= 39.0 HM= 0.370 GAUSS REFERENCE HEIGHT = 78.200

PHI	COS(PHI)	T E E		T E M		T M E		T M M		HBOT	HREF
		AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE		
65.00	0.4226	0.38160	3.373	0.26620	1.117	0.23430	1.404	0.20810	2.862	74.0	77.8
73.00	0.2924	0.25960	4.215	0.37860	2.279	0.34040	2.327	0.20470	2.697	74.0	75.3
78.00	0.2079	0.13940	4.096	0.40350	2.769	0.35960	2.749	0.32110	2.611	74.0	74.6
80.00	0.1736	0.12700	3.589	0.39770	2.934	0.35330	2.896	0.39090	2.647	74.0	76.2
81.00	0.1564	0.14640	3.321	0.39020	3.011	0.34620	2.966	0.43030	2.675	74.0	77.3
82.00	0.1392	0.18270	3.135	0.37910	3.086	0.33600	3.034	0.47290	2.709	74.0	78.2

DISTANCE		TOTAL	GR WAVE	HOP 1	HOP 2	HOP 3	HOP 4	HOP 5	HOP 6	HOP 7	HOP 8	HOP 9
600	AMP	4.15-004	3.44-004	2.17-004	2.19-004							
	PHI C	0.16	0.40	-1.96	0.64							
800	AMP	3.33-004	2.08-004	1.27-004	1.00-004							
	PHI C	1.26	0.59	2.22	1.51							
1000	AMP	3.20-004	1.32-004	9.49-005	9.82-005							
	PHI C	0.91	0.78	1.16	0.85							
1200	AMP	1.72-004	8.50-005	8.04-005	8.95-005	3.01-005	3.30-005	2.66-139				
	PHI C	1.29	0.98	0.72	2.71	-1.15	1.31	-1.89				
1400	AMP	2.06-004	5.57-005	7.55-005	7.79-005	2.97-005	1.21-005	3.30-005				
	PHI C	0.55	1.19	0.52	0.14	-2.45	0.34	0.74				
1600	AMP	2.84-005	3.68-005	6.86-005	7.04-005	3.13-005	8.33-006	7.93-006				
	PHI C	-1.19	1.39	0.43	-2.13	-1.76	2.81	-2.64				
1800	AMP	1.01-004	2.45-005	5.99-005	6.47-005	2.47-005	7.97-006	4.07-006				
	PHI C	1.35	1.59	0.42	2.56	0.36	1.27	-2.39				
2000	AMP	1.06-004	1.65-005	5.08-005	5.78-005	2.03-005	8.77-006	3.34-006				
	PHI C	1.23	1.80	0.46	1.44	-2.87	1.33	0.37				
2200	AMP	1.08-004	1.11-005	3.97-005	5.29-005	1.83-005	1.00-005	3.40-006				
	PHI C	0.96	2.00	0.78	0.61	1.42	2.59	-1.36				
2400	AMP	7.70-005	7.51-006	3.13-005	4.98-005	1.76-005	7.36-006	3.76-006				
	PHI C	0.13	2.21	1.01	0.01	-0.75	-1.29	-1.75				
2600	AMP	3.84-005	5.11-006	2.48-005	4.74-005	1.71-005	6.12-006	4.23-006				
	PHI C	-0.02	2.41	1.14	-0.39	-2.44	1.72	-1.07				
2800	AMP	3.55-005	3.48-006	1.94-005	4.51-005	1.54-005	5.50-006	4.31-006				
	PHI C	0.10	2.62	1.29	-0.63	2.52	-0.23	0.53				

Figure 8. Printed output from ANIHOP sample case, page 4.

3000	AMP	2.40-005	2.38-006	1.50-005	4.24-005	1.40-005	5.19-006	3.41-006	
	PHI C	-0.02	2.82	1.45	-0.75	1.45	-2.77	2.92	
3200	AMP	4.04-005	1.63-006	1.15-005	3.90-005	1.32-005	5.03-006	2.93-006	
	PHI C	-0.11	3.03	1.63	-0.77	0.58	1.41	-0.45	
3400	AMP	4.17-005	1.12-006	8.74-006	3.50-005	1.28-005	4.98-006	2.64-006	
	PHI C	-0.44	-3.05	1.80	-0.73	-0.12	-0.38	2.90	
3600	AMP	3.81-005	7.70-007	6.59-006	3.08-005	1.27-005	4.80-006	2.50-006	
	PHI C	-0.64	-2.85	1.99	-0.64	-0.68	-1.87	1.23	
3800	AMP	3.18-005	5.30-007	4.95-006	2.65-005	1.28-005	4.43-006	2.39-006	
	PHI C	-0.80	-2.64	2.17	-0.51	-1.11	-3.11	-1.12	
4000	AMP	2.14-005	3.66-007	3.69-006	2.23-005	1.32-005	4.18-006	2.32-006	
	PHI C	-0.76	-2.44	2.36	-0.36	-1.43	2.09	3.11	
4200	AMP	1.96-005	2.53-007	2.75-006	1.80-005	1.39-005	4.04-006	2.28-006	
	PHI C	-0.57	-2.23	2.55	-0.22	-1.63	1.15	1.32	
4400	AMP	2.12-005	1.75-007	2.04-006	1.42-005	1.46-005	3.97-006	2.26-006	
	PHI C	-0.73	-2.03	2.75	-0.08	-1.73	0.34	-0.26	
4600	AMP	2.02-005	1.21-007	1.50-006	1.10-005	1.51-005	4.01-006	2.05-006	
	PHI C	-1.03	-1.82	2.94	0.07	-1.75	-0.36	-1.60	
4800	AMP	1.77-005	8.42-008	1.11-006	8.59-006	1.51-005	4.15-006	1.89-006	
	PHI C	-1.23	-1.62	3.14	0.28	-1.70	-0.95	-2.80	
5000	AMP	1.53-005	5.84-008	8.14-007	6.67-006	1.46-005	4.30-006	1.77-006	
	PHI C	-1.31	-1.41	-2.95	0.49	-1.61	-1.43	2.42	
5200	AMP	1.38-005	4.06-008	5.97-007	5.14-006	1.38-005	4.45-006	1.69-006	
	PHI C	-1.30	-1.21	-2.75	0.70	-1.48	-1.83	1.47	
5400	AMP	1.29-005	2.83-008	4.37-007	3.94-006	1.26-005	4.73-006	1.64-006	
	PHI C	-1.27	-1.00	-2.55	0.92	-1.32	-2.17	0.62	
5600	AMP	1.19-005	1.97-008	3.20-007	3.00-006	1.13-005	5.22-006	1.63-006	
	PHI C	-1.29	-0.80	-2.36	1.14	-1.15	-2.44	-0.12	
5800	AMP	1.05-005	1.37-008	2.34-007	2.28-006	9.92-006	5.91-006	1.63-006	
	PHI C	-1.37	-0.59	-2.16	1.36	-0.97	-2.60	-0.78	
6000	AMP	9.02-006	9.57-009	1.70-007	1.72-006	8.54-006	6.63-006	1.65-006	
	PHI C	-1.52	-0.39	-1.96	1.58	-0.77	-2.68	-1.37	
6200	AMP	6.88-006	6.68-009	1.24-007	1.30-006	7.19-006	7.24-006	1.72-006	6.04-007 3.12-007
	PHI C	-1.77	-0.18	-1.76	1.80	-0.56	-2.67	-1.88	1.78 1.70
6400	AMP	6.23-006	4.67-009	9.03-008	9.74-007	5.95-006	7.62-006	1.81-006	5.97-007 2.88-007
	PHI C	-1.88	0.02	-1.56	2.02	-0.35	-2.60	-2.30	0.92 0.35

Figure 9. Printed output from ANIHOP sample case, page 8.

6600	AMP	6.39-006	3.26-009	6.56-008	7.30-007	4.86-006	7.74-006	1.88-006	5.93-007	2.70-007		
	PHI C	-2.00	0.22	-1.35	2.24	-0.13	-2.49	-2.65	0.13	-0.90		
6800	AMP	6.57-006	2.28-009	4.77-008	5.46-007	3.91-006	7.61-006	1.96-006	5.99-007	2.55-007		
	PHI C	-2.12	0.43	-1.15	2.46	0.10	-2.35	-2.96	-0.59	-2.05		
7000	AMP	6.33-006	1.60-009	3.46-008	4.07-007	3.13-006	7.26-006	2.10-006	6.21-007	2.45-007		
	PHI C	-2.20	0.63	-0.95	2.68	0.32	-2.19	3.03	-1.23	-3.11		
7200	AMP	5.65-006	1.12-009	2.51-008	3.04-007	2.49-006	6.75-006	2.38-006	6.43-007	2.39-007		
	PHI C	-2.25	0.84	-0.75	2.90	0.54	-2.01	2.80	-1.79	2.20		
7400	AMP	4.76-006	7.84-010	1.82-008	2.26-007	1.97-006	6.13-006	2.80-006	6.60-007	2.34-007		
	PHI C	-2.29	1.04	-0.55	3.12	0.76	-1.82	2.66	-2.30	1.30		
7600	AMP	3.92-006	5.50-010	1.32-008	1.68-007	1.54-006	5.46-006	3.27-006	6.89-007	2.34-007		
	PHI C	-2.37	1.25	-0.35	-2.94	0.98	-1.62	2.62	-2.76	0.47		
7800	AMP	3.34-006	3.86-010	9.55-009	1.25-007	1.21-006	4.79-006	3.71-006	7.48-007	2.38-007		
	PHI C	-2.52	1.45	-0.14	-2.73	1.20	-1.42	2.65	3.12	-0.28		
8000	AMP	3.11-006	2.71-010	6.91-009	9.24-008	9.38-007	4.13-006	4.05-006	8.16-007	2.42-007		
	PHI C	-2.70	1.66	0.06	-2.51	1.41	-1.21	2.74	2.81	-0.97		
8200	AMP	3.18-006	1.90-010	5.00-009	6.85-008	7.26-007	3.53-006	4.25-006	8.66-007	2.48-007	9.79-008	4.92-008
	PHI C	-2.90	1.86	0.26	-2.29	1.63	-0.99	2.86	2.54	-1.61	2.56	2.27
8400	AMP	3.10-006	1.34-010	3.62-009	5.07-008	5.60-007	2.96-006	4.32-006	9.04-007	2.63-007	9.65-008	4.62-008
	PHI C	-3.00	2.07	0.46	-2.07	1.85	-0.77	3.02	2.28	-2.20	1.64	1.00
8600	AMP	2.97-006	9.39-011	2.62-009	3.75-008	4.30-007	2.44-006	4.26-006	9.77-007	2.81-007	9.68-008	4.38-008
	PHI C	-3.03	2.27	0.67	-1.85	2.07	-0.55	-3.09	2.02	-2.70	0.79	-0.20
8800	AMP	2.75-006	6.61-011	1.89-009	2.77-008	3.30-007	1.99-006	4.08-006	1.14-006	2.89-007	9.67-008	4.21-008
	PHI C	-3.03	2.48	0.87	-1.63	2.29	-0.32	-2.90	1.79	-3.14	0.01	-1.32
9000	AMP	2.44-006	4.65-011	1.37-009	2.05-008	2.52-007	1.60-006	3.83-006	1.39-006	2.98-007	9.81-008	4.06-008
	PHI C	-3.04	2.68	1.07	-1.41	2.50	-0.10	-2.70	1.66	2.71	-0.73	-2.37
9200	AMP	2.06-006	3.27-011	9.89-010	1.51-008	1.92-007	1.27-006	3.51-006	1.69-006	3.22-007	1.03-007	3.98-008
	PHI C	-3.12	2.89	1.28	-1.20	2.72	0.13	-2.50	1.63	2.30	-1.40	2.92
9400	AMP	1.70-006	2.31-011	7.15-010	1.11-008	1.46-007	1.01-006	3.17-006	1.99-006	3.65-007	1.07-007	3.96-008
	PHI C	3.00	3.09	1.48	-0.98	2.94	0.35	-2.28	1.68	1.96	-2.01	2.00
9600	AMP	1.50-006	1.63-011	5.17-010	8.22-009	1.11-007	8.05-007	2.81-006	2.25-006	4.09-007	1.10-007	3.92-008
	PHI C	2.77	-2.99	1.68	-0.76	-3.13	0.58	-2.07	1.79	1.72	-2.58	1.14
9800	AMP	1.47-006	1.15-011	3.73-010	6.06-009	8.40-008	6.36-007	2.46-006	2.45-006	4.37-007	1.16-007	3.96-008
	PHI C	2.56	-2.78	1.89	-0.54	-2.91	0.81	-1.85	1.93	1.52	-3.14	0.31
10000	AMP	1.50-006	8.09-012	2.70-010	4.46-009	6.36-008	5.01-007	2.14-006	2.56-006	4.51-007	1.26-007	4.10-008
	PHI C	2.45	-2.58	2.09	-0.33	-2.70	1.03	-1.63	2.09	1.30	2.67	-0.44

Figure 10. Printed output from ANIHOP sample case, page 9.

WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR 1000 WATTS RADIATED POWER
 FREQUENCY = 30.00KHZ, EARTH CONDUCTIVITY = 4.0000 MHO/M, RELATIVE PERMITIVITY = 80.00

IONOSPHERIC MODEL

SAN FRAN SEAPATH		NIGHT		PHIA= 299.2 DIP= 66.9 HM= 0.510 GAUSS				REFERENCE HEIGHT = 79.400				
PHI	COS(PHI)	T E E		T E M		T M E		T M M		H BOT	H REF	
		AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE			
65.00	0.4225	0.30900	3.299	0.22240	1.728	0.23890	1.467	0.43740	-1.781	74.0	79.1	
73.00	0.2924	0.32130	3.413	0.33840	2.759	0.36820	2.737	0.32150	-0.118	74.0	78.7	
78.00	0.2079	0.39860	3.238	0.34870	3.187	0.38770	3.225	0.34800	1.074	74.0	79.0	
80.00	0.1736	0.45690	3.177	0.33570	3.335	0.37550	3.388	0.40150	1.500	74.0	79.2	
81.00	0.1564	0.49220	3.153	0.32470	3.407	0.36420	3.466	0.43710	1.695	74.0	79.3	
82.00	0.1392	0.53160	3.134	0.31030	3.477	0.34890	3.542	0.47820	1.878	74.0	79.4	
DISTANCE		TOTAL	GR WAVE	HOP 1	HOP 2	HOP 3	HOP 4	HOP 5	HOP 6	HOP 7	HOP 8	HOP 9
600	AMP	5.83-004	3.44-004	3.41-004	9.51-005							
	PHI C	0.01	0.40	-0.72	1.48							
800	AMP	2.49-004	2.08-004	3.29-004	1.06-004							
	PHI C	2.90	0.59	-2.86	2.80							
1000	AMP	4.50-004	1.32-004	3.05-004	9.15-005							
	PHI C	1.79	0.78	2.10	1.99							
1200	AMP	3.59-004	8.50-005	2.86-004	6.32-005	3.57-005	6.19-006	8.66-140				
	PHI C	1.47	0.98	1.42	-2.58	1.41	2.72	2.90				
1400	AMP	3.72-004	5.57-005	2.62-004	3.67-005	4.24-005	5.09-006	5.99-007				
	PHI C	0.96	1.19	1.05	0.84	0.00	-2.94	1.47				
1600	AMP	3.13-004	3.68-005	2.33-004	2.01-005	4.32-005	1.04-005	1.94-006				
	PHI C	0.79	1.39	0.88	-0.75	0.48	-0.04	0.33				
1800	AMP	2.04-004	2.45-005	2.01-004	1.05-005	3.90-005	1.47-005	2.10-006				
	PHI C	1.03	1.59	0.82	-1.67	2.41	-1.69	1.36				
2000	AMP	1.45-004	1.65-005	1.69-004	1.36-005	3.28-005	1.70-005	3.41-006				
	PHI C	0.61	1.80	0.84	-2.01	-0.94	-1.81	-1.71				
2200	AMP	1.12-004	1.11-005	1.41-004	1.73-005	2.78-005	1.79-005	5.08-006				
	PHI C	1.19	2.00	0.97	-2.58	-3.04	-0.74	2.82				
2400	AMP	1.56-004	7.51-006	1.15-004	1.75-005	2.47-005	1.76-005	6.31-006				
	PHI C	1.30	2.21	1.11	-3.13	0.92	1.40	2.30				
2600	AMP	6.87-005	5.11-006	9.12-005	1.53-005	2.25-005	1.65-005	6.99-006				
	PHI C	1.39	2.41	1.25	2.54	-0.94	-2.01	2.81				
2800	AMP	8.33-005	3.48-006	7.15-005	1.50-005	2.07-005	1.52-005	7.20-006				
	PHI C	1.70	2.62	1.40	1.76	-2.46	1.41	-2.10				

Figure 11. Printed output from ANIHOP sample case, page 7.

3000	AMP	8.24-005	2.38-006	5.54-005	2.02-005	1.93-005	1.44-005	7.19-006
	PHI C	1.44	2.82	1.56	1.16	2.59	-0.32	0.20
3200	AMP	7.50-005	1.63-006	4.25-005	2.80-005	1.85-005	1.35-005	6.86-006
	PHI C	1.68	3.03	1.73	0.88	1.59	-2.47	3.02
3400	AMP	8.41-005	1.12-006	3.23-005	3.49-005	1.80-005	1.28-005	6.38-006
	PHI C	1.25	-3.05	1.91	0.79	0.79	1.99	0.02
3600	AMP	6.67-005	7.70-007	2.44-005	3.96-005	1.72-005	1.23-005	6.02-006
	PHI C	0.88	-2.85	2.09	0.80	0.16	0.44	-1.62
3800	AMP	4.89-005	5.30-007	1.83-005	4.17-005	1.61-005	1.19-005	5.53-006
	PHI C	0.80	-2.64	2.28	0.87	-0.34	-0.88	2.26
4000	AMP	3.24-005	3.66-007	1.37-005	4.13-005	1.51-005	1.16-005	5.08-006
	PHI C	0.75	-2.44	2.46	0.97	-0.77	-2.00	0.17
4200	AMP	2.00-005	2.53-007	1.02-005	3.83-005	1.49-005	1.14-005	4.69-006
	PHI C	1.49	-2.23	2.66	1.16	-1.07	-2.97	-1.66
4400	AMP	3.02-005	1.75-007	7.54-006	3.43-005	1.52-005	1.13-005	4.35-006
	PHI C	1.88	-2.03	2.85	1.39	-1.23	2.49	3.02
4600	AMP	3.12-005	1.21-007	5.57-006	3.00-005	1.54-005	1.12-005	4.10-006
	PHI C	1.72	-1.82	3.04	1.63	-1.23	1.79	1.63
4800	AMP	2.38-005	8.42-008	4.11-006	2.63-005	1.54-005	1.11-005	3.93-006
	PHI C	1.37	-1.62	-3.04	1.80	-1.10	1.18	0.42
5000	AMP	1.63-005	5.84-008	3.02-006	2.26-005	1.54-005	1.13-005	3.80-006
	PHI C	0.90	-1.41	-2.85	1.97	-0.87	0.66	-0.65
5200	AMP	1.28-005	4.06-008	2.22-006	1.90-005	1.55-005	1.15-005	3.73-006
	PHI C	0.43	-1.21	-2.65	2.15	-0.58	0.24	-1.59
5400	AMP	1.20-005	2.83-008	1.62-006	1.58-005	1.59-005	1.15-005	3.71-006
	PHI C	0.20	-1.00	-2.45	2.33	-0.28	-0.10	-2.42
5600	AMP	1.22-005	1.97-008	1.19-006	1.30-005	1.63-005	1.13-005	3.71-006
	PHI C	0.23	-0.80	-2.25	2.52	0.02	-0.38	3.12
5800	AMP	1.26-005	1.37-008	8.67-007	1.05-005	1.66-005	1.11-005	3.78-006
	PHI C	0.35	-0.59	-2.05	2.71	0.30	-0.64	2.46
6000	AMP	1.30-005	9.57-009	6.32-007	8.45-006	1.65-005	1.13-005	3.91-006
	PHI C	0.43	-0.39	-1.85	2.90	0.57	-0.85	1.89
6200	AMP	1.29-005	6.68-009	4.60-007	6.73-006	1.52-005	1.19-005	4.05-006
	PHI C	0.26	-0.18	-1.65	3.10	0.83	-1.00	1.39
6400	AMP	8.95-006	4.67-009	3.35-007	5.32-006	1.36-005	1.28-005	4.18-006
	PHI C	0.09	0.02	-1.45	-2.99	1.09	-1.06	0.94
								-1.96
								-1.87

Figure 12. Printed output from ANIHOP sample case, page 5.

6600	AMP	6.99-006	3.26-009	2.44-007	4.18-006	1.19-005	1.36-005	4.39-006	1.23-006	1.12-006		
	PHI C	0.00	0.22	-1.25	-2.79	1.37	-1.04	0.53	-2.75	3.10		
6800	AMP	8.00-006	2.28-009	1.77-007	3.27-006	1.02-005	1.42-005	4.76-006	1.21-006	1.09-006		
	PHI C	-0.12	0.43	-1.05	-2.60	1.65	-0.95	0.18	2.84	1.90		
7000	AMP	1.06-005	1.60-009	1.29-007	2.54-006	8.89-006	1.44-005	5.20-006	1.20-006	1.06-006		
	PHI C	-0.25	0.63	-0.85	-2.40	1.93	-0.80	-0.09	2.24	0.80		
7200	AMP	1.28-005	1.12-009	9.32-008	1.97-006	8.03-006	1.43-005	5.54-006	1.19-006	1.04-006		
	PHI C	-0.31	0.84	-0.65	-2.20	2.17	-0.62	-0.29	1.69	-0.21		
7400	AMP	1.36-005	7.84-010	6.76-008	1.51-006	7.12-006	1.39-005	5.70-006	1.24-006	1.03-006		
	PHI C	-0.32	1.04	-0.45	-2.00	2.41	-0.40	-0.46	1.21	-1.13		
7600	AMP	1.31-005	5.50-010	4.90-008	1.16-006	6.21-006	1.32-005	5.75-006	1.32-006	1.02-006		
	PHI C	-0.28	1.25	-0.24	-1.80	2.63	-0.17	-0.62	0.82	-1.96		
7800	AMP	1.20-005	3.86-010	3.55-008	8.90-007	5.35-006	1.25-005	5.88-006	1.39-006	1.01-006		
	PHI C	-0.19	1.45	-0.04	-1.60	2.86	0.08	-0.78	0.50	-2.72		
8000	AMP	1.09-005	2.71-010	2.57-008	6.79-007	4.55-006	1.16-005	6.27-006	1.45-006	1.01-006		
	PHI C	-0.09	1.66	0.16	-1.40	3.08	0.33	-0.92	0.22	2.87		
8200	AMP	1.04-005	1.90-010	1.86-008	5.17-007	3.83-006	1.06-005	6.93-006	1.53-006	1.02-006	5.89-007	3.13-007
	PHI C	0.03	1.86	0.36	-1.20	-2.99	0.57	-1.00	-0.07	2.24	0.98	1.48
8400	AMP	1.03-005	1.34-010	1.35-008	3.92-007	3.19-006	9.51-006	7.73-006	1.72-006	1.03-006	5.88-007	3.04-007
	PHI C	-0.06	2.07	0.57	-1.00	-2.77	0.81	-1.01	-0.33	1.68	0.06	0.15
8600	AMP	9.89-006	9.39-011	9.73-009	2.97-007	2.64-006	8.34-006	8.51-006	1.99-006	1.04-006	5.91-007	2.95-007
	PHI C	-0.21	2.27	0.77	-0.79	-2.56	1.06	-0.96	-0.53	1.19	-0.79	-1.08
8800	AMP	9.48-006	6.61-011	7.04-009	2.24-007	2.16-006	7.18-006	9.13-006	2.29-006	1.04-006	5.94-007	2.89-007
	PHI C	-0.33	2.48	0.97	-0.59	-2.34	1.31	-0.85	-0.64	0.73	-1.57	-2.23
9000	AMP	9.33-006	4.65-011	5.09-009	1.69-007	1.76-006	6.07-006	9.53-006	2.52-006	1.06-006	6.02-007	2.83-007
	PHI C	-0.37	2.68	1.17	-0.39	-2.13	1.57	-0.70	-0.71	0.32	-2.30	2.98
9200	AMP	9.46-006	3.27-011	3.68-009	1.27-007	1.43-006	5.05-006	9.67-006	2.64-006	1.10-006	6.15-007	2.79-007
	PHI C	-0.33	2.89	1.38	-0.19	-1.92	1.84	-0.53	-0.76	-0.02	-2.97	1.98
9400	AMP	9.50-006	2.31-011	2.66-009	9.57-008	1.15-006	4.32-006	9.59-006	2.68-006	1.13-006	6.27-007	2.78-007
	PHI C	-0.26	3.09	1.58	0.01	-1.71	2.12	-0.34	-0.83	-0.30	2.71	1.05
9600	AMP	9.30-006	1.63-011	1.92-009	7.18-008	9.18-007	3.72-006	9.30-006	2.73-006	1.13-006	6.39-007	2.77-007
	PHI C	-0.19	-2.99	1.78	0.22	-1.50	2.40	-0.13	-0.94	-0.54	2.15	6.19
9800	AMP	8.74-006	1.15-011	1.39-009	5.38-008	7.32-007	3.20-006	8.87-006	2.91-006	1.13-006	6.61-007	2.78-007
	PHI C	-0.14	-2.78	1.99	0.42	-1.29	2.67	0.09	-1.06	-0.77	1.62	-0.61
10000	AMP	7.88-006	8.09-012	1.00-009	4.03-008	5.81-007	2.74-006	8.32-006	3.29-006	1.17-006	6.90-007	2.82-007
	PHI C	-0.12	-2.58	2.19	0.62	-1.08	2.94	0.32	-1.15	-0.99	1.16	-1.36

Figure 13. Printed output from ANIHOP sample case, page 6.

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PROGRAM ANIHOP

```

C      A PROGRAM TO COMPUTE LF/VLF RADIO PROPAGATION USING WAVE HOP
C      THEORY.
C      REFERENCE. A WAVE HOP PROPAGATION PROGRAM FOR AN ANISOTROPIC
C      IONOSPHERE BY L.A. BERRY AND J.E. HERMAN, (ITS, BOULDER, COLO,
C      80302) TELECOMMUNICATIONS RESEARCH REPORT NO. 11.

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      DIMENSION AMP(9),ATI(12),ATR(12),CPHI(13),E(9),FTX(2),G(48)
      1,HOP(9,9),HREF(12),ID(10),JGO(9),NH(4),O(96),PH(9),PHI(12)
      2,PT(45,96),Q(2),R(2,5,5),SD(96),T(40),TA(9,4)
      3,TAMP(4,13),TPHA(4,13),TPR(4,13),TPI(4,13),W(48),XD(4)

```

```

      COMMON/PAIN/X,Y,Z,DMIN,XDIST,DSAV,A1,A2,Q,KMAX,T
      COMMON/WGW/QKA1,QKA2,FK,AK1,V,V2, THETA,STH

```

```

      COMPLEX B,C,C1,C2,DT1,DT2,E,ETA,F1,F2,F3,GW,HOP,O,PT,Q,QKA2,R,RE,
      1RM,RR,R1,R12,R2,R21,R3,R4,R5,S,SD,SQ2,SUM,S1,S2,T,TA,T1,T2,TY1,TY2

```

```

1  FORMAT(F5.1,F4.1,2F6.1,F7.1,I2,I3,F7.1,2I2,4(F7.1,I2))
2  FORMAT(2A8,A3,A8,4X,A5,A6,A4,4A8, * REFERENCE HEIGHT =*,F8.3)
3  FORMAT(F6.1,F4.1,4(E9.3,F6.3),2F5.1)
4  FORMAT(2A8,A3,A8,4X,A5,A6,A4,4A8)
5  FORMAT(1H1,10X,* WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD S
      1TRENGTH (V/M) FOR *,F8.0,* WATTS RADIATED POWER*//10X,
      211HFREQUENCY = ,F9.2,*KHZ, EARTH CONDUCTIVITY =*,F8.4,* MHO/M,
      3RELATIVE PERMITIVITY =*,F7.2//40X,*IONOSPHERIC MODEL*//)
7  FORMAT(1H1,8X,5HW(49),15X,5HW(50)/(2E20.8))
8  FORMAT(1H0F7.0, * AMP *, 11E10.2)
9  FORMAT(9X,*PHI C *, F6.2, 10F10.2)

```

C ABCISSA (G) AND WEIGHTS (W) FOR GAUSSIAN INTEGRATION

```

      G(1)=-.9987710073 $ G(2)=-.9935301723 $ G(3)=-.9841245837
      G(4)=-.9705915925 $ G(5)=-.9529877032 $ G(6)=-.9313866907
      G(7)=-.9058791367 $ G(8)=-.8765720203 $ G(9)=-.8435882616
      G(10)=-.8070662040 $ G(11)=-.7671590325 $ G(12)=-.7240341309
      G(13)=-.6778723796 $ G(14)=-.6288673968 $ G(15)=-.5772247261
      G(16)=-.5231609747 $ G(17)=-.4669029048 $ G(18)=-.4086864820
      G(19)=-.3487558863 $ G(20)=-.2873624874 $ G(21)=-.2247637904
      G(22)=-.1612223561 $ G(23)=-.09700469921 $ G(24)=-.03238017096
      G(25)=-.03238017096 $ G(26)=-.09700469921 $ G(27)=-.1612223561
      G(28)=-.2247637904 $ G(29)=-.2873624874 $ G(30)=-.3487558863
      G(31)=-.4086864820 $ G(32)=-.4669029048 $ G(33)=-.5231609747
      G(34)=-.5772247260 $ G(35)=-.6288673968 $ G(36)=-.6778723796
      G(37)=-.7240341309 $ G(38)=-.7671590325 $ G(39)=-.8070662040
      G(40)=-.8435882616 $ G(41)=-.8765720203 $ G(42)=-.9058791367
      G(43)=-.9313866907 $ G(44)=-.9529877032 $ G(45)=-.9705915925
      G(46)=-.9841245837 $ G(47)=-.9935301723 $ G(48)=-.9987710073

```

W(1)=.003153346052 \$ W(2)=.007327553901 \$ W(3)=.01147723458
 W(4)=.01557931572 \$ W(5)=.01961616046 \$ W(6)=.02357076084
 W(7)=.02742650971 \$ W(8)=.03116722783 \$ W(9)=.03477722256
 W(10)=.03824135107 \$ W(11)=.04154508294 \$ W(12)=.04467456086
 W(13)=.04761665849 \$ W(14)=.05035903555 \$ W(15)=.05289018949
 W(16)=.05519950370 \$ W(17)=.05727729210 \$ W(18)=.05911483970
 W(19)=.06070443917 \$ W(20)=.06203942316 \$ W(21)=.06311419229
 W(22)=.06392423858 \$ W(23)=.06446616444 \$ W(24)=.06473769681
 W(25)=.06473769681 \$ W(26)=.06446616444 \$ W(27)=.06392423858
 W(28)=.06311419229 \$ W(29)=.06203942316 \$ W(30)=.06070443917
 W(31)=.05911483970 \$ W(32)=.05727729210 \$ W(33)=.05519950370
 W(34)=.05289018949 \$ W(35)=.05035903555 \$ W(36)=.04761665849
 W(37)=.04467456086 \$ W(38)=.04154508294 \$ W(39)=.03824135107
 W(40)=.03477722256 \$ W(41)=.03116722783 \$ W(42)=.02742650971
 W(43)=.02357076084 \$ W(44)=.01961616046 \$ W(45)=.01557931572
 W(46)=.01147723458 \$ W(47)=.007327553901 \$ W(48)=.003153346052
 A1=6.36739E6

C
C

INPUT DATA

10 READ 1,CNDK,EPS2,DMIN,DELTA,DMAX,NHOP,NT,POWER,KASE,ICOND,
 1(XD(I),NH(I),I=1,ICOND)

C CNDK = GROUND CONDUCTIVITY IN MHOS/METER
 C EPS2 = DIELECTRIC CONSTANT OF THE GROUND RELATIVE TO FREE SPACE
 C NOTE. THE GROUND WAVE AND THE FIRST NHOP HOPS WILL BE
 C CALCULATED EVERY DELTA KM. FROM DMIN KM. TO DMAX KM USING NHOP HOPS,
 C THEN ON TO XD(1) KM USING NH(1) HOPS,
 C THEN ON TO XD(2) KM USING NH(2) HOPS,
 C AND SO ON TO ICOND SECTIONS. THE MAX OF NHOP OR NH IS 9.
 C NT = NUMBER OF ANGLES OF INCIDENCE FOR WHICH THE REFLECTION
 C COEFFICIENTS ARE READ INTO THE PROGRAM
 C POWER IS THE RADIATED POWER IN WATTS RELATIVE TO DIPOLE IN HALF SPACE
 C KASE = 1, THE AMPLITUDE AND PHASE OF THE WAVE HOPS ARE
 C PUNCHED ON DATA CARDS - KASE = 0, NO DATA CARDS ARE OBTAINED

DS=-1.
 IF = 1

C CONTROL VARIABLES. DS IS DISTANCE MEMORY IN INTEGRATION LOOP.
 C IF IS USED TO INDICATE FIRST OR SUBSEQUENT ENTRY TO CWGW.
 C DSAV AND KMAX ARE USED IN LPAINR TO PREVENT RECALCULATION OF
 C AVAILABLE NUMBERS.

DSAV=-1. \$ KMAX=0
 ICD=0
 MHOP=NHOP
 XDIST=DMAX
 IF(EOF,60)999,20
 20 IF(ICOND .LE. 0) GO TO 25
 MHOP=NH(ICOND)
 XDIST=XD(ICOND)

C
C

IDENTIFICATION OF THE PROFILE

```

25 READ 4, ID(1), ID(2), ID(3), ITIM, IPHI, PHIA, ID(4), ID(5), ID(6), ID(7),
   7 ID(8)

C      THESE PARAMETERS ARE FOR IDENTIFICATION ONLY AND USUALLY
C      INCLUDE A NAME, TIME, GEOGRAPHIC AZIMUTH, DIP ANGLE,
C      AND MAGNETIC FIELD INTENSITY.
C
C      READ ANISOTROPIC REFLECTION COEFFICIENTS

DO 30 N=1, NT
READ 3, FREQ, PHI(N), (TAMP(L, N), TPHA(L, N), L=1, 4), HBOT, HREF(N)

C      FREQ = FREQUENCY IN KHZ
C      PHI = ANGLE OF INCIDENCE IN DEGREES
C      TAMP = AMPLITUDE OF REFLECTION COEFFICIENT
C      TPHA = PHASE OF REFLECTION COEFFICIENT
C      NOTE. THE REFLECTION COEFFICIENTS SHOULD BE GIVEN IN THE
C      FOLLOWING ORDER- TEE, TEM, TME AND TMM
C      HBOT = BOTTOM OF THE PROFILE IN KM
C      HREF = REFLECTION HEIGHT IN KM

30 CPHI(N)=COSF(PHI(N)*.01745329252)
   OMEGA=FREQ*6283.185307
   QKA1=WAVE=OMEGA/2.997925E8
   NPI=(CPHI(NT-1)*TPHA(1, NT)-CPHI(NT)*TPHA(1, NT-1))/
   1(6.283185307*(CPHI(NT-1)-CPHI(NT)))
   H=HREF(NT)+NPI*6.283185307/(2.*WAVE*CPHI(NT))*0.001

C
C      CALCULATION OF THE VARIABLES THAT ARE NOT A FUNCTION
C      OF DISTANCE

AK1=A1*QKA1
A2=A1+H*1.E3
SQ2=CMPLX(WAVE*WAVE*EPS2, -12.56637061E-7*OMEGA*CN DK)
QKA2=CSQRT(SQ2)
R12=QKA1/QKA2
R21=QKA2/QKA1
V=CUBERTF(AK1/2.)
V2=V*V
Y=QKA1*H*1000./V
33 FK= 30.* SQRT(3.141592653*POWER/120.)
   Z=1.25/V2
   Q(1)=V*R12*(0., -1.)*CSQRT(1.-R12*R12)
   ETA=R21*R21
   Q(2)=ETA*Q(1)

C
C      INITIALIZE
C      NN=1      SIGNALS FIRST TIME IN NUMERICAL INTEGRATION

NN=1
DO 35 J=1, MHOP
AMP(J)=0.
35 PH(J)=0.

```

```

C          PRINT LABELS AND VALUES OF CONSTANTS

      PRINT 5,POWER,FREQ,CNDK,EPS2
      PRINT 2,ID(1),ID(2),ID(3),ITIM,IPHI,PHIA,ID(4),ID(5),ID(6),ID(7),
      7ID(8),H
      PRINT 36
36  FORMAT( *0   PHI  COS(PHI)*, 7X, *T E E*, 15X, *T E M*, 15X, *T M
      1E*,15X, *T M M*, 12X, *HBOT*, 5X, *HREF*/
      215X, 4(*   AMPLITUDE  PHASE *) )

C
C          ADJUST PHASE OF T TO REFERENCE HEIGHT
      DO 55 L=1,4
      DO 40 N=1,NT
      TPHA(L,N)=TPHA(L,N)+2.*WAVE*(H-HBOT)*CPHI(N)*1000.
      IF(TPHA(L,N).GT. 6.283185307) TPHA(L,N)=MODF(TPHA(L,N),
      1 6.283185307)
40  CONTINUE

C          MAKE PHASE OF REFLECTION COEFFICIENT CONTINUOUS AS A FUNCTION
C          OF COS(PHI)

      CALL GUDFAZ(L,CPHI,TPHA,NT)

C
C          COMPUTE A FOR THIS ANGLE

      DO 50 N=1,NT
      IF( L .EQ. 2 .OR. L .EQ. 3) GO TO 47
      IF ( L .EQ. 4)
      1PRINT 37, PHI(N), CPHI(N),(TAMP(I,N),TPHA(I,N),I=1,4),HBOT,HREF(N)
37  FORMAT( F7.2, F8.4, 4(F10.5, F10.3), 2F10.1)
45  TPR(L,N)=LOGF(TAMP(L,N))/CPHI(N)
      TPI(L,N)=(TPHA(L,N)-3.141592653)/CPHI(N)
      GO TO 50
47  TPR(L,N) = TAMP(L,N)
      TPI(L,N) = TPHA(L,N)
50  CONTINUE
55  CONTINUE

C          PRINT 39
39  FORMAT(*0DISTANCE*,8X,*TOTAL*, 5X, *GR WAVE*, 3X, *HOP 1*, 5X,
      1*HOP 2*, 5X, *HOP 3*, 5X, *HOP 4*, 5X, *HOP 5*, 5X, *HOP 6*, 5X,
      2*HOP 7*, 5X, *HOP 8*, 5X, *HOP 9*)

C
C          INITIALIZE JGO,      METHOD SELECTOR FLAG FOR CALCULATING I SUB J,M
C
C          GEOMETRICAL OPTICS FOR SHORTEST DISTANCES      JGO=4
C          SADDLE POINT FOR SHORT DISTANCES                JGO=3
C          NUMERICAL INTEGRATION FOR LONG DISTANCES        JGO=2
C          RESIDUE SERIES FOR LONGEST DISTANCES            JGO=1
C          (UNLESS NHOP .GT. 5, THEN USE NUM. INTEG.)

      DO 79 J=1,MHOP

```

```

      JGO(J)=4
      IF(COSF(DMIN/(.002*J*A1)) .LE. (A1/A2)) JGO(J)=1
79  CONTINUE
C
C      BEGINNING OF THE LOOP DMIN (DELTA) DMAX - LOOP ENDS
C      AT 995

80  THETA=DMIN*1.E3/A1
      X=V*THETA
      X2=X*X
      STH=SIN(THETA)
      AK1D=QKA1*DMIN*1000.
      F3=FK/SQRT(A1*A1*THETA*STH)
      F2= SQRT(2.*X/3.141592653)*F3*(-1.,1.)
      F1= (0., 6.283185307) *F2
      DO 85 J=1,MHOP
      DO 85 M=1,J
85  HOP(J,M)=0
C
C      CALCULATION OF E SUB 0, THE GROUND WAVE

      CALL CWGW( IF, DMIN, 0.,0.,1., GW , Q(1), X,Z)
      SUM = GW
C
C      BEGIN HOP LOOP
C
      DO 980 J=1,NHOP
      TUJ=2*J
      TOTJ=THETA/TUJ
      GO TO (700,500,300,100),JGO(J)
C
C
C      BEGIN GEOMETRICAL OPTICS METHOD OF CALCULATING I SUB J,M
C
100 DJ=TUJ *SQRTF(2.*A1*A2*OMCOS(TOTJ)+H*H*1.E6)
      SINTAU=TUJ *A2*SINF(TOTJ)/DJ
      COSSQT=1.-SINTAU*SINTAU
      COSTAU=SQRTF(COSSQT)
      COSPHI =(DJ+TUJ*A1*COSTAU)/(TUJ*A2)
      COS5=COSSQT*COSSQT*COSTAU
C
      IS COS TAU SMALL ENOUGH TO USE SADDLEPOINT FORMULA

      IF(COS5 .LE. 1./AK1) GO TO 295
150 BJ=A2/A1*SQRTF(DJ*SINTAU*COSPHI/(A2*STH*COSTAU))
      S=(0.,1.)*V*COSTAU
      C1 = CMPLX(COSPHI,0.)
      RM=(S+Q(2))/(S-Q(2))
      RE=(S+Q(1))/(S-Q(1))
      P= QKA1*DJ-AK1D
      S1=BJ*SINTAU*SINTAU* FK/DJ*(1.+RE)**2
      I * CMPLX(COS(P),-SIN(P))
      HOP(J,1) = S1*RE**(J-1)
      DO 175 M=2,J

```

```

175 HOP(J,M) = HOP(J,M-1)*RM/RE
GO TO 900
C
295 JGO(J)=JGO(J)-1
C
C
C
C BEGIN SADDLE POINT METHOD OF CALCULATING I SUB J,M
C
300 P=(4.*J*J*Y-X2)/(4.*X*J)
C
C IS P SO SMALL WE MUST INTEGRATE NUMERICALLY
C
IF(P .LT. 2.+(J-1.)*.15) GO TO 495
301 P2=P*P
DJ=SQRT(Y+P2)
C1= CMPLX (DJ/V,0.)
OMEGA = -X*P2+1.333333*J*((Y+P2)*DJ-P2*P)
S2= CMPLX(0.,P)
RE=(S2+Q(1))/(S2-Q(1))
RM=(S2+Q(2))/(S2-Q(2))
S = SQRT((1.+X/(2*J*P))*(1.-.5*P2/V2)**5)*(1.+RE)**2
1 *CMPLX(COS(OMEGA), -SIN(OMEGA))*RE**(J-1)
HOP(J,1) = F3*S
DO 475 M=2,J
475 HOP(J,M) = HOP(J,M-1)*RM/RE
GO TO 900
C
495 JGO(J)=JGO(J)-1
C
C
C THE PATH INTEGRALS ARE CALCULATED USING NUMERICAL
C INTEGRATION. GAUSSIAN INTEGRATION
C IS USED WITH 48 ABSCISSAS AND WEIGHTS. THE CONTOURS
C OF INTEGRATION CONSIST OF TWO SEGMENTS, T GOING
C FROM 4 TO 0 AND THEN INTO THE LOWER HALF PLANE AT A
C SLOPE OF 1/4
C
500 IF(COSF(THETA/TUJ) .LE. A1/A2) GO TO 695
C
C IF WE ARE IN THE SHADOW REGION (TAU .GE. 90 DEGREES)
C USE RESIDUE SERIES.
C
501 IF(NN.EQ.2)GO TO 600
C
C THE FIRST TIME A HOP IS CALCULATED WITH NUMERICAL
C INTEGRATION THE COEFFICIENT OF E**(-IXT) IS
C DETERMINED FOR THE GAUSSIAN ABSCISSAS AND STORED FOR
C FUTURE USE
C
TOP=0.
BOT=4.
KK=0
DO 590 L=1,2
DO 575 K=1,48
KK=KK+1

```

```

      IF(L.EQ.2)GO TO 520
      OI=0.
      OR=((TOP-BOT)*G(K)+TOP+BOT)*.5
      GO TO 525
520  OI=((TOP-BOT)*G(K)+TOP+BOT)*.5
      OR= OI*4.
525  O(KK)=CMPLX(OR,OI)
      CALL CWAIRY(1,O(KK),T1,M1,T2,M2)
      CALL CWAIRY(2,O(KK),DT1,MD1,DT2,MD2)
      CALL CWAIRY(1,O(KK)-Y,TY1,MTY1,TY2,MTY2)
      S2=DT1*(2.718281828**MD1)-Q(1)*T1*(2.718281828**M1)
      R1=(1.+Z*O(KK))/(S2*S2)
      R2=2.718281828**((MTY1-MTY2)*TY1/TY2)
      R3=2.718281828**((M2-M1)*T2/T1)
      S1=2.718281828**((MD2-M2)*DT2/T2)
      S2=2.718281828**((MD1-M1)*DT1/T1)
      R4=(S1-Q(1))/(S2-Q(1))
      R5=(S1-Q(2))/(S2-Q(2))
      S1= (1.,0.)
      RR= R1/R3
      JM=0
      DO 575  JJ=1,MHOP
      RR =-RR*R2*R3
      JM=JM+1
      PT(JM,KK)=RR*S1
C
C      PT(JM,KK) = THE ARRAY OF COEFFICIENTS OF E**(-IXT)
C      FOR THE 48 GAUSSIAN ABSCISSAS
C
C      NOW CALCULATE PT  FOR ALL HOPS
C
      S1= S1*R4
      DO 574 M=2,JJ
      MJ=JM
      JM=JM+1
574  PT(JM,KK)=PT(MJ,KK)*R5/R4
575  CONTINUE
      FTX(L)=.5*(TOP-BOT)
      BOT=0.
590  TOP=-9./(2.*SQRTF(4.+Y)-4.)
      NN=2
600  KK=0
C
C      CALCULATION OF THE INTEGRAL
C
      DO 650 L=1,2
      DO 650 K=1,48
      HH=W(K)*FTX(L)
      KK=KK+1
      IF(DS .EQ. DMIN) GO TO 625
      IF(L.EQ.2)GO TO 620
      ARG=X*REAL(O(KK))
      SD(KK)=CMPLX(COSF(ARG),-SINF(ARG))
      GO TO 625

```



```

620 SD(KK)=CEXP(CMPLX(0.,-X)*O(KK))
625 CONTINUE
    JM= (J*(J-1))/2
    DO 650 M=1,J
    JM=JM+1
    IF(L.EQ.2)GO TO 630
C
C          SUMMING OF THE INTEGRAND FOR THE FIRST INTERVAL
C          REAL PLANE
    HOP(J,M)=HOP(J,M)+HH*PT (JM,KK)*SD(KK)*F2
    GO TO 650
C
C          SUMMING OF THE INTEGRAND FOR THE SECOND INTERVAL
C          COMPLEX PLANE
630 B=PT (JM,KK)*SD(KK)
    S1=B*(4.,1.)
    HOP(J,M) = HOP(J,M) +HH*S1*F2
650 CONTINUE
    ALP=(4.*J*J*Y-X2)/(4.*X*J)
    C1= CMPLX(SQRT(Y+ALP*ALP)/V,0.)
    DS=DMIN
    GO TO 900
C
695 IF(J .GT. 5)      GO TO 501
    JGO(J)=JGO(J)-1
C
C          THE PATH INTEGRALS ARE CALCULATED WITH THE RESIDUE
C          SERIES
700 MO=0
800 DO 850 K=1,20
C          LPAINR COMPUTES THE RESIDUES FOR ALL M FOR THIS J.
    CALL LPAINR(J,K,R)
    DO 825 M=1,J
    HOP(J,M) = HOP(J,M)-(R(1,J,M)+R(2,J,M))*F1
    IF(K.EQ.1)GO TO 825
    IF(CABS(((R(1,J,M)+R(2,J,M))*F1)/HOP(J,M)) .GT. 0.0005) GO TO 825
    MO=MO+1
    IF(MO .NE. J)      GO TO 825
    GO TO 885
825 CONTINUE
850 CONTINUE
885 CONTINUE
    DCRIT = SQRT( .008*A1*H)
    S1= MIN1F( 1., (DMIN-J*DCRIT)/(300*J))*T(1)
    C1= CSQRT(((2000.*H/A1-S1      /V2)/(1.+2000.*H/A1))
C
C          COMPUTE TEE,TEM,TME AND TMM

```

```

900 CONTINUE
    IF(JGO(J) .EQ. 1) GO TO 904
895 C1= CMPLX( (A1*OMCOS(TOTJ)+H*1000.)/SQRT(2.*A1*A2*OMCOS(TOTJ) +
    1 H*H*1.E6),0.)

C      IF PHI IS MUCH SMALLER THAN ANY INPUT VALUE, SET TA (AND HENCE
C      THIS HOP) TO A VERY SMALL VALUE AS AN INDICATOR.

904 CONTINUE
    IF(REAL(C1) .LT. 1.2*CPHI(1)) GO TO 905
    TA(J,1)=TA(J,2)=TA(J,3)=TA(J,4)=1.E-25
    GO TO 940
905 C2=C1*C1
    NL=NT-1
    NI=1      $ NIP = NI+1
    CY = REAL(C1)

C      DETERMINE WHICH VALUES OF T(COS(PHI)) TO USE IN INTERPOLATION

    IF( CY .GT. CPHI(1)) GO TO 908
906 IF((CY .LE. CPHI(NI)) .AND. (CY .GT. CPHI( NIP))) GO TO 908
    IF(NI .GE. NL) GO TO 908
    NI = NIP      $ NIP = NIP +1
    GO TO 906
908 CX = CPHI (NI) - CPHI(NIP)

C      INTERPOLATE TO GET  A SUB 1  AND  A SUB 2

    DO 925 L=1,4
    ATR( 2) = ( TPR(L,NI) - TPR( L, NIP))/CX
    ATI( 2) = ( TPI(L,NI) - TPI( L, NIP))/CX
    ATR( 1) = TPR(L,NIP) -ATR( 2)*CPHI(NIP)
    ATI( 1) = TPI(L,NIP) -ATI( 2)*CPHI(NIP)
    IF(L.EQ.1.OR.L.EQ.4)GO TO 920
    DY=ATI( 1)+ATI( 2)*REAL(C1)
    DX=EXP(-ATI( 2)*AIMAG(C1))
    TA(J,L)=(ATR( 1)+ATR( 2)*C1)*DX*CMPLX(COS(DY),SIN(DY))
    GO TO 925
920 S=CMPLX(ATR( 1),ATI( 1))*C1+CMPLX(ATR( 2),ATI( 2))*C2
    TA(J,L) = -CEXP(S)
925 CONTINUE

C
C      CALCULATION OF C SUB J,M

940 E(J)=HOP(J,1)*TA(J,1)**J
    DO 960 M=2,J
    C=0.
    AJMK=J-M+1
    M1=M-1
    DO 950 K=1,M1
    IF(K.EQ.1)GO TO 950
    AJMK=(J+2-M-K)*(M-K)/(K*(K-1))*AJMK
950 C      =C      +AJMK*TA(J,1)**(J+1-M-K)*(TA(J,2)*TA(J,3))**K*TA(J,4)
    1** (M-1-K)

```

```

C
C          CALCULATION OF E SUB J, THE WAVE HOPS.
960 E(J)=E(J)+HOP(J,M)*C
C
C          CALCULATION OF E SUB R, THE VERTICAL ELECTRIC FIELD
      SUM=SUM+E(J)
      AMP(J)=CABS(E(J))
980 PH(J) = -CANG(E(J))
C
C      END OF J LOOP
C
      GAMP=CABS(GW)
      GPH = -CANG(GW)
      SAMP=CABS(SUM)
      SPH = - CANG(SUM)
C
C          PRINT AND PUNCH OUTPUT
C
      PRINT 8, DMIN, SAMP, GAMP, (AMP(J), J=1,NHOP)
      PRINT 9, SPH, GPH, (PH(J), J=1,NHOP)
990 IF(KASE.EQ.0)GO TO 995
      PUNCH 12,FREQ,ID(1),ID(2),ITIM,PHIA,DMIN,SAMP,CNDK
12  FORMAT(F7.1,2A8,2X,A8,A6,F9.2,E20.8,F6.1)
C
C          END OF LOOP DMIN (DELTA) DMAX
995 DMIN=DMIN+DELTA
      IF(DMIN-DMAX)80,80,1000
999 CALL EXIT
1000 IF(ICD .GE. ICOND) GO TO 10
      ICD=ICD+1
      DMAX=XD(ICD)
      NHOP=NH(ICD)
      GO TO 80
      END

```

```

SUBROUTINE LPAINR(J,K,R)

C   SUBROUTINE FOR ANIHOP.  COMPUTES RESIDUES FOR GIVEN J,K, AND
C   DISTANCES.

C   IMPORTANT
C   ARRAYS E AND A MUST HAVE THE SAME FIRST DIMENSION FOR USE
C   IN DEPROD

TYPE INTEGER TUJ
COMPLEX Q,T,W,TE,E,A,BJM,AJM,SUM,DA,ZX,ARG,EX,AHAT,R
DIMENSION Q(2),T(40),W(8,2,2),MW(20,2,4),TE(2),E(7,2,2),A(7,15)
1,BJM(6,20,6),AJM(7,20,15),DA(7),EX(6,40),AHAT(6),R(2,5,5)
COMMON/PAIN/X,Y,Z,DMIN,XDIST,DSAV,A1,A2,Q,KMAX,T
COMMON/PROD/C
COMPLEX C(7,15)
DATA (EC=2.718281828)
NP(N) = IZ*5+IS*N
LP(L,N)= L+IZ*(IZ*5+IS*N)

C   IF THIS IS THE FIRST ENTRY FOR THIS MODEL, COMPUTE JRES

IF(DSAV .GT. 0.) GO TO 5
JRES=MINOF(5,XFIXF(XDIST/(.002*A1*ACOSF(A1/A2))))
KEX=0

5 IQ=1

C   IQ=1 FOR Q SUB E POLES.  IQ=2 FOR Q SUB M POLES.

IZ=0
IS=1
NDER=JRES+1
KK=K $ JN=J+1
IW=1 $ IY=2
SIGN=(-1)**(J-1)

C   IF WE HAVE COMPUTED A S AND B S FOR THIS K, GO TO 240

10 IF(K .LE. KMAX) GO TO 240

C   FROM HERE TO 200, COMPUTE AJM AND BJM FOR THIS K.
C   FIRST, FIND T(K) AND AIRY FUNCTIONS OF T.
C   TW FINDS SOLUTIONS OF E1(T,Q)=0.

20 CALL TW(K-1,Q(IQ),T(KK),W(1,1,1),MW(K,1,IW),W(2,1,1),MD1,
AW(1,2,1),MW(K,2,IW),W(2,2,1),MD2)
TE(1)=T(KK)
TE(2)=TE(1)-Y
W(2,1,1)=(EC**((MD1-MW(K,1,IW)))*W(2,1,1)
W(2,2,1)=(EC**((MD2-MW(K,2,IW)))*W(2,2,1)
CALL CWAIRY(1,TE(2),W(1,1,2),MW(K,1,IY),W(1,2,2),MW(K,2,IY))
CALL CWAIRY(2,TE(2),W(2,1,2),MD1,W(2,2,2),MD2)
W(2,1,2)=(EC**((MD1-MW(K,1,IY)))*W(2,1,2)

```

```

W(2,2,2)=(EC*(MD2-MW(K,2,IY)))*W(2,2,2)

C      NOW GET HIGHER DERIVATIVES OF W AND E

      NTOP=NDER+2
      DO 30 N=3,NTOP
      DO 29 I=1,2
      DO 28 KIND=1,2
      W(N,KIND,I)=TE(I)*W(N-2,KIND,I)+(N-3)*W(N-3,KIND,I)
      E(N-1,KIND,I)=W(N,KIND,1)-Q(I)*W(N-1,KIND,1)
      IF(N .GT. 3) GO TO 28
      E(1,KIND,I)=W(2,KIND,1)-Q(I)*W(1,KIND,1)
28 CONTINUE
29 CONTINUE
30 A(N-2,1)=W(N-2,1,2)

C      W SUB ONE (T-Y) AND DERIVATIVES ARE IN COL. 1 OF A.

C      NOW COMPUTE BJM
40 BJM(LP(1,1),K,NP(1))=E(2,1,IQ)
      DO 50 N=2,NDER
      BJM(LP(1,N),K,NP(N)) = BJM(LP(1,N-1),K,NP(N-1))*E(2,1,IQ)
      DO 48 L=2,N
      SUM=0.
      KF=1
      KKT=L-1
      DO 46 KI=1,KKT
      KF=KF*(KI+1)
      SUM=(KI*(N+1)-KKT)/(KKT*KF)*BJM(LP(L-KI,N),K,NP(N)) *
1 E(KI+2,1,IQ)+SUM
46 CONTINUE
48 BJM(LP(L,N),K,NP(N)) = SUM/E(2,1,IQ)
50 CONTINUE

C      DERINV FINDS DERIVATIVES OF 1/F.

      CALL DERINV(W(1,2,2),NDER-1,A(1,2))

C      GET W/W AND DERIVATIVES AND PUT IN COL. 2 OF A.

C      DEPROD COMPUTES DERIVATIVES OF A PRODUCT OF N FUNCTIONS.

      CALL DEPROD(A,2,NDER-1,A(1,2))

C      NOW PUT D(2) AND DERIVATIVES IN COL. 1 OF A.

      A(1,1)=1.+Z*TE(1)
      A(2,1)=Z
      A(3,1)=A(4,1)=A(5,1)=A(6,1)=0.

      GO TO (70,150),IQ

```

C AJM FOR QE POLES

```
70 CONTINUE
   CALL DERINV(E(1,1,2),NDER-1,E(1,1,2))
   CALL DEPROD(E(1,1,2),2,NDER-1,A(1,15))
```

C E2/E1 AND ITS DERIVATIVES ARE NOW IN COL 15 OF A.
C NOW CALCULATE AJM/EXP(-IXT)

```
      JRES2 = JRES+JRES
      DO 75 JX=4,JRES2,2
      DO 74 I=1,NDER
        A(I,JX)= E(I,2,1)
74    A(I,JX-1) = A(I,2)
75    CONTINUE
      DO 90 M=1,JRES
        CALL DEPROD (A,JRES2,JRES-M+1,DA)
      DO 80 JX=M,JRES
        JM= JX-5+(M*(11-M))/2
        IM= JX-M+2
      DO 78 I=1,IM
78    AJM(I,K,JM) = C(I,JX+JX)
80    CONTINUE
      IF(M .EQ. JRES) GO TO 240
      JP = M+M+2
      IM=NDER-M
      DO 82 I=1,IM
82    A(I,JP) = A(I,15)
90    CONTINUE
      GO TO 240
```

C AJM FOR QM POLE

```
150 AJM(3,K,6) = A(1,1)*E(1,2,2)*(A(1,2)/E(1,1,1))*2
   IF(JRES .LT. 3) GO TO 200
   ZX=A(1,2)*E(1,2,1)/E(1,1,1)
   CALL DERINV(E(1,1,1), NDER-1, E(1,1,1))
   DO 160 N=1,NDER
     A(N,4)=A(N,3)=A(N,2)
     A(N,6)=A(N,5)=E(N,1,1)
     A(N,8)=A(N,7)=E(N,2,2)

     JX=N+2
     IF(N .GE. NDER) GO TO 161
```

C STATEMENT 160 GETS AJM SUB ZERO FOR J=N+1, M=2

```
160 AJM(1+JX, K, N+6) = AJM(JX,K,N+ 5) *ZX
161 CONTINUE
```

C GET AJM SUB ZERO AND ONE FOR J=3, M=3

```
      CALL DEPROD(A,8,NDER-1,A(1,1))
      AJM(3,K ,10)=A(1,1)
```

```

    AJM(4, K, 10) = A(2, 1)
    IF( JRES .LT. 4) GO TO 200
    CALL DEPROD(E(1,1,1), 2, NDER-1, A(1,5))
    DO 170 N=1, NDER
    A(N,3) = A(N,5)
170 A(N,4) = A(N,2)
    DO 190 M=3, JRES
    JM = (M*(11-M))/2-1
    M1 = M-1
175 CALL DEPROD(A, 5, M-2, AJM(8-M, K, JM+1))
    IF(M .EQ. 5) GO TO 200
    DO 176 I=1, M1
176 AJM(I+6-M, K, JM) = C(I,3)
178 MF = 2*M-3
    DO 180 N=1, NDER
180 A(N, MF) = E(N, 2, 2)
190 CONTINUE
200 CONTINUE
    KMAX = K
240 IF(DSAV .NE. DMIN) GO TO 250
    IF(K .LE. KEX) GO TO 300

C      COMPUTE EXP(-IXT) AND DERIVATIVES.

250 ARG = CMPLX(0., -X)
    EX(1, KK) = CEXP(ARG*T(KK))
    DO 260 N=2, NDER
260 EX(N, KK) = ARG*EX(N-1, KK)
    IF ( IQ .EQ. 2) KEX = K
300 SCALE = 2.718281828**((J*(MW(K,1,IY)-MW(K,2,IY))+(J-1)*(MW(K,2,IW)
    A-MW(K,1,IW)))*SIGN

C      COMPUTE RESIDUES FOR I-SUB-JM FOR THIS POLE.

    DO 330 M=1, J
    JB = J-M+2
    IN = 0
    IF(IQ .EQ. 1) GO TO 301
    IN = JB
    JB = M-1
301 JM = J-5+(M*(11-M))/2
    IF(JB-1) 302, 303, 304
302 R(IQ, J, M) = 0.
    GO TO 330
303 R(IQ, J, M) = SCALE*AJM(1+IN, K, JM) *EX(1, KK)/BJM(LP(1,1), K, NP(1))
    GO TO 330
304 DO 305 N=1, JB
    A(N, 1) = AJM(N+IN, K, JM)
305 A(N, 2) = EX(N, KK)

C      COMPUTES A-SUB-N, EQ(36).

    CALL DEPROD(A, 2, JB-1, DA)
    NPB = NP(JB)

```

FACT=1.

C NOW COMPUTE A-HATS USING (38).

```
      AHAT(1)=DA(1)/BJM(LP(1,JB),K,NPB)
      DO 315 N=2,JB
      DA(N)=DA(N)/FACT
      SUM=DA(N)
      IM=N-1
      DO 312 L=1,IM
312  SUM=SUM-BJM(LP(N-L+1,JB),K,NPB)      )*AHAT(L)
      AHAT(N)=SUM/BJM(LP(1,JB),K,NPB)
315  FACT=FACT*N
      R(IQ,J,M)=SCALE*AHAT(JB)
330  CONTINUE
      IF(IQ .EQ. 2) GO TO 350
      IQ=2
      IZ=1
      IS=-1
      KK=K+20
      IW=3
      IY=4
      JN=J-1
      NDER=JRES-1
      GO TO 10
350  DSAV=DMIN
      RETURN
      END
```



```

SUBROUTINE DEPROD(A,JF,JN,DA)

C      INPUT
C      A(I,J)=THE (I-1)TH DERIVATIVE (I=1,JN+1) OF FUNCTION A(I,J),(J=1,JF)

C      OUTPUT
C      DA(I)=THE (I-1)TH DERIVATIVE OF THE PRODUCT A(1,1)*A(1,2)*...A(1,JF)
C              IMPORTANT
C      INPUT AND OUTPUT ARRAYS IN DEPROD MUST HAVE SAME FIRST
C      DIMENSION FOR USE WITH LPAINR

C      MAKES REPEATED USE OF LEIBNITZS RULE.

COMMON/PROD/C
COMPLEX A(7,15),DA(15),C(7,15)
N1=JN+1
DO 25 N=1,N1
25 C(N,1)=A(N,1)
DO 50 M=2,JF
50 C(1,M)=C(1,M-1)*A(1,M)
DO 75 M=2,JF
DO 74 N=2,N1
C(N,M)=C(1,M-1)*A(N,M)
FACN=1.
FACL=1.
DO 73 L=2,N
FACN=FACN*(N-L+1)
FACL=FACL*(L-1)
73 C(N,M)=C(N,M)+FACN/FACL*C(L,M-1)*A(N-L+1,M)

C      C(N,M)=N TH DERIVATIVE OF PRODUCT (A(1,L),L=1,M)

74 CONTINUE
75 CONTINUE
DO 100 I=1,N1
100 DA(I)=C(I,JF)
RETURN
END

```

```

SUBROUTINE CWGW ( MM, DMIN, H1, H2, ALFA, GW , Q, X,Z)

C   CALCULATION OF THE GROUND WAVE
C   INPUT
C       DMIN = DISTANCE BETWEEN TRANSMITTER AND RECEIVER IN KM
C       H1 = HEIGHT OF THE TRANSMITTER IN KM
C       H2 = HEIGHT OF THE RECEIVER IN KM
C       ALFA = EFFECTIVE EARTH RADIUS
C       Q = -I*V*(NORMALIZED SURFACE IMPEDANCE)
C       X = V*THETA
C       Z = 1.25/V**2

C   OUTPUT
C       GW = THE GROUND WAVE

COMMON/WGW/QKA1,QKA2,FK,AK1,V,V2, THETA,STH
COMPLEX T(200),W(200),G,QKA2,R12,S,D,Q,W1,DW1,WY1,WY2,GW,S2,TS
Y1=QKA1*H1*1000./V
Y2=QKA1*H2*1000./V
COTH=COS(THETA)/STH
FAC=FK*SQRT(6.283185307 *X/((6.36739E6*ALFA)**2*THETA*STH))
GW=0.

C       IF THIS IS THE FIRST ENTRY COMPUTE T-SUB-S AND
C       COEFFICIENT OF EXP(-IXT)
GO TO(60,125),MM
60 J2=1
MM=2
65 DO 100 J=J2,200

C       FIND LOCATION OF POLE.

CALL TW( J-1 , Q, T(J), W1, MW1, DW1, MD1, S,M,S,M)
IF(H1.GT.0)GO TO 80
IF(H2.GT.0)GO TO 75
W(J)=1.
GO TO 85

C       HEIGHT GAIN FUNCTIONS.

75 CALL CWAIRY(1,T(J)-Y2,WY2,MY2,S,M)
W(J)=2.7182818**((MY2-MW1)*WY2/W1
GO TO 85
80 CALL CWAIRY(1,T(J)-Y1,WY1,MY1,S,M)
W(J)=2.7182818**((MY1-MW1)*WY1/W1
IF(H2.GT.0)GO TO 85
CALL CWAIRY(1,T(J)-Y2,WY2,MY2,S,M)
S=2.7182818**((MY2-MW1)*WY2/W1
W(J)=W(J)*S
85 W(J)=W(J)*(1.+Z*T(J))/(T(J)-Q*Q)

C       COMPUTE TERM OF RESIDUE SERIES AND ADD.

```

```

      G=(1.+(CMPLX(3.,COTH)))/(8.*(AK1+V*T(J)))*CEXP((0.,-1.)*X*T(J))
      G=W(J)*G
      GW=GW+G
      IF(J.EQ.1)GO TO 100
      IF( CABS(G/GW) .GT. 0.0005) GO TO 100
      J1=J
      GO TO 110
100  CONTINUE
      J2=200
      GO TO 165
110  IF(J1.LE.J2)GO TO 165
      J2=J1
      GO TO 165
125  DO 140 J=1,J2
      G=(1.+(CMPLX(3.,COTH)))/(8.*(AK1+V*T(J)))*CEXP((0.,-1.)*X*T(J))
      G=W(J)*G
      GW=GW+G
      IF(J.EQ.1)GO TO 140
      IF( CABS(G/GW) .LT. 0.0005) GO TO 165
140  CONTINUE
      IF(J2.GE.200)GO TO 165
      J2=J2+1
      GO TO 65
165  GW=GW*FAC *(1.,-1.)
      RETURN
      END

```

SUBROUTINE ZEXP(A,B,X,Y,MAGTUD)

```

C   SCALED EVALUATION OF THE EXPONENTIAL FUNCTION IN THE COMPLEX PLANE.
C   INPUT.
C   (A+IB) = THE COMPLEX EXPONENT.

C   OUTPUT.
C   EXP(A+IB) = (X+IY)*(E**MAGTUD)

```

```

      MAGTUD=A
      SCALE=MAGTUD
      E=EXP(A-SCALE)
      X=E*COS(B)
      Y=E*SIN(B)
      RETURN
      END

```

```

FUNCTION OMCOS(X)
C   OMCOS(X)= 1 - COS(X)
C   IS ACCURATE FOR ALL X INCLUDING X NEAR 0.

IF(ABS(X).GT..15) GO TO 40
IF(X.EQ.0.) GO TO 50

C       IF X IS SMALL, SUM TAYLORS SERIES FOR 1-COS(X)

S = X*X
OMCOS = T = .5*S
R = 4.
10 T=-T*S/(R*(R-1.))
OMCOS=OMCOS + T
IF(ABS(T/OMCOS) .LE. .5E-9) GO TO 51
R=R+2.
GO TO 10
40 OMCOS = 1.-COS(X)
RETURN
50 OMCOS = 0.
51 RETURN
END

```

```

SUBROUTINE DERINV(F,N,EF)

C   INPUT
C   F(K) = THE (K-1)TH DERIVATIVE OF F, K=1,N+1

C   OUTPUT
C   EF(K) = THE (K-1)TH DERIVATIVE OF 1/F, K=1,N+1

COMPLEX F(15),DF(15),EF(15)
DF(1)=1./F(1)
DO 50 K=1,N
K1=K+1
FACK=1.
FACM=1.
DF(K1)=0.
DO 50 M=1,K
FACK=FACK*(K1-M)
FACM=FACM*M
50 DF(K1)=DF(K1)-((FACK/FACM*F(M+1)*DF(K1-M))/F(1))
M=N+1
DO 60 K=1,M
60 EF(K)=DF(K)
RETURN
END

```

```

SUBROUTINE TW( I, Q, T, W1,MW1, DW1,MD1, W2,MW2, DW2,MD2)
C
C T IS THE I-TH ROOT OF W-SUB-ONE-PRIME - Q*W-SUB-1 =0.(W IS THE AIRY FUNCTION)
C THE ROOTS ARE COUNTED IN ORDER OF INCREASING MAGNITUDE.
C W-SUB-ONE(T) = EXP(MW1)*W1, W-SUB-ONE-PRIME = EXP(MD1)*DW1,
C W-SUB-TWO = EXP(MW2)*W2, ETC.
C
C DIMENSION TZERO(11), TINFIN(11)
C COMPLEX Q,W1,DW1,W2,DW2, PH, A ,T
C
C W-SUB-ONE-PRIME(TZERO(I)) =0.
C
C DATA (TZERO= 1.018793, 3.2481975, 4.8200992, 6.1633074,
1 7.3721773, 8.4884868, 9.5354490, 10.52766, 11.475057, 12.384788,
2 13.262219)
C
C W-SUB-ONE(TINFIN(I)) =0.
C
C DATA (TINFIN = 2.3380997, 4.0879494, 5.5205598, 6.7867081,
1 7.9441336, 9.0226508, 10.040174, 11.008524, 11.936016, 12.828777,
2 13.691489)
C DATA (PH =(0.5, -0.8660254)), ( CON= 1.17809724)
C
C IF( REAL(Q)**2 + AIMAG(Q)**2 .GT. 1.) GO TO 50
C IF(I .GT. 10) GO TO 10
C TZ = TZERO(I+1)
C GO TO 20
10 YS = ((4*I+1)*CON)**2
C TZ = CUBERTF(YS) *(1.-.1458333/YS)
20 T = TZ*PH
C
C T IS NOW SOLUTION FOR Q =0. THE NEXT STEP IS THE FIRST NEWTON ITERATION.
C
C T = T+Q/T
C GO TO 100
50 IF(I .GT. 10) GO TO 60
C TZ = TINFIN(I+1)
C GO TO 70
60 YS = ((4*I+3) *CON)**2
C TZ= CUBERTF(YS)*(1.+.1041667/YS)
70 T = TZ*PH
C
C T IS SOLUTION FOR Q=INFINITY. NEXT STEP IS THE FIRST NEWTON ITERATION.
C
C T = T+1./Q
100 K=0
C
C NOW, USE NEWTONS ITERATION TO CONVERGE ON SOLUTION
C CWAIRY COMPUTES W(T) AND W PRIME (T)
C
101 CALL CWAIRY(1,T,W1,MW1,W2, MW2)
C CALL CWAIRY(2,T,DW1,MD1,DW2,MD2)
C A=(2.718281828**((MD1-MW1))*DW1/W1
C A = (A-Q)/( T -A*Q)

```

```

T = T-A
K=K+1
IF(K .GT. 15) GO TO 150
IF(CABS(A/T) .GT. 0.5E-6) GO TO 101
RETURN
150 PRINT 155 , I, T, A
155 FORMAT(* FAILED TO CONVERGE ON T(*, I2, *), T = *,
1 C(E14.6,E14.6),* LAST CORRECTION =*, C(E14.6,E14.6))
RETURN
END

```

```

C
C SUBROUTINE GUDFAZ(K,C,FAZ,N)
C
C     SMOOTHS PHASE OF IONO REFLECTION COEFFICIENTS FOR ANIHOP
C     K=1,2,3,4 FOR TEE,TEM,TME,TMM, RESPECTIVELY
C     C=COS(PHI), ARRANGED IN ORDER OF INCREASING PHI
C     (=ANGLE OF INCIDENCE)
C     FAZ=PHASE OF REFLECTION COEFFICIENT, ARRANGED IN
C     SAME ORDER AS C.
C     N=NUMBER OF DATA POINTS
C
C
C     DIMENSION C(20),FAZ(4,20)
C     DATA (PI=3.141592653)
C     C(N+1)=0.
C     FAZ(K,N+1)=PI
3  IF(ABS(PI-FAZ(K,N)) .LE. PI) GO TO 5
   FAZ(K,N)=FAZ(K,N)-SIGNF(2.*PI,FAZ(K,N))
   GO TO 3
5  DO 10 I=2,N
     J=N+2-I
     TEST=PI+(FAZ(K,J+1)-FAZ(K,J))*C(J-1)/(C(J+1)-C(J))
7  TRY=TEST-FAZ(K,J-1)
   IF(ABS(TRY) .LE. PI) GO TO 8
   FAZ(K,J-1)=FAZ(K,J-1)+SIGNF(2.*PI,TRY)
   GO TO 7
8  TRY = FAZ(K,J-1) -FAZ(K,J)
   IF(ABS(TRY) .LT. 2.*PI) GO TO 10
   FAZ(K,J-1) = FAZ(K,J-1) - SIGNF( 2.*PI, TRY)
   GO TO 8
10 CONTINUE
   RETURN
   END

```

```

SUBROUTINE CWAIRY(KK,T,F1,M1,F2,M2)

C   CALCULATION OF THE W(T) AIRY FUNCTIONS
C   INPUT
C       KK=1, W(T) OF KIND 1 AND W(T) OF KIND 2 ARE COMPUTED
C       KK=2, THE DERIVATIVE OF W(T) OF KIND 1 AND THE DERIVATIVE OF
C       W(T) OF KIND 2 ARE COMPUTED
C       T = THE COMPLEX ARGUMENT
C   OUTPUT
C       F1*(E**M1) = W(T) OF KIND 1 OR THE DERIVATIVE OF W(T) OF KIND 1
C       AS INDICATED BY KK
C       F2*(E**M2) = W(T) OF KIND 2 OR THE DERIVATIVE OF W(T) OF KIND 2
C       AS INDICATED BY KK
C   NOTE. F1 AND F2 ARE COMPLEX, E=2.718281828..., AND M1 AND M2 ARE
C   EXPONENTS

COMMON/MEXP/M
COMPLEX F1,F2,WI1,WI2,WI1P,WI2P,T
GO TO(100,200),KK
100 F2=WI1(T)
   M2=M
   F1=WI2(T)
   M1=M
   GO TO 300
200 F2=WI1P(T)
   M2=M
   F1=WI2P(T)
   M1=M
300 F1=1.7724538509*(0.,-1.)*F1
   F2=1.7724538509*(0.,+1.)*F2
RETURN
END

```

COMPLEX FUNCTION AIRY(ZZ)

C SUBROUTINE TO CALCULATE AIRY FUNCTIONS. USES TAYLORS EXPANSIONS
C AROUND VARIOUS COMPLEX CONSTANTS FOR SMALL T AND ASSYMPTOTIC
C SERIES FOR LARGE T. WRITTEN BY DR. GEORGE HUFFORD, ITS,
C BOULDER, COLORADO 80302.

COMPLEX ZZ
COMMON/MEXP/M

C

COMPLEX Z,Z1,A,AP,U,ZT,ZA,ZB,ZE,ZR,B0,B1,B2,B3
DIMENSION X(2),X1(2),XT(2) \$ EQUIVALENCE (X,Z),(X1,Z1),(XT,ZT)
DATA (LG=3),(Z1=(0.,0.)),(A=(0.35502805389,0.))
DATA (AP=(-0.25881940379,0.))

DIMENSION	AV(70) \$ COMPLEX	AV \$ DATA (	AV=	AV	
X	(-3.2914517363E-001, 0.0000000000E+000),			AV	0
X	(-2.6780035625E+000, 1.4774589547E+000),			AV	1
X	(3.5076100903E-001, 0.0000000000E+000),			AV	2
X	(2.4122262158E+000, 6.9865124448E-001),			AV	3
X	(3.3635531189E+001,-3.4600959696E+000),			AV	4
X	(3.4449739613E+002,-3.3690890250E+002),			AV	5
X	(-7.0265532950E-002, 0.0000000000E+000),			AV	6
X	(-5.4818219290E-001,-1.9207365909E+000),			AV	7
X	(-1.3383395342E+001,-1.6022590802E+001),			AV	8
X	(-2.2967795901E+002,-3.2072452637E+001),			AV	9
X	(-1.8040780476E+003, 2.1917675036E+003),			AV	10
X	(-3.7881429368E-001, 0.0000000000E+000),			AV	11
X	(-1.3491836060E+000, 8.4969077213E-001),			AV	12
X	(-6.0453339320E+000, 1.0623175540E+001),			AV	13
X	(3.1169621695E+001, 9.8813517650E+001),			AV	14
X	(9.8925349347E+002, 1.3905286008E+002),			AV	15
X	(2.2740742820E-001, 0.0000000000E+000),			AV	16
X	(7.1857403459E-001, 9.7809094170E-001),			AV	17
X	(6.0621088063E+000, 2.7203014866E+000),			AV	18
X	(3.6307084828E+001,-2.0961355813E+001),			AV	19
X	(-6.7139789190E+001,-3.0904638708E+002),			AV	20
X	(-2.8001653691E+003, 4.6649365984E+002),			AV	21
X	(5.3556088329E-001, 0.0000000000E+000),			AV	22
X	(9.2407365385E-001,-1.9106560052E-001),			AV	23
X	(1.8716185961E+000,-2.5743310394E+000),			AV	24
X	(-7.2188436328E+000,-1.2924200190E+001),			AV	25
X	(-8.1787377840E+001, 3.2087013839E+001),			AV	26
X	(2.9933948552E+002, 5.6922179258E+002),			AV	27
X	(3.5502805389E-001, 0.0000000000E+000),			AV	28
X	(3.1203438104E-001,-3.8845385098E-001),			AV	29
X	(-5.2839999360E-001,-1.0976411220E+000),			AV	30
X	(-4.2009351585E+000, 1.1940151191E+000),			AV	31
X	(7.1858832892E+000, 1.9600912513E+001),			AV	32
X	(1.0129121011E+002,-7.5951233292E+001),			AV	33
X	(1.3529241631E-001, 0.0000000000E+000),			AV	34
X	(3.2618478398E-002,-1.7084872785E-001),			AV	35
X	(-3.4215381085E-001,-8.9067646330E-002),			AV	36
X	(-1.4509641493E-001, 1.0328015748E+000),			AV	37
X	(4.1001968523E+000,-6.8936911760E-001),			AV	38
X				AV	39

X	(-1.3030124036E+001,-1.6910541453E+001),	AV	40
X	(3.4924130423E-002, 0.0000000000E+000),	AV	41
X	(-8.4464726625E-003,-4.2045154421E-002),	AV	42
X	(-6.9313268963E-002, 3.5364798705E-002),	AV	43
X	(1.5227622646E-001, 1.2848454470E-001),	AV	44
X	(1.0681373184E-001,-6.7766153503E-001),	AV	45
X	(-2.6193432727E+000, 1.5699859905E+000),	AV	46
X	(6.5911393574E-003, 0.0000000000E+000),	AV	47
X	(-3.9443985580E-003,-6.8060106117E-003),	AV	48
X	(-5.9820131079E-003, 1.1799010149E-002),	AV	49
X	(2.9922498406E-002,-5.9772930737E-003),	AV	50
X	(-7.7464130231E-002,-5.2292402759E-002),	AV	51
X	(1.1276585896E-001, 3.5112442431E-001),	AV	52
X	(9.5156385121E-004, 0.0000000000E+000),	AV	53
X	(-8.0842995655E-004,-7.6590132690E-004),	AV	54
X	(1.6147816065E-004, 1.7661755136E-003),	AV	55
X	(2.0138718363E-003,-3.1976716632E-003),	AV	56
X	(-9.5086784440E-003, 4.5377832492E-003),	AV	57
X	(3.7560191819E-002, 5.7361916864E-004),	AV	58
X	(1.0834442814E-004, 0.0000000000E+000),	AV	59
X	(-1.0968606480E-004,-5.9902329668E-005),	AV	60
X	(1.0778191327E-004, 1.5771596227E-004),	AV	61
X	(-6.8980937889E-005,-3.7626457370E-004),	AV	62
X	(-1.6166126174E-004, 9.7457773280E-004),	AV	63
X	(9.9476943603E-006, 0.0000000000E+000),	AV	64
X	(-1.0956823939E-005,-2.9508799638E-006),	AV	65
X	(1.4709074502E-005, 8.1042089702E-006),	AV	66
X	(-2.4446015180E-005,-2.0638143108E-005),	AV	67
X	(7.4921288640E-007, 0.0000000000E+000),	AV	68
X	(-8.4619068946E-007,-3.6807338399E-008),	AV	69
X	(1.2183963384E-006, 8.3589199402E-008),	AV	70
DIMENSION APV(70) \$ COMPLEX APV \$ DATA (APV=		APV	0
X	(3.4593548728E-001, 0.0000000000E+000),	APV	1
X	(4.1708876594E+000, 6.2414437707E+000),	APV	2
X	(3.2719281855E-001, 0.0000000000E+000),	APV	3
X	(1.0828742735E+000,-5.4928302529E+000),	APV	4
X	(-2.3363517933E+001,-7.4901848140E+001),	APV	5
X	(-1.0264877579E+003,-5.6707940802E+002),	APV	6
X	(-7.9062857537E-001, 0.0000000000E+000),	APV	7
X	(-3.8085833358E+000, 1.5129605192E+000),	APV	8
X	(-2.6086379081E+001, 3.5540709915E+001),	APV	9
X	(1.0761838222E+002, 5.1239944904E+002),	APV	10
X	(6.6597797197E+003, 1.8096186253E+003),	APV	11
X	(3.1458376921E-001, 0.0000000000E+000),	APV	12
X	(1.8715425470E+000, 2.0544836557E+000),	APV	13
X	(2.2591736932E+001, 4.8562995474E+000),	APV	14
X	(1.6162997879E+002,-1.4335597185E+002),	APV	15
X	(-8.0047161665E+002,-2.1527454270E+003),	APV	16
X	(6.1825902074E-001, 0.0000000000E+000),	APV	17
X	(1.3019603890E+000,-1.2290774954E+000),	APV	18
X	(1.5036118745E-001,-1.1008092874E+001),	APV	19
X	(-7.0116800393E+001,-4.0480822759E+001),	APV	20
X	(-4.8317166902E+002, 4.9692755718E+002),	APV	21
X	(4.8970655652E+003, 4.8627290801E+003),	APV	22

X (-1.0160567116E-002, 0.0000000000E+000),	APV 23
X (-5.4826636454E-001,-7.1365288463E-001),	APV 24
X (-4.6749134088E+000,-1.1924245293E-001),	APV 25
X (-1.0536397828E+001, 2.4943711387E+001),	APV 26
X (1.6333770696E+002, 9.0394910688E+001),	APV 27
X (5.6449455285E+002,-1.4248324426E+003),	APV 28
X (-2.5881940380E-001, 0.0000000000E+000),	APV 29
X (-4.8620754109E-001, 1.5689924913E-001),	APV 30
X (-4.7348131897E-001, 1.7093438130E+000),	APV 31
X (7.0373840765E+000, 3.6281824918E+000),	APV 32
X (1.7739586372E+001,-4.0360422402E+001),	APV 33
X (-2.9791511956E+002,-3.8408892977E+001),	APV 34
X (-1.5914744130E-001, 0.0000000000E+000),	APV 35
X (-1.1340423572E-001, 1.9730504925E-001),	APV 36
X (4.0126209154E-001, 3.9222995820E-001),	APV 37
X (1.3348652430E+000,-1.4377272421E+000),	APV 38
X (-7.9022494720E+000,-4.2063644605E+000),	APV 39
X (-1.3892752107E+000, 5.1229416787E+001),	APV 40
X (-5.3090384434E-002, 0.0000000000E+000),	APV 41
X (-1.6832965528E-003, 6.8366967859E-002),	APV 42
X (1.3789401334E-001,-1.1613804016E-002),	APV 43
X (-1.4713730621E-001,-3.7151985747E-001),	APV 44
X (-1.0070196495E+000, 1.1591348425E+000),	APV 45
X (7.5045049133E+000, 4.6913115383E-001),	APV 46
X (-1.1912976706E-002, 0.0000000000E+000),	APV 47
X (5.1468574932E-003, 1.3660891236E-002),	APV 48
X (1.8309710537E-002,-1.8808588497E-002),	APV 49
X (-6.4461593156E-002,-1.3611794718E-002),	APV 50
X (1.0516239905E-001, 1.9313053560E-001),	APV 51
X (2.0520046212E-001,-9.1772617372E-001),	APV 52
X (-1.9586409502E-003, 0.0000000000E+000),	APV 53
X (1.4695649526E-003, 1.8086384633E-003),	APV 54
X (5.9709947951E-004,-3.8332699216E-003),	APV 55
X (-6.8910893004E-003, 5.4467425272E-003),	APV 56
X (2.6167927738E-002,-8.4092000294E-004),	APV 57
X (-8.8284474192E-002,-4.6475312179E-002),	APV 58
X (-2.4741389087E-004, 0.0000000000E+000),	APV 59
X (2.3707837404E-004, 1.6461109527E-004),	APV 60
X (-1.7465569860E-004,-4.2026783985E-004),	APV 61
X (-1.0394516180E-004, 9.4761844375E-004),	APV 62
X (1.3004110522E-003,-2.2446656813E-003),	APV 63
X (-2.4765200397E-005, 0.0000000000E+000),	APV 64
X (2.6714870982E-005, 9.8691565027E-006),	APV 65
X (-3.3539774178E-005,-2.7113284942E-005),	APV 66
X (4.9197843140E-005, 6.9349092091E-005),	APV 67
X (-2.0081508947E-006, 0.0000000000E+000),	APV 68
X (2.2671244519E-006, 2.7848508382E-007),	APV 69
X (-3.2692132725E-006,-7.3943488682E-007),	APV 70
DIMENSION ASLT(17) \$ DATA (ASLT= 1.1407E+002, 1.1549E+002,	
X 1.1779E+002, 1.2124E+002, 1.2619E+002, 1.3319E+002,	ASLT 0
X 1.4307E+002, 1.5716E+002, 1.7774E+002, 2.0884E+002,	ASLT 1
X 2.5832E+002, 3.4294E+002, 5.0339E+002, 8.5678E+002,	ASLT 2
X 1.8336E+003, 5.7270E+003, 3.5401E+004)	ASLT 3
DIMENSION ASV(21) \$ DATA (ASV= 1.8335766942E+010,	
	ASV 4
	ASV 0

```

X 1.9293755496E+009, 2.1428803701E+008, 2.5198919876E+007, ASV 1
X 3.1482574185E+006, 4.1952487519E+005, 5.9892513580E+004, ASV 2
X 9.2072066015E+003, 1.5331694323E+003, 2.7846508084E+002, ASV 3
X 5.5622785377E+001, 1.2341573335E+001, 3.0794530307E+000, ASV 4
X 8.7766696967E-001, 2.9159139927E-001, 1.1609906404E-001, ASV 5
X 5.7649190421E-002, 3.7993059132E-002, 3.7133487657E-002, ASV 6
X 6.9444444448E-002, 1.0000000000E+000 ASV 7
DIMENSION APSV( 21) $ DATA (APSV=-1.8643931093E+010, APSV 0
X -1.9635237894E+009,-2.1829342088E+008,-2.5697908389E+007, APSV 1
X -3.2145365220E+006,-4.2895240048E+005,-6.1335706678E+004, APSV 2
X -9.4463548250E+003,-1.5763573037E+003,-2.8703323717E+002, APSV 3
X -5.7508303524E+001,-1.2807293083E+001,-3.2104935853E+000, APSV 4
X -9.2047999257E-001,-3.0825376496E-001,-1.2410589605E-001, APSV 5
X -6.2662163500E-002,-4.2462830794E-002,-4.3885030868E-002, APSV 6
X -9.722222227E-002, 1.0000000000E+000 APSV 7
DIMENSION NQTT(15),NQT(8) $ EQUIVALENCE (NQTT(8),NQT(1))
DATA(NQTT=1,3,7,12,17,23,29,35,41,47,53,59,64,68,71)
ANM(Z)=ABSF(REAL(Z))+ABSF(AIMAG(Z))
ENTRY AI $ LA=0 $ GO TO 1
ENTRY AIP $ LA=0 $ GO TO 2
ENTRY WI $ ENTRY WI1 $ LA=1 $ GO TO 1
ENTRY WIP $ ENTRY WI1P $ LA=1 $ GO TO 2
ENTRY WI2 $ LA=-1 $ GO TO 1
ENTRY WI2P $ LA=-1 $ GO TO 2
1 LB=0 $ GO TO 3
2 LB=1 $ GO TO 3
3 Z=ZZ
IF(LA) 5,7,4
4 U=(-0.5,0.86602540378) $ GO TO 6
5 U=(-0.5,-0.86602540378)
6 Z=U*Z
7 LC=0 $ IF(X(2)) 8,9,10
8 LC=1 $ X(2)=-X(2) $ GO TO 10
9 X(2)=0.
10 CONTINUE COMPARE WITH PREVIOUS
IF(X(1) .NE. X1(1) .OR. X(2) .NE. X1(2)) GO TO 20
I=LB+1 $ I=LG.AND.I $ IF(I) GO TO 400
IF(LB) 220,210
400 CONTINUE EXIT
IF(LB) 402,401
401 ZT=A $ IF(LC)XT(2)=-XT(2)
IF(LA)404,411,403
402 ZT=AP $ IF(LC)XT(2)=-XT(2)
IF(LA)403,411,404
403 U=(1.,-1.7320508076) $ GO TO 410
404 U=(1.,1.7320508076) $ GO TO 410
410 ZT=U*ZT
411 AIRY=ZT $ RETURN
20 CONTINUE AFFINE COORDINATES
M=0
Z1=Z $ LG=0
IF(X(1).LE.-7. .OR. X(1).GT.7. .OR. X(2).GT.6.928203232) GO TO 200
IP=7.-X(1) $ P=IP-7-IP
Q=IQ=0.86602540378*X(2)+0.5*(P-X(1))

```

```

      N=NQT(IP)+IQ
      IF(N.GE.NQT(IP+1)) GO TO 200
100  CONTINUE          SERIES
      XT(1)=P $ XT(2)=1.1547005384*Q
      U=Z-ZT $ B1=AV(N) $ B3=B1*ZT*U $ AP=APV(N) $ B2=AP*U
      A=B2+B1 $ AP=AP+B3
      AN=1.
      DO 110 I=2,3
      AN=AN+1.
      B3=B3*U/AN
      A=B3+A
      B0=B1 $ B1=B2 $ B2=B3
      B3=(ZT*B1+U*B0)*U/AN
      AP=B3+AP
      IF(ANM(B2).GT.0.5E-10*ANM(A) .AND. ANM(B3).GT.0.5E-10*ANM(AP)) I=0
110  CONTINUE
      LG=3
      GO TO 400
200  CONTINUE          ASYMPTOTICS
      ZA=CSQRT(Z)
      ZB=0.28209479177/CSQRT(ZA)
      ZT=-0.66666666667*Z*ZA
      T=XT(1)**2+XT(2)**2
      CALL ZEXP(XT(1),XT(2),SX,SY,M)
      ZE=CMPLX(SX,SY)
      ZM=2.718281828***(M+M)
      ZR=1./ZT
      IF(XT(2) .GT. 0. .AND. XT(1) .LT. 11.8595) LG=4
      DO 201 NT=2,18
      IF(T .LT. ASLT(NT-1)) GO TO 202
201  CONTINUE $ NT=19
202  IF(LB) 220,210
210  CONTINUE          A
      ZT=ASV(NT-1)
      DO 211 I=NT,21
211  ZT=ASV(I)+ZT*ZR
      A=ZT*ZE
      I=4.AND.LG $ IF(I) 212,216
212  ZT=ASV(NT-1)
      DO 213 I=NT,21
213  ZT=ASV(I)-ZT*ZR
      A=A+(0.,1.)*ZT/(ZE*ZM)
216  A=ZB*A $ LG=1.OR.LG $ GO TO 401
220  CONTINUE          AP
      ZT=APSV(NT-1)
      DO 221 I=NT,21
221  ZT=APSV(I)+ZT*ZR
      AP=-ZT*ZE
      I=4.AND.LG $ IF(I) 222,226
222  ZT=APSV(NT-1)
      DO 223 I=NT,21
223  ZT=APSV(I)-ZT*ZR
      AP=AP+(0.,1.)*ZT/(ZE*ZM)
226  AP=ZA*ZB*AP $ LG=2.OR.LG $ GO TO 402
      END

```

7. APPENDIX: AN IONOSPHERIC REFLECTION COEFFICIENT PROGRAM

7.1 General Description

The propagation program described in the main body of this report requires ionospheric reflection coefficients as input. We will now describe briefly a computer program that can be used to calculate such reflection coefficients for any profile of ionospheric electron density.

We assume a plane ionosphere whose electron density and electron-neutral collision frequency vary only with altitude and a static magnetic field with constant dip angle and magnitude. For an infinite plane wave incident from below on the ionosphere at an angle, φ , we compute the reflection coefficient matrix defined by

$$\begin{bmatrix} \underline{r}^{\underline{E}}_{\underline{e}} \\ \underline{r}^{\underline{E}}_{\underline{m}} \end{bmatrix} = \begin{bmatrix} T_{ee} & T_{me} \\ T_{em} & T_{mm} \end{bmatrix} \cdot \begin{bmatrix} \underline{i}^{\underline{E}}_{\underline{e}} \\ \underline{i}^{\underline{E}}_{\underline{m}} \end{bmatrix} \quad (\text{A1})$$

where the pre-subscript indicates the incident or reflected wave and the post-subscript indicates whether the electric or magnetic vector is in the plane of incidence (e is "vertical polarization," m is "horizontal polarization").

The solution is accomplished by (1) dividing the ionosphere into thin homogeneous slabs whose properties are those of the continuous profile at the center of the slab, (2) solving Maxwell's equations in each slab, and (3) satisfying the boundary conditions at the slab interfaces. The solution is derived by Johler and Harper (1962).

The user must determine the height profile of electron density, the magnetic field parameters (strength, dip angle, azimuth relative to direction of propagation), the radio frequency, and the angle of incidence. By modifying the appropriate subroutine, he may specify the collision frequency.

The magnetic field parameters are well known (certainly to the precision required for this application), and maps of them can be found for instance in Davies (1965).

The collision frequency is also fairly well known--it decreases almost exponentially with height with a slope that the experts agree on. There is about a factor of 3 disagreement on the absolute magnitude at a given height (or an uncertainty of about 3 km in the height for a given value). The collision frequency computed in the subroutine CFEO is that recommended by Gambill and Knapp (1969). You can change subroutine CFEO to return any collision frequency profile you want.

We do not know the height distribution of electrons in the lower ionosphere very well yet; nor do we know how the electron density varies with season, sunspot cycle, latitude, etc. So we have included two versions of subroutine ENN to define electron density as a function of height. One simply interpolates in a table supplied as input by the user. The other contains a crude model ionosphere, which varies with the sun's zenith angle. This latter model (described by Berry and Jones (1970)) does a fair job of predicting the LF and VLF field strength variations during the day, especially at middle latitudes. You can easily incorporate any model of the ionosphere you want into the calculation by writing an appropriate version of ENN and substituting it for the version listed in this report.

Figure 14 shows a large block diagram of the organization of the program ANIREF. Details of the implementation can be determined from the comments cards in the FORTRAN listing at the end of this appendix.

7.2 A Sample Case

Table 2 lists the input parameters required by the program. The input for a sample case is shown in figure 15. The block diagram in

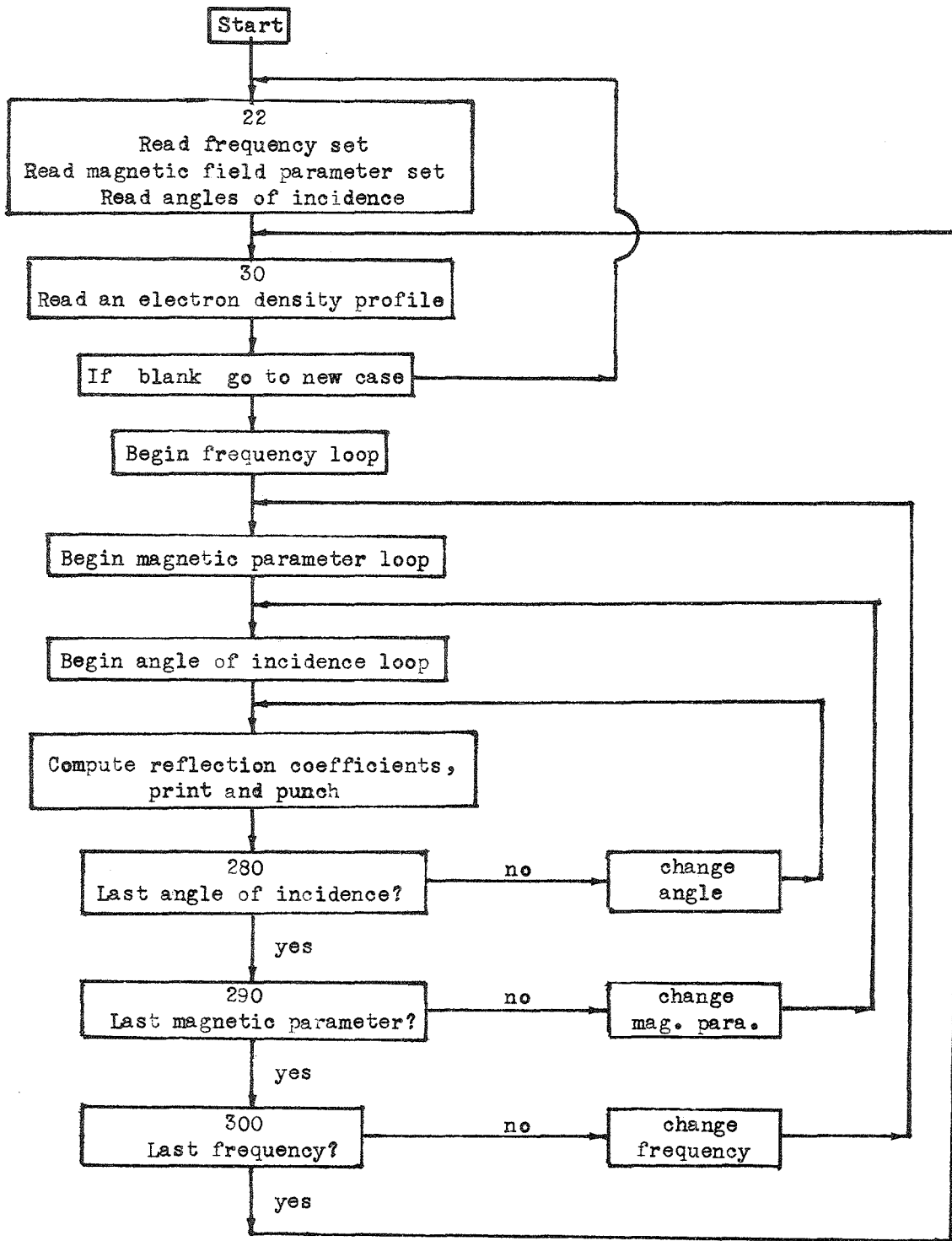


Figure 14. Flow chart for Program ANIREF.

Table 2. Input Data for Program ANIREF

Variable	Physical Quantity	Units	Comments
IFR	Number		Number of frequencies to be read in; maximum of 9.
FRE(I)	Radio frequency	kHz	
NPA	Number		Number of sets of magnetic field parameters; maximum of 8.
PHA(I)	Angle	Degrees	Azimuth of propagation of incident wave measured clockwise from north.
HA(I)	Magnetic strength	Gauss	Earth's magnetic field intensity.
DAP(I)	Dip	Degrees	Magnetic field dip angle, measured from horizontal.
NPI	Number		Number of angles of incidence; maximum of 10.
SPHI(I)	Angle	Degrees	Angle of incidence of radio wave on ionosphere; measured from vertical.
PROFLE(I)			Alphanumeric identification of ionospheric model.
HITE	Height	km	Height of bottom of ionosphere. Should equal Z(1) if ionosphere is defined by a table.
LAYR	Number		Number of data cards defining ionospheric model.
Z(L)	Height	km	} Usually a table defining electron density as a function of height. Can also be used to define parameters of an analytic model of electron density.
EN(L)	Electron density	electrons per cc	

ANIREF INPUT

```

1  no. frequencies
  30.0 frequency
3  no. magnetic parameter triples          azimuth
   213.5      .31      6.5      angle
   258.1      .37      39.0      dip
   299.2      .51      66.9      angle
6  no. angles of incidence                field
   65.      73.      78.      80.      81.      82. angles of incidence
SAN FRAN SEAPATH  NIGHT      H = 74.  KM,  19 PTS
74.00  1.63E+00
76.00  7.67E+00
78.00  2.31E+01
80.00  5.10E+01
82.00  9.70E+01
84.00  1.57E+02
86.00  2.25E+02
88.00  2.96E+02
90.00  3.69E+02
92.00  4.58E+02
94.00  5.58E+02
96.00  6.74E+02
98.00  8.11E+02
100.00 9.74E+02
105.00 1.20E+03
110.00 1.37E+03
115.00 1.58E+03
120.00 1.82E+03
125.00 2.09E+03
END OF FILE

```

identification
 ht of iono
 bottom
 no. cards to follow
 table of electron density
 as a function of height

Figure 15. List of input data deck for ANIREF sample case.

figure 16 shows possibilities of data deck set up when several cases are to be run at one time.

The printed output for this sample case is listed in figures 17 and 18. The punched cards produced by this sample case are listed in figure 19. These cards are in the right format to be used as input to ANIHOP and were used as input for the sample case in the main body of this report (compare fig. 4).

The input electron density profile is printed on the first page of output (fig. 17). Then a table of computed reflection coefficients is printed under a heading which contains the other relevant input information. Comments have been superimposed on the printout to help explain the various entries.

7.3 Mathematical Details of Reflection Coefficient Calculation

The main subroutine of ANIREF solves the set of equations

$$\begin{bmatrix}
 a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} & a_{1,6} \\
 b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} & b_{1,5} & b_{1,6} \\
 c_{1,1} & c_{1,2} & c_{1,3} & c_{1,4} & c_{1,5} & c_{1,6} \\
 d_{1,1} & d_{1,2} & d_{1,3} & d_{1,4} & d_{1,5} & d_{1,6} \\
 \\
 a_{2,3} & a_{2,4} & a_{2,5} & a_{2,6} & a_{2,7} & a_{2,8} & a_{2,9} & a_{2,10} \\
 b_{2,3} & b_{2,4} & b_{2,5} & b_{2,6} & b_{2,7} & b_{2,8} & b_{2,9} & b_{2,10} \\
 c_{2,3} & c_{2,4} & c_{2,5} & c_{2,6} & c_{2,7} & c_{2,8} & c_{2,9} & c_{2,10} \\
 d_{2,3} & d_{2,4} & d_{2,5} & d_{2,6} & d_{2,7} & d_{2,8} & d_{2,9} & d_{2,10} \\
 \\
 a_{3,7} & a_{3,8} & a_{3,9} & a_{3,10} & a_{3,11} & a_{3,12} & a_{3,13} & a_{3,14} \\
 b_{3,7} & b_{3,8} & b_{3,9} & b_{3,10} & b_{3,11} & b_{3,12} & b_{3,13} & b_{3,14} \\
 c_{3,7} & c_{3,8} & c_{3,9} & c_{3,10} & c_{3,11} & c_{3,12} & c_{3,13} & c_{3,14} \\
 d_{3,7} & d_{3,8} & d_{3,9} & d_{3,10} & d_{3,11} & d_{3,12} & d_{3,13} & d_{3,14} \\
 \\
 & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 \\
 a_{p,4p-5} & \dots & \dots & \dots & \dots & \dots & \dots & a_{p,4p} \\
 b_{p,4p-5} & \dots & \dots & \dots & \dots & \dots & \dots & b_{p,4p} \\
 c_{p,4p-5} & \dots & \dots & \dots & \dots & \dots & \dots & c_{p,4p} \\
 d_{p,4p-5} & \dots & \dots & \dots & \dots & \dots & \dots & d_{p,4p}
 \end{bmatrix}
 +
 \begin{bmatrix}
 T_{ee'} & T_{mm} \\
 T_{ee'} & T_{me} \\
 U_{eio'}^{(1)} & U_{mio}^{(1)} \\
 U_{eie'}^{(1)} & U_{mie}^{(1)} \\
 U_{ero'}^{(1)} & U_{mro}^{(1)} \\
 U_{ere'}^{(1)} & U_{mre}^{(1)} \\
 U_{eio'}^{(2)} & U_{mio}^{(2)} \\
 U_{eie'}^{(2)} & U_{mie}^{(2)} \\
 U_{ero'}^{(2)} & U_{mro}^{(2)} \\
 U_{ere'}^{(2)} & U_{mre}^{(2)} \\
 \\
 U_{eio'}^{(p-1)} & U_{mio}^{(p-1)} \\
 U_{eie'}^{(p-1)} & U_{mie}^{(p-1)} \\
 U_{ero'}^{(p-1)} & U_{mro}^{(p-1)} \\
 U_{ere'}^{(p-1)} & U_{mre}^{(p-1)} \\
 U_{eio'}^{(p)} & U_{mio}^{(p)} \\
 U_{eie'}^{(p)} & U_{mie}^{(p)}
 \end{bmatrix}
 = 0,
 \quad (A2)$$

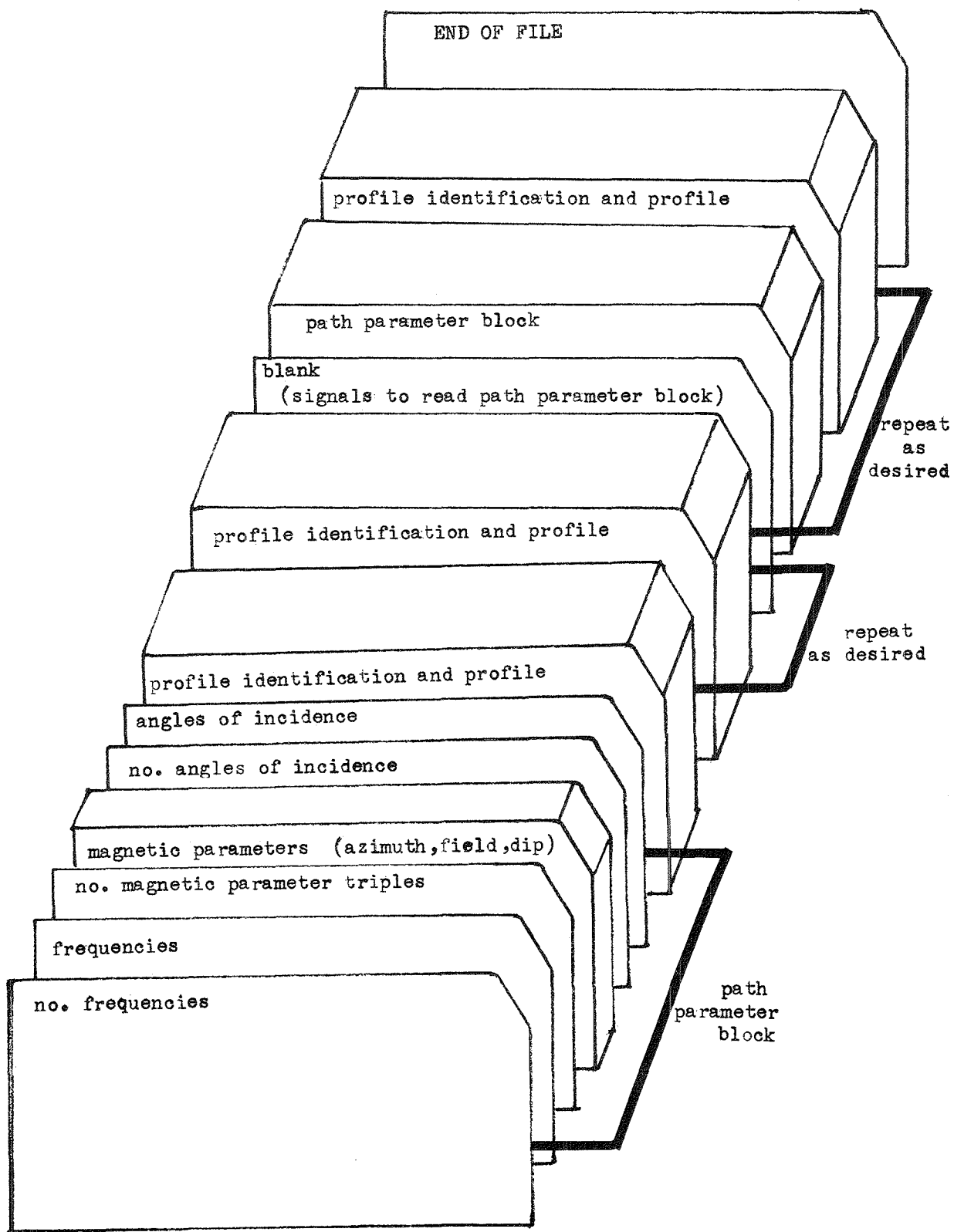


Figure 16. Data deck setup for Program ANIREF.

print out of input profile

SAN FRAN SEAPATH	NIGHT	H =74.00 KM,	19 PTS
------------------	-------	--------------	--------

74.00	1.63+000
76.00	7.67+000
78.00	2.31+001
80.00	5.10+001
82.00	9.70+001
84.00	1.57+002
86.00	2.25+002
88.00	2.96+002
90.00	3.69+002
92.00	4.58+002
94.00	5.58+002
96.00	6.74+002
98.00	8.11+002
100.00	9.74+002
105.00	1.20+003
110.00	1.37+003
115.00	1.58+003
120.00	1.82+003
125.00	2.09+003



h



N(h), per cc

Figure 17. Printed output from ANIREF sample case, page 1.

magnetic azimuth, PHA of Table 2

magnetic field strength

layer thickness

incidence angle

bottom of ionosphere

quasi-stationary phase height

calculation time, ms

SAN FRAN SEAPATH NIGHT H = 74.00 KM, 19 PTS

PHI SUB A = 213.50 DEG, DIP = 6.50 DEG, HM = 0.310 GAUSS DELTA Z = 0.10 LAMBDA

FREQ	PHII	TEE AMPL	TEE PHASE	TEM AMPL	TEM PHASE	TME AMPL	TME PHASE	TMM AMPL	TMM PHASE	HT	KLOCK
30.00	65.00	0.23856519	1.92681192	0.18302223	-1.01805620	0.07001212	1.54136447	0.33321719	-1.04922457	76.3	1408
30.00	73.00	0.16266621	2.82468960	0.20930955	0.93520164	0.07233439	-1.85135796	0.48378107	0.56485196	74.9	1405
30.00	78.00	0.15273893	2.57590069	0.21869476	1.89230384	0.09881201	-0.78770553	0.59805832	1.40249094	76.2	1438
30.00	80.00	0.20311441	2.47479954	0.21352332	2.23087025	0.10278956	-0.44324662	0.65025711	1.71344939	77.1	1408
30.00	81.00	0.24095183	2.47544341	0.20811945	2.39210712	0.10271450	-0.28142231	0.67813333	1.86489684	77.4	1408
30.00	82.00	0.28655988	2.50399238	0.20046239	2.54830795	0.10104579	-0.12547458	0.70733399	2.01397227	77.6	1413

PHI SUB A = 258.10 DEG, DIP = 39.00 DEG, HM = 0.370 GAUSS DELTA Z = 0.10 LAMBDA

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FREQ	PHII	TEE AMPL	TEE PHASE	TEM AMPL	TEM PHASE	TME AMPL	TME PHASE	TMM AMPL	TMM PHASE	HT	KLOCK
30.00	65.00	0.38155164	1.14126944	0.26623194	-1.11495209	0.23429944	-0.82783711	0.20806923	0.63034112	77.8	1388
30.00	73.00	0.25956327	2.67116816	0.37855115	0.73488144	0.34043288	0.78285144	0.20473602	1.15340141	75.3	1386
30.00	78.00	0.13936897	2.99774770	0.40347995	1.67116360	0.35962760	1.65099236	0.32105013	1.51263570	74.6	1388
30.00	80.00	0.12702794	2.67196456	0.39765747	2.01688815	0.35332814	1.97888532	0.39085256	1.72958226	76.2	1408
30.00	81.00	0.14643622	2.49519245	0.39022334	2.18509825	0.34624633	2.13974279	0.43028605	1.84870269	77.3	1389
30.00	82.00	0.18273117	2.40044694	0.37914027	2.35067062	0.33599701	2.29890345	0.47287972	1.97353155	78.2	1392

PHI SUB A = 299.20 DEG, DIP = 66.90 DEG, HM = 0.510 GAUSS DELTA Z = 0.10 LAMBDA

FREQ	PHII	TEE AMPL	TEE PHASE	TEM AMPL	TEM PHASE	TME AMPL	TME PHASE	TMM AMPL	TMM PHASE	HT	KLOCK
30.00	65.00	0.30895406	0.42889414	0.22238814	-1.14200831	0.23888201	-1.40264562	0.43741572	1.63167652	79.1	1419
30.00	73.00	0.32133278	1.42769882	0.33839888	0.77401499	0.36818328	0.75173612	0.32153268	-2.10332744	78.7	1404
30.00	78.00	0.39863447	1.82595990	0.34873673	1.77533154	0.38771328	1.81292608	0.34801681	-0.33840968	79.0	1430
30.00	80.00	0.45694583	1.99797248	0.33568951	2.15637183	0.37554602	2.20929661	0.40150820	0.32145615	79.2	1401
30.00	81.00	0.49222184	2.09085258	0.32471278	2.34473475	0.36422063	2.40395604	0.43708498	0.63293077	79.3	1407
30.00	82.00	0.53159478	2.18860970	0.31034911	2.53234626	0.34891375	2.59704933	0.47817761	0.93301088	79.4	1397

Figure 18. Printed output from ANIREF sample case, page 2.

ANIREF PUNCHED OUTPUT

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identification		azimuth		dip angle		field intensity			
SAN FRAN SEAPATH NIGHT		PHIA= 213.5		DIP= 6.5		HM= 0.310		GAUSS	
30.065.02.386-001	1.9271.830-001-1.0187.001-002	1.5413.332-001-1.049	74.0	76.3					
30.073.01.627-001	2.8252.093-001 0.9357.233-002-1.8514.838-001	0.565	74.0	74.9					
30.078.01.527-001	2.5762.187-001 1.8929.881-002-0.7885.981-001	1.402	74.0	76.2					
30.080.02.031-001	2.4752.135-001 2.2311.028-001-0.4436.503-001	1.713	74.0	77.1					
30.081.02.410-001	2.4752.081-001 2.3921.027-001-0.2816.781-001	1.865	74.0	77.4					
30.082.02.866-001	2.5042.005-001 2.5481.010-001-0.1257.073-001	2.014	74.0	77.6					
SAN FRAN SEAPATH NIGHT		PHIA= 258.1		DIP= 39.0		HM= 0.370		GAUSS	
30.065.03.816-001	1.1412.662-001-1.1152.343-001-0.8282.081-001	0.630	74.0	77.8					
30.073.02.596-001	2.6713.786-001 0.7353.404-001 0.7832.047-001	1.153	74.0	75.3					
30.078.01.394-001	2.9984.035-001 1.6713.596-001 1.6513.211-001	1.513	74.0	74.6					
30.080.01.270-001	2.6723.977-001 2.0173.533-001 1.9793.909-001	1.730	74.0	76.2					
30.081.01.464-001	2.4953.902-001 2.1853.462-001 2.1404.303-001	1.849	74.0	77.3					
30.082.01.827-001	2.4003.791-001 2.3513.360-001 2.2994.729-001	1.974	74.0	78.2					
SAN FRAN SEAPATH NIGHT		PHIA= 299.2		DIP= 66.9		HM= 0.510		GAUSS	
30.065.03.090-001	0.4292.224-001-1.1422.389-001-1.4034.374-001	1.632	74.0	79.1					
30.073.03.213-001	1.4283.384-001 0.7743.682-001 0.7523.215-001-2.103	74.0	78.7						
30.078.03.986-001	1.8263.487-001 1.7753.877-001 1.8133.480-001-0.338	74.0	79.0						
30.080.04.569-001	1.9983.357-001 2.1563.755-001 2.2094.015-001 0.321	74.0	79.2						
30.081.04.922-001	2.0913.247-001 2.3453.642-001 2.4044.371-001 0.633	74.0	79.3						
30.082.05.316-001	2.1893.103-001 2.5323.489-001 2.5974.782-001 0.933	74.0	79.4						
frequency	angle of incidence	amp	phase	amp	phase	amp	phase	amp	phase
		T _{ee}		T _{em}		T _{me}		T _{mm}	
								ht of iono bottom	reference height

Figure 19. List of punched card output from ANIREF sample case.

The elements of the matrix are defined by Johler and Harper (1962) in their equations (33-35) (note an error in the general form of their subscripts). They depend on the roots of a quartic equation ((8) through (20) of Johler and Harper (1962)), which in turn depends on the ionospheric model, radio frequency, and direction of propagation.

Instead of solving (A2) directly, we divide the matrix into block matrices:

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & . & . & . \\ A_{21} & A_{22} & A_{23} & 0 & . & . & . \\ 0 & A_{32} & A_{33} & A_{34} & . & . & . \\ 0 & 0 & . & . & . & 0 & . \\ . & . & . & . & A_{p-1,p-2} & A_{p-1,p-1} & A_{p-1,p} \\ 0 & . & . & . & 0 & A_{p,p-1} & A_{pp} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ . \\ . \\ U_p \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ 0 \\ . \\ . \\ 0 \end{bmatrix} \quad (A3)$$

A_{ij} is 4×4 , U_i and K are 4×2 .

Consider the rows as separate equations and solve the last one for U_p

$$U_p = -A_{pp}^{-1} A_{p,p-1} U_{p-1} \quad (A4)$$

Substitute this value into the next-to-last equation, so it has only two unknowns and solve for U_{p-1} . Repeat the process until U_1 is found.

The algorithm is: Let

$$B_p = A_{pp} \quad (A5)$$

and let

$$B_k = A_{kk} - A_{k,k+1} B_{k+1}^{-1} A_{k+1,k}, \text{ for } 1 \leq k < p. \quad (A6)$$

Then

$$B_1 U_1 = K, \text{ or } U_1 = B_1^{-1} K. \quad (A7)$$

Since only columns 3 and 4 of $A_{k+1,k}$ and columns 1 and 2 of $A_{k,k+1}$ are nonzero, the elements of the A_{ij} are stored in a $4p \times 8$ array arranged as follows:

$$\begin{array}{ccc} \text{Column No.} & 1, 2 & 3, 4, 5, 6 & 7, 8 \\ \left[\begin{array}{ccc} 0 & A_{11} & A_{12} \\ A_{21} & A_{22} & A_{23} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ A_{p,p-1} & A_{pp} & 0 \end{array} \right] & & (A8) \end{array}$$

The real and imaginary parts of (A8) are stored in separate arrays. The rows a_{ik} and b_{ik} of each layer (see (A2)) have been interchanged for convenience. If the elements of (A8) are denoted α_{ij} , then

$$\alpha_{4p+1,j} = b_{p,m}, \alpha_{4p+2,j} = a_{p,m}, \quad (A9)$$

$$\alpha_{4p+3,j} = c_{p,m}, \text{ and } \alpha_{4p+4,j} = d_{p,m},$$

where $m = 4p + j - 6$, and a_{ij} , b_{ij} , c_{ij} , and d_{ij} are the elements of (A2).

After the array (A8) has been generated, the B_k are computed as follows:

- (1) $B_k^{-1} A_{k,k-1}$ is computed and stored in A_{kk} . B_k^{-1} is found by solving $B_k B_k^{-1} = I$.
- (2) Then B_{k-1} is calculated using (A6) and stored in $A_{k-1,k-1}$, and the process is repeated.

When B_1 has been calculated, columns 1 and 2 are interchanged with columns 3 and 4 (to prevent zeros on the diagonal in certain cases) and (A7) is solved. The reflection coefficients have now been found.

The phase of the reflection coefficient is referenced to the bottom of the ionospheric profile. We also compute the quasi-stationary phase height of T_{ee} as follows: There is a height where the phase of T_{ee} is nearly π over a considerable range of angles of incidence near grazing. In general the phase of T_{ee} at height h_r in terms of the phase at h_o is

$$p_r = p_o + \frac{4\pi}{\lambda} \cos \varphi (h_r - h_o) , \quad (A10)$$

where p_i is the phase at height h_i . So let $p_r = \pi$ and solve for h_r :

$$h_r = h_o + \frac{\pi - p_o}{\frac{4\pi \cos \varphi}{\lambda}} . \quad (A11)$$

We call h_r the quasi-stationary phase height.

7.4 FORTRAN Listing of Program ANIREF and Its Subroutines

PROGRAM ANI REF

```

C
C COMPUTES IONOSPHERIC REFLECTION COEFFICIENTS FOR STRATIFIED IONOSPHERE
C BY L. A. BERRY AND J. E. HERMAN, OFFICE OF TELECOMMUNICATIONS
C REFERENCE IS TELECOMMUNICATIONS RESEARCH REPORT NO. 11 ,
C ITS, BOULDER COLORADO, 80302
C
  DIMENSION FRE(9),SPHI(10),PHA(8),HA(8),DAP(8)
  DIMENSION PROPLE(5),EN(80),Z(80)
  DIMENSION PR(4),PI(4),QR(4),QI(4),ZR(4),ZI(4)
  DIMENSION AR(600,8), AI( 600,8)
  DIMENSION ROOT(8), COF(10)
  COMMON AR,AI,PR,PI,QR,QI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
1  CI,SI,HM,OMEGA
2  ,ZMA,WAVE,COF
  COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
1  TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
1  FORMAT(F10.2,E10.2)
2  FORMAT(1H )
3  FORMAT(*  FREQ      PHII      TEE AMPL  TEE PHASE  TEM AMPL  TE
1M PHASE  TME AMPL  TME PHASE  TMM AMPL  TMM PHASE HT
2  KLOCK *)
4  FORMAT(1H1)
6  FORMAT(2F8.2,8F12.8,F6.1,7X ,I9)
7  FORMAT(5A6,5H H = F5.2,5H KM, ,15,8H POINTS. )
8  FORMAT(8F10.0)
9  FORMAT(1H0,9X,12HPhi SUB A = F7.2,11H DEG, DIP = F7.2,
110H DEG, HM = F6.3,18H GAUSS DELTA Z=F5.2, 7H LAMBDA )
19  FORMAT(I2)
25  FORMAT(3F10.0)

C          RAT IS THE SLAB THICKNESS, HERE = .1 WAVELENGTH

  DATA (RAT=.1)
22  READ 19,IFR

C
C IFR IS NUMBER OF FREQUENCIES
C
  IF(EOF,60)400,20
20  READ 8,(FRE(I),I=1,IFR)

C
C FRE IS THE FREQUENCY IN KHZ
  READ 19,NPA
C NPA IS NUMBER OF MAGNETIC FIELD CONDITIONS
C
  READ 25,(PHA(I),HA(I),DAP(I),I=1,NPA)
C PHA = PHI A = MAGNETIC AZIMUTH
C HA = HM = MAGNETIC FIELD STRENGTH IN GAUSS
C DAP = DIP = MAGNETIC FIELD DIP ANGLE IN DEGREES
C CAUTION---( (DIP=0) .AND.(PHIA=90.OR.270) ) IS NOT ALLOWED
C
  READ 19,NPI
C NPI = NUMBER OF ANGLES OF INCIDENCE, PHI I , IN DEGREES
C

```

```

      READ 8, (SPHI(I),I=1,NPI)
30 READ 7,(PROFLE(I),I=1,5),HITE,LAYR
C
C  PROFLE  IS ALPHANUMERIC DESCRIBING IONO PROFILE, HITE IS HT OF BOTTOM OF
C  IONOS IN KM,  LAYR IS THE NUMBER OF DATA CARDS IN IONO PROFILE TO FOLLOW
C  A BLANK CARD RETURNS THE PROGRAM TO 22 TO READ A NEW SET OF PARAMETERS
C
      IF(EOF,60)400,31
31 CONTINUE
      IF (LAYR) 22,22,32
32 PRINT 4
      PRINT 4
      PRINT 7,(PROFLE(I),I=1,5),HITE,LAYR
      DO 33 L=1,LAYR
      READ 1,Z(L),EN(L)
C
C  Z(L) IS THE HEIGHT IN KM,  EN(L) IS THE ELECTRONS/CC AT HT Z(L)
C
      PRINT 1,Z(L),EN(L)
33 CONTINUE
      PRINT 4
      PRINT 7,(PROFLE(I),I=1,5),HITE,LAYR
C
C  BEGINNING OF FREQUENCY LOOP
C
      DO 300 J=1,IFR
      F=FRE(J)
      DELTA = RAT*300./F
C
C  BEGINNING OF MAGNETIC PARAMETERS LOOP
C
      DO 290 K=1,NPA
      PHIA=PHA(K) $ HM=HA(K) $ DIP=DAP(K)
      PRINT 9,PHIA,DIP,HM,RAT
      LINE =0
      PRINT 2
      PRINT 2
      PRINT 3
      RDG=.017 453 292 519 943
      FIA=RDG*PHIA
      DD= RDG*DIP
      CFIA=COSF(FIA)
      SFIA=SINF(FIA)
      SI =SINF(DD)
      CI =COSF(DD)
      100 CONTINUE
      77 FORMAT(5A6,* PHIA=*,F6.1,* DIP=*,F6.1,* HM=*,F6.3,* GAUSS*)
      PUNCH 77, (PROFLE(I),I=1,5), PHIA,DIP,HM
C
C  BEGINNING OF ANGLE OF INCIDENCE LOOP
C
      DO 280 L=1,NPI
      PHII=SPHI(L)
      110 FII=RDG*PHII

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CFII=COSF(FII)	MOD UP
SFII=SINF(FII)	MOD UP
AT=SFII*SFIA	MOD UP
AL=SFII*CFIA	MOD UP
K1=KLOCK(0)	

```

150 CALL REFCOF
C
C   REFCOF COMPUTES THE REFLECTION COEFFICIENTS.  INPUT AND OUTPUT ARE IN
C   COMMON/1/
C
160 K2=KLOCK(1)

C   COMPUTE QUASI STATIONARY PHASE HEIGHT

HREF =HITE+(3.1416-TEEP)/(2.*WAVE*CFII)
PRINT 6,F,PHII,TEEA,TEEP,TEMA,TEMP,TMEA,TMEP,TMMA,MMMP
1,HREF      ,K2
PUNCH 1111 , F,PHII,TEEA,TEEP,TEMA,TEMP,TMEA,TMEP,TMMA,MMMP,HITE,
1 HREF
1111 FORMAT(F6.1,F4.1,4(E9.3,F6.3),2F5.1)
170 LINE=LINE+1
    IF(LINE.LE.50) GO TO 200
    PRINT 4
    PRINT              7,(PROFLE(I),I=1,5),HITE,LAYR
    PRINT 9,PHIA,DIP,HM,RAT
    LINE =0
    PRINT 2
    PRINT 3
    PRINT 2
200 CONTINUE

C
C   END OF ANGLE OF INCIDENCE LOOP
C
280 CONTINUE
290 CONTINUE

C END OF FREQUENCY LOOP

300 CONTINUE
GO TO 30
400 CALL EXIT
END

```

```

C      SUBROUTINE REFCOF
C
C      MAIN SUBROUTINE FOR ANIREF.  COMPUTES REFLECTION COEFFICIENTS.
C      REFERENCE IS TELECOMMUNICATIONS RESEARCH REPORT NO. 11 ,
C      BY L. A. BERRY AND J. E. HERMAN, OFFICE OF TELECOMMUNICATIONS
C      ITS, BOULDER COLORADO, 80302
C
C      COMMON AR, AI, PR, PI, QR, QI, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1  CI, SI, HM, OMEGA
2  , ZMA, WAVE, COF
C      COMMON/1/ Z, EN, LAYR, TEER, TEEI, TEEA, TEEP, TEMR, TEMI, TEMA, TEMP,
1  TMER, TMEI, TMEA, TMEP, TMMR, TMMI, TMMA, TMMP
C      COMMON/4/ BAR, BAI
C      DIMENSION PR(4), PI(4), QR(4), QI(4)
C      DIMENSION BAR(4,8), BAI(4,8), COF(10), ROOT(8), P(8), Q(8)
C      DIMENSION AR(600,8), AI(600,8), Z(80), EN(80)
C      OMEGA=6283.1853*F
C      WAVE=OMEGA/2.997925 E 5
C      NLAY = (Z(LAYR) - Z(1)) / DELTA
C
C      THERE ARE NLAY SLABS
C
C      IF( NLAY .LE. 149) GO TO 10
C      NLAY = 149
C      DELTA = (Z(LAYR) - Z(1)) / 149.
C      DEL = WAVE * DELTA / 6.2831853
C      PRINT 9, DEL
C      9 FORMAT ( * CHANGED DELTA Z TO *, F6.2, * LAMBDA.*)
C      10 CISA=CFII*SFIA
C      CDEL=3.1415927/(2.*WAVE)
C      CICA=CFII*CFIA
C
C      INITIAL GUESS FOR ROOTS OF BOOKER QUARTIC AT BOTTOM OF IONOSPHERE
C
C      ROOT(1)=CFII
C      ROOT(2)=0.
C      ROOT(3)=CFII
C      ROOT(4)=0.
C      ROOT(5)=-CFII
C      ROOT(6)=0.
C      ROOT(7)=-CFII
C      ROOT(8)=0.
C      ZK=WAVE*DELTA
C      ZM=Z(1)+.5*DELTA
C      ELEC=ENN(ZM)
C      COL=CFO(ZM)
C
C      ZM IS HEIGHT AT MIDDLE OF SLAB
C      ELEC IS ELECTRON DENSITY OF SLAB, COL IS COLLISION FREQUENCY OF SLAB.
C
C      CALL CQPZ(ELEC,COL)
C      CALLFOLEST(8,1)
C
C      THE CALL TO Q9EXUN SURPRESSES THE SYSTEM EXPONENT UNDERFLOW ERROR

```

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```

C          CHECK, DIAGNOSIS, AND HALT. (EXPONENT UNDERFLOWS ARE IGNORED)
      CALL Q9EXUN
C
C      FILL UP COEFFICIENT MATRIX.  SEE EQ(A2) AND (A8).
C
      DO 60 L=2,NLAY
      LP=4*L-3
      DO 40 J=1,4
      TX=ZK*ROOT(2*J)
35  EX=EXP(TX)
      TY=-ZK*ROOT(2*J-1)
      AR(LP,J)=-EX*COSF(TY)
      AI(LP,J)=-EX*SINF(TY)
      DO 40 K=1,3
      LPK=LP+K
      AR(LP,K)=AR(LP,J)*AR(LP,K-4,J+4)-AI(LP,J)*AI(LP,K-4,J+4)
      AI(LP,K)=AI(LP,J)*AR(LP,K-4,J+4)+AR(LP,J)*AI(LP,K-4,J+4)
      IF(AR(LP,K)**2+AI(LP,K)**2 .GE. 1.E-100) GO TO 40
      NLAY=L
40  CONTINUE
      IF(L-NLAY)42,61,61
42  ZM=ZM+DELTA
      ELEC=ENN(ZM)
      COL=CFEO(ZM)
      CALL CQPZ(ELEC,COL)
60  CALLFOLEST(8,LP)
61  HT = ZM + DELTA
      CALL CQPZ( ENN(HT), CFEO(HT))
      CALLFOLEST(6,LP)
      NTOP=NLAY-1
C
C      BEGIN RECURSIVE SOLUTION OF MATRIX EQ(A5),(A6).
C
      DO 150 L=1,NTOP
71  LA=NLAY+1-L
      LP=4*LA-4
      DO 90 I=1,4
      LPI=LP+I
      DO 90 J=1,4
      BAR(I,J)=AR(LPI ,J+2)
      BAI(I,J)=AI(LPI ,J+2)
90  CONTINUE
C      MINVERT INVERTS A 4X4 COMPLEX MATRIX
98  CALL MINVERT
C
C      THIS LOOP IS STEP 1, FOLLOWING EQ(A9).
C
      DO 140 I=1,4
      LPA=LP+I
      DO 140 J=5,6
      JA=J-4
      AR(LPA,J)=BAR(I,5)*AR(LP+1,JA)-BAI(I,5)*AI(LP+1,JA)
      AI(LPA,J)=BAI(I,5)*AR(LP+1,JA)+BAR(I,5)*AI(LP+1,JA)

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DO 140 K=2,4
LPK=LP+K
TX=BAR(I,K+4)*AR(LPK,JA)-BAI(I,K+4)*AI(LPK,JA)
TY=BAI(I,K+4)*AR(LPK,JA)+BAR(I,K+4)*AI(LPK,JA)
AR(LPA,J)=AR(LPA,J)+TX
140 AI(LPA,J)=AI(LPA,J)+TY
C
C   THIS LOOP IS STEP 2, FOLLOWING EQ(A9)
C
DO 150 I=1,4
LPA=LP-4+I
DO 150 J=5,6
JA=J+2
TX=AR(LPA,7)*AR(LP+1,J)-AI(LPA,7)*AI(LP+1,J)
TY=AI(LPA,7)*AR(LP+1,J)+AR(LPA,7)*AI(LP+1,J)
SX=AR(LPA,8)*AR(LP+2,J)-AI(LPA,8)*AI(LP+2,J)
SY=AI(LPA,8)*AR(LP+2,J)+AR(LPA,8)*AI(LP+2,J)
AR(LPA,J)=AR(LPA,J)-TX-SX
150 AI(LPA,J)=AI(LPA,J)-TY-SY
91 DO 95 J=1,2
DO 95 I=1,4
BAR(I,J)=AR(I,J+4)
BAI(I,J+2)=0.
95 BAI(I,J)=AI(I,J+4)
BAR(1,3)=SFIA
BAR(1,4)=CICA
BAR(2,3)=-CFIA
BAR(2,4)=CISA
BAR(3,3)=CISA
BAR(3,4)=CFIA
BAR(4,3)=CICA
BAR(4,4)=-SFIA
C
C   SOLVE EQ (A7).
C
CALL MINVERT
TEMR = BAR(3,5)*CICA+BAR(3,6)*CISA-BAR(3,7)*CFIA+BAR(3,8)*SFIA
TEMI = BAI(3,5)*CICA+BAI(3,6)*CISA-BAI(3,7)*CFIA+BAI(3,8)*SFIA
TEER = BAR(4,5)*CICA+BAR(4,6)*CISA-BAR(4,7)*CFIA+BAR(4,8)*SFIA
TEEI = BAI(4,5)*CICA+BAI(4,6)*CISA-BAI(4,7)*CFIA+BAI(4,8)*SFIA
TMMR = -BAR(3,5)*SFIA + BAR(3,6)*CFIA+BAR(3,7)*CISA+BAR(3,8)*CICA
TMMI = -BAI(3,5)*SFIA + BAI(3,6)*CFIA+BAI(3,7)*CISA+BAI(3,8)*CICA
TMER = -BAR(4,5)*SFIA + BAR(4,6)*CFIA+BAR(4,7)*CISA+BAR(4,8)*CICA
TMEI = -BAI(4,5)*SFIA + BAI(4,6)*CFIA+BAI(4,7)*CISA+BAI(4,8)*CICA
TEEA=CABS(CMPLX(TEER,TEEI))
TEEP=CANG(CMPLX(TEER,TEEI))
TEMA=CABS(CMPLX(TEMR,TEMI))
TEMP=CANG(CMPLX(TEMR,TEMI))
TMEA=CABS(CMPLX(TMER,TMEI))
TMEP=CANG(CMPLX(TMER,TMEI))
TMMA=CABS(CMPLX(TMMR,TMMI))
TMMP=CANG(CMPLX(TMMR,TMMI))
RETURN
END

```

MOD UP

```

      FUNCTION ENN(ZM)
C
C  USES LOGARITHMIC INTERPOLATION IN TABLE OF N(H) TO GET ELECTRON DENSITY AT ZM

```

```

      DIMENSION Z(80),EN(80)
      COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
1  TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
      KID=1
      IF(Z(LAYR)-ZM)70,70,25
70  ENN=EN(LAYR)
      GO TO 71
25  IF(Z(KID+1)-ZM)26,27,28
26  KID=KID+1
      GO TO 25
27  ENN=EN(KID+1)
      GO TO 71
28  TX=(ZM-Z(KID))/(Z(KID+1)-Z(KID))
      ENN= EN(KID)*EXPF(TX*LOGF(EN(KID+1)/EN(KID)))
71  CONTINUE
      RETURN
      END

```

```

      FUNCTION CFEO(H)
C
C  DASA COLLISION FREQUENCY MODEL FIT WITH EXP(P9),
C  WHERE P9 IS A NINTH DEGREE POLYNOMIAL.
C

```

```

      DIMENSION A(10)
      DATA((A(I),I=1,9 )=2.587803463E1, -1.210027715E-1, -1.462645167E-3
1  , -1.172264046E-5, 1.749042668E-6, -2.948406644E-8, 1.351055095E-
2  10, 4.11118378E-13, -3.289391577E-15 )
      SUM =A(9)
      DO 10 J=1,8
10  SUM = H*SUM +A(9-J)
      CFEO = EXPF(SUM)
      RETURN
      END

```


SUBROUTINE CQPZ(ELEC,COL)

MOD ONE

C
C SUBROUTINE FOR REFCOF. RETURNS THE ROOTS OF THE QUARTIC, AND THE
C FIELD RATIOS P AND Q, SEE JOHLER AND HARPER, JAN 1961 RADIO SCIENCE.
C

```
COMMON AR, AI, PR, PI, QR, QI, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1 CI, SI, HM, OMEGA
2 , ZMA, WAVE, COF
DIMENSION COF(10), PR(4), PI(4), QR(4), QI(4), ZR(4), ZI(4)
DIMENSION AR(600,8), AI( 600,8)
DIMENSION ROOT(8), TOOT(4), ZOOTR(4), ZOOTI(4), ZZR(4), ZZI(4)
JU=1
JD=4
Y=1.75888 E 7 *HM/OMEGA
YT= Y*CI
YL=-Y*SI
X=(3.1824858 E 9*ELEC)/(OMEGA*OMEGA)
Z=+COL/OMEGA
ATSQ=AT*AT
ALSQ=AL*AL
CSQ =1. -ATSQ-ALSQ
XSQ=X*X
YSQ =Y*Y
ZSQ=Z*Z
YLSQ=YL*YL
YTSQ=YT*YT
UR=1.
UI=-Z
U2R=1. -ZSQ
U2I=-2. *Z
TAR=1.-ZSQ-YSQ
TAI=2.*Z
TX=TAR*TAR+TAI*TAI
TX=-X/TX
TAR=TAR*TX
TAI=TAI*TX
PXXR=TAR -TAI*UI
PXXI=TAR*UI+TAI
PXYR=-YL*TAI
PXYI= YL*TAR
PXZR= YT*TAI
PXZI=-YT*TAR
PZXR=-PXZR
PZXI=-PXZI
TX=1.+ZSQ
TBR=(TAR-TAI*Z)/TX
TBI=(TAI+TAR*Z)/TX
TCR=U2R-YLSQ
TCI=U2I
PZZR=TBR*TCR-TBI*TCI
PZZI=TBR*TCI+TBI*TCR
PZYR=-YL*YT*TBR
PZYI=-YL*YT*TBI
TCR=U2R-YTSQ
```

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PYYR=TBR*TCR-TBI*TCI
PYYI=TBR*TCI+TBI*TCR
TCR=PXXR*(1.-ATSQ)+PYYR*(1.-ALSQ)+PZZR
TCI=PXXI*(1.-ATSQ)+PYYI*(1.-ALSQ)+PZZI
C
C COEFFICIENTS OF BOOKER QUARTIC
C
COF(9)=1.+PZZR
COF(10)=PZZI
COF(7)=2.*AL*PZYR
COF(8)=2.*AL*PZYI
COF(5)=-2.*CSQ+2.*X*TAR-TCR-CSQ*PZZR
COF(6)=2.*X*TAI-TCI-CSQ*PZZI
COF(3)=-CSQ*COF(7)
COF(4)=-CSQ*COF(8)
COF(1)=CSQ*CSQ+CSQ*TCR-(2.*CSQ+1.)*X*TAR+XSQ*TBR
COF(2)=CSQ*TCI-(2.*CSQ+1.)*X*TAI+XSQ*TBI
CALL ZROOT
DO 9 I=1,4
II=I+I
ZR(I)=ROOT(II-1)
ZI(I)=ROOT(II)
9 CONTINUE
R12R=AL*AT+PXYR
R12I=PXYI
R33R=CSQ+PZZR
R33I=PZZI
C
C COMPUTE P AND Q IN THE 40 LOOP
C
DO 40 I=1,4
ZRSQ=ZR(I)*ZR(I)-ZI(I)*ZI(I)
ZISQ=2.*ZR(I)*ZI(I)
R11R=1.-ALSQ-ZRSQ+PXXR
R11I=-ZISQ+PXXI
R13R=AT*ZR(I)+PXZR
R13I=AT*ZI(I)+PXZI
R31R=AT*ZR(I)+PZXR
R31I=AT*ZI(I)+PZXI
R32R=AL*ZR(I)+PZYR
R32I=AL*ZI(I)+PZYI
DENR=R33R*R11R-R33I*R11I - R13R*R31R+R13I*R31I
DENI=R13I*R31R+R13R*R31I - R33I*R11R-R33R*R11I
TX=DENR*DENR+DENI*DENI
PPR=R12R*R31R-R12I*R31I -R32R*R11R+R32I*R11I
PPI=R12I*R31R+R12R*R31I -R32I*R11R-R32R*R11I
QQR=R13R*R32R-R13I*R32I -R33R*R12R+R33I*R12I
QQI=R13I*R32R+R13R*R32I -R33I*R12R-R33R*R12I
TX1=PPR*DENR-PPI*DENI
PPI=PPI*DENR+PPR*DENI
PPR=TX1
TX1=QQR*DENR-QQI*DENI
QQI=QQI*DENR+QQR*DENI
QQR=TX1

```

```

PR(I)=PPR/TX
PI(I)=PPI/TX
QR(I)=QQR/TX
QI(I)=QQI/TX
40 CONTINUE
RETURN
END

```

```

SUBROUTINE FOLEST(JJ,LP)

```

C
C
C

```

SUBROUTINE FOR REFCOF. FILLS UP COL 5 - 8 OF EQ (A8).

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```

DIMENSION PR(4), PI(4), QR(4), QI(4), ROOT(8), P(8), Q(8)
DIMENSION AR(600,8), AI( 600,8)
DIMENSION COF(10)
COMMON AR, AI, PR, PI, QR, QI, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1 CI, SI, HM, OMEGA
2 , ZMA, WAVE, COF
DO 100 K=1,4
KK=2*K
P(KK-1) = PR(K)
P(KK) = PI(K)
Q(KK-1) = QR(K)
100 Q(KK) = QI(K)
DO 20 J=5, JJ
JP=2*J-9
AR(LP, J)=1.
AI(LP, J)=0.
AR(LP+1, J)=Q(JP)
AI(LP+1, J)=Q(JP+1)
AR(LP+2, J)=AL*P(JP)-ROOT(JP)
AI(LP+2, J)=AL*P(JP+1)-ROOT(JP+1)
TX=ROOT(JP)*Q(JP)-ROOT(JP+1)*Q(JP+1)
TY=ROOT(JP+1)*Q(JP)+ROOT(JP)*Q(JP+1)
AR(LP+3, J)=TX-AT*P(JP)
20 AI(LP+3, J)=TY-AT*P(JP+1)
RETURN
END

```

```

C      SUBROUTINE ZROOT
C
C      ZROOT FINDS THE ROOTS OF A QUARTIC WITH COMPLEX COEFFICIENTS
C      USING THE NEWTON-RAPHSON METHOD TO FIND THE FIRST TWO ROOTS,
C      AND THE QUADRATIC FORMULA FOR THE OTHER TWO. COF(1)+I*COF(2) IS
C      THE CONSTANT TERM, COF(3)+I*COF(4) IS THE COEFFICIENT OF Z ,ETC
C      THE ROOTS ARE ROOT(2*K-1)+I*ROOT(2*K),K=1,4 WRITTEN BY B. WEIDER
C
      DIMENSION COF(10),ROOT(8)
      DIMENSION AR(600,8), AI(600,8), VR(4), VI(4), WR(4), WI(4)
      COMMON AR,AI,VR,VI,WR,WI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
1  CI,SI,HM,OMEGA
2  ,ZMA,WAVE,COF
      J=1
      JRT=2
      JCOF=10
1000 EPSILON=1.E-8
999  JROOT=JRT+JRT
      L=0
C
C      USE ROOT FOR PREVIOUS CASE AS FIRST GUESS
C
      XR=ROOT(JROOT-1)
      XI=ROOT(JROOT)
1  I=JCOF-J
      L=L+1
      YR=XR
      YI=XI
      FR=COF(I)
      FI=COF(I+1)
2  I=I-2
      TX=XR*FR-XI*FI+COF(I)
      FI=XR*FI+XI*FR+COF(I+1)
      FR=TX
      IF(I.GT.1) GO TO 2
      A=FR
      B=FI
C
C      A+I*B=P(XR+I*XI)
C
C      I=JCOF-J
      CCOF=(I-1)/2
      GR=CCOF*COF(I)
      GI=CCOF*COF(I+1)
4  I=I-2
      CCOF=CCOF-1.
      TX=XR*GR-XI*GI+CCOF*COF(I)
      GI=XR*GI+XI*GR+CCOF*COF(I+1)
      GR=TX
      IF(I.GT.3) GO TO 4
      C=GR
      D=GI
C
C      C+I*D=DERIVATIVE OF P(XR+I*XI)

```

```

C      ASSIGN 5 TO LSW
      GO TO 400
5 EPSR=X
  EPSI=Y
C
C      EPSR+I*EPSI=CORRECTION TO ROOT.
C
      A=EPSR
      B=EPSI
      C=XR
      D=XI
      ASSIGN 6 TO LSW
      GO TO 400
6 X=ABSF(X)
  Y=ABSF(Y)
C
C      HAVE WE CONVERGED
C
      IF(X.GT.EPSILON) GO TO 7
      IF(Y.LT.EPSILON) GO TO 60
7 XR=XR-EPSR
  XI=XI-EPSI
  IF(L.LE.10) GO TO 1
  EPSILON=10.*EPSILON
  L=0
  IF(EPSILON .GT. 1.1E-6) PRINT 800,EPSILON,ZMA
800 FORMAT(1H0,* INCREASED EPSILON TO*,E9.1,* IN ZROOT AT HEIGHT*,
1 F8.2//)
  GO TO 1
60 ROOT(JROOT-1)=XR
  ROOT(JROOT)=XI
  N=JCOF-3
61 A=COF(N+2)
  B=COF(N+3)
  X=A*XR-B*XI
  Y=A*XI+B*XR
  COF(N )=COF(N )+X
  COF(N+1)=COF(N+1)+Y
  N=N-2
  IF(N.GT.1)GO TO 61
  N=JCOF-2
  DO 62 K=1,N
  COF(K)=COF(K+2)
62 CONTINUE
  JCOF=JCOF-2
  JRT=JRT+2
  IF(JRT.LE.4) GO TO 1000
C
C      FROM HERE TO 72+4 IS QUADRATIC FORMULA
C
      X=COF(3)*COF(3)-COF(4)*COF(4)
      A=COF(3)*COF(4)
      Y=A+A

```

```

A=4.*(COF(1)*COF(5)-COF(2)*COF(6))
B=4.*(COF(1)*COF(6)+COF(2)*COF(5))
A=X-A
B=Y-B
ASSIGN 70 TO LSW
GO TO 500
70 A=X
B=Y
C=2.*COF(5)
D=2.*COF(6)
ASSIGN 71 TO LSW
GO TO 400
71 TX=X
TY=Y
A=COF(3)
B=COF(4)
ASSIGN 72 TO LSW
GO TO 400
72 A=-X+TX
B=-Y+TY
C=-X-TX
D=-Y-TY
IF(B) 73,75,74
73 ROOT(1)=A
ROOT(2)=B
ROOT(5)=C
ROOT(6)=D
GO TO 78
74 ROOT(1)=C
ROOT(2)=D
ROOT(5)=A
ROOT(6)=B
GO TO 78
75 IF(A.GT.0.) GO TO 76
ROOT(1)=C
ROOT(5)=A
GO TO 77
76 ROOT(1)=A
ROOT(5)=C
77 ROOT(2)=0.
ROOT(6)=0.
78 CONTINUE
RETURN
C
C THIS IS A COMPLEX DIVISION BRANCH.  $X+I*Y=(A+I*B)/(C+I*D)$ 
C
400 IF(C)410,405,410
405 IF(D)406,440,406
406 X=B/D
Y=-A/D
GO TO LSW
410 IF(D)415,411,415
411 X=A/C
Y=B/C

```

```

      GO TO LSW
415 IF(ABSF(C)-ABSF(D))416,430,430
416 AX=C/D
      DEN=AX*C+D
      X=(A*AX+B)/DEN
      Y=(B*AX-A)/DEN
      GO TO LSW
430 AX=D/C
      DEN=C+AX*D
      X=(A+B*AX)/DEN
      Y=(B-A*AX)/DEN
      GO TO LSW
440 PRINT 401
401 FORMAT(* ZERO DIVISOR IN ZMROOT DIVIDE*)
      CALL EXIT
C
C      THIS IS A COMPLEX SQUARE ROOT BRANCH.  $X+I*Y=\text{SQRT}(A+I*B)$ .
C
500 S=1.
      IF(B)501,505,505
501 S=-1.
505 IF(A)502,504,502
502 X=SQRTF(.5*(SQRTF(A*A+B*B)+ABSF(A)))
      D=B/(2.*X)
      Y=D
      IF(A.GE.0.)GO TO LSW
      Y=S*X
      X=S*D
      GO TO LSW
504 X=SQRTF(S*B*.5)
      Y=S*X
      GO TO LSW
      END

```

```

SUBROUTINE MINVERT
C
C INVERTS 4X4 COMPLEX MATRIX
C
COMMON /4/ BR,AR,BI,AI
DIMENSION IPVOT(4),INDEX(8),PIVOTR(4),PIVOTI(4)
DIMENSION AR(16),AI(16),BR(16),BI(16)
N=4 $ NSQ=16
DO 888 I=1,NSQ
AR(I)=BR(I)
AI(I)=BI(I)
888 CONTINUE
DETR=1.
DETI=0.
DO 17 J=1,N
IPVOT(J)=0
17 CONTINUE
DO 135 I=1,N
TR=0.
TI=0.
DO 9 J=1,N
IF(IPVOT(J).EQ.1)GO TO 9
NK=-N
DO 23 K=1,N
NK=NK+N
JK=J+NK
IF(IPVOT(K)-1)43,23,81
43 IF((TR*TR+TI*TI)-(AR(JK)*AR(JK)+AI(JK)*AI(JK)))83,23,23
83 IROW=J
ICOL=K
TR=AR(JK)
TI=AI(JK)
23 CONTINUE
9 CONTINUE
IPVOT(ICOL)=IPVOT(ICOL)+1
IF(IROW.EQ.ICOL)GO TO 109
DETR=-DETR
DETI=-DETI
NL=-N
DO 12 L=1,N
NL=NL+N
IROWL=IROW+NL
ICOLL=ICOL+NL
TR=AR(IROWL)
TI=AI(IROWL)
AR(IROWL)=AR(ICOLL)
AI(IROWL)=AI(ICOLL)
AR(ICOLL)=TR
AI(ICOLL)=TI
12 CONTINUE
109 I1=I
I2=I+N
ICOLICOL=ICOL+N*(ICOL-1)
INDEX(I1)=IROW

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INDEX(I2)=ICOL	XIRTAM
PIVOTR(I)=AR(ICOLICOL)	XIRTAM
PIVOTI(I)=AI(ICOLICOL)	XIRTAM
TXR=DETR*PIVOTR(I)-DETI*PIVOTI(I)	XIRTAM
TXI=DETR*PIVOTI(I)+DETI*PIVOTR(I)	XIRTAM
DETR=TXR	XIRTAM
DETI=TXI	XIRTAM
AR(ICOLICOL)=1.	XIRTAM
AI(ICOLICOL)=0.	XIRTAM
NL=-N	
DO 205 L=1,N	
NL=N+NL	
ICOLL=ICOL+NL	XIRTAM
DXR=PIVOTR(I)*PIVOTR(I)+PIVOTI(I)*PIVOTI(I)	XIRTAM
TXR=AR(ICOLL)*PIVOTR(I)+AI(ICOLL)*PIVOTI(I)	XIRTAM
TXI=AI(ICOLL)*PIVOTR(I)-AR(ICOLL)*PIVOTI(I)	XIRTAM
AR(ICOLL)=TXR/DXR	XIRTAM
AI(ICOLL)=TXI/DXR	XIRTAM
205 CONTINUE	
DO 135 LI=1,N	
IF(LI-ICOL)21,135,21	XIRTAM
21 LIICOL=LI+N*(ICOL-1)	XIRTAM
TR=AR(LIICOL)	XIRTAM
TI=AI(LIICOL)	XIRTAM
AR(LIICOL)=0.	XIRTAM
AI(LIICOL)=0.	XIRTAM
NL=-N	XIRTAM
DO 89 L=1,N	
NL=N+NL	XIRTAM
ICOLL=ICOL+NL	XIRTAM
LIL=LI+NL	XIRTAM
TXR=AR(ICOLL)*TR-AI(ICOLL)*TI	XIRTAM
TXI=AI(ICOLL)*TR+AR(ICOLL)*TI	XIRTAM
AR(LIL)=AR(LIL)-TXR	XIRTAM
AI(LIL)=AI(LIL)-TXI	XIRTAM
89 CONTINUE	
135 CONTINUE	
DO 3 I=1,N	
L1=N-I+1	XIRTAM
L2=L1+N	XIRTAM
IF(INDEX(L1)-INDEX(L2))19,3,19	XIRTAM
19 JROW=INDEX(L1)	XIRTAM
JCOL=INDEX(L2)	XIRTAM
NJROW=N*(JROW-1)	
NJCOL=N*(JCOL-1)	
DO 549 K=1,N	
KJROW=K+NJROW	
KJCOL=K+NJCOL	
TR=AR(KJROW)	XIRTAM
TI=AI(KJROW)	XIRTAM
AR(KJROW)=AR(KJCOL)	XIRTAM
AI(KJROW)=AI(KJCOL)	XIRTAM
AR(KJCOL)=TR	XIRTAM
AI(KJCOL)=TI	XIRTAM

549 CONTINUE
3 CONTINUE
81 CONTINUE
RETURN
END

XIRTAM
XIRTAM
-

If no electron density-height function is known, use the subroutine. The electron density profile is a function of the sun's zenith angle, hence a function of month, day, and hour.

```

FUNCTION ENN(H)
C      MOD VIII OF THE D REGION
C      H IS HEIGHT IN KM, ENN IS ELECTRON DENSITY PER CC.
C      CHI IS SUNS ZENITH ANGLE IN DEGREES
C      USE ABSOLUTE VALUE OF CHI AS IT IS SYMMETRIC ABOUT ZERO
      DIMENSION Z(80),EN(80)
      COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
1 TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
      DATA (CHIO=0.), (EN1=80.), (H1=65.), (B=.15), (H2=72.), (SCALH=3.3)
      EQUIVALENCE (Z(2),CHI)
      IF (CHI .EQ. CHIO) GO TO 50
      CHIO =CHI
      H1=65.
      IF(CHI .GE. 90.3) H1=65.+(CHI-90.3)*1.03
      EN1=80.
      IF( CHI .GT. 97.) EN1=0.
      IF(CHI.LE.97. .AND. CHI.GT.95.) EN1=4.*(97.-CHI)
      IF(CHI.LE. 95. .AND. CHI.GT.90.) EN1=80.*10.**((90.-CHI)/5.)
      B=.15
      IF( CHI .GE. 100. ) B=.35
      IF(CHI.GT.70. .AND. CHI.LT. 100.) B= .15+(CHI-70.)/30.*.20
      IF(CHI .GE. 90.) H2=74.
      IF(CHI .LT. 90.) H2=74.-22.*COS(CHI*.01745329252)
50  Q=(H-H1)/SCALH
      IF(CHI.GT. 97.) 55,52
52  CN=EN1*EXP(1.-Q-EXP(-Q))
      GO TO 60
55  CN=0.
60  DN=1000.*EXP(B*(H-H2))
      ENN=CN+DN
      RETURN
      END

```

Given the month, day, hour, latitude, and longitude, this subroutine calculates the sun's zenith angle which is used in the alternate Function ENN listed.

```

      FUNCTION SOL ZEN(MONTH,DAY,TIME,LAT,LONG)
      CALCULATES THE SOLAR ZENITH ANGLE AT THE GIVEN MONTH, DAY,TIME,LONG,LAT
      C
      C      MONTH, 1-12
      C      DAY,    1-31
      C      TIME,   HOUR= 0-24
      C      LAT=LATITUDE, RADIANS,  +=NORTH,  -=SOUTH
      C      LONG = LONGITUDE, RADIANS EAST

      REAL MONTH,LAT,LONG
      DATA (PI=3.141592654)
      GLAT=PI/180.*23.5*SIN(2.*PI*(30.5*(MONTH-3.))+DAY-21.)/365.)
      GLON =-PI/180.*15.*TIME+PI
      SOL ZEN=ACOS(SIN(LAT)*SIN(GLAT)+COS(LAT)*COS(GLAT)*COS(GLON-LONG))
      RETURN
      END

```