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METRICS FOR SPECTRUM-SPACE USAGE

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U.S. DEPARTMENT OF COMMERCE Office of Telecommunications

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FOREWORD

As a nation, we have come to regard the electromagnetic spectrum as an important national resource. Because the spectrum has become crowded in recent years, it is increasingly important that it be well managed.

This report discusses three possible approaches to the problem of measuring spectrum-space usage, and relates them to previously proposed metrics. Although each of the metrics proposed has disadvantages as well as advantages, it is hoped that some insight is gained by their discussion. If the report serves to stimulate dialogue on these matters, it will serve a useful purpose.

This report was prepared by the Policy Support Division of the Office of Telecommunications, U.S. Department of Commerce. It is one of a series of such reports prepared in support of the Office of Telecommunications Policy, Executive Office of the President.

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METRICS FOR SPECTRUM-SPACE USAGE

by

Donald R. Ewing* and Leslie A. Berry†

ABSTRACT

A generally accepted spectrum use metric (unit of spectrum-space use) would expedite efficient allocation and management of the spectrum resource. It would be useful for quantitative studies of spectrum congestion, and for economic analysis of the resource. Such a metric would benefit the operation of a market or quasi-market in spectrum rights.

Several metrics have previously been proposed, but none has been generally adopted.

In this paper the position is taken that use of the spectrum-space means denial of frequency bandwidth, geographic area, and time to other prospective users. This denial may be physical or administrative in nature. Further, both transmitters and receivers use spectrum-space in this sense.

Three alternative spectrum-space metrics are defined based on physical denial of spectrum-space. Their relative merits and their relationship to previously proposed metrics are discussed.

The situation-specific metric calculates the space a system denies to actual existing systems competing for the same space. It is the most realistic metric for a completely specified situation, but because of this it is not persistent.

The uniform metric computes the spectrum-space a system denies to standard reference systems. It is computable, persistent, and objective; however, the choice of some of the reference parameters is apparently arbitrary.

The autonomous metric depends only on the characteristics of the evaluated system. It is persistent, and non-arbitrary, and is proportional to the spectrum-space denied in an important special case. However, we do not know how to compute it for multiple-transmitter or multiple-receiver systems, so it is not yet practical.

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1. THE NEED FOR A UNIT OF MEASURE OF SPECTRUM USE

Efficient allocation and management of the radio spectrum resource calls for a unit of measure of the resource -- a spectrum space metric (Gifford, 1966; JTAC, 1968; Powers, 1968; Hinchman, 1969). A metric would expedite quantitative studies of relative congestion, studies of economic value of the resource, and assignment of priorities. Such a metric would be an important adjunct to a market (DeVaney et al., 1969), or quasi-market (Gifford, 1966; Levin, 1971) in spectrum rights.

It is widely accepted that power, bandwidth, antenna directivity, and area of service are all factors of spectrum-space use. A significant problem is to properly combine these and other factors into a metric.

Several metrics have been proposed -- Gifford's (1966) PODAF (Power Density over an Area within a Frequency band), Powers' (1968) spectrum acre (bandwidth × area × time), and the TAS (time, area, spectrum) package of DeVaney et al. (1969) -- but none has been generally adopted. Perhaps this is because they oversimplify the actual physical situation. We believe that part of this oversimplification is that they are defined primarily in terms of the transmitter portion of the radio system, and do not give equal visibility to the legitimate use of spectrum-space by receivers.

Other authors have been concerned with developing measures of relative value or efficiency of spectrum use rather than amount. Norton's Effective Service Sum (JTAC, 1968, p.S8-94), Hoxie's Relative Value Index (NAE, 1970), and Cohn's Spectrum Utilization Efficiency (JTAC, 1968, p.S8-85) are examples of this approach. Of these only the Effective Service Sum does not include an implicit spectrum metric. The Effective Service Sum takes service as the basic commodity, and tries to maximize the dollar value of services provided using spectrum.

Hoxie's Relative Value Index (NAE, 1970) is the product of factors representing economic efficiency, technical efficiency and sociopolitical priority. How technical efficiency is to be evaluated is not specified, but the value index is proportional to time/bandwidth.

Cohn's Spectrum Utilization Efficiency (JTAC, 1968, p. S8-85) is the ratio of the spectrum space used by an "ideal" system to the spectrum space used by the evaluated system. Spectrum-space is defined to be the bandwidth × time × physical volume that the radio system denies to other users. It is recognized that receivers as well as transmitters deny space to others. The objection to this measure of spectrum efficiency is the difficulty of defining an "ideal" system. However, our proposals for a measure of spectrum-space use are closely related to the spectrum-space measure Cohn used to compute Spectrum Utilization Efficiency.

At any rate, the absence of agreement on a unit of measure of spectrum-space usage indicates that further development is necessary. In this paper, we develop several alternatives for such a metric. One feature of all the metrics we propose is balanced consideration of the use of space by both transmitters and receivers. Another feature is that the basic definitions incorporate realistic information about the emission and selectivity characteristics of the systems, yet reduce to some of the simple measures suggested before if perfect transmitter and receiver characteristics are assumed.

After discussion of the desirable characteristics of spectrum use metrics, and explanation of the complementary nature of use of the spectrum by transmitters and receivers, we define three possible metrics which range from a situation-specific metric whose computation requires detailed information about all relevant systems, to a generalized, easily calculated metric with somewhat limited flexibility. Each attempts to measure physical use of the spectrum, in contrast to administrative use of the spectrum. Although several conceptual problems remain, we present the metrics in the hope that they might add insight to the problem of finding useful ways to measure spectrum usage.

2. DESIRABLE CHARACTERISTICS OF SPECTRUM-SPACE METRICS

A metric of spectrum-space usage should be realistic, practical, and relevant. These adjectives are meant to imply that the metric describes the situation as it is understood (perhaps only intuitively) by people experienced in the field. In this sense, a good metric is not likely to seem novel nor surprising; but merely a systematic, structured exposition of the conventional wisdom.

Frequency allocation and assignment have long dealt with frequency, bandwidth, and physical space, so these must be the fundamental dimensions (JTAC, 1964, p. 5) of spectrum-space. Figure 1 depicts a transmitter "occuping" these three dimensions of the spectrum-space.

For spectrum allocation purposes, it is the interference region that is of interest rather than the service region. That is, assignments are made far enough apart to avoid harmful interference, rather than close enough together so that service regions are immediately adjacent to each other. So a realistic metric is one that measures the space denied to other users, rather than the space that is served by the system. Combined, these criteria imply that a spectrum-space metric should be proportional to the bandwidth × physical area(or volume) × time that a specific assignment denies to other potential users. This is analogous to the economic concept of "opportunities forgone."

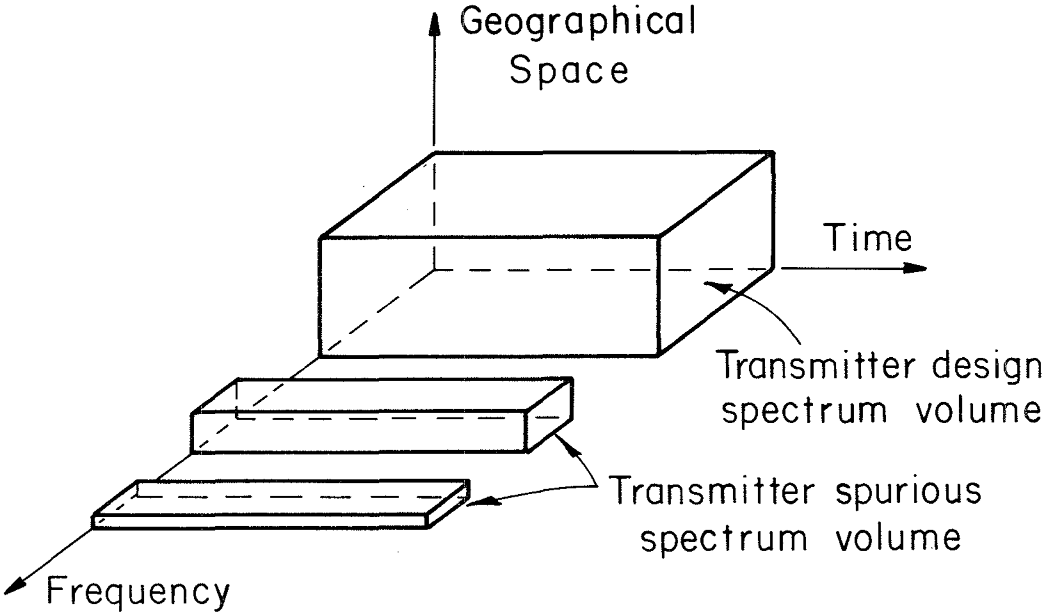

Fig. 1 A TRANSMITTER "OCCUPYING" SPECTRUM - SPACE

A metric should be objective and specific, so that any party at interest can compute its value for a present or proposed radio system. Simplicity of calculation is desirable, although it is understood that simple calculations may be less accurate.

Ideally, a metric definition should recognize that the parameters involved are random variables that exhibit statistical variation. In this paper, we do not explicitly include this statistical character; but there are no conceptual difficulties in extending the formulations to allow for statistical variation.

For some purposes, a measure of a system's use of the spectrum-space should be persistent. That is, it should not change with time, frequency, or space. Neither should it change as a result of the actions of others. One of the metrics we propose in this paper does not satisfy this criteria; the other two do. We will discuss the difference after the measures have been defined.

Finally, a metric should recognize the dual nature of spectrum-space usage by transmitters and receivers. Although this dual usage has been recognized, previous suggestions for spectrum-space metrics have been stated in terms of transmitters, thus failing to reflect the realities of spectrum allocation. It is in the formulation of complementary transmitter and receiver metrics that this paper makes it contribution.

3. THE COMPLEMENTARY NATURE OF SPECTRUM-SPACE USE BY RECEIVERS AND TRANSMITTERS

What constitutes "use" of spectrum-space? There are at least two points of view about this question: one shows up explicitly in spectrum assignments and previously suggested spectrum metrics, and the other is implicit in standard procedures. The first position is, "Transmitters use the space by filling it with power", and the second is, "Receivers use the space by denying radiating rights to others". These two views will be developed in detail below.

3.1 Transmitters as users

Traditionally, radio transmitters have been considered the users of the spectrum resource. They use the spectrum-space by filling some portion of it with radio power -- so much power that receivers of other systems cannot operate in certain locations, times, and frequencies because of unacceptable interference. Notice that the transmitter denies the space to receivers only. The mere fact that the space contains power in no way prevents another transmitter from emitting power into the same location; that is, the transmitter does not deny operation of another transmitter.

The developments in radio broadcasting in the United States before government control illustrate the thesis that transmitters are the users. Since there were no administrative restraints, receivers used no spectrum-space; that is, no system was denied because of the presence of reception rights. Transmitters used the resource by radiating power. If another potential user wanted the space, he tried to fill it with an overwhelming amount of power. The result was the power race and the chaos which the United States sought to abolish with the creation of the Federal Communications Commission, which was given the authority to regulate and set transmitter standards, but was not given comparable authority over receivers.

Even today there are service bands where the transmitter is the only recognized user. The amateur bands are the clearest example. Anyone who meets the requirements for an amateur transmitter license is allowed to transmit. No attempt is made to control the number or location of stations so as to prevent interference. Receiver owners have no explicit nor implicit rights to good reception of a particular transmitter.

Some recent attempts to define spectrum usage and spectrum rights are based on the concept "transmitters are the spectrum users". The "PODAF", a measure of spectrum usage proposed by Gifford (1966), is based entirely on transmitter characteristics; it is the power density over an area and frequency bandwidth produced by the transmitter. Power's (1968) definition of the "spectrum acre" begins "Assume that the spectral measure of a given transmission..." (emphasis added). Similarly, the spectrum rights proposed by DeVany, et al. (1969) are defined in terms of transmission. The rights proposed were that within a given location-frequency area, a user would be allowed to do anything as long as he did not allow power above a certain level to escape across the boundary of this area. The emphasized words are essentially a restriction on transmitters.

None of these proposed measures has been generally accepted even though they are consonant with current U. S. licensing procedures which allow the license holder to operate a transmitter of given characteristics at a given frequency and location. We think one reason for the non-acceptance is that they do not account for all of the spectrum-space available and used, even in a congested region. For example, a map of the service areas of broadcast transmitters would show large vacant spaces between the service areas. These are called "spectrum wastelands" by Hinchman (President's Task Force on Communications Policy, 1969). The "wastelands" exist because the frequency managers implicitly grant reception rights to many existing receivers by rejecting applications for transmitters that would interfere with them. The "wastelands" are in fact the spectrum-space used by the receivers.

3.2 Receivers as users

Receivers use spectrum because they deny it to transmitters (JTAC, 1968, p.S8-85). At the present time this denial is implemented administratively. The mere physical operation of the receiver interferes with no one except as it inadvertently acts as a transmitter or power source. Even then the space used physically is relatively small. Rather, it is the administrative or regulatory protection given to the receiver that "uses" the spectrum resource. The authorities deny licenses to transmitters in an attempt to guarantee interference-free reception within a certain space. This denial constitutes use of the space by the receiver. The radio astronomy bands are a familiar example of the recognition of receiver use of the spectrum space.

Receiver rights are typified as "defensive" rights or "protection" rights. In simple form the holder of the rights would be guaranteed that no signals greater than some specified level will penetrate his boundary more than some small fraction of the time (DeVaney et al., 1969). The boundary may be in frequency, geography, or any of the other possible dimensions of the spectrum-space. A spectrum metric for receiver usage of spectrum would involve these same factors.

However, receiver use is seldom stated explicitly in the United States (although the Government Master File does contain data on receiver locations and characteristics). No certificate is issued showing the specific protection given. Rather, each transmitter application is examined to see whether it will infringe on the unwritten rights of protected receivers. If it does, the written rights of the transmitter are redefined so that the protection rights of the receiver are guaranteed. In some cases, such as the television allocation table, the receiver protection is built into the specifications of the transmitter rights even before transmitter applications are accepted.

The standard AM broadcasting band provides an exceptional example of explicit definition of these defensive rights. No new transmitter is allowed to put a signal above a specified power into an area already assigned (JTAC, 1968, p.S7-11). The defensive barrier is erected at multiple boundaries graduated in a stepwise fashion. There are two geographical boundaries, the A and B contours, and the frequency boundaries extend three channels on either side. The signal strength that another transmitter can put across each of these boundaries is limited to specific amounts which vary with frequency. The applications for a new transmitter must show that these barriers will not be violated. If, after operation actually begins, it can be shown that the barriers have been violated, the transmitter owner can be required to change the characteristics of the transmitter to protect the previous reception rights.

Even in this case, however, the rights are not couched in terms of reception. The barriers are not erected around a market area chosen by the licensee, but instead are defined by contours of specified signal strength produced by his transmitter.

Now, let's look at some implicit grants of reception rights. One of the common ways of granting these is by setting up "guard bands" between assigned transmitter channels. For example, in the U.S., adjacent television channels are not assigned in the same city because receivers do not have sufficient selectivity to reject strong signals in adjacent channels. Thus, TV receivers are using (by denying to others) a wider bandwidth than transmitters. Similarly, co-channel assignments are separated far more in geographical space than the coverage area of a transmitter to protect the quality of reception.

The unassigned bandwidth and geographic space are unaccounted for "wastelands" if the measure of usage depends only on transmitter characteristics. They imply the need for a unit of measure of receiver use of the spectrum-space. The complementary nature of transmitter and receiver use of the spectrum-space has been recognized in discussion of radio communications rights: "...There must be emission rights to radiate energy on specified bandwidth, at a specified time, within a designated area, at some maximum power level; admission rights that exclude others from radiating energy greater than some predetermined level on a frequency at a time, and in an area to which one's rights pertain..." (Levin, 1971, p. 90).

3.3 There are two spectrum-spaces

The preceeding discussion indicates that receiver and transmitter usage of the spectrum resource results in complementary denial;

transmitters deny use of a time-frequency-geographic region to receivers wishing to receive another signal and a protected receiver denies a time-frequency-geographic region to transmitters whose operation would interfere with it. An obvious way to incorporate these facts into a unit of measure of spectrum space is to partition the resource into two spaces -- the transmitter space and the receiver space -- and define dual units to measure the usage of each space.

4. ASSUMPTIONS AND BASIC FACTORS USED IN THE DEFINITION OF SPECTRUM METRICS

We have emphasized that spectrum-space use results from either physical or administrative denial to other users. However, we should be aware that there is not always a clear distinction between these two types of use. An illustration of this point may be made by a pulse radar with a rotating antenna, where the transmitter and receiver are co-located. The pulse duration may be measured in microseconds, so that the transmitter emits electromagnetic power only a small fraction of the time. However, the receiver requires the interpulse periods to process the radar returns. Similiarly, in the space domain, the transmitter directs energy in a given direction only a portion of the time. Although it is clear that the spectrum-space denied is not entirely denied physically, there may be confusion as to what space is denied physically and what space is denied administratively.

The remainder of this paper is devoted to spectrum-space metrics which emphasize <u>physical</u> denial. Hopefully, their definition and discussion will give insight into the nature of spectrum-space use and its measurement.

As a result of the discussion in Sections 2 and 3, we take the term, "spectrum-space" to imply the dimensions of geographic area (or volume), frequency bandwidth, and time, and "spectrum-space used" to mean "spectrum-space denied to other users" (as contrasted to "spectrum space in which a service is provided"). It has long been accepted that the former space ("the interference region") is larger than the latter ("the service region") and that it is the one more appropriate for spectrum assignment purposes.

Within these general specifications, we consider three ways of determining the amount of space used:

The Situation Specific Denial Metric. The measure can be the volume of spectrum space that a system denies actual existing systems that are competing for the space.

The Uniform Denial Metric. The measure can be the volume of the spectrum space denied by a transmitter (receiver) to an idealized, reference receiver (transmitter). Once the reference is chosen, this measure depends only on the characteristics of the station being evaluated.

The Autonomous Metric. A measure might depend only on the characteristics of the particular station (receiver or transmitter) which is denying space to others. The most important characteristics are the emission or admission characteristics of the station, and the grade of service required. Such a simple measure is especially appealing if it is proportional to the amount of spectrum space denied.

In sections 5, 6, and 8, we develop spectrum metrics for both the transmitter and receiver spaces for all three of these alternatives, and show the relationship of the metrics to each other. All the metrics are defined in terms of the basic characteristics of transmitters and receivers that we will now describe.

For the purpose of determining its spectrum use, a transmitter can be characterized by its location in geography and frequency, and by its emission power density function $\epsilon(\varphi, f; f_T)$ which has the units watts per radian per Hertz. This function shows the spectral power density at frequency f radiated in the azimuthal direction φ, when the transmitter is tuned to frequency f_T. We are confining ourselves to a two-dimensional surface for simplicity of exposition. The extension to a third spatial dimension is given in section 7.

In most practical problems the power density emission function of the transmitter can be approximately separated into the product of a function of frequency and a function of azimuth:

$$\epsilon(\varphi, f; f_T) \approx p(f; f_T)\, g_T(\varphi) \tag{1}$$

where $p(f; f_T)$ is the spectral power density at the antenna terminals and $g_T(\varphi)$ is the transmitting antenna gain function (at its nominal design frequency). Figure 2 shows an illustrative plot of a transmitter spectral power density function. Most of the power is in a narrow band centered at the tuned frequency, f_T. There are spurious emissions at the harmonic frequencies, $2f_T$, $3f_T$, ...

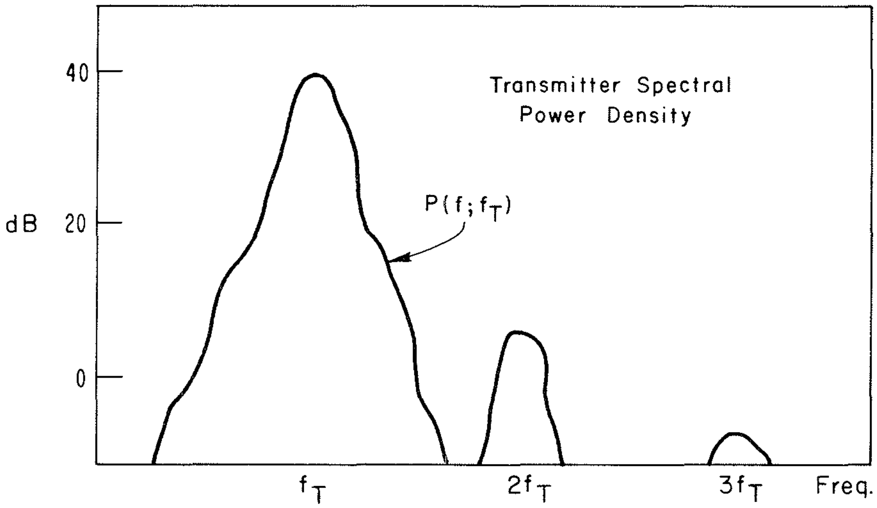

Fig. 2 TRANSMITTER SPECTRAL POWER DENSITY

Similarly, a receiver can be characterized by its location, and its admission function $\alpha(\varphi', f; f_R)$, which shows what fraction of the power density at frequency f arriving from direction φ' will reach the demodulator of a receiver tuned to frequency f_R. It has no units.

The receiver admission function can be approximately separated

$$\alpha(\varphi', f; f_R) \approx \frac{g_R(\varphi')}{q(f; f_R)} \tag{2}$$

where $q(f; f_R)$ is the selectivity function of the receiver and $g_R(\varphi')$ is the receiver antenna gain pattern. Figure 3 shows an example of a receiver's selectivity function.

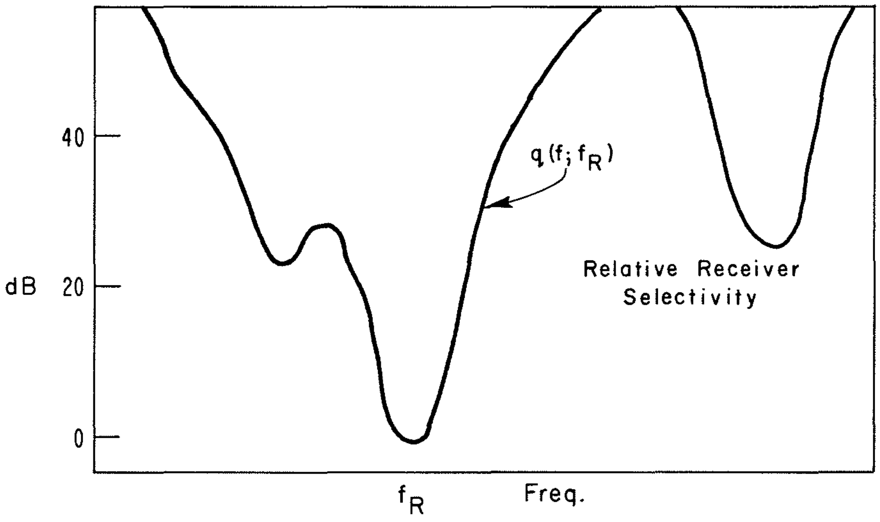

Fig.3 RELATIVE RECEIVER SELECTIVITY

Much of the power emitted by the transmitter is lost before it reaches the receiver. This is usually described in terms of the basic transmission loss. "Basic transmission loss" is defined to be the loss between isotropic antennas. We will denote it by $\ell(f, d)$ where d is the distance between the transmitter and receiver. In practical situations, the basic transmission loss will depend on the terrain along the path, and other factors such as atmospheric conditions, ionospheric conditions, and the presence of rain, dust, or fog. It will vary with time because these conditions vary with time. We will assume that $\ell(f, d)$ represents the basic transmission loss for average conditions for the frequency of interest.

A general expression for the power coupling between a transmitter T and a receiver R is

$$\begin{aligned} P &= \int_0^\infty \left[\frac{\epsilon(\varphi, f; f_T)}{\ell(f, d)} \right] \left[\alpha(\varphi', f; f_R) \right] df \\ &\approx g_T(\varphi)\, g_R(\varphi') \int_0^\infty \frac{p(f; f_T)}{\ell(f, d)\, q(f; f_R)}\, df \quad . \end{aligned} \tag{3}$$

In this equation, φ is the azimuth angle from T to R and φ' is the azimuth angle from R to T. The first factor in brackets is the spectral power density arriving at the receiver having suffered basic transmission loss $\ell(f, d)$. Figure 4 shows a sample plot of transmission loss as a function of distance (Rice et al., 1967). The second factor in brackets shows what fraction of that power density affects the receiver. By integrating over all frequencies with non-zero power density, we get the total received power, P.

Suppose now that transmitter T and receiver R are not in the same system. Thus there is a potential for an interference situation. For any receiver there is some threshold amount of power in an unwanted signal that will interfere with acceptable reception of the wanted signal. We will assume that this threshold power level is known for each receiver of interest, and will denote the threshold power level of receiver R by P_R. In real situations, P_R usually depends on the power available in the wanted signal, but we will neglect this dependence.

If we set $P = P_R$, equation (3) can be used to determine the minimum non-interfering separation of T and R, assuming all other characteristics of T and R are fixed (including the orientations φ and φ'). In the denial type measures we will define, we will find this minimum separation for each azimuth, and for each tuned frequency of the denied system. The minimum separation that T requires a receiver like R in direction φ is denoted by $d(\varphi, f_R)$, and the minimum separation that R requires a transmitter like T in direction φ' is denoted by $d(\varphi', f_T)$.

Finally, either a transmitter or a receiver may want to operate only part of the time. We will denote the time used by τ.

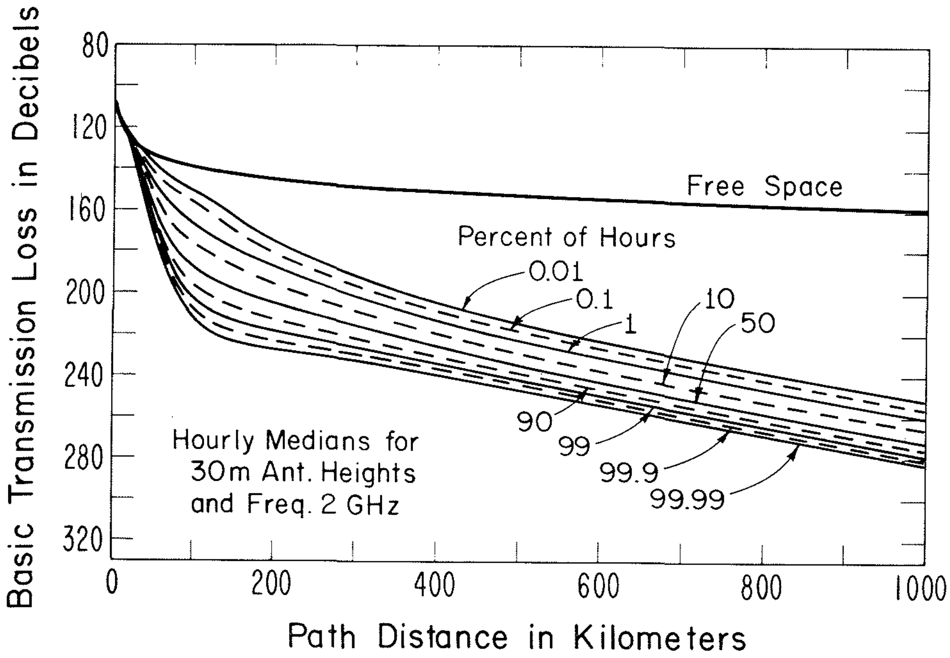

Fig. 4 BASIC TRANSMISSION LOSS vs DISTANCE AND TIME AVAILABILITY

The denial metrics which we define based on (3) assume that the characteristics of the stations remain fixed. This is not always true. For example, a radar may have a rotating antenna (so that φ varies periodically with time) or an airborne transmitter may rove all over the globe. In these cases, we revert to our basic guideline that the metric must measure the bandwidth × area × time denied others, rather than mechanically evaluating formulas.

5. THE SITUATION SPECIFIC DENIAL METRIC

Often, there are only a few types of systems competing for assignments in a specific spectral location and geographic region. The spectrum-space that an authorized system denies to each competing system can be calculated. This is the approach to the <u>situation-specific</u> metric. The combined spectrum-space that a system denies to all competing systems is the measure of its spectrum-space usage.

5.1 Spectrum-space used by the transmitter

The transmitter is characterized by its spectral density function $p(f;f_T)$ and its antenna pattern $g_T(\varphi)$. Suppose the typical receiver R in a competing system has selectivity function $q(f;f_R)$, antenna pattern $g_R(\varphi')$, and power threshold P_R. Solve equation (3) for $d(\varphi,f_R)$, the minimum non-interfering distance separation in direction φ for receiver tuned frequency f_R. Figure 5 is a plot of $d(\varphi,f_R)$ for an illustrative transmitter with a directional antenna. It can be seen from figure 5 that the geographical area that T denies R is given by

$$A(f_R) = \int_0^{2\pi} \tfrac{1}{2}\, d^2(\varphi,f_R)\; d\varphi \qquad (4)$$

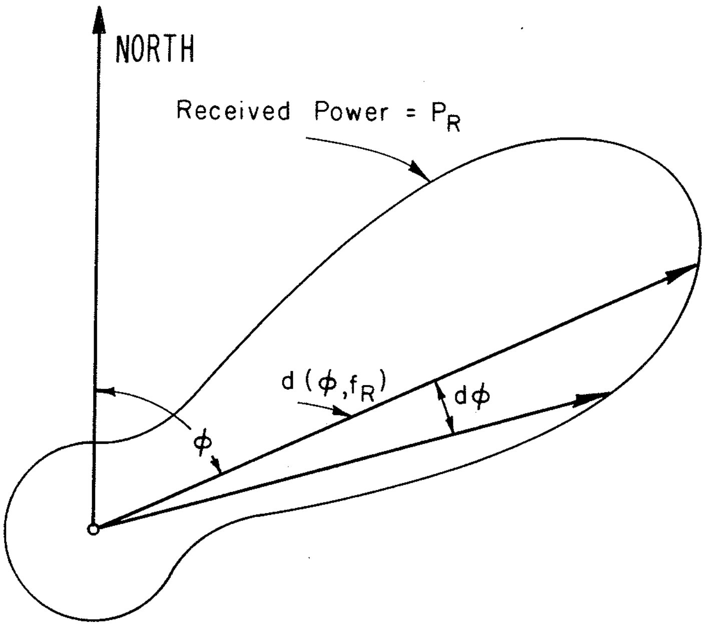

Fig. 5 PLOT OF MINIMUM NON-INTERFERING DISTANCE FROM A TRANSMITTER TO A RECEIVER.

The area in (4) can be computed for each frequency f_R (hence for all frequency separations $f_T - f_R$), and the resulting areas are summed. The result multipled by τ is our definition of the situation specific metric M_T for the spectrum-space used by transmitter T,

$$M_T = \tau \int_0^\infty A(f_R) \, df_R = \tau \int_0^\infty \int_0^{2\pi} \tfrac{1}{2} d^2(\varphi, f_R) \, d\varphi \, df_R \qquad (5)$$

where $d = d(\varphi, f_R)$ satisfies (3).

Notice that this measure is correct only for a fixed receiver pointing angle φ' (relative to T). This is one reason that we call this metric "situation specific". Another reason is that it is the measure of the space denied to a receiver with a specific selectivity function. There are often other receivers with different selectivity functions which are potential users of the spectrum-space. Thus the situation specific metric for T with respect to another receiver would likely result in a different value.

5.2 Spectrum-space used by the receiver

To ensure that receivers have satisfactory reception, frequency assignment authorities will deny applications for transmitters too near these receivers in frequency, geography, and time. The space within which applications are denied is the space used by the evaluated system's receivers. Its calculation is analogous to the calculation of the transmitters use.

In this case, transmitter T is chosen to be a typical transmitter in a system competing with R. The metric is defined by

$$M_R = \tau \int_0^\infty \int_0^{2\pi} \tfrac{1}{2} d^2(\varphi', f_T)\, d\varphi'\, df_T \qquad (6)$$

where $d = d(\varphi', f_T)$ must satisfy (3) for the admission function of the evaluated receiver and the emission function of the denied transmitter.

Formally, the only difference between the metrics M_R and M_T is the interchange of admission and emission functions but it is likely that the numerical value is different. At any rate, the space measured is different; it is the receiver space denied to transmitters.

As before, there may be several kinds of transmitters denied which results in several values of M_R. A way of making the metric single-valued (i.e., well-defined) will be discussed after we have interpreted the calculations graphically.

5.3 Graphical interpretation and the case of more than one competing system

We will consider only the transmitter metric in this section. For the receiver metric, there is an analogous discussion.

For simplicity assume that the receiving antenna is isotropic, so that $g_R(\varphi') = 1$. The minimum separation between the transmitter T and the "typical" receiver in direction φ from T can be computed using figures 2, 3, and 4 in equation (3). If this is done for all directions, the result will be a plot such as that shown in Figure 5. The area inside the boundary can then be estimated for each receiver tuned frequency f_R, and the result $A(f_R)$ plotted against f_R as shown in Figure 6 (f_T remains fixed). The area under the curve in figure 6 multiplied by the time τ is then the measure M_T of spectrum resource denied by the system being evaluated with respect to the competing system receiver R.

The calculations of the measures are sufficiently complicated that a computer would usually be used to evaluate the metrics. This is especially true if there are several competing systems. Figure 7 shows an array of plots (such as Figure 6) that might result from an evaluation of the space denied to four competing systems. The column on the left is the transmitter space denied by the evaluated system's transmitter to the four competing systems' receivers. Notice that both the shape and the magnitude of the space may be different for different systems. The column on the right shows the receiver space denied by the evaluated system's receiver to the transmitters of other potential users. Although the information in this form is useful for studying potential interference situations, we would like the measure to be single-valued for most purposes.

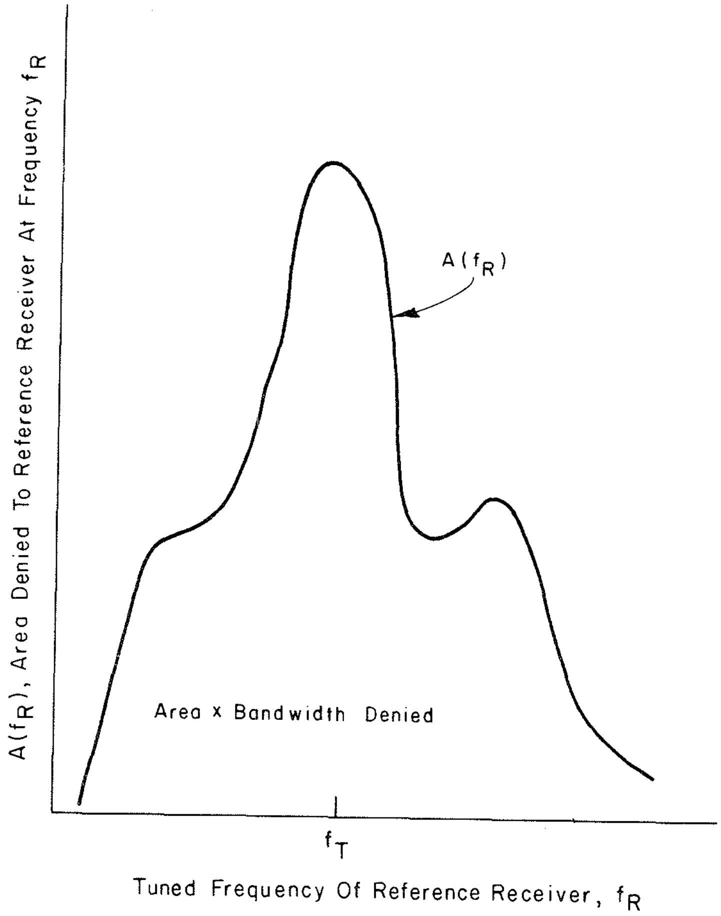

Fig. 6 AREA THAT TRANSMITTER T DENIES RECEIVER (R)

We will maintain the concept that the measure is the spectrum-space denied to competing systems. The choice of $A(f_R)$ can be made as illustrated in figure 8 where the transmitter space curves of figure 7 are plotted together. The envelope of these different curves represents the outer limits of the resource denial to the competing system's receivers. It is the area bounded by this envelope that would represent the single-valued measure of the spectrum resource denied. The effect

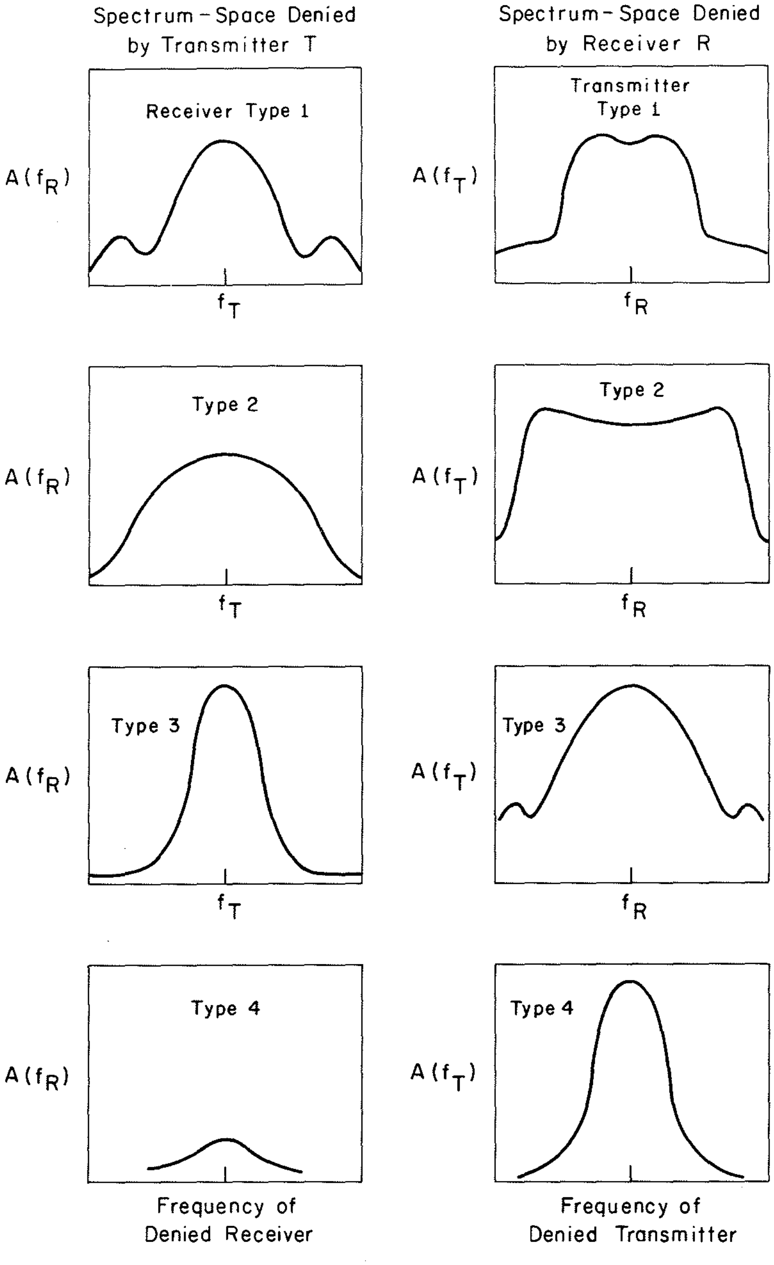

Fig. 7 FAMILY OF SPECTRUM - SPACE DENIAL PLOTS FOR VARIOUS COMBINATIONS OF RECEIVERS AND TRANSMITTERS

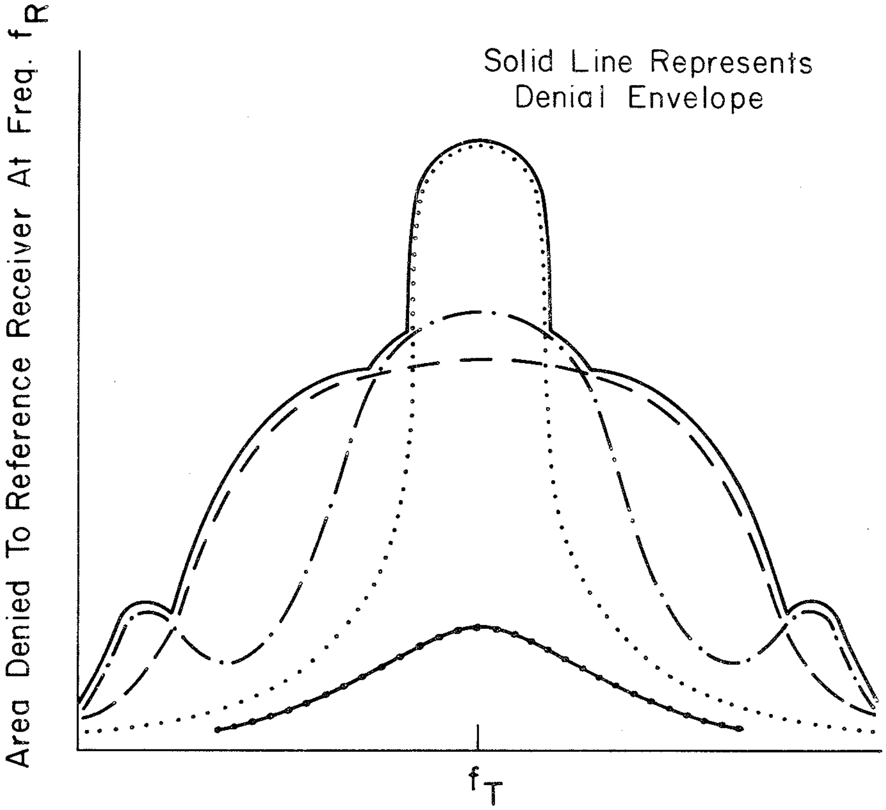

Fig. 8 MEASURE OF COMPOSITE DENIAL OF SPECTRUM - SPACE BY TRANSMITTER T TO THE FOUR RECEIVER TYPES OF Fig. 7.

of this choice for $A(f_R)$ is the same as by defining $d(\varphi, f_R) = \max_{1 \leq i \leq n} \left\{ d(\varphi, f_{R_i}) \right\}$ where there are n competing systems with receivers $R_1, \dots, R_n$. Thus it is possible for the measure to be completely determined by its denial to some one system that is especially susceptable to interference; this will be the case, for example, when $d(\varphi, f_R) > d(\varphi, f_{R_j})$ for $1 < j \leq n$.

5.4 Limitations of the situation-specific metric

The situation-specific metric would be very useful for frequency management or spectrum engineering because it is a calculation of the interference expected between existing systems in the region of interest. However, there are limitations to its use as a general-purpose spectrum metric because it does not completely satisfy two of the criteria listed in Section 2. These limitations are stated below in terms of the transmitter metric, but similar statements could be made for the receiver metric.

- the metric is somewhat subjective, since the person computing it determines the list of competing systems to be evaluated.
- the reference receiver's orientation with respect to T is fixed. This forces an assumption about the reference receiver's antenna gain in the direction of T.
- the value of the metric can be dominated by one competing system--one particularly susceptible to interference from T.
- the metric can change whenever a new system is developed for the same spectrum-space region.

In short, the metric is not persistent. This limitation suggests the concept of using a standard reference receiver (or transmitter) which is the basis of the uniform metric discussed in the next section.

6. THE UNIFORM DENIAL METRIC

We can develop a spectrum metric which is persistent, and inherently single-valued, by using the equations in the previous section to compute the spectrum-space denied to idealized reference receivers and transmitters. The underlying idea of the uniform metrics is that of reference receivers and transmitters which can be used for all regions of the spectrum-space thus avoiding the limitations of the situation-specific metrics. Spectrum-space used is now considered to be the spectrum-space denied to such reference receivers and transmitters. Equations (5) and (6) are still used to define the uniform metrics; however, equation (3) now has simplified forms. Evaluation of the metric for a sample radar system appears in Appendix A.

6.1 The uniform metric for a transmitter

For the transmitter metric, we will define an idealized "probe" receiver which has an isotropic, loss-free antenna $(g_R(\varphi') = 1)$, and a perfect, narrow selectivity function, so that $q(f; f_R) = 1$ if $f_R - b/2 \le f \le f_R + b/2$ and $q(f; f_R) = \infty$ otherwise. The receiver bandwidth b is chosen to be much smaller than the bandwidth of the transmitter being evaluated; indeed, small enough that the spectral density $p(f; f_T)$ of the transmitter is essentially constant over the bandwidth of the reference receiver.

With these assumptions on the reference receiver, the power coupling equation (3) becomes

$$P_R = g_T(\varphi)\ p(f_R; f_T)\ b / \ell(f_R, d) \quad . \tag{7}$$

where P_R is the interference power threshold of the reference receiver. Recall that (7) must be solved for $d = d(\varphi, f_R)$ to evaluate (5) for the transmitter metric.

The power threshold P_R of the reference receiver in (7) is somewhat arbitrary. However, it may logically be related to the average ambient noise power density since this is the power that would "use" the space in the absence of any system. Specifically, we may choose P_R / b to be the average ambient noise power density. The average ambient noise power density varies with frequency approximately as shown in Figure 9 [13]. The fact that this definition of P_R/b varies with frequency is not critical since it will be the same for all systems in the same part of the frequency spectrum.

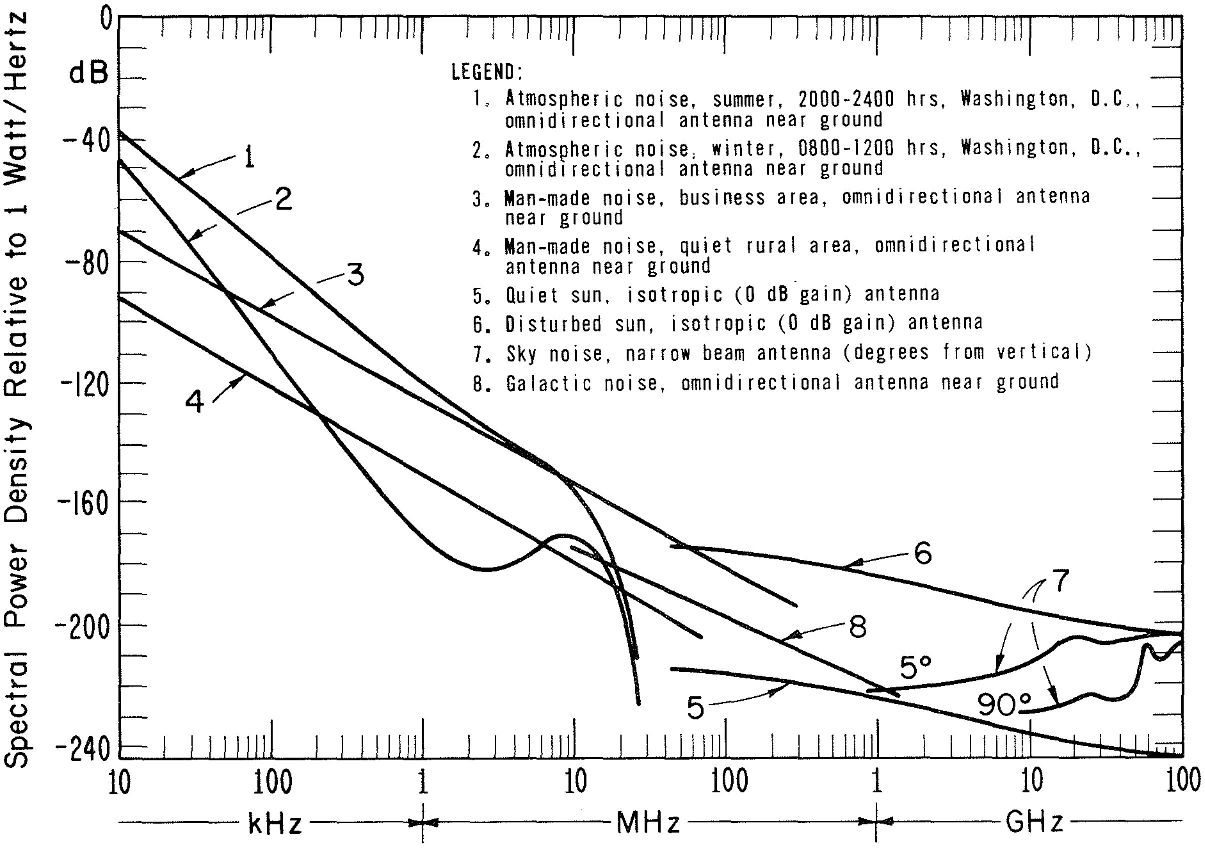

Fig. 9 MEDIAN RADIO NOISE POWER SPECTRAL DENSITY FROM VARIOUS SOURCES

6.2 The uniform metric for a receiver.

An analogous definition can be made of the space denied to a reference transmitter by a particular receiver R. In this case, we assume that the reference transmitter has an isotropic antenna ($g_T(\varphi) = 1$), and a perfect, narrow spectral density function. Specifically, $p(f, f_T) = 0$ unless $f_T - b/2 \leq f \leq f_T + b/2$, and within this interval $p(f; f_T) = P_T/b$, where P_T is the emitted power of the reference transmitter.

With these assumptions equation (3) becomes

$$P_R = \frac{P_T \; g_R(\varphi')}{q(f_T;f_R) \; \ell(f_T, d)} \tag{8}$$

where P_R is the interference threshold of the evaluated receiver. Again, equation (8) must be solved for $d = d(\varphi', f_T)$ to evaluate equation (6) for the uniform metric for receiver R.

Equation (8) shows explicitly what is intuitively obvious -- that the space denied by a receiver to a transmitter depends on the power P_T emitted by the reference transmitter. In this case there is no "natural" reference level to use as there was in the case of a reference receiver, so the choice will have to be arbitrary. How important is this choice?

For many applications, we are really interested in the amount of spectrum-space used by a receiver relative to the spectrum-space used by other receivers. It can be shown (Appendix B) that if the transmission loss $\ell(f, d)$ is proportional to a power of distance d, then the relative value of the uniform metric is independent of the choice of P_T. An important case where this condition holds is the case of free-space loss where the loss is proportional to d^2 (for frequency fixed). However, in general, even the relative value of the spectrum-space used by receivers depends on the choice of P_T because loss near the earth's surface is not proportional to a power of d (especially not for all distances). Thus the "best" choice for P_T remains an open question. Once a choice has been made, however, this measure of spectrum space use is persistent because it depends only on the characteristics of the evaluated receiver, including its power threshold.

6.3 The uniform metric, the spectrum acre, and the TAS

The uniform metric for a transmitter assumes that the reference receiver has a rectangular bandpass (i.e., selectivity function). Similarly, the uniform metric for a receiver assumes that the reference transmitter had a rectangular power spectral density function. In both cases the emission (or admission) function of the evaluated equipment (transmitter or receiver) was arbitrary; it did not need to be "rectangular".

Suppose on the other hand that the equipment which we wish to evaluate has such "perfect" characteristics. Suppose that a transmitter has a perfect power spectral density function of bandwidth B. Then we can show (Appendix C) that the uniform metric $M_T = \tau BA(f_T)$ where $A(f_T)$ is the area (4) denied to a competing receiver with tuned frequency $f_R = f_T$. Analogously, if we want to evaluate the metric for a receiver with a perfect rectangular bandpass of bandwidth B, then the uniform metric is $M_R = \tau BA(f_R)$ where $A(f_R)$ is the area denied to a competing transmitter.

Thus the uniform transmitter metric M_T for a perfect transmitter is Power's spectrum acre, the product of time, area, and frequency [3]. The TAS package is a similar product except it was proposed that the area involved be some simple, arbitrary shape (for example, a square) rather than the actual area denied to others [5]. Neither the spectrum acre nor the TAS explicitly recognize the receiver metric, however. It appears that the area involved in the TAS was the interference area of the transmitter rather than the service area. (Formally, this is just a difference in the threshold value P_R of the reference receiver.) Thus, the TAS appears to account for receiver use of the spectrum-space by incorporating receiver use into the transmitter use, much the same as frequency managers currently define rights to the spectrum-space.

6.4 Adjustments to the uniform metric.

Our measures of spectrum use above are descriptive of the amount of spectrum-space denied by individual transmitters and receivers. These measures will describe the amount of spectrum used by a system consisting of one transmitter and one receiver, even if there is overlap of the geographical areas denied to the reference receiver and transmitter, respectively. However, there is a problem in measuring spectrum use by a system consisting of multiple transmitters and/or receivers. If a system has say, multiple receivers and the spectrum-space volume denied by these receivers are not disjoint, then the amount of spectrum used by the system is not simply the sum of use by its component parts. Rather, it should be the union of spectrum-space volume denied, and the measure of system use should be less than the sum of the use by component receivers. An analogous situation may occur with a system having multiple transmitters.

As an example of a multireceiver system, let's consider television receivers in an urban area. Nearly every home has a VHF receiver. Each receiver uses a bandwidth-area (in addition to that assigned to local transmitters) in that it denies this area to transmitters. But the frequency-area denied to transmitters by your television receiver is nearly the same frequency-area as that which your neighbor's receiver denies. Thus the spectrum used by these receivers overlaps to a large extent.

A mobile system is another common type of system that can be thought of in terms of the overlap problem. Here the area denied to a reference transmitter or receiver includes, but is larger than, the area denied by a single stationary transmitter or receiver. From the viewpoint of frequency-space denial, it doesn't matter whether the system has one or one thousand mobile units. The amount of spectrum used is that amount denied to reference transmitters and receivers assuming that the geographical area is filled with many imaginary fixed transmitters and receivers. Thus the problem of measuring the use of the spectrum by a mobile system is a special case of the overlap problem.

The procedure for calculating the total spectrum-space used by receivers with overlapping protection areas illustrates the general principle. For example, suppose the evaluated system has three receivers located at R_1, R_2, and R_3 as shown in figure 10. For a particular frequency separation, the geographic area denied by each

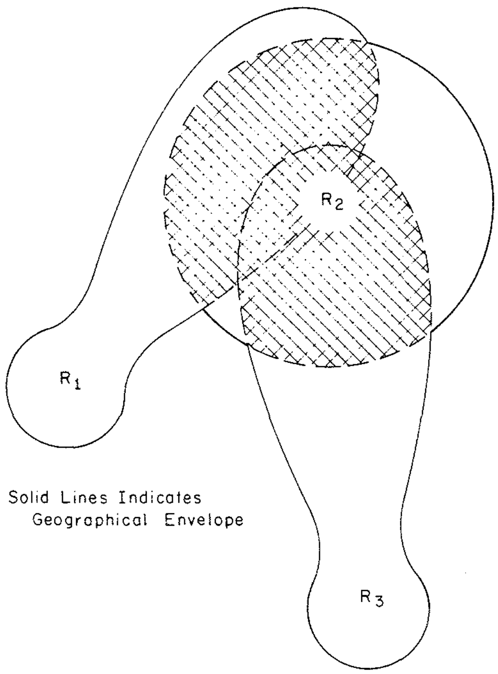

Fig. 10 OVERLAPPING RECEIVER DENIAL AREAS.

receiver is indicated by the light lines, with overlapping protection areas shaded. If equation (6) were evaluated separately for each of the receivers, and the results added together to get the total system use of receiver space, the shaded areas would each be counted twice. To avoid this requires an extra step. After $d(\varphi, f_R)$ has been found for each receiver, the envelope of the boundaries of protected areas (shown by the heavy line in figure 10) is used as the boundary of the area denied.

Similarly, Figure 11 illustrates overlap in the frequency dimension. In this case a system may have two receivers tuned to different frequencies with the selectivity functions shown. The guard band protection required by the two receivers overlaps as shown. To avoid adding this area twice, a composite selectivity function as indicated by the heavy line should be used when evaluating equation (6).

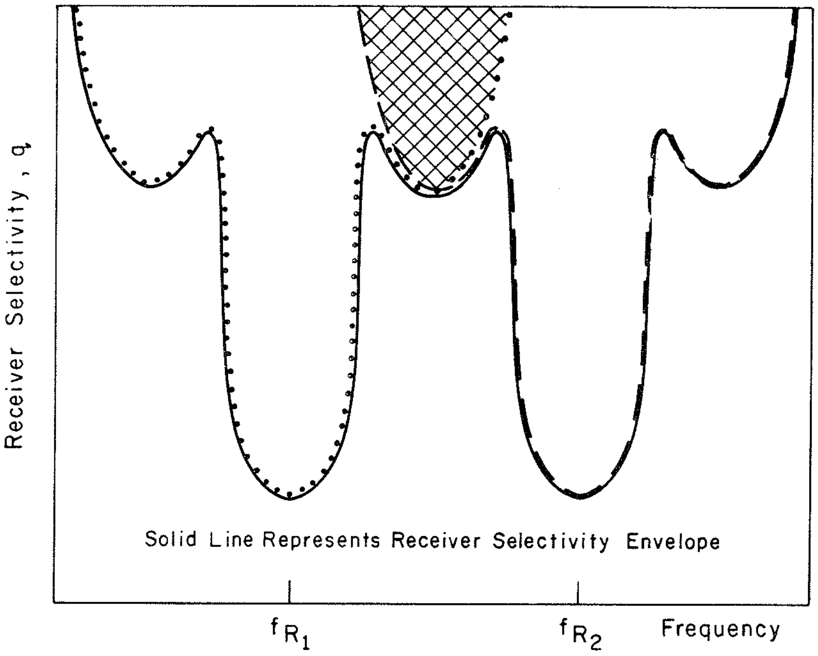

Fig. 11 OVERLAPPING FREQUENCY GUARD BANDS OF TWO RECEIVERS

Another adjustment which must be made in our definition of the uniform metric arises from the fact that for some equipments the emission or admission functions are not constant, but vary with time. One example is a narrow-beam radar which sweeps over an area. The metric as defined by equations (5) and (6) evaluated for one position of the beam does not adequately reflect the spectrum-space denied. A closer approximation may be obtained if we assume that the antenna gain is the maximum gain in all directions (simultaneously). Appendix A shows a sample calculation for this type of system.

Furthermore, with radar, the time when pulses are actually emitted is less than the time needed to receive and process returns. We reemphasize that the time τ in equations (5) and (6) is the time denied to others rather than the time that power is emitted. In general, the metrics should be interpreted as measuring spectrum space denied rather than spectrum space occupied.

This example brings us to investigating the distinction between a spectrum-space metric and rights to spectrum-space [5]. We have noticed that possession of rights to the spectrum (i.e., holding a license) does not imply the "use" of the space except in the sense that competing systems are denied access. However, any system of rights to the spectrum-space is intimately connected with denial of space to competing systems. Thus a rights system and our metrics have the denial concept in common. There is an important distinction, however. A spectrum-space right, say for a transmitter, requires detailed specification as to the portion and extent of spectrum-space which can be accessed. On the other hand, a metric for that transmitter consists only of one number which reflects the amount of spectrum space denied because of the spectrum-space right [5].

7. DENIAL METRICS FOR THREE SPATIAL DIMENSIONS

The spectrum-space resource actually is not limited to 2-dimensions in geographical space as we have heretofore assumed. Rather, the resource has three spatial dimensions. This is especially important when we consider the growth in space communications and aircraft use of the spectrum resource.

If we wish our equations to reflect this third spatial dimension, we must make some changes in the formulation of the equations defining the denial metrics. Let θ be the "elevation angle" of a reference receiver with respect to a transmitting antenna, and let θ' be the elevation angle of a reference transmitter with respect to a receiving antenna. If the antenna is on the earth's surface this has the usual meaning. Otherwise θ and θ' simply represent the third dimension in a spherical coordinate system. In this case, however, θ and θ' are measured from $-\pi/2$ to $\pi/2$ rather than from 0 to π in order to be consistent with the concept of elevation angle.

Several parameters in our equations now depend on the elevation angle, such as the antenna gains, the emission and admission functions, and the distance to the reference receiver or transmitter. Below, we reformulate the equations (3), (4), (5), and (6) to account for three spatial dimensions. The necessary notational changes in (1), (2), (7), and (8) are easily made.

$$\begin{aligned} P_R &= \int_0^{\infty} \frac{\epsilon(\theta,\varphi,f;f_T)}{\ell(f,d)} \quad \alpha(\theta',\varphi',f;f_T) \; df \\ &\approx g_T(\theta,\varphi) \; g_R(\theta',\varphi') \int_0^{\infty} \frac{p(f;f_T)}{\ell(f,d) \;\; q(f;f_R)} \; df \end{aligned} \qquad (3')$$

In Equations (4'), (5'), and (6'), d must satisfy (3').

$$V(f_R) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} 1/3\ d^3(\theta, \varphi, f_R)\ d\varphi\ d\theta \tag{4'}$$

$$M_T = \tau \int_0^{\infty} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} 1/3\ d^3(\theta, \varphi, f_R)\ d\varphi\ d\theta\ df_R \tag{5'}$$

$$M_R = \tau \int_0^{\infty} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} 1/3\ d^3(\theta, \varphi, f_T)\ d\varphi\ d\theta\ df_T \tag{6'}$$

8. THE AUTONOMOUS METRIC

The spectrum metrics developed in sections 5 and 6 both have objections. The situation-specific metric is not persistent, since its value depends critically on the characteristics of the most interference-prone competing system. Furthermore, its evaluation depends on the solution of an integral equation (3). These objections are removed in the uniform metric by using measures of the space denied by idealized reference systems. However, in this case, the objections center on the somewhat arbitrary nature of the reference systems.

Both of these metrics depend on the transmission loss ℓ, which in reality is only known statistically, and which varies greatly depending on the terrain, climate and other geographic and temporal features.

These limitations are probably an inherent part of any metric which is intended to compute the actual amount of spectrum-space denied to other users.

In this section, we use the insight gained by developing the above metrics to define a generalized metric called the *autonomous metric* because it depends only on the characteristics of the evaluated system, and does not require arbitrary choices. Although it is not inherently a calculation of spectrum-space denied to other systems, this metric is proportional to the amount of spectrum space used for the important special case of free space transmission loss and two spatial dimensions.

In making the definition for the receiver metric, we assume that the receiver user can define the protection he requires from interfering power spectral density as a function of frequency and direction of arrival of the interfering signal. We denote this *protection power spectral density* as $\beta(\varphi', f)$, with units watts/radian/hertz. The receiver admission function $\alpha(\varphi', f; f_R)$ and the threshold of interference, P_R would probably be used to determine β.

8.1 Definition of the metric

As indicated in the previous section, the transmitter use will be directly proportional to the emission power spectral density; the more energy radiated, the more the resource is used. Receiver use will be *inversely* proportional to the protection power spectral density. For a receiver, the larger the protection power spectral density, the less the receiver is "protected", hence the amount of spectrum used is less.

The measures are defined now by summing the differential spectrum use over all angles and frequencies.

$$M_T = T \int_0^{\infty} \int_0^{2\pi} \epsilon(\varphi, f) \, d\varphi \, df \qquad (9)$$

$$M_R = \tau \int_0^\infty \int_0^{2\pi} \frac{1}{\beta(\varphi', f)} \, d\varphi' \, df \tag{10}$$

8.2 Simplified expressions for the metrics

Our next objective is to simplify these general expressions. The transmitter and receiver measures will be handled separately. Substituting equation (1) into (9) yields

$$M_T = \tau \int_0^\infty p(f;f_T) \, df \int_0^{2\pi} g_T(\varphi) \, d\varphi$$

$$= \tau P \int_0^{2\pi} g_T(\varphi) \, d\varphi \tag{11}$$

where P is the transmitter power.

If the antenna gain function is symmetric about the main axis, $\varphi = 0$, and there is only one main lobe, then a good approximation to the gain integral is $2\sqrt{2 g_o}$ where g_o is the maximum gain. See Appendix D where the approximation is shown. Even when the pattern does not have one main lobe, this expression is not grossly inadequate. In the case of an isotropic antenna where $g(\varphi) = 1$ for all φ, we have

$$\int_0^{2\pi} g(\varphi) d\varphi = 2\pi, \quad \text{whereas} \quad 2\sqrt{2 g_o} = 2\sqrt{2}$$

Using this approximation we have $M_T \approx 2\sqrt{2}\, \tau P \sqrt{g_o}$ where g_o is the maximum antenna gain. Notice that the use of the spectrum-space by a transmitter increases as the square root of the maximum antenna gain. Thus, for a given power, the more directive the antenna, the larger is the autonomous metric. This effect is reversed however, for a point-to-point service, where the power required with a directive antenna with gain g_o can be reduced by a factor of g_o below the power needed by an isotropic radiation. The net result is that for a point-to-point transmitter, the measure of spectrum use varies inversely with the square root of g_o.

If a receiver has an isotropic antenna and a rectangular relative selectivity function, then the required protection can be specified by $\beta(\varphi', f) = r$ where r is a constant representation of the maximum amount of interfering power spectral density allowed at the receiver. However, if the receiver has a directional antenna, or filters the incoming signals, the protection power spectral density is $\beta(\varphi', f) = rq(f; f_R) g_o' / g(\varphi')$ where $q(f; f_R)$ is the selectivity function, $g_R(\varphi')$ is the antenna gain function, and g_o' is the gain in the direction of the desired transmitter. Here, as in the case of the transmitter, we are assuming the independence of the two functions $q(f; f_R)$ and $g_R(\varphi')$ representing the selectivity function and the antenna gain function of the receiver respectively.

If the above assumptions are applicable, then

$$M_R = \tau \int_0^\infty \int_0^{2\pi} \frac{d\varphi' \, df}{\beta(\varphi', f)} = \frac{1}{r} \int_0^\infty \frac{df}{q(f;f_R)} \quad \frac{1}{g_o'} \int_0^{2\pi} g_R(\varphi') \, d\varphi' \qquad (12)$$

Now, by applying the approximation of Appendix D and defining

$$\frac{1}{Q} = \frac{1}{r} \int_0^\infty \frac{df}{q(f;f_R)} \qquad (13)$$

we have, $M_R = \dfrac{2\sqrt{2}\,\tau}{Q\sqrt{g_o'}}$.

Thus, for point-to-point service a directive antenna allows the spectrum used to be decreased by a factor of $\sqrt{g_o'}$ over the spectrum used by an isotropic antenna.

Since only relative values of these spectrum measures are important, the constants in our simplified expressions are not critical, and we could simplify the measures to

$$M_T = \tau P \sqrt{g_o} \qquad M_R = \frac{\tau}{Q\sqrt{g'_o}} \tag{14}$$

8.3 Relation of the autonomous metric and uniform denial metric

The autonomous metrics defined by (9) and (10) are proportional to the uniform denial metrics for an important special case -- the case in which free space transmission loss and two spatial dimensions are appropriate. We will verify this only for the transmitter metric; verification for the receiver metric is analogous.

In this case, $\ell(d) = Hd^2$ where H is not a function of d. Substituting this into (7) we get

$$d^2(\omega, f_R) = \left[\frac{b}{H\ P_R}\right] p(f_R;f_T)\ g_T(\omega) \tag{15}$$

which is to be used in (5) to get the uniform metric. The resulting uniform metric is

$$M_T = \tau\left[\frac{1}{2}\,\frac{b}{H\ P_R}\right] \int_o^{\infty} p(f_R;f_T)\ df_R \int_o^{2\pi} g_T(\omega)\ d\omega\ . \tag{16}$$

The right side of (16) differs from the autonomous metric for a transmitter (11) only by the constant $1/2\ (b/HP_R)$. Therefore, the autonomous metric for a transmitter is proportional to the uniform metric.

For other functional forms of the transmission loss or if one must use three spatial dimensions, then autonomous and uniform metrics are not proportional. However, the autonomous metric is persistent. This and its relative ease of computation (since a loss function is not involved) are its advantages.

8.4 The overlap problem

The autonomous metric is defined for individual transmitters and receivers, but, just as in section 6.4, there is the possibility that a system will have multiple receivers or transmitters whose spectrum-spaces denied overlap. Clearly, the sum of the metrics for several overlapping cases will not be proportional to the total space denied. The solution to the overlap problem for this generalized metric is not obvious, since we do not deal directly with space denied anywhere in its calculation. So, despite the appealing simplicity of the metric, and its independence from arbitrary choices, we cannot recommend it at this time.

9. COMPARISONS OF THE THREE PROPOSED METRICS

The situation-specific metric defined in section 5 is very realistic in terms of potential interference of existing systems -- that is, it accurately computes the bandwidth X area X time denied by the system to competing systems. But it is not persistent, since its value can be changed by introduction of a new system that is more susceptible to interference. Indeed, it is not entirely objective, because the values computed by different persons might depend on the completeness of their knowledge of competing systems. A related problem is that the value of the metric may be dominated by one (perhaps unimportant, or rare) system.

The uniform metric eliminates some of these objections by substituting reference receivers and transmitters for actual competing systems. It is therefore objective and persistent. The main objection to this metric is the apparent arbitrariness of the specifications of some parameters of the reference systems. These may seem unrealistic considering the wide range of systems in use in some regions of the spectrum-space.

The autonomous metric defined in section 8 is persistent, objective, easy to compute, and requires no arbitrary reference standards. Nor do we need to know the transmission loss to compute it. Moreover, it is proportional to spectrum-space denied for the important special case when transmission loss is approximately that of free space and only two spatial dimensions are considered.

However, we do not yet know how to handle the "overlap" problem for the autonomous metric. Perhaps more thought will result in modifications which will make this metric practical.

10. CONCLUDING REMARKS

In this paper, we have taken the position that use of the spectrum space means denying bandwidth, geographic area, and time to other prospective users. We have defined methods of calculating the volume metrics (or units of measure) which compute the amount of such denial using the fundamental system characteristics: the transmitter emission function, the receiver admission function, and the transmission loss.

An important extension of previous formulations is the recognition that receivers, as well as transmitters, use spectrum space, and the definition of metrics for receivers that are complementary or dual to those for transmitters.

We recommend increased public discussion and refinement of the various alternative metrics, because some generally accepted metric is necessary for advancement in studies of economic value of spectrum resources, objective application of social priorities, and even more intensive use of the resource via spectrum engineering.

11. ACKNOWLEDGMENT

Our interest in this problem was stimulated by Walter Hinchman, then Assistant Director of the Office of Telecommunications Policy (OTP). As the ideas developed we had fruitful, and sometimes heated, discussions with many people; especially Dave Colby of OTP and Stan Cohn of the Office of Telecommunications in the U.S. Department of Commerce. In addition, we thank Gene Ax and Douglas Crombie of the Office of Telecommunications for their review and criticism. Although the comments and criticisms have improved the manuscript, we alone are responsible for any of its shortcomings.

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APPENDIX A: EXAMPLE CALCULATION OF UNIFORM METRIC

The purpose of this appendix is to give an example of the evaluation of the uniform metric for a radar system. A numerical integration procedure was used. In this example, only "typical" transmitter and receiver characteristics are used for transmitter power spectral density receiver selectivity, and antenna pattern; the data entered is shown in figures A1, A2, and A3. The 50% of time curve of figure 4 was used for the loss function.

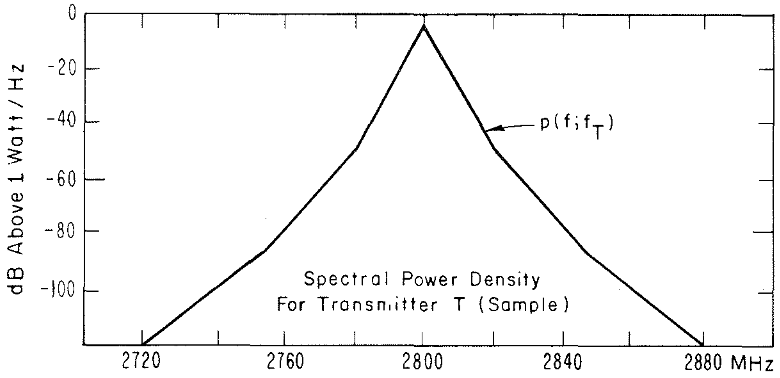

Fig. A1 SPECTRAL POWER DENSITY OF A TRANSMITTER (SAMPLE)

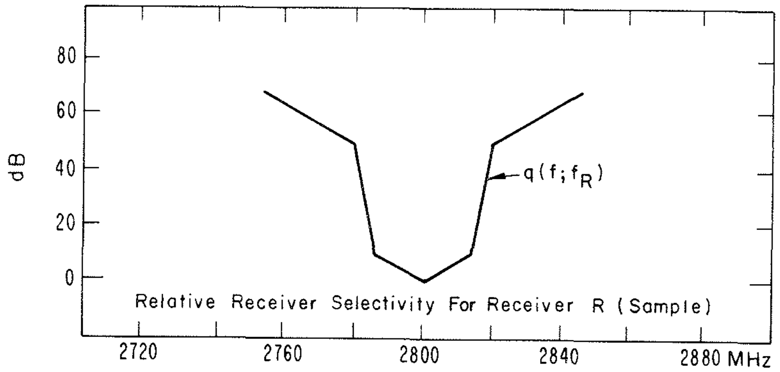

Fig. A2 RELATIVE SELECTIVITY OF A RECEIVER (SAMPLE)

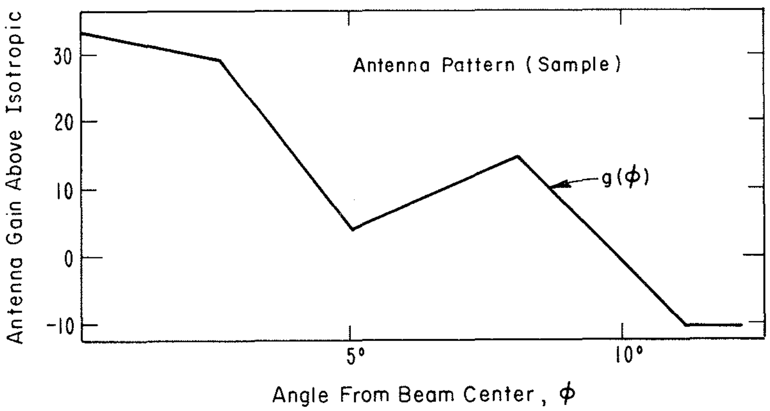

Fig. A3 ANTENNA PATTERN (SAMPLE)

The reference receiver is characterized by $P_R/b = -190$ dB which approximates the ambient noise at 2.8 GHz. The reference transmitter is characterized by $P_T/P_R = 170.6$ dB which was chosen so that the area denied at 2.8 GHz is the same for both transmitter and receiver. The basic transmission loss which was used for this example is the median loss for 2 GHz.

The computations are done in two ways. First, we assume the antenna is fixed, so that the metric shows how much of the spectrum-space is used at a given instant. Second, we assume that the antenna is rotating in the horizontal plane, so that the system must deny more of the spectrum-space. Here we assume the maximum gain in all directions because this is representative of what must be denied.

The evaluated metrics are shown in tables A1-A4. Intermediate steps show the geographical areas denied to a reference receiver (or transmitter) at various frequencies.

The metrics have units of "spectral" meters^2, since the factors are time (seconds) × bandwidth (Hertz) × area (meters^2).

Table A-1.

Transmitter Metric(Fixed Antenna)

frequency(MHz)	area denied(km^2)
2760	.05 × 10^3
2770	.15
2780	.70
2790	4.80
2800	11.80
2810	4.80
2820	.70
2830	.15
2840	.05

$M_T = 18.9 \times 10^{21}$ "spectral" m^2 (for one day)

Table A-3.

Transmitter Metric(Rotating Antenna)

frequency(MHz)	area denied(km^2)
2760	3.05 × 10^3
2770	6.73
2780	10.61
2790	22.89
2800	106.18
2810	22.89
2820	10.61
2830	6.73
2840	3.05

$M_T = 146.6 \times 10^{21}$ "spectral" m^2 (for one day)

Table A-2.

Receiver Metric (Fixed Antenna)

frequency(MHz)	area denied(km^2)
2760	.07 × 10^3
2780	.20
2790	9.20
2795	10.42
2800	11.81
2805	10.42
2810	9.20
2820	.20
2840	.07

$M_R = 28.5 \times 10^{21}$ "spectral" m^2 (for one day)

Table A-4.

Receiver Metric (Rotating Antenna)

frequency(MHz)	area denied(km^2)
2760	3.92 × 10^3
2780	7.88
2790	65.75
2795	82.23
2800	106.35
2805	82.23
2810	65.75
2820	2.88
2840	3.92

$M_R = 253.4 \times 10^{21}$ "spectral" m^2 (for one day)

APPENDIX B: DEPENDENCE OF UNIFORM METRIC ON REFERENCE POWER.

The purpose of this appendix is to verify the following statement.

> If basic transmission loss is proportional to a power of distance, then the relative value of the uniform receiver metric is independent of the choice of P_T (the reference transmitter power).

Let R_1 and R_2 be two receivers. We want to show that M_{R_1} / M_{R_2} is independent of choice of the reference transmitter power P_T, when $\ell(d) = H\, d^n$ where H is independent of d. Equation (8) in the text yields

$$d^2 = \left[\frac{P_T}{H\, P_R} \qquad \frac{g_R(\varphi')}{q(f_T;f_R)} \right]^{2/n}$$

Thus the uniform metric for R_i $(i = 1, 2)$ becomes by (6)

$$M_{R_i} = \frac{1}{2} \left[\frac{P_T}{H} \right]^{2/n} \mathrm{T} \int_0^{\infty} \int_0^{2\Pi} \frac{g_{Ri}(\varphi')}{P_{Ri}\; q(f_T;f_{Ri})} \, d\varphi' \, df_T \, .$$

We see that $1/2[P_T/H]^{2/n}$ is a factor in both M_{R_1} and M_{R_2}, so that the ratio of M_{R_1}, to M_{R_2} is independent of P_T.

We note also that the analogous statement for transmitters is true.

> If the basic transmission loss is proportional to a power of distance, the relative value of the uniform transmitter metric is independent of the choice of P_R/b (where P_R is the threshold power of the reference receiver, and b is its bandwidth).

APPENDIX C: UNIFORM METRIC AND TAS.

The purpose of this appendix is to show that for a "perfect" transmitter (or receiver), the uniform metric is proportional to the TAS, and the spectral acre.

The "perfect" transmitter is characterized by a rectangular spectral density function $p(f)$ which is zero outside the band of width B and constant $p(f_T)$ within the band. Outside the band, where the spectral density is zero, the distance $d = d(\varphi, f_R)$ to a reference receiver is zero by equation (7) of the text. Within the band the distance to a reference receiver is independent of the frequency, i.e., $d(\varphi, f_R) = d(\varphi, f_T)$. When we make these substitutions into (5) we have

$$M_T = \tau \int_{f_T - B/2}^{f_T + B/2} \int_0^{2\pi} \frac{1}{2} d^2(\varphi, f_T) \, d\varphi \, df_R$$

$$= \tau B \int_0^{2\pi} \frac{1}{2} d^2(\varphi, f_T) \, d\varphi$$

$$= \tau B A(f_T)$$

by (4), where $A(f_T)$ is the area the transmitter denies a reference receiver at frequency f_T. This is time × bandwidth × area, just as the TAS, and spectral acre are.

A "perfect" receiver is characterized by a rectangular selectivity function, which is infinite outside a band of width B and 1 within the band. Outside the band the distance to a reference transmitter will be zero by (8). Within the band the distance to a reference transmitter is independent of frequency so that $d(\varphi', f_T) = d(\varphi, f_R)$. When we make these substitutions into (6), we have

$$
\begin{aligned}
M_R &= \tau \int_{f_R - B/2}^{f_R + B/2} \int_0^{2\pi} \frac{1}{2} \; d^2 (\varphi', f_R) \; d\varphi' \; df_T \\
&= \tau \; B \int_0^{2\pi} \frac{1}{2} \; d^2 (\varphi', f_R) \; d\varphi' \\
&= \tau \; B \; A(f_R)
\end{aligned}
$$

where $A(f_R)$ is the area denied a reference receiver at frequency f_R.

APPENDIX D: NARROW BEAM ANTENNA APPROXIMATION.

The purpose of this appendix is to establish the approximation $\int_0^{2\pi} g(\varphi)d\varphi = 2\sqrt{2g_0}$ where $g(\varphi)$ is the antenna pattern for a symmetric narrow beam antenna and g_0 is the maximum gain.

Let φ and θ be the spherical coordinates used in the text, and suppose the main beam of an antenna is in the direction $\varphi = 0$. Any antenna pattern $g(\varphi, \theta)$ must satisfy

$$\int_0^{2\pi} \int_0^{\pi} g(\varphi, \theta) \sin\varphi \, d\varphi \, d\theta = 4\pi$$

by the definition of "gain". If the antenna is symmetric about the main beam axis, as is generally the case with narrow beam antennas, then $g(\varphi, \theta) = g(\varphi)$. That is, the gain depends only on the angle φ from the main beam axis. In this case, the above equation can be simplified to

$$\int_0^{\pi} g(\varphi) \sin\varphi \, d\varphi = 2$$

The antenna patterns of figure D1 were "designed" so that the above equation is satisfied. One can compare these patterns with some actual patterns shown in figure D2.

Our formulations for measures of spectrum use involve the expression

$$\int_0^{\pi} g(\varphi) \, d\varphi$$

if we are assuming that the pattern is also symmetric about the main beam axis.

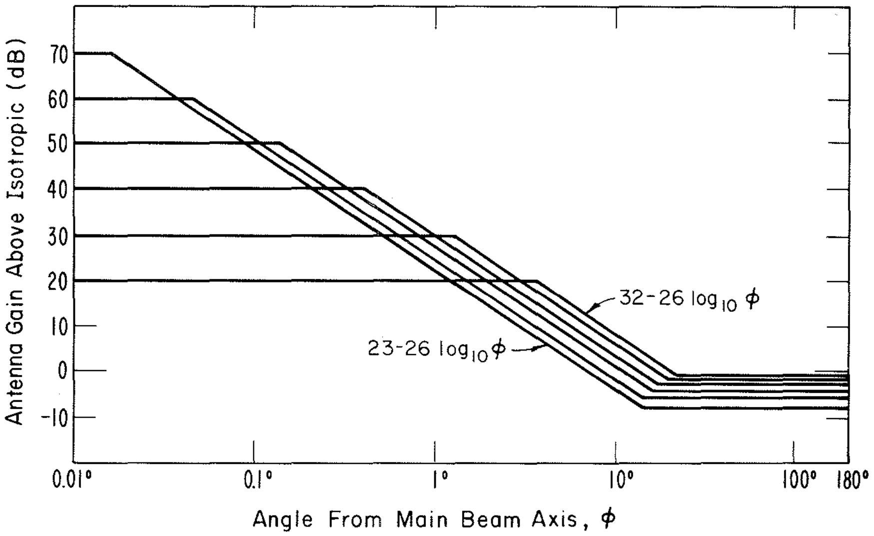

Fig. D1 "DESIGNED" ANTENNA PATTERNS

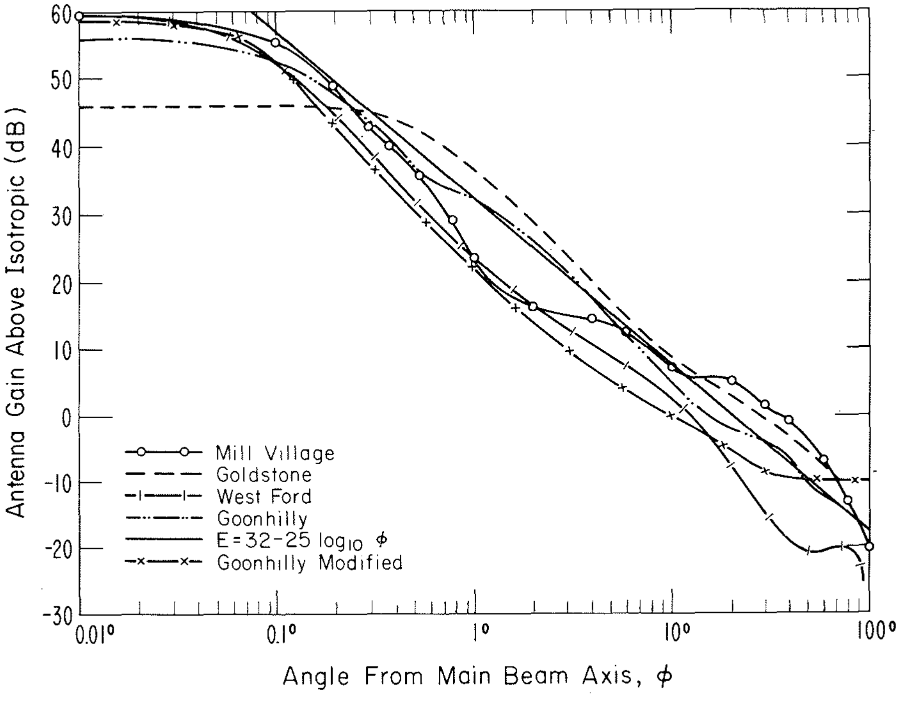

Fig. D2 SOME ACTUAL ANTENNA PATTERNS

This expression was evaluated by numerical integration procedures for the antenna patterns of figure D1 along with the Mill Village pattern of figure D2. These points are plotted in figure D3 as a function of the maximum antenna gain in decibels, G_o. (The point representing the 60 dB pattern and the Mill Village pattern coincide.)

For the purpose of comparison, the function $\sqrt{2 g_o}$ is also plotted, where g_o is the gain (not in decibels). We conclude that for narrow beam antennas, say $G_o = 10 \log_{10} g_o \geq 20$ dB, $\sqrt{2 g_o}$ is a reasonable approximation to $\int_0^{\pi} g(\varphi)\, d\varphi$. Thus for a symmetric antenna pattern, we have the approximation $\int_0^{2\pi} g(\varphi)\, d\varphi = 2 \sqrt{2 g_o}$

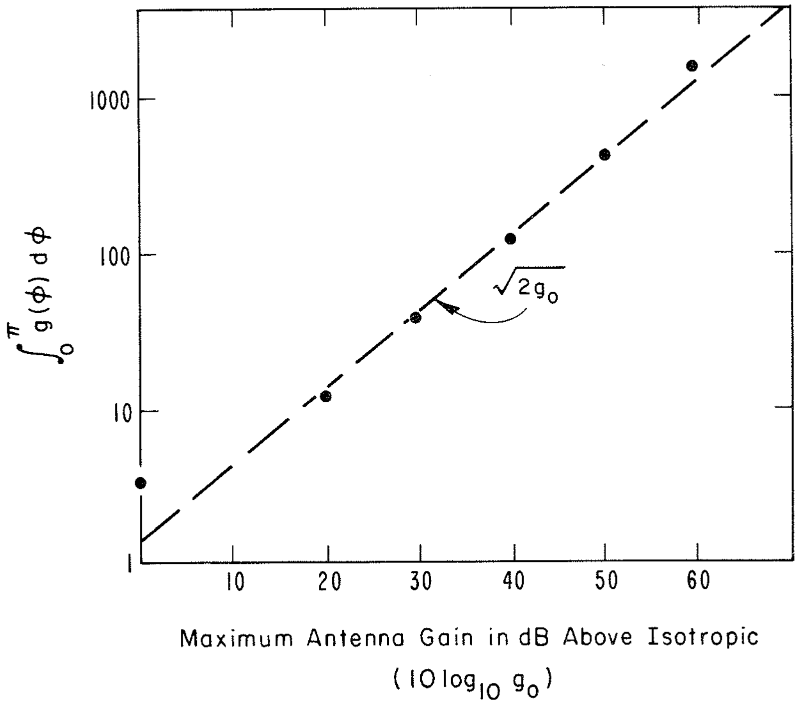

Fig. D3 PLOT SHOWING THE GAIN INTEGRAL APPROXIMATION

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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

A generally accepted spectrum use metric (unit of spectrum-space use) would expedite efficient allocation and management of the spectrum resource. It would be useful for quantitative studies of spectrum congestion, and for economic analysis of the resource. Such a metric would benefit the operation of a market or quasi-market in spectrum rights.

In this paper the position is taken that use of the spectrum-space means denial of frequency bandwidth, geographic area, and time to other prospective users. This denial may be physical or administrative in nature. Further, both transmitters and receivers use spectrum-space in this sense.

Three alternative spectrum-space metrics are defined based on physical denial of spectum-space. Their relative merits and their relationship to previously proposed metrics are discussed.

16. Key words *(Alphabetical order, separated by semicolons)*

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