FIXED RADAR SPECTRUM UTILIZATION MODEL

Prepared by the Frequency Management Support Division Office of Telecommunications U.S. Department of Commerce





 $TN-76-3$ 

# FIXED RADAR SPECTRUM

# UTILIZATION MODEL

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JUNE, 1976

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# **ABSTRACT**

This report contains a description of the Fixed Radar Spectrum Utilization Model. This model is automated and can be used to obtain: (1) a measure of spectrum crowding in a specific geographic area and radar frequency band, (2) a list of preferable frequencies at which a new radar system may operate with the least objectionable consequences, and (3) an indication as to the suitability of a proposed location as the site for a new radar system. The program has been written in Fortran for the UNIVAC 1108 computer.

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#### SECTION 1

#### INTRODUCTION

#### **BACKGROUND**

The Office of Telecommunications Policy in accordance with Executive Order 11556 has tasked (Reference 1) the Spectrum Management Support Division of the Office of Telecommunications, U.S. Department of Commerce, to perform a number of spectrum resource assessments (References 2-5). The intent of these assessments is to provide a quantitative understanding of potential problems in each frequency band of concern as well as to identify options available for dealing with these problems. Considerable effort at OT is directed towards investigating the government use of the radar frequency bands. In support of these investigations, a program is currently under way to make available an automated procedure that is to be used to assist in the determination of the congestion of the radar spectrum.

**Cohn** (March, 1968) proposed a measure of efficient use of the spectrum by taking the ratio of the spectrum space used by an ideal system to the spectrum space used by the system being evaluated. In this conception, spectrum space was defined as the bandwidth x time x physical volume that the radio system denies to other users. This initial concept of efficient use of the spectrum has since been broadened and applied to an environment. The capability developed as a

result of expanding the initial concept is the subject of this report.

#### OBJECTIVE

The objective of this task is the development of an automated capability that can be utilized:

1) to assess the spectrum congestion of a specified radar frequency band in a particular geographic area,

2) to determine if the designated spectrum does have the capacity to absorb the planned expansion of the electromagnetic systems in a particular geographic area,

3) to assist in choosing an operating frequency for a new radar system that is to be an addition to the existing electromagnetic environment, and

4) to assist in modifying an existing frequency assignment in a specified environment for those situations where the present assignment results in unacceptable interference conditions.

#### APPROACH

The development of this automated model for determining a measure of the effectiveness of radar spectrum utilization is based upon the area denial concept. This concept acknowledges the fact that within a given geographic area both transmitters and receivers contribute to the total denial area. Any government agency which is planning to add a new or additional radar system to the existing electromagnetic environment must recognize the existance of the denial areas and proceed accordingly. The calculations for determining the denial area take into account basic transmission loss, receiver and transmitter characteristics, antenna gains and consideration of an interference threshold value.

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The work reported by Cohn (March, 1968) was modified so the calculation of efficient use of the spectrm considers all the systems within a given geographic area as opposed to the original concept of determining how efficiently any one system uses the spectrum. This modification provides a means by which the degree of congestion in a given frequency band and location can be computed. This broadening of the original concept also allows for the projection as to the ability of the spectrum resource to support the planned expansion of the number of electromagnetic systems in a particular area.

In Cohn's original work the spectrum efficiency attributed to a specific radar system was determined by taking the ratio of required spectrum used by the radar system and an ideal radar system. In the modified procedure which determines the congestion of the spectrum, the parameters of an ideal radar system are not employed.

The determination of the deni al area versus frequency, .<br>. which is analogous to spectrum congestion, is computed by

calculating the separation distance required between an existing radar system and one that is planned for deployment. The separation distance referred to is the physical separation required between radar systems so that neither will suffer interference effects due to the other. The denial area is then computed by finding the area of a circle.having a radius equal to the required separation distance having its center at the location of the existing system. The denial · area is then computed at convienent increments across the frequency band. The overall result is a measure of spectrum congestion which can be illustrated graphically as a plot of denial area versus frequency.

Characteristics of a typical radar system is used for those cases where an indication of spectrum congestion is required but no definite plans exist for deployment of a new or additional system in the existing environment.

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#### SECTION 2

#### DESCRIPTION OF THE SPECTRUM UTILIZATION MODEL

#### GENERAL

For any given radar receiver and any given radar transmitter, there is some minimum required distance separation between the equipments at which the degradation to the performance of the radar receiver, due to the interfering signal from the transmitter, is at a threshold level. If the two equipments are separated by less than the minimum required distance separation, the degradation to the receiver peformance is unacceptable. If the equipments are separated by more than the minimum required distance, the degradation to the receiver performance is tolerable. The minimum required distance separation is a function of transmitter power, receiver sensitivity, antenna gains of the two equipments, transmitter emission spectrum characteristics, receiver transfer function, propagation, and a subjective determination of the threshold at which the degradation to receiver performance becomes unacceptable. In particular, the propagation loss is a function of the terrain profile along the propagation path.

When terrain is irregular, site elevation can be as \_ significant a factor in the propagation loss as distance

separation. At higher site elevations, receiver degradation may be unacceptable even though the distance separation is great. At lower site elevations, receiver degradation may be tolerable even at relatively close distances. When the terrain is irregular, there may be many different distance separations for which the receiver performance degradation equals the threshold value.

For smooth terrain, propagation loss always increases with increasing distance separation. There is one distance corresponding to the performance degradation threshold. This distance is independent of bearing to the interfering transmitter. For smooth terrain, the area in which a transmitter can be located without causing unacceptable degradation can be described as the area outside a circle with radius equal the minimum required distance separation centered at the receiver location.

### SPECTRUM UTILIZATION MODEL STRUCTURE

The Spectrum Utilization Model comprises two separate computer programs. These two programs are referred to as the pre-processor and the processor. It is the function of the pre-processor to take the input data describing the systems in the environment and the data describing the new or reference system and generate frequency-distance curves. These curves are generated considering the potential inter-

actions between the systems in the environment and the new or reference system. The processor then has the function of utilizing these frequency-distance characteristics to determine the denial area as a function of frequency. The processor also may be used to assist in determining a frequency assignment for a new radar system and to made an assessment of spectrum congestion. The primary function for which this model has be en .<br>. designed is the preliminary analysis for siting new radar systems.

### PRE-PROCESSOR DESCRIPTION

The pre-processor has been designed to perform the functions that occur only once for a given environment, and those that are time consuming and require a large portion of the computer core during execution time. The output data file, once it has been generated, can be accessed repeatedly by the processor. The pre-processor thus reduces the amount of computer memory required to execute the processor program and allow the processor to have a shorter run time.

Figure 2-1 is an illustration of the salient features of the pre-processor. Once the input data has been assimilated into the computer, frequency dependent rejection characteristics are computed for potential interactions between the existing radar systems and the system being proposed for deployment. The program then proceeds to generate frequency-distance curves. These curves are for the potential

READ DATA ON EXISTING SITES AND PROPOSED SITES LOCATION RX CHARACTERISTICS TX CHARACTERISTICS ANTENNA GAIN ANTENNA HEIGHT

CALCULATE THE FREQUENCY-DEPENDENT-REJECTION CHARACTERISTICS (FDR)

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CONSTRUCT FREQUENCY VERSUS DISTANCE CURVES

CREATE AN OUTPUT FILE FOR THE PROCESSOR PROGRAM

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Figure 2-1. Simplified Diagram of the Pre-Processor Program

interaction between the proposed system and those radars comprising the environment. These curves are generated using an interference threshold of 0 dB INR. The basic transmission loss used in the generation of these F-D curves is obtained by using the smooth curve-smooth earth mode of the inverse Integrated Propagation System Model.

The last major operation of the pre-processor is the creation of an output file containing the frequency-distance curves and the locations of the radars in the environment.

### PROCESSOR DESCRIPTION

The processor portion of the Spectrum Utilization Model determines the extent of the denial areas, calculates the spectrum utilization factor, and makes an evaluation as to the suitability of a particular frequency and a specified location for the operation of a new radar system. Figure 2-2 contains a simplified flow diagram of the operations contained within the processor program.

The input data is obtained from the output file created by the pre-processor program and from the user. The data contained in the pre-processor output file includes the locations of the radar sites, the frequencies on which the radars operate and the frequency-distance curves. The user supplies the possible operating frequencies for the proposed system and the location or locations at which the new radar

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**Figure 2-2. Simplified Flow Diagram for the Processor Program** 

system is likely to be deployed. At execution time, the user has the option as to whether the spectrum utilization factor is to be calculated.

Once the input requirements have been satisfied, the processor program then establishes a reference point at the center of the geographic area under investigation. The locations of all the radar sites in the environment are then computed with respect to this reference location. This is done to simplify calculation of the spectrum utilization factor and the suitability of the proposed locations for the new radar system.

The next step is then the determination of the denial area surrounding each site in the environment. The denial area is defined as that geographic area surrounding an existing radar site such that if the proposed system were to be located withir. this designated area, electromagnetic interference would be experienced by either the existing system, or the proposed system, or both. This points out an important feature of the spectrum utilization model. This model recognizes the fact that the existence of a receiver in the environment as well as a transmitter can deny spectrum space to another system. The concern in the model involves the realization that a receiver must operate interference free to perform its designed function. Therefore, the calculation for determining the radius of the denial area considers the possible interference

effects to the existing system due to the proposed system's transmitter. The separation between the carrier frequencies is used to obtain the required separation distance for interference free operation. When this has been completed, the reverse process is also calculated. In order to insure interference free operation for the proposed system, it is necessary to determine the required separation between the transmitter. of the existing site and the receiver of the proposed system. The final determination of the denial area is made by choosing the larger of the two separation distances. It should be noted at this point that a smooth curve-smooth earth model was employed to compute the basic transmission loss. This factor tends to place the predictions of the spectrum utilization model on the conservative side of the scale. This factor is recognized by the authors and is reflected partially in the output of the program.

The Spectrum Utilization Model may also be employed to assist in determining an operating frequency for a new radar system which will have the least impact on the electromagnetic environment and spectrum. In order to accomplish this, the program must have as an input the possible locations for the new system. The distance separation between the proposed locations and the existing radar site is computed and a comparison made to the radius of the denial area associated with each site. A table is constructed indicating at which

frequencies proposed for the new system does the postulated location fall outside of the denial areas of the equipment in the environment. It is possible to subsequently order a list of frequencies at which the postulated location for a new system falls outside the denial areas. This is accomplished by modifying the input data so that it appears that the new system is part of the existing environment. When this has been complete4 the denial areas are then recalculated and the frequency list ordered on the incremental increase in denial area that is attributed to the existence of the new system. The frequency at which the increase in denial area is the smallest is the most acceptable from a spectrum management viewpoint.

The Spectrum Utilization Model has been designed to estimate the congestion of a radar frequency band in a specific geographic area. This is accomplished by implementing equation 2-1 reported by Hinkle and Mayher, May, 1975. This spectrum utilization factor is one measure of congestion of a radar frequency band for a given geographic region. In addition to being a measure of congestion, this spectrum utilization factor can also be used to choose the frequency for a new radar system which will provide the most efficient use of the spectrum. Using this concept, the spectrum utilization factor would be calculated with and without the proposed system as part of the environment. In this manner the

effective increase in denial area as a function of frequency can be assessed due to the inclusion of the new system in the environment. This process can then be repeated for each proposed operating frequency for the new system. The smallest increase in the spectrum utilization factor would be the frequency tc choose for the new system to operate at.

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M = \sum_{i=1}^{i=n} A_i B_i
$$
 (2-1)  
A. B

Where:

A<sub>j</sub> = Area denied to the proposed system by the j<sup>th</sup> radar in the environment – this denial area occurs at frequency 
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f_j
$$
 in square miles  
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B_j =
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 Tuning increment between  $f_j$  and  $f_{j-1}$  in MHz  
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B =
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 Width of the radar frequency band in MHz  
\nA = Total geographic area under investigation  
\nin square miles  
\nM = spectrum Utilization factor

Appendix C contains detailed flow charts describing the pre-processor and processor computer programs .

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#### SECTION 3

#### CONCLUSIONS AND RECOMMENDATIONS

#### CONCLUSIONS

A capability has been developed and automated that provides a means for evaluation of the congestion of a radar frequency band in a specified geographic area. An adjunct to this capability is the provision for obtaining a list of frequencies at which another radar system can be added to the existing environment with the least number of electromagnetic interference problems. Although the development of this capability is a step forward, considerable headway can be achieved in revising the existing program to insure that it runs more efficiently and provides a wider flexibility in the output. Changes definitely need to be made so that the program is user oriented.

#### RECOMMENDATIONS

Three recommendations covering areas where additional effort should be directed are presented. The first and the one that should be given priority is that a task should be established with the specific objective of making the Spectrum Utilization Model a user oriented computer program. Detailed recommendations along this line comprise Appendixes A and B. The second recommendation is that a task be established to determine the feasibility of extending this technique of

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evaluating spectrum congestion using the concept of area denial to other than the radar frequency bands. The third recommendation involves determining the feasibility of automating a frequency assignment procedure that will generate an assignment for a particular environment.

### REFERENCES

- 1. IEEE, The Joint Technical Advisory Committee, "Spectrum Engineering--The Key To Progress, " March 1968, pp 58-85
- 2. S.I. Cohn, "Memorandum to JTAC Task Group 63.1.2," November, 1967
- 3. R. Hinkle and R. Mayher, "Spectrum Recourse Assessment In The  $2.7-2.9$  GHz Band, Phase I," OT Report Number 1/73-P 1, May 1975

# APPENDIX A

PROPOSALS FOR MODIFYING

THE RADAR SPECTRUM

UTILIZATION MODEL

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This Appendix contains detailed proposals for modifying the existing Radar Spectrum Utilization computer program. The recommendations cover both the pre-processor and the processor programs. These proposals are intended to make the program user oriented, and to have the program utilize the capabilities of the computer in a more efficient manner.

1) Establish a radar system data base for the 2.7-2.9 GHz frequency band to be used with the Spectrum Utilization Model.

2) Generate within the pre-processor a distance separation array for all the radar sites in the environment.

3) Develop a new routine to calculate the denial area in order to speed up the calculation. An example of such a routine is included in Appendix B.

4) Construct a processor program which will be interactive with the user. This program should be able to do the following as a minimum:

(a) display the denial area by frequency,

(b) allow the user to choose the frequencies at which the calculations are to be done,

(c) display denial areas for any frequency the user chooses,

(d) include provisions for the user to include a new system in the environment and evaluate its effects on the denial areas, and

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(e) determine the spectrum utilization factor for a given environment.

These items are to be included in addition to the existing capabilities.

# APPENDIX B

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A PROPOSED APPROACH TO CALCULATING THE DENIAL AREAS

The Spectrum Utilization Model employs a graphical technique to determine the denial area of a given environment. This approach was taken because of complexity of the problem of calculating the area of overlapping circles (See Figure  $B-1$ ). The proposal being made here does not replace the graphical technique but merely supplements it.

The graphical approach is time consuming when a large area is being investigated and the calculations are being made with a reasonable amount of accuracy. This proposed routine is designed to reduce the size of the area that 1s being considered.

Examination of Figure B-1 will indicate several conditions exist that can be taken advantage of in order to reduce the size of the denial area that must be calculated graphically. These are denial areas associated with a particular radar site which are not overlapped by the denial area of any other site. Obviously these areas can be calculated mathematically and the site eliminated from further consideration. Another condition exists when the denial area of one site is totally overlapped by another site. This site may also be removed from further consideration in the calculations. Still another condition exists, and that involves the situation when the denial area of a particular site (1) is partially overlapped by the



#### FIGURE B-1. DENIAL AREA PLOT

denial area from only one other site (2). In this case sufficient information exists to employ the law of cosines in order to calculate the area not overlapped by the denial area of site #2. When this has been completed then site #1 can be eliminated from further consideration. The overall result is a considerably smaller area over which the graphical technique for determining the denial area must be used.

The following flow charts present considerable detail as to the method that may be employed to implement this concept in the program.

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RADIUS  $(K) = AN ARRY COV TAINING The$ RADIUS of The donial AREA for each site in the ENVIRON MENT

 $DISSEP(T,T)$  - A DISTANCE SEPARATION ARRAY for each of the<br>sites in the expirement. THIS ARRAY IS symmetrical

SUBROUTINE STOTAL (RIR, D, TOTAL, KG)  $A = D^2 + R1^2 - R2^2$ R1- RAdius of circle MI  $2*0*RI$  $(T^{th})$  $K9 = 1$ R2 = RAdius of circle #2  $(J \nightharpoonup k)$ D = separation distance  $B = D^{2} R 2 - R 1$ Between the contric  $2 * D x R 2$  $of$   $R1 \le R2$  $A = \cos \circ f$  Angle between RIYD.  $A1 = ABCCos(A) * Q$  $B1 = AACcos(B) \nless Z$  $B = \cos$  of purche refuger  $R$  2 and  $D$  $D < R1$ <br> $D < R2$ <br> $D < R2$  $15$ YES YES  $R1 > R2$ No No  $AT = \pi R1^2$ TOTAL=TOTAL +  $-\frac{1}{\vartheta}Rt^{2}(A1-su\sqrt{h})$  $TOTAL = TOTAL$  $\pi R_2$ <sup>2</sup>  $+$   $\frac{1}{2}$  R1<sup>2</sup>(AI-SIN(AI))  $-\frac{1}{2}R_{1}^{3}(A1-sw(h))$  $-\frac{1}{3}R2(B)-SIN(B))$  $-\frac{1}{2}R2^{2}(81-51N(8))$ AT=  $AT = TRT^2$  $-523(81-5116)$  $\sqrt{67}$ AL=TOTAL + AT RETURN RETURN Return  $END$ 

SUBROUTINE<br>STOTL2 (R1, K9)  $K9=1$  $\frac{Torac}{HeC}$  is  $\rho$  ve  $VARf_{RBCE}$  $e$ ither Soline  $P1$  $\int_{1}^{b} 5$  $\frac{1}{\sqrt{2\pi}}$ hir Routine  $\mathcal{R}$ holf in common  $A = R1 + 2$  $A1 = P I * A$ TOTAL = TOTAL  $+A1$ RETURN END

 $B-10$ 

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# APPENDIX C

FLOW DIAGRAMS FOR THE SPECTRUM UTILIZATION MODEL COMPUTER PROGRAM

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PROGRAM DRIVER READ  $Dise$  $FilE$  $CALL$  $CGEN$ END

THIS PROGRAM  $1<sup>5</sup>$ THE DRIVER ROUTINE  $F \circ R$ THE PROCESSOR PROGRAM

THIS DISC FILE WAS CENERATED BY THE PRE-PROCESSOR. IT F-D CHRVES CONTAINS THE AND DATA for the SITE LOCATIONS

CGEN DETERMINES THE EXTENT OF THE DENIAL AREA AssociateD with EACH site & CALLS THE ROUTINES TO DETERMINE THE TOTAL DeNine  $A R E A$  $THE$ فليحيض SPECTRUM UTILIZATION FACTOR.



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XMIN, XMAX, YMIN, YMAX

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INCREMENT IN THE  $X - A X Z S$ 

I NCREMENT IN THE  $Y - A X I S$ 

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cohumN Location in GRID

X- Axis center point of present LOCATION

J INCREMENTS ACROSS THE GRID NETWORK IN THE  $\mathcal{Y}$  $D$ *nccrion* 

Row Location in GRID.

 $y - a x_i s$  center point of PRESENT LOCATION

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DESIGNED ROUTINE SUBROUTINE THIS  $2<sup>1</sup>$  $LAP$ TO DETERMINE IF A PROPOSEd LOCATION FALLS Within THE deNIAL AREA OF INITIALIZE RACAR AN EXISTING PARAMETERS USED IN SYSTEM SUBROUTINE yes  $TSSW1>0$ 7 No Loop for ARRANGING Roposed sites in Sematusenble By<br>Cordis SAME AS ABOVE for RADAR Sites in The EXIVIRONMENT SET THE COUNTER ARRAY TO  $Z$  $\epsilon$ RO

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