NTIA REPORT 93-298

DIGITAL EMISSION SPECTRUM MODEL

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NTIA Report 93-298

DIGITAL EMISSION SPECTRUM MODEL

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ABSTRACT

A computer program was developed to calculate the power spectral density (PSD) and the fractional power containment bandwidth for various digital modulation techniques. The power containment capability was used to provide guidance for determination of necessary bandwidth in support of Annex J of the "NTIA Manual of Regulations and Procedures for Federal Radio Frequency Management." This report documents the models contained in the computer program and illustrates the models with various sample problems. The report also shows the verification of the computer program implementation.

The digital modulation techniques that are presently included in the computer program are:

Phase shift keying (PSK) with non-return-to-zero modulation
Quadrature amplitude modulation (QAM)
Frequency shift keying (FSK)
Minimum shift keying (MSK)
Bi-phase shift keying with raised cosine modulation
Bi-phase shift keying with Manchester modulation.

KEY WORDS

Bandwidth
Digital Modulation
Emission Spectrum
Necessary Bandwidth
Power Spectral Density
Spectrum
Transmitter Spectrum

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SECTION 1
INTRODUCTION

BACKGROUND

The National Telecommunications and Information Administration (NTIA) is responsible for managing the Federal Government's use of the radio frequency spectrum. NTIA's responsibilities include establishing policies concerning spectrum assignment, allocation and use, and providing various departments and agencies with guidance to ensure that their conduct of telecommunications activities is consistent with these policies.[1] In discharging these responsibilities, NTIA assesses spectrum utilization, identifying existing and/or potential compatibility problems among the telecommunications systems that belong to various departments and agencies, provides recommendations for resolving any compatibility conflicts that may exist in the use of the radio frequency spectrum, and recommends changes to promote spectrum efficiency and improve spectrum management procedures.

The Technical Subcommittee (TSC) of the Interdepartment Radio Advisory Committee (IRAC) is concerned with the technical aspects of the use of the electromagnetic spectrum. One purpose of the TSC is to develop and recommend new standards and revise existing standards pertinent to the use of the radio spectrum. The TSC was tasked by the IRAC to prepare updated tables of necessary bandwidth formulas for the "NTIA Manual of Regulations and Procedures for Federal Radio Frequency Management." It was determined that formulas for calculating necessary bandwidths are generally available for most analog modulation techniques and that a few formulas for calculating necessary bandwidth for some digital modulation techniques are available. The National Telecommunications and Information Administration (NTIA) agreed to examine the spectral and performance characteristics of certain common digital modulation techniques and develop methodologies for calculating the necessary bandwidth for these modulation types. The results of the NTIA effort are reported in "Necessary Bandwidth and Spectral Properties of Digital Modulation," NTIA Report 84-168. As a result of the analyses documented in NTIA Report 84-168, Annex J of the "NTIA Manual of Regulations and Procedures for Federal Radio Frequency Management" was revised to provide additional guidance for determination of necessary bandwidth. As part of the Report 84-168 study, a computer program was developed to calculate the power spectral density (PSD) and the fractional power containment bandwidth for various digital modulation techniques. The digital modulation techniques that are presently included in the computer program are:

Phase Shift Keying (PSK) with Non-Return-to-Zero (NRZ) Modulation

BPSK	8 PSK
QPSK	16 PSK

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[1] NTIA, Manual of Regulations and Procedures for Federal Radio Frequency Management, National Telecommunications and Information Administration, Washington, D.C., Revised May 1992.

Quadrature Amplitude Modulation (QAM)

4 QAM	64 QAM
16 QAM	256 QAM

Two-Level Frequency Shift Keying (FSK)
Minimum Shift-Keying (MSK)
BPSK with Raised Cosine Modulation
BPSK with Manchester Modulation

Since the PSD models are applicable to other spectrum management activities, NTIA decided to develop the existing computer program into a user-friendly capability that is available on the personal computer (PC). As an initial step in this development, it was decided to verify and document the spectrum models represented in the computer program as these models are not all discussed in NTIA Report 84-168.

OBJECTIVE

The objective of this study was to verify and document with sample problems the models contained in the computer program[2] named "Emission Spectrum Model for Digital Signals."

APPROACH

To meet the objective of this study, the following approach was taken:

1. The PSD formulations used in the computer program were determined and substantiated by comparison with appropriate references.

2. The PSD models were applied to several sample problems and the results verified through comparison with manual calculations.

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[2] This computer program also includes the optional capability to determine the effect of passing the computed emission spectrum through a filter. The filter functions that are available are,

Chebyshev
Butterworth
Nyquist
Boxcar

In addition, the program includes the capability to plot the FCC spectrum mask (FCC Rules and Regulations Section 21.106) along with the computed spectrum (with or without filtering) to examine comparison with the FCC requirements.

Neither the filter or spectrum mask capabilities were validated by this study and thus these capabilities are not discussed in this report. However, examples of these capabilities are included in Appendix A.

SECTION 2
POWER SPECTRAL DENSITY MODELS

GENERAL

In general, the PSD for the digital modulation techniques that were modeled can be represented as:

$$\mathrm{PSD(f)} = \alpha' S'(f) \tag{2-1}$$

where

$$S'(f) = \textit{shape function}$$

$$\alpha' = \frac{P_T}{\int S'(f)df}$$

$$P_T = \textit{transmitter power}$$

$$f = \textit{frequency variable}$$

thus

$$\int PSD(f)df = \alpha' \int S'(f)df = P_T \tag{2-2}$$

For a specified transmitter power

$$PSD(f) = \alpha' S'(f) \tag{2-3}$$

The computer model that is discussed here uses the formulations of S'(f) and α' to obtain the required PSD information. The S'(f) formulations that are used to model the various digital modulation techniques are discussed in this section of the report. Sample PSD plots and computations verifying the PSD data are also presented.

Phase Shift Keying (PSK)

The computer program models the PSD for PSK wherein a non-return-to-zero (NRZ) modulation signal shifts the carrier phase at the symbol rate between signal states (S) representing distinct, equally-spaced angular-positions. The computer program includes the capability to model

BPSK (Binary PSK)
QPSK (Quaternary PSK)
8 PSK
16 PSK

The PSD for M-ary PSK modulation is independent of the number of signal states (S) and is a function of the symbol rate ($1/T_s$). It is of the form[3]

$$S'(f) = \left[\frac{\mathit{Sin}\ \pi T_s (f-f_c)}{\pi T_s (f-f_c)}\right]^2 \tag{2-4}$$

where

T_s = M-ary symbol length, $T_s = T_B (\log_2 S)$
T_B = digital bit length, $T_B = 1/B_R$
B_R = digital bit rate
f = frequency variable
f_c = carrier frequency

The digital bit rate (B_R) is the bit rate applied to the modulating process. This bit rate includes bits required for net management, addressing, error correction, etc. In the case of spread spectrum the bit rate would be the psuedo-noise chip rate. This parameter is denoted as modulating bit rate on the various spectrum plots in this report.

Example PSDs for BPSK (S=2) are shown in Figures 2-1 through 2-3.[4]
Figure 2-1 is the PSD for a bit rate of 1 MHz and 2 watts transmitter power. Figure 2-2 shows a bit rate of 3 kHz with 300 watts and Figure 3 is a bit rate of 5 MHz and 150 watts. Examination of equation 2-4 shows that S'(f) would have a maximum at $(f-f_c) = 0$ where it can be shown that in the limit

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3 Feher, K., Digital Communications, 1981.

4 The PSD plots also show a "99% band" value. This is the bandwidth that contains 99% of the transmitter power.

$$S'(0) = 1$$

and subsequent maxima would occur at

$$(f\text{-}f_c)\ T_s = \pm\ 3/2,\ \pm\ 5/2,\ \pm 7/2, \text{ etc.}$$

The minima would occur at

$$(f\text{-}f_c)\ T_s = \pm 1,\ \pm 2,\ \pm 3, \text{ etc.}$$

For BPSK where $S=2$, $T_s = T_B$ and $B_R = 1/T_S$, the maxima and minima of Figures 2-1 through 2-3 are at the correct multiples of the bit rate. Equation 2-4 shows the spectral envelope fall-off expressed in dB is

$$20 \log \frac{1}{\pi T_s\ (f\text{-}f_c)}$$

That is, the first maximum would be down 13.5 dB from the maximum at $(f\text{-}f_c) = 0$. The second maximum would be down 17.9 dB, the third down 20.8 dB, and the fourth down 23.0 dB. The figures for BPSK show the expected spectral decay. The 3 dB bandwidth for BPSK can be shown to be approximately B_R. (A more exact figure would be 0.88 B_R.) Figure 2-1 is based on 2.0 watts; the PSD at $(f\text{-}f_c) = 0$ should be

$$10 \log \frac{2.0\ watts}{1.0\ MHz} = 3.0\ dBW/MHz$$

Figure 2-2, the maximum PSD is

$$10 \log \frac{300\ watts}{3\ kHz} = 20.0\ dBW/kHz$$

Figure 2-3, the maximum PSD is

$$10 \log \frac{150\ watts}{5\ MHz} = 14.7\ dBW/MHz$$

The PSDs are shown for QPSK (S=4) in Figures 2-4 through 2-6. For QPSK: S = 4, $T_S = 2/T_B$, $T_B = 1/B_R$ so that the maxima of the PSDs should occur at

$$(f\text{-}f_c) = 0,\ \pm 3/2\ (B_R/2),\ \pm 5/2\ (B_R/2),\ \pm 7/2\ (B_R/2), \text{ etc.}$$

The minima should occur at

$$(f\text{-}f_c) = \pm B_R/2,\ \pm 2B_R/2,\ \pm 3B_R/2, \text{ etc.}$$

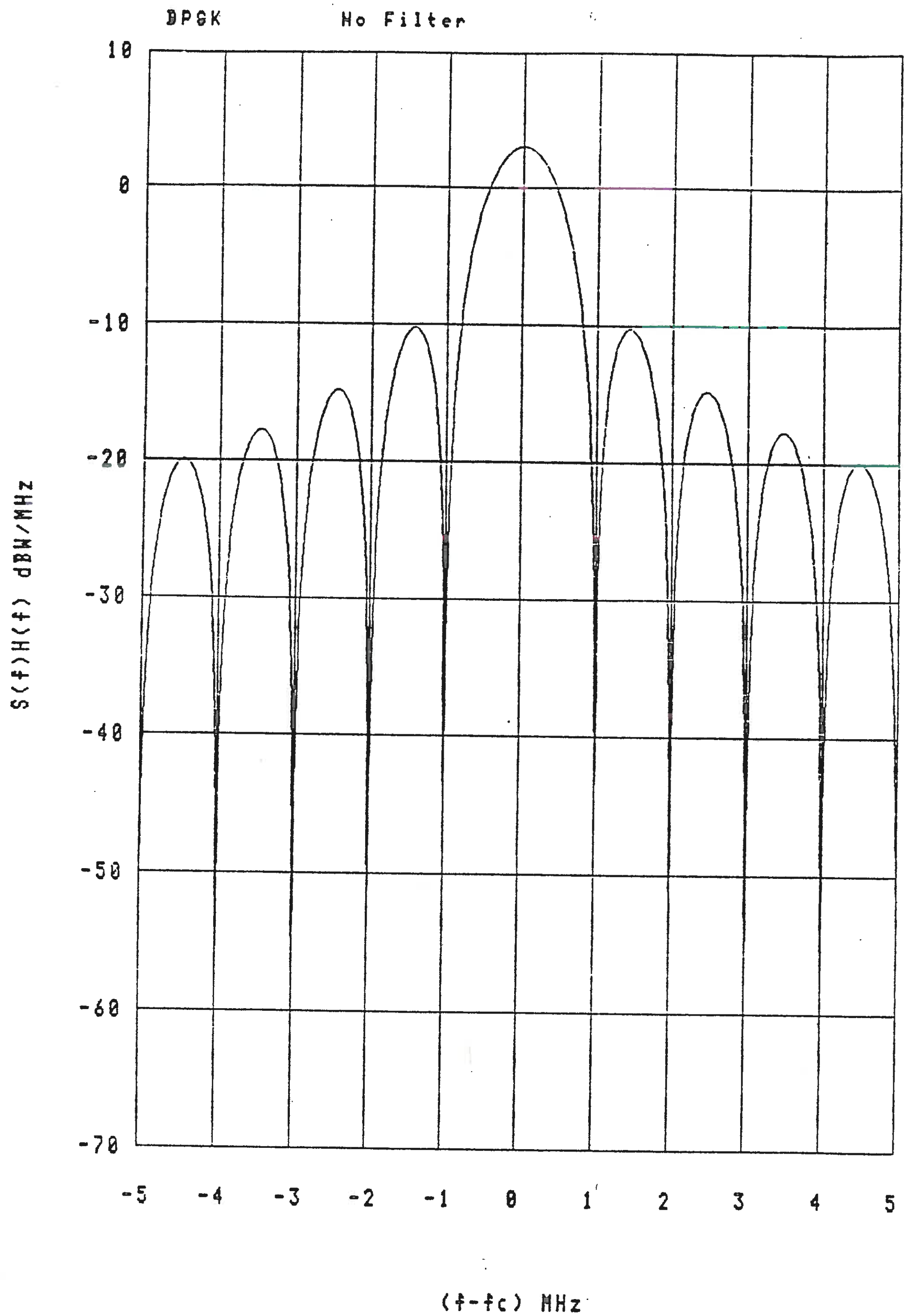
The spectral fall-off of the PSD maxima should be the same as that of the BPSK case, that is, the first maximum should be down 13.5 dB, relative to the maximum at the carrier frequency. Figure 2-4 is based on 10 watts and for QPSK, the 3 dB bandwidth is approximately 1/2 B_R, thus the PSD at $(f\text{-}f_c) = 0$ should be

$$10 \log \frac{10\ watts}{0.5\ (100\ Hz)} = -6.9\ dBW/Hz$$

For Figure 2-5, the maximum PSD should be 23.9 dBW/MHz, and for Figure 2-6, it should be 20.0 dBW/kHz.

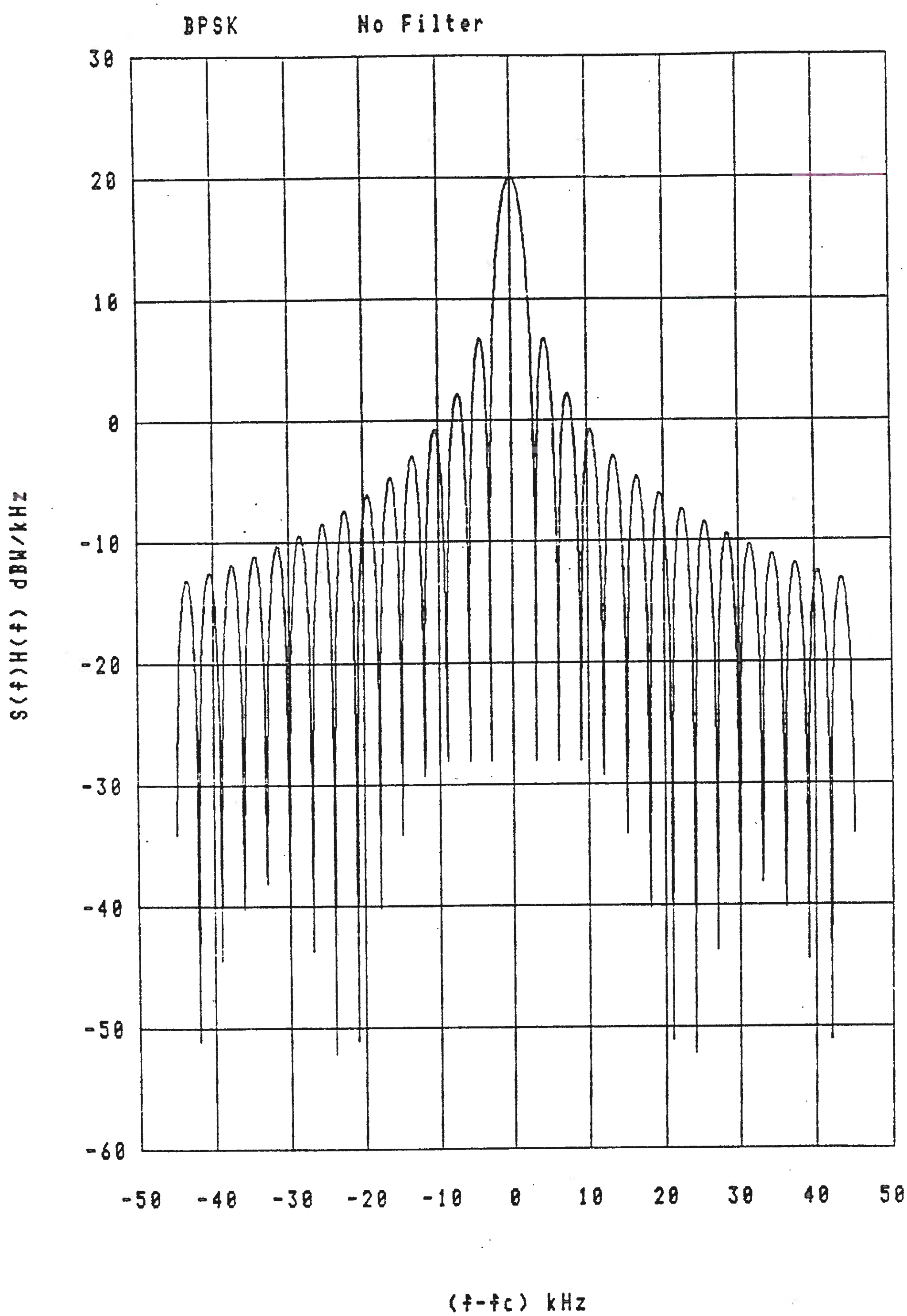
Similarly, it can be shown that Figure 2-7, an example of 8 PSK (S=8), is correct and Figure 2-8 for 16 PSK (S=16) is also correct.

Thus, the computer program is developing the PSD for PSK as expected.



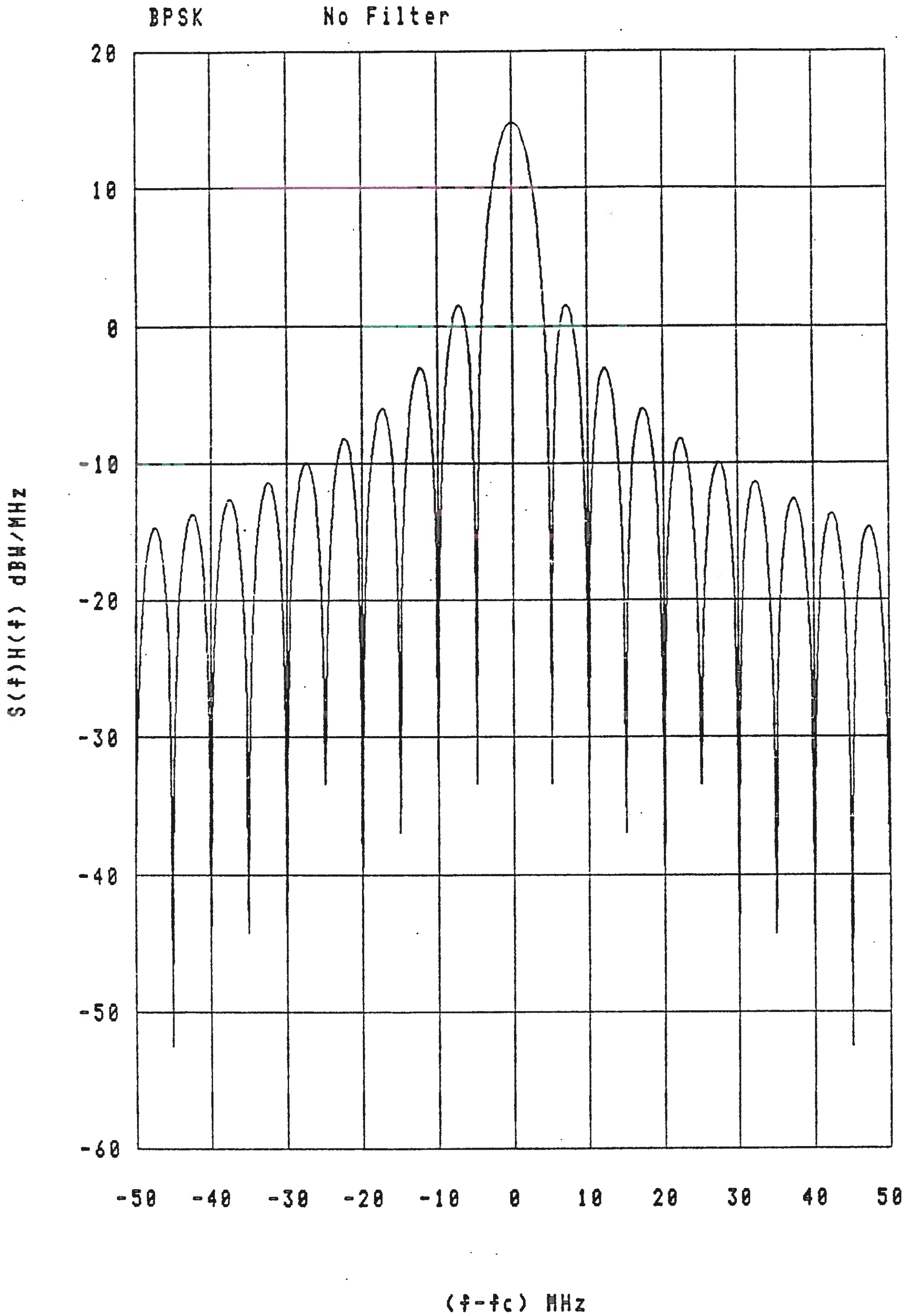
Modulating bit rate = 1.00 mps Power = 2.00 watts 99% band = 15.63 MHz

Figure 2-1. Power spectral density BPSK with NRZ modulation.



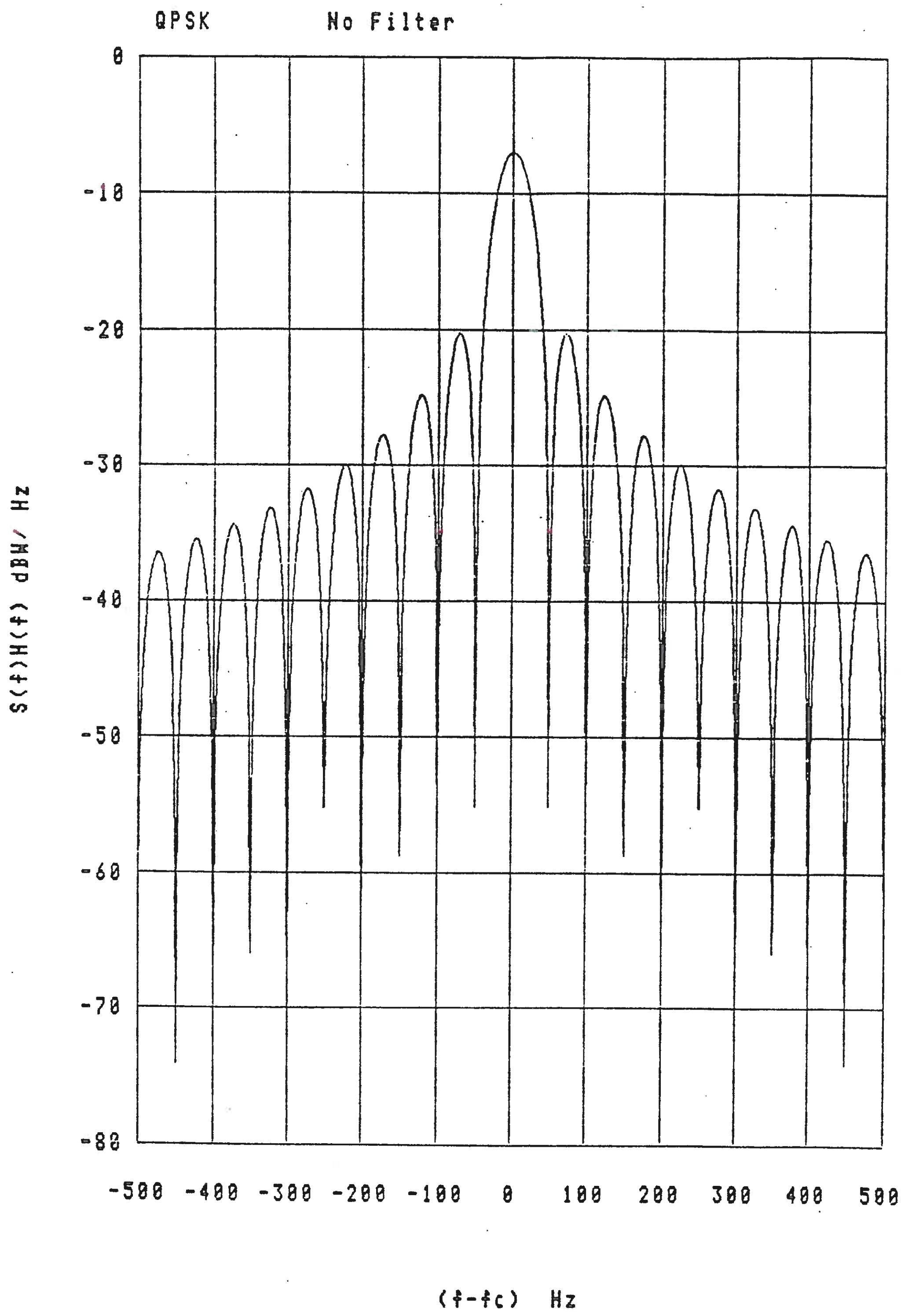
Modulating bit rate = 3.00 kps Power = 300.00 watts 99% band = 46.88 kHz

Figure 2-2. Power spectral density BPSK with NRZ modulation.



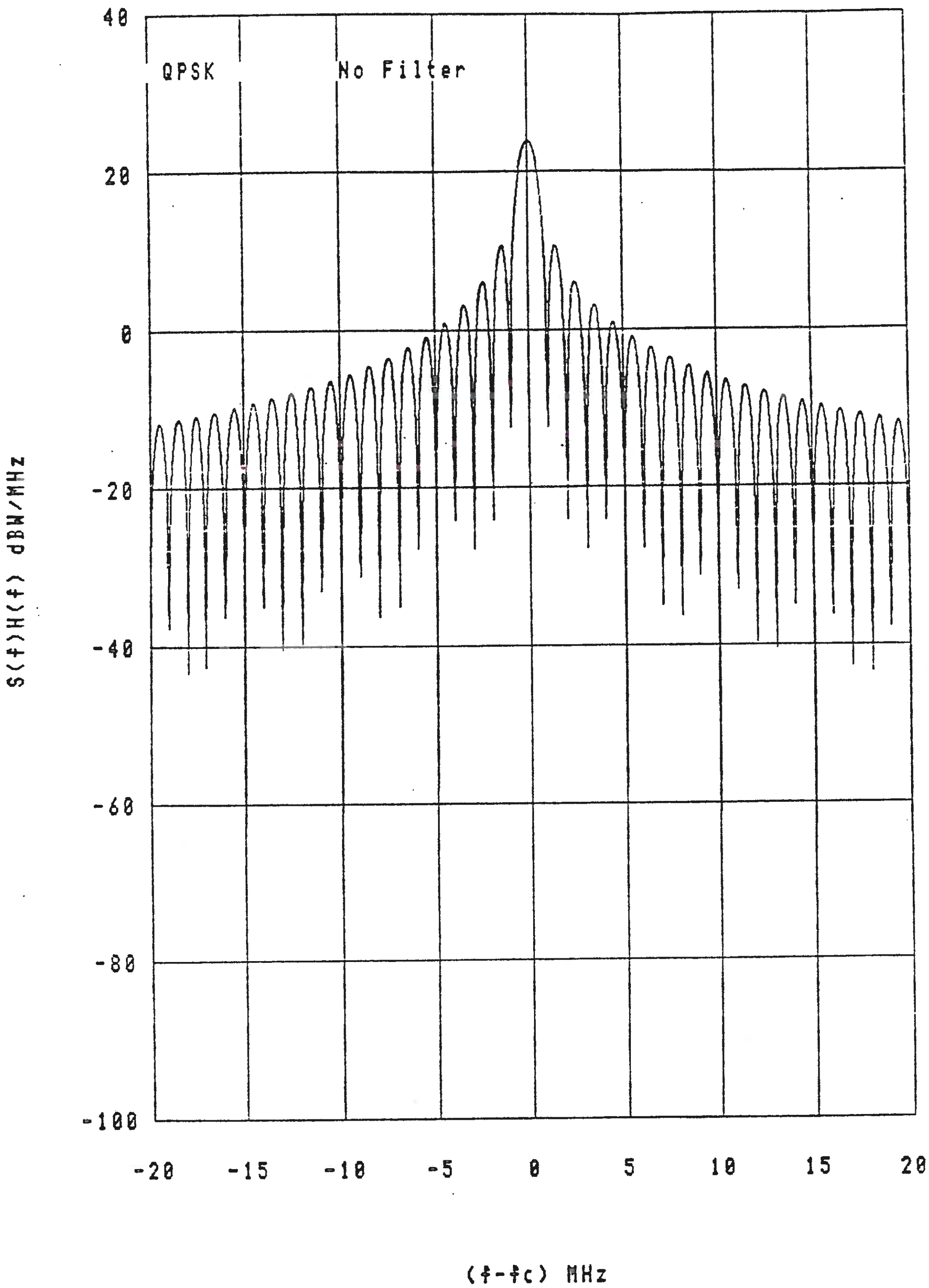
Modulating bit rate = 5.00 mps   Power = 150.00 watts   99% band = 78.13 MHz

Figure 2-3. Power spectral density BPSK with NRZ modulation.



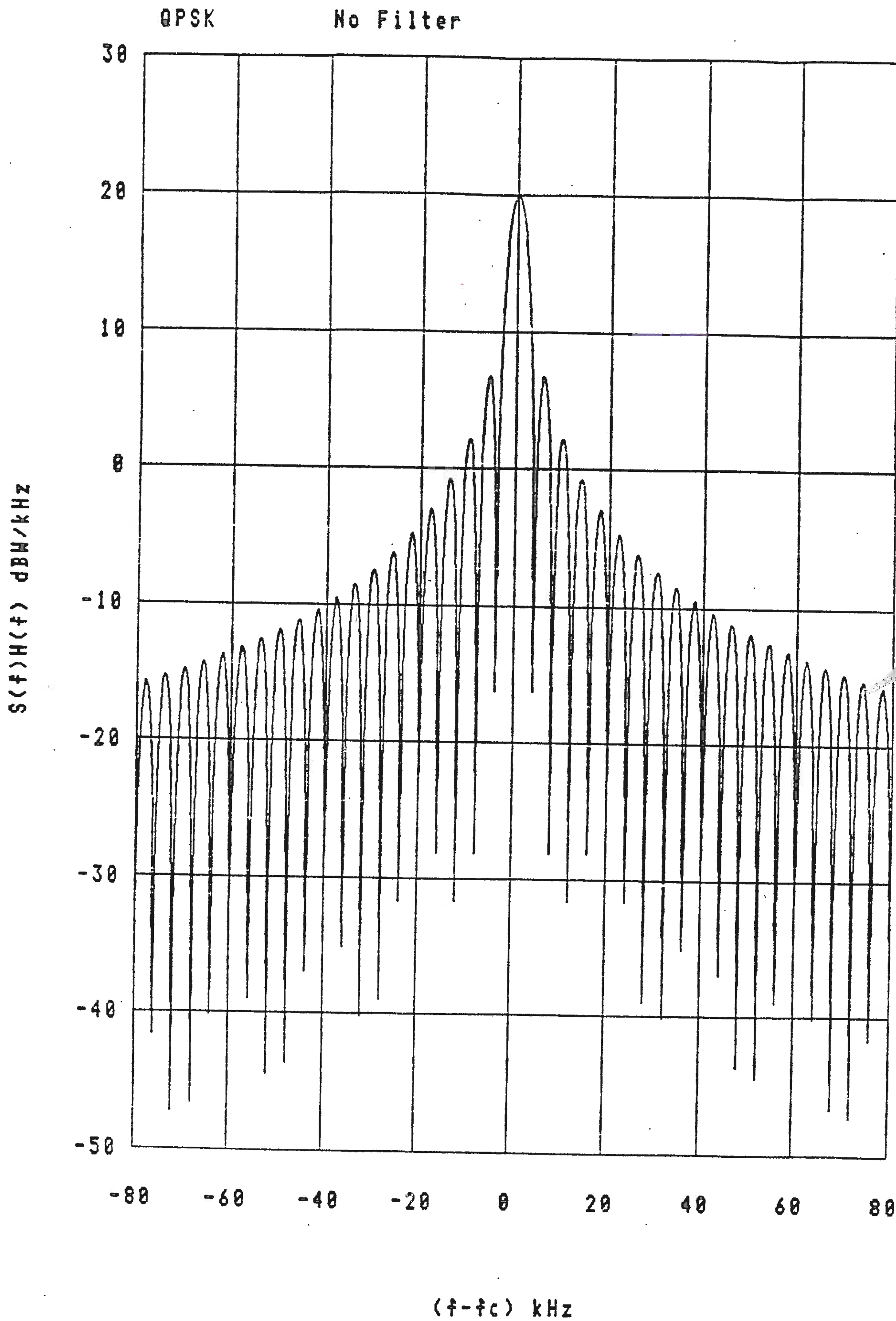
Modulating bit rate = 100.00 bps Power = 10.00 watts 99% band = 937.50 Hz

Figure 2-4. Power spectral density QPSK with NRZ modulation.



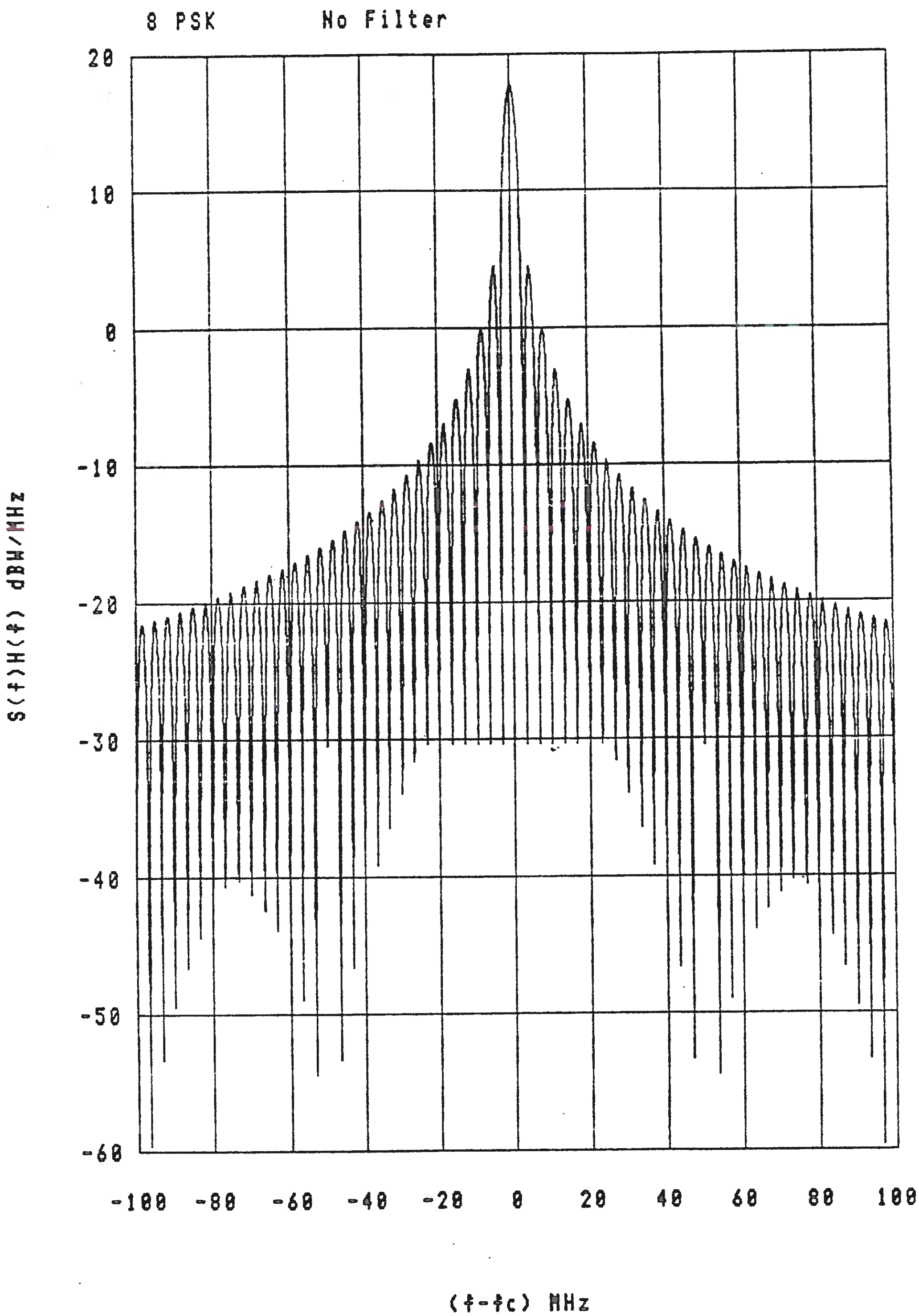
Modulating bit rate = 2.00 mps Power = 250.00 watts 99% band = 18.75 MHz

Figure 2-5. Power spectral density QPSK with NRZ modulation.



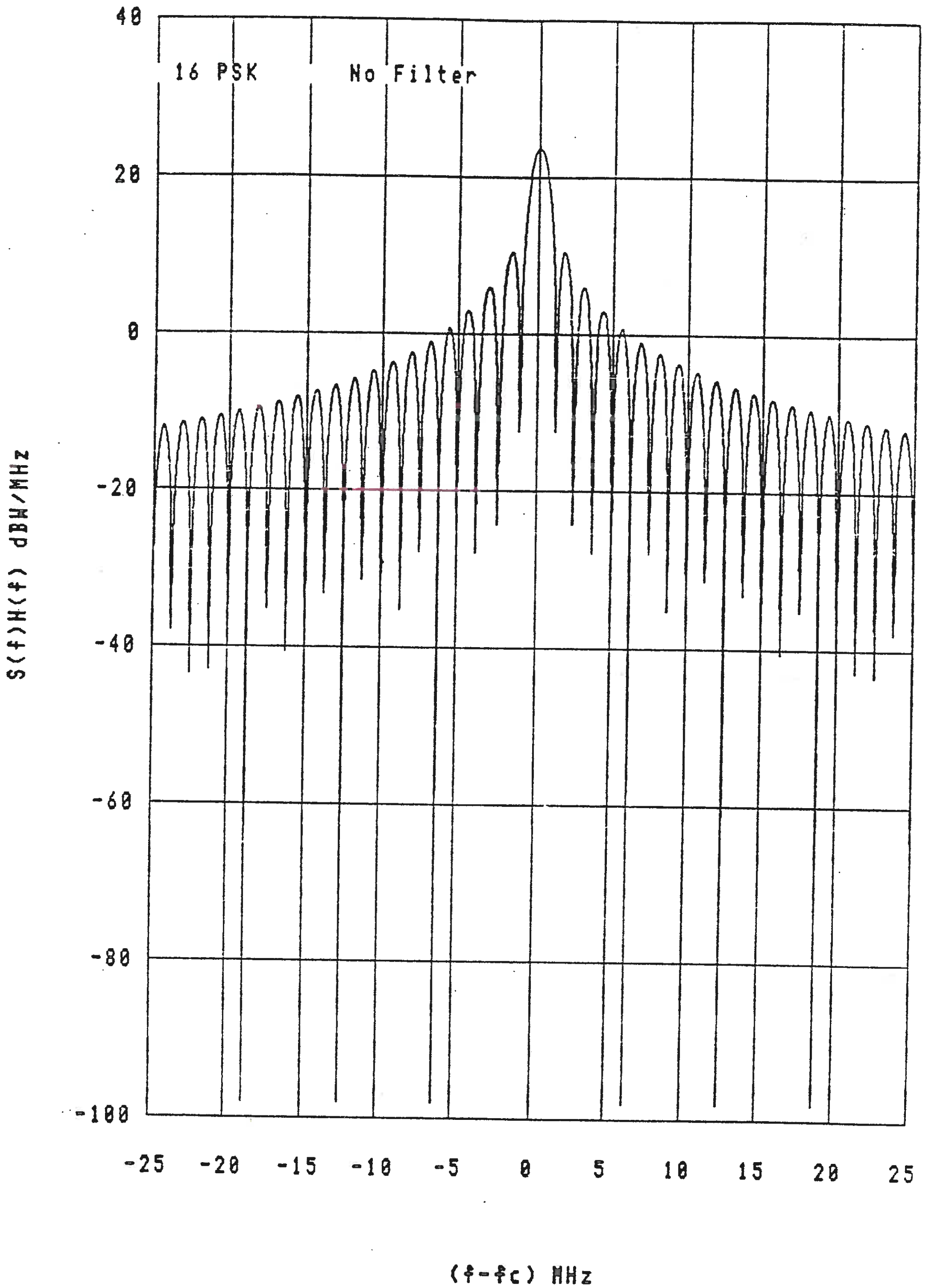
Modulating bit rate = 8.00 kps Power = 400.00 watts 99% band = 75.00 kHz

Figure 2-6. Power spectral density QPSK with NRZ modulation.



Modulating bit rate = 10.00 mps Power = 200.00 watts 99% band = 62.50 MHz

Figure 2-7. Power spectral density 8 PSK with NRZ modulation.



Modulating bit rate = 5.00 mps Power = 300.00 watts 99% band = 23.44 MHz

Figure 2-8. Power spectral density 16 PSK with NRZ modulation.

Quadrature Amplitude Modulation (QAM)

The computer program models the PSD for QAM. With QAM, the modulation signal shifts both the carrier phase and amplitude at the symbol rate between distinct signal states (S). The computer program includes the capability to model:

4 QAM
16 QAM
64 QAM
256 QAM

The PSD for M-ary QAM[5] is modeled using the same formulation as that of PSK (equation 2-4).

An example PSD for 4 QAM ($S=4$) is shown in Figure 2-9 for a bit rate of 2 MHz and 250 watts transmitter power. With $S=4$, $T_S = 2/T_B$, $T_B = 1/B_R$ so the maxima of the PSD should occur at

$$(f-f_c) = 0, \pm 3/2(B_R/2), \pm 5/2(B_R/2), \pm 7/2(B_R/2), \text{etc.}$$

The minima should occur at

$$(f-f_c) = \pm(B_R/2), \pm 2(B_R/2), \pm 3(B_R/2), \text{etc.}$$

Examination of Figure 2-9 (B_R = 2 MHz) shows the maxima and minima to be as expected. The expected spectral fall-off is as previously stated for PSK (i.e., 13.5 dB, 17.9 dB, 20.8 dB, 23.0 dB). With a transmitter power of 250 watts, the PSD at $(f-f_c) = 0$ should be 24 dBw/MHz. The spectral fall-off and peak PSD shown in Figure 2-9 are as expected.

Figure 2-10 is an example PSD for 16 QAM ($S=16$) for a bit rate of 5 MHz and 300 watts transmitter power. Examination of this PSD shows it is as expected.

Figures 2-11 through 2-13 are example PSDs for 64 QAM ($S=64$). These figures show the one-sided spectrum plot that is an output capability of the computer program. Figure 2-11 illustrates a bit rate of 1 MHz and a transmitter power of 1 watt. Figure 2-12 is for a bit rate of 5 MHz and 5 watts of transmitter power; Figure 2-13 represents a 10 MHz bit rate and 10 watts of transmitter power. These spectral plots can also be shown to be correct.

Figures 2-14 through 2-16 are examples of 256 QAM ($S=256$). Figure 2-14 is the PSD for a bit rate of 1 MHz and 1 watt transmitter power. Figure 2-15 shows a bit rate of 5 MHz with 5 watts and Figure 2-16 shows a 10 MHz bit rate and 10 watts. These spectral plots are also as expected.

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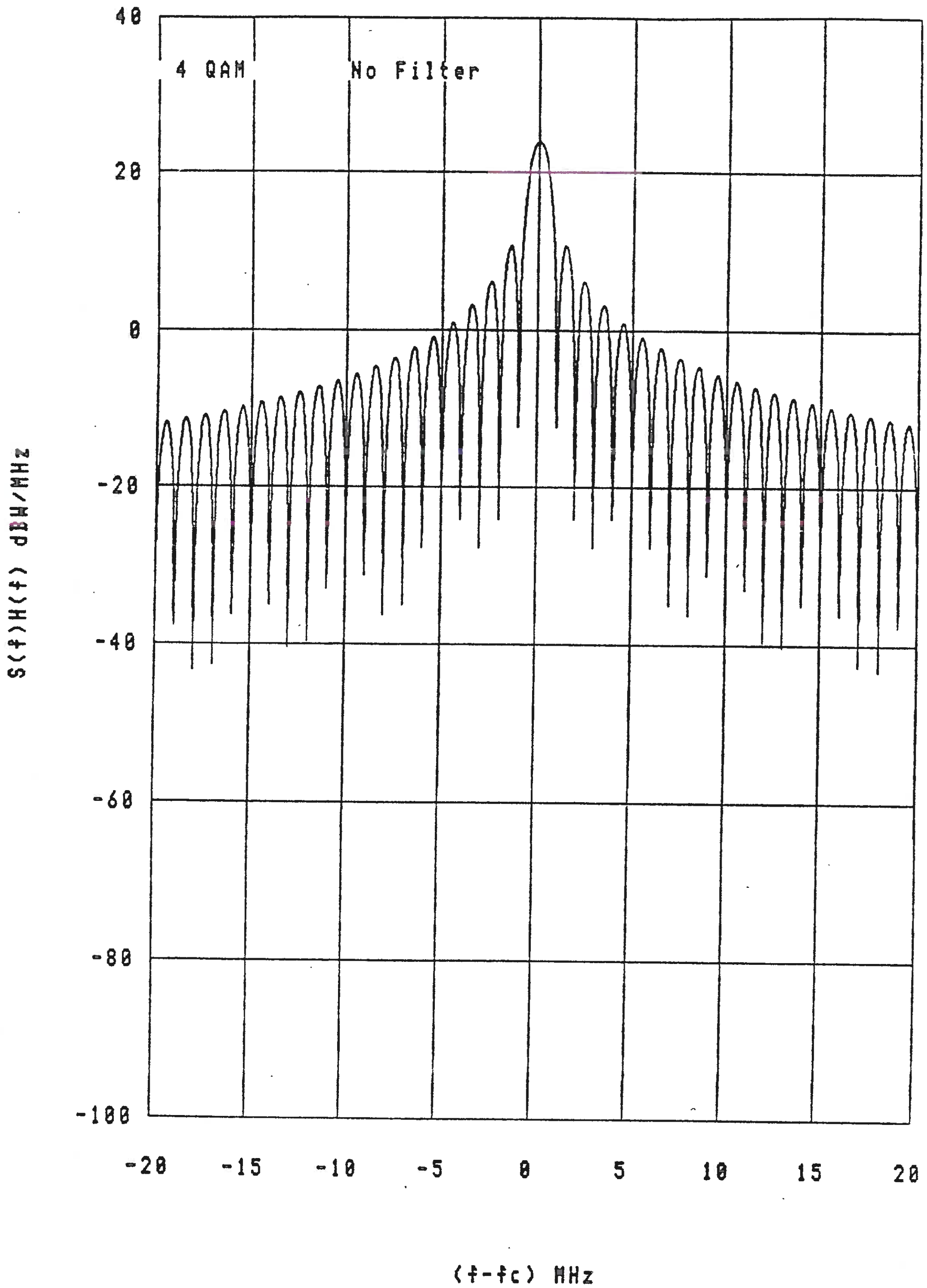
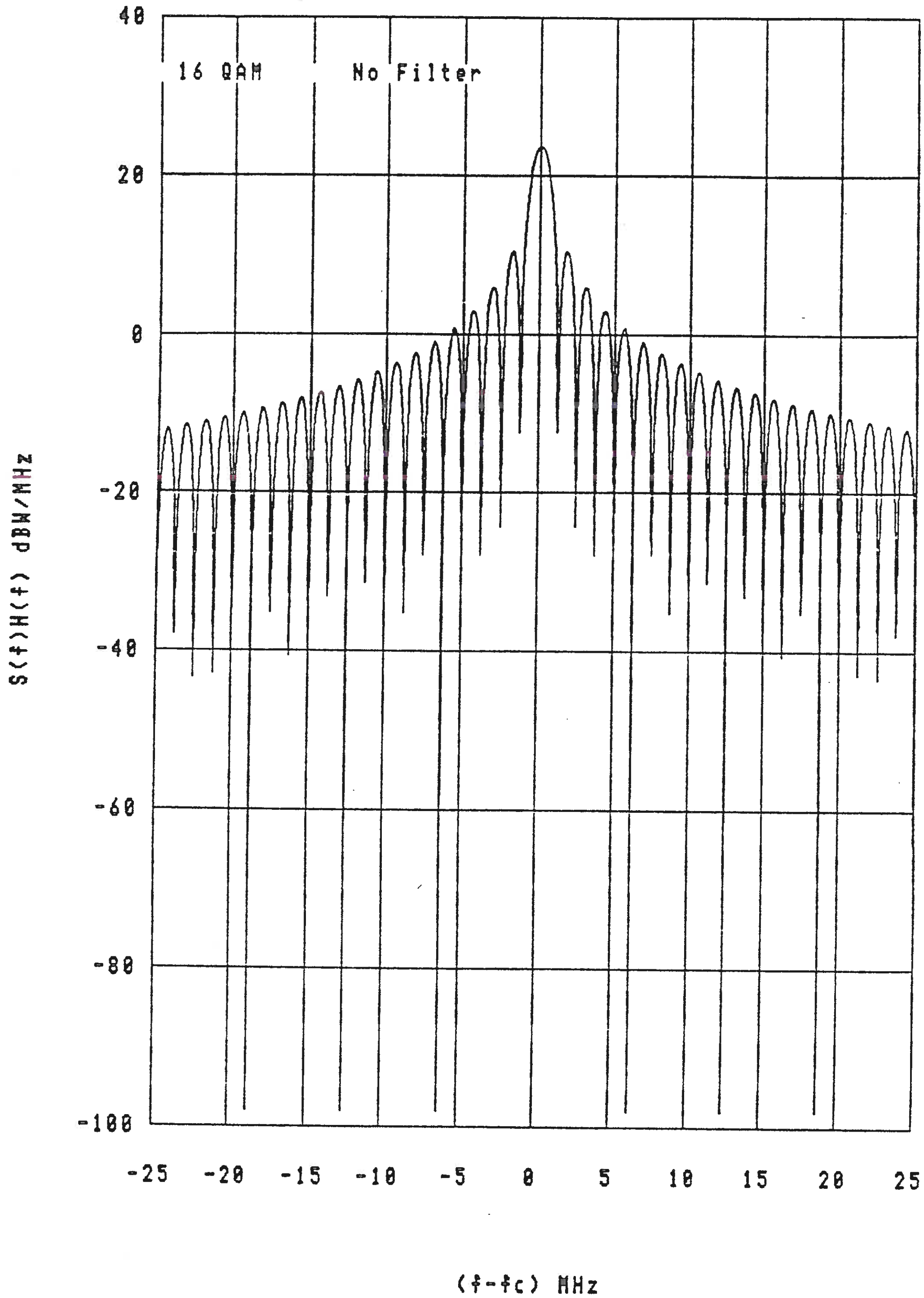
5 Amoroso, F., The Bandwidths of Digital Signals, IEEE Communications Magazine, November 1980.



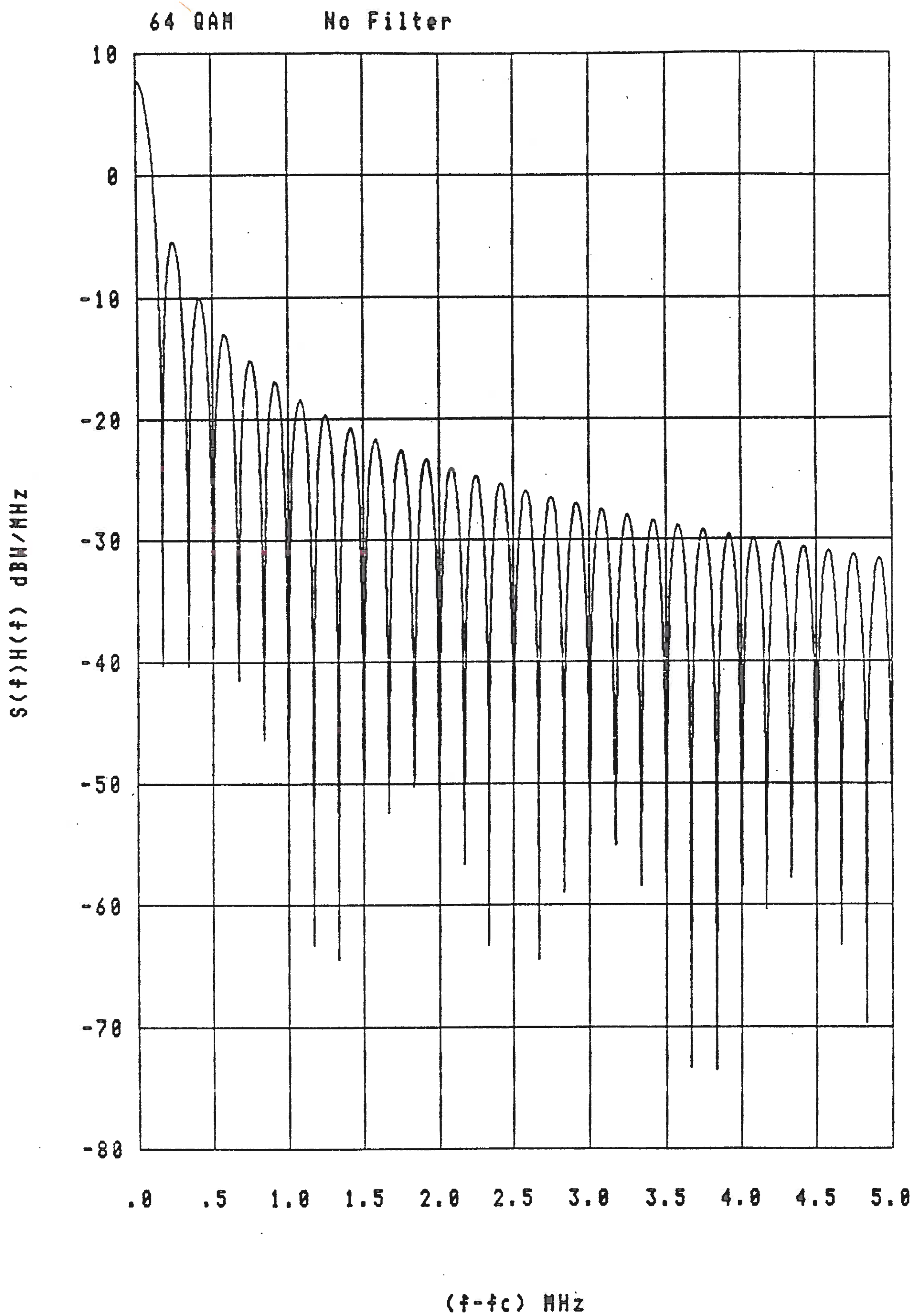
Modulating bit rate = 2.00 mps Power = 250.00 watts 99% band = 18.75 MHz

Figure 2-9. Power spectral density 4 QAM.



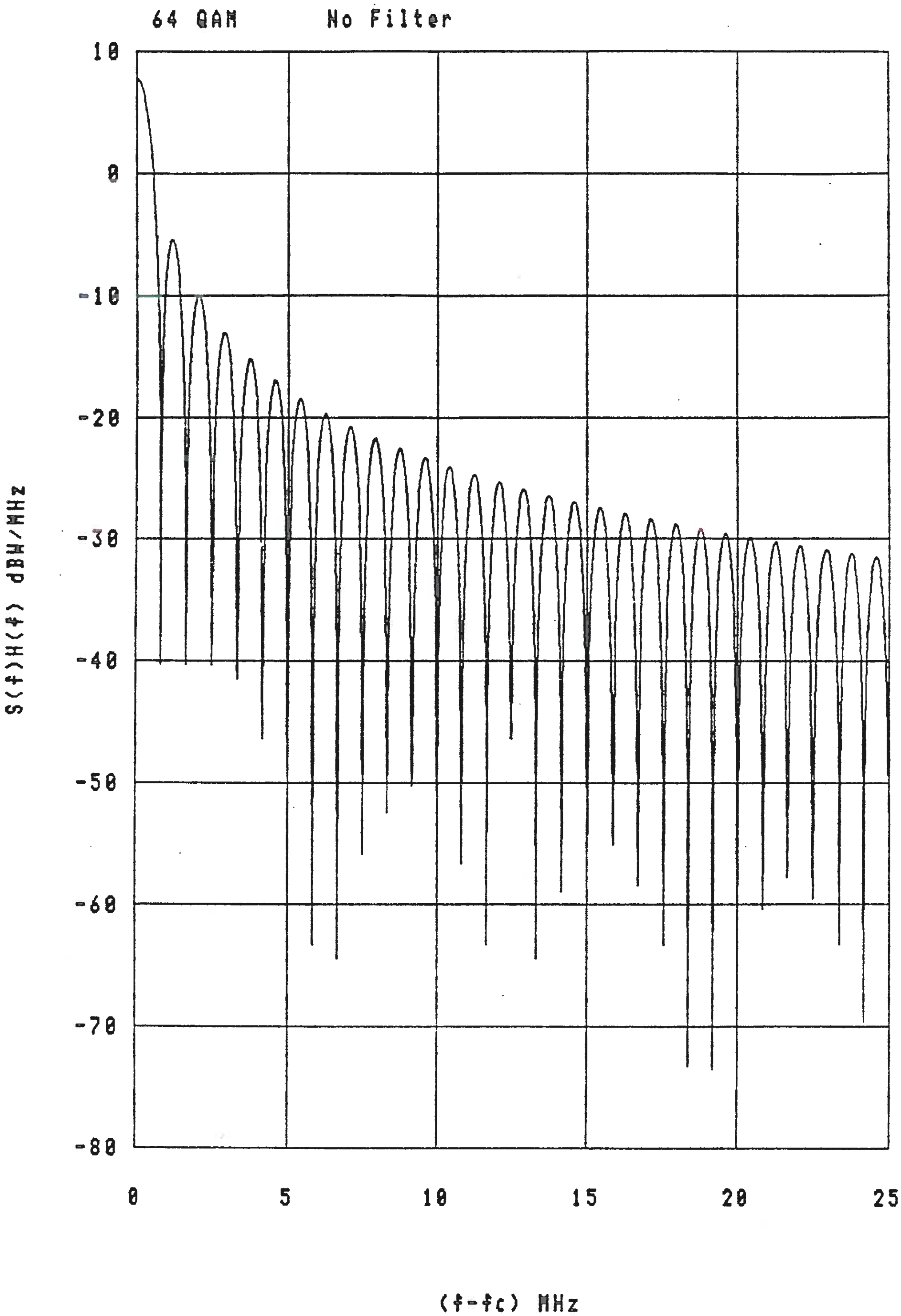
Modulating bit rate = 5.00 mps Power = 300.00 watts 99% band = 23.44 MHz

Figure 2-10. Power spectral density 16-QAM.



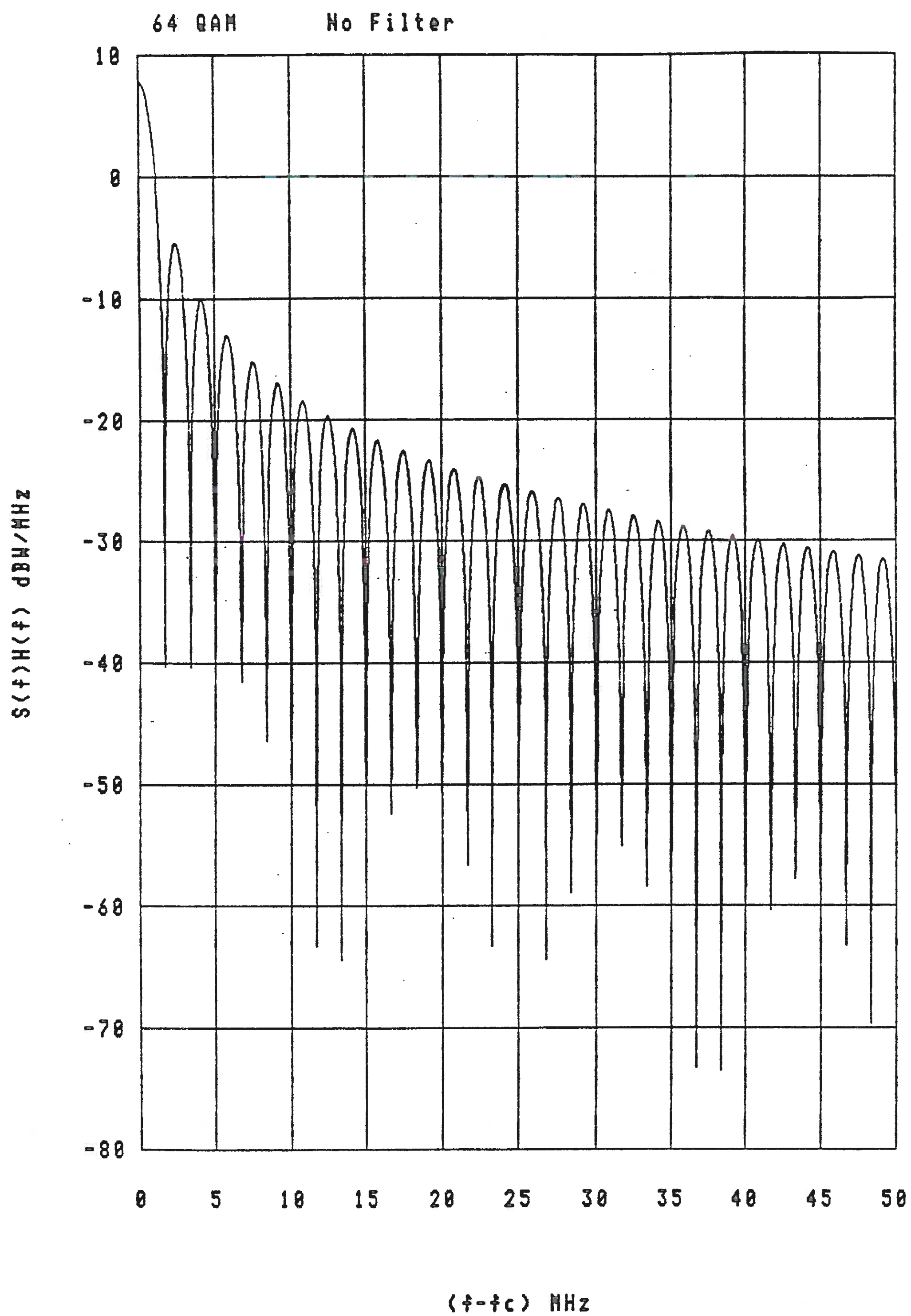
Modulating bit rate = 1.00 mps   Power = 1.00 watts   99% band = 3.13 MHz

Figure 2-11. Power spectral density 64-QAM.



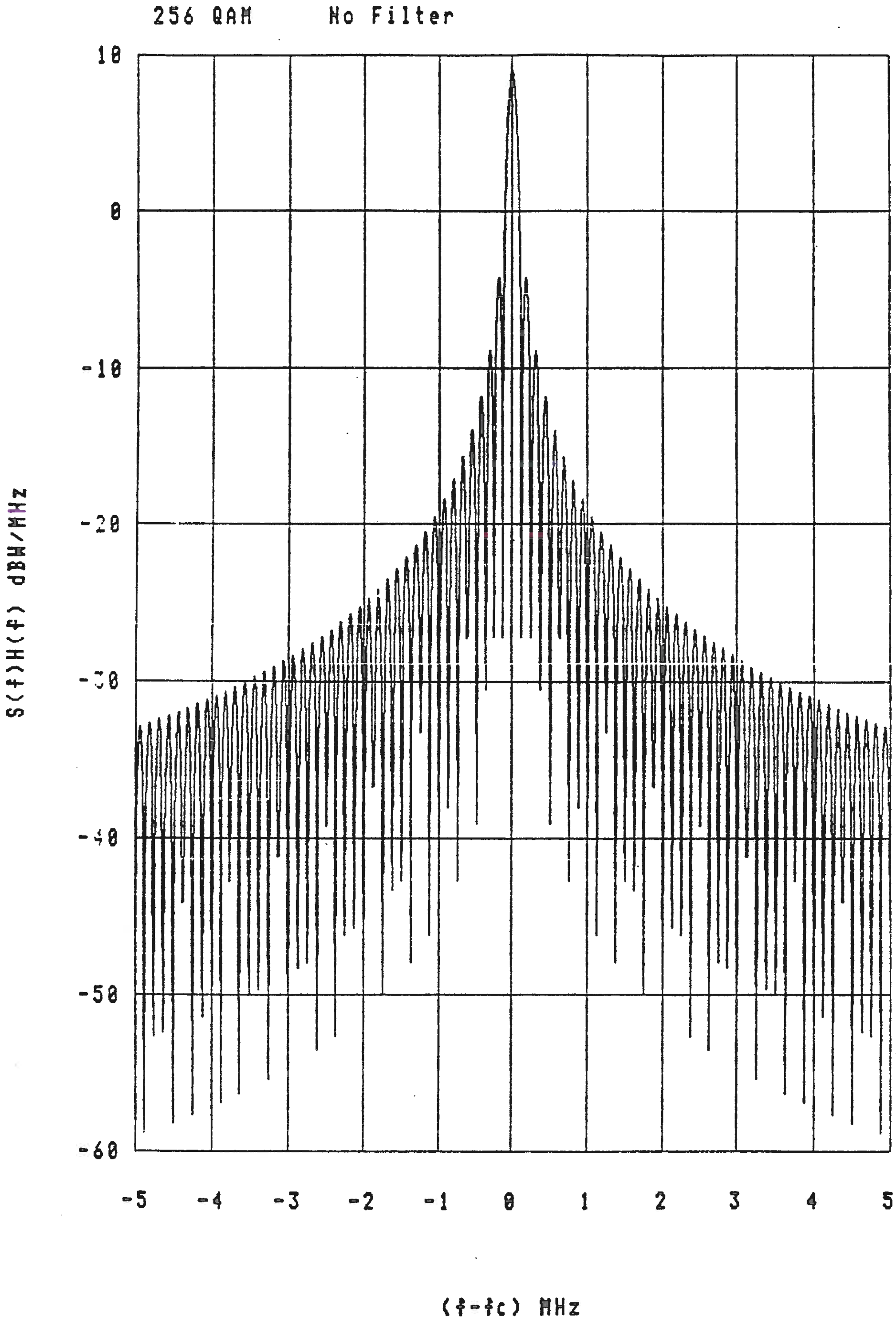
Modulating bit rate = 5.00 mps Power = 5.00 watts 99% band = 15.63 MHz

Figure 2-12. Power spectral density 64-QAM.



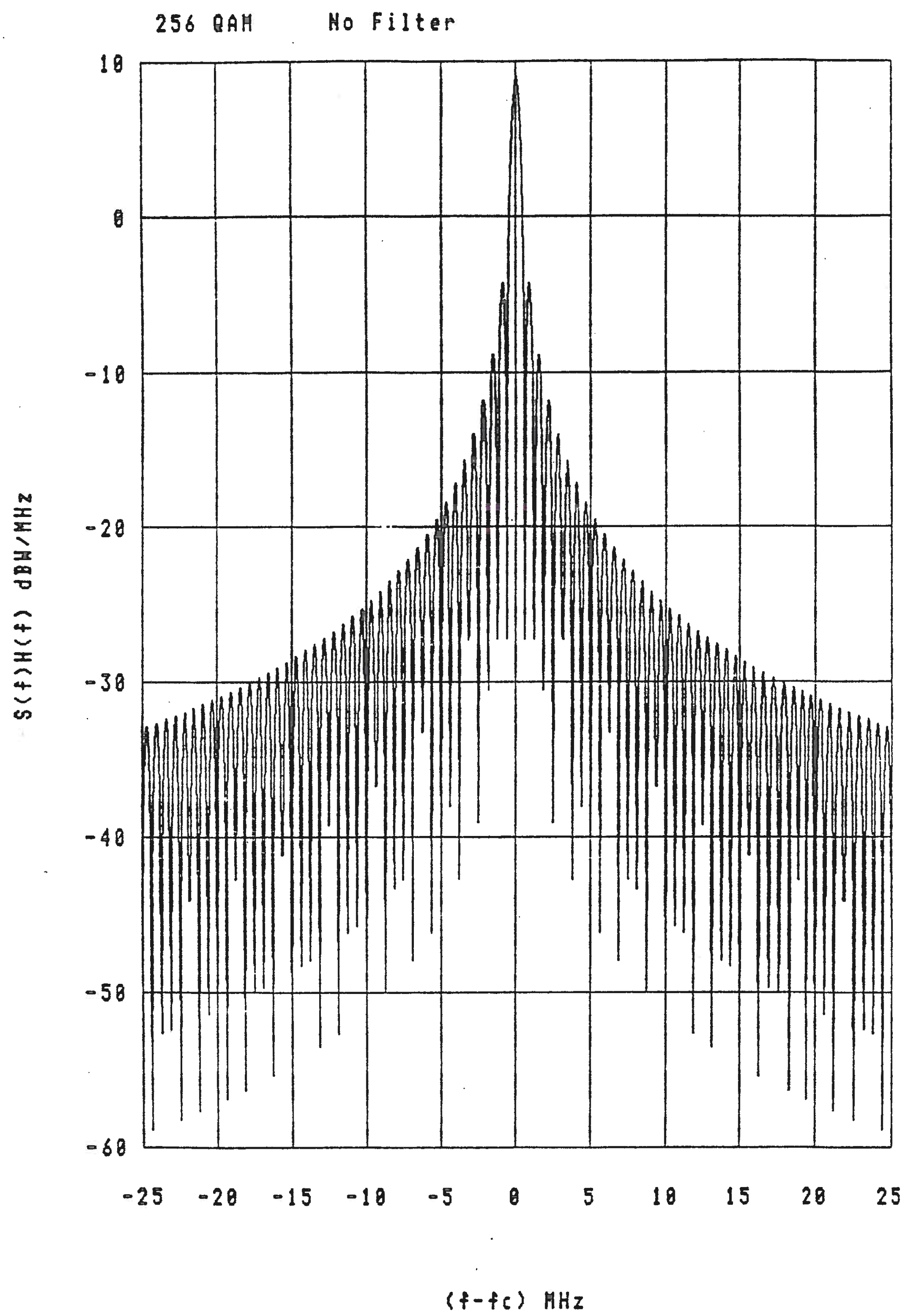
Modulating bit rate = 10.00 mps Power = 10.00 watts 99% band = 31.25 MHz

Figure 2-13. Power spectral density 64-QAM.



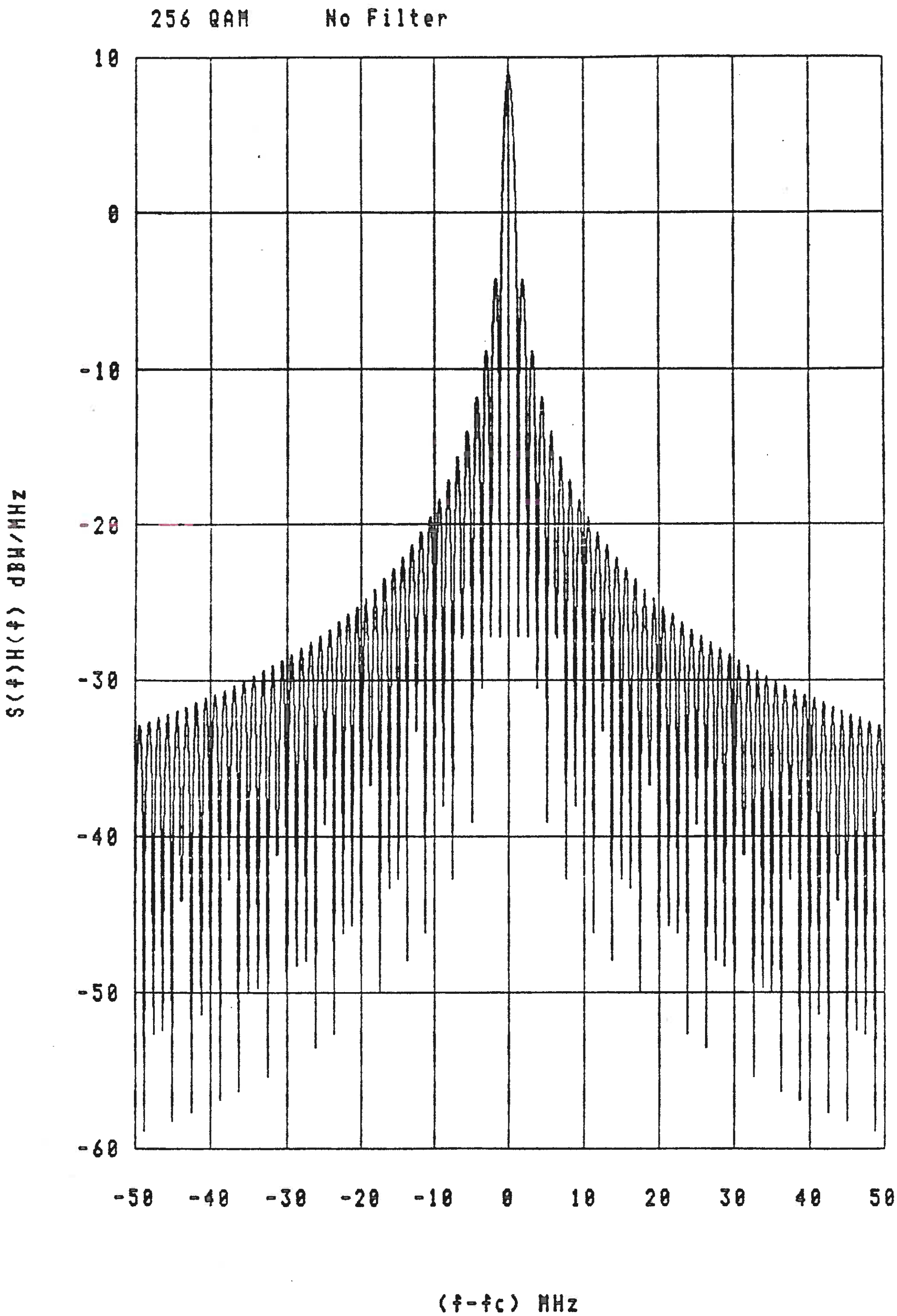
Modulating bit rate = 1.00 mps Power = 1.00 watts 99% band = 2.34 MHz

Figure 2-14. Power spectral density 256-QAM.



Modulating bit rate = 5.00 mps Power = 5.00 watts 99% band = 11.72 MHz

Figure 2-15. Power spectral density 256-QAM.



Modulating bit rate = 10.00 mps Power = 10.00 watts 99% band = 23.44 MHz

Figure 2-16. Power spectral density 256-QAM.

Two-Level Frequency Shift Keying (FSK)

The computer program includes the capability to model the PSD for two-level, continuous-phase, frequency-shift keying. The PSD formulation[6] is

$$S'(f) = \left[\frac{1}{D^2 - (f-f_c)^2}\right]^2 \frac{\left[Cos\left(\frac{2\pi D}{B_R}\right) - Cos\,(\pi\, x)\right]^2}{1 - 2\,Cos\left(\frac{2\pi D}{B_R}\right) Cos\,(\pi\, x) + Cos^2\left(\frac{2\pi D}{B_R}\right)} \tag{2-5}$$

where

D = 1/2 (difference between mark and space frequencies)

$$x = \frac{2(f-f_c)}{B_R}$$

The PSD modeled by equation 2-5 is a continuum and is valid if B_R and D are not related such that

$$\left| Cos\,\frac{2\pi D}{B_R} \right| = 1 \tag{2-6}$$

If the relation shown in equation 2-6 is true, the PSD for FSK will include two carrier spikes and must be modeled by a formulation that is different than the S'(f) of equation 2-5. The formulation for the other case is not currently included in the computer model. The computer model includes a check of equation 2-6 and prints out a warning

SPIKE CONDITION ENCOUNTERED

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[6] Pelchat, M.G., The Autocorrelation Function, and Power Spectrum of PCM/FM with Random Binary Modulating Waveforms, IEEE Transactions on Space Electronics and Telemetry, March 1964.

if the expression is satisfied. The computer program will not compute S'(f) when the spike condition is encountered.

Figures 2-17 through 2-19 are example PSDs for FSK. Figure 2-17 is for a bit rate of 1 MHz, a transmitter power of 2 watts and a mark/ space frequency difference of 0.4 MHz. Maxima of S'(f) occur when the term

$$\left[Cos\,\frac{2\pi D}{B_R} - Cos\,\pi x\right]^2$$

is a maximum. For a value of D = 0.2 and B_R = 1 MHz, the maxima will occur at

$$Cos\,2\pi\,(f-f_c) = -1$$

or

$$(f-f_c) = \pm 1/2,\ \pm 3/2,\ \pm 5/2,\ \pm 7/2,\ \text{etc. (MHz)}$$

Additional maxima occur when

$$1-2\,Cos\,\frac{2\pi D}{B_R}\,Cos\,\pi x + Cos^2\left(\frac{2\pi D}{B_R}\right)$$

is a minimum. For Figure 2-17, this would occur at

$$(f-f_c) = 0,\ \pm 1,\ \pm 2,\ \pm 3,\ \text{etc. (MHz)}$$

However, the maxima at $(f-f_c) = \pm 1/2$ MHz does not appear as a maximum because for values of $|f-f_c| < 1/2$ MHz, the term

$$\left[\frac{1}{D^2 - (f-f_c)^2}\right]^2$$

is so large that it causes the S'(f) function to continue to increase as $(f-f_c)$ goes from $\pm 1/2$ to 0. A cursory examination of S'(f) might indicate maxima at $D = |f-f_c|$ due to the denominator going to zero. However, further examination shows that

$$Cos\,\frac{2\pi D}{B_R} - Cos\,\pi x$$

also goes to zero at $D = |f-f_c|$. It can be shown that S'(f) at $D = |f-f_c|$ is

$$S'(f) = \left(\frac{\pi}{DB_R}\right)^2$$

Thus, the maxima are expected to occur at

$$(f-f_c) = 0,\ \pm 1,\ \pm 3/2,\ \pm 2,\ \pm 5/2,\ \pm 3,\ \text{etc. (MHz)}$$

Figure 2-17 shows this effect. The minima of S'(f) would occur when

$$Cos\,\frac{2\pi D}{B_R} = Cos\,\frac{2\pi(f-f_c)}{B_R}$$

For the example problem, this would occur at

$$2\pi\,(f\text{-}f_c) = \pm 0.4\pi,\ \pm(2\pi\text{-}0.4\pi),\ \pm(2\pi+0.4\pi),\ \ \pm(4\pi\text{-}0.4\pi),\ \text{etc.}$$

$$(f\text{-}f_c) = \pm 0.2,\ \pm 0.8,\ \pm 1.2,\ \pm 1.8,\ \pm 2.2,\ \text{etc. (MHz)}$$

When $f\text{-}f_c = \pm\ 0.2$, then $D = |f\text{-}f_c|$ which as previously stated are not true maxima or minima. Thus, the minima occur at

$$(f\text{-}f_c) = \pm 0.8,\ \pm 1.2,\ \pm 1.8,\ \pm 2.2,\ \text{etc. (MHz)}$$

Again a cursory examination might indicate a minimum in S'(f) when

$$1 - 2\ Cos\ \frac{2\pi D}{B_R}\ Cos\ \pi x + Cos^2\left(\frac{\pi D}{B_R}\right)$$

is a maximum. This would be when

$$\text{Cos}\ \pi\,x\ = -1$$

At these values, however, the numerator is a maximum and in fact, the function S'(f) is a maximum. This can be shown through trial calculations of S'(f).

Thus, the maxima for Figure 2-17 are expected to occur at

$$(f\text{-}f_c) = 0,\ \pm 1,\ \pm 3/2,\ \pm 2,\ \pm 5/2,\ \pm 3,\ \text{etc. (MHz)}$$

and minima at

$$(f\text{-}f_c) = \pm 0.8,\ \pm 1.2,\ \pm 1.8,\ \pm 2.2,\ \text{etc. (MHz)}$$

which are the values shown in the figure.

The spectral fall-off of the maxima of the PSD for FSK can be computed from

$$20 \log \frac{1}{D^2 - (f\text{-}f_c)^2}$$

The fall-off is relative to the value at $(f\text{-}f_c) = 0$ and for Figure 2-17 is

$(f\text{-}f_c)$ (MHz)	Relative Attenuation (dB)
0	0
1	-27.65
3/2	-34.9
2	-40.0
5/2	-43.9
3	-47.0
7/2	-49.7

This is the fall-off shown in Figure 2-17.

The peak PSD at $(f\text{-}f_c) = 0$ can be approximated by

$$10 \log \frac{\textit{Transmitter Power}}{\textit{3 dB Spectrum Bandwidth}}$$

For Figure 2-17,

$$10 \log \frac{2.00 \ watts}{0.4 \ MHz} = 7 \ dBW/MHz$$

where the 3 dB bandwidth was estimated from the PSD plot. This peak is in agreement with the value of Figure 2-17.

Figure 2-18 is the PSD for a 3 kHz bit rate, 300 watts transmitter power and a mark/space frequency difference of 5 kHz. For these conditions, maxima would be expected at

$$(f\text{-}f_c) = 0, \pm 1/2\ (B_R), \pm\ B_R, \pm 3/2\ (B_R), \pm\ 2\ B_R, \text{etc.}$$

$$= 0, \pm 1.5, \pm 3, \pm 4.5, \pm 6, \text{etc. (kHz)}$$

However, because the term

$$\left[\frac{1}{D^2 - (f-f_c)^2}\right]^2$$

is increasing so rapidly from $(f-f_c) = 0$ to $(f-f_c) = D$, the potential maxima at $(f-f_c) = \pm 1.5$ kHz do not occur. Also, the potential maxima at ± 3 kHz and ± 4.5 kHz are shifted to slightly lower frequencies by the rapid decrease in

$$\left[\frac{1}{D^2 - (f-f_c)^2}\right]^2$$

as $(f-f_c)$ increases from D. The maxima at $(f-f_c) = \pm 6$ kHz and higher values occur at the expected frequencies. The minima would be expected at

$$Cos\,\frac{2\pi D}{B_R} = Cos\,\frac{2\pi(f-f_c)}{B_R}$$

or

$$2\pi/3\ (f-f_c) = \pm\pi/3,\ \pm 5\pi/3,\ \pm 7\pi/3,\ \pm 11\pi/3,\ \pm 13\pi/3,\ \text{etc.}$$

$$(f-f_c) = \pm 0.5,\ \pm 2.5,\ \pm 3.5,\ \pm 5.5,\ \text{etc. (kHz)}$$

The possible minima at ± 2.5 kHz are really $|f-f_c| = D$ which as previously explained are not minima. The spectral fall-off of the maxima of the PSD is determined by

$$20 \log \frac{1}{D^2 - (f-f_c)^2}$$

For Figure 2-18, the expected fall-off relative to the value at $(f-f_c) = 0$ is

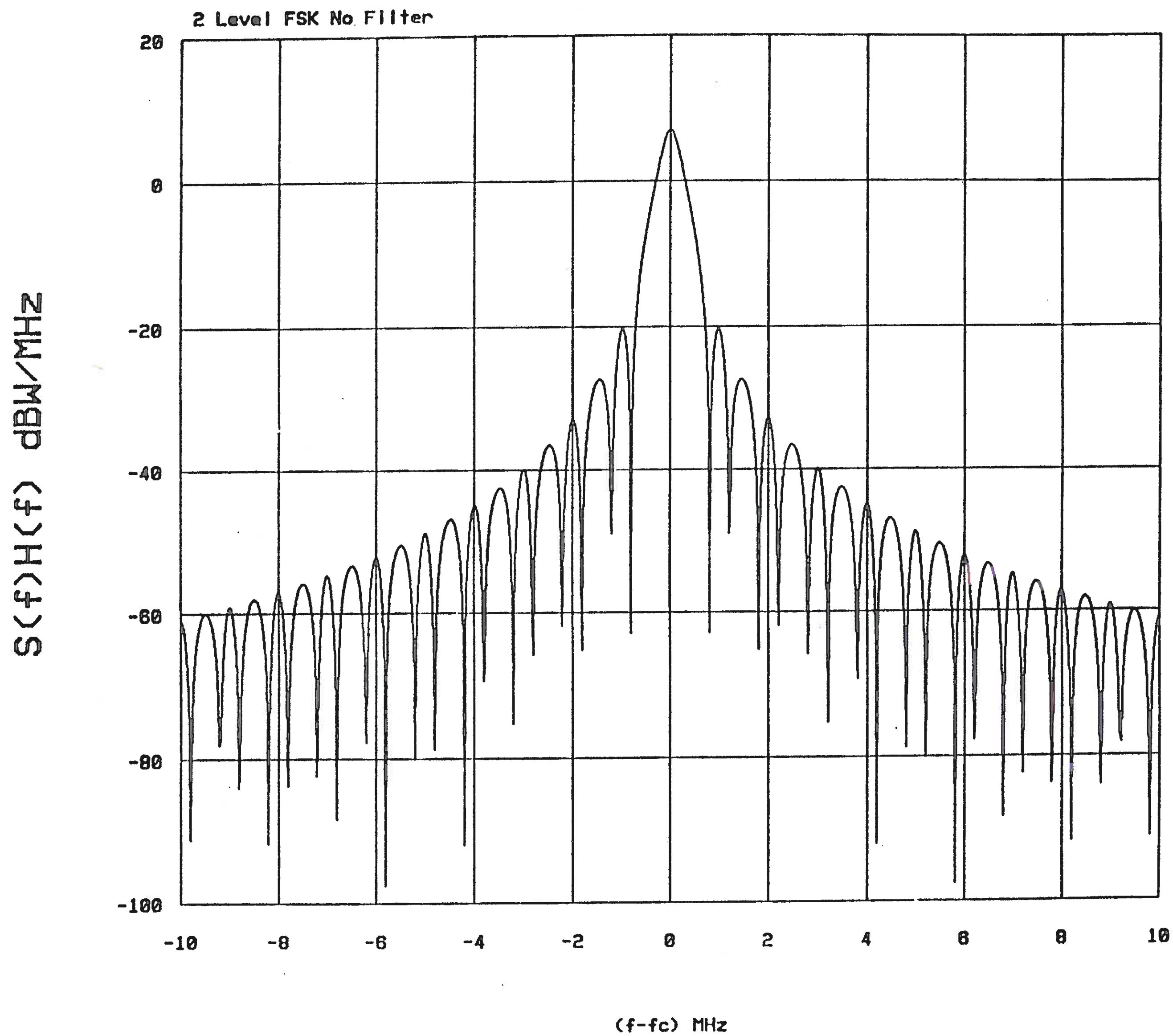
$(f-f_c)$ (kHz)	Relative Attenuation (dB)
0	0
2.9	+9.2
4.2	-5.2
6	-13.6
7.5	-18.1

The expected peak value of the PSD is

$$10 \log \frac{300\ watts}{2.4\ kHz} = 21\ dBW/kHz$$

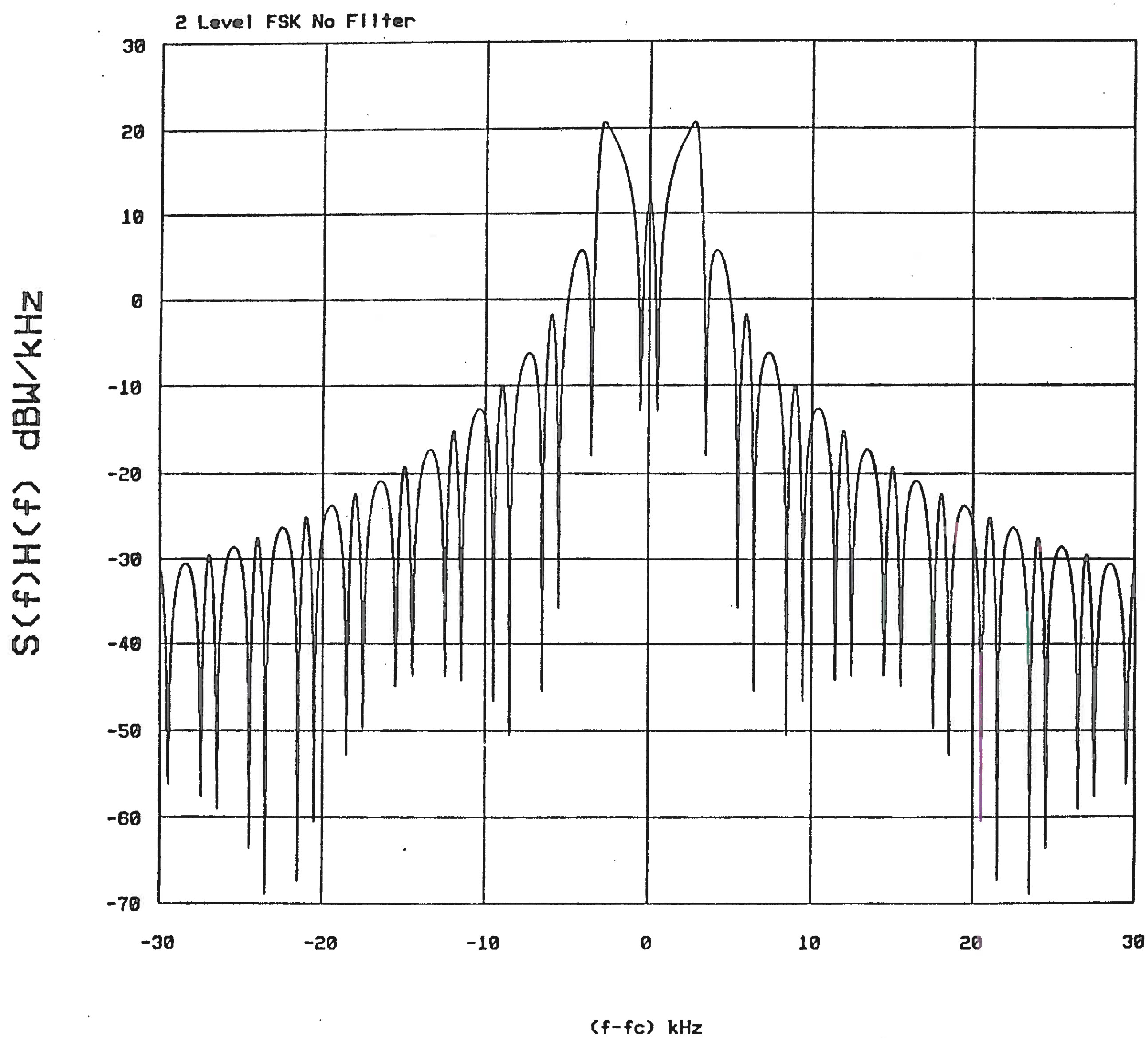
The PSD plot shown in Figure 2-18 is as expected.

Figure 2-19 is the PSD for FSK with a 5 MHz bit rate, 150 watts power and a mark/space frequency difference of 3 MHz. Comparisons similar to those of Figure 2-17 and 2-18 show the PSD of Figure 2-19 to be as expected.



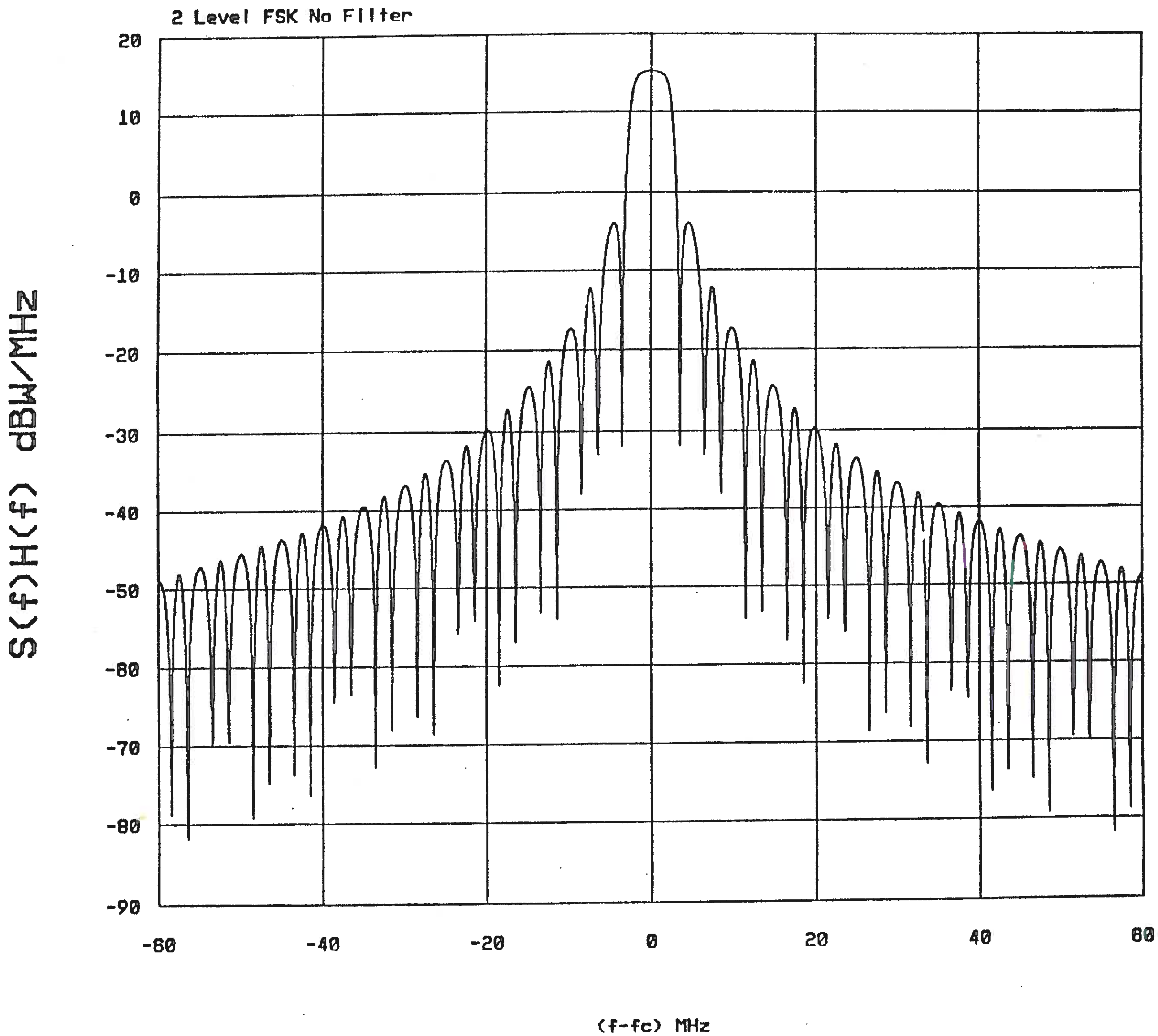
Modulating bit rate = 1.00 mps Power = 2.00 watts Delta = .40 MHz 99% band = 1.07 MHz

Figure 2-17. Power spectral density two level FSK.



Modulating bit rate = 3.00 kps Power = 300.00 watts Delta = 5.00 kHz 99% band = 9.38 kHz

Figure 2-18. Power spectral density two level FSK.



Modulating bit rate = 5.00 mps Power = 150.00 watts Delta = 3.00 MHz 99% band = 7.81 MHz

Figure 2-19. Power spectral density two level FSK.

Minimum Shift Keying (MSK)

The capability to model the PSD for minimum shift keying[7] is included in the computer program. The PSD formulation is

$$S'(f) = \left[\frac{\cos 2\pi T_B (f-f_c)}{1-16\ T_B^2\ (f-f_c)^2}\right]^2 \tag{2-7}$$

Figure 20 shows the spectrum for MSK with a bit rate of 1 MHz and 2 watts of transmitter power. A bit rate of 1 MHz results in a bit length of 1 μsec. For these conditions, one would expect possible maxima at

$$\cos 2\pi T_B (f-f_c) = \pm 1$$

$$(f-f_c) = 0,\ \pm 1/2,\ \pm 1,\ \pm 3/2,\ \pm 2,\ \text{etc. (MHz)}$$

However, as explained below there are no minima between 0 and $\pm 1/2$ MHz. Thus , $\pm 1/2$ MHz do not appear as maxima since S' (f) continues to increase from $\pm 1/2$ MHz to a maximum at $(f-f_c) = 0$. Examination of equation 2-7 might indicate a potential maximum at $1 = 16\ T_B\ (f-f_c)^2$ because the denominator goes to zero. However, the numerator also goes to zero and application of L' Hospital's Rule will show that S' (f) $= \pi^2/16$ at $(f-f_c) = 1/4T_B$. Therefore, maxima would be expected at

$$(f-f_c) = 0,\ \pm 1,\ \pm 3/2,\ \pm 2,\ \pm 5/2,\ \text{etc. (MHz)}$$

The actual maxima for small Δf values (i.e., ± 1 MHz and $\pm 3/2$ MHz) are shifted to slightly lower frequencies. This is caused by the rapid increase of the denominator of S'(f) as Δf deviates away from 0.

Minima values for S'(f) would be expected at

$$\cos 2\pi T_B (f-f_c) = 0$$

$$(f-f_c) = \pm 1/4,\ \pm 3/4,\ \pm 5/4,\ \pm 7/4,\ \text{etc. (MHz)}$$

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[7] Prabhu, V.K., MSK and Offset QPSK Modulation with Bandlimiting Filters, IEEE Transactions on Aerospace and Electronic Systems, January 1981.

However, as previously stated $(f-f_c) = \pm 1/4$ MHz are not minima. Thus, minima are expected at

$$(f-f_c) = \pm 3/4,\ \pm 5/4,\ \pm\ 7/4, \text{ etc. (MHz)}$$

The PSD at $(f-f_c) = 0$ can be estimated from

$$10 \log \frac{\textit{Transmitter Power}}{\textit{3dB Spectrum Bandwidth}} = 10 \log \frac{2\,watts}{0.65\,MHz} = 4.9\ \frac{dBW}{MHz}$$

The fall-off of the spectra maxima can be determined by considering

$$20 \log \frac{1}{1-16T_B^2(f-f_c)^2}$$

For Figure 2-20,

$(f-f_c)$ (MHz)	Relative Attenuation (dB)
0	0
1	-23.6
1.5	-30.8
2	-35.9
2.5	-40.0
3	-43.0
4	-48.0

Similarly, Figure 2-21 illustrates the PSD for MSK modulation with a bit rate of 3 kHz and 300 watts transmitter power. Using the same consideration as the previous example, one would expect maxima at

$$(f-f_c) = 0, \pm 3, \pm 9/2, \pm 6, \pm 15/2, \text{etc.} \quad (\text{kHz})$$

and minima at

$$(f-f_c) = \pm 9/4, \pm 15/4, \pm 21/4, \pm 27/4, \text{etc.} \quad (\text{kHz})$$

Again, the expected maxima at ± 3 kHz are shifted to slightly lower frequencies by the rapid change in the value of the denominator.

The spectral density at $(f-f_c) = 0$ is

$$10 \log \frac{300}{1.6} = 22.7 \frac{dBW}{kHz}$$

and the spectral fall-off should be

$(f-f_c)$ (kHz)	Relative Attenuation (dB)
0	0
3	-23.6
4.5	-30.8
6	-35.9
7.5	-40.0
9	-43.0

Figure 2-22 is the PSD for MSK with a 5 MHz bit rate and power output of 150 watts. For these conditions, maxima are expected at

$$(f-f_c) = 0, \pm 5, \pm 15/2, \pm 10, \pm 25/2, \text{etc.,} \quad (\text{MHz})$$

and minima at

$$(f\text{-}f_c) = \pm 15/4,\ \pm 25/4,\ \pm 35/4,\ \pm\ 45/4,\ \text{etc.} \qquad \text{(MHz)}$$

The maxima expected at ±5 MHz are shifted to slightly lower frequencies as previously explained.

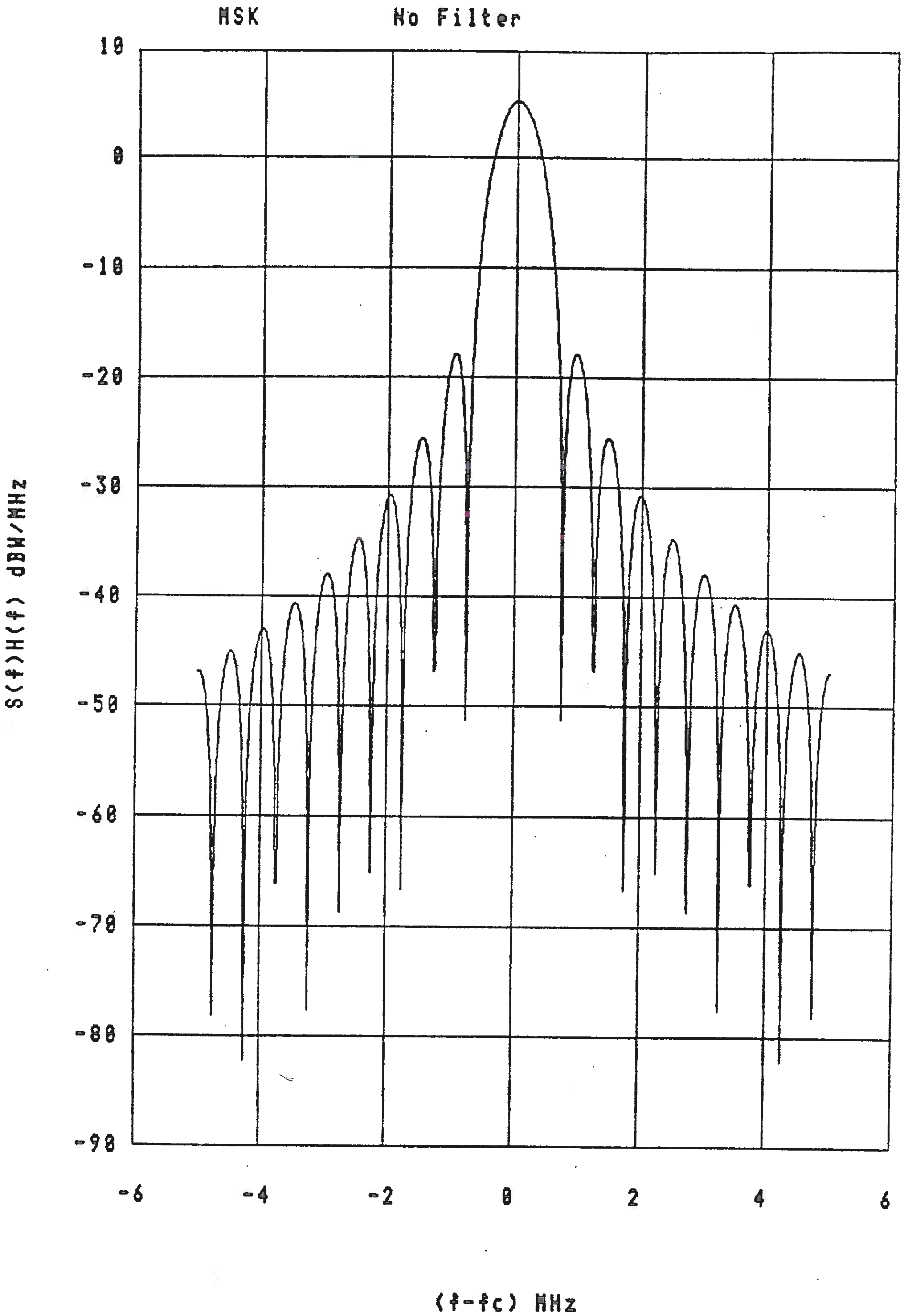
The spectral density at $(f\text{-}f_c) = 0$ should be

$$10 \log \frac{150 watts}{3 MHz} = 17.0 \frac{dBW}{MHz}$$

and the spectral fall-off should be

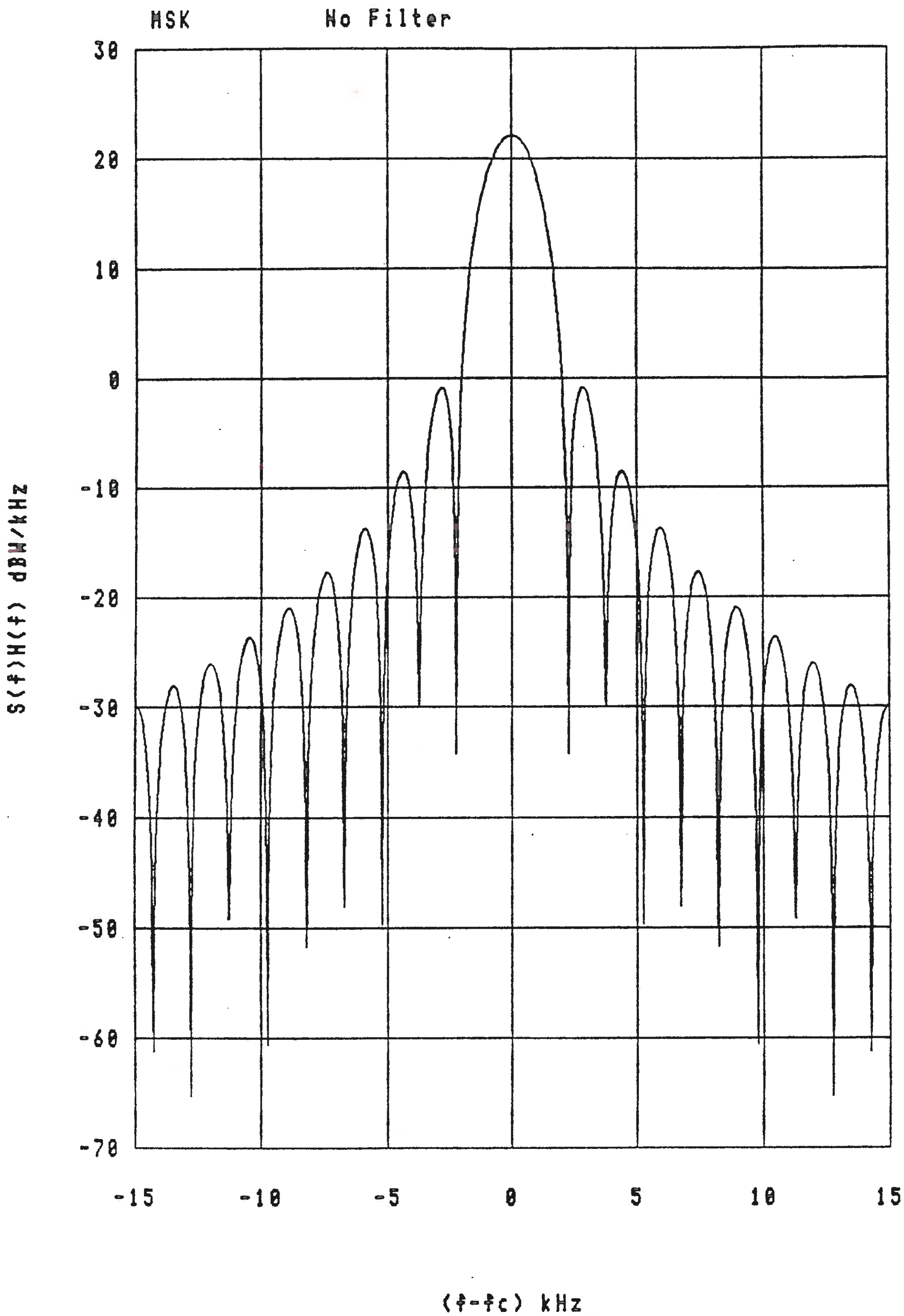
$(f\text{-}f_c)$ (MHz)	Relative Attenuation (dB)
0	0
5	-23.6
7.5	-30.8
10	-35.9
12.5	-40.0
15	-43.0

Comparison of the expected PSD values and the computer generated PSD plots for MSK are in agreement.



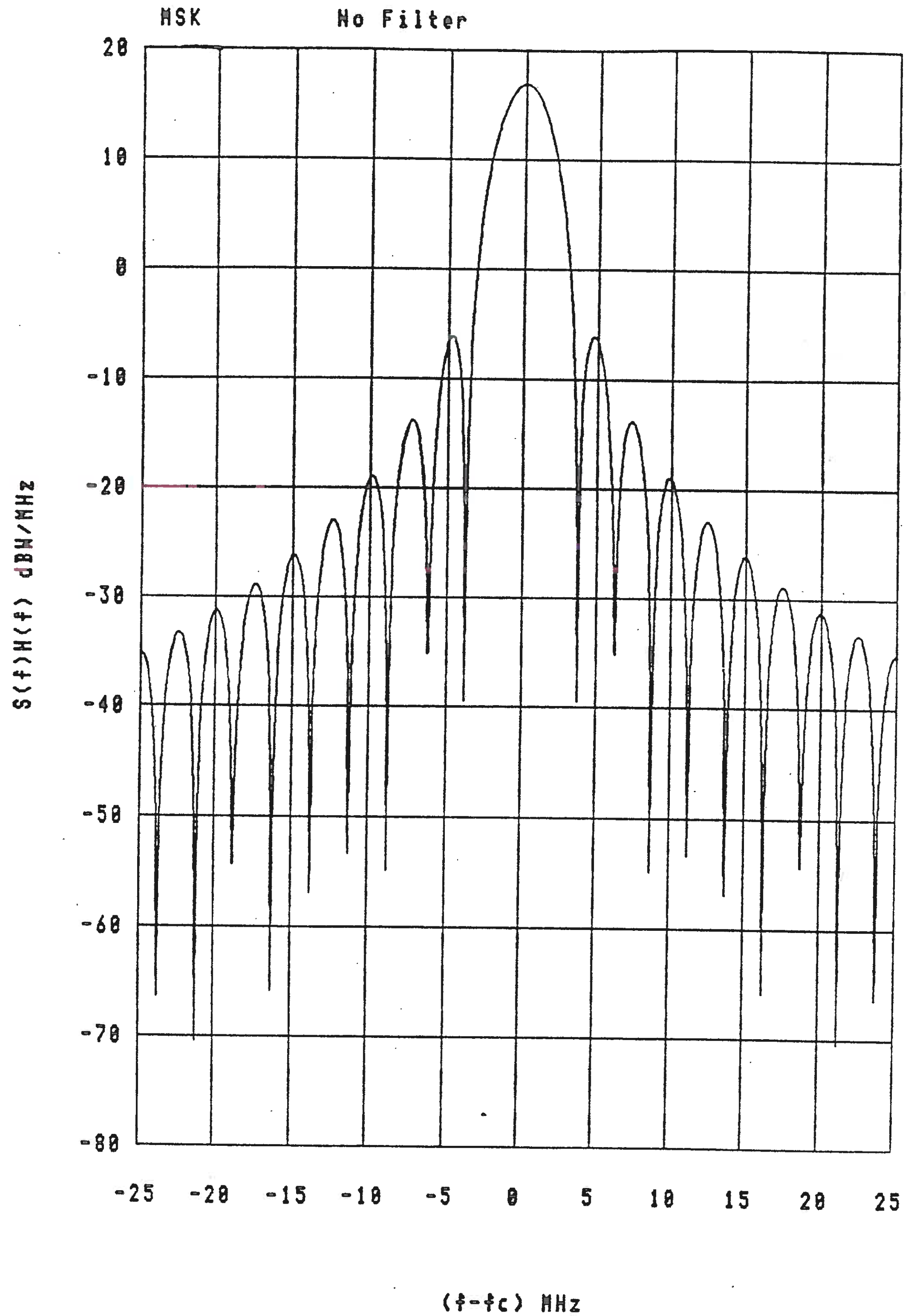
Modulating bit rate = 1.00 mps Power = 2.00 watts 99% band = 1.17 MHz

Figure 2-20. Power spectral density MSK.



Modulating bit rate = 3.00 kps   Power = 300.00 watts 99% band = 3.52 kHz

Figure 2-21. Power spectral density MSK.



Modulating bit rate = 5.00 mps Power = 150.00 watts 99% band = 5.86 MHz

Figure 2-22. Power spectral density MSK.

BPSK with Raised Cosine Modulation

This emission spectrum is the result of using a modulating waveform where the amplitude of the modulating signal is shaped to a raised cosine. The modulating signal then bi-phase shift modulates a carrier. The spectrum generated in the computer model should not be confused with the class of signal where the modulated signal is filtered by a filter with a raised-cosine response.

The PSD for BPSK with raised cosine modulation[8] is determined using the formulation

(2-8)

$$S'(f) = \left[\frac{Sin\ \pi(f-f_c)\frac{T_B}{2}}{\pi(f-f_c)\frac{T_B}{2}}\right]^2 \left[\frac{Cos\ \pi(f-f_c)\frac{T_B}{2}}{1-[(f-f_c)T_B]^2}\right]^2$$

The indeterminate points of the form zero-over-zero occur at

$(f-f_c)\ T_B = 1$ and $(f-f_c) = 0$

application of L'Hospital's Rule yields

$S'(f) = 0.25$ when $(f-f_c)\ T_B = 1$ and

$S'(f) = 1.0$ when $(f-f_c) = 0$

The PSD for BPSK with raised-cosine modulation is illustrated in Figure 2-23. This is the spectrum for a 1 MHz bit rate and 2 watts transmitter power. The maxima are determined by combining the Sin^2 and Cos^2 terms in the numerator using Sin 2x = 2 Sin x Cos x. Thus the maxima could occur at

$Sin^2\pi(f-f_c)T_B = 1$ or

$\pi(f-f_c)T_B = \pm\pi/2,\ \pm 3\pi/2,\ \pm 5\pi/2,\ \pm 7\pi/2$

For Figure 2-23

$(f-f_c) = \pm 1/2,\ \pm 3/2,\ \pm 5/2,\ \pm 7/2,$ etc., (MHz)

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[8] Gagliardi, R.M., Satellite Communications, 1984.

However, $\pi(f-f_c)\ T_B = \pm\pi/2$ are not true maxima because the term

$$1-[(f-f_c)T_B]^2$$

becomes relatively small for small values of $(f-f_c)$ so that S'(f) continues to increase from the minima at $\pi(f-f_c)T_B = \pm\pi$ through the maxima at $(f-f_c) = 0$. The indeterminate value at $(f-f_c) = 0$ is a maximum. Thus, the maxima would be expected at

$$(f-f_c) = 0,\ \pm 5/2,\ \pm 7/2,\ \pm 9/2,\ \text{etc.} \qquad \text{(MHz)}$$

The actual maxima for small Δf values are shifted to slightly lower frequencies. This is caused by the rapid change of the denominator of S'(f) as Δf approaches 0.

The minima could occur at

$$\sin^2 \pi(f-f_c)\ T_B = 0$$

$$\pi(f-f_c)\ T_B = 0,\ \pm\pi,\ \pm 2\pi,\ \pm 3\pi,\ \text{etc.}$$

However, the PSD at $\pi(f-f_c)\ T_B = 0$ is indeterminate and is, in fact, a maximum. At $\pi(f-f_c)\ T_B = \pm\pi$, S'(f) is also indeterminate and is not a minimum with S'(f) = 0.25. Thus, the expected minima, for Figure 2-23, should occur at

$$(f-f_c) = \pm 2,\ \pm 3,\ \pm 4,\ \pm 5,\ \text{etc.} \quad \text{(MHz)}$$

The PSD at the carrier frequency is

$$10 \log \frac{2.0 watts}{1.5 MHz} = 1.2 \frac{dBW}{MHz}$$

The expected spectral fall-off of the maxima can be shown to be

$(f-f_c)$ (MHz)	Relative Attenuation (dB)
0	0
2.5	-32.4
3.5	-41.9
4.5	-48.9

Figure 2-24 is the PSD for BPSK with raised cosine modulation with a 3 kHz bit rate and 300 watts of output power. For these conditions, the maxima are expected at

$$\pi(f-f_c)\,T_B = 0,\ \pm 5\pi/2,\ \pm 7\pi/2,\ \pm 9\pi/2,\ \pm 11\pi/2,\ \text{etc.}$$

$$(f-f_c) = 0,\ \pm 15/2,\ \pm 21/2,\ \pm 27/2,\ \pm 33/2,\ \text{etc.} \quad \text{(kHz)}$$

However, the maxima at $\pm 15/2$ kHz are shifted to slightly lower Δf values by the rapid change in the denominator.

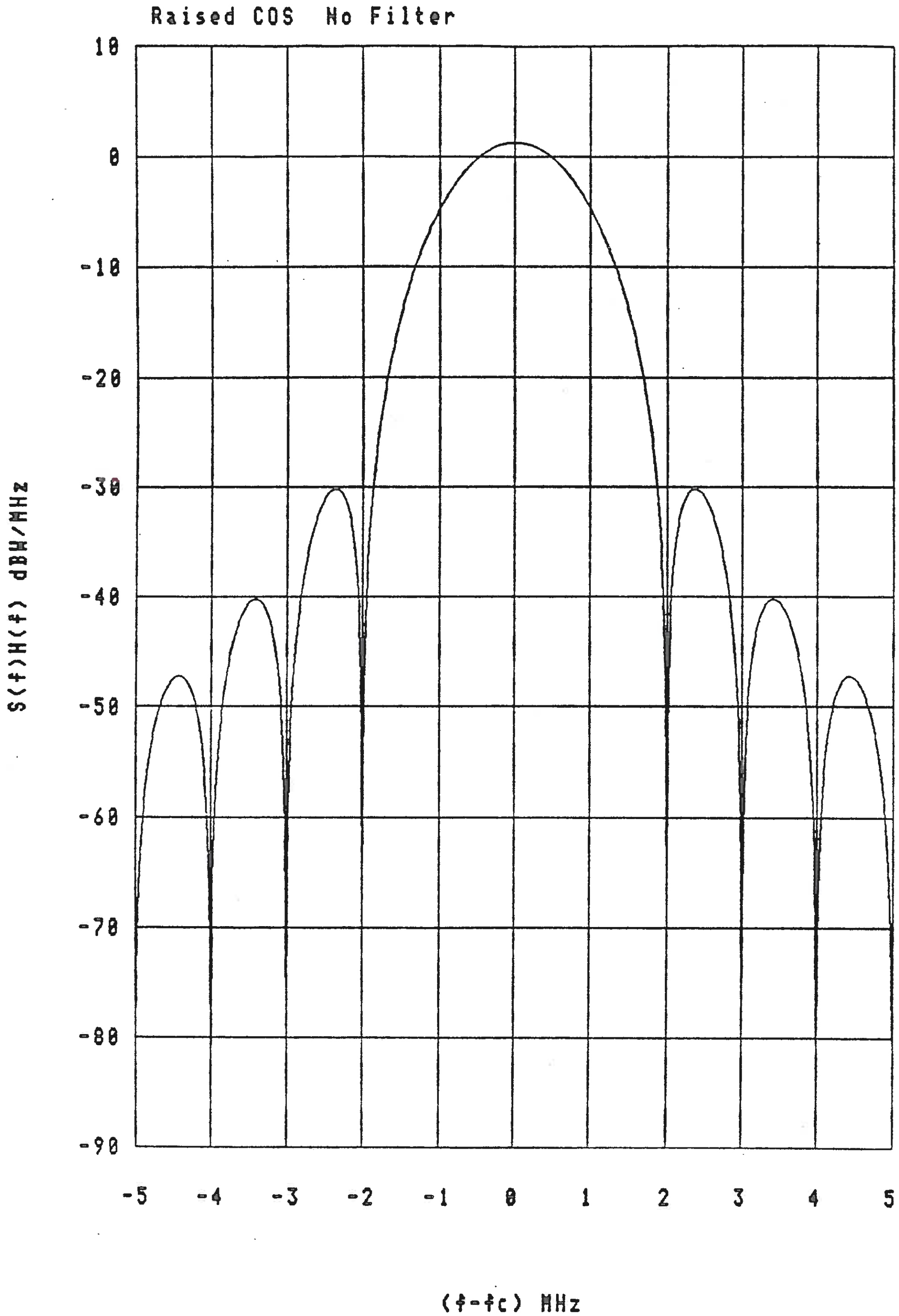
Minima are expected at

$$\pi(f-f_c)\,T_B = \pm 2\pi,\ \pm 3\pi,\ \pm 4\pi,\ \pm 5\pi,\ \text{etc.}$$

$$(f-f_c) = \pm 6,\ \pm 9,\ \pm 12,\ \pm 15,\ \text{etc.} \quad \text{(kHz)}$$

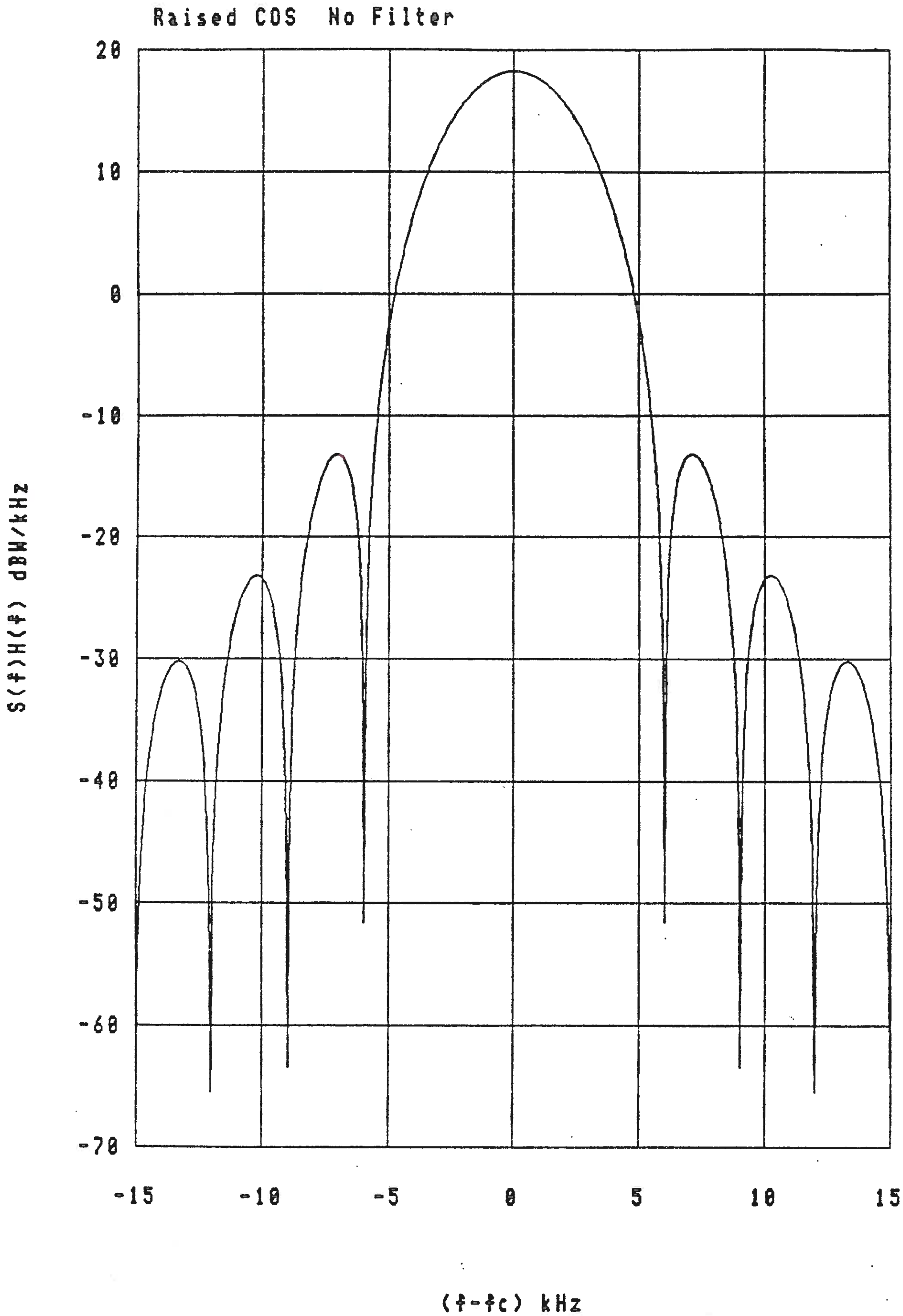
The PSD at the carrier frequency is

$$10 \log \frac{300\,watts}{4.5\,MHz} = 18.2\ \frac{dBW}{MHz}$$



Modulating bit rate = 1.00 mps Power = 2.00 watts 99% band = 2.83 MHz

Figure 2-23. Power spectral density BPSK with raised cosine modulation.



Modulating bit rate = 3.00 kps Power = 300.00 watts 99% band = 8.50 kHz

Figure 2-24. Power spectral density BPSK with raised cosine modulation.

The spectral fall-off should be

$(f-f_c)$ (kHz)	Relative Attenuation (dB)
0	0
7.5	-32.4
11.5	-41.9
13.5	-48.9

Figure 2-25 is the PSD for BPSK with raised cosine modulation with a 5 MHz bit rate and 150 watts. The maxima are expected at

$$(f-f_c) = 0, \pm 25/2, \pm 35/2, \pm 45/2, \pm 55/2 \text{ etc. (MHz)}$$

The maxima at $\pm 25/2$ MHz are shifted to slightly lower Δf values, as previously explained. The minima are expected at

$$(f-f_c) = \pm 10, \pm 15, \pm 20, \pm 25, \text{etc.} \quad \text{(MHz)}$$

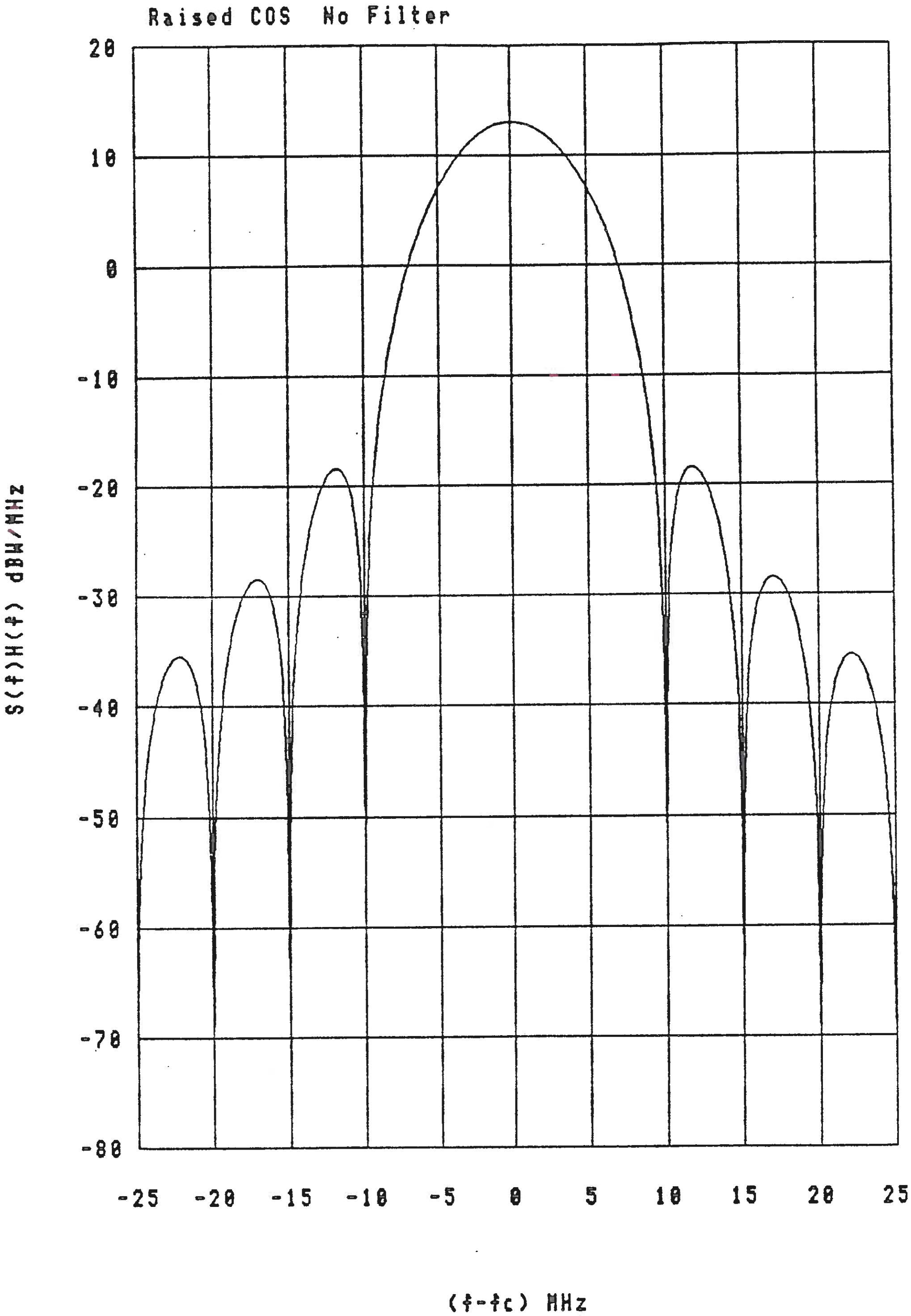
The PSD at the carrier frequency is

$$10 \log \frac{150 watts}{7.3 MHz} = 13.1 \frac{dBW}{MHz}$$

The fall-off should be

$(f-f_c)$ (MHz)	Relative Attenuation (dB)
0	0
12.5	-32.4
17.5	-41.9
22.5	-48.9

The PSD plots for BPSK with raised-cosine modulation are as expected.



Modulating bit rate = 5.00 mps Power = 150.00 watts 99% band = 14.16 MHz

Figure 2-25. Power spectral density BPSK with raised cosine modulation.

BPSK with Manchester Modulation

The capability to model BPSK with Manchester coding as the modulating signal (see reference 8) is also included in the computer program. The PSD formulation is

$$S'(f) = \frac{Sin^4 \dfrac{\pi T_B (f-f_c)}{2}}{\left[\dfrac{\pi T_B (f-f_c)}{2}\right]^2} \tag{2-9}$$

The expression for S'(f) is indeterminate at $(f-f_c) = 0$ and by application of L'Hospital's Rule it can be shown that

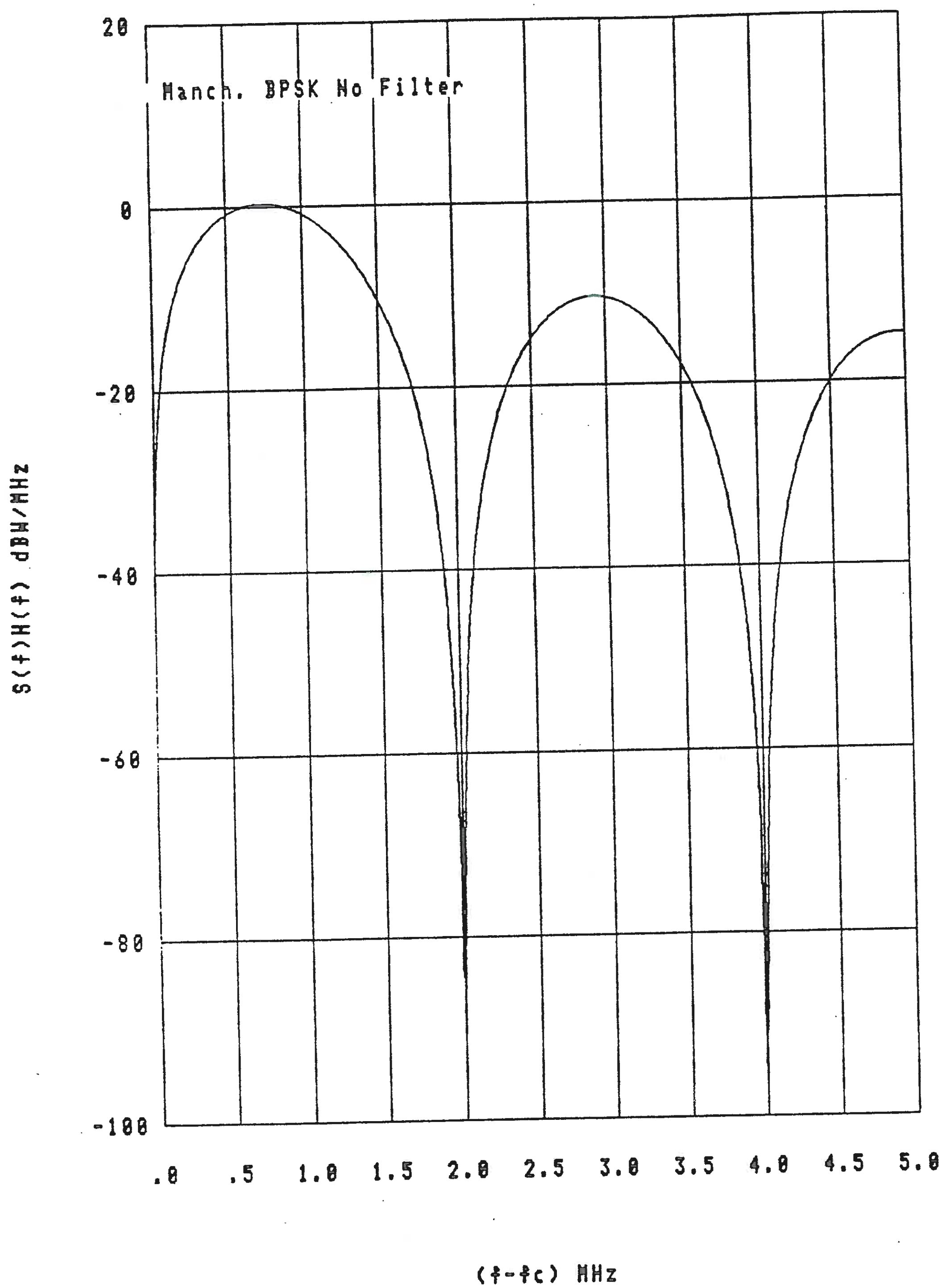
$$S'(f) = 0 \text{ at } (f-f_c) = 0$$

Figure 2-26 is the one-sided spectrum for BPSK with Manchester modulation with a 1 MHz bit rate and 2 watts of power. The maxima should occur at

$$Sin^4 \frac{\pi T_B (f-f_c)}{2} = 1 \ or$$

$$\frac{\pi T_B (f-f_c)}{2} = \pm\pi/2, \pm3\pi/2, \pm5\pi/2, \pm7\pi/2, etc.$$

$$(f-f_c) = \pm 1, \pm 3, \pm 5, \pm 7, \text{etc. (MHz)}$$



Modulating bit rate = 1.00 mps Power = 2.00 watts 99% band = 28.13 MHz

Figure 2-26. Power spectral density BPSK with Manchester modulation.

All the example PSDs for BPSK with Manchester modulation show the actual maxima at small Δf values shifted to slightly lower Δf values. This can be attributed to the rapid change of the denominator for small Δf values.

The minima should occur at

$$Sin^4 \frac{\pi T_B (f-f_c)}{2} = 0 \ or$$

$$(f-f_c) = 0, \pm 2, \pm 4, \pm 6, \pm 8, \text{etc. (MHz)}$$

The maximum PSD which occurs at $(f-f_c) = 1$ should be

$$10 \log \frac{2.0 watts}{1.7 MHz} = 0.7 \frac{dBW}{MHz}$$

The spectral fall-off can be determined by evaluating

$$20 \log \frac{2}{\pi T_B (f-f_c)}$$

which results in

$(f-f_c)$ (MHz)	Relative Attenuation (dB)
1	0
3	-9.0
5	-14.0
7	-16.9
9	-19.1

Figure 2-27 is the one-sided spectrum for BPSK with Manchester modulation with a 3 kHz bit rate and 300 watts. The maxima should occur at

$$\frac{\pi T_B (f-f_c)}{2} = \pm\pi/2,\ \pm 3\pi/2,\ \pm 5\pi/2,\ \pm 7\pi/2,\ etc.$$

$$(f-f_c) = \pm 3,\ \pm 9,\ \pm 15,\ \pm 21,\ \text{etc.}\ \ (\text{kHz})$$

and the minima at

$$\frac{\pi T_B (f-f_c)}{2} = 0,\ \pm\pi,\ \pm 2\pi,\ \pm 3\pi,\ \pm 4\pi,\ etc.$$

$$(f-f_c) = \pm 0,\ \pm 6,\ \pm 12,\ \pm 18,\ \pm 24,\ \text{etc.}\ \ (\text{kHz})$$

The peak PSD value should be

$$10 \log \frac{300 \, watts}{4.9 \, MHz} = 17.8 \, \frac{dBW}{MHz}$$

and the spectral fall-off is expected to follow

$(f-f_c)$ (kHz)	Relative Attenuation (dB)
3	0
9	-9.6
15	-14.0
21	-16.9
27	-19.1

Figure 2-28 is another example of PBSK with Manchester modulation. The bit rate is 5 MHz and the transmitter power is 150 watts. The maxima are expected at

$(f-f_c) = \pm 5, \pm 15, \pm 25, \pm 35,$ etc. (MHz)

and the minima are expected at

$(f-f_c) = 0, \pm 10, \pm 20, \pm 30, \pm 40,$ etc. (MHz)

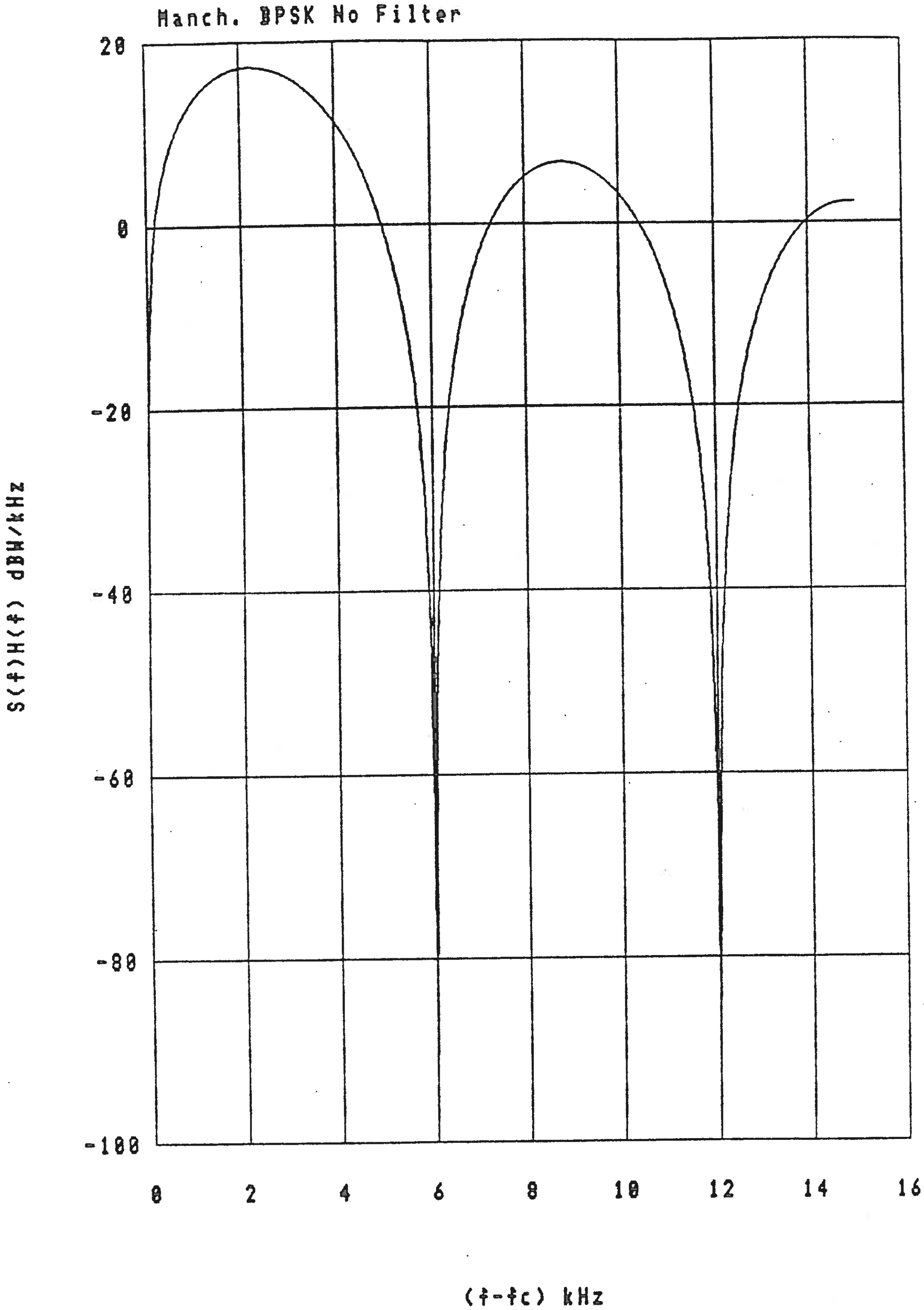
The peak PSD should be

$$10 \log \frac{150 \, watts}{8.3 \, MHz} = 12.6 \, \frac{dBW}{MHz}$$

and the spectral fall-off should be

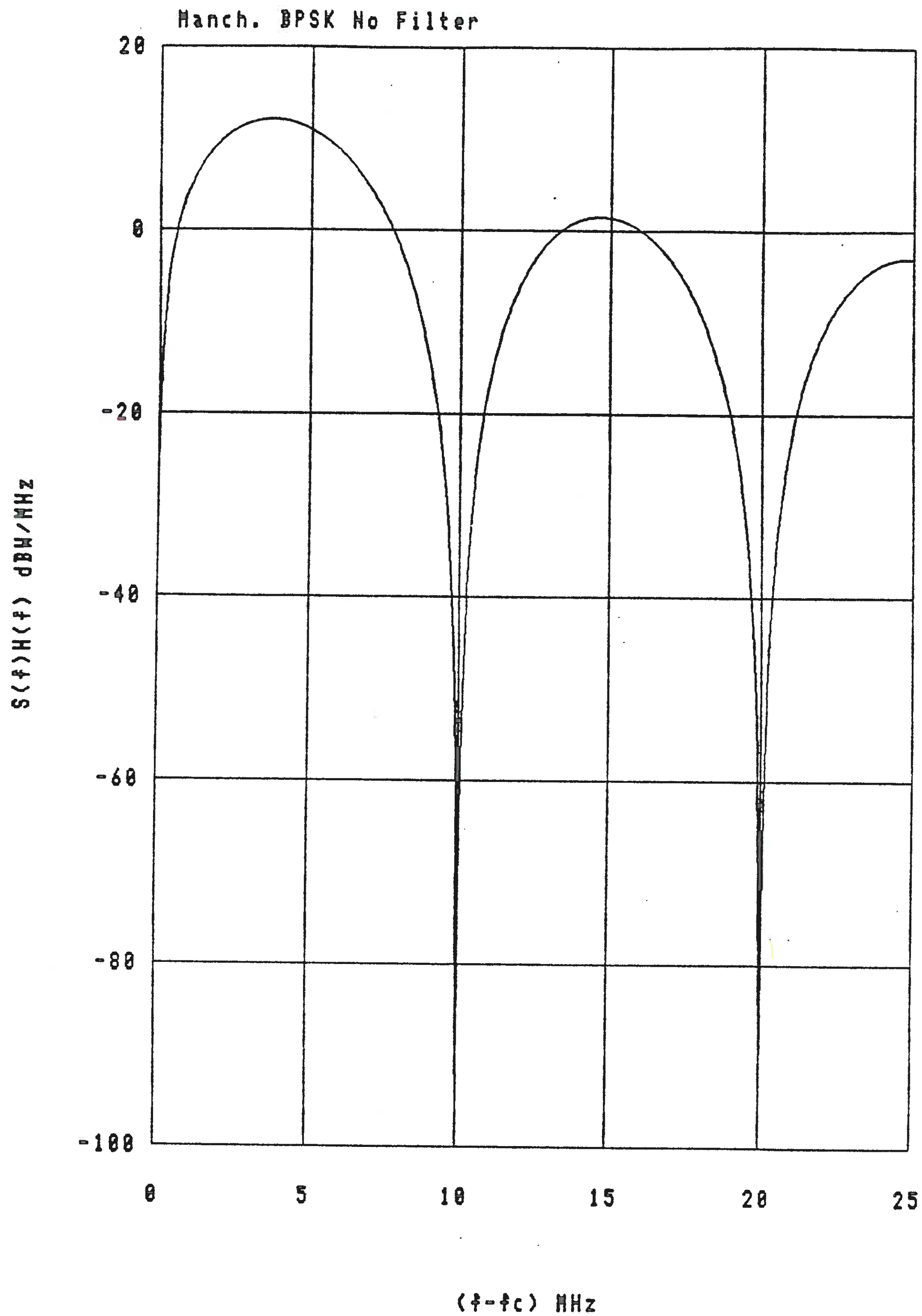
$(f-f_c)$	Relative Attenuation
5 MHz	0 dB
15	-9.6
25	-14.0
35	-16.9
45	-19.1

Thus, the PSD plots for BPSK with Manchester modulation are as expected.



Modulating bit rate = 3.00 kps   Power = 300.00 watts   99% band = 84.38 kHz

Figure 2-27. Power spectral density BPSK with Manchester modulation.



Modulating bit rate = 5.00 mps Power = 150.00 watts 99% band = 140.63 MHz

Figure 2-28. Power spectral density BPSK with Manchester modulation.

SECTION 3

CONCLUSION

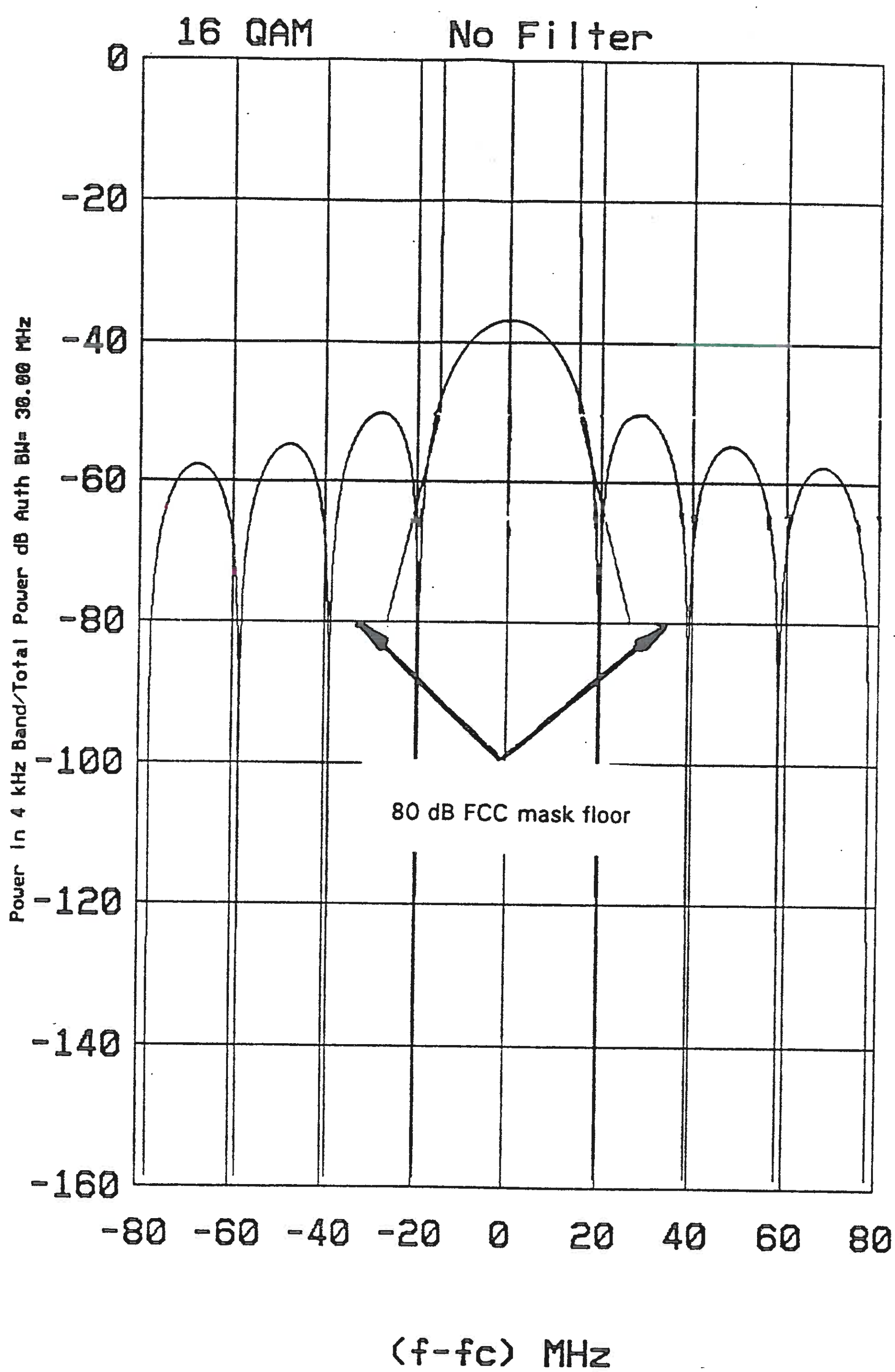
The power spectral density data developed by the emission spectra computer model have been verified to conform with the theoretical formulations. The computer model is a user-friendly capability that is available for operation on a PC.

APPENDIX A

The computer program, "Emission Spectrum Model for Digital Signals," contains additional capabilities that were not validated as part of this study. These capabilities include the FCC spectrum mask and various filter options. Several graphs are presented in this Appendix to illustrate these capabilities.

Figure A-1 is a plot of the power spectral density for 16 QAM along with the FCC spectrum mask. The PSD is for power level of one watt and a data rate of 78 Mb/s. The FCC mask is for the 6 GHz band and an authorized bandwidth of 30 MHz.

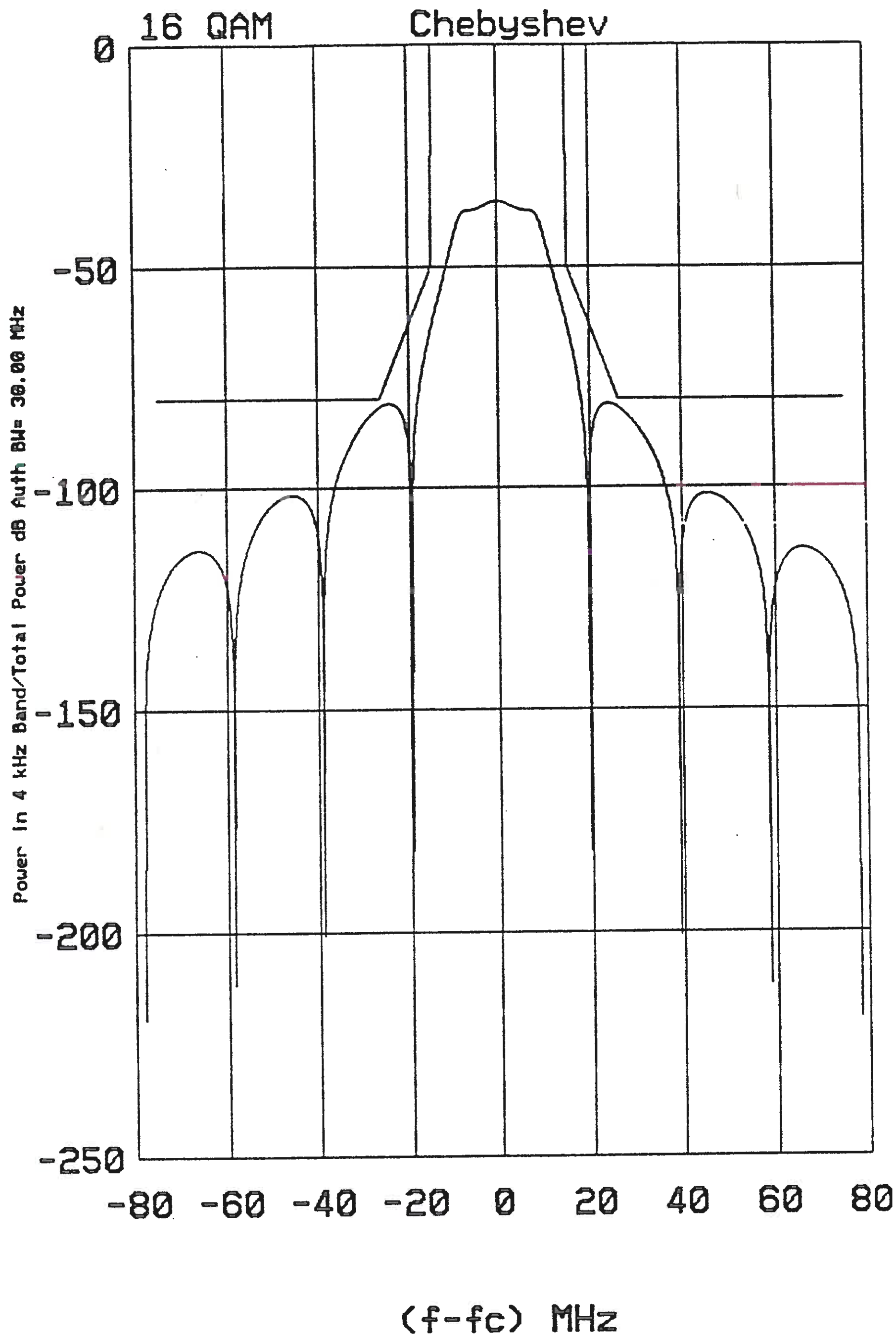
Figure A-2 is a plot of the PSD of the 16 QAM function of Figure A-1 however the PSD is modified to show the effect of passing the emission spectrum through a filter. The filter used for Figure A-2 was a third order Chebyshev with a 20 MHz bandwidth and 1 dB ripple. The FCC mask for the 6 GHz band with a 30 MHz authorized bandwidth is also shown.



Modulating bit rate = 78.0 mps Power = 1.00 watts 99% band = 365.63 MHz

FCC Mask for 6 GHz band and 30 MHz authorized bandwidth.

Figure A-1. Power spectrum density with FCC Mask for 16 QAM.



Modulating bit rate = 78.0 mps Power = 1.00 watts 99% band = 21.42 MHz

Chebyshev Filter - 20 MHz bandwidth, 1 dB ripple, 3rd order

FCC Mask for 6 GHz band and 30 MHz authorized bandwidth.

Figure A-2. Filter power spectrum density with FCC Mask for 16 QAM.

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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

A computer program was developed to calculate the power spectral density (PSD) and the fractional power containment bandwidth for various digital modulation techniques. The power containment capability was used to provide guidance for determination of necessary bandwidth in support of Annex J of the "NTIA Manual of Regulations and Procedures for Federal Radio Frequency Management." This report documents the models contained in the computer program and illustrates the models with various sample problems. The report also shows the verification of the computer program implementation.

The digital modulation techniques that are presently included in the computer program are:

Phase shift keying (PSK) with non-return-to-zero modulation
Quadrature amplitude modulation (QAM)
Frequency shift keying (FSK)
Minimum shift keying (MSK)
Bi-phase shift keying with raised cosine modulation
Bi-phase shift keying with Manchester modulation.

16. Key Words *(Alphabetical order, separated by semicolons)*

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Necessary Bandwidth
Power Spectral Density
Spectrum
Transmitter Spectrum

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