Enhanced Location Estimation via Pattern Matching and Motion Modelling

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ABSTRACT

Mobile location based applications are in the first place designed for urban areas where providers find a high density of customers. Exactly in these environments conventional trilateration location techniques often lack performance due to multipath propagation. Attention has thus be drawn to fingerprint localization methods allowing to localize mobile users in such areas. These have already proved that accuracies below 100 meters are possible in heavy urban areas, but can further be improved significantly when using several fingerprints than relying only on a single one. In this paper we will present a method to optimally combine consecutive position estimates utilizing a motion model for the targeted user. We show trial results from the city of Vienna where we have successfully applied the method and compare it with the single fingerprint case. The achieved accuracy in 90% of all cases has improved from above 100 meters with the single fingerprint method to below 70 meters using the proposed method. This is adequate for most location applications.

I. INTRODUCTION

Fingerprint methods [1], [2], [3] estimate the position of a target (e.g. mobile user) by comparing location dependent parameters (e.g. received power levels) with beforehand measured samples. Accuracy for a single snapshot ranges from about 300 to below 100 meters in urban and heavy urban areas. Most of the location based applications and E911 in the US however do require a significantly higher accuracy.

One strategy of improving the accuracy is to increase the number of pre-measured samples in the database. This is undesired since a larger number of samples requires a higher effort to deploy and maintain the fingerprint based location system. A more promising approach is therefore to rely on a sequence of position estimates and compute the most probable one. This does not effect the size of the database, but does unfortunately increase the required localization time. We will see however that already three consecutive snapshots significantly increase the accuracy even for slow moving pedestrians.

In this paper we propose an algorithm similar to Kalman filtering to utilize several consecutive snapshots instead of relying on a single one. The algorithm combines uncertain position information from several snapshots with a mobility model for the targeted user to enhance the final position estimate. We avoid using a mobility model which assumes a deterministic realization of the velocity and direction [4], but instead combine deterministic behavior with randomness to mimic actual human behavior.

The rest of the paper is organized as follows. In section II we shortly review a pattern matching based localization method which will serve as position estimator for a single measured fingerprint. In section III we show the mobility model used to simulate the motion of the user and apply it to a single position estimate. In section IV we finally update the propagated position estimate with a new position estimate to improve the overall accuracy. We further present results where we have applied the method in the city of Vienna. Finally we conclude in section V.

II. SINGLE POSITION ESTIMATES

Before we start to improve the localization accuracy by combining several estimates we first define an estimator for the probability of being at a position over all considered positions. We therefore briefly review the method proposed in [3] which will serve as a simple single fingerprint estimator. Bayesian networks[1] are used there to represent a position by describing dependencies between the different measured Cell IDs (serving cells and ordered neighboring cells according to the received power levels) at a position. The Bayesian networks are then trained with pre-measured data. In our test area in the city of Vienna we used equally spaced measurements every 5 meters. The final search of the mobile's position is then a comparison of the mobile's current fingerprint $\boldsymbol{f}$ containing the received serving and neighboring cells and all models in the expected target area (e.g. the area of the serving cell). For the comparison we use the marginal likelihood as a scoring method to identify the optimal model according to

$$\mathcal{L}(\lambda_{\boldsymbol{i}}) := p(\boldsymbol{f}|\lambda_{\boldsymbol{i}}) = \int p(\boldsymbol{f}|\lambda_{\boldsymbol{i}}, \boldsymbol{\theta_i})p(\boldsymbol{\theta_i})d\boldsymbol{\theta_i} \qquad (1)$$

with $\lambda_{\boldsymbol{i}}$ being the Bayesian network at position $\boldsymbol{i} = (x, y)$, $\boldsymbol{f}$ the fingerprint of the mobile we want to localize and $\boldsymbol{\theta_i}$ the parameters of the Bayesian network which are updated during the training with the measured samples.

Maximization over all Bayesian networks within the area of the serving cells results in the best matching Bayesian network and thus in the best estimate for a single position. The resulting accuracy within our target area is shown in Fig. 5 (dashed line).

We should note at this point that the considerations concerning the fingerprint method address GSM in this paper. We would like to stress however that this the all the methods introduced here can be applied to any location dependent parameter of the mobile system in general.

III. User Mobility model

The mobility model we propose here attempts to mimic human movement behavior to predict the new position of a mobile user. This is important, since we avoid the approach of estimating the position of a user simply as the mean position computed from several single localization estimates. The result would suffer from a systematically increasing error for increasing velocity of the target, caused by the larger spatial separation of the different position estimates.

Instead we initially rely on the probability density distribution of the positions given the measured location dependent parameter of the mobile resulting from (1):

$$p(\boldsymbol{i}|\boldsymbol{f}) := p(\lambda_{\boldsymbol{i}}|\boldsymbol{f}) = \frac{p(\lambda_{\boldsymbol{i}})}{p(\boldsymbol{f})}\tilde{\mathcal{L}}(\lambda_{\boldsymbol{i}}) = \gamma\mathcal{L}(\lambda_{\boldsymbol{i}}), \qquad (2)$$

[1] For an introduction see e.g. [5], [6]

with γ being a constant, if we assume no prior knowledge about the occurrence of either a certain position or a certain fingerprint.

In order to combine this information with information from the next fingerprint at time $t+T$ we use a mobility model which will change our believes about the initial position taken at time t. The variance of the first estimate will thus increase since the user might move ahead during the time T. We are not so sure anymore where the user actually is located.

For the mobility model we make three assumptions:

1) The user will normally move with constant velocity $\boldsymbol{u}$ for the time under consideration. (This is about a few seconds).
2) Physical obstacles, other persons, etc. are viewed as perturbations $\boldsymbol{v}$ upon the constant velocity trajectory from assumption one.
3) The user will try to reestablish it's constant velocity (equal to $\boldsymbol{v} = 0$), once he was perturbed.

In general these assumptions result from the tendency of a person to maximize his personal utility, which includes to avoid deceleration and acceleration processes [7].

For a single physical dimension we therefore model the targeted user's motion as a dynamic linear system and write:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{G}\boldsymbol{w}(t) \qquad (3)$$

with

$$\begin{aligned}\boldsymbol{x} &= \begin{bmatrix} x \\ v \end{bmatrix} \\ &= \begin{bmatrix} \text{user's position at time t} \\ \text{user's velocity variation around its constant speed at time t} \end{bmatrix}.\end{aligned} \qquad (4)$$

The vector $\boldsymbol{u} = [u, 0]$ is a deterministic vector and addresses assumption one by denoting the constant velocity. The random vector $\boldsymbol{w} = [0, w]$ represent white Gaussian noise and models our second assumption where the user is perturbed by obstacles and suddenly has to change his velocity. The resulting speed difference between his current speed and his desired velocity is denoted by the variable v. In such a case the user will try to reach its personal optimal speed u again and thus will change his speed until the term v becomes zero. This indicates the speed difference v to be correlated in time; if the user does not move with his desired speed u at time t, it is likely that he still moves with different speed than u at time $t + \tau$ for sufficient small τ.

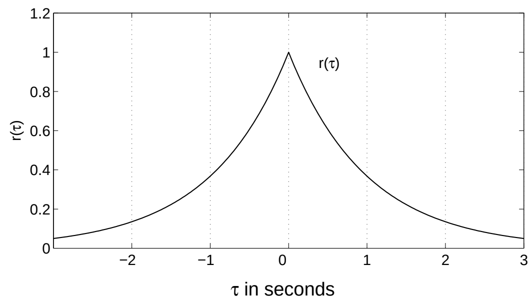


Fig. 1. Correlation function $r_w(\tau)$ of user's perturbation caused velocity v.

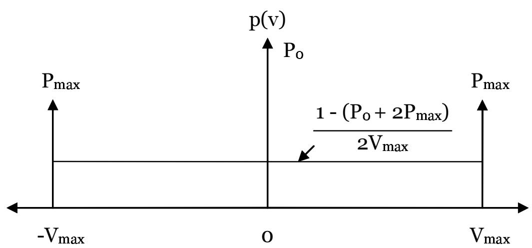


Fig. 2. Probability density model of the user's velocity

Singer used in his paper about tracking [8] a similar model but incorporates acceleration also. We have omitted to model acceleration here, since the acceleration period of the users under consideration (mainly pedestrians) is assumed to be small compared to the systems time constants. For the time correlation of v we assume (refer also to Fig. 1):

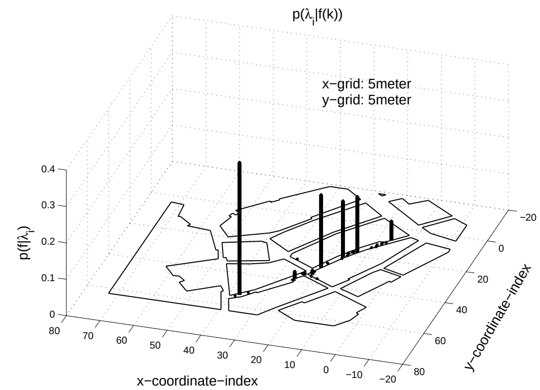
$$r_v(\tau) = E\{v(t)v(t+\tau)\} = \sigma_m^2 e^{-\alpha|\tau|} \quad (5)$$

where σ_m^2 is the variance of the difference speed v and α is the reciprocal of the random difference speed time constant. We assume $\alpha = \frac{1}{0.2}$ and for σ_m^2 we use the same approach as in [8]: We construct the variance assuming that the user may increase or decrease his speed due to perturbation by a maximum value V_{max} ($-V_{max}$). He will do so with a probability P_{max}. The user will not change his velocity with probability P_0 and will speed up or down between the limits according to a uniform distribution (Fig. 2). We can then write for the variance

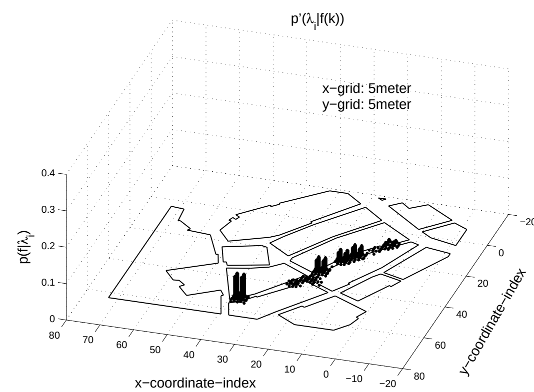
$$\sigma_m^2 = \frac{V_{max}^2}{3}(1 - 4P_{max} - P_0). \quad (6)$$

Deriving further the power density spectrum from (5) and interpreting the result as being produced by a shaping filter driven by white Gaussian noise w we get for the corresponding differential equation:

$$\dot{v}(t) = -\alpha v + w(t) \quad \text{with} \quad \sigma_w^2(\tau) = 2\alpha\sigma_m^2\delta(\tau) \quad (7)$$



(a) probability density of the position estimate given the fingerprint $\boldsymbol{f}$ at time k



(b) probability density of the position estimate given the fingerprint $\boldsymbol{f}$ at time k+1.

Fig. 3. Impact of the mobility model on a position estimate. The probability of a single position is not so sure anymore. The possible movement of a user broadens the variance of $p(\lambda_{\boldsymbol{i}}|\boldsymbol{f}(k))$

The remaining desired velocity u we model as a random variable with its density constructed by the superposition of three Gaussian shaped curves:

$$\begin{aligned} f(u) &= \frac{(1-\mathrm{w})}{2}\mathcal{N}(-u_m, \sigma_{u_m}^2) + \mathrm{w}\mathcal{N}(0, \sigma_{u_0}^2) + \\ &+ \frac{(1-\mathrm{w})}{2}\mathcal{N}(u_m, \sigma_{u_m}^2) \end{aligned} \quad (8)$$

The two Gaussian shaped curves with mean $-u_m$ and u_m represent the users moving forward or backwards. The curve in the middle denotes a motionless (or almost motionless) user. The weighting factor w $\in [0, 1]$ allows to control the percentage of motionless users.

The dynamic linear system equation (3) is now specified completely by

$$\boldsymbol{F} = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{G} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

and represents the motion of a user in a single physical dimension.The extension into a second dimension is straight forward if we assume independence between the cartesian coordinates. For convenience we keep the same names for the variables, but introduce the indices x and y to describe the physical dimension.

Assuming a new position estimate every T seconds and applying the state-space-method to (8) we write for the discrete mobility equation

$$\boldsymbol{X}(k+1) = \boldsymbol{\Theta}(T,\alpha)\boldsymbol{X}(k) + \boldsymbol{B_d}(k)\boldsymbol{U}(k) + \boldsymbol{W}(k) \quad (9)$$

where

$$\begin{aligned} \boldsymbol{X} &= [x, v_x, y, v_y]^T \\ \boldsymbol{U} &= [u_x, 0, u_y, 0]^T \\ \boldsymbol{B_d}(T) &= \int_t^{t+T} \boldsymbol{\Theta}(t-\tau,\alpha)\boldsymbol{B}d\tau \end{aligned}$$

$\boldsymbol{X}$ is the dynamic state vector containing the position and the velocity for both cartesian dimensions. $\boldsymbol{U}$ is the desired deterministic speed of the user and $\boldsymbol{W}(k)$ is a discrete-time zero-mean white Gaussian noise with statistics according to

$$E\{\boldsymbol{W}(k)\} = \boldsymbol{0}$$

$$E\{\boldsymbol{W}(k)\boldsymbol{W}^T(j)\} = \left\{ \begin{array}{cc} \boldsymbol{Q}(k) & j = k \\ \boldsymbol{0} & j \neq k \end{array} \right\}.$$

and

$$\boldsymbol{Q}(k) = \boldsymbol{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\alpha{\sigma_m}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\alpha{\sigma_m}^2 \end{pmatrix}$$

The matrices $\boldsymbol{\Theta}$ and $\boldsymbol{B_d}$ are the state transition matrices to link the system at the times k and $k+1$.

Since $\boldsymbol{F}$ is time invariant the state transition matrix $\boldsymbol{\Theta}(T,\alpha)$ can easily be obtained by [9]

$$\boldsymbol{\Theta}(T,\alpha) = \mathcal{L}^{-1}\{(s\boldsymbol{I} - \boldsymbol{F})^{-1}\} \quad (10)$$

where $\mathcal{L}$ denotes the Laplace transformation. This results in

$$\boldsymbol{\Theta}(T,\alpha) = \begin{pmatrix} 1 & \frac{1}{\alpha}(1-e^{-\alpha T}) & 0 & 0 \\ 0 & e^{-\alpha T} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{\alpha}(1-e^{-\alpha T}) \\ 0 & 0 & 0 & e^{-\alpha T} \end{pmatrix} \quad (11)$$

We see from (9) that $\boldsymbol{X}(k+1)$ is Gaussian if $\boldsymbol{X}(k)$ is either Gaussian or deterministic and since we assume a known initial position at time $k=0$ the density function $p_{\boldsymbol{X}(k+1)}(.)$ is completely determined by the mean and covariance given by [9]:

$$\boldsymbol{m_X}(k+1) = \boldsymbol{\Theta}(T,\alpha)E\{\boldsymbol{X}(k)\} + \boldsymbol{B_d}(T)\boldsymbol{U}(k) \quad (12)$$

$$\begin{aligned} \boldsymbol{P_{XX}}(k+1) &= \boldsymbol{\Theta}(T,\alpha)E\{\boldsymbol{X}(k)\boldsymbol{X}^T(k)\}\boldsymbol{\Theta}(T,\alpha)^T + \\ &+ \int_0^T \boldsymbol{\Theta}(T-\tau,\alpha)\boldsymbol{Q}\boldsymbol{\Theta}(T-\tau,\alpha)^T d\tau \end{aligned} \quad (13)$$

Letting the mobile user start at the initial position $\boldsymbol{X}(k=0)$ and with velocity $\boldsymbol{U}(k=0), v_x = 0, v_y = 0$ the mean results according to (12) in

$$\boldsymbol{m_X}(k+1) = \boldsymbol{X}(k=0) + T\boldsymbol{U}(k=0) \quad (14)$$

The covariance computes to

$$\boldsymbol{P_{XX}}(k+1) = 2\alpha{\sigma_m}^2 \begin{pmatrix} p11 & p12 & 0 & 0 \\ p12 & p22 & 0 & 0 \\ 0 & 0 & p11 & p12 \\ 0 & 0 & p12 & p22 \end{pmatrix} \quad (15)$$

where

$$p11 = \frac{1}{2\alpha^3}\left(-e^{-2\alpha T} + 4e^{-\alpha T} - 3 + 2\alpha T\right)$$

$$p12 = \frac{1}{2\alpha^2}\left(e^{-2\alpha T} - 2e^{-\alpha T} + 1\right)$$

$$p22 = \frac{1}{2\alpha}\left(1 - e^{-2\alpha T}\right).$$

IV. Combined Position Estimate and User Mobility Model

We are now able to propagate the optimal position estimate $\hat{\boldsymbol{i}}(k)$ at time k into the estimate $\hat{\boldsymbol{i}}'(k+1)$ at time $k+1$.

We therefore treat the position $\hat{\boldsymbol{i}}(k)$ of the user as random variable and use (2) to describe its probability density. By adding the distance $\boldsymbol{X}(k+1)$ which the user has moved during time period T we receive the new position to be:

$$\hat{\boldsymbol{i}}'(k+1) = \hat{\boldsymbol{i}}(k) + \boldsymbol{A}\boldsymbol{X}(k+1). \quad (16)$$

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Thus we are able to compute $\hat{\boldsymbol{i}}'$ from the position estimate $\hat{\boldsymbol{i}}(k)$ at time k and the mobility model's contribution $\boldsymbol{X}(k+1)$.

Figure 3 illustrates the impact of the motion model. It shows a section of the target area in the inner city of Vienna. The x- and y-coordinates are given as indices of a 5 times 5 meter measurement grid. The z-axis shows the probability $p(\lambda_{\boldsymbol{i}}|\boldsymbol{f})$ of being at a certain position $\boldsymbol{i}$. In Fig. 3(a) a first estimate for the fingerprint $\boldsymbol{f}$ at time k is shown. A simple maximum likelihood estimator would localize the user at the position with the indices $(38, 52)$. Fig. 3(b) shows the situation at time $k+1$ after $T = 3$ seconds. The reliability of the first estimate is reduced by the possible movement of the user. The probability of being at the position $(38, 52)$ is reduced by about 80% compared to time t.

Let us now consider the incorporation of the position estimate $\hat{\boldsymbol{i}}(k+1)$ which becomes available at time $k+1$. We now combine this estimate with the one $\hat{\boldsymbol{i}}'(k+1)$ propagated over time.

Since we still do not know whether to trust the propagated position estimate or the newly available estimate more, we combine them according to

$$\hat{\boldsymbol{i}}^*(k+1) = (\boldsymbol{I} - \boldsymbol{K})\hat{\boldsymbol{i}}'(k+1) + \boldsymbol{K}\hat{\boldsymbol{i}}(k+1). \quad (17)$$

whit $\boldsymbol{K}$ denoting a blending factor and $\boldsymbol{I}$ being the identity matrix.

To find the blending factor $\boldsymbol{K}$ we chose to minimize the estimator's variance and limit $\boldsymbol{K}$ to be between $\boldsymbol{0}$ and $\boldsymbol{I}$. This is equal to a minimization of the major diagonal of the covariance matrix of the estimator $\hat{\boldsymbol{i}}^*(k+1)$ and we write:

$$\frac{d(tr(\boldsymbol{P}_{KK}))}{d\boldsymbol{K}} = 0 \quad (18)$$

with

$$\begin{aligned} \boldsymbol{P}_{KK} &= E\{\hat{\boldsymbol{i}}^*(k+1)\hat{\boldsymbol{i}}^*(k+1)^T\} - \\ &- E\{\hat{\boldsymbol{i}}^*(k+1)\}E\{\hat{\boldsymbol{i}}^*(k+1)\}^T \quad (19) \\ & \quad (20) \end{aligned}$$

Assuming $\hat{\boldsymbol{i}}'$ and $\hat{\boldsymbol{i}}$ uncorrelated and applying a straightforward differential calculus approach utilizing

$$\frac{d\,(tr(\boldsymbol{AB}))}{d\boldsymbol{A}} = \boldsymbol{B^T} \quad \boldsymbol{A}, \boldsymbol{B} \text{ square}$$

$$\frac{d\left(tr(\boldsymbol{ACA^T})\right)}{d\boldsymbol{A}} = 2\boldsymbol{AC} \quad \boldsymbol{C} \text{ symmetric}$$

we find for the blending factor

$$\boldsymbol{K} = \frac{2(\boldsymbol{Q^T} - \bar{\boldsymbol{i}}'\bar{\boldsymbol{i}}'^{\boldsymbol{T}}) + \bar{\boldsymbol{i}}'\bar{\boldsymbol{i}}^{\boldsymbol{T}} + \bar{\boldsymbol{i}}\bar{\boldsymbol{i}}'^{\boldsymbol{T}}}{2(\boldsymbol{Q} + \boldsymbol{R} - (\bar{\boldsymbol{i}}' - \bar{\boldsymbol{i}})(\bar{\boldsymbol{i}}' - \bar{\boldsymbol{i}})^{\boldsymbol{T}})} \quad (21)$$

TABLE I

TEST CAMPAIGN'S PARAMETER SETTING IN THE CITY OF VIENNA

Parameter	Description	Value
α	reciprocal difference speed time constant	5
V_{max}	maximal speed increase due to perturbation	1.5 meter/s
$-V_{max}$	maximal speed decrease due to perturbation	1.5 meter/s
P_{max}	probability of maximal speed increase	0.1
P_0	probability of no perturbation	0.6
$\|um\|$	mean speed of moving user	1.5 meter/s
σ_{ym}^2	variance of forward/backward moving user	0.25
σ_{u0}^2	variance of motionless user	0.0025
w	proportion of motionless user	0.1
T	time period between measured fingerprints	3s

Fig. 4. Map of test area in the city of Vienna. Source of the map: [10]

with

$$\bar{\boldsymbol{i}} = E\{\boldsymbol{i}\}, \quad \bar{\boldsymbol{i}'} = E\{\boldsymbol{i}'\}$$

$$\text{and} \quad \boldsymbol{Q} = E\{\boldsymbol{i}'\boldsymbol{i}'^T\}, \quad \boldsymbol{R} = E\{\boldsymbol{i}\boldsymbol{i}^T\}.$$

The time propagated measurement $\hat{\boldsymbol{i}}'(k+1)$ can now be updated according to (17) and we receive a final estimator for the position at time $k+1$. The same procedure can easily be applied for following time periods. It has to be noted however that we compute for every time step sums of random variables which involves a convolution. A simple tracking will thus be inefficient in terms of computational effort. For the improvement of position estimates however, where only a few time steps are considered the method is suitable.

To test our method we use a heavy urban area in the downtown area of Vienna. A map is shown in Fig. 4. For the initial training of the Bayesian networks to perform the single position estimate we use 10 samples per position. For the localization we choose the time period T between two consecutive measurements to be 3 seconds to allow a pedestrian to move at least several meters. For a sum of all parameters chosen refer to Tab. I. The resulting accuracy is shown in Fig. 5 (solid line). The error $e = \|\hat{\boldsymbol{i}}^* - \boldsymbol{i}\|$ is defined as the difference between the true and the estimated position. We can see, that the 90% margin is below an error of 70

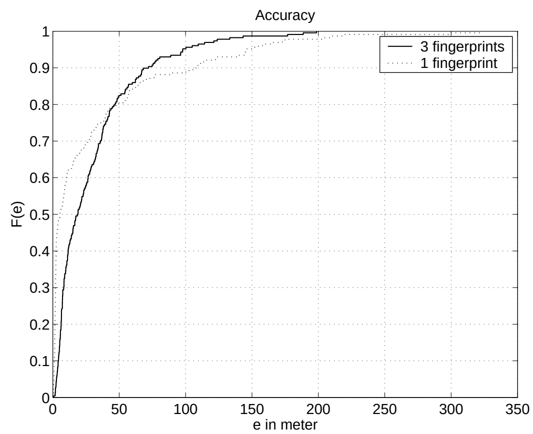


Fig. 5. Accuracy of the method utilizing one single fingerprint (dashed) and three consecutive fingerprints (solid). Total sample size: 280

meters compared to more than 100 meters for the single estimation case (dashed line). On the other side, due to the combination of several position estimates, positioning errors up to about 50 meters are more likely to occur. The main achievement however is the reduction of outliers which classifies the method to be suitable for most location based services, especially if a deployment in densely populated heavy urban areas is intended.

V. Conclusions

In this paper we have presented a method to improve the accuracy of a simple pattern matching based position estimate by applying a motion model and combining several consecutive fingerprints. We have then applied the method to trail measurements taken in the city of Vienna and have achieved an accuracy of about 70 meters in 90% of all cases and less than 40 meters in 67% of all cases. Limitations to the method apply if the target user is very slowly moving and the underlying localization method show the same probability densities of the positions for all three consecutive fingerprints. In this case no accuracy improvement can be expected.

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References

[1] S. Ahonen and H. Laitinen, “Database correlation method for UMTS location,” *IEEE Vehicular Technology Confererence*, 2003.

[2] H. Laitinen, J. Lähteenmäki, and T. Nordström, “Database correlation method for GSM location,” *IEEE VTC 2001 Spring Conf.*, May 2001.

[3] H. Kunczier and H. Anegg, “Enhanced cell id based terminal location for urban area location based applications,” *to be presented at IEEE Consumer Communications and Networking Conference*, January 2004.

[4] D. Hong and S. S. Rappaport, “Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonpriorirized handoff procedures,” *IEEE Transactions On Vihicular Technology*, vol. 35, no. 3, August 1986.

[5] D. Heckerman, D. Geiger, and D. M. Chickering, “Learning bayesian networks: The combination of knowledge and statistical data,” Microsoft Research, Advanced Technology Division, Microsoft Corporation, One Microsoft Way, Redmond, WA 98052, Technical Report MSR-TR-94-09, 1995.

[6] R. G. Cowell, A. P. Dawid, S. L. Lauritzen, and D. J. Spiegelhalter, *Probabilistic Networks and Expert Systems*, ser. Statistics for Engineering and Information Science. 175 Figth Avenue, New York, NY 10010, USA: Springer Verlag New York, Inc., 1999.

[7] D. Helbing, “A mathematical model for the behavior of pedestrians,” *Behavioral Science*, vol. 36, pp. 298–310, 1991.

[8] R. A. Singer, “Estimating optimal tracking filter performance for manned maneuvering targets,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-6, no. 4, July 1970.

[9] P. S. Maybeck, *Stochastic Models, Estimation, and Control*, ser. Mathematics in Science and Engineering. Academic Press New York, 1979, vol. 1.

[10] Wien-Grafik Redaktion, “Stadtplan mit Adressensuche,” www.wien.at ©1995-2001 wien.at: Magistrat der Stadt Wien, Rathaus, A-1082 Wien, 2003.