Sample Size and Precision in Communication Performance Measurements

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SAMPLE SIZE AND PRECISION IN COMMUNICATION PERFORMANCE MEASUREMENTS

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This report describes an interactive computer program that facilitates efficient measurement of communication system performance parameters. The program performs three primary functions: (1) determines the minimum sample size required to achieve a desired precision in estimating delay, rate, or failure probability parameters; (2) analyzes measurement results to determine the precision achieved; and (3) tests independent sets of measured data for statistical homogeneity. The report discusses statistical concepts underlying the program, shows how each function is performed, and provides comprehensive program documentation in the form of mathematical formulas, structured design diagrams, and the program listing. The program was written specifically to facilitate measurement of the performance parameters defined in a newly approved Data Communication Standard, American National Standard X3.102. The statistical techniques implemented in it may also be applied to any other delay, rate, or failure probability measurement. The program is written in ANSI (1977) standard FORTRAN to enhance its portability. It is available from the author at duplication cost.

Key words: American National Standards; communication system; performance parameters; sample sizes; statistical analysis

1. INTRODUCTION

Rapid growth of distributed computing and the trend toward competition and deregulation in the U.S. telecommunications industry have created a need to uniformly specify and measure the performance of data communication services as seen by the end user. Over the past several years, the Federal government and industry organizations have been working together to meet that need through the development of user-oriented, system-independent performance parameters and measurement methods. Their results are being promulgated

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within the Federal government in the form of Federal Telecommunication Standards and in industry in the form of American National Standards.

Two related data communication performance standards have been developed. The first specifies a set of user-oriented performance parameters. That standard was approved as Interim Federal Standard 1033 in 1979 and has subsequently been adapted for proposal as an American National Standard by a task group of the American National Standards Institute (ANSI Task Group X3S35). The proposed ANSI standard, designated X3.102, was formally approved by ANSI's Board of Standards Review in February of 1983 (ANSI, 1983). It is expected to replace Interim Federal Standard 1033, probably as a mandatory Federal Standard. During its trial period, Interim 1033 was applied successfully in several Federal procurements of public packet switching services.

The second standard, proposed Federal Standard 1043, specifies uniform methods of measuring the standard performance parameters. An initial 1043 draft was completed in 1980 and an ANSI adaptation, designated X3S35/135, is expected to be completed in 1984. It will follow a review and approval path similar to that of Interim 1033.

American National Standard (ANS) X3.102 and its Federal counterpart are unique in providing a set of performance parameters that may be used to describe any digital communication service, irrespective of features such as topology and control protocol. Because the performance parameters are system--independent, they are useful in relating the performance needs of data communications users to the offered services. The measurement standard will exploit this property by enabling users to compare performance among competing services.

Table 1a summarizes the 21 user-oriented performance parameters defined in ANS X3.102. The parameters express performance relative to three primary communication functions: access, user information transfer, and disengagement. These functions correspond to connection, data transfer, and disconnection in connection-oriented services, but are also applicable to connectionless services (e.g., electronic mail). They divide a data communication session according to the user's perception of service and provide a structure for performance description.

In defining the parameters, each function was considered with respect to three categories of results: successful performance, incorrect performance, and nonperformance. These possible results correspond closely with the three

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Table 1. Summary of ANS X3.102 Performance Parameters.

a. Organization by function and performance criterion.

		PERFORMANCE CRITERION			PERFORMANCE TIME ALLOCATION
		SPEED	ACCURACY	RELIABILITY	
FUNCTION	ACCESS	ACCESS TIME	INCORRECT ACCESS PROBABILITY	ACCESS DENIAL PROBABILITY ACCESS OUTAGE PROBABILITY	USER FRACTION OF ACCESS TIME
	USER INFORMATION TRANSFER	BLOCK TRANSFER TIME	BIT ERROR PROBABILITY BIT MISDELIVERY PROBABILITY EXTRA BIT PROBABILITY BLOCK ERROR PROBABILITY BLOCK MISDELIVERY PROBABILITY EXTRA BLOCK PROBABILITY	BIT LOSS PROBABILITY BLOCK LOSS PROBABILITY	USER FRACTION OF BLOCK TRANSFER TIME
		USER INFORMATION BIT TRANSFER RATE	TRANSFER DENIAL PROBABILITY		USER FRACTION OF INPUT/OUTPUT TIME
	DISENGAGEMENT	DISENGAGEMENT TIME	DISENGAGEMENT DENIAL PROBABILITY		USER FRACTION OF DISENGAGEMENT TIME

Legend:

☐ Primary Parameters

▨ Ancillary Parameters

b. Organization by function and performance parameter type.

		PERFORMANCE PARAMETER TYPE		
		DELAY (IF COMPLETED)	RATE (IF COMPLETED)	FAILURE PROBABILITY
FUNCTION	ACCESS	• ACCESS TIME • USER FRACTION OF ACCESS TIME		• INCORRECT ACCESS • ACCESS OUTAGE • ACCESS DENIAL
	USER INFORMATION TRANSFER	• BLOCK TRANSFER TIME • USER FRACTION OF BLOCK TRANSFER TIME • USER FRACTION OF INPUT/OUTPUT TIME	• USER INFORMATION BIT TRANSFER RATE	• BIT ERROR • BIT MISDELIVERY • EXTRA BIT • BIT LOSS • BLOCK ERROR • BLOCK MISDELIVERY • EXTRA BLOCK • BLOCK LOSS • TRANSFER DENIAL
	DISENGAGEMENT	• DISENGAGEMENT TIME • USER FRACTION OF DISENGAGEMENT TIME		• DISENGAGEMENT DENIAL

performance criteria most frequently expressed by data communications users: speed, accuracy, and reliability.

One or more primary parameters were defined to express performance of each function-criterion pair. As an example, four primary parameters were defined for the access function: one access-speed parameter (Access Time), one access-accuracy parameter (Incorrect Access Probability), and two access-reliability parameters (Access Denial Probability and Access Outage Probability). Access failures attributable to the user (e.g., called user does not answer) were excluded.

The X3.102 parameters also include four ancillary parameters. Each ancillary parameter relates to a primary speed parameter and expresses the fraction of the performance time attributable to user delays. As an example, the primary parameter, Access Time, normally includes delays attributable to the users (e.g., dialing time, answer time, etc.) as well as delays attributable to the system (e.g., switching time). The ancillary parameter, User Fraction of Access Time, expresses the average fraction of total Access Time that is attributable to user delays. The ancillary parameters remove user influence on the primary speed parameters and allow the entity (user or system) responsible for nonperformance to be identified (e.g., access timeouts).

For statistical estimation, the X3.102 parameters are most naturally classified as: time delay, time rate, and failure probability parameters. This classification is shown in Table 1b. Note that the ancillary performance parameters are classified with the delays.

Figure 1 illustrates the structure of the proposed measurement standard, X3S35/135. The standard is divided into four parts. The first defines a procedure to design experiments to measure the ANS X3.102 parameters. The second specifies functional requirements for extraction of performance data. The third specifies functional requirements for reduction of the data. The fourth specifies methods of analyzing and reporting the ANS X3.102 performance data.

1.1 Purpose of Report

The major purpose of this report is to specify and implement statistical procedures to be used with the experiment design and test data analysis (parts 2 and 4 of the proposed measurement standard). The measurement of the performance of a communication system always involves a conflict between precision

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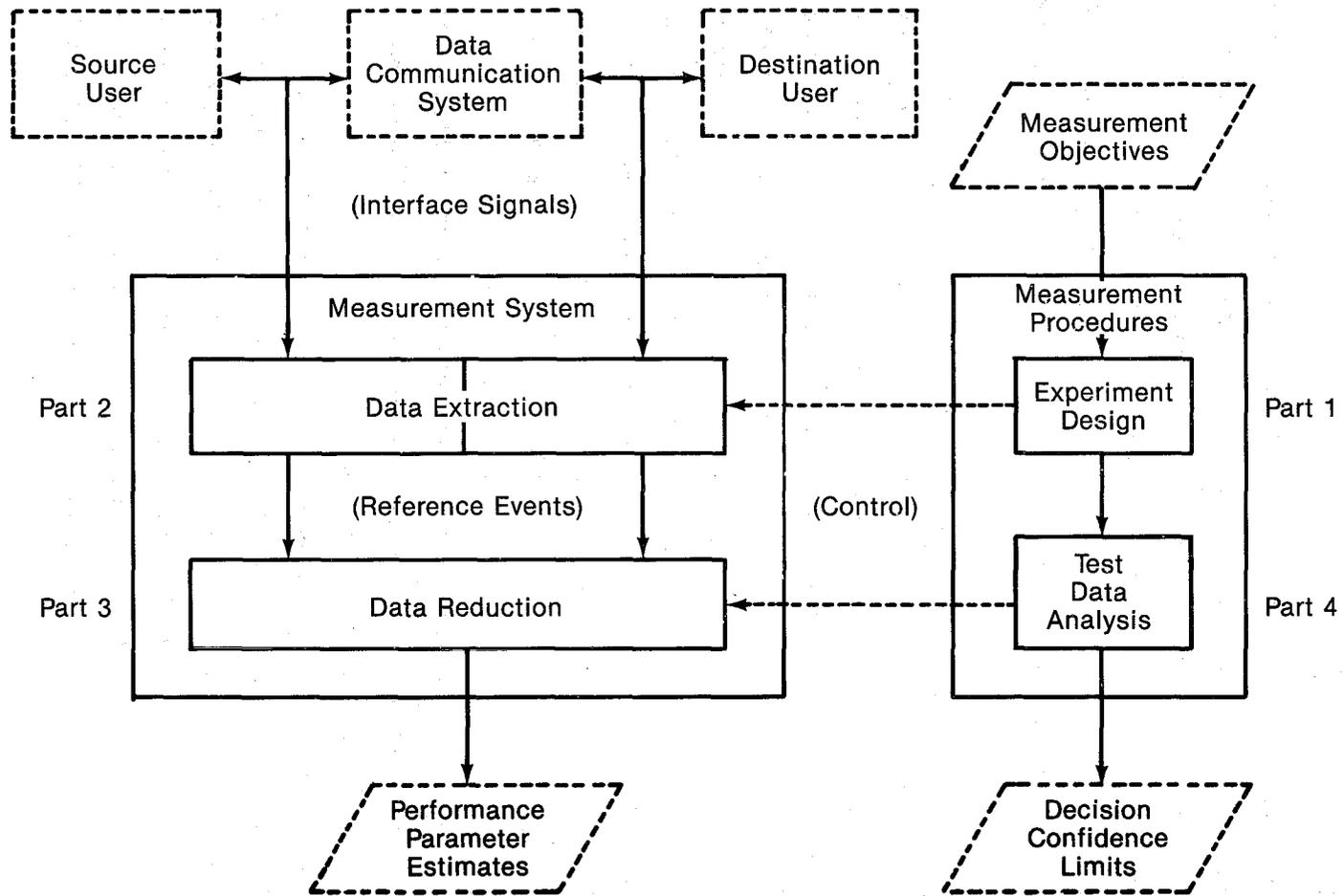
Figure 1. Structure of proposed measurement standard.

and cost. Because the quantities that measure performance are random variables, precision requires many trials, time, personnel, equipment, travel, etc. A practical test should be designed using the theory of mathematical statistics to determine the minimum number of trials required to provide a specified precision.

Historically, the theory of statistics has been developed for and applied to problems in agriculture, biology, etc. Its use in the sampling procedures for communication testing is rather new, and the literature is correspondingly sparse. Hence, the problem of relating measurement precision to sample size is often difficult for communication test engineers. The absence of straightforward statistical procedures for sample size determination and test data analysis can result in either excessive or insufficient testing, no testing at all (because testing is viewed as too costly), or incomplete reporting of test results.

This report describes the design and use of an inter-active computer program that implements such procedures. Statistical theory is used to determine the minimum sample size necessary to achieve a desired precision (from knowledge of the dispersion and the dependence among sample values). The sample is analyzed by calculating its mean value and determining an interval about this estimate within which the true mean can, with a certain level of confidence, be expected to lie.

Often the use of this program will show that a smaller sample size than expected can achieve the desired precision. Costs saved from such information can be substantial. Although developed specifically for the American National Standard X3.102 parameters, this program can be used to estimate values for any delay, rate, or failure probability parameters; the principles are the same. Copies of the program may be obtained from the author at duplication cost.

1.2 Organization of Report

This report is organized in the order in which the statistical procedures would normally be applied in measuring the performance of a communication system. Section 2 discusses some statistical concepts to help the user make decisions about the optimal sample size for his/her tests and interpret the test results. Section 3 discusses sample size determination. Section 4 discusses the analysis of the test data. Very likely more than one test will

be conducted to estimate the value of a performance parameter. If so, the estimate of the population mean should be more precise if the data can be combined (and they can be if they come from the same statistical population). Section 5 discusses the procedure for determining if data from multiple tests come from the same population; it provides the subsequent analysis if they are combined. Appendix A provides some theory and lists the formulas used by the computer program. Appendix B is the set of diagrams of the logic of the main program and each subroutine. Appendix C is the listing of the program. The relationship among these three appendices is documented by equation numbers and subroutine names.

When the program is accessed, the introductory statements in Figure 2 are listed:
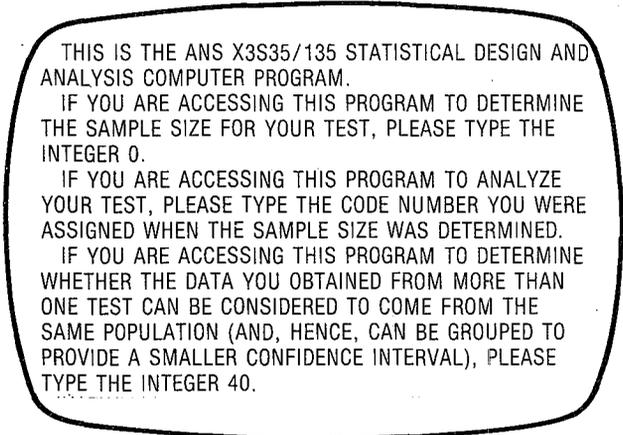
THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
  IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

Figure 2. Introductory message from the computer program.

These statements show how to initiate each of the three procedures of the program: sample size determination, analysis of a test, and analysis of multiple tests. Subsequent statements issued by the program are shown in the appropriate sections.

Figure 3 is a diagram of the operator's interaction with the computer program to determine the sample size for a test and, later, to analyze the test. This diagram shows only those decisions and activities required of the operator. The top part shows how a sequence of three to seven decisions results in one of nine tests (labeled A through I). The bottom part shows how each test is analyzed. For delay and rate parameters, the test results can be entered from a keyboard or a file. In three cases (tests B, E, and H), it is

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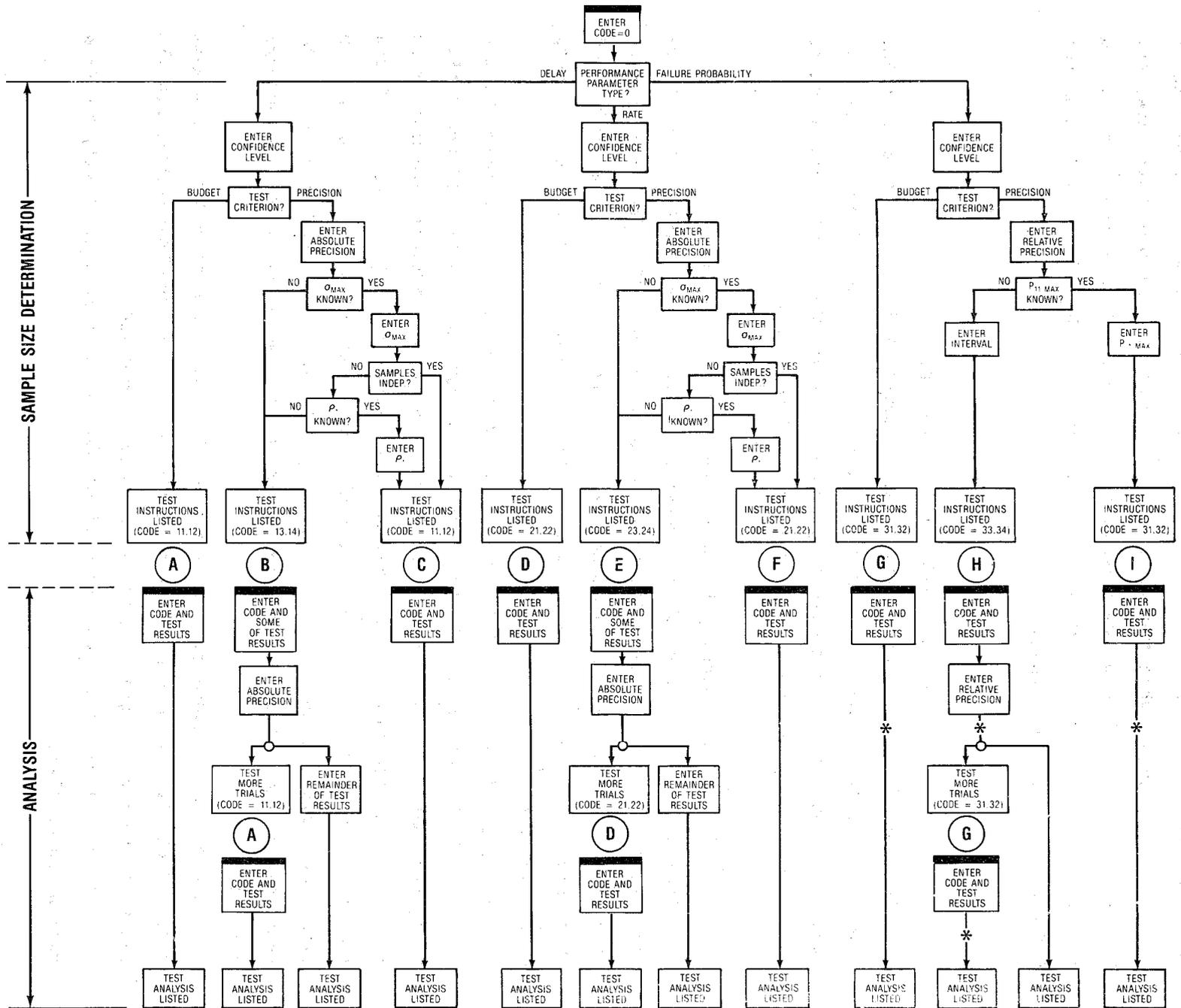
Figure 3. Operator-decision diagram.

possible that the sample size was insufficient, and more data must be obtained. This can happen when the values in the sample varied more than expected. The bar at the top of some boxes means that the program is accessed there, and the small circles mean that the program makes a decision based upon the information previously entered.

2. STATISTICAL CONCEPTS

This section introduces statistical concepts such as populations, their characteristics, precision, confidence levels, confidence limits, etc. A review of these concepts should help answer some questions that are posed by the computer program when the sample size is determined.

2.1 Density Functions and Their Parameters

Drawing conclusions about the general (i.e., the population) from knowledge of the specific (i.e., the observed sample) is called inductive inference. Even though this procedure results in uncertain success, the methods of statistics allow us to measure the uncertainty and reduce it to a known, tolerable level.[1]

The population is the totality of elements that have one or more characteristics to be measured. If the number of elements is finite, the population is said to be real. If the number of elements is infinite, the population is said to be hypothetical. For example, the males in a particular school are elements from a real population, and the access attempts to a communication system are elements from a hypothetical population. The number of elements is said to be countable if they can correspond one-to-one with the natural numbers.

The number of elements observed from the population is called the sample size. Each element in the sample is called a trial or an observation, and the value of the subject characteristic is called the outcome.

If one is to infer something about the population from a sample, care must be taken that:

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[1]In contrast, deductive inference always yields a correct conclusion because it results from a chain of proved conclusions (e.g., mathematical theorems are proved by deductive inference).

Elements in the sample come from the intended (target) population. (Otherwise the conclusions drawn from the sample are probably representative of another population.)

The sample is a random sample (i.e., a sample whose outcomes are independent). For real (finite) populations, a random sample can be obtained only if the elements are replaced after each sampling. For hypothetical populations, the randomness of the sample is not influenced by replacement.

A random variable, X, is a function that assigns a real value to each element in the set of all possible random samples (i.e., the sample space). For most purposes, random variables are either of the discrete type or of the continous type:[2]

Random variables of the discrete type. Suppose the number of values, x_i, is either finite or countable, and the random variable can assume the value x_i (i=1,2,...) with probability $P(X=x_i)=p_i$.

The random variable, X, is of the discrete type if

- $p_i \geqslant 0$ for all i,

and

$$\begin{cases} \sum_{i=1}^{n} p_i = 1 & \text{(for a finite sample of size n)} \\ \\ \sum_{i=1}^{\infty} p_i = 1 & \text{(for a countable sample).} \end{cases}$$

The probability function is $P(X=x_i)=p_i$, and the distribution function is $F(x)=P(X \leqslant x)= \sum_{x_i \leqslant x} p_i$ for all x for which $x_i \leqslant x$.

Random variables of the continous type. A random variable, X, is of the continous type if there exists a function, f, such that

- $f(x) \geq 0,$ for all real x,

and

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[2]Very rarely, they are both discrete and continuous.

$$\cdot F(x) = P(x \leqslant x) = \int_{-\infty}^{x} f(u)\,du.$$

The density function is f(x), and the distribution function is F(x).

The values obtained from a continous sample have a pattern called the sample density. To determine the sample density from such a sample, several steps are required:

1. Order the values.

2. Determine the range of values.

3. Partition the range into appropriate intervals (as indicated by the number and range of values).

4. Record the number of values occurring in each interval.

5. Determine the relative frequency of occurrence in each interval (by dividing the number of values in each subinterval by the total number of values).

The shape of the sample density can then be seen by plotting the relative frequencies for each interval.

EXAMPLE:The following sixteen values have been sampled from a population:

5.1, 7.9, 3.2, 0.9, 1.2, 2.1, 3.3, 6.7, 6.5, 5.1, 6.2, 5.4, 3.4, 6.3, 6.9, and 8.2.

Determine the sample density.

SOLUTION:

1. Order the values:

0.9, 1.2, 2.1, 3.2, 3.3, 3.4, 5.1, 5.1, 5.4, 6.2, 6.3, 6.5, 6.7, 6.9, 7.9, and 8.2.

2. The values range from 0.9 to 8.2.

3. Appropriate intervals seem to be

[0,2), [2,4), [4,6), [6,8), and [8,10).

4. and 5. are demonstrated by Table 2.

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Table 2. Example of a Sample Density

	Step 4	Step 5
Interval	Number of Values	Relative Frequency
[0,2)	2	0.1250
[2,4)	4	0.2500
[4,6)	3	0.1875
[6,8)	6	0.3750
[8,10)	<u>1</u>	<u>0.0625</u>
	16	1.0000

Since sample density functions are composed of relative frequencies, they are non-negative functions that sum to 1. Figure 4 is a graph of this sample density. If the sample had been much larger, the range could have been divided into many smaller intervals. Then the sample density would probably have more nearly approximated the population density.

Population density (or probability) functions may have one or more parameters whose values specify their shape. Similar to a family of curves, the formula of the function may be known, but a different curve exists for each value of the parameter(s). A simple example of a family of curves with two parameters is the family of straight lines,

$$y = ax + b,$$

where the parameter a is the slope and the parameter b is the y-intercept. A different value of either a or b defines a different straight line.

The true (population) value of a parameter can be estimated by a function of the values obtained from a random sample. These functions are called statistics. The most common statistic is the sample mean. Using the previous example, the sample mean is

$$\bar{x} = \frac{1}{16} (5.1 + 7.9 + \ldots + 8.2) = 4.9.$$

When the mean is computed from the entire population, the statistic is called the expected value or, simply, the population mean (usually denoted by μ). Another important statistic is the sample variance. It estimates the dispersion of the population values about the mean. The population variance is
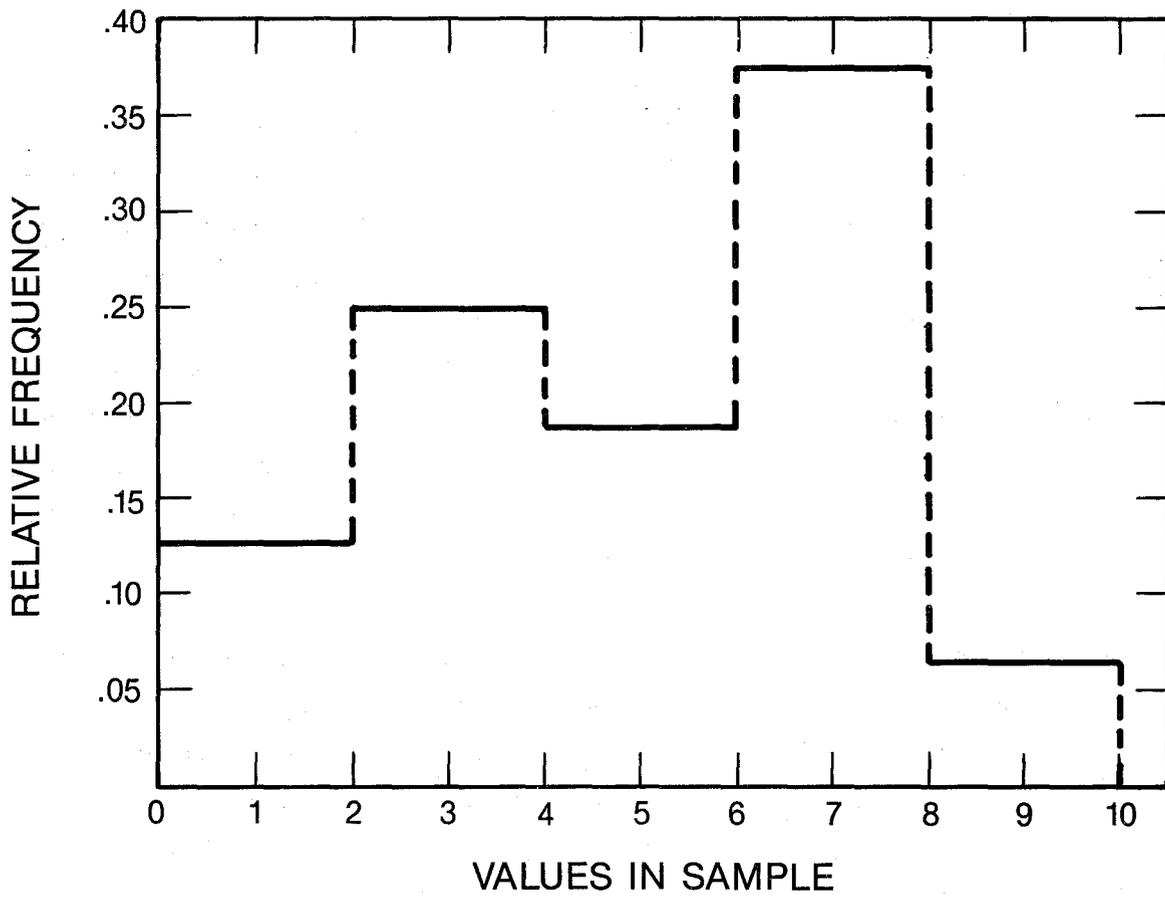
Figure 4. Example of a density.

usually denoted by σ^2; the square root of the variance is called the standard deviation and is denoted by σ.

The normal density is an example of a two-parameter density (whose parameters also happen to be the population mean and variance). Figure 5 shows the normal density function and the gamma density function (both for a random variable of the continuous type). It also shows the binomial probability function and the Poisson probability function (both for a random variable of the discrete type). The figure lists their parameters and their mean and variance (expressed in terms of these parameters).

2.2 Estimating the Population Mean

The random sample can indicate something about the value of the population mean.[3] This can be demonstrated from the following two important observations of sampling:

Suppose we want to obtain a sample for which the sample mean deviates from the population mean by less than a given amount. We can determine the sample size required to bring the probability of this as close to 1 as is desired. Moreover, the required sample size is independent of the shape of the population density function (but dependent on the population variance).

Suppose random samples, each of size n, are repeatedly drawn from a population. After each draw, the sample mean is determined. The set of sample means so obtained have their own sample density (called a sampling density). In fact, regardless of the density of the parent (original) population, the sample mean has approximately the normal density: Its mean equals the parent population mean, but its variance is only 1/n as large as the parent population variance. Hence, as, n, the sample size, becomes larger, the variance of the sample mean becomes proportionally smaller. Thus the sample means are closely clustered about the population mean when n is large. Figure 6 shows a gamma density (with mean 4 and variance 2). The sample mean, obtained from samples drawn from this population, is shown as having a density that is very close to the normal density for samples of size 10, 20, and 30.

These two observations of sampling show that knowledge of the sample mean allows us to infer knowledge of the population mean, and the inference is less uncertain when the population variance is small. However, in order to benefit from these observations, it is necessary to quantify them.

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[3]Provided only that the population has a mean and a finite variance.

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BINOMIAL PROBABILITY FUNCTION

$P(X = k) = \dbinom{n}{k} \; p^k(1-p)^{n-k} \quad k=0, 1, \cdots, n$

Parameters: $1 \leq n$, $o < p < 1$

Mean: $\mu = np$

Variance: $\sigma^2 = np\,(1-p)$

[Plot: $n = 9$, $p = \frac{1}{3}$, $\mu = 3$, $\sigma^2 = 2$; horizontal axis k]

POISSON PROBABILITY FUNCTION

$P(X = k) = \dfrac{\lambda}{k!} \cdot e^{-\lambda} \quad k=0, 1, 2, \cdots$

Parameters: $o < \lambda$

Mean: $\mu = \lambda$

Variance: $\sigma^2 = \lambda$

[Plot: $\lambda = 3$, $\mu = 3$, $\sigma^2 = 3$; horizontal axis k]
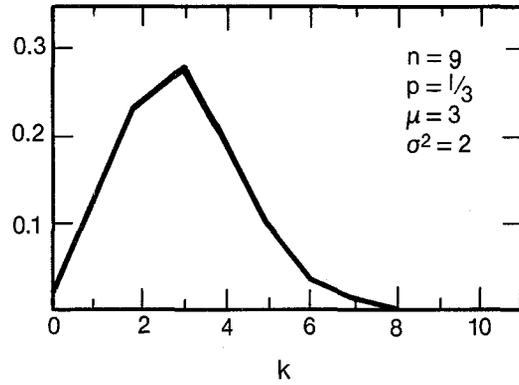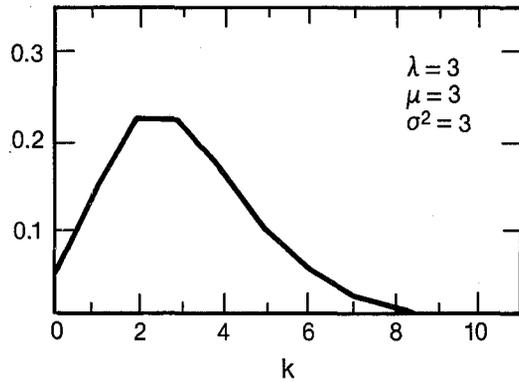
NORMAL DENSITY

$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\dfrac{1}{2} \cdot \left(\dfrac{x - \mu}{\sigma} \right)^2 \right]$

Parameters: μ, σ (positive)

Mean: $\mu = \mu$

Variance: $\sigma^2 = \sigma^2$

[Plot: $\mu = 3$, $\sigma^2 = 2$; horizontal axis x]
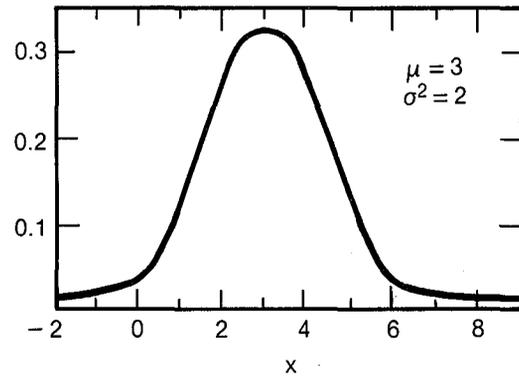
GAMMA DENSITY

$f(x) = \begin{cases} \dfrac{b^p}{\Gamma(p)} \cdot x^{p-1} e^{-bx} & 0 \leq x \\[2mm] 0 & \text{Otherwise} \end{cases}$

Parameters: b, p (both positive)

Mean: $\mu = \dfrac{p}{b}$

Variance: $\sigma^2 = \dfrac{p}{b^2}$

[Plot: $p = \dfrac{9}{2}$, $b = \dfrac{3}{2}$, $\mu = 3$, $\sigma^2 = 2$]
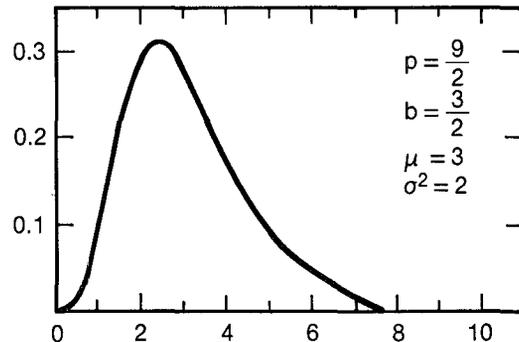
Figure 5. Common probability functions and density functions.

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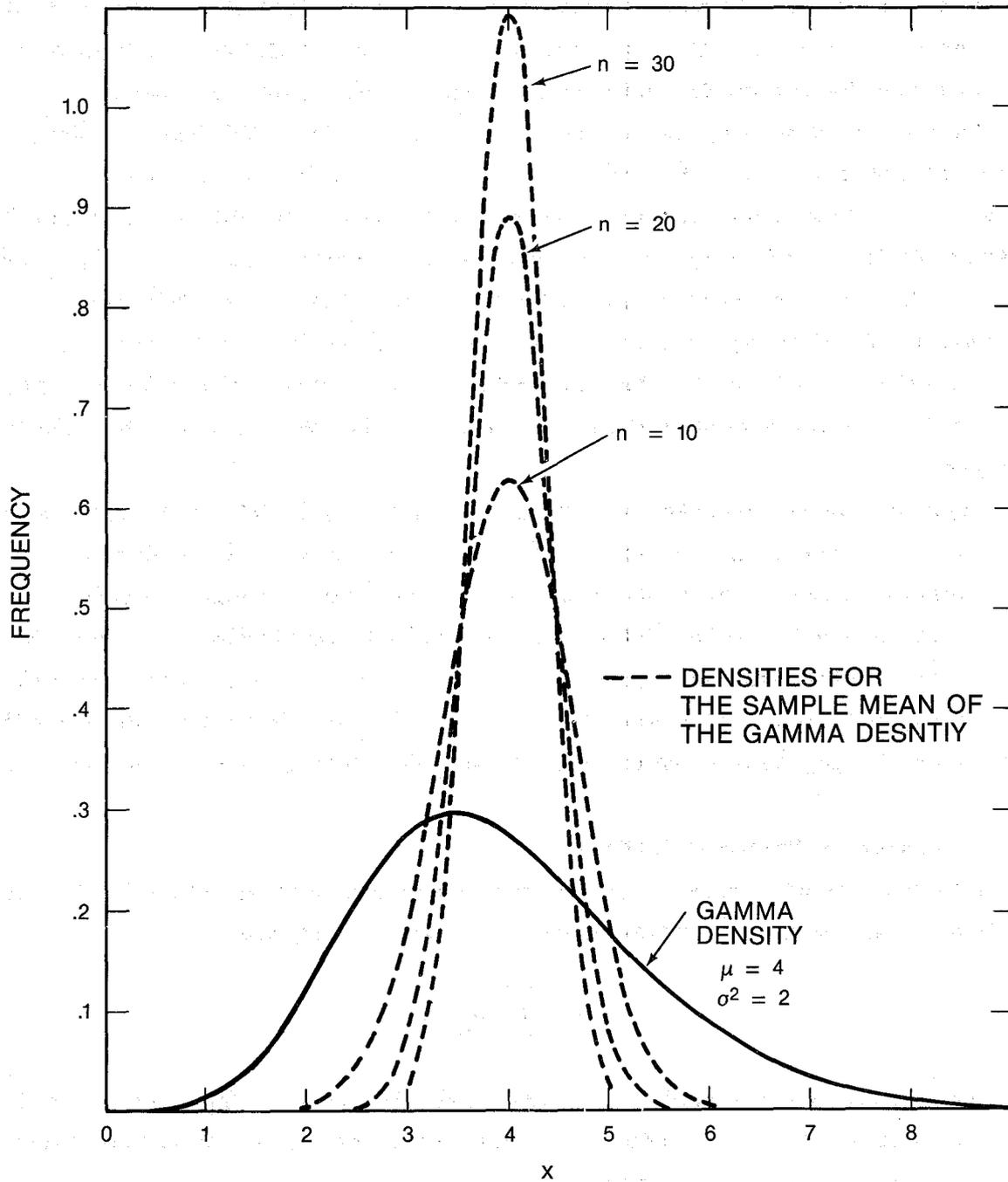
Figure 6. Gamma density and its sample mean densities.

2.3 Confidence Intervals

The estimate of the mean should be accompanied by some interval about the estimate together with some assurance that the population mean is in the interval. Such an interval is called a confidence interval. For example, a confidence interval for the population mean is called a 95% confidence interval if we can be 95% confident that it contains the population mean.

In theory, many samples are drawn and a confidence interval is determined from each sample. The end points of each confidence interval depend upon the values in the sample. Then we expect that, say, 95% of all the intervals determined from the samples include the population mean (a single fixed point). However, in practice, a single confidence interval is determined from the values in the sample, and we say that we have a certain confidence that the population mean is in that interval. Confidence intervals can also be defined for parameters other than the population mean (e.g., the population variance).

Suppose random samples are obtained from a population (not necessarily normal) with mean μ and variance σ^2. Suppose also that $\bar{x}$ and s^2 are the estimates of these population values obtained from a sample of size n. The confidence interval can be determined for populations having the normal density and for populations having a non-normal density if the number of samples is large (say, $n > 30$).[4] The confidence intervals for these two cases are identical when the population variance is known, but differ when it is not known:

2.3.1 Population Variance is Known

From the second observation, above, $\bar{x}$ has approximately a normal density with mean μ and variance σ^2/n. Hence, the random variable,

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad ,$$

has the normal density with mean zero and variance one. This particular normal density is called the standard normal density and is tabulated in virtually every statistics book.

Suppose z_1 and z_2 are the end points of a probability interval for z, and $z_1 < z < z_2$. If we want to determine, say, a 95% confidence interval for μ,

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[4]Provided the density has a mean and a finite variance.

we choose z_1 and z_2 so that 95% of the probability of the density (i.e., 95% of the area under the curve) is in the interval. An infinite number of intervals can be defined that include the specified probability. However, z_1 and z_2 should be chosen so that the interval is shortest, because the shortest interval provides the most information about the value of the population mean. For densities symmetric about the mean, the values of z_1 and z_2 that provide the shortest interval are those that are also symmetric about the mean.

It is customary to refer to the confidence level as a $100(1-2\alpha)$% confidence level because the α, so specified, can be used to relabel z_1 and z_2 in terms of the chosen confidence level. For the standard normal density, $z_1 = -z_\alpha$ and $z_2 = z_\alpha$. For example, a 95% confidence level is defined by $\alpha = 0.025$ (i.e., $100(1-2 \times 0.025)$% = 100×0.95% = 95%). The end points, $-z_\alpha = -z_{0.025}$, and $z_\alpha = z_{0.025} = 1.960$, exclude 2.5% of the probability (area) from each tail of the density.

From

$$-z_\alpha < z < z_\alpha$$

we obtain

$$-z_\alpha < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < z_\alpha \quad ,$$

and then

$$\bar{x} - \frac{\sigma}{\sqrt{n}} \cdot z_\alpha < \mu < \bar{x} + \frac{\sigma}{\sqrt{n}} \cdot z_\alpha \quad .$$

This is the $100(1-2\alpha)$% confidence interval for the population mean when the population variance is known. Notice that the length of the interval,

$$\frac{2\sigma}{\sqrt{n}} \cdot z_\alpha \quad ,$$

is fixed because it does not depend upon the values from the sample; however, the end points of the interval will vary because $\bar{x}$ depends upon the values from the sample.

2.3.2 Population Variance Is Not Known

It is somewhat unrealistic to suppose that σ is known. (If it were, μ would also probably be known.) However, if σ is not known, it is reasonable to estimate it by s (determined from a sample of size n).

However, the random variable,

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}},$$

does not have the standard normal density as does z (particularly for small n). This is because s replaces σ in the expression, and s is a random variable that can have different values in different random samples. The density for it was determined by Gosset (under the pseudonym Student). The parameter, n, provides only n-1 degrees of freedom (choices) in determining s because the values in the sample have the constraint that they must also determine $\bar{x}$ (an independent linear relation). Hence, t is said to have Student's t density with n-1 degrees of freedom. The end-point is labelled, $t_{\alpha,n-1}$. This density has the same mean as does the standard normal density (i.e., 0), but, because s is only an estimate of σ, there is more uncertainty, and its variance is larger. However, as n increases, the uncertainty decreases and the variance approaches that of the standard normal (i.e., 1).[5]

For the 100 (1-2α)% confidence level and n-1 degrees of freedom,

$$-t_{\alpha,n-1} < t < t_{\alpha,n-1}.$$

From this inequality we obtain:

$$\bar{x} - \frac{s}{\sqrt{n}} t_{\alpha,n-1} < \mu < \bar{x} + \frac{s}{\sqrt{n}} \cdot t_{\alpha,n-1}.$$

The length of the interval,

$$\frac{2s}{\sqrt{n}} t_{\alpha,n-1},$$

is dependent on s, and the end points are dependent on both $\bar{x}$ and s; hence, both the length and the end points vary with the values from the sample. The formulas for the confidence limits are summarized in Table 3.

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[5] $t_{\alpha,n-1} > z_\alpha$ for all n, and $\lim\limits_{n \to \infty} \left(\dfrac{t_{\alpha,n-1}}{z_\alpha} \right) = 1$. For $\alpha = .025$ and n = 10, $t_{.025,9} = 2.262$, so the ratio is 1.154.

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Table 3. Confidence Limits for the Population Mean Depending
on Either the Density of the Parent Population or the
Number of Random Samples and Knowledge of the Population
Variance

	DENSITY OF THE PARENT POPULATION		
	NORMAL	OTHER $n \geq 30$	OTHER $n < 30$
σ KNOWN	$\bar{x} \pm z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm z_\alpha \cdot \frac{\sigma}{\sqrt{n}}$	(not definable)
σ NOT KNOWN	$\bar{x} \pm t_{\alpha,n-1} \cdot \frac{s}{\sqrt{n}}$	$\bar{x} \pm z_\alpha \cdot \frac{s}{\sqrt{n}}$	(not definable)

2.4 Sample Size and Precision

To determine the confidence limits for the population mean, we can specify any two of the following three quantities, and the remaining quantity will be determined from formulas:

* the confidence level,
* the sample size, and
* the length of the confidence interval.

However, it is customary to specify the confidence level. Then either the sample size or the length of the confidence interval must be specified. The sample size is specified when the budget (i.e., sampling time) is important, and the length of the confidence interval is specified when the precision of the estimate is important.

When precision is important, the required sample size is determined by specifying either the absolute precision or the relative precision:[6]

2.4.1 Absolute Precision

If the population mean is roughly known, say to an order of magnitude, the half-length of the confidence interval can be stated as the maximum acceptable error with which the sample mean estimates the population mean. This error corresponds to the absolute precision. Suppose m_L and m_U are lower and upper confidence limits for μ, and $\bar{x}$ is the sample mean. The absolute error, a, is then

$$a = \frac{m_U - m_L}{2} .$$

In the case of samples from a normal population in which the population standard deviation is known, the length of the confidence interval is

$$m_U - m_L = \frac{2 \sigma z_\alpha}{\sqrt{n}} .$$

---

[6]Mathematical statisticians distinguish between precision and accuracy. Precision describes the closeness of trial values to each other (as measured by the standard error, Equation A-7). Accuracy describes the closeness of an estimate to the population value. Precision and accuracy should be related; however, a high degree of precision can coexist with poor accuracy if the sample is not random or is not drawn from the target population. See Section 2.1.

Then

$$a = \frac{\sigma z_{\alpha}}{\sqrt{n}} \quad ,$$

or

$$n = \left(\frac{\sigma \, z_{\alpha}}{a}\right)^2 \quad .$$

In this case, specifying the length of the confidence interval directly determines the sample size. If the sample is not a random sample, some dependence exists between pairs of observations, and the formula is more complicated.

2.4.2 Relative Precision

If the population mean is not roughly known, it is prudent to require the length of the confidence interval to be proportional to the mean. The half-length, specified in this way, is called the relative error. The percent of relative error, r, is then

$$r = \left(\frac{m_U - m_L}{2 \, \bar{x}}\right) 100\%.$$

For example, if $r = 50\%$, then $m_U - m_L = \bar{x}$. If the density is symmetric about the mean, the interval is centered on the mean, and $m_L = (1/2)\bar{x}$ and $m_U = (3/2)\bar{x}$.

3. SAMPLE SIZE DETERMINATION FOR A TEST

Figure 7 is the operator-decision diagram for sample size determination; it is the top part of Figure 3. As seen in the diagram, nine possible tests result from the sequence of decisions by the operator.[7] The number of decisions required varies from three to seven.

When the computer program is accessed to determine the sample size, the statements in Figure 8 are listed. Since the sample size is to be determined, type the integer zero.

---

[7]Since two confidence levels (90% and 95%) can be selected, there are actually 18 possible tests. However, the tests resulting from this selection (see Figure 10) differ only in the required sample size, not in the method of analyzing the data.

Figure 7. Operator-decision diagram for sample size determination.
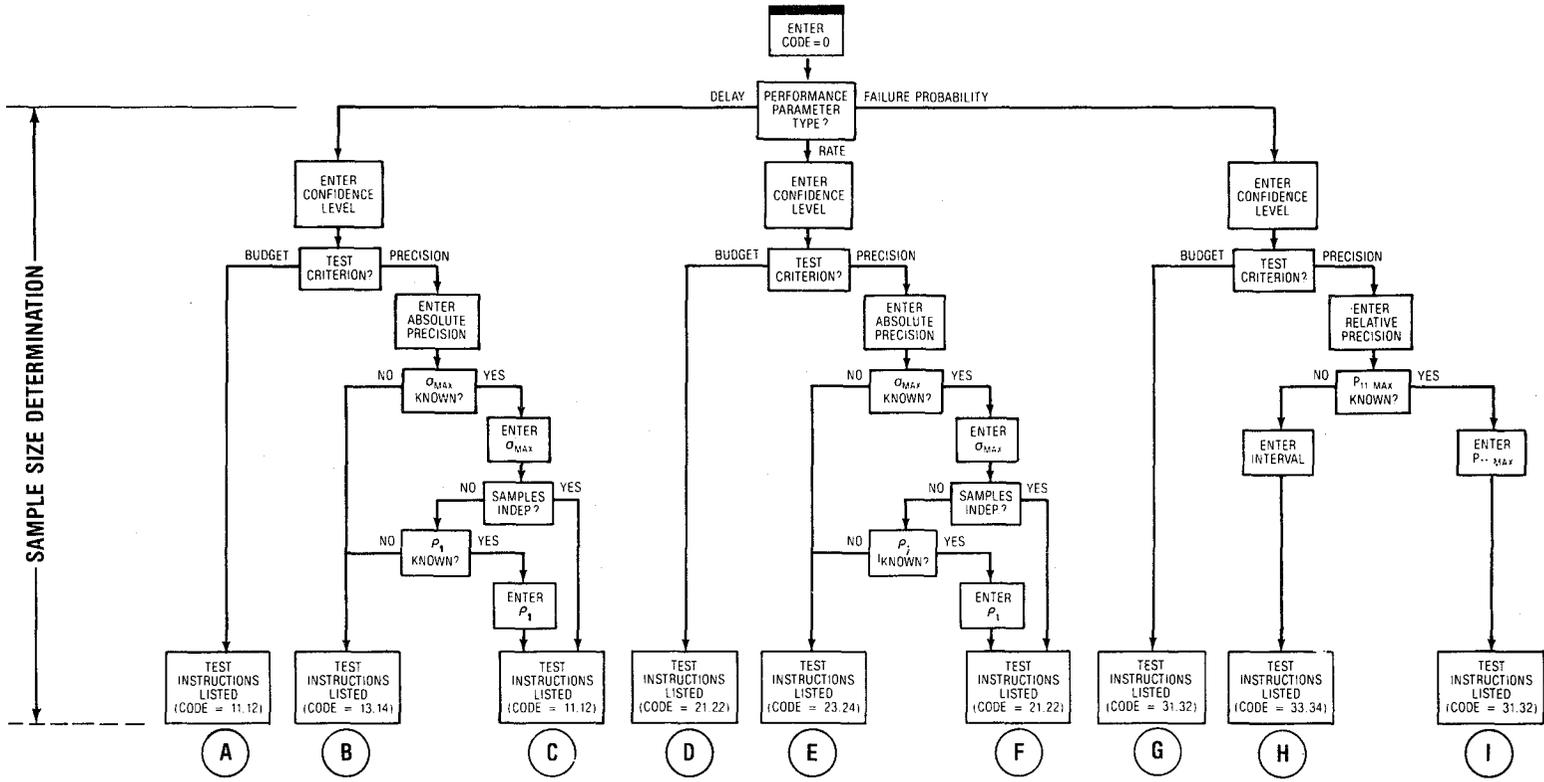
23

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND
ANALYSIS COMPUTER PROGRAM.
    IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE
THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE
INTEGER 0.
    IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE
YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE
ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
    IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE
WHETHER THE DATA YOU OBTAINED FROM MORE THAN
ONE TEST CAN BE CONSIDERED TO COME FROM THE
SAME POPULATION (AND, HENCE, CAN BE GROUPED TO
PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE
TYPE THE INTEGER 40.

Figure 8.   Introductory message from the computer program.

Then select the type of performance parameter to be tested. All perform-
ance parameters fall into one of three types: delays, rates, and failure
probabilities   The statements in Figure 9 show how the type of performance
parameter is indicated.

    YOU CAN TEST THE SYSTEM WITH RESPECT
TO
    1. DELAYS,
    2. RATES,
OR
    3. FAILURES
    PLEASE TYPE THE INTEGER LISTED AT THE
LEFT OF THE TYPE OF PARAMETER THAT YOU
WISH TO ANALYZE.

Figure 9.   Performance parameter selection.

Although delays and rates are different types of parameters, the sample
size for each is determined in the same way; both are referred to as time
parameters.

The statements in Figure 10 instruct the operator to select a confidence
level:

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```
     THE PERFORMANCE OF THE PARAMETER
     THAT YOU SELECTED CAN BE MEASURED TO
     PROVIDE ONE OF THE FOLLOWING LEVELS OF
     CONFIDENCE:
       1. 90%
       2. 95%
       PLEASE TYPE THE INTEGER LISTED AT THE
     LEFT OF THE CONFIDENCE LEVEL THAT YOU
     HAVE SELECTED.
```
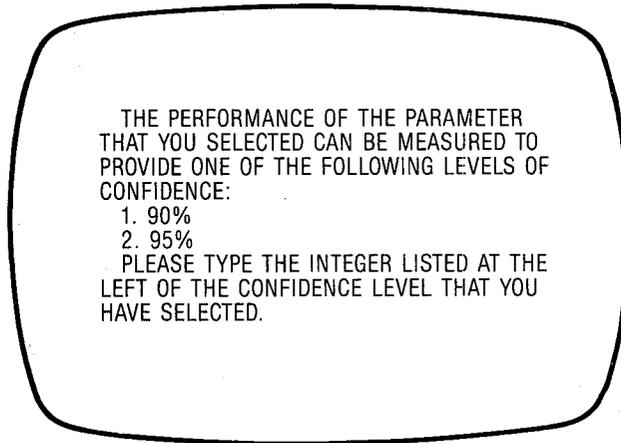
Figure 10   Confidence level selection.

The next two sections show how the sample size is determined for the time
and failure probability performance parameters.

3.1   Time Parameters

The two time parameters are the delay and rate parameters. The sample
size required for both is determined from the same random variable, the time
to accomplish. For delays the time to accomplish is the delay itself. For
rates the time to accomplish is the input/output time; the rate is then the
number of elements (such as bits) transferred divided by the input/output
time. Each trial in the sample results in an input/output time and a number
of bits transferred. An individual trial is called a "transfer sample" in ANS
X3.102. Figures 11 and 12 are the operator-decision subdiagrams of Figure 3
(or Figure 7). They describe the procedure of sample size determination for
delays and rates, respectively. The text in these two figures is the text
that is displayed or printed during execution of the computer program. It can
be seen that the procedure for delays and rates is identical; hence, it is
convenient to discuss the sample size determination for delays and mention
rates only parenthetically. The sequence of decisions results in one of three
tests for delays: A, B, and C (D, E, and F for rates).

To begin, the test criterion is the operator's decision to test a sample
whose size is determined by either budget or precision. If the sample size is
determined by budget, no further decision is required, and the computer pro-
gram lists the test instructions. This test is called test A (test D for
rates).

25
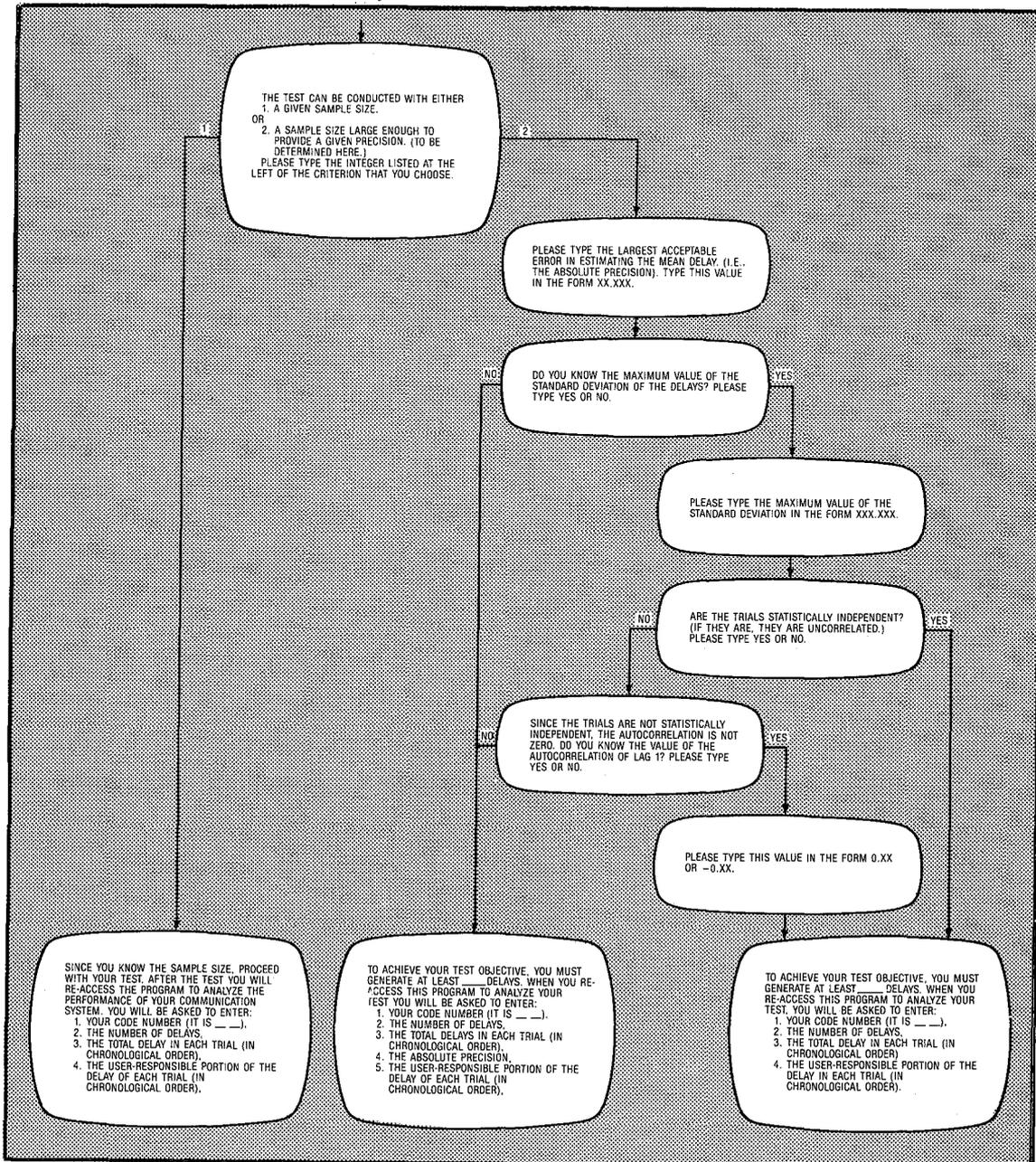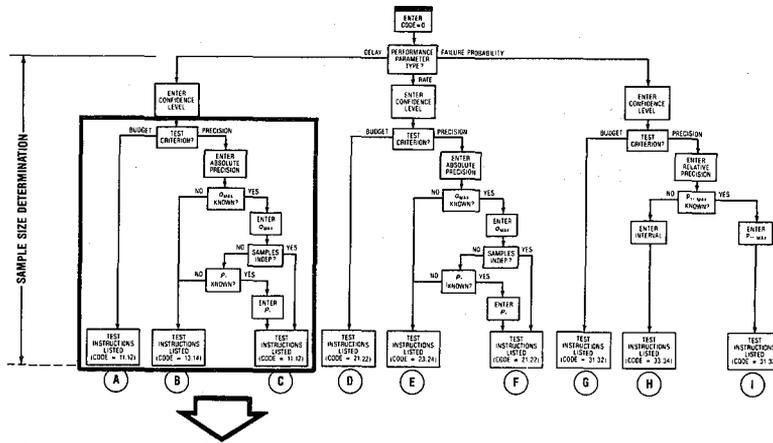
Figure 11. Program messages for sample size determination for delays.
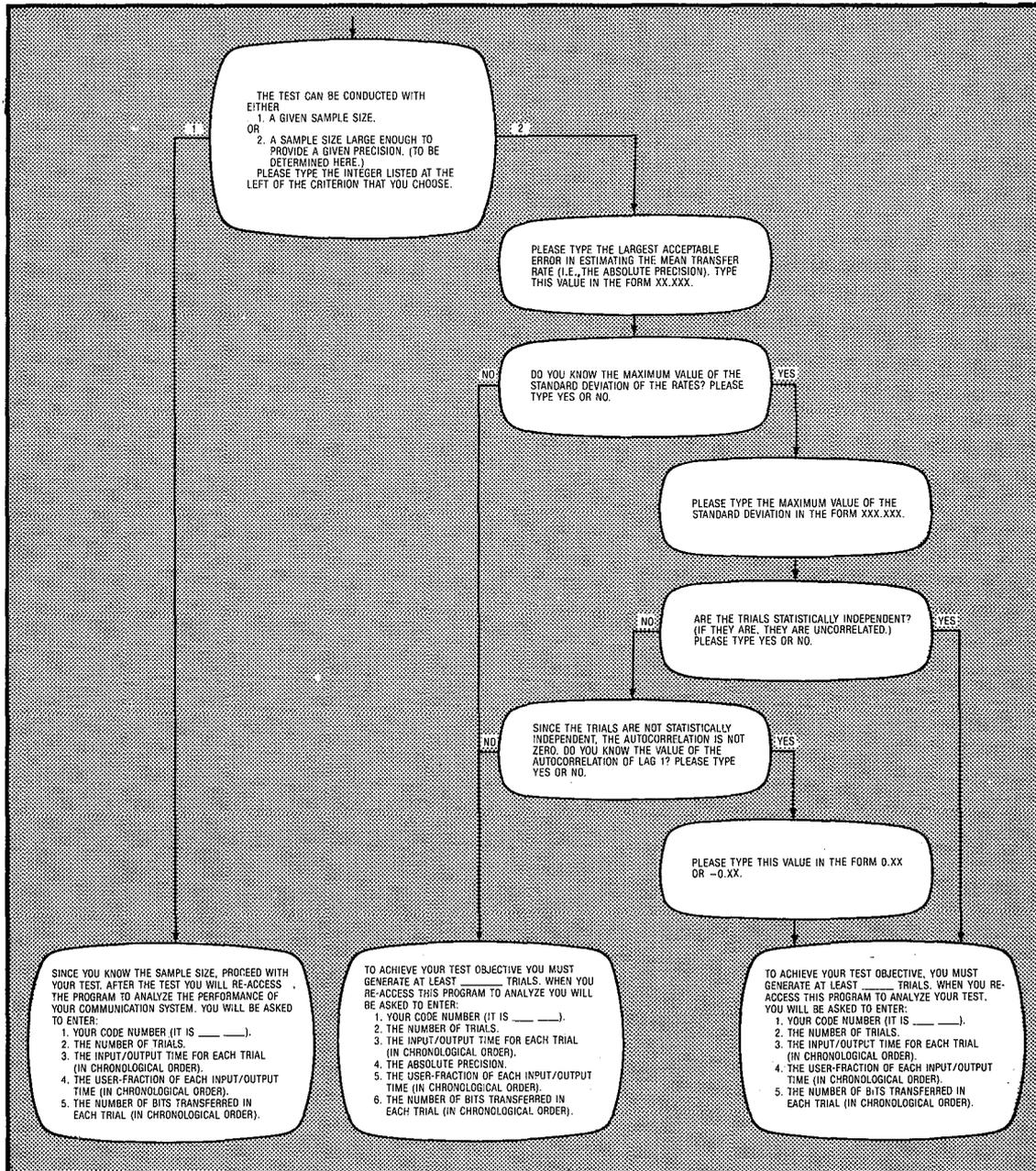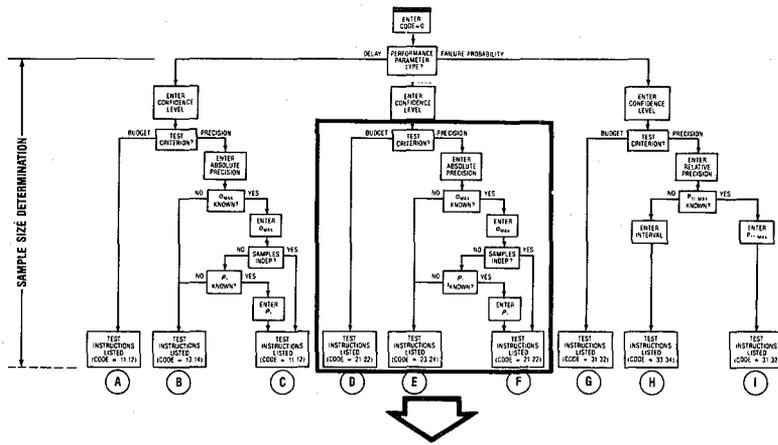
Figure 12. Program messages for sample size determination for rates.

On the other hand, if the sample size is to be large enough to provide a given precision (with a specified confidence level), it is necessary to specify the largest acceptable error in estimating the true mean delay (rate). This error corresponds to the absolute precision (see Section 2.4.1).

The next statement asks if the maximum value of the population standard deviation is known. As mentioned earlier, the standard deviation (the square root of the variance) is a measure of the dispersion of the values. If x_1, x_2, ..., x_n are values from a sample of size n, and $\bar{x}$ is the sample mean, the unbiased estimate of the population standard deviation is

$$s = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right]^{1/2} .$$

(If all outcomes are equal, the standard deviation is zero.) If the maximum value of the population standard deviation is not known, the instructions are listed for test B (test E for rates). If the maximum value is known, that value is to be entered.

Then the program asks if the trials are statistically independent. Two trials are statistically independent if the previous outcome of one trial has nothing to do with the probable outcome of the other. Strictly, consider the events A and B. Suppose the probability that A will occur is P(A), and the conditional probability that A will occur, given that B has occurred, is P(A|B). Events A and B are statistically independent if

$$P(A|B) = P(A) .$$

If the trials are statistically independent, the instructions for test C (test F for rates) are listed.

If they are not statistically independent, some serial dependence exists among the trials. Trials that occur close in time or space are more likely to be either more similar or more dissimilar than are trials that are not close. Hence, if serial dependence exists it probably occurs between adjacent trials (i.e., trials of lag 1).

Serial dependence for lag k=1, 2,..., n-1 is measured by the sample autocorrelation function,

$$r_k = \frac{\sum\limits_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum\limits_{i=1}^{n} (x_i - \bar{x})^2} .$$

This function can have a value between -1 and 1. In the case of dependence, the function could assume a value near these extremes. But, in the case of independence, the value will tend to be near zero. Serial dependence for lag k can be observed by plotting, in the x-y plane, the n-k points whose coordinates are $x_i = x_i$ and $y_i = x_{i+k}$. Positive autocorrelation is indicated by points that tend to approximate a curve with a continuously positive slope. Negative autocorrelation is indicated by points that tend to approximate a curve with a continuously negative slope; and zero autocorrelation is indicated by points that have no systematic tendency.

If the autocorrelation of lag 1 is known, at least approximately, this value should be entered; then the instructions for test C (test F for rates) are listed. Otherwise, instructions for test B (test F for rates) are listed.

Instructions for tests A, B, and C (tests D, E, and F for rates) consist of a list of information that will be required when the computer program is re-accessed to analyze the test results.[8] Notice that test B (test E for rates) resulted because not much was known about the population; hence, it is possible that the suggested sample size is too small to achieve the desired precision. If so, the analysis portion of the computer program will determine the number of additional trials required to achieve this precision.

The code numbers required for re-access are either 11 or 13 (21 or 23 for rates) for the 90% confidence level and either 12 or 14 (22 or 24 for rates) for the 95% confidence level. The computer program is designed to analyze as many as 200 trials.

EXAMPLE: Determine the sample size necessary to estimate the mean access time within 0.7 seconds and be 90% confident of this. Assume that the maximum value of the standard deviation of the delays is known to be 1.5 seconds, and the trials are statistically independent.

SOLUTION:

1. Gain access to the computer program.

2. Type 0 (the code number to determine the sample size), and

---

[8]If the test results will be entered through a file (instead of through a keyboard) the number of delays (or transfers) needn't be entered as indicated in Item 2 of the test instructions of Figure 11 (or Figure 12).

press the return key.

3. Type 1 (for delays), and press the return key.

4. Type 1 (for the 90% confidence level), and press the return key.

5. Type 2 (to obtain precision), and press the return key.

6. Type 0.7 (the largest acceptable error), and press the return key.

7. Type YES (the maximum standard deviation is known), and press the return key.

8. Type 1.5 (the maximum value), and press the return key.

9. Type YES (the trials are statistically independent), and press the return key.

Now, the following instructions are listed for the test:

"To achieve your test objective, you must generate at least 13 delays. When you re-access this program to analyze your test, you will be asked to enter: 1. Your code number (It is 11).

2. The number of delays.

3. The total delay in each trial (in chronological order).

4. The user-fraction of the delay in each trial (in chronological order)."

Sections 4 and 4.1 discuss analysis of the test results for time parameters. The analysis part of this example, which resulted in Test C, is shown at the end of Section 4.1.

3.2 Failure Probability Parameters

Figure 13 is an operator-decision subdiagram of Figure 3. It shows the sequence of decisions that results in one of three possible tests of failure probability (Tests G, H, and I).

If the sample size is dictated by budget or time, the test instructions are listed for Test G.

If failure probability is to be tested for a given precision, the desired relative precision must be specified (see Section 2.4.2).

Suppose P is the probability of failure on a particular trial, and P_{11} is the conditional probability of a failure given that a failure occurred in the previous trial. Then,

- $P_{11} > P$ means that failures and successes tend to cluster,

30

The flowchart contains the following labeled elements:

Top flowchart (SAMPLE SIZE DETERMINATION):

ENTER CODE=0

DELAY / PERFORMANCE PARAMETER TYPE? / FAILURE PROBABILITY

RATE

ENTER CONFIDENCE LEVEL

BUDGET / TEST CRITERION? / PRECISION

ENTER ABSOLUTE PRECISION

NO / q_{max} KNOWN? / YES

ENTER q_{max}

NO / SAMPLES INDEP? / YES

NO / p_c KNOWN? / YES

ENTER p_c

TEST INSTRUCTIONS LISTED (CODE = 11,12)   (A)

INSTRUCTIONS LISTED (CODE = 13,14)   (B)

TEST INSTRUCTIONS LISTED (CODE = 11,12)   (C)

TEST INSTRUCTIONS LISTED (CODE = 21,22)   (D)

TEST INSTRUCTIONS LISTED (CODE = 23,24)   (E)

TEST INSTRUCTIONS LISTED (CODE = 21,22)   (F)

ENTER CONFIDENCE LEVEL

BUDGET / TEST CRITERION? / PRECISION

ENTER RELATIVE PRECISION

NO / $p_{c,max}$ KNOWN? / YES

ENTER INTERVAL

ENTER $p_{c,max}$

TEST INSTRUCTIONS LISTED (CODE = 31,32)   (G)

TEST INSTRUCTIONS LISTED (CODE = 33,34)   (H)

TEST INSTRUCTIONS LISTED (CODE = 31,32)   (I)

Screen messages:

THE TEST CAN BE CONDUCTED WITH EITHER
1. A GIVEN SAMPLE SIZE.
OR
2. A SAMPLE SIZE LARGE ENOUGH TO PROVIDE A GIVEN PRECISION. (TO BE DETERMINED HERE.)
PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE CRITERION THAT YOU CHOOSE.

PLEASE TYPE THE DESIRED RELATIVE PRECISION AS A PERCENT (I.E., A ONE OR TWO-DIGIT NATURAL NUMBER.)

CAN YOU ESTIMATE THE MAXIMUM VALUE OF THE CONDITIONAL PROBABILITY OF A FAILURE GIVEN THAT A FAILURE OCCURRED IN THE PREVIOUS TRIAL? PLEASE TYPE YES OR NO.

SINCE YOU CANNOT ESTIMATE THE MAXIMUM VALUE OF THE CONDITIONAL PROBABILITY, SPECIFY THE INTERVAL ABOVE THE CONDITIONAL PROBABILITY THAT YOU WANT IT TO EXCEED ONLY 5% OF THE TIME. PLEASE TYPE THIS VALUE IN THE FORM 0.XXX.

PLEASE TYPE THIS DECIMAL VALUE IN THE FORM 0.XXX.

SINCE YOU KNOW THE SAMPLE SIZE, PROCEED WITH YOUR TEST. AFTER THE TEST YOU WILL RE-ACCESS THE PROGRAM TO ANALYZE THE PERFORMANCE OF YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO ENTER:
1. YOUR CODE NUMBER (IT IS __ __),
2. THE SAMPLE SIZE,
3. THE NUMBER OF FAILURES IN THE SAMPLE
4. THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN THE SAMPLE.

TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE AT LEAST ____ FAILURES. AFTER THE TEST YOU WILL RE-ACCESS THIS PROGRAM TO ANALYZE THE PERFORMANCE OF YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO ENTER:
1. YOUR CODE NUMBER (IT IS __ __),
2. THE SAMPLE SIZE,
3. THE NUMBER OF FAILURES IN THE SAMPLE,
4. THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN THE SAMPLE,
5. THE SPECIFIED RELATIVE PRECISION

TO ACHIEVE YOUR TEST OBJECTIVES, YOU MUST GENERATE AT LEAST ____ FAILURES. AFTER THE TEST YOU WILL RE-ACCESS THIS PROGRAM TO ANALYZE THE PERFORMANCE OF YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO ENTER:
1. YOUR CODE NUMBER (IT IS __ __),
2. THE SAMPLE SIZE,
3. THE NUMBER OF FAILURES IN THE SAMPLE,
4. THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN THE SAMPLE,
5. YOUR ESTIMATE OF THE CONDITIONAL PROBABILITY OF A FAILURE, GIVEN THAT A FAILURE OCCURRED IN THE PREVIOUS TRIAL.

Figure 13. Program messages for sample size determination for failure probabilities.

- $P_{11} = P$ means that the trials are independent, and
- $P_{11} < P$ means that failures and successes tend to alternate.

The operator is now asked to estimate the maximum value of the conditional probability, called P_{11max}. Like all probabilities, the conditional probability is between 0 and 1. The larger the estimate of the maximum value of P_{11}, the larger the required sample size. If this value can be estimated, it is entered, and the instructions for Test I are listed.[9]

If the maximum value cannot be estimated, estimate the amount above the conditional probability that the conditional probability is desired to exceed only 5% of the time. This amount is the absolute precision for P_{11}. The instructions for Test H are then listed.

Instructions for Tests G, H, and I list the information that will be required when the computer program is re-accessed to analyze the test results. Notice that Test H results when not much is known about the population; hence, it is possible that the suggested sample size is too small to achieve the desired precision. If so, the analysis portion of the computer program will state the number of additional trials required to achieve this precision.

The re-access code numbers are either 31 or 33 for the 90% confidence level and either 32 or 34 for the 95% confidence level.

EXAMPLE: Determine the sample size necessary to estimate the failure probability within 30% of its true value and be 90% confident of this. Assume that you cannot estimate the maximum value of the conditional probability (of a failure, given that a failure occurred in the previous trial). However, you want the conditional probability to be exceeded by 0.2 only 5% of the time.

SOLUTION:

1. Gain access to the computer program.

2. Type 0 (the code number to determine the sample size), and press the return key.

3. Type 1 (for failure probability), and press the return key.

4. Type 1 (for the 90% confidence level), and press the return key.

5. Type 2 (to obtain precision), and press the return key.

6. Type 30 (the percent of relative precision), and press the return key.

---

[9] The computer program cannot use a value equal to 1.

7. Type NO (since the maximum value of the conditional probability is not known), and press the return key.

8. Type 0.2 (since the conditional probability is to be exceeded by 0.2 with 5% probability), and press the return key.

Now, the following instructions are listed for the test:

"To achieve your test objectives, you must generate at least 17 failures. After the test you will re-access this program to analyze the performance of your communication system. You will be asked to enter:

1. Your code number (It is 33),

2. The sample size,

3. The number of failures in the sample,

4. The number of pairs of consecutive failures in the sample,

5. The specified relative precision."

Sections 4 and 4.2 discuss analysis of the test results for failure probability parameters. The analysis part of this example, which resulted in Test H, is shown at the end of Section 4.2.

4. ANALYSIS OF A TEST

Figure 14 is a subdiagram of Figure 3. It shows the operator-decisions required for analysis of each of the nine tests (labelled A through I). When the program is re-accessed for analysis, the introductory message in Figure 8 is listed again. The test instructions assigned a code number to be entered after the program is re-accessed for analysis of the test results. The code number directs the computer program to the proper formulas to analyze the test data. Table 4 contains code numbers and test labels for each type of performance parameter, the adequacy of the sample size (resulting from the degree of accuracy achieved in predicting the population density), and the confidence level.

It should be noted that the precision obtained from the test can differ from the desired precision. This can happen when

the finite random samples vary among one another,

the population density is not as believed, or

some other assumption, like independence, is violated.

Section 4.1 discusses analysis of the time parameters (delays and rates). Section 4.2 discusses analysis of the failure probability parameters.

34
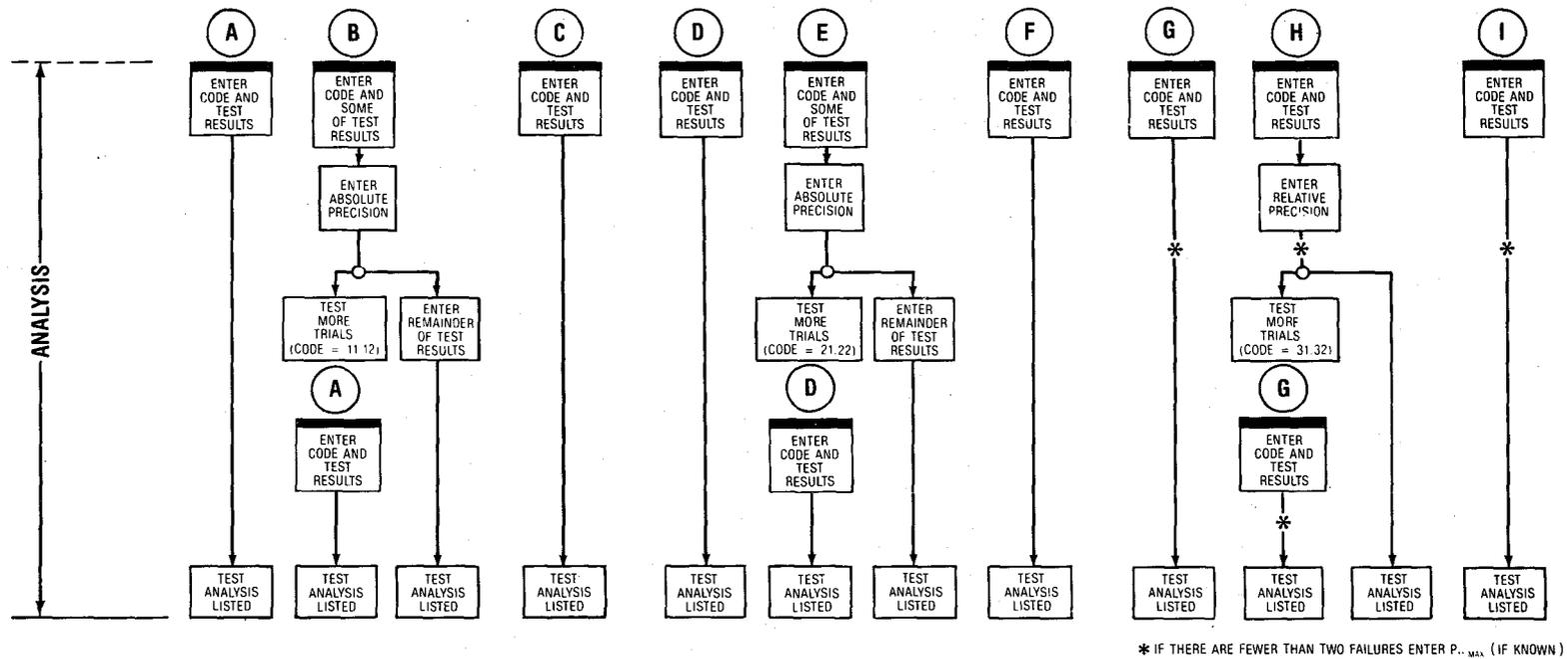
Figure 14. Operator-decision diagram for analysis.

Table 4. Code Numbers and Corresponding Test Labels
Resulting From Sample Size Determination

		Sample Size Known To be Adequate Before Test		Sample Size Not Known To be Adequate Before Test	
		Confidence Level		Confidence Level	
		90%	95%	90%	95%
PERFORMANCE PARAMETERS	Delays	11	12	13	14
		A or C	A or C	B	B
	Rates	21	22	23	24
		D or F	D or F	E	E
	Failure Probability	31	32	33	34
		G or I	G or I	H	H

4.1 Time Parameters

There are three possible tests of delay parameters (tests A, B, and C) and three possible tests of rate parameters (tests D, E, and F). Tests A and D result from specifying a given sample size; Tests B and E result from specifying a desired absolute precision and either not knowing the maximum standard deviation of the delays (input/output times for rates) or realizing some statistical dependence exists but not knowing the autocorrelation of lag 1; and Tests C and F result from specifying a desired absolute precision, knowing the maximum standard deviation, and knowing the trials are statistically independent. Figures 15 and 16 are subdiagrams of the operator-decision diagram for analysis of a test; they show the sequence of events that results in analysis of delays and rates.[10]

Test A

Enter the Following:

- Code Number. Enter 11 if the 90% confidence level was selected when the sample size was determined or 12 if the 95% confidence level was selected.

- Mode of Data Entry. Enter 1 if data are to be entered from a keyboard or 2 if data are to be entered from a file.

□ If data are to be entered from a keyboard, enter:

- Number of Total delays. This is an integer greater than 1. The computer program is designed for as many as 200 total delays.

- Total delay in each trial. This is a positive decimal number in the form XXXXXX.XXX; not all delays can be equal. Enter the delays in chronological order, and include the decimal point.[11]

- User-fraction of the total delay in each trial. The same restrictions apply as for the total delay.[11] The units must be the same as those of the total delay.

□ If data are to be entered from a file, enter:

---

[10]Due to space limitations in Figures 15 and 16, the first response after the introductory messages is not shown. It asks whether test data is to be entered from a keyboard or a file. If it is to be entered from a keyboard, the remaining responses proceed as in these figures. If it is to be entered from a file, the program asks the operator to enter the file name; since this file contains all test data, the remaining response is simply the test results. The symbol is used below to distinguish the two modes of data entry.

[11]Immediately after entering a sequence of numbers, the computer program allows you to correct any number entered in error.

Flowchart (top)

ANALYSIS

A — ENTER CODE AND TEST RESULTS
B — ENTER CODE AND SOME OF TEST RESULTS → ENTER ABSOLUTE PRECISION
C — ENTER CODE AND TEST RESULTS
D — ENTER CODE AND TEST RESULTS
E — ENTER CODE AND SOME OF TEST RESULTS → ENTER ABSOLUTE PRECISION
F — ENTER CODE AND TEST RESULTS
G — ENTER CODE AND TEST RESULTS
H — ENTER CODE AND TEST RESULTS → ENTER RELATIVE PRECISION
I — ENTER CODE AND TEST RESULTS

TEST MORE TRIALS (CODE · 11 '2) — ENTER REMAINDER OF TEST RESULTS
ENTER CODE AND TEST RESULTS

TEST MORE TRIALS (CODE 21 224) — ENTER REMAINDER OF TEST RESULTS
ENTER CODE AND TEST RESULTS

TEST MORE TRIALS (CODE 31 3.4)
ENTER CODE AND TEST RESULTS

TEST ANALYSIS LISTED (repeated across columns)

*IF THERE ARE FEWER THAN TWO FAILURES ENTER P   (IF KNOWN)

Program messages (A)

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE NUMBER OF DELAYS.

PLEASE TYPE THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE USER-FRACTION OF THE DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

YOUR TEST RESULTED IN AN ESTIMATED MEAN DELAY OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN DELAY IS BETWEEN _____ AND _____.
YOUR TEST RESULTED IN AN ESTIMATED MEAN USER-FRACTION DELAY OF _____.
YOU CAN BE ____ PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN _____ AND _____.

Program messages (B)

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE NUMBER OF DELAYS.

PLEASE TYPE THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE AMOUNT OF TIME WITHIN WHICH YOU WISH TO KNOW THE AVERAGE DELAY (I.E. THE ABSOLUTE PRECISION). TYPE THIS VALUE IN THE FORM XX.XXX.

TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE AT LEAST '_____' MORE DELAYS. WHEN YOU RE-ACCESS THIS PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER:
1. YOUR CODE NUMBER (IT IS _____.)
2. THE NUMBER OF DELAYS.
3. THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER).
4. THE USER-FRACTION OF THE DELAY IN EACH TRIAL TIME (IN CHRONOLOGICAL ORDER).

PLEASE TYPE THE USER-FRACTION OF THE DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

YOUR TEST HAS RESULTED IN AN ESTIMATED MEAN DELAY OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN DELAY IS BETWEEN _____ AND _____.
YOUR TEST HAS RESULTED IN AN ESTIMATED MEAN USER-FRACTION DELAY OF _____.
YOU CAN BE ____ PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN _____ AND _____.

Program messages (C)

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE NUMBER OF DELAYS.

PLEASE TYPE THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE USER-FRACTION OF THE DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

YOUR TEST RESULTED IN AN ESTIMATED MEAN DELAY OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN DELAY IS BETWEEN _____ AND _____.
YOUR TEST HAS RESULTED IN AN ESTIMATED MEAN USER-FRACTION DELAY OF _____.
YOU CAN BE ____ PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN _____ AND _____.

Figure 15. Program messages for analysis of delays.

ANALYSIS

A   B   C   D   E   F   G   H   I

ENTER CODE AND TEST RESULTS

ENTER CODE AND SOME OF TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER CODE AND SOME OF TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER ABSOLUTE PRECISION

ENTER ABSOLUTE PRECISION

ENTER RELATIVE PRECISION

TEST MORE TRIALS (CODE = 11,12)

ENTER REMAINDER OF TEST RESULTS

TEST MORE TRIALS (CODE = 21,22)

ENTER REMAINDER OF TEST RESULTS

TEST MORE TRIALS (CODE = 31,32)

A

D

G

ENTER CODE AND TEST RESULTS

ENTER CODE AND TEST RESULTS

ENTER CODE AND TEST RESULTS

TEST ANALYSIS LISTED (× several)

Box D

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
  IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE NUMBER OF TRIALS.

PLEASE TYPE THE INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE USER-FRACTION OF THE INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL. PLEASE PRESS THE RETURN KEY AFTER EACH ENTRY.

YOUR TEST RESULTED IN AN ESTIMATED MEAN INPUT/OUTPUT TIME OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN INPUT/OUTPUT TIME IS BETWEEN _____ AND _____
  YOUR TEST HAS RESULTED IN AN ESTIMATED MEAN USER-FRACTION INPUT/OUTPUT TIME OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN _____ AND _____.
  THE ESTIMATED MEAN TRANSFER RATE IS _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE RATE IS BETWEEN _____ AND _____

Box E

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
  IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE NUMBER OF TRIALS.

PLEASE TYPE THE INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE LARGEST ABSOLUTE ERROR ACCEPTABLE FOR THE TRANSFER RATE. TYPE THIS VALUE IN THE FORM XX.XXX

TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE AT LEAST _____ MORE TRIALS. WHEN YOU RE-ACCESS THIS PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER:
  1. YOUR CODE NUMBER (IT IS _____)
  2. THE NUMBER OF TRIALS
  3. THE INPUT/OUTPUT TIME FOR EACH TRIAL (IN CHRONOLOGICAL ORDER);
  4. THE USER-FRACTION OF EACH INPUT/OUTPUT TIME (IN CHRONOLOGICAL ORDER);
  5. THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL (IN CHRONOLOGICAL ORDER).

PLEASE TYPE THE USER-FRACTION OF THE INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL. PLEASE PRESS THE RETURN KEY AFTER EACH ENTRY.

YOUR TEST RESULTED IN AN ESTIMATED MEAN INPUT/OUTPUT TIME OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN INPUT/OUTPUT TIME IS BETWEEN _____ AND _____
  YOUR TEST HAS RESULTED IN AN ESTIMATED MEAN USER-FRACTION INPUT/OUTPUT TIME OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN _____ AND _____.
  THE ESTIMATED MEAN TRANSFER RATE IS _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE RATE IS BETWEEN _____ AND _____

Box F

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
  IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE NUMBER OF TRIALS.

PLEASE TYPE THE INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE USER-FRACTION OF THE INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL. PLEASE PRESS THE RETURN KEY AFTER EACH ENTRY.

YOUR TEST RESULTED IN AN ESTIMATED MEAN INPUT/OUTPUT TIME OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN INPUT/OUTPUT TIME IS BETWEEN _____ AND _____
  YOUR TEST HAS RESULTED IN AN ESTIMATED MEAN USER-FRACTION INPUT/OUTPUT TIME OF _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN _____ AND _____.
  THE ESTIMATED MEAN TRANSFER RATE IS _____. YOU CAN BE _____ PERCENT CONFIDENT THAT THE TRUE RATE IS BETWEEN _____ AND _____

Figure 16. Program messages for analysis of rates.

- File Name. The name should be a character name of the form AAAAAA. The file format must be 2F16.3. Columns one and two contain the total delay and the user-fraction of the total delay, respectively. The data are to be listed in chronological order. The program is designed for as many as 200 records.

Analysis consists of:

- The estimation of the mean delay and its confidence limits.
- The estimation of the mean user-fraction of the total delay and its confidence limits.

Test B

Enter the following:

- Code number. Enter 13 if the 90% confidence level was selected when the sample size was determined or 14 if the 95% confidence level was selected.
- Mode of Entry. Enter 1 if data are to be entered from a keyboard or 2 if data are to be entered from a file.
- □ If data are to be entered from a keyboard, enter:
- Number of Total delays. This is an integer greater than 1. The computer program is designed for as many as 200 total delays.
- Total delay in each trial. This is a positive decimal number in the form XXXXXX.XXX; not all delays can be equal. Enter the delays in chronological order, and include the decimal point.[11]
- Absolute precision. This is a positive decimal number as defined in Section 2.4. Depending upon the total delays, the confidence level, and the absolute precision, the computer program determines whether more trials are required.

If no more trials are required, enter:

- User-fraction portion of the total delay in each trial. The same restrictions apply as for the total delay.[11] The units must be the same as those of the total delay.
- □ If data are to be entered from a file, enter:
- File Name. The name should be a character name of the form AAAAAA. The file format must be 2F16.3. Columns one and two contain the total delay and the user-fraction of the total delay, respectively. The data are to

---

[11]Immediately after entering a sequence of numbers, the computer program allows you to correct any number entered in error.

be entered in chronological order. The program is designed for as many as 200 records.

- Absolute precision. This is a positive decimal number as defined in Section 2.4.1. Depending upon the total delays, the confidence level, and the absolute precision, the computer program determines whether more trials are required.

If no more trials are required, analysis consists of:

- The estimation of the mean delay and its confidence limits.
- The estimation of the mean user-fraction of the total delay and its confidence limits.

On the other hand, if more trials are required, analysis consists of:

- Determining the number of additional trials required.
- Assigning a new code number for the next analysis (11 for the 90% confidence level or 12 for the 95% confidence level). The number of required trials is now known (i.e., the total from the preliminary Test B and this one). Hence, after this test, this re-entry code will cause analysis to proceed as after Test A, which results from knowing, initially, that the sample size is sufficient.

Test C

This is the same as Test A.

Test D

Enter the following:

- Code number. Enter 21 if the 90% confidence level was selected when the sample size was determined or 22 if the 95% confidence level was selected.
- Mode of entry. Enter 1 if data are to be entered from a keyboard or 2 if data are to be entered from a file.
- ☐ If data are to be entered from a keyboard, enter:
- Number of trials. This is an integer greater than 1. The computer program is designed for as many as 200 trials.
- Total input/output time in each trial. This is a positive decimal number in the form XXXXXX.XXX, not all equal. Enter the input/output times in chronological order, and include the decimal point.[11]
- User-fraction of the total input/output time in each trial.

---

[11]Immediately after entering a sequence of numbers, the computer program allows you to correct any number entered in error.

The same restrictions apply as for the total input/output time.[11] The units must be the same as those of the total input/output time.

- Number of bits transferred in each trial. This is a positive integer having from 1 to 10 digits.

☐ If data are to be entered from a file, enter:

- File name. The name should be a character name of the form AAAAAA. The file format must be 2F16.3, F16.0. Columns one, two, and three contain the total input/output time in each trial, the user-fraction of the total input/output time in each trial, and the number of bits transferred in each trial, respectively. The data are to be entered in chronological order. The program is designed for as many as 200 records.

Analysis consists of:

- The estimation of the mean input/output time and its confidence limits.
- The estimation of the mean user-fraction of the input/output time and its confidence limits.
- The estimation of the mean transfer rate and its confidence limits.

Test E

Enter the following:

- Code number. Enter 23 if the 90% confidence level was selected when the sample size was determined or 24 if the 95% confidence level was selected.
- Mode of entry. Enter 1 if data are to be entered from a keyboard or 2 if data are to be entered from a file.

☐ If data are to be entered from a keyboard, enter:

- Number of trials. This is an integer greater than 1. The computer program is designed for as many as 200 trials.
- Total input/output time in each trial. This is a positive decimal number in the form XXXXXX.XXX, not all equal. Enter the input/output times in chronological order, and include the decimal point.[11]
- Absolute precision. This is a positive decimal number as defined in Section 2.4.1. Depending upon the total input/output times, the confidence level, and the absolute precision, the computer program determines whether more trials are required.

---

[11]Immediately after entering a sequence of numbers, the computer program allows you to correct any number entered in error.

If no more trials are required, enter:

- User-fraction of the total input/output time in each trial. The same restrictions apply as for the total input/output time. The units must be the same as those of the total input/output time.
- Number of bits transferred in each trial. This is a positive integer having from 1 to 10 digits.

□ If data are to be entered from a file, enter:

- File name. The name should be a character name of the form AAAAAA. The file format must be 2F16.3, F16.0. Columns one, two, and three contain the total input/output time in each trial, the user-fraction of the total input/output time in each trial, and the number of bits transferred in each trial, respectively. The data are to be listed in chronological order. The program is designed for as many as 200 records.
- Absolute precision. This is a positive decimal number as defined in Section 2.4. Depending upon the total delays, the confidence level, and the absolute precision, the computer program determines whether more trials are required.

If no more trials are required, analysis consists of:

- The estimation of the mean input/output time and its confidence limits.
- The estimation of the mean user-fraction of the input/output time and its confidence limits.
- The estimation of the mean transfer rate and its confidence limits.

On the other hand, if more trials are required, analysis consists of:

- Determining the number of additional trials required.
- Assigning a new code number for the next analysis (21 for the 90% confidence level or 22 for the 95% confidence level). The number of required trials is now known (i.e., the total from the preliminary Test E and this one). Hence, after the test, this re-entry code will cause analysis to proceed as after Test D, which results from knowing, initially, that the sample size is sufficient.

<u>Test F</u>

This is the same as Test D.

The following example is the analysis portion of the example in Section 3.1.

> EXAMPLE: The test from the example in Section 3.1 (Test C) has been conducted. It produced the following 13 delays: 5., 7., 6., 5., 4., 5., 8., 5., 6., 7., 6., 6., and 5. It also produced the following 13 user-fraction delays 3., 4., 4., 4., 2., 3., 5., 3., 3., 4., 5., 4., and 3. Enter the data from a keyboard, and analyze the test.

> SOLUTION:

> 1.   Gain access to the computer program.

> 2.   Type 11 (the assigned code number), and press the return key.

> 3.   Type 1 (for keyboard entry), and press the return key.

> 4.   Type 13 (the number of delays tested), and press the return key.

> 5.   Type 5., and press the return key.
>
>   Type 7., and press the return key.
>
>   Type 6., and press the return key.
>
>   Type 5., and press the return key.
>
>   Type 4., and press the return key.
>
>   Type 5., and press the return key.
>
>   Type 8., and press the return key.
>
>   Type 5., and press the return key.
>
>   Type 6., and press the return key.
>
>   Type 7., and press the return key.
>
>   Type 6., and press the return key.
>
>   Type 6., and press the return key.
>
>   Type 5., and press the return key.

> 6.   Type 0 (since all delays were entered correctly), and press the return key.

> 7.   Type 3., and press the return key.

> 8.   Type 4., and press the return key.
>
>   Type 4., and press the return key.
>
>   Type 4., and press the return key.
>
>   Type 2., and press the return key.
>
>   Type 3., and press the return key.
>
>   Type 5., and press the return key.

Type 3., and press the return key.

Type 3., and press the return key.

Type 4., and press the return key.

Type 5., and press the return key.

Type 4., and press the return key.

Type 3., and press the return key.

The following analysis of the test is listed:

"Your test resulted in an estimated mean delay of .57692E+01. You can be 90 percent confident that the true mean delay is between .52757E+01 and .62627E+01.

Your test has resulted in an estimated mean user-fraction delay of .62664E+00. You can be 90 percent confident that the true mean is between .58465E+00 and .66864E+00."[12]

(Execution for this example required .22 central processing seconds on a main frame, 60-bit word computer.)

4.2 Failure Probability Parameters

There are three possible tests of failure probability (Tests G, H, and I). Test G resulted from specifying a given sample size; Test H resulted from specifying a desired relative precision but not knowing the maximum value of the conditional probability, P_{11}; and Test I resulted from specifying a desired relative precision and knowing the maximum value of P_{11}. Figure 17 is an operator-decision subdiagram of Figure 14. It shows the sequence of events leading to analysis of failure probability. Since analysis of the failure probability requires entry of the code number and only three other numbers, there is no provision for entry from a file.

Test G

Enter the following:

• Code number. Enter 31 if the 90% confidence level was selected when the sample size was determined or 32 if the 95% confidence level was selected.

---

[12] The confidence limits are closer to the estimate of the mean than the specified 0.7 seconds (i.e., 0.49) because the sample standard deviation of the delays is 1.05 (smaller than the 1.5 maximum entered when the sample size was to be determined).

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Top flowchart — columns A through I:

(A) ENTER CODE AND TEST RESULTS

(B) ENTER CODE AND SOME OF TEST RESULTS → ENTER ABSOLUTE PRECISION → TEST MORE TRIALS (CODE = 11 12) / ENTER REMAINDER OF TEST RESULTS → (A) ENTER CODE AND TEST RESULTS

(C) ENTER CODE AND TEST RESULTS

(D) ENTER CODE AND TEST RESULTS

(E) ENTER CODE AND SOME OF TEST RESULTS → ENTER ABSOLUTE PRECISION → TEST MORE TRIALS (CODE = 21 22) / ENTER REMAINDER OF TEST RESULTS → (D) ENTER CODE AND TEST RESULTS

(F) ENTER CODE AND TEST RESULTS

(G) ENTER CODE AND TEST RESULTS

(H) ENTER CODE AND TEST RESULTS → ENTER RELATIVE PRECISION → TEST MORE TRIALS (CODE = 31 32) → (G) ENTER CODE AND TEST RESULTS

(I) ENTER CODE AND TEST RESULTS

TEST ANALYSIS LISTED (for each column)

Lower section — three columns G, H, I:

(G)

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE SAMPLE SIZE FROM YOUR TEST.

PLEASE TYPE THE NUMBER OF FAILURES IN THE SAMPLE.

PLEASE TYPE THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN THE SAMPLE.

CAN YOU ESTIMATE THE MAXIMUM VALUE OF THE CONDITIONAL PROBABILITY OF A FAILURE, GIVEN THAT A FAILURE HAS OCCURRED IN THE PREVIOUS TRIAL? IF YOU CANNOT, THE VALUE 0.8 WILL BE USED. PLEASE TYPE YES OR NO.

YOUR TEST RESULTED IN AN ESTIMATED FAILURE RATE OF _____. YOU CAN BE __ PERCENT CONFIDENT THAT THE TRUE FAILURE RATE IS BETWEEN _____ AND _____

(H)

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE SAMPLE SIZE FROM YOUR TEST.

PLEASE TYPE THE NUMBER OF FAILURES IN THE SAMPLE.

PLEASE TYPE THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN THE SAMPLE.

PLEASE TYPE THE DESIRED RELATIVE PRECISION AS A PERCENT (I.E., A ONE OR TWO-DIGIT NATURAL NUMBER).

CAN YOU ESTIMATE THE MAXIMUM VALUE OF THE CONDITIONAL PROBABILITY OF A FAILURE, GIVEN THAT A FAILURE HAS OCCURRED IN THE PREVIOUS TRIAL? IF YOU CANNOT, THE VALUE 0.8 WILL BE USED. PLEASE TYPE YES OR NO.

TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE AT LEAST _____ MORE FAILURES. AFTER THE TEST YOU WILL RE-ACCESS THIS PROGRAM TO ANALYZE THE PERFORMANCE OF YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO TYPE:
1. THE TOTAL SAMPLE SIZE
2. THE TOTAL NUMBER OF FAILURES
3. THE TOTAL NUMBER OF PAIRS OF CONSECUTIVE FAILURES
4. YOUR CODE NUMBER (IT IS __ __.)

(G)

YOUR TEST RESULTED IN AN ESTIMATED FAILURE RATE OF _____. YOU CAN BE __ PERCENT CONFIDENT THAT THE TRUE FAILURE RATE IS BETWEEN _____ AND _____

(I)

THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

PLEASE TYPE THE SAMPLE SIZE FROM YOUR TEST.

PLEASE TYPE THE NUMBER OF FAILURES IN THE SAMPLE.

PLEASE TYPE THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN THE SAMPLE.

CAN YOU ESTIMATE THE MAXIMUM VALUE OF THE CONDITIONAL PROBABILITY OF A FAILURE, GIVEN THAT A FAILURE HAS OCCURRED IN THE PREVIOUS TRIAL? IF YOU CANNOT, THE VALUE 0.8 WILL BE USED. PLEASE TYPE YES OR NO.

YOUR TEST RESULTED IN AN ESTIMATED FAILURE RATE OF _____. YOU CAN BE __ PERCENT CONFIDENT THAT THE TRUE FAILURE RATE IS BETWEEN _____ AND _____
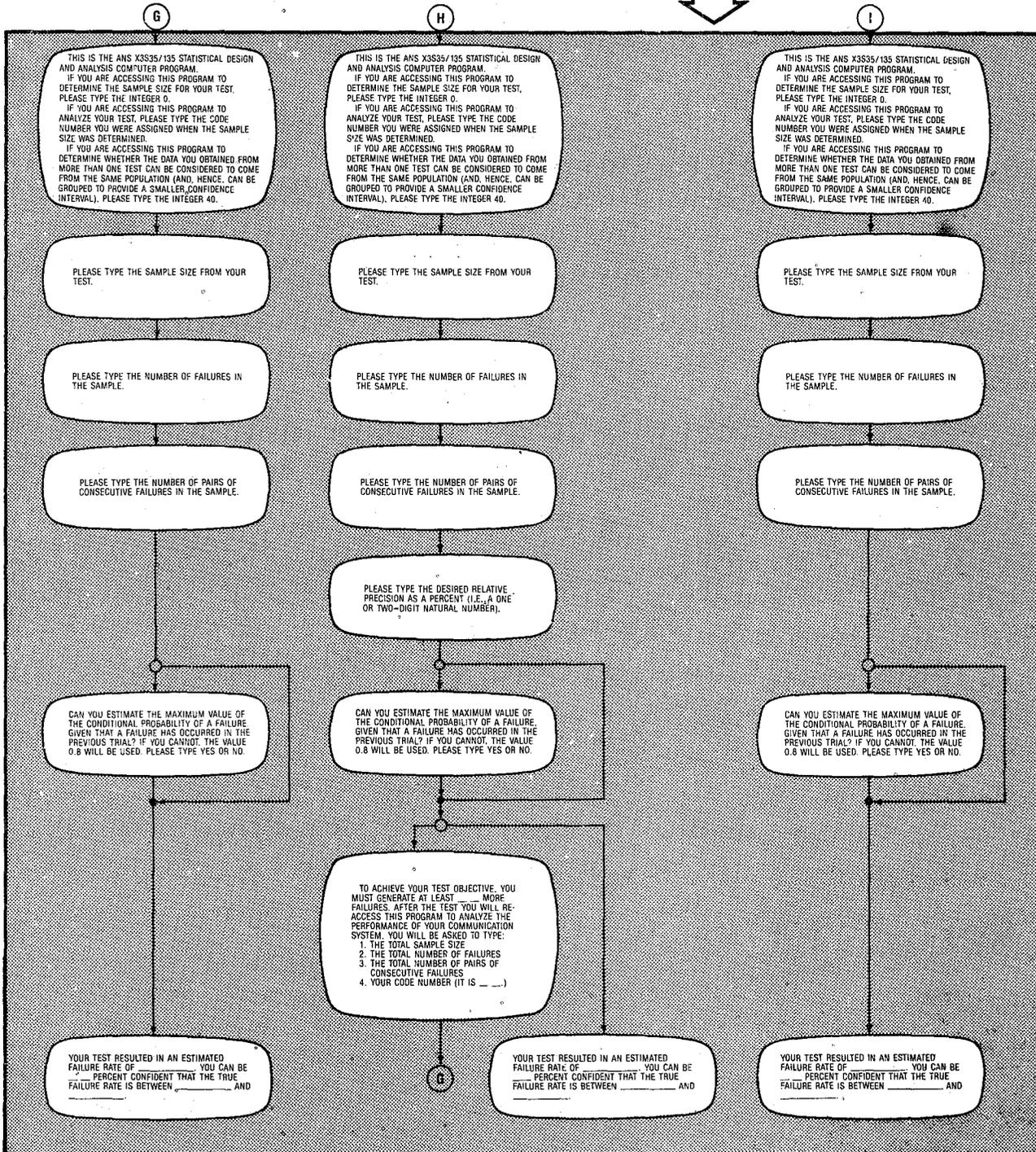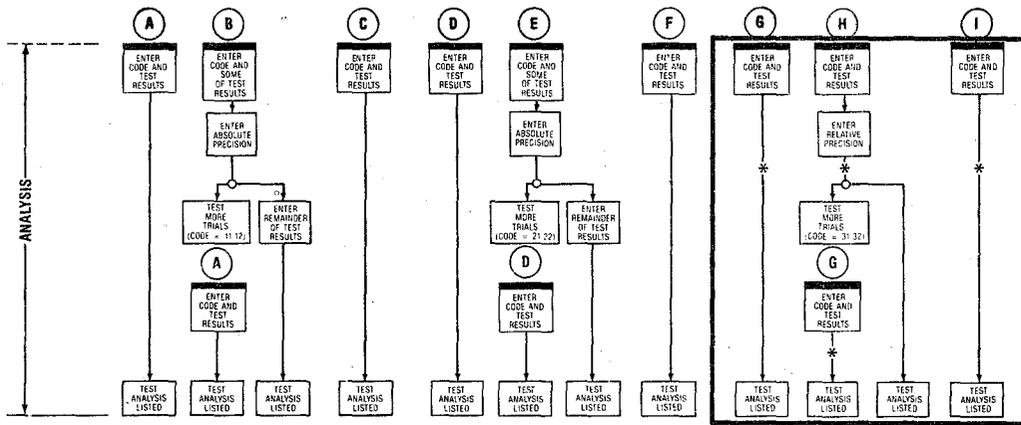
Figure 17. Program messages for analysis of failure probabilities.

- Sample size (the number of trials). This is a positive integer having from 1 to 10 digits.[13]

- Number of failures in the sample. This number is an integer from zero to one less than the sample size.

- Number of pairs of consecutive failures. This is an integer from zero to one less than the number of failures. (However, enter zero if there are also zero failures.)

Analysis consists of:

The estimation of the mean failure rate and its confidence limits.

Test H

Enter the following:

- Code number. Enter 33 if the 90% confidence level was selected when the sample size was determined or 34 if the 95% confidence level was selected.

- Sample size (the number of trials). This is a positive integer having from 1 to 10 digits.[13]

- Number of failures in the sample. This number is an integer from zero to one less than the sample size.

- Number of pairs of consecutive failures. This is an integer from zero to one less than the number of failures. (However, enter zero if there are also zero failures.)

- Relative precision. This is a one- or two-digit positive integer as defined in Section 2.4.2.

If the number of failures is less than 2, enter (if known):

- Maximum value of the conditional probability of a failure given that a failure occurred in the previous trial. (See Section 3.2.)

If no more trials are required, analysis consists of:

- The estimation of the mean failure rate and its confidence limits.

On the other hand, if more trials are required, analysis consists of:

- Determining the number of additional trials required.

- Assigning the new code number (31 for the 90% confidence level or 32 for

---

[13]If the number of failures in the sample is zero or one, the sample size must have a minimum value (to preclude an arithmatic error in computing the upper confidence limit). This minimum value depends on the number of failures, the confidence level, and the maximum value of the conditional probability (to be entered below). This minimum sample size is listed in Table 5.

Table 5.   Minimum Sample Sizes When the Number
of Failures is Zero or One

MAXIMUM VALUE OF CONDITIONAL PROBABILITY	NUMBER OF FAILURES			
	0		1*	
	90%	95%	90%	95%
0.99	25	13	26	26
0.95	7	9	13 (14)	17 (19)
0.90	6	8	9 (11,12)	11 (15)
0.80	5	6	5 (8,9)	6 (11)
0.70	4	5	4 (7)	4 (9)
0.60	4	4	4 (6)	4 (7,8)
0.50	3	4	4 (5)	4 (6,7)
0.40	3	3	6	4 (5,6)
0.30	3	3	5	6
0.20	2	3	5	5
0.10	2	3	4	5
0	2	2	4	4

*EXCLUDE THE SAMPLE SIZES IN PARENTHESIS; THEY ARE NOT ACCEPTABLE.

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the 95% confidence level). The number of required trials is now known (i.e., the total from the preliminary Test H and this one). Hence, after this test, this re-entry code will cause analysis to proceed as after Test G, which results from knowing, initially, that the sample size is sufficient.

Test I

This is the same as Test G.

EXAMPLE: The test from the example in Section 3.2 has been conducted (Test H). It resulted in 752650 trials, 17 failures, and three pairs of consecutive failures. The specified relative precision was 30%. Analyze the test data.

SOLUTION:

1. Gain access to the computer program.

2. Type: 33 (the assigned code number), and press the return key.

3. Type: 752650 (the number of trials), and press the return key.

4. Type: 17 (the number of failures), and press the return key.

5. Type: 3 (the number of pairs of consecutive failures), and press the return key.

6. Type: 30 (the percentage relative precision), and press the return key.

7. The following analysis of the test is listed:

"To achieve your test objective, you must generate at least 50 more failures. After the test you will re-access this program to analyze the performance of your communication system. You will be asked to enter:

1. Your code number (it is 31),

2. The total sample size,

3. The total number of failures,

4. The total number of pairs of consecutive failures."

EXAMPLE: (continued): The second test has been conducted (now Test G because the sample size is known). It resulted in 2249012 additional trials, 50 additional failures, and 8 additional pairs of consecutive failures. Analyze the combined data from both tests.

SOLUTION:

1. Gain access to the computer program.

2. Type: 31 (the assigned code number), and press the return key.

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3. Type: 3001662 (the total number of trials), and press the return key.

4. Type: 67 (the total number of failures), and press the return key.

5. Type: 11 (the total number of pairs of consecutive failures), and press the return key.

6. The following analysis of the test is listed:

   "Your test resulted in an estimated failure rate of .22321E-04. You can be 90 percent confident that the true failure rate is between .17340E-04 and .28342E-04." The relative precision achieved is 24.6%, better than the specified 30%.

5. ANALYSIS OF MULTIPLE TESTS

When multiple tests have been conducted to measure a performance parameter, it is possible that the trials came from the same population. If so, they may be grouped to provide a single larger sample. The larger sample should provide a narrower confidence interval than do any of the individual samples and, hence, more information about the population mean.

Figure 18 shows the initial three decisions required to analyze multiple tests. After gaining access to the computer program, the code number 40 must be entered. Then select the performance parameter and the confidence level. Section 5.1 discusses the test of homogeneity for delays and rates, and Section 5.2 discusses the test of homogeneity for failure probability.

5.1 Time Parameters

The procedures to analyze multiple tests of delays and rates are similar.

To analyze multiple tests of delays proceed as in Figure 19, and enter the following:

- Number of tests. This is an integer from 2 to 6. The computer program could be altered to accommodate more tests.

- Mode of Data Entry. Enter 1 if data are to be entered from a keyboard or 2 if data are to be entered from files.

☐ If data are to be entered from a keyboard, enter:

- The number of delays during test 1. This is a positive integer. The computer program is currently designed for as many as 200 delays (the total from all tests).

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THIS IS THE ANS X3S35/135 STATISTICAL DESIGN AND ANALYSIS COMPUTER PROGRAM.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.
  IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE SAMPLE SIZE WAS DETERMINED.
  IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE TYPE THE INTEGER 40.

DO YOUR TESTS MEASURE
  1. DELAYS,
  2. RATES,
OR
  3. FAILURES?
PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE APPROPRIATE PARAMETER.

THE PERFORMANCE OF THE PARAMETER THAT YOU SELECTED CAN BE MEASURED TO PROVIDE ONE OF THE FOLLOWING LEVELS OF CONFIDENCE:
  1. 90%
  2. 95%
PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE CONFIDENCE LEVEL THAT YOU HAVE SELECTED.

Figure 18. Initial program messages for analysis of multiple tests.

PLEASE TYPE THE NUMBER OF TESTS. THIS
NUMBER MUST BE GREATER THAN 1 AND LESS
THAN 7.

DO YOU WISH TO ENTER THE TEST DATA
   1. FROM A KEYBOARD
OR
   2. FROM A FILE?
PLEASE TYPE THE INTEGER AT THE LEFT OF THE
DESIRED ENTRY MODE.

PLEASE TYPE THE NUMBER OF DELAYS
OBSERVED DURING TEST 1.

PLEASE ENTER THE NAME OF THE FILE CONTAIN-
ING DATA FROM TEST 1. THIS NAME SHOULD
BE A CHARACTER NAME OF THE FORM
AAAAAA.

PLEASE TYPE (IN CHRONOLOGICAL ORDER) THE
DELAYS OBSERVED DURING TEST 1. PRESS THE
RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE NUMBER OF DELAYS
OBSERVED DURING TEST 2.

PLEASE ENTER THE NAME OF THE FILE CONTAIN-
ING DATA FROM TEST 2. THIS NAME SHOULD
BE A CHARACTER NAME OF THE FORM
AAAAAA.

PLEASE TYPE (IN CHRONOLOGICAL ORDER) THE
DELAYS OBSERVED DURING TEST 2. PRESS THE
RETURN KEY AFTER EACH ENTRY.

WITH 95% CONFIDENCE THE TRIALS CAN BE
CONSIDERED TO COME FROM THE SAME
POPULATION. HENCE, THEY CAN ALL BE
GROUPED TO DETERMINE A SMALLER
CONFIDENCE INTERVAL.

PLEASE TYPE THE USER-RESPONSIBLE PORTION
OF EACH OF THE __ TRIALS (IN CHRONOLOGICAL
ORDER). PRESS THE RETURN KEY AFTER EACH
ENTRY.

WITH 95% CONFIDENCE THE TRIALS CANNOT
BE CONSIDERED TO COME FROM THE SAME
POPULATION. HENCE, THEY CANNOT BE
GROUPED TO DETERMINE A SMALLER
CONFIDENCE INTERVAL.
   IF YOU WISH, OMIT DATA FROM ONE OR
MORE TESTS THAT ARE THOUGHT TO CAUSE
REJECTION, AND RE-ACCESS THIS PROGRAM.
YOUR CODE WILL REMAIN 40.

YOUR TESTS RESULTED IN AN ESTIMATED
AVERAGE DELAY OF _____ YOU CAN
BE ____ PERCENT CONFIDENT THAT THE TRUE
AVERAGE DELAY IS BETWEEN _____
AND _____
   YOUR TESTS RESULTED IN AN ESTIMATED
AVERAGE USER-RESPONSIBLE FRACTIONAL
DELAY OF _____ YOU CAN BE
____ PERCENT CONFIDENT THAT THE TRUE
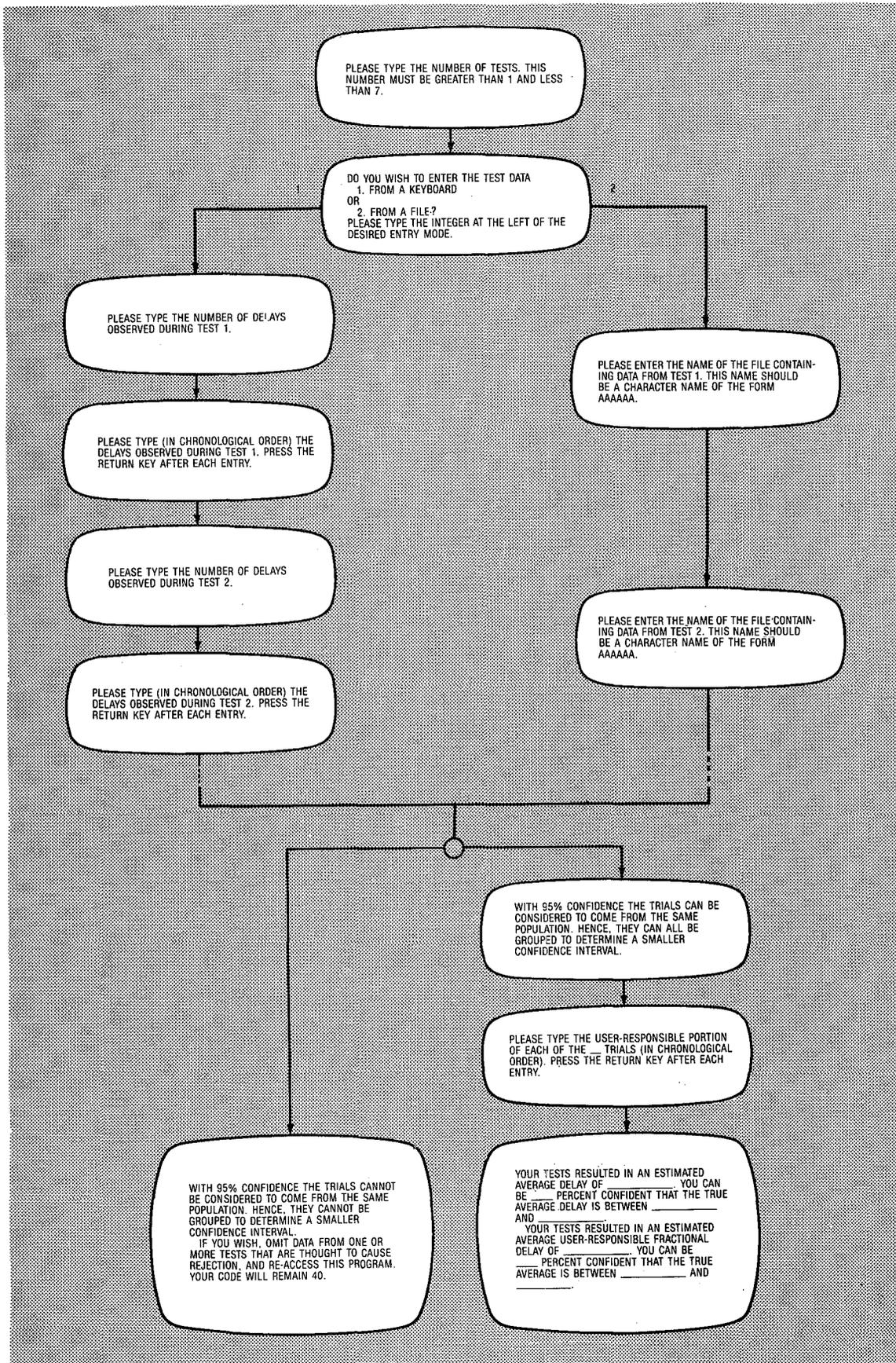AVERAGE IS BETWEEN _____ AND
_____

Figure 19.   Program messages for analyses of multiple tests of delays.

- The delays observed during test 1. These are positive decimal numbers in the form XXXXXX.XXX. Enter the delays in chronological order, and include the decimal point.[14]

- Repeatedly, the last two types of data for each remaining test. Not all delays can be equal (or division by zero will occur).[15]

If the trials cannot be combined,

- analysis is halted; the program states that data from one or more tests thought to cause rejection can be omitted, and the test for homogeneity can be repeated.

If the trials can be combined, enter:

- User-fraction of the delay, in each trial. These are positive decimal numbers in the form XXXXXX.XXX. Enter the delays in chronological order and include the decimal point.[14] Not all of the user-fraction delays can be equal (or division by zero will occur).

☐ If data are to be entered from files, enter:

- File name for test 1. The name should be a character name of the form AAAAAA. The file format must be 2F16.3. Columns one and two contain the total delay and the user-fraction portion of the total delay, respectively. The data are to be listed in chronological order. The program is designed for as many as 200 records (the total from all tests).

- File name for test 2. The same comments apply as for test 1. Repeatedly, the file names for all remaining tests. Not all delays can be equal (or division by zero will occur).[15]

If the trials cannot be combined,

- analysis is halted; the program states that data from one or more tests thought to cause rejection can be omitted, and the test for homogeneity can be repeated.

On the other hand if the trials can be combined, analysis consists of:

- The estimation of the mean delay and its confidence limits.

- The estimation of the mean user-fraction of the delay and its confidence limits.

_____

[14]Immediately after entering a sequence of numbers, the computer program allows you to correct any number entered in error.

[15]The computer program determines, at the 95% confidence level, whether the trials from the tests can be combined. This confidence level should not be confused with the confidence level you chose for the interval about the estimate of the population mean.

To analyze multiple tests of rates proceed as in Figure 20, and enter the following:

- Number of tests. This is an integer from 2 to 6. The computer program could be altered to accommodate more tests.

- Mode of data entry. Enter 1 if data are to be entered from a keyboard or 2 if data are to be entered from files.

☐ If data are to be entered from a keyboard, enter:

- Number of trials during test 1. This is a positive integer. The computer program is currently designed for as many as 200 trials (the total from all tests).

- Input/output times during test 1. These are positive decimal numbers in the form XXXXXX.XXX. Enter the input/output times in chronological order, and include the decimal point.[16]

- Repeatedly, the last two types of data for each remaining test. Not all input/output times can be equal (or division by zero will occur).[17]

If the trials cannot be combined,

- analysis is halted; the program states that data from one or more tests thought to cause rejection can be omitted, and the test for homogeneity can be repeated.

If the trials can be combined, enter:

- User fraction of the input/output time in each trial. These are positive decimal numbers in the form XXXXXX.XXX. Enter the input/output times in

---

[16]Immediately after entering a sequence of numbers, the computer program allows you to correct any number entered in error.

[17]The computer program determines, at the 95% confidence level, whether the trials from the tests can be combined. This confidence level should not be confused with the confidence level you chose for the interval about the estimate of the population mean.
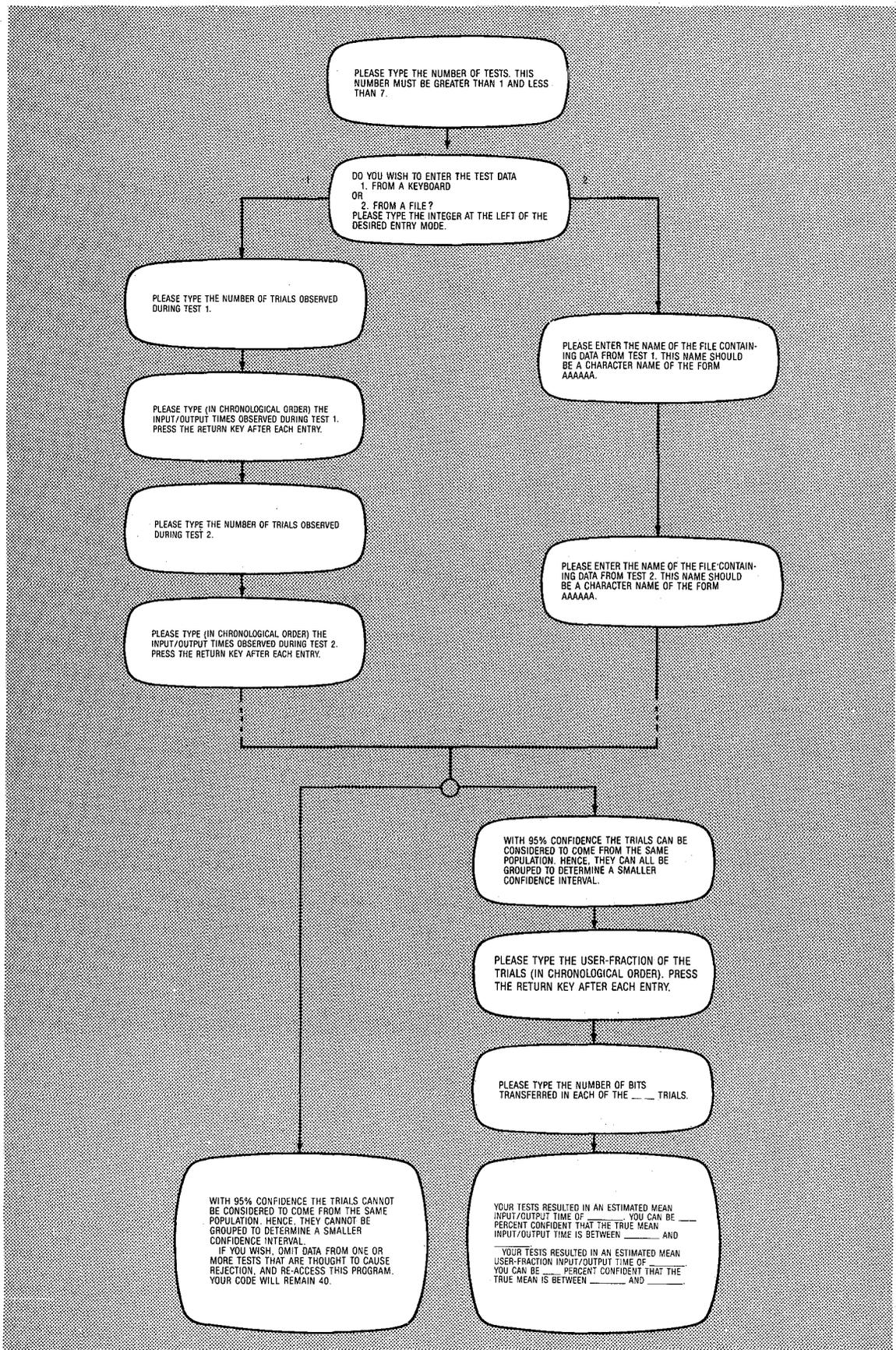
Figure 20. Program messages for analysis of multiple tests of rates.

chronological order, and include the decimal point.[18]   Not all of the
user-fraction input/output times can be equal (or division by zero will
occur).

- Number of bits transferred in each trial.   These are positive integers
  having from 1 to 10 digits.

▢ If data are to be entered from files, enter:

- File name for test 1.   The name should be a character name of the form
  AAAAAA.   The format must be 2F16.3, F16.0.   Columns one, two, and three
  contain the total input/output time in each trial, the user-fraction of
  the total input/output time in each trial, and the number of bits
  transferred in each trial, respectively.   The data are to be listed in
  chronological order.   The program is designed for as many as 200 records
  (the total from all tests).

- File name for test 2.   The same comments apply as for test 1.

- Repeatedly, the file names for all remaining tests.   Not all input/output
  times can be equal (or division by zero will occur).[19]

If the trials cannot be combined,

analysis is halted; the program states that data from one or more tests
thought to cause rejection can be omitted, and the test for homogeneity
can be repeated.

Analysis consists of:

- The estimation of the mean input/output time and its confidence limits.

- The estimation of the mean user-fraction of the input/output time and its
  confidence limits.

- The estimation of the mean transfer rate and its confidence limits.

5.2   Failure Probability Parameters

It is useful to determine whether trials of more than one test of failure
probability can be combined to form a larger sample size.   If they can be
combined, the larger sample size should cause the confidence limits to become

---

[18]Immediately after entering a sequence of numbers, the computer program
allows you to correct any number entered in error.

[19]The computer program determines, at the 95% confidence level, whether the
trials from the tests can be combined.   This confidence level should not be
confused with the confidence level you chose for the interval about the
estimate of the population mean.

closer, thereby providing more information about the population mean failure probability.

To analyze multiple tests of a failure probability parameter, proceed as in Figure 21 and enter the following:

- Number of tests. This is a positive integer from 2 to 6. The computer program could be altered to accommodate more tests. A test that resulted in zero failures cannot be included.

- Number of trials in each test. This is a positive integer having from 1 to 10 digits.

- Number of failures in each test. This is a positive integer having from 1 to 10 digits.

- Number of pairs of consecutive failures in each test. This is a non-negative integer having from 1 to 10 digits.

There are three possible results:

- The trials cannot be combined because they fail the test for homogeneity. Analysis is halted and the program states that data from one or more tests thought to cause rejection can be omitted, and the test for homogeneity can be repeated.

- The trials cannot be tested for homogeneity because the total number of pairs of consecutive failures is zero. A statement to this effect is listed. However, for the operator's information, the data are combined; analysis proceeds and consists of:

  - The estimation of the mean failure rate and its confidence limits (as if the trials were from the same population).

- The trials can be considered to come from the same population. A statement is listed; analysis proceeds and consists of:

  - The estimation of the mean failure rate and its confidence limits.

6. ACKNOWLEDGMENTS

PLEASE TYPE THE NUMBER OF TESTS.

PLEASE TYPE THE NUMBER OF TRIALS IN EACH TEST. PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE NUMBER OF FAILURES IN EACH TEST. PRESS THE RETURN KEY AFTER EACH ENTRY.

PLEASE TYPE THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES IN EACH TEST. PRESS THE RETURN KEY AFTER EACH ENTRY.

WITH 95% CONFIDENCE THE TRIALS CANNOT BE CONSIDERED TO COME FROM THE SAME POPULATION. HENCE, THEY CANNOT BE GROUPED TO DETERMINE A SMALLER CONFIDENCE INTERVAL.
    IF YOU WISH, OMIT DATA FROM ONE OR MORE TESTS THAT ARE THOUGHT TO CAUSE REJECTION, AND RE-ACCESS THIS PROGRAM. YOUR CODE WILL REMAIN 40.

SINCE THERE ARE NO PAIRS OF CONSECUTIVE FAILURES, THE HOMOGENEITY OF THE SAMPLES CANNOT BE DETERMINED. EVEN SO, THEY WILL BE GROUPED TO DETERMINE A SMALLER CONFIDENCE INTERVAL.
    THE ____ TESTS RESULTED IN AN ESTIMATED MEAN FAILURE RATE OF _____ YOU CAN BE __ __ CONFIDENT THAT THE TRUE MEAN FAILURE RATE IS BETWEEN _____ AND _____.

WITH 95% CONFIDENCE THE TRIALS CAN BE CONSIDERED TO COME FROM THE SAME POPULATION. HENCE, THEY CAN ALL BE GROUPED TO DETERMINE A SMALLER CONFIDENCE INTERVAL.
    THE ____ TESTS RESULT IN AN ESTIMATED FAILURE RATE OF _____, YOU CAN BE __ __ PERCENT CONFIDENT THAT THE TRUE FAILURE RATE IS BETWEEN _____ AND _____.
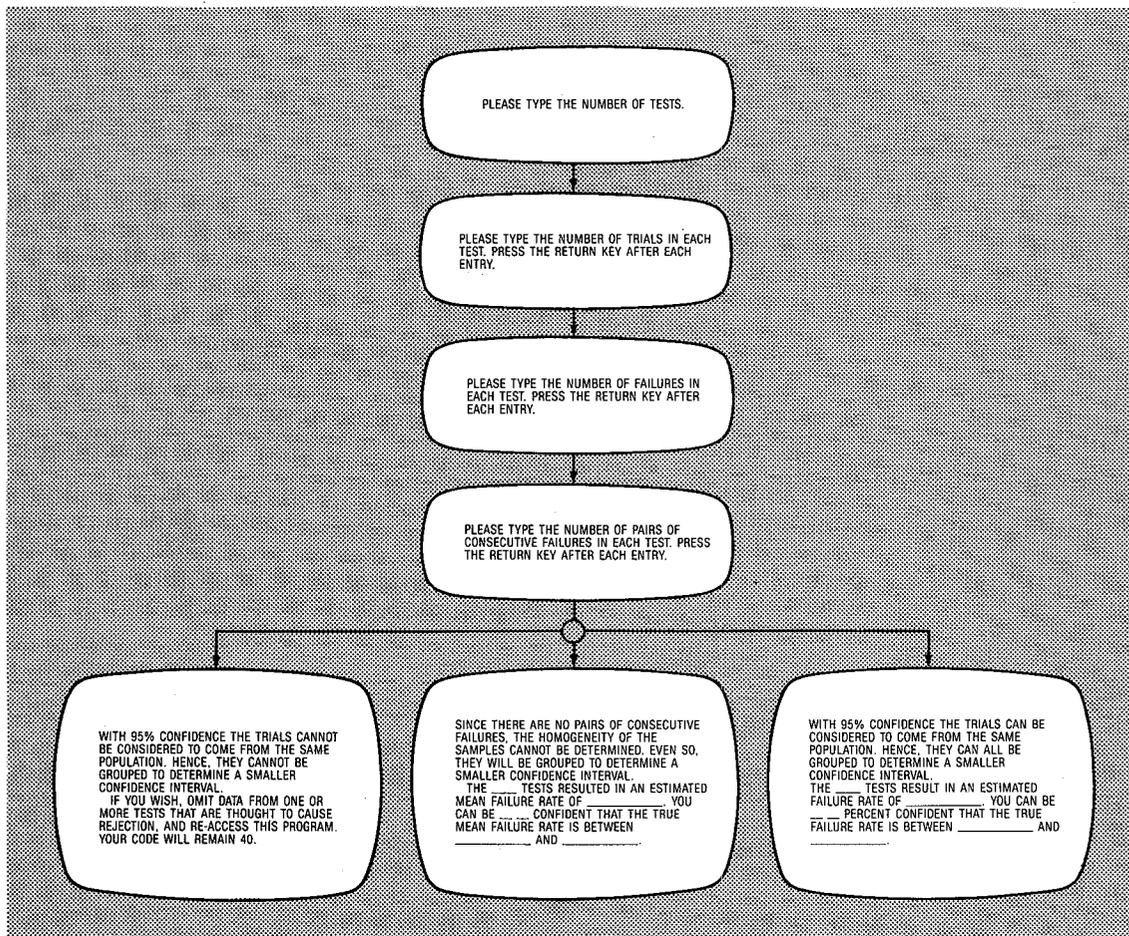
Figure 21.   Program messages for analysis of probabilities of failure.

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APPENDIX A: MATHEMATICAL FORMULAS

This Appendix contains the mathematical theory used to determine the sample size, an estimate of the mean, and its confidence limits.[1] The formulas are numbered here, in the structured design diagrams (Appendix B), and in the computer program listing (Appendix C). The name of the subroutine using the formula is also listed beneath the formula number.

Section A.1 states the formulas to determine the sample size; Section A.2 states the formulas for the analysis (i.e., the estimate of the population mean and the smallest confidence interval about that estimate); and Section A.3 states the formulas used to determine whether trials from multiple tests can be grouped to provide a better estimate of the population mean.

A.1 Sample Size Determination for a Test

Sometimes the sample size is determined by time or budget. However, it is usually determined by the precision required for the application of the communication system.

If precision is required, the sample size is determined by either the relative or the absolute precision and the confidence level. The following equations determine the sample size required to achieve a given precision at a given confidence level. In one type of problem, the equations determine the sample size from knowledge of certain maximum values of the population. In a second type, they determine a minimum preliminary sample size. Statistical information from the preliminary sample is then used to determine the number of additional observations (if any) that is necessary to achieve the required precision.

A.1.1 Time Parameters

Even though time parameters consist of both delays and rates, the required sample size is determined from delays (i.e., the time to accomplish functions such as access, information transfer, and disengagement).

Delays are non-negative values. Since they are bounded below by zero but unbounded above, they cannot be normally distributed. Data indicate they are

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[1]Only a sketch of the theory is given here. The detailed theory was obtained from the referenced reports and books, other texts, and extensive private communication with E. L. Crow.

asymmetric, but not far from normally distributed (possibly log-normally or gamma distributed).

The sample mean has been selected to estimate the population mean. It is sometimes not used because it can be contaminated by outlying values. This objection has been removed since ANS X3.102 defines a delay to be a failure if it exceeds three times the nominal delay. The sample mean is an unbiased estimate[2] for any distribution and efficient for the normal and gamma distribution. The sample median is another common estimate of the population mean. However, it is inefficient for the normal and gamma distributions.

To estimate the population mean of the delays, specify two measures of precision:

● The absolute precision with which the estimate must approximate the mean delay.

● The level of confidence that the absolute precision has been achieved.

The required sample size can be determined if we know both the maximum value of the population standard deviation of the delays and the extent of the statistical dependence among them. Otherwise a preliminary test of at least ten delays is recommended to estimate the sample standard deviation and the statistical dependence (as measured by the autocorrelation of lag 1).

The required sample size can be determined from one of the following four cases:

A.1.1.1. The upper bound of the population standard deviation of the delays is known, and the delays are independent:

If σ_{max} is the upper bound of the population standard deviation, u_α is the upper $100\alpha\%$ point of the normal density, and a is the absolute precision as specified by the half-length of the $100(1-2\alpha)$ confidence interval, the required number of delays is

$$n = \left(\frac{u_\alpha \sigma_{max}}{a}\right)^2 .$$

(A-1)

[SSDTIM]

---

[2] An estimate of a parameter is said to be an unbiased estimate if it's expected value equals the parameter.

A.1.1.2. The upper bound of the population standard deviation is known, and the delays are dependent with known autocorrelation of lag 1:

If ρ_1 is the autocorrelation of lag 1, the required number of delays is

$$n = \left(\frac{u_\alpha \sigma_{max}}{a}\right)^2 \cdot \left(\frac{1 + \rho_1}{1 - \rho_1}\right).$$

(A-2)

[SSDTIM]

A.1.1.3. The upper bound, σ_{max}, of the population standard deviation is known, and the delays are dependent with unknown autocorrelation of lag 1:

A.1.1.3.1. Conduct a preliminary test of n' delays, w_i, (at least 10 delays) in order to estimate the autocorrelation of lag 1.

A.1.1.3.2. From the preliminary test, compute, $r_1(w)$, the estimate of the autocorrelation of lag 1:

$$r_1(w) = \frac{1}{s^2(n'-1)} \sum_{i=1}^{n'-1} (w_i - \bar{w})(w_{i+1} - \bar{w})$$

(A-3)

[SSDTIM]

where

$$\bar{w} = \frac{1}{n'} \sum_{i=1}^{n'} w_i \ ,$$

(A-4)

[SSDTIM]

and

$$s^2 = \frac{1}{n'-1} \sum_{i=1}^{n'} (w_i - \bar{w})^2 .$$

(A-5)

[SSDTIM]

A.1.1.3.3. Determine $t_{n'-1,\alpha}$, the upper $100\alpha\%$ point of the Student t distribution corresponding to n'-1 degrees of freedom. (This value is determined in subroutine STUDNT.)

A.1.1.3.4. Compute

$$A(r_1) = t_{n'-1,\alpha} \cdot \frac{\sigma_{max}}{n'} \cdot \sqrt{\frac{1 + r_1(w)}{1 - r_1(w)}} .$$

(A-6)

[SSDTIM]

The quantity,

$$s_{\bar{w}} = \frac{s}{\sqrt{n'}} \cdot \sqrt{\frac{1 + r_1(w)}{1 - r_1(w)}} \quad , \qquad \text{(A-7)}$$

[-]

is a measure of the uncertainty of $\bar{w}$ called the standard error.

A.1.1.3.5. Now,

• if $A(r_1) > a$, test

$$n = n' \left(\frac{A(r_1)}{a} \right)^2 - n' \qquad \text{(A-8)}$$

[SSDTIM]

additional delays.

• Otherwise no more delays need to be tested.

A.1.1.4. The upper bound of the population standard deviation is not known. Proceed as in A.1.1.3, except use s instead of σ_{max} in (A-6).

A.1.2 Failure Probability Parameters

If trials result in either success or failure and are statistically independent they have the binomial distribution with failure probability, P. However, successive trials are usually dependent. In this case, one can model the failure probability with a Markov chain of order m (where a large m indicates a high order of dependence between successive trials). For instance, the Markov chain of order m=0 assumes independence, and a Markov chain of order m=1 assumes that the occurrence of an error depends (to some extent) on the occurrence of an error on only the previous trial.

This report models the trials by a stationary first-order Markov chain. A stationary Markov chain is one that is independent of time. The first-order model results from defining the parameter, P_{11}, the conditional probability of a failure given that a failure occurred in the previous trial.

Suppose x_1, x_2, ..., x_n is a sequence of identically distributed random variables each of which can assume one of two values (0 for a success and 1 for a failure).

Then

$$P = P[x_i = 1] \text{ for } i = 1, 2, ..., n$$

and,

$$P_{11} = P[x_i = 1 | x_{i-1} = 1] \text{ for } i = 2, 3, ..., n.$$

We see that

- $P_{11} > P$ means 1's and 0's tend to cluster,

- $P_{11} = P$ means the trials are independent,

and

- $P_{11} < P$ means 1's and 0's tend to alternate.

Let s be the number of failures and r be the number of pairs of consecutive failures. For a sample of size n,

$$s = \sum_{i=1}^{n} x_i, \text{ and } r = \sum_{i=2}^{n} x_{i-1} x_i .$$

To determine the sample size required to estimate the mean of the failure probability parameter with the desired precision, it is necessary to specify two measures of precision:

- The relative precision with which the estimate should approximate the failure probability mean.

- The level of confidence that the relative precision has been achieved.

The method to determine the required sample size depends upon whether or not the maximum value of P_{11} (called P_{11max}) is known:[3]

A.1.2.1. P_{11max} is known: The number of failures to be tested is

$$s_0 = s_{ind} \cdot \frac{1 + P_{11max}}{1 - P_{11max}}$$

(A-9)

[SSDFLR]

where s_{ind} is a function of the specified relative precision and confidence level. This function is called SINDF in subroutine SSDFLR.

A.1.2.2. P_{11max} is not known:

A.1.2.2.1. Specify, b, the absolute precision with which P_{11} is to be determined. For example, if P_{11} is needed only to a rough approximation, specify b=0.5. We are concerned only with P_{11} being too large. For example, we might estimate P_{11} to be 0.2 when it is actually 0.7. On the other hand, if P_{11} is needed quite precisely (or thought to be near zero), specify b=0.1. The smaller is b, the larger is the sample size required to estimate P_{11}.

_____

[3]Since testing is halted because a prescribed number of failures (not trials) has been achieved, failures have the negative binomial distribution.

A.1.2.2.2. Specify the one-sided confidence level for P_{11}, and determine its associated upper 100α percentage point, u_α, of the normal density.[4]

A.1.2.2.3. The number of failures necessary to estimate P_{11} as close as or closer than b with the desired level of confidence is

$$s' = \left(\frac{u_\alpha}{2b}\right)^2.$$ (A-10)

[SSDFLR]

A.1.2.2.4. Conduct a preliminary test that generates at least s' failures.

A.1.2.2.5. If the test resulted in s failures (i.e., $s \geq s'$) and r pairs of consecutive failures, compute

$$P_{11U} = \frac{2s\hat{P}_{11} + u_\alpha^2 + \sqrt{(2s\hat{P}_{11} + u_\alpha^2)^2 - 4s\hat{P}_{11}^2(s + u_\alpha^2)}}{2(s + u_\alpha^2)}$$ (A-11)

[ANZFLR]

where

$$\hat{P}_{11} = \frac{r}{s - \frac{s}{n}}.$$ (A-12)

[ANZFLR]

A.1.2.2.6. Determine

$$s_0 = s_{ind} \cdot \frac{1 + P_{11U}}{1 - P_{11U}}.$$ (A-13)

[SSDFLR]

A.1.2.2.7. Now,

- If $s_0 > s'$, test

  $s = s_0 - s'$ (A-14)

  additional failures. [SSDFLR]

- If $s_0 \leq s'$, no more observations are required.

A.2 Analysis of a Test

Analysis consists of estimating the population mean and determining the smallest confidence interval about that estimate for the given confidence level.

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[4]The confidence level of 95% is used in the computer program, so $u_\alpha = 1.645$.

A.2.1 Time Parameters

There are two types of time parameters: delay and rate parameters. The delay can be the total delay or the fraction of the total delay for which the user is responsible. The rate is the number of elements[5] transferred during a certain period of time (Referred to in ANS X3.102 as the input/output time).

A.2.1.1. Total delay. The population mean delay, W, is estimated from n delays by $\bar{w}$ in (A-18), and the $100(1-2\alpha)\%$ confidence limits for W are

$$W_L = \bar{w} - t_{n-1,\alpha} \cdot (s/\sqrt{n})\left(\frac{1 + r_1(w)}{1 - r_1(w)}\right)^{1/2} ,$$

and   (A-15)

                                                                                        [ANZTIM]

$$W_U = \bar{w} + t_{n-1,\alpha} \cdot (s/\sqrt{n})\left(\frac{1 + r_1(w)}{1 - r_1(w)}\right)^{1/2} .$$

where r_1 is defined in (A-3), but replace n' by n. The parameters s and $r_1(w)$ are computed in equations (A-5) and (A-3) respectively, and $t_{n-1,\alpha}$ is the upper $100\alpha\%$ point of the Student t distribution for $n-1$ degrees of freedom.

A.2.1.2. User-responsible fraction of the delay. If W is the population mean total delay and T is the population mean user-responsible delay, the population mean of the user-responsible fraction of the delay is

$$p = \frac{T}{W} .$$ (A-16)

                                                                                        [-]

An unbiased estimate of p is

$$p'' = \frac{\bar{t}}{\bar{w}} \cdot \left[1 + \frac{1}{n}\left(\frac{s_{tw}}{\bar{t}\bar{w}} - \frac{s_w^2}{\bar{w}^2}\right)\right]$$ (A-17)

                                                                                        [ANZTIM]

where

$$\bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i ,$$ (A-18)

                                                                                        [ANZTIM]

---

[5]These elements are usually (but need not be) user information bits or blocks.

$$\bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_i \ , \qquad \text{(A-19)}$$

$$s_t^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} (t_i - \bar{t})^2 \right] \cdot \left(\frac{1 + r_1(t)}{1 - r_1(t)} \right) \ , \qquad \text{(A-20)}$$

$$s_w^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} (w_i - \bar{w})^2 \right] \cdot \left(\frac{1 + r_1(w)}{1 - r_1(w)} \right) \ , \qquad \text{(A-21)}$$

$$s_{tw} = \frac{1}{n-1} \left[\sum_{i=1}^{n} (w_i - \bar{w})(t_i - \bar{t}) \right] \cdot \left(\frac{1 + r_1(t)}{1 - r_1(t)} \right)^{1/2} \cdot \left(\frac{1 + r_1(w)}{1 - r_1(w)} \right)^{1/2} , \qquad \text{(A-22)}$$

and

$$s_{p''}^2 = \frac{p''^2}{n} \left(\frac{s_t^2}{\bar{t}^2} + \frac{s_w^2}{\bar{w}^2} - 2 \frac{s_{tw}}{\overline{tw}} \right) \ . \qquad \text{(A-23)}$$

The confidence limits for p are

$$P_L = p'' - u_\alpha s_{p''} \ ,$$

and

$$\text{(A-24)}$$

$$P_U = p'' + u_\alpha s_{p''}$$

where u_α is the upper $100\alpha\%$ point of the normal density.

A.2.1.3.   Rate.   If B is the number of elements (e.g., bits) successfully transferred during a performance measurement period and w is the duration of the period (the input/output time, a delay), the transfer rate for a particular period is

$$r = \frac{B}{w} \ . \qquad \text{(A-25)}$$

[-]

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The transfer rate of the communication system is

$$R = \lim_{w \to \infty} \frac{B}{w} \quad .$$

(A-26)

[-]

It can be estimated by

$$\hat{R} = \frac{\bar{B}}{\bar{w}}$$

(A-27)

[ANZTIM]

where

$$\bar{B} = \frac{1}{n} \sum_{i=1}^{n} B_i \quad ,$$

(A-28)

[ANZTIM]

and $\bar{w}$ is determined from (A-18). Each B_i should be nearly equal and each w_i should be allowed to vary.

The confidence limits for the system transfer rate, R, are

$$R_L = \frac{\bar{B}}{\bar{W}_U} \quad ,$$

and

(A-29)

[ANZTIM]

$$R_U = \frac{\bar{B}}{\bar{W}_L}$$

where W_L and W_U are determined in (A-15).

A.2.2 Failure Probability Parameters

Suppose P_L and P_U are the lower and upper confidence limits for P. We seek a 100(1-2α) percent confidence interval for P such that

$$\sum_{i=s}^{n} f(i|P_L, P_{11}, n) = \alpha, \quad \text{and} \quad \sum_{i=0}^{s} f(i|P_U, P_{11}, n) = \alpha$$

where $f(i|P, P_{11}, n)$ is the probability function of s with parameters P, P_{11}, and n.

If P_{11} is known, these sums determine the exact confidence limits for P.

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However, the procedure requires excessive computer time and storage for n over, say, 150. Furthermore, for a large sample size and small probabilities, exact confidence limits are unnecessary.

When the number of failures exceeds one, the confidence limits can be approximated satisfactorily by using the normal approximation and the Poisson approximation; these two approximations are then averaged. To obtain the normal approximation, the sums (above) are replaced by the normal integral with the mean and variance of s (Crow and Miles, 1977). For small P, the binomial distribution (for the number of failures) can be well approximated by the Poisson distribution as modified by Anderson and Burstein (Crow and Miles, 1977).

Analysis of the failure probability parameters involves estimating the mean failure rate, P, and the upper and the lower confidence limits, P_U and P_L. The unbiased estimate of the mean failure probability is

$$\hat{P} = \frac{s}{n}$$

(A-30)

[ANZFLR]

where s is the number of failures, and n is the sample size.

Formulas for the $100(1-2\alpha)\%$ confidence limits for P depend upon whether the number of failures exceeds 1 or not.

A.2.2.1. Number of failures exceeds 1.

In this case, the confidence limits for P are the average of the $100(1-2\alpha)\%$ confidence limits for an approximation to the normal density and an approximation to the Poisson density:

$$P_L = \frac{P_{LN} + P_{LP}}{2} ,$$

and

(A-31)

[ANZFLR]

$$P_U = \frac{P_{UN} + P_{UP}}{2} .$$

As seen in (A-37) and (A-41), both approximations utilize the quantity, $\hat{F}$, which is given by the formula

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$$\hat{F} = \left[1 + \frac{2\hat{\rho}}{n(1-\hat{\rho})} \cdot \left(n - \frac{1-\hat{\rho}^n}{1-\hat{\rho}} \right) \right]^{1/2}$$ (A-32)

[ANZFLR]

where

$$\hat{Q} = 1 - \hat{P} ,$$ (A-33)

[ANZFLR]

$$\hat{P}_{11} = \frac{r}{s - \hat{P}} ,$$ (A-34)

[ANZFLR]

and

$$\hat{\rho} = \frac{(\hat{P}_{11} - \hat{P})}{\hat{Q}} .$$ (A-35)

[ANZFLR]

A.2.2.1.1. Normal approximation confidence limits. These limits are given by

$$P_{LN} = \frac{nV + 2s - 1 - R_-}{2n(1+V)} ,$$

and

$$P_{UN} = \frac{nV + 2s + 1 + R_+}{2n(1+V)}$$

(A-36)

[ANZFLR]

where

$$\hat{\sigma}_{\hat{P}} = \sqrt{\frac{\hat{P}\hat{Q}}{n}} \cdot \hat{F} ,$$ (A-37)

[ANZFLR]

$$V = \frac{(nu_\alpha \hat{\sigma}_{\hat{P}})^2}{s(n-s)} ,$$ (A-38)

[ANZFLR]

---

[6] Often ρ is very small and n is very large. In such cases, ρ^n would be small enough to cause a fatal error in execution of the computer program. To avoid this, the program assigns to ρ^n the maximum of ρ^n or 1×10^{-20}. See subroutine LIMIT in Appendices B and C.

$$R_+ = [(nV + 2s + 1)^2 - (2s + 1)^2(1 + V)]^{1/2},$$ (A-39)

$$R_- = [(nV + 2s - 1)^2 - (2s - 1)^2(1 + V)]^{1/2},$$ (A-40)

and u_α is the upper $100\alpha\%$ point of the normal density.

A.2.2.1.2. Poisson approximation confidence limits. These confidence limits depend upon whether P_{LP} exceeds zero or not:

• if $P_{LP} \geq 0$,

$$P_{LP} = \hat{P} - (\hat{P} - P_{LI}) \cdot \hat{F} ,$$

and (A-41)

$$P_{UP} = \hat{P} + (P_{UI} - \hat{P}) \cdot \hat{F} .$$

• if $P_{LP} < 0$,

$$P_{LP} = 0 ,$$

and (A-42)

$$P_{UP} = (P_{UI} - P_{LI}) \cdot \hat{F} .$$

The confidence limits, P_{LI} and P_{UI}, are approximate confidence limits for P assuming the trials are independent (i.e., assuming $P_{11} = P$):

$$P_{LI} = \frac{L}{n - (\frac{s - 1 - L}{2})} ,$$

and (A-43)

$$P_{UI} = \frac{U}{n + d + (\frac{U - s}{2})}$$

where L and U are confidence limits for the mean of a Poisson distribution and d is a numerical adjustment. (U, L, and d are determined from tables in subroutine POISS.)

A.2.2.2. The number of failures is 0 or 1. In this case the confidence limits are obtained from the cumulative probability function of s

(i.e., $f(0|P,P_{11},n) = \alpha$, and $\sum_{i=0}^{1} f(i|P,P_{11},n) = \alpha$):

$$P_L = 0 \ ,$$

and

$$P_U = \frac{1 - x}{2 - P_{11} - x} \quad {}^{7}$$

where

$$x = \begin{cases} \left(\dfrac{\alpha}{Q_U}\right)^{\frac{1}{n-1}} & \text{for } s=0 \\[4ex] \left(\dfrac{\alpha \, Q_U}{1 + z_2 P_U - z_1 P_U^2}\right)^{\frac{1}{n-3}} & \text{for } s=1 \ , \end{cases}$$

$$z_1 = (n - 1)(1 - P_{11})^2 - 1 \ ,$$

and

$$z_2 = (n - 2)(1 - P_{11})^2 - 2 \ .$$

The value of P_U is obtained by iteration. The first value of Q_U is

$$Q_U = 1 - P_{UP}$$

where

$$P_{UP} = (P_{UI} - P_{LI}) \cdot \hat{F} \ .$$

(This is the upper Poisson approximate confidence limit when $s > 1$, and $P_{LP} < 0$, but $\hat{P}_{11}$ is replaced by P_{11} in (A-35) when computing F.) Subsequently,

---

[7] See Table 5.

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$$Q_U = 1 - P_U \qquad\qquad\qquad (A-49)$$

where P_U was obtained in the previous iteration (i.e., if indices are used, $Q_{U(i)} = 1 - P_{U(i-1)}$ for i=2, 3,...). Iteration continues until

$$1 - 10^k < \frac{P_{U(i)}}{P_{U(i-1)}} < 1 + 10^{-k} \qquad\qquad (A-50)$$

where k > 0. In this computer program, k = 4.

A.3 Analysis of Multiple Tests

If a performance parameter is measured more than once, it may be possible to group the data from the multiple tests, and, by virtue of the larger sample size, obtain a better estimate of the population mean. However, the data can be grouped only if there is no reason to reject the hypothesis that they come from the same population.

The following two sections provide the formulas necessary to test this hypothesis for time and failure probability parameters, respectively.

A.3.1 Time Parameters

To determine if the data from multiple tests can be combined, determine the F-statistic and compare it with the corresponding percentage point of the F distribution.

Suppose there are m tests and n_i delays from each test. The total number of delays is

$$N = \sum_{i=1}^{m} n_i . \qquad\qquad\qquad (A-51)$$

The jth observation from the ith test is denoted by w_{ij}; the estimate of the mean delay during the ith test is

$$w_i \cdot = \frac{1}{n_i} \sum_{j=1}^{n_i} w_{ij} . \qquad\qquad\qquad (A-52)$$

Compute

$$s_e^2 = \frac{1}{N-m} \cdot \sum_{i=1}^{m} \sum_{j=1}^{n_i} (w_{ij} - w_{i\cdot})^2 .$$

(A-53)

[FTEST]

The estimate of the population mean for all i and j is

$$\bar{w} = \frac{1}{N} \sum_{i=1}^{m} \sum_{j=1}^{n_i} w_{ij} .$$

(A-54)

[FTEST]

Compute

$$s_1^2 = \frac{1}{m-1} \sum_{i=1}^{m} n_i (w_{i\cdot} - \bar{w})^2 .$$

(A-55)

[FTEST]

Now, the F-statistic,

$$F = \frac{s_1^2}{s_e^2} ,$$

(A-56)

[FTEST]

is compared with the F distribution (at the upper 95% in this computer program) with m-1 and N-m degrees of freedom.

If the F-statistic is less than the F distribution percentage point, there is no reason to reject the hypothesis that the samples come from the same population. Hence, the samples are grouped and the methods of Section A.2.1 employed to obtain a better estimate of the population mean delay or rate.

If the F-statistic exceeds the corresponding value of the F distribution, the F test can be conducted again while omitting data from one or more tests that are thought to cause rejection of the hypothesis (of course, data from at least two tests must remain).

A.3.2 Failure Probability Parameters

If there are k tests of a failure probability parameter, the number of observations, failures, and pairs of consecutive failures can be denoted by n_j, s_j, and r_j, respectively, where j=1,2,...,k. Moreover, the total number of each can be denoted by

$$N = \sum_{j=1}^{k} n_j, \quad S = \sum_{j=1}^{k} s_j, \quad \text{and} \quad R = \sum_{j=1}^{k} r_j ,$$

(A-57)

[CHISQR]

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respectively.

Then

$$X^2_{k-1}(P_{11}) = \frac{\sum\limits_{j=1}^{k} (s_j - \bar{P}) \cdot (\hat{P}_{11j} - \hat{P}_{11ave})^2}{\hat{P}_{11ave}(1 - \hat{P}_{11ave})}, \quad 0 < \hat{P}_{11ave} < 1$$

(A-58)

[CHISQR]

is the chi-squared statistic, where

$$\bar{P} = \frac{S}{N},$$

(A-59)

[CHISQR]

$$\hat{P}_{11ave} = \frac{R}{S - k\bar{P}}, \quad ^8$$

(A-60)

[CHISQR]

and

$$\hat{P}_{11j} = \frac{r_j}{s_j - \hat{P}_j}.$$

(A-61)

[CHISQR]

If $X^2_{k-1}(P_{11})$ exceeds the 5% point of the chi-squared distribution for k-1 degrees of freedom, then there is a question that the data from the k tests should be combined. In this case the chi-squared test can be conducted again while omitting data from one or more tests that are thought to cause rejection of the hypothesis. (Of course, data from at least two tests must remain.)

If $X^2_{k-1}(P_{11})$ does not exceed the 5% point of the chi-squared distribution with k-1 degrees of freedom, compute

$$X^2_{k-1}(P) = \sum\limits_{j=1}^{k} \frac{(\hat{P}_j - \bar{P})^2}{\hat{\sigma}_{P_j}^2},$$

(A-62)

[CHISQR]

where

$$\hat{P}_j = \frac{s_j}{n_j},$$

(A-63)

[CHISQR]

_____

[8]The chi-squared test cannot be used when R=0.

$$\bar{F}_j = \left[1 + \frac{2\bar{\rho}}{n_j(1 - \bar{\rho})} \cdot \left(n_j - \frac{1 - \bar{\rho}^{n_j}}{1 - \bar{\rho}} \right) \right]^{1/2}$$ (A-64)

[CHISQR]

$$\hat{\sigma}_{\hat{P}_j}^{2} = \frac{\bar{P}\bar{Q}}{n_j} \cdot \bar{F}_j^2$$ (A-65)

[CHISQR]

$$\bar{Q} = 1 - \bar{P} ,$$ (A-66)

[CHISQR]

and

$$\bar{\rho} = \frac{\hat{P}_{11ave} - \bar{P}}{\bar{Q}} .$$ (A-67)

[CHISQR]

The chi-squared statistic for P is compared with the chi-squared distribution just as it was for P_{11}. If neither $X_{k-1}^2(P_{11})$ nor $X_{k-1}^2(P)$ exceeds the 5% point, there is no reason to reject the hypothesis that the data from the k tests come from the same population.

Compute the confidence limits exactly as in Section A.2.2, substituting

$\bar{P}$	for	$\hat{P}$,
$\hat{P}_{11ave}$	for	$\hat{P}_{11}$,
$\bar{\rho}$	for	ρ ,
R	for	r ,
S	for	s ,

and

N	for	n .

REFERENCES

Crow, E. L., and M. J. Miles (1977), Confidence limits for digital error rates from dependent transmissions, Office of Telecommunications Report 77-118, March. Extension of Crow's 1974 report to the case of dependent errors. Two-state (Markov) error mode assumed.

APPENDIX B: STRUCTURED DESIGN DIAGRAMS OF THE COMPUTER PROGRAM

This Appendix is the set of structured design diagrams that describes the logic of the computer program, STATDA. Subroutines involved with Sample Size Determination begin with SSD. Those involved with Analyzing the data begin with ANZ. Those involved with Time parameters end with TIM, and those involved with Failure probability parameters end with FLR. Subroutine MULTIP analyzes data from multiple tests.

Figure B-1 shows the relationship among the subroutines (with the exception of subroutines that allow entry of data and responses and allow correction of data incorrectly entered from a keyboard). Figure B-2 is the diagram of the main program. The remaining figures are diagrams of the subroutines listed alphabetically by name.[1] In these figures, diamonds indicate decisions, rectangles indicate arithmetic operations, and parallelograms indicate input (output is omitted). The tables in the figures link the parameter notation of the computer program (Appendix C) with that of the mathematical formulas (Appendix A).

---

[1]Three subroutines (called ENTERA, ENTERI, and ENTERX) that allow entry of data and responses from a keyboard are omitted. Also, if test data for time parameters are entered from a file (instead of a keyboard) only trivial changes occur in Figures B-2 and B-7.
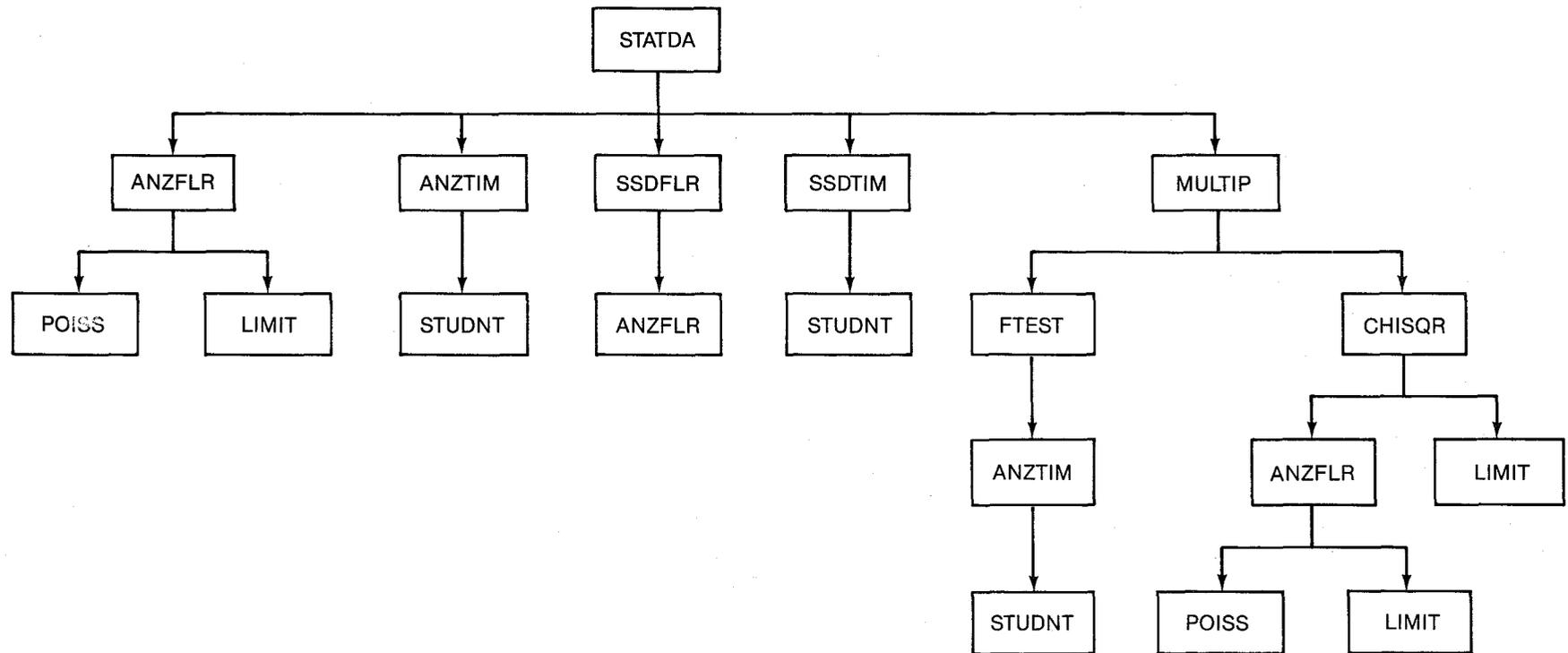
Figure B-1.   Organization of the subroutine used by the main program, STATDA.
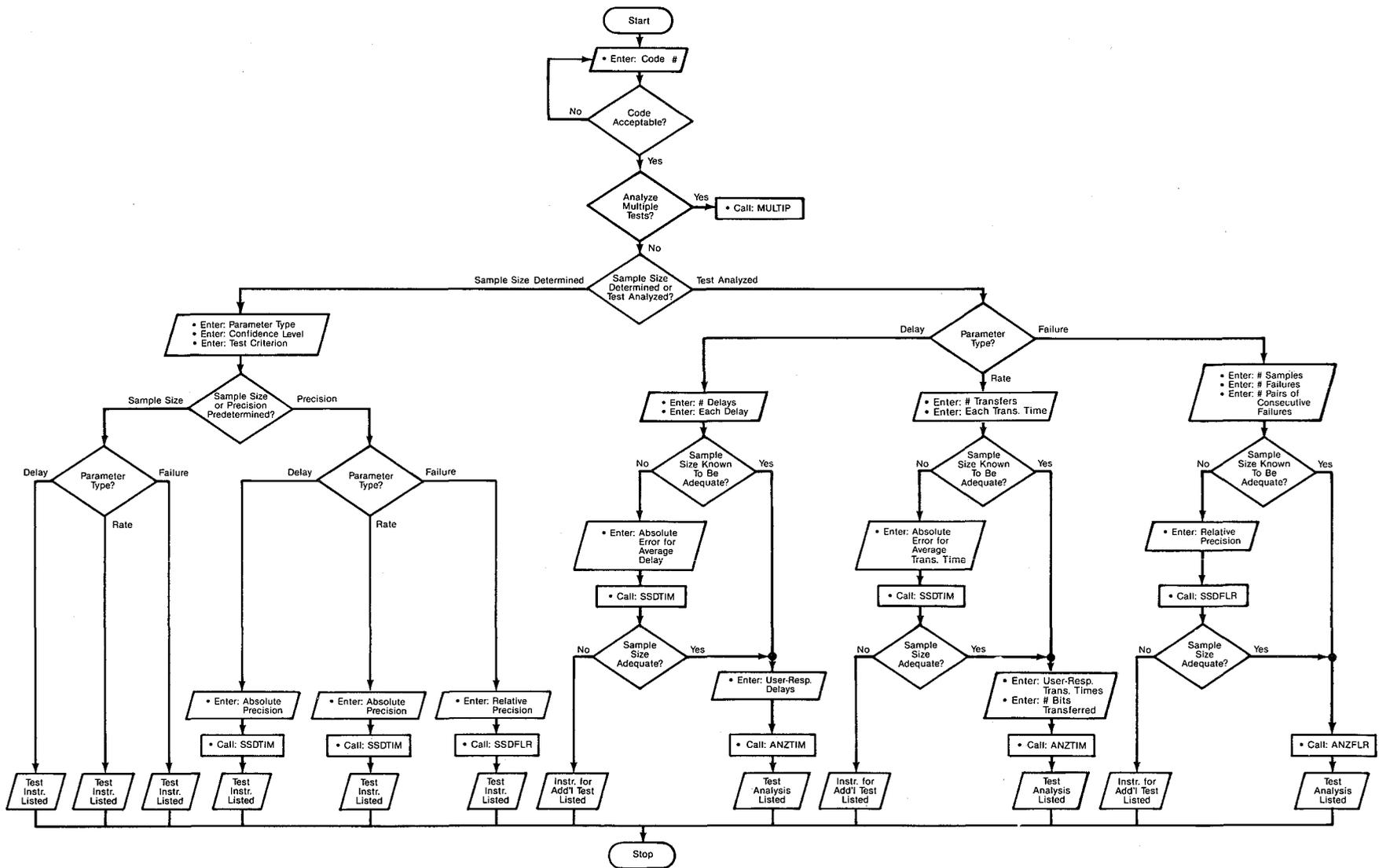
Figure 3-2. Structured design diagram of the main program, STATDA.
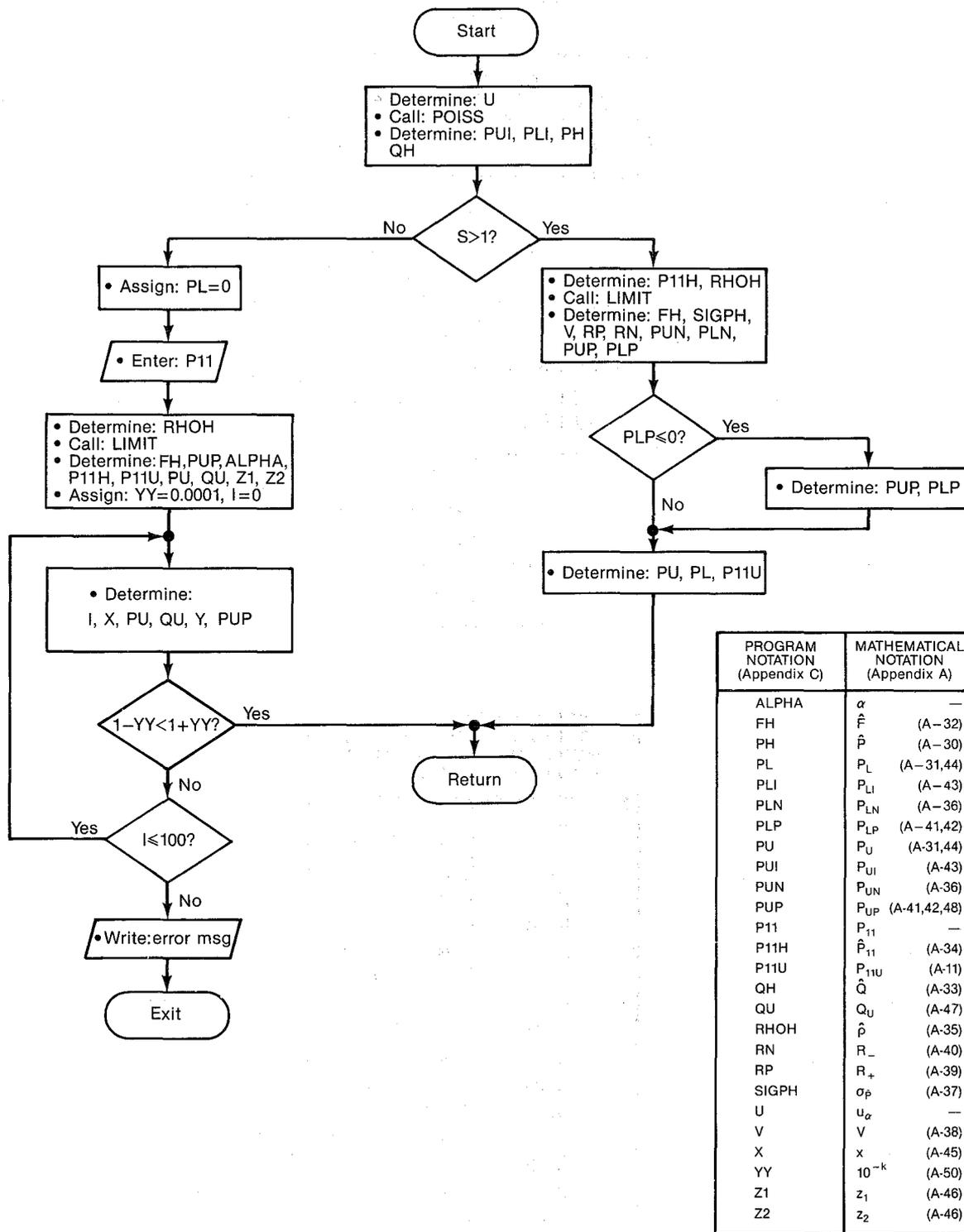
Figure B-3. Structured design diagram of ANZFLR.
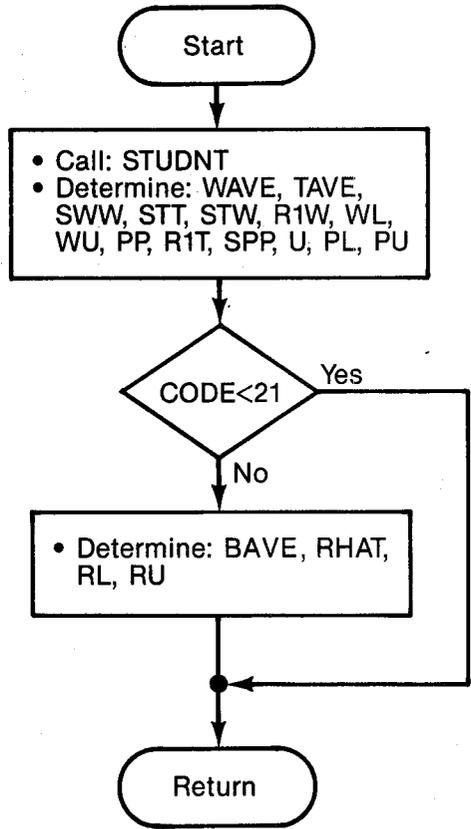
PROGRAM NOTATION (Appendix C)	MATHEMATICAL NOTATION (Appendix A)	
BAVE	$\overline{B}$	(A-28)
PL	p_L	(A-24)
PP	p''	(A-17)
PU	p_U	(A-24)
RHAT	$\hat{R}$	(A-27)
RL	R_L	(A-29)
RU	R_U	(A-29)
R1T	$r_1(t)$	(A-3)
R1W	$r_1(w)$	(A-3)
SPP	$s_{p''}$	(A-23)
STT	s_t	(A-20)
STW	s_{tw}	(A-22)
SWW	s_w	(A-21)
TAVE	$\overline{t}$	(A-19)
U	u_α	—
WAVE	$\overline{w}$	(A-18)
WL	w_U	A-15)
WU	w_L	A-15)

Figure B-4. Structured design diagram of ANZTIM.
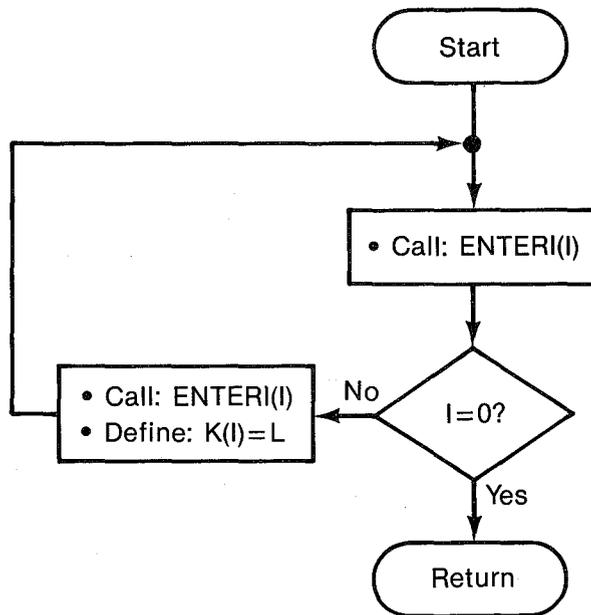
Figure B-5.   Structured design diagrams of CHECKI and CHECKX.
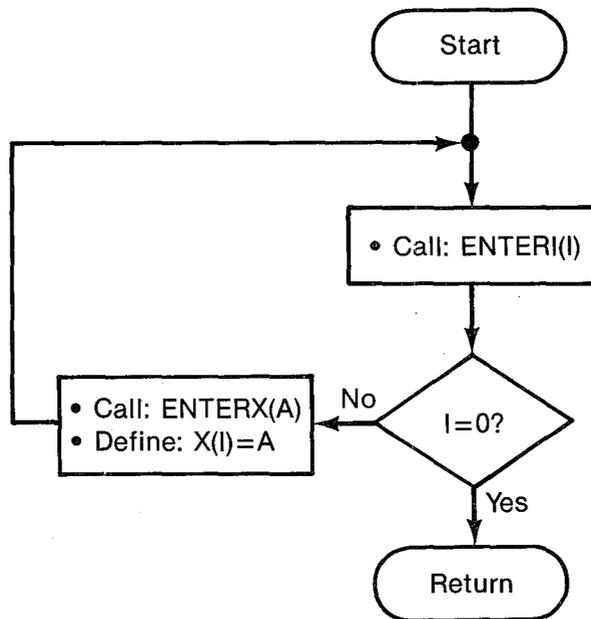
Figure B-5. Continued.
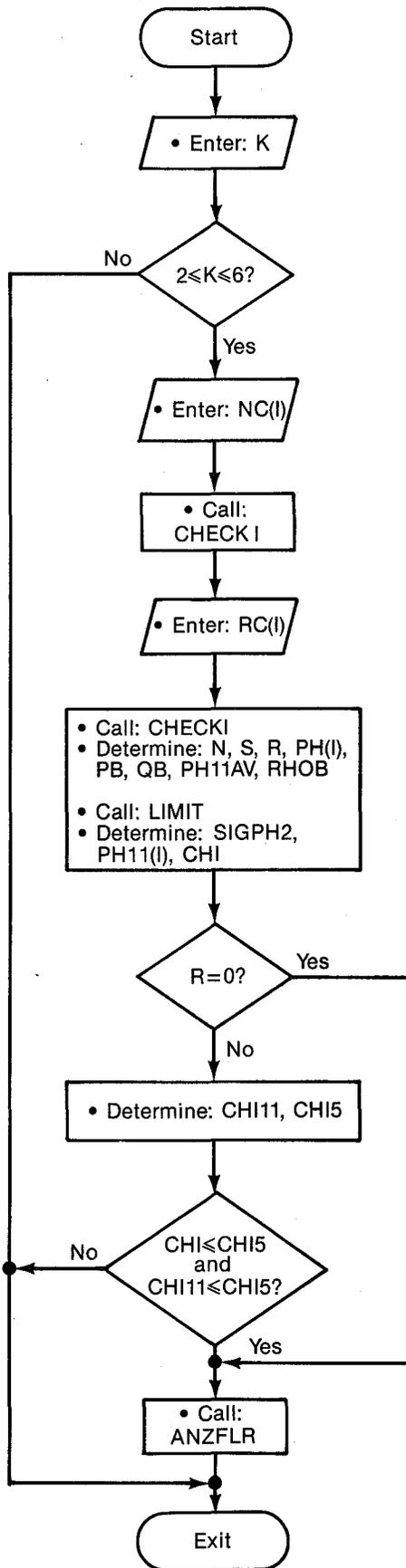
PROGRAM NOTATION (Appendix C)	MATHEMATICAL NOTATION (Appendix A)	
CHI	$\chi^2_{k-1}(P)$	(A-62)
CHI11	$\chi^2_{k-1}(P_{11})$	(A-58)
CHI5	—	—
FBJ	$\overline{F}_j$	(A-64)
K	k	—
N	N	(A-57)
NC(I)	n_j	—
PB	$\overline{P}$	(A-59)
PH(I)	$\hat{P}_j$	(A-63)
PH11(I)	$\hat{P}_{11j}$	(A-61)
PH11AV	$\hat{P}_{11AVE}$	(A-60)
QB	$\overline{Q}$	(A-66)
R	R	(A-57)
RC(I)	r_j	—
RHOB	$\overline{\rho}$	(A-67)
S	S	(A-57)
SC(I)	s_j	—
SIGPH2	$\hat{\sigma}^2_{P_j}$	(A-65)

Figure B-6. Structured design diagram of CHISQR.

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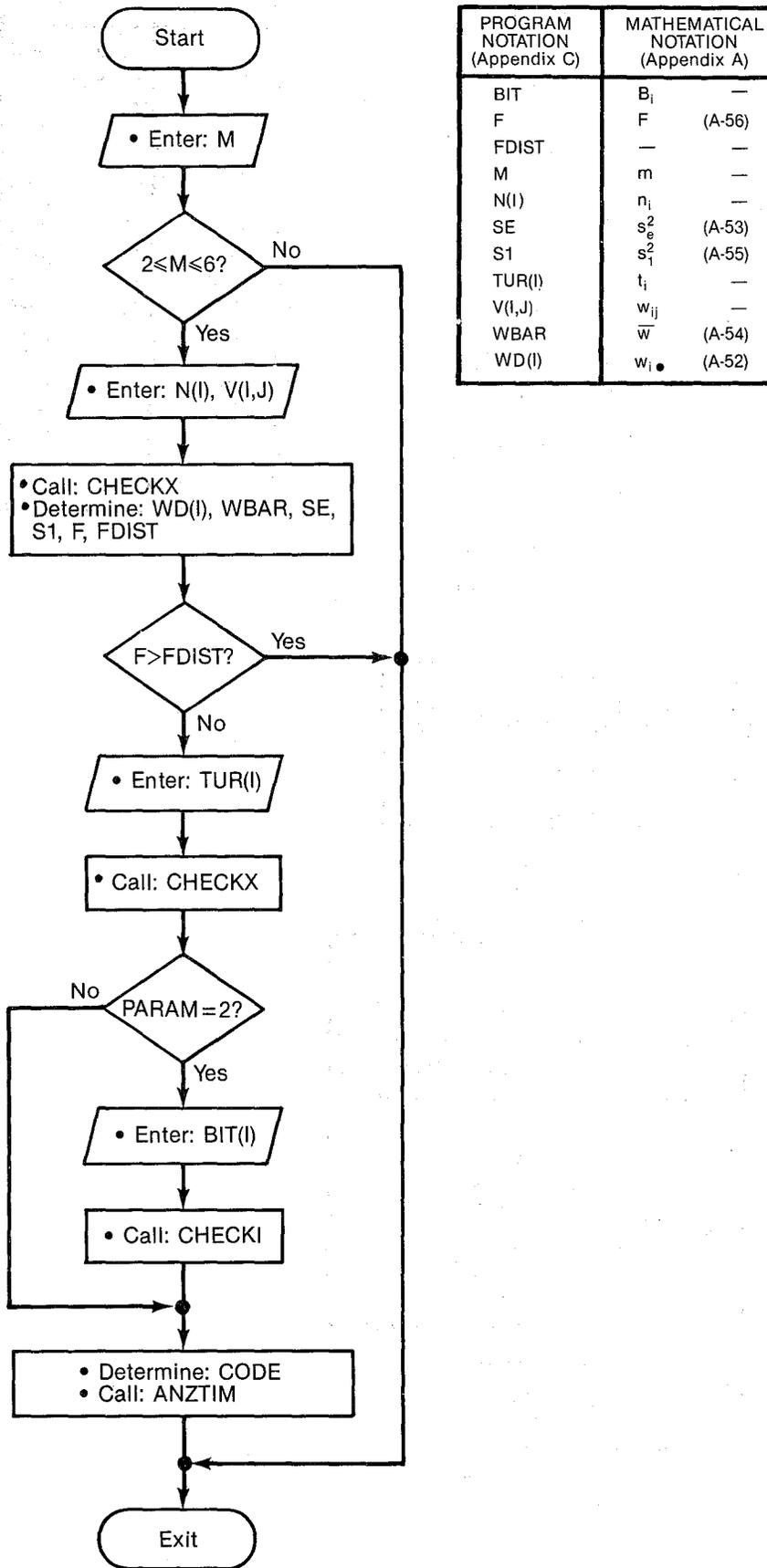
PROGRAM NOTATION (Appendix C)	MATHEMATICAL NOTATION (Appendix A)	
BIT	B_i	—
F	F	(A-56)
FDIST	—	—
M	m	—
N(I)	n_i	—
SE	s_e^2	(A-53)
S1	s_1^2	(A-55)
TUR(I)	t_i	—
V(I,J)	w_{ij}	—
WBAR	$\overline{w}$	(A-54)
WD(I)	$w_{i \bullet}$	(A-52)

Start

• Enter: M

$2 \leqslant M \leqslant 6$? — No

Yes

• Enter: N(I), V(I,J)

• Call: CHECKX
• Determine: WD(I), WBAR, SE, S1, F, FDIST

F>FDIST? — Yes

No

• Enter: TUR(I)

• Call: CHECKX

PARAM=2? — No

Yes

• Enter: BIT(I)

• Call: CHECKI

• Determine: CODE
• Call: ANZTIM

Exit

Figure B-7. Structured design diagram of FTEST.
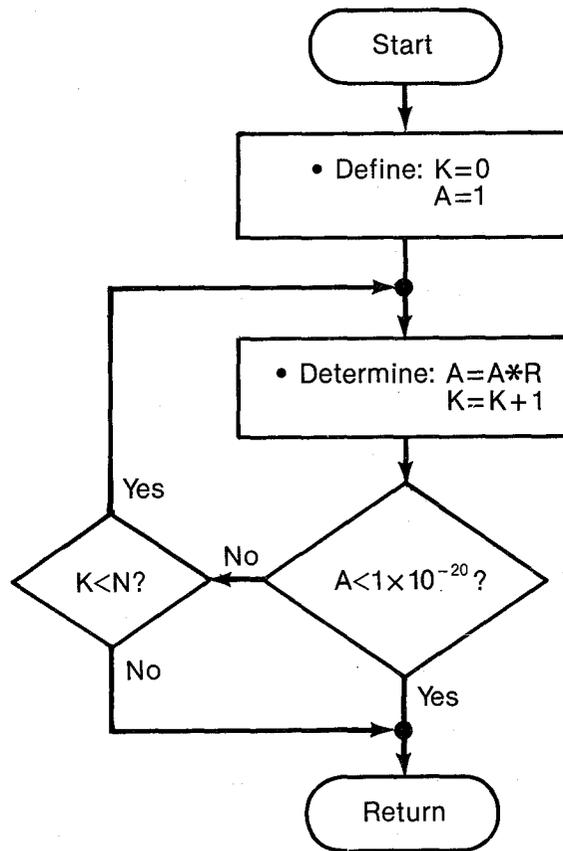
PROGRAM NOTATION (Appendix C)	MATHEMATICAL NOTATION (Appendix A)
N	$\left\{ \begin{array}{l} n \text{ in ANZFLR (A-32)} \\ n_i \text{ in CHISQR (A-64)} \end{array} \right.$
R	$\left\{ \begin{array}{l} \hat{p} \text{ in ANZFLR (A-32)} \\ \bar{p} \text{ in CHISQR (A-64)} \end{array} \right.$

Figure B-8. Structured design diagram of LIMIT.
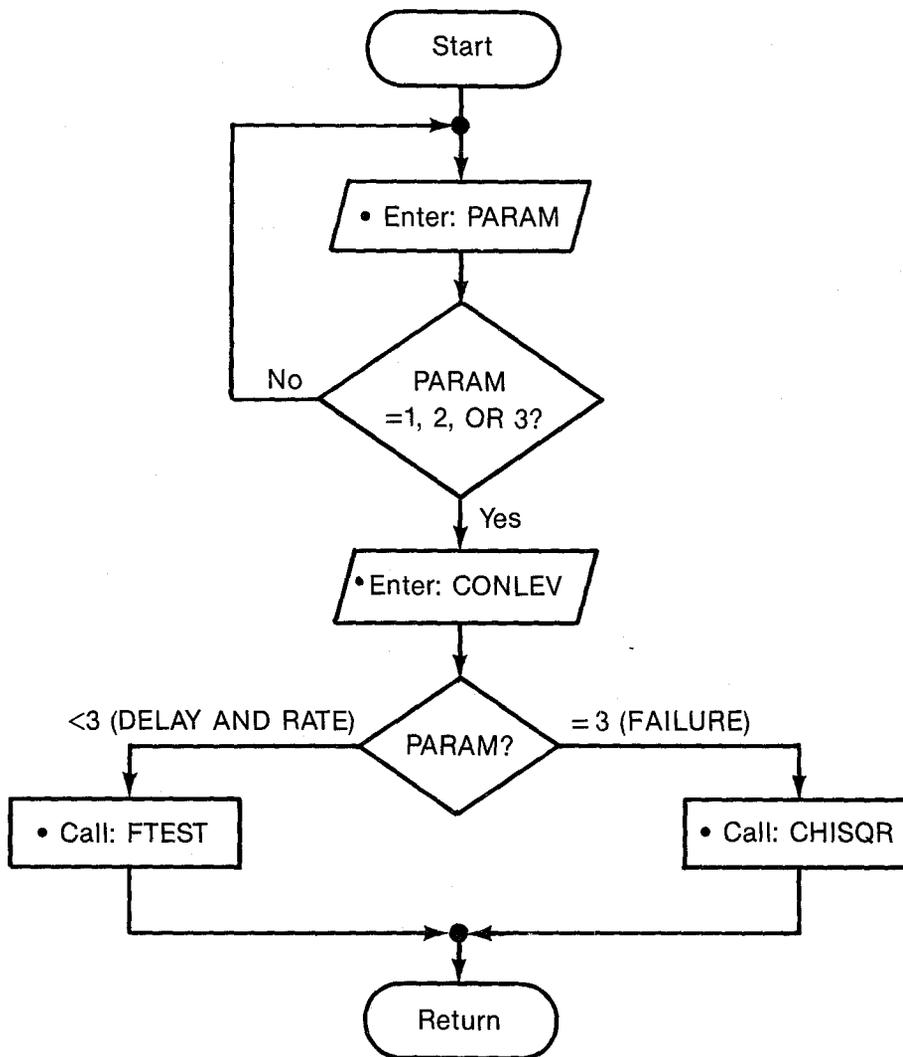
Figure B-9. Structured design diagram of MULTIP.
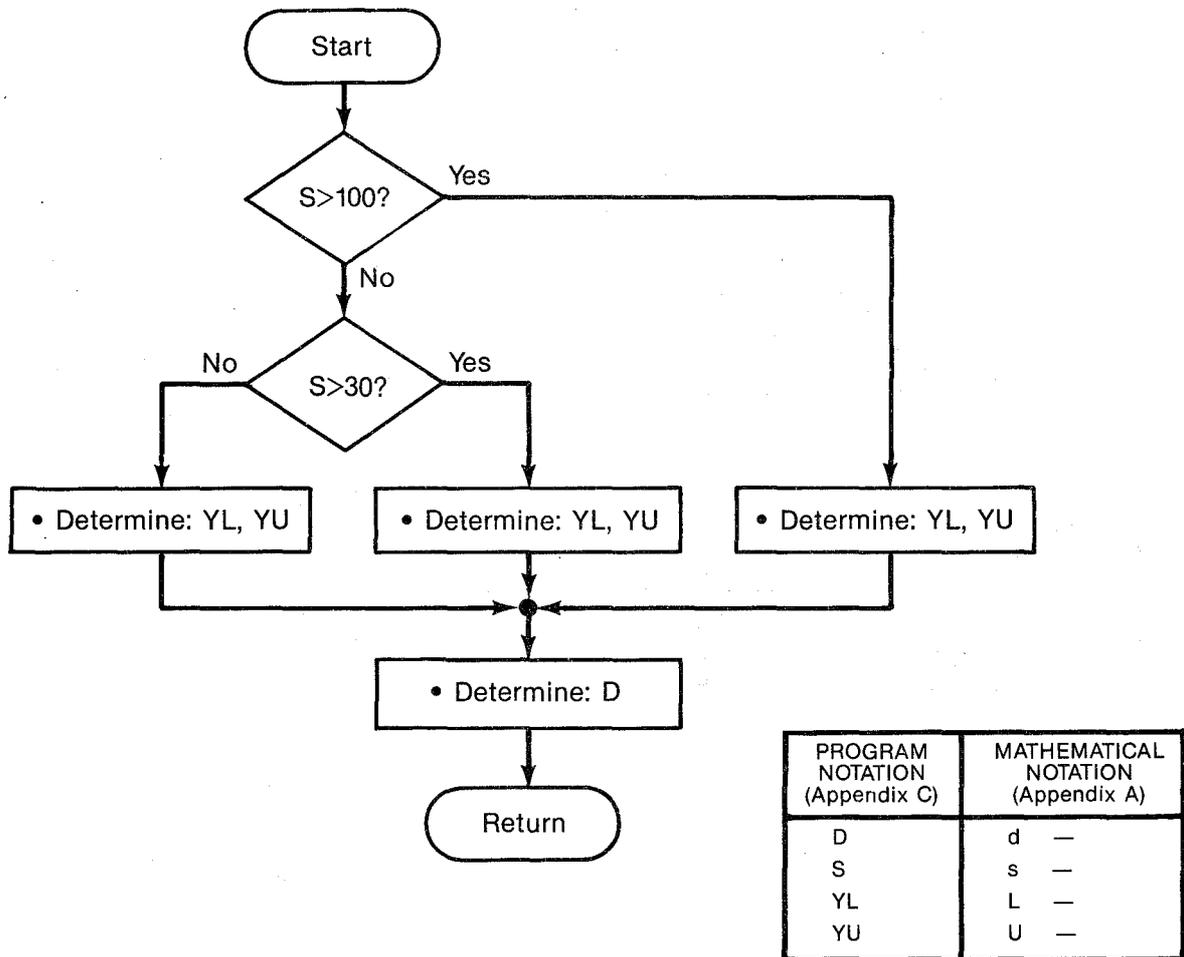
Figure B-10. Structured design diagram of POISS.
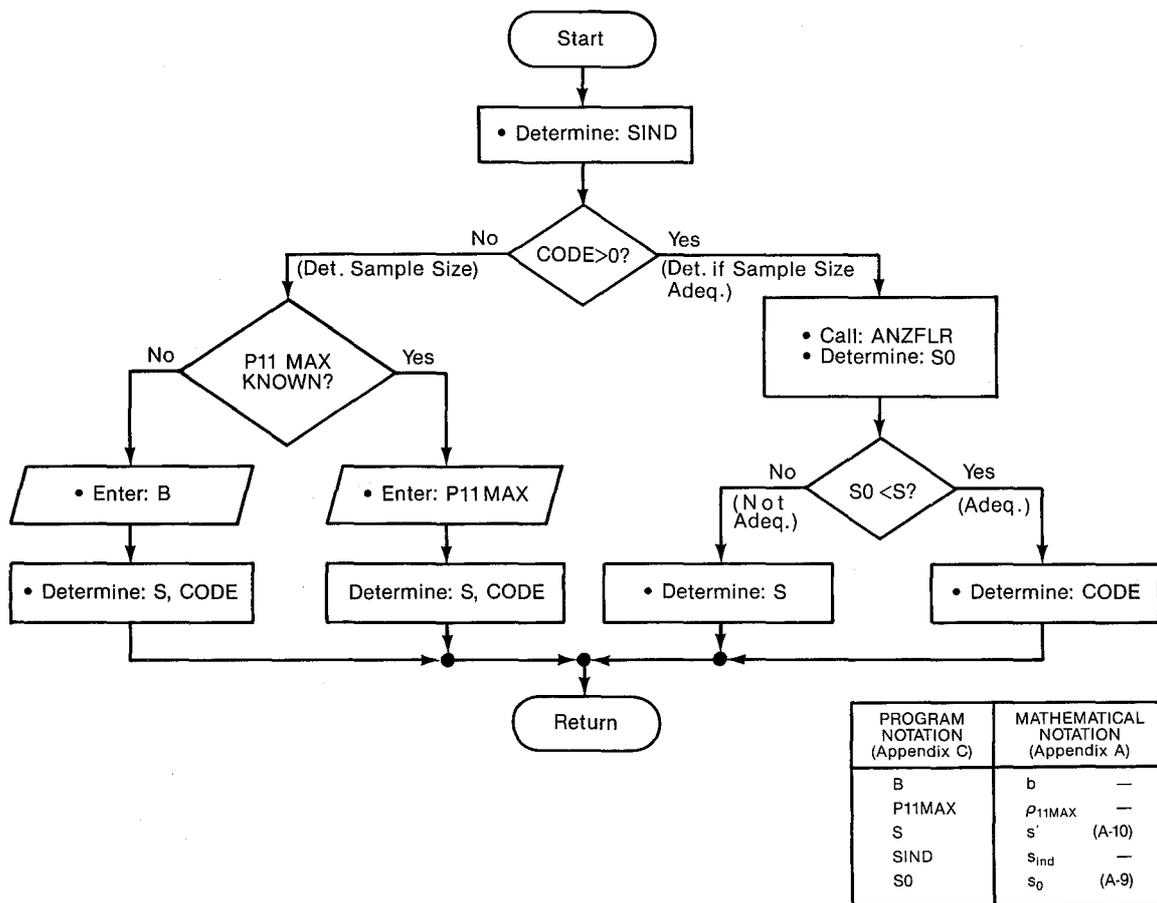
Figure B-11. Structured design diagram of SSDFLR.
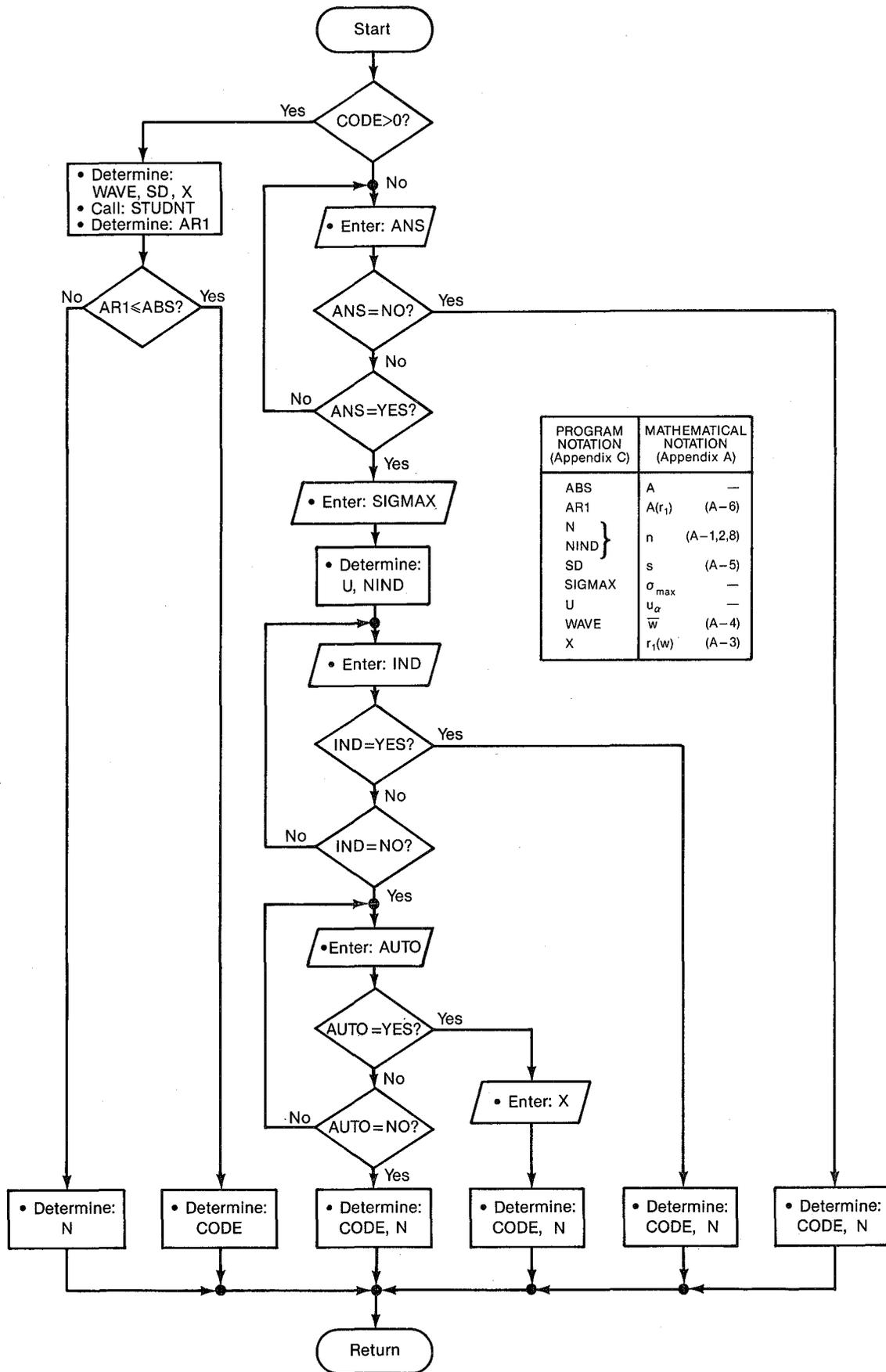
Figure B-12. Structured design diagram of SSDTIM.

Start

• Define: I=0

• Determine: I=I+1

$NP(I) \leqslant N-1 \leqslant NP(I+1)?$

No

Yes

• Determine: T

Return
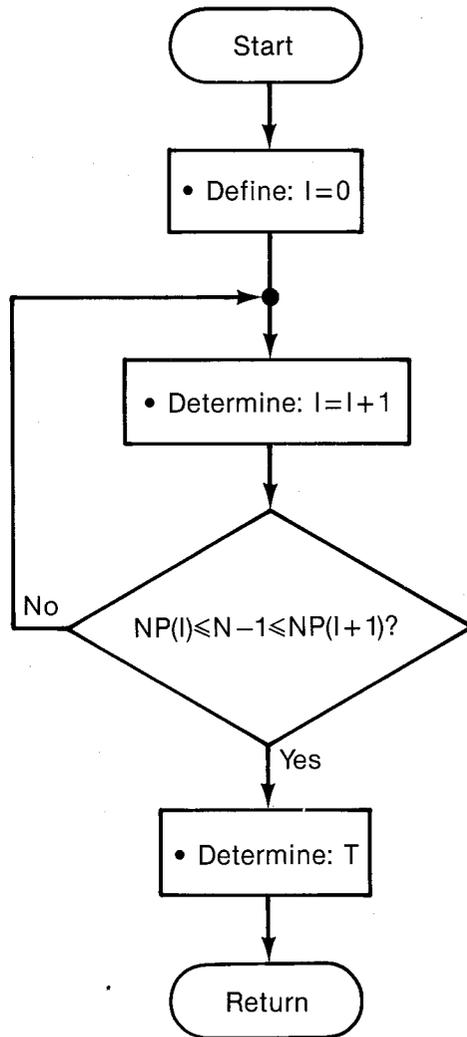
PROGRAM NOTATION (Appendix C)	MATHEMATICAL NOTATION (Appendix A)
N	n (A-1,2,8)
T	$t_{n-1,\alpha}$ —

Figure B-13. Structured design diagram of STUDNT.
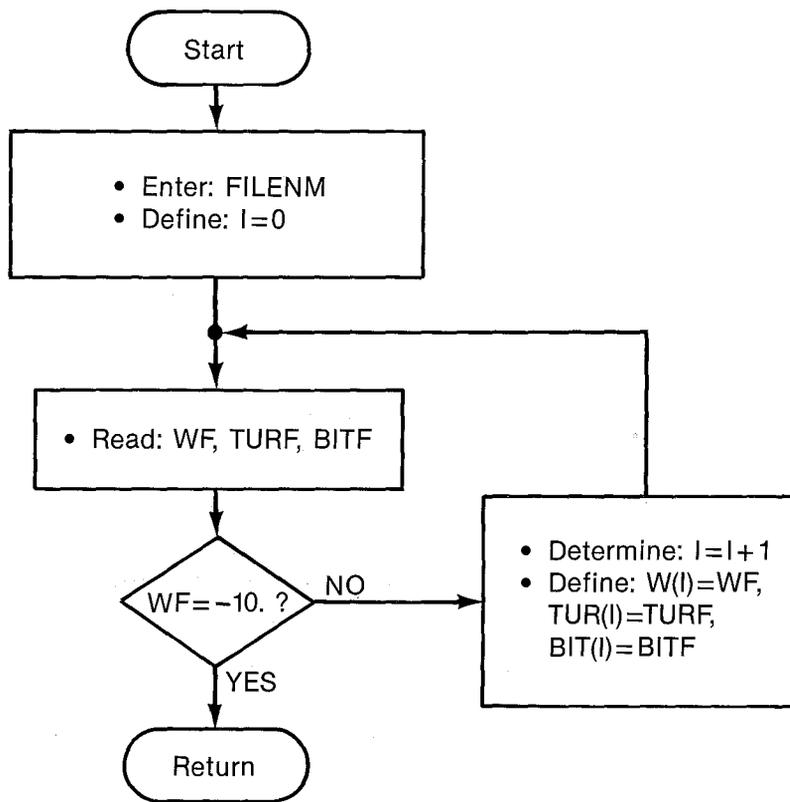
Figure B-14. Structured design diagram of XFILE and YFILE.

APPENDIX C: LISTING OF THE COMPUTER PROGRAM

This Appendix is the listing of the computer program for Statistical Design and Analysis (called STATDA).   It is written in FORTRAN 77 (but does not include the extensions).   Since the program does not use special library routines or peripheral devices, it is portable.   However, one possible exception has been found: Although the CLOSE and OPEN statements in subroutines ENTERA, ENTERI, and ENTERX are FORTRAN 77, they must be omitted when used by some systems.

STATDA is an interactive program, and both data and responses are entered through a terminal keyboard.   It contains 1382 lines of code and compiles in less than three seconds on a main frame, 60-bit word computer.   A copy of the program is available from the author at duplication cost.

The main program is listed first (Figure C-1).   It is followed by the subroutines listed in alphabetical order.   The number of each mathematical formula from Appendix A is shown in columns 73-78.   Table C-1 briefly describes the function of each subroutine.

Table C-1. Functions of Subroutines

Subroutine Name	Figure Number	Function of Subroutine
ANZFR	C-2	Analyzes failure probability parameter data
ANZTIM	C-3	Analyzes time parameter data
CHECKI CHECKX	C-4	Allows changing incorrect entries in a sequence of entered data to be corrected.
CHISQR	C-5	Computes chi squared test for multiple tests of the failure probability parameters
ENTERA ENTERI ENTERX	C-6	Allows entry of data and responses from a keyboard
FTEST	C-7	Computes F test for multiple tests of the time parameters
LIMIT	C-8	Prevents arithemetic underflow (and termination of execution)
MULTIP	C-9	Coordinates the tests of homogeneity of data from multiple tests
POISS	C-10	Determines Poisson approximation confidence limits
SSDFLR	C-11	Determines sample size for failure probability parameters
SSDTIM	C-12	Determines sample size for time parameters
STUDNT	C-13	Determines Student t statistic
XFILE YFILE	C-14	Allows entry of time parametr data from a file

```
      PROGRAM STATDA

C     THIS PROGRAM CAN BE ACCESSED FOR THREE PURPOSES:
C          *    TO DETERMINE THE SAMPLE SIZE FOR A TEST
C          *    TO ANALYZE A TEST
C          *    TO ANALYZE MULTIPLE TESTS
C     THIS PROGRAM ANALYZES THREE TYPES OF PARAMETERS:
C          *    DELAY PARAMETERS
C          *    RATE PARAMETERS
C          *    FAILURE PROBABILITY PARAMETERS
C     THIS PROGRAM ALLOWS YOU TO CHOOSE TWO LEVELS OF CONFIDENCE:
C          *    90%
C          *    95%
C
C     THE FOLLOWING IS A DIRECTORY OF CODE NUMBERS THAT ARE TO BE ENTERED
C     DEPENDING UPON THE PURPOSE OF ACCESS.
C
C           PURPOSE OF ACCESS                              CODE NUMBER
C
C     TO DETERMINE THE SAMPLE SIZE FOR A TEST                   0
C
C     TO ANALYZE A TEST
C        DELAY PARAMETERS
C           SAMPLE SIZE KNOWN TO BE ADEQUATE
C              90%                                             11
C              95%                                             12
C           TEST ADEQUACY OF THE SAMPLE SIZE
C              90%                                             13
C              95%                                             14
C        RATE PARAMETERS
C           SAMPLE SIZE KNOWN TO BE ADEQUATE
C              90%                                             21
C              95%                                             22
C           TEST ADEQUACY OF THE SAMPLE SIZE
C              90%                                             23
C              95%                                             24
C        FAILURE PROBABILITY PARAMETERS
C           SAMPLE SIZE KNOWN TO BE ADEQUATE
C              90%                                             31
C              95%                                             32
C           TEST ADEQUACY OF THE SAMPLE SIZE
C              90%                                             33
C              95%                                             34
C
C     TO ANALYZE MULTIPLE TESTS                                40

      COMMON/SQ/CODE,PARAM,W(200),TUR(200),BIT(200)
      INTEGER  PARAM, CODE, CONLEV, CRIT,   REL, S, R
  100 WRITE(*,505)
      CALL ENTERI(CODE)

C     ************    CHECK THE ACCEPTABILITY OF THE CODE    ************

      LODE=0
      IF(CODE.EQ.0)LODE=1
      IF((CODE.GE.11).AND.(CODE.LE.14))LODE=1
      IF((CODE.GE.21).AND.(CODE.LE.24))LODE=1
      IF((CODE.GE.31).AND.(CODE.LE.34))LODE=1
      IF(CODE.EQ.40)LODE=1
      IF(LODE.EQ.0)THEN
      WRITE(*,510)
      GO TO 100
      END IF
```

Figure C-1.   Listing of the main program, STATDA.

```
      IF(CODE.EQ.40)CALL MULTIP
      IF(CODE.GT.0)GO TO 300

C     **********   DETERMINE THE SAMPLE SIZE FOR THE TEST   ***********

      WRITE(*,525)
      CALL ENTERI(PARAM)
      WRITE(*,550)
      CALL ENTERI(CONLEV)
      NODE=10*PARAM+CONLEV
      WRITE(*,530)
      CALL ENTERI(CRIT)
      IF(CRIT.EQ.2)GO TO 200
      IF(PARAM.EQ.1)WRITE(*,535)NODE
      IF(PARAM.EQ.2)WRITE(*,540)NODE
      IF(PARAM.EQ.3)WRITE(*,545)NODE
      CALL EXIT

200   IF(PARAM.EQ.3)GO TO 250
      IF(PARAM.EQ.1)WRITE(*,556)
      IF(PARAM.EQ.2)WRITE(*,557)
      CALL ENTERX(ABS)
      CALL SSDTIM(NODE,CONLEV,N,ABS)
      IF(CODE.EQ.11)WRITE(*,613)N,11
      IF(CODE.EQ.12)WRITE(*,613)N,12
      IF(CODE.EQ.13)WRITE(*,615)N,13
      IF(CODE.EQ.14)WRITE(*,615)N,14
      IF(CODE.EQ.21)WRITE(*,618)N,21
      IF(CODE.EQ.22)WRITE(*,618)N,22
      IF(CODE.EQ.23)WRITE(*,620)N,23
      IF(CODE.EQ.24)WRITE(*,620)N,24
      CALL EXIT

250   WRITE(*,555)
      CALL ENTERI(REL)
      A=1.*REL
      CALL SSDFLR(CODE,NODE,CONLEV,N,S,R,A,PH,PL,PU)
      IF(CODE.EQ.31)WRITE(*,558)S,31
      IF(CODE.EQ.32)WRITE(*,558)S,32
      IF(CODE.EQ.33)WRITE(*,559)S,33
      IF(CODE.EQ.34)WRITE(*,559)S,34
      CALL EXIT

C     **************   ANALYZE THE TEST   ****************

300   CONLEV=1
      IF(CODE.EQ.2*(CODE/2))CONLEV=2
      IF(CODE.GE.31)GO TO 340
      WRITE(*,581)
      CALL ENTERI(MODE)
      IF(MODE.EQ.2)CALL XFILE(W,TUR,BIT,N)
      IF(MODE.EQ.2)GO TO 306
      IF(CODE.LE.14)WRITE(*,585)
      IF(CODE.GE.21)WRITE(*,586)
      CALL ENTERI(N)
      IF(CODE.LE.14)WRITE(*,590)
      IF(CODE.GE.21)WRITE(*,591)
      DO 305 I=1,N
      CALL ENTERX(W(I))
305   CONTINUE
      CALL CHECKX(W)
306   IF(CODE.GE.21)GO TO 320
      IF(CODE.LE.12)GO TO 310
      WRITE(*,556)
```

Figure C-1. Continued.

```
      CALL ENTERX(ABS)
      CALL SSDTIM(MODE,CONLEV,N,ABS)
      IF(CODE.GE.13)THEN
      CODE=CODE-2
      WRITE(*,640)N,CODE
      CALL EXIT

      END IF
310   IF(MODE.EQ.2)GO TO 317
      WRITE(*,595)
      DO 315 I=1,N
      CALL ENTERX(TUR(I))
315   CONTINUE
      CALL CHECKX(TUR)
317   CALL ANZTIM(CONLEV,N,WAVE,WL,WU,RHAT,RL,RU,PP,PL,PU)
      IF(CODE.EQ.11)WRITE(*,610)WAVE,90,WL,WU,PP,90,PL,PU
      IF(CODE.EQ.12)WRITE(*,610)WAVE,95,WL,WU,PP,95,PL,PU
      CALL EXIT

320   IF(CODE.LE.22)GO TO 330
      WRITE(*,557)
      CALL ENTERX(ABS)
      CALL SSDTIM(NODE,CONLEV,N,ABS)
      IF(CODE.GE.23)THEN
      CODE=CODE-2
      WRITE(*,645)N,CODE
      CALL EXIT

      END IF
330   IF(MODE.EQ.2)GO TO 338
      WRITE(*,596)
      DO 335 I=1,N
      CALL ENTERX(TUR(I))
335   CONTINUE
      CALL CHECKX(TUR)
      WRITE(*,600)
      DO 336 I=1,N
      CALL ENTERX(BIT(I))
336   CONTINUE
      CALL CHECKX(BIT)
338   CALL ANZTIM(CONLEV,N,WAVE,WL,WU,RHAT,RL,RU,PP,PL,PU)
      IF(CODE.EQ.21)WRITE(*,636)WAVE,90,WL,WU,PP,90,PL,PU,RHAT,90,
     *RL,RU
      IF(CODE.EQ.22)WRITE(*,636)WAVE,95,WL,WU,PP,95,PL,PU,RHAT,95,
     *RL,RU
      CALL EXIT

340   WRITE(*,565)
      CALL ENTERI(N)
      WRITE(*,570)
      CALL ENTERI(S)
      WRITE(*,575)
      CALL ENTERI(R)
      IF(CODE.LE.32)GO TO 345
      WRITE(*,555)
      CALL ENTERI(REL)
      A=1.*REL
      CALL SSDFLR(CODE,NODE,CONLEV,N,S,R,A,PH,PL,PU)
      IF(CODE.GE.33)THEN
      CODE=CODE-2
      WRITE(*,560)S,CODE
      CALL EXIT

      END IF
```

Figure C-1. Continued.

```
345 CALL ANZFLR(CONLEV,N,S,R,PH,PL,PU,P11U)
    IF((CODE.EQ.31).AND.(PH.EQ.0.))WRITE(*,577)PH,90,PL,PU
    IF((CODE.EQ.32).AND.(PH.EQ.0.))WRITE(*,577)PH,95,PL,PU
    IF((CODE.EQ.31).AND.(PH.NE.0.).AND.(PL.EQ.0.))WRITE(*,578)PH,90,
   *PL,PU
    IF((CODE.EQ.32).AND.(PH.NE.0.).AND.(PL.EQ.0.))WRITE(*,578)PH,95,
   *PL,PU
    IF((CODE.EQ.31).AND.(PH*PL.NE.0.))WRITE(*,580)PH,90,PL,PU
    IF((CODE.EQ.32).AND.(PH*PL.NE.0.))WRITE(*,580)PH,95,PL,PU

C      **************   FORMAT STATEMENTS    ****************

510 FORMAT(//'THIS IS AN UNACCEPTABLE CODE.')
505 FORMAT(/'     THIS IS THE ANS X3535/135 STATISTICAL DESIGN AND'/
   *'ANALYSIS COMPUTER PROGRAM.'/
   *'    IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE THE SAMPLE '/
   *'SIZE FOR YOUR TEST, PLEASE TYPE THE INTEGER 0.'/
   *'    IF YOU ARE ACCESSING THIS PROGRAM TO ANALYZE YOUR TEST, '/
   *'PLEASE TYPE THE CODE NUMBER YOU WERE ASSIGNED WHEN THE'/
   *'SAMPLE SIZE WAS DETERMINED.'/
   *'    IF YOU ARE ACCESSING THIS PROGRAM TO DETERMINE'/
   *'WHETHER THE DATA YOU OBTAINED FROM MORE THAN ONE TEST CAN BE'/
   *'CONSIDERED TO COME FROM THE SAME POPULATION (AND, HENCE, CAN'/
   *'BE GROUPED TO PROVIDE A SMALLER CONFIDENCE INTERVAL), PLEASE '/
   *'TYPE THE INTEGER 40.')
525 FORMAT(//'     YOU CAN TEST THE SYSTEM WITH RESPECT TO'/
   *'    1. DELAYS,'/
   *'    2. RATES,'/
   *'OR'/
   *'    3. FAILURES.  '/
   *'  PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE TYPE'/
   *' OF PERFORMANCE PARAMETER THAT YOU WISH TO ANALYZE.')
530 FORMAT(//'     THE TEST CAN BE CONDUCTED WITH'/
   *'EITHER'/
   *'    1.  A GIVEN SAMPLE SIZE,'/
   *'OR'/
   *'    2.  A SAMPLE SIZE LARGE ENOUGH TO PROVIDE'/
   *'        A GIVEN PRECISION. (TO BE DETERMINED HERE.)'/
   *'     PLEASE TYPE THE INTEGER LISTED AT THE LEFT '/
   *'OF THE TEST CRITERION THAT YOU CHOOSE.')
535 FORMAT(//'   SINCE YOU KNOW THE SAMPLE SIZE, PROCEDE WITH'/
   *'YOUR TEST. AFTER THE TEST YOU WILL RE-ACCESS THE PROGRAM'/
   *'TO ANALYZE THE PERFORMANCE OF YOUR COMMUNICATIION SYSTEM.'/
   *'YOU WILL BE ASKED TO ENTER:'/
   *'      1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
   *'      2. THE NUMBER OF DELAYS,'/
   *'      3. THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER),'/
   *'      4. THE USER-FRACTION OF THE DELAY OF EACH'/
   *'         TRIAL (IN CHRONOLOGICAL ORDER)')
540 FORMAT(//' SINCE YOU KNOW THE SAMPLE SIZE, PROCEDE WITH'/
   *'YOUR TEST. AFTER THE TEST YOU WILL RE-ACCESS THE PROGRAM'/
   *'TO ANALYZE THE PERFORMANCE OF YOUR COMMUNICATION SYSTEM.'/
   *'YOU WILL BE ASKED TO ENTER:'/
   *'      1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
   *'      2. THE NUMBER OF TRIALS,'/
   *'      3. THE INPUT/OUTPUT TIME FOR EACH TRIAL'/
   *'         (IN CHRONOLOGICAL ORDER),'/
   *'      4. THE USER-FRACTION OF EACH INPUT/OUTPUT'/
   *'         TIME. (IN CHRONOLOGICAL ORDER),'/
   *'      5. THE NUMBER OF BITS TRANSFERED IN EACH TRIAL'/
   *'         (IN CHRONOLOGICAL ORDER).')
545 FORMAT(//'   SINCE YOU KNOW THE SAMPLE SIZE, PROCEDE WITH YOUR'/
   *'TEST. AFTER THE TEST YOU WILL RE-ACCESS THE PROGRAM TO'/
   *'ANALYZE THE PERFORMANCE OF YOUR COMMUNICATION SYSTEM.'/
```

Figure C-1. Continued.

```
      *'YOU WILL BE ASKED TO ENTER:'/
      *'     1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
      *'     2. THE SAMPLE SIZE,'/
      *'     3. THE NUMBER OF FAILURES IN THE SAMPLE,'/
      *'     4. THE NUMBER OF  PAIRS OF CONSECUTIVE FAILURES'/
      *'           IN THE SAMPLE.')
550 FORMAT(//'   THE MEAN OF THE PERFORMANCE PARAMETER THAT YOU '/
      *'SELECTED CAN BE MEASURED TO PROVIDE ONE OF THE FOLLOWING '/
      *'LEVELS OF CONFIDENCE:'/
      *'     1. 90%'/
      *'     2. 95%'/
      *'    PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE '/
      *'CONFIDENCE LEVEL THAT YOU HAVE SELECTED.')
555 FORMAT(//'    PLEASE TYPE THE DESIRED RELATIVE PRECISION AS A'/
      *'PERCENT (I.E., A ONE OR TWO-DIGIT NATURAL NUMBER.)')
556 FORMAT(//'PLEASE TYPE THE LARGEST ACCEPTABLE ERROR IN ESTIMATING'/
      *'THE MEAN DELAY. (I.E., THE ABSOLUTE PRECISION).'/
      *'TYPE THIS VALUE IN THE FORM XX.XXX')
557 FORMAT(//'PLEASE TYPE THE LARGEST ACCEPTABLE ERROR IN ESTIMATING'/
      *'THE MEAN TRANSFER RATE. (I.E., THE ABSOLUTE PRECISION).'/
      *'TYPE THIS VALUE IN THE FORM XX.XXX')
558 FORMAT(//'    TO ACHIEVE YOUR TEST OBJECTIVES, YOU MUST GEN-'/
      *'ERATE AT LEAST ',I4,' FAILURES. AFTER THE TEST YOU WILL '/
      *'RE-ACCESS THIS PROGRAM TO ANALYZE THE PERFORMANCE OF'/
      *'YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO ENTER:'/
      *'     1. YOUR CODE NUMBER. (IT IS ',I2,'.),'/
      *'     2. THE SAMPLE SIZE,'/
      *'     3. THE NUMBER OF FAILURES IN THE SAMPLE,'/
      *'     4. THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES'/
      *'         IN THE SAMPLE,'/
      *'     5. YOUR ESTIMATE OF THE CONDITIONAL PROBABILITY'/
      *'         OF A FAILURE, GIVEN THAT A FAILURE OCCURRED'/
      *'         IN THE PREVIOUS TRIAL.')
559 FORMAT(//'    TO ACHIEVE YOUR TEST OBJECTIVES, YOU MUST GEN-'/
      *'ERATE AT LEAST ',I4,' FAILURES. AFTER THE TEST YOU WILL '/
      *'RE-ACCESS THIS PROGRAM TO ANALYZE THE PERFORMANCE OF'/
      *'YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO ENTER:'/
      *'     1. YOUR CODE NUMBER. (IT IS ',I2,'.),'/
      *'     2. THE SAMPLE SIZE,'/
      *'     3. THE NUMBER OF FAILURES IN THE SAMPLE,'/
      *'     4. THE NUMBER OF PAIRS OF CONSECUTIVE FAILURES'/
      *'         IN THE SAMPLE,'/
      *'     5. THE SPECIFIED RELATIVE PRECISION.')
560 FORMAT(//'    TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
      *'AT LEAST ',I3,' MORE FAILURES. AFTER THE TEST YOU WILL'/
      *'RE-ACCESS THIS PROGRAM TO ANALYZE THE PERFORMANCE OF'/
      *'YOUR COMMUNICATION SYSTEM. YOU WILL BE ASKED TO ENTER:'/
      *'     1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
      *'     2. THE TOTAL SAMPLE SIZE,'/
      *'     3. THE TOTAL NUMBER OF FAILURES,'/
      *'     4. THE TOTAL NUMBER OF PAIRS OF CONSECUTIVE FAILURES.')
565 FORMAT(//'   PLEASE TYPE THE SAMPLE SIZE FROM YOUR TEST.')
570 FORMAT(//'   PLEASE TYPE THE NUMBER OF FAILURES IN THE SAMPLE.')
575 FORMAT(//'   PLEASE TYPE THE NUMBER OF PAIRS OF CONSECUTIVE'/
      *'FAILURES IN THE SAMPLE.')
577 FORMAT(//'YOUR TEST RESULTED IN AN ESTIMATED FAILURE RATE OF'/
      *,F2.0,'. YOU CAN BE ',I2,' PERCENT CONFIDENT THAT THE TRUE'/
      *'FAILURE RATE IS BETWEEN ',F2.0,' AND ',E10.5,'.')
578 FORMAT(//'YOUR TEST RESULTED IN AN ESTIMATED FAILURE RATE OF'/
      *,E10.5,'. YOU CAN BE ',I2,' PERCENT CONFIDENT THAT THE TRUE'/
      *'FAILURE RATE IS BETWEEN ',F2.0,' AND ',E10.5,'.')
580 FORMAT(//'YOUR TEST RESULTED IN AN ESTIMATED FAILURE RATE OF'/
      *,E10.5,'. YOU CAN BE ',I2,' PERCENT CONFIDENT THAT THE TRUE'/
      *'FAILURE RATE IS BETWEEN ',E10.5,' AND ',E10.5,'.')
```

Figure C-1. Continued.

```
581 FORMAT(///'DO YOU WISH TO ENTER THE TEST DATA'/
   *'    1. FROM A KEYBOARD'/
   *'    OR'/
   *'    2. FROM A FILE?'/
   *'PLEASE ENTER THE INTEGER AT THE LEFT '/
   *'OF THE DESIRED ENTRY MODE.')
585 FORMAT(///'PLEASE TYPE THE NUMBER OF DELAYS.')
586 FORMAT(///'PLEASE TYPE THE NUMBER OF TRIALS.')
590 FORMAT(///'PLEASE TYPE THE TOTAL DELAY IN EACH TRIAL '/
   *'(IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY '/
   *'AFTER EACH ENTRY.')
591 FORMAT(///'PLEASE TYPE THE INPUT/OUTPUT TIME IN EACH TRIAL '/
   *'(IN CHRONOLOGICAL ORDER). PRESS THE RETURN KEY '/
   *'AFTER EACH ENTRY.')
595 FORMAT(///'PLEASE TYPE THE USER-FRACTION OF THE'/
   *'DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER). '/
   *'PRESS THE RETURN KEY AFTER EACH ENTRY.')
596 FORMAT(///'PLEASE TYPE THE USER-FRACTION OF THE'/
   *'INPUT/OUTPUT TIME IN EACH TRIAL (IN CHRONOLOGICAL ORDER). '/
   *'PRESS THE RETURN KEY AFTER EACH ENTRY.')
600 FORMAT(1X,/   'PLEASE TYPE THE NUMBER OF BITS TRANSFERED'/
   *'IN EACH TRIAL. PLEASE PRESS THE '/
   *'RETURN KEY AFTER EACH ENTRY.')
610 FORMAT(///'YOUR TEST RESULTED IN AN ESTIMATED MEAN DELAY OF'/
   *,E10.5,' . YOU CAN BE ',I2,' PERCENT CONFIDENT THAT THE '/
   *'TRUE MEAN DELAY IS BETWEEN ',E10.5,' AND ',E10.5,' .'/
   *'   YOUR TEST RESULTED IN AN ESTIMATED MEAN '/
   *'USER-FRACTION DELAY OF ',E10.5,'. YOU CAN BE'/
   *,I2,' PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN ',E10.5,/
   *'AND ',E10.5,'.')
613 FORMAT(///' TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
   *'AT LEAST ',I4,' DELAYS. WHEN YOU RE-ACCESS THIS'/
   *'PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER:'/
   *'     1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
   *'     2. THE NUMBER OF DELAYS,'/
   *'     3. THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER),'/
   *'     4. THE USER-FRACTION OF THE DELAY'/
   *'        IN EACH TRIAL (IN CHRONOLOGICAL ORDER).')
615 FORMAT(///'TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
   *'AT LEAST ',I4,'  DELAYS. WHEN YOU RE-ACCESS THIS '/
   *'PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER'/
   *'     1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
   *'     2. THE NUMBER OF DELAYS,'/
   *'     3. THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER),'/
   *'     4. THE ABSOLUTE PRECISION,'/
   *'     5. THE USER-FRACTION OF THE DELAY OF'/
   *'        EACH TRIAL (IN CHRONOLOGICAL ORDER)')
618 FORMAT(///'TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
   *'AT LEAST ',I4,' TRIALS. WHEN YOU RE-ACCESS THIS'/
   *'PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER:'/
   *'     1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
   *'     2. THE NUMBER OF TRIALS,'/
   *'     3. THE INPUT/OUTPUT TIME FOR EACH TRIAL'/
   *'        (IN CHRONOLOGICAL ORDER),'/
   *'     4. THE USER-FRACTION OF EACH INPUT/OUTPUT TIME'/
   *'        (IN CHRONOLOGICAL ORDER),'/
   *'     5. THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL'/
   *'        (IN CHRONOLOGICAL ORDER).')
620 FORMAT(///'TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
   *'AT LEAST ',I4,' TRIALS. WHEN YOU RE-ACCESS THIS '/
   *'PROGRAM TO ANALYZE YOU WILL BE ASKED TOO ENTER:'/
   *'     1. YOUR CODE NUMBER (IT IS ',I2,'.)'/
   *'     2. THE NUMBER OF TRIALS,'/
   *'     3. THE INPUT/OUTPUT TIME FOR EACH TRIAL'/
```

Figure C-1. Continued.

```
      *'           (IN CHRONOLOGICAL ORDER),'/
      *'      4. THE ABSOLUTE PRECISION,'/
      *'      5. THE USER-FRACTION OF EACH INPUT/OUTPUT'/
      *'         TIME (IN CHRONOLOGICAL ORDER),'/
      *'      6. THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL'/
      *'         (IN CHRONOLOGICAL ORDER).')
  636 FORMAT('    YOUR TEST RESULTED IN AN ESTIMATED MEAN INPUT/OUTPUT '/
      *'TIME OF ',E10.5,' . YOU CAN BE ',I2,' PERCENT CONFIDENT '/
      *'THAT THE TRUE MEAN INPUT/OUTPUT TIME IS BETWEEN ',E10.5,' AND' /
      *E10.5,' .'/
      *'  YOUR TEST RESULTED IN AN ESTIMATED MEAN USER-FRACTION '/
      *' INPUT/OUTPUT TIME OF',E10.5,'. YOU CAN BE ',I2,/
      *'PERCENT CONFIDENT THAT THE TRUE MEAN IS BETWEEN ',E10.5, /
      *'AND ',E10.5,'.'/
      *'    THE ESTIMATED MEAN TRANSFER RATE IS ',E10.5,'. YOU CAN BE'/
      *,I2,' PERCENT CONFIDENT THAT THE TRUE RATE IS BETWEEN ',E10.5,/
      *'AND ',E10.5,'.')
  640 FORMAT(///' TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
      *'AT LEAST ',I4,' MORE DELAYS. WHEN YOU RE-ACCESS THIS'/
      *'PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER:'/
      *'      1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
      *'      2. THE NUMBER OF DELAYS,'/
      *'      3. THE TOTAL DELAY IN EACH TRIAL (IN CHRONOLOGICAL ORDER),'/
      *'      4. THE USER-FRACTION OF THE DELAY'/
      *'         IN EACH TRIAL (IN CHRONOLOGICAL ORDER).')
  645 FORMAT(///'TO ACHIEVE YOUR TEST OBJECTIVE, YOU MUST GENERATE'/
      *'AT LEAST ',I4,' MORE TRIALS. WHEN YOU RE-ACCESS THIS'/
      *'PROGRAM TO ANALYZE YOUR TEST, YOU WILL BE ASKED TO ENTER:'/
      *'      1. YOUR CODE NUMBER (IT IS ',I2,'.),'/
      *'      2. THE NUMBER OF TRIALS,'/
      *'      3. THE INPUT/OUTPUT TIME FOR EACH TRIAL'/
      *'         (IN CHRONOLOGICAL ORDER),'/
      *'      4. THE USER-FRACTION OF EACH INPUT/OUTPUT TIME'/
      *'         (IN CHRONOLOGICAL ORDER),'/
      *'      5. THE NUMBER OF BITS TRANSFERRED IN EACH TRIAL'/
      *'         (IN CHRONOLOGICAL ORDER).')
      END
```

Figure C-1.   Continued.

```
      SUBROUTINE ANZFLR(CONLEV,N,S,R,PH,PL,PU,P11U)
      INTEGER CONLEV,S,R
C     THIS SUBROUTINE DETERMINES THE CONFIDENCE LIMITS, PL AND PU,
C     FOR THE ESTIMATE OF THE MEAN NUMBER OF FAILURES.
      CHARACTER MY*3, MN*2
      DATA MY/↑YES↑/,MN/↑NO↑/
      IF(CONLEV.EQ.1)U=1.645
      IF(CONLEV.EQ.2)U=1.960
      CALL POISS(CONLEV,U,S,AL,AU,D)
      PUI=AU/(N+D+(AU-S)/2.)                                           (A-43)
      PLI=AL/(N-(S-1.-AL)/2.)                                          (A-43)
      PH=1.*S/N                                                        (A-30)
      QH=1.-PH                                                         (A-33)
      IF(S.GT.1)GO TO 6


C     ************* DETERMINE PL OR PU FOR S=0 AND S=1.  **************


      PL=0.                                                           (A-44)
      WRITE(*,100)
      CALL ENTERX(P11)
    7 RHOH=(P11-PH)/QH                                                (A-35)
      CALL LIMIT(N,RHOH,APPR)
      FH=(1.+(2.*RHOH)/(N*(1.-RHOH))*(N-(1.-APPR)/                    (A-32)
     *(1.-RHOH)))**0.5
      PUP=(PUI-PLI)*FH                                                (A-48)
      IF(CONLEV.EQ.1)ALPHA=0.10
      IF(CONLEV.EQ.2)ALPHA=0.05
      P11H=P11
C     P11U IS REQUIRED IN SUBROUTINE SSDFLR
      P11U=(2.*S*P11H+U*U+SQRT((2.*S*P11H+U*U)**2.                    (A-11)
     *-4.*S*P11H*P11H*(S+U*U)))/(2.*(S+U*U))
      PU=PUP
      QU=1.-PUP                                                       (A-47)
      Z1=(N-1.)*(1.-P11)**2.-1.                                       (A-46)
      Z2=(N-2.)*(1.-P11)**2.-2.                                       (A-46)
      YY=.0001
      I=0
C     ITERATE UNTIL THE RATIO OF TWO CONSECUTIVE VALUES OF PU IS BETWEEN 1.-YY
C     AND 1.+YY
    2 I=I+1
      IF(S.EQ.0.)X=(ALPHA/QU)**(1./(N-1.))                            (A-45)
      IF(S.EQ.1.)X=((ALPHA*QU)/(1.+PU*Z2-PU*PU*Z1))**(1./(N-3.))      (A-45)
      PU=(1.-X)/(2.-P11-X)                                            (A-44)
      QU=1.-PU                                                        (A-49)
      Y=PUP/PU                                                        (A-50)
      PUP=PU
      IF((Y.GT.1.-YY).AND.(Y.LT.1.+YY))RETURN                        (A-50)
      IF(I.LE.100)GO TO 2
      PRINT105,100,YY
      CALL EXIT


C     ***********   DETERMINE PL AND PU FOR S GREATER THAN 1   ***********

    6 P11H=R/(S-PH)                                                   (A-34)
      RHOH=(P11H-PH)/QH                                               (A-35)
      CALL LIMIT(N,RHOH,APPR)
      FH=(1.+(2.*RHOH)/(N*(1.-RHOH))*(N-(1.-APPR)/                    (A-32)
     *(1.-RHOH)))**0.5
      SIGPH=FH*(PH*QH/N)**0.5
```

Figure C-2. Listing of subroutine ANZFLR.

```
                                                                    (A-37)
    V=((N*U*SIGPH)**2.)/(S*(N-S))                                   (A-38)
    RP=((N*V+2.*S+1.)**2.-(2.*S+1.)**2.*(1.+V))**0.5                (A-39)
    RN=((N*V+2.*S-1.)**2.-(2.*S-1.)**2.*(1.+V))**0.5                (A-40)
    PUN=(N*V+2.*S+1.+RP)/(2.*N*(1.+V))                              (A-36)
    PLN=(N*V+2.*S-1.-RN)/(2.*N*(1.+V))                             (A-36)
    PUP=PH+FH*(PUI-PH)                                              (A-41)
    PLP=PH+FH*(PLI-PH)                                              (A-41)
    IF(PLP.LE.0.)THEN
    PUP=FH*(PUI-PLI)                                               (A-42)
    PLP=0.                                                          (A-42)
    END IF
    PU=(PUN+PUP)/2.                                                 (A-31)
    PL=(PLN+PLP)/2.                                                 (A-31)
    P11U=(2.*S*P11H+U*U+SQRT((2.*S*P11H+U*U)**2.                    (A-11)
   *-4.*S*P11H*P11H*(S+U*U)))/(2.*(S+U*U))
    RETURN

C   ***************  FORMAT STATEMENTS   *****************

 99 FORMAT(//*PLEASE TYPE THIS VALUE IN THE FORM 0.XXX.*)
100 FORMAT(//*CAN YOU ESTIMATE THE MAXIMUM VALUE OF THE CON-*/
   **DITIONAL PROBABILITY OF A FAILURE, GIVEN THAT A*/
   **FAILURE  OCCURRED IN THE PREVIOUS TRIAL* */
   **IF YOU CAN, ENTER IT IN THE FORM 0.XXX.*/
   **IF YOU CANNOT, ENTER THE VALUE 0.8 .*)
101 FORMAT(*   YOU FAILED TO TYPE EITHER YES OR NO.*)
105 FORMAT(//*EVEN AFTER *,I3,* ITERATIONS, NO TWO CONSECUTIVE*/
   **VALUES OF THE UPPER CONFIDENCE LIMIT ARE  WITHIN *,F8.6,/
   **OF EACH OTHER.*)
    END
```

Figure C-2. Continued.

```
      SUBROUTINE ANZTIM(CONLEV,N,WAVE,WL,WU,RHAT,RL,RU,PP,PL,PU)
      INTEGER BIT, CONLEV, CODE
      COMMON/SQ/CODE,PARAM,W(200),TUR(200),BIT(200)

C     **********  DETERMINE STATISTICS FOR THE DELAYS.  **********

      CALL STUDNT(CONLEV,N,T)
      WAVE=0.
      TAVE=0.
C     W(I)=TOTAL ACCESS TIME IN ITH SAMPLE
C     TUR(I)=USER-RESPONSIBLE ACCESS TIME IN ITH SAMPLE
      DO 10 I=1,N
      WAVE=WAVE+W(I)
      TAVE=TAVE+TUR(I)
   10 CONTINUE
      WAVE=WAVE/N                                                    (A-18)
      TAVE=TAVE/N                                                    (A-19)
      PH=TAVE/WAVE
      WA=0.
      TA=0.
      TWA=0.
      DO 15 I=1,N
      WA=WA+(W(I)-WAVE)*(W(I)-WAVE)
      TA=TA+(TUR(I)-TAVE)*(TUR(I)-TAVE)
      TWA=TWA+(TUR(I)-TAVE)*(W(I)-WAVE)
   15 CONTINUE
      IF(WA*TA.EQ.0.)THEN
      WRITE(*,90)
      CALL EXIT

      END IF
      NA=N-1
      SWW=SQRT(WA/NA)
      STT=SQRT(TA/NA)
      STW=TWA/NA
      WAL=0.
      TAL=0.
      DO 20 I=1,NA
      WAL=WAL+(W(I)-WAVE)*(W(I+1)-WAVE)
      TAL=TAL+(TUR(I)-TAVE)*(TUR(I+1)-TAVE)
   20 CONTINUE
      R1W=WAL/(SWW*SWW*NA)
      AW=SQRT((1.+R1W)/(1.-R1W))
      SDN=SWW/SQRT(1.*N)
C     CONFIDENCE LIMITS FOR THE DELAY, W.
      WL=WAVE-T*SDN*AW                                               (A-15)
      WU=WAVE+T*SDN*AW                                               (A-15)

C     *********  DETERMINE STATISTICS FOR THE TIME FRACTION  *********

C     ESTIMATE OF THE MEAN TIME FRACTION.
      PP=PH*(1.+(STW/(TAVE*WAVE)-SWW*SWW/(WAVE*WAVE))/N)             (A-17)
      R1T=TAL/(STT*STT*NA)
      AT=SQRT((1.+R1T)/(1.-R1T))
      STT=STT*AT                                                     (A-20)
      SWW=SWW*AW                                                     (A-21)
      STW=STW*AW*AT                                                  (A-22)
      SPP=(PP*PP/N)*(STT*STT/(TAVE*TAVE)+SWW*SWW/(WAVE*WAVE)-2.*STW/ (A-23)
     *(TAVE*WAVE))
      IF(CONLEV.EQ.1)U=1.645
      IF(CONLEV.EQ.2)U=1.960
C     CONFIDENCE LIMITS FOR THE MEAN TIME FRACTION.
      PL=PP-U*SQRT(SPP)                                              (A-24)
      PU=PP+U*SQRT(SPP)                                              (A-24)
      IF(CODE.LT.21)RETURN
```

Figure C-3. Listing of subroutine ANZTIM.

104

```
C      *****   DETERMINE STATISTICS FOR THE BIT TRANSFER RATE.   ******

       BAVE=0.
       DO 35 I=1,N
       BAVE=BAVE+BIT(I)
    35 CONTINUE
       BAVE=BAVE/N                                                    (A-28)
C      ESTIMATE OF THE MEAN BIT TRANSFER RATE.
       RHAT=BAVE/WAVE                                                 (A-27)
C      CONFIDENCE LIMITS FOR THE MEAN BIT TRANSFER RATE.
       RL=BAVE/WU                                                     (A-29)
       RU=BAVE/WL                                                     (A-29)
       RETURN

C      **************   FORMAT STATEMENT   ************

    90 FORMAT(↑ANALYSIS IS NOT ATTEMPTED WHEN ALL DELAYS ARE EQUAL.↑/
      *↑(I.E. THE ESTIMATED STANDARD DEVIATION IS ZERO.)↑)
       END
```

Figure C-3. Continued.

105

```
      SUBROUTINE CHECKI(K)
      DIMENSION K(200)
      WRITE(*,300)
      WRITE(*,400)
    4 CALL ENTERI(I)
      IF(I.EQ.0)RETURN
      WRITE(*,300)
      CALL ENTERI(L)
      K(I)=L
      GO TO 4
  300 FORMAT(//)
  350 FORMAT(3X,I4,3X,E12.5)
  400 FORMAT(↑   IF ALL DATA WAS ENTERED CORRECTLY,↑/
     *↑   *   TYPE THE INTEGER 0↑/
     *↑   *   PRESS RETURN↑/
     *↑OTHERWISE,↑/
     *↑   *   TYPE THE ORDINAL NUMBER OF THE INCORRECT ENTRY.↑/
     *↑.  *   PRESS RETURN↑/
     *↑   *   TYPE THE CORRECT VALUE OF THE ENTRY↑/
     *↑   *   PRESS RETURN↑/
     *↑REPEAT THIS PROCESS UNTIL ALL ENTRIES ARE CORRECT.↑/
     *↑THEN↑/
     *↑   *   TYPE THE INTEGER 0↑/
     *↑   *   PRESS RETURN↑)
      END




      SUBROUTINE CHECKX(X)
      DIMENSION X(200)
      WRITE(*,300)
      WRITE(*,400)
    4 CALL ENTERI(I)
      IF(I.EQ.0)RETURN
      WRITE(*,300)
      CALL ENTERX(A)
      X(I)=A
      GO TO 4
  300 FORMAT(//)
  350 FORMAT(3X,I4,3X,E12.5)
  400 FORMAT(↑   IF ALL DATA WAS ENTERED CORRECTLY,↑/
     *↑   *   TYPE THE INTEGER 0↑/
     *↑   *   PRESS RETURN↑/
     *↑OTHERWISE,↑/
     *↑   *   TYPE THE ORDINAL NUMBER OF THE INCORRECT ENTRY.↑/
     *↑   *   PRESS RETURN↑/
     *↑   *   TYPE THE CORRECT VALUE OF THE ENTRY↑/
     *↑   *   PRESS RETURN↑/
     *↑REPEAT THIS PROCESS UNTIL ALL ENTRIES ARE CORRECT.↑/
     *↑THEN↑/
     *↑   *   TYPE THE INTEGER 0↑/
     *↑   *   PRESS RETURN↑)
      END
```

Figure C-4. Listing of subroutines CHECKI and CHECKX.

```
      SUBROUTINE CHISQR(CONLEV)
      INTEGER  CONLEV
C     THIS SUBROUTINE DETERMINES THE CONFIDENCE THAT THE DATA FROM
C     MULTIPLE FAILURE TESTS COMES FROM ONE POPULATION. THE 5% POINT
C     OF THE CHI SQUARED TEST IS USED TO TEST THIS HYPOTHESIS.
      CHARACTER MY*3, MN*2
      INTEGER SC(10),RC(10),S,R
      DIMENSION NC(10),PH(10),PH11(10)
      DATA MY/+YES+/,MN/+NO+/
      WRITE(*,108)
      CALL ENTERI(K)
      IF((K.LT.2).OR.(K.GT.6))THEN
      WRITE(*,103)
      CALL EXIT


C     ****   DETERMINE THE CHI SQUARED STATISTIC FOR BOTH P AND P11.   ****

      END IF
      WRITE(*,111)
      DO 3 I=1,K
      CALL ENTERI(NC(I))
 3    CONTINUE
      CALL CHECKI(NC)
      WRITE(*,104)
      DO 6 I=1,K
      CALL ENTERI(SC(I))
      IF(SC(I).EQ.0)THEN
      WRITE(*,109)
      CALL EXIT
      END IF
 6    CONTINUE
      CALL CHECKI(SC)
      WRITE(*,105)
      DO 7 I=1,K
      CALL ENTERI(RC(I))
 7    CONTINUE
      CALL CHECKI(RC)
      N=0
      S=0
      R=0
      DO 8 I=1,K
      N=N+NC(I)                                                         (A-57)
      S=S+SC(I)                                                         (A-57)
      R=R+RC(I)                                                         (A-57)
      PH(I)=1.*SC(I)/NC(I)                                              (A-63)
 8    CONTINUE
      PB=1.*S/N                                                         (A-59)
      QB=1.-PB                                                          (A-66)
      PH11AV=R/(S-K*PB)                                                 (A-60)
      RHOB=(PH11AV-PB)/QB                                              (A-67)
      CHI=0.
      H=0.
      DO 9 I=1,K
      NA=NC(I)
      CALL LIMIT(NA,RHOB,APPR)
      FBJ=SQRT((1.+(2.*RHOB/(NC(I)*(1.-RHOB)))*(NC(I)-                  (A-64)
     *(1.-APPR)/(1.-RHOB))))
      SIGPH2=(PB*QB/NC(I))*FBJ*FBJ                                      (A-65)
      PH11(I)=1.*RC(I)/(SC(I)-PH(I))                                    (A-61)
C     CHI=CHI SQUARED VALUE OF P FOR K-1 DEG. OF FREEDOM
      CHI=CHI+(PH(I)-PB)*(PH(I)-PB)/SIGPH2                              (A-62)
      H=H+(SC(I)-PB)*(PH11(I)-PH11AV)*(PH11(I)-PH11AV)
 9    CONTINUE
```

Figure C-5. Listing of subroutine CHISQR.

```
        IF(R.EQ.0)THEN
        WRITE(*,106)
        GO TO 35
        END IF
C       CHI11=CHI SQUARED VALUE OF P11 FOR K-1 DEG. OF FREEDOM
        CHI11=H/(PH11AV*(1.-PH11AV))                                    (A-58)
        IF(K-1.EQ.1)CHI5=3.841
        IF(K-1.EQ.2)CHI5=5.991
        IF(K-1.EQ.3)CHI5=7.815
        IF(K-1.EQ.4)CHI5=9.488
        IF(K-1.EQ.5)CHI5=11.070

C       ****    COMPARE BOTH CHI AND CHI11 WITH 5% POINT OF CHI SQ. DIST.   ****

        IF((CHI.LE.CHI5).AND.(CHI11.LE.CHI5))GO TO 30
        WRITE(*,116)
        CALL EXIT

C       ****    DETERMINE THE CONFIDENCE LIMITS FOR THE DATA FROM ALL TESTS.   ****

   30 WRITE(*,114)
   35 CALL ANZFLR(CONLEV,N,S,R,PPH,PL,PU,P11U)
        IF(CONLEV.EQ.1)WRITE(*,107)K,PPH,90,PL,PU
        IF(CONLEV.EQ.2)WRITE(*,107)K,PPH,95,PL,PU
        CALL EXIT

C       *************    FORMAT STATEMENTS    ***************

  103 FORMAT(//*THE NUMBER OF TESTS MUST BE 2,3,4,5, OR 6.*)
  104 FORMAT(//*PLEASE TYPE THE NUMBER OF FAILURES IN EACH TEST.*/
       **PRESS THE  RETURN KEY AFTER EACH ENTRY.*)
  105 FORMAT(//*PLEASE TYPE THE NUMBER OF PAIRS OF CONSECUTIVE */
       **FAILURES IN EACH TEST. PRESS THE RETURN KEY AFTER*/
       **EACH ENTRY.*)
  106 FORMAT(//*SINCE THERE ARE NO PAIRS OF CONSECUTIVE FAILURES, THE*/
       **HOMOGENEITY OF THE SAMPLES CANNOT BE DETERMINED. EVEN SO, */
       **THEY WILL BE GROUPED AND TREATED AS IF THEY WERE */
       **HOMOGENEOUS.*)
  107 FORMAT(*THE *,I1,* TESTS RESULT IN AN ESTIMATED FAILURE RATE*/
       **OF *,E12.5,*. YOU CAN BE *,I2,* PERCENT CONFIDENT THAT THE */
       **TRUE FAILURE RATE IS BETWEEN *,E12.5,* AND *,E12.5,*.*)
  108 FORMAT(//*PLEASE TYPE THE NUMBER OF TESTS. DO NOT INCLUDE  */
       **A TEST THAT HAD ZERO FAILURES.*)
  109 FORMAT(//*  YOU INCLUDED A TEST THAT HAD ZERO FAILURES. PLEASE*/
       **OMIT IT WHEN YOU RE-ACCESS THIS PROGRAM. YOUR CODE WILL*/
       **REMAIN 40.*)
  111 FORMAT(//*PLEASE TYPE THE NUMBER OF TRIALS IN EACH TEST.*/
       **PRESS THE RETURN KEY AFTER EACH ENTRY.*)
  114 FORMAT(/*WITH 95% CONFIDENCE THE TRIALS CAN BE CONSIDERED TO*/
       **COME FROM THE SAME POPULATION. HENCE, THEY CAN ALL BE */
       **GROUPED TO DETERMINE A SMALLER CONFIDENCE INTERVAL*)
  116 FORMAT(//*WITH 95% CONFIDENCE THE TRIALS CANNOT BE CONSIDERED*/
       **TO COME FROM THE SAME POPULATION. HENCE, THEY CANNOT BE */
       **GROUPED TO DETERMINE A SMALLER CONFIDENCE INTERVAL. */
       ** IF YOU WISH, OMIT DATA FROM ONE OR MORE TESTS THAT ARE THOUGHT*/
       **TO CAUSE REJECTION . THEN RE-ACCESS THE PROGRAM. YOUR CODE*/
       **WILL REMAIN 40.*)
        END
```

Figure C-5. Continued.

```
      SUBROUTINE ENTERA(A)
      CHARACTER A*6
    2 FORMAT(A6)
    3 CONTINUE
C     CLOSE(UNIT=5)
C     OPEN(UNIT=5,FILE=↑INPUT↑)
      READ(5,2,END=3)A
      RETURN
      END




      SUBROUTINE ENTERI(I)
    2 FORMAT(I10)
    3 CONTINUE
C     THE CLOSE AND OPEN STATEMENTS ARE FORTRAN 77. IF YOUR SYSTEM DEFAULTS
C     INPUT TO THE KEYBOARD AND OUTPUT TO THE CRT, THEY ARE UNNECESSARY.
C     CLOSE(UNIT=5)
C     OPEN(UNIT=5,FILE=↑INPUT↑)
      READ(5,2,END=3)I
      RETURN
      END




      SUBROUTINE ENTERX(X)
    2 FORMAT(F16.3)
    3 CONTINUE
C     CLOSE(UNIT=5)
C     OPEN(UNIT=5,FILE=↑INPUT↑)
      READ(5,2,END=3)X
      RETURN
      END
```

Figure C-6.   Listing of subroutines ENTERA, ENTERI, and ENTERX.

```
      SUBROUTINE FTEST(CONLEV)
C     THIS SUBROUTINE DETERMINES WHETHER THE DATA FROM SEVERAL
C     TIME MEASUREMENTS COME FROM THE SAME POPULATION.
      CHARACTER MY*3,MN*2
      DIMENSION N(6),WD(6),V(6,200),F951(17),F952(17),F953(17),
     *F954(17),F955(17),YA(17),NQ(17),SS(200),TT(200),RR(200)
      COMMON/SQ/CODE,PARAM,W(200),TUR(200),BIT(200)
      INTEGER   CONLEV, PARAM, CODE
      DATA MY/'YES'/, MN/'NO'/
C     THE VARIABLES F951, F952,....,F955 CONTAIN THE F DISTRIBUTION
C     FOR 95% AND 1, 2,....,5 DEGREES OF FREEDOM.

      DATA F951/161.,18.5,10.1,7.71,6.61,5.99,5.59,5.32,5.12,
     *4.96,4.75,4.54,4.35,4.17,4.00,3.92,3.84/
      DATA F952/200.,19.0,9.55,6.94,5.79,5.14,4.74,4.46,4.26,
     *4.10,3.89,3.68,3.49,3.32,3.15,3.07,3.00/
      DATA F953/216.,19.2,9.28,6.59,5.41,4.76,4.35,4.07,3.86,
     *3.71,3.49,3.29,3.10,2.92,2.76,2.68,2.60/
      DATA F954/225.,19.2,9.12,6.39,5.19,4.53,4.12,3.84,3.63,
     *3.48,3.26,3.06,2.87,2.69,2.53,2.45,2.37/
      DATA F955/230.,19.3,9.01,6.26,5.05,4.39,3.97,3.69,3.48,
     *3.33,3.11,2.90,2.71,2.53,2.37,2.29,2.21/
      DATA NQ/1,2,3,4,5,6,7,8,9,10,12,15,20,30,60,120,1000/

C     **************  COMPUTE THE F STATISTIC   ***************

C     M=NUMBER OF DAYS
C     N(I)=NUMBER OF TRIALS IN THE ITH TEST
C     NN=TOTAL NUMBER OF TRIALS
      WRITE(*,100)
      CALL ENTERI(M)
      IF((M.LT.2).OR.(M.GT.6))THEN
      WRITE(*,98)
      CALL EXIT

      END IF
      NN=0
      WBAR=0.
      K=0
      WRITE(*,90)
      CALL ENTERI(MODE)
      IF(MODE.EQ.2)THEN
      DO 17 I=1,M
      CALL YFILE(RR,TT,SS,I,NA)
      N(I)=NA
      WD(I)=0.
      DO 16 J=1,NA
      V(I,J)=RR(J)
      WBAR=WBAR+V(I,J)
      WD(I)=WD(I)+V(I,J)
      K=K+1
```

Figure C-7. Listing of subroutine FTEST.

```fortran
      W(K)=V(I,J)
      TUR(K)=TT(J)
      BIT(K)=SS(J)
   16 CONTINUE
      NN=NN+NA
      WD(I)=WD(I)/NA
   17 CONTINUE
      GO TO 19
      END IF
      DO 20 I=1,M
      IF(PARAM.EQ.1)WRITE(*,101)I
      IF(PARAM.EQ.2)WRITE(*,91)I
      CALL ENTERI(N(I))
      WD(I)=0.
      NA=N(I)
      IF(PARAM.EQ.1)WRITE(*,102)I
      IF(PARAM.EQ.2)WRITE(*,92)I
      DO 18 J=1,NA
      CALL ENTERX(V(I,J))
      WBAR=WBAR+V(I,J)
      WD(I)=WD(I)+V(I,J)
      K=K+1
      W(K)=V(I,J)
      RR(J)=V(I,J)
   18 CONTINUE
      CALL CHECKX(RR)
      NN=NN+N(I)                                                   (A-51)
      WD(I)=WD(I)/N(I)                                             (A-52)
   20 CONTINUE
   19 WBAR=WBAR/NN                                                 (A-54)
      A=0.
      B=0.
      DO 22 I=1,M
      NA=N(I)
      DO 21 J=1,NA
      A=A+(V(I,J)-WD(I))*(V(I,J)-WD(I))
   21 CONTINUE
      B=B+N(I)*(WD(I)-WBAR)*(WD(I)-WBAR)
   22 CONTINUE
      SE=A/(NN-M)                                                  (A-53)
      S1=B/(M-1.)                                                  (A-55)
C     F IS THE F STATISTIC
      F=S1/SE                                                      (A-56)

C     *****   DETERMINE, FDIST, THE VALUE OF THE CUMULATIVE F DISTRIB-   *****
C     *****   UTION (AT 95%) FOR M-1 AND NN-M DEGREES OF FREEDOM.   **********

      I=-1
      L=NN-M
      DO 40 J=1,16
      IF(M-1.EQ.1)YA(J)=F951(J)
      IF(M-1.EQ.2)YA(J)=F952(J)
      IF(M-1.EQ.3)YA(J)=F953(J)
      IF(M-1.EQ.4)YA(J)=F954(J)
      IF(M-1.EQ.5)YA(J)=F955(J)
      IF((L.GE.NQ(J)).AND.(L.LT.NQ(J+1)))I=J
      IF(J.EQ.I+1)GO TO 41
   40 CONTINUE
      IF(L.GE.NQ(17))THEN
      I=16
      NQ(17)=L
      END IF
   41 IF(L.LE.10)FDIST=YA(I)
      IF((L.GT.NQ(I)).AND.(L.LE.NQ(I+1)))FDIST=YA(I)*(NQ(I+1)-L)
```

Figure C-7.   Continued.

111

```
      */(NQ(I+1)-NQ(I))+YA(I+1)*(L-NQ(I))/(NO(I+1)-NQ(I))

C     *******      COMPARE THE F DISTRIBUTION WITH THE F STATISTIC.   *******

      IF(F.GT.FDIST)THEN
      WRITE(*,104)
      CALL EXIT

C     ***********   SINCE THE DELAYS COME FROM ONE POPULATION,   *******
C     ***********   ENTER THE REMAINDER OF THE DATA AND ANALYZE   ******
C     ***********   THE DATA.   ****************************************

      END IF
      WRITE(*,103)
      IF(MODE.EQ.1)THEN
      WRITE(*,107)K
      DO 52 I=1,K
      CALL ENTERX(TUR(I))
   52 CONTINUE
      CALL CHECKX(TUR)
      IF(PARAM.EQ.2)THEN
      WRITE(*,108)K
      DO 53 I=1,K
      CALL ENTERX(BIT(I))
   53 CONTINUE
      CALL CHECKX(BIT)
      END IF
      END IF
      CODE=10*PARAM+CONLEV
      CALL ANZTIM(CONLEV,K,WAVE,WL,WU,RHAT,RL,RU,PP,PL,PU)
      IF(CODE.EQ.11)WRITE(*,109)WAVE,90,WL,WU,PP,90,PL,PU
      IF(CODE.EQ.12)WRITE(*,109)WAVE,95,WL,WU,PP,95,PL,PU
      IF(CODE.EQ.21)WRITE(*,110)WAVE,90,WL,WU,PP,90,PL,PU,RHAT,90
     *,RL,RU
      IF(CODE.EQ.22)WRITE(*,110)WAVE,95,WL,WU,PP,95,PL,PU,RHAT,95
     *,RL,PU
      CALL EXIT

C     ***************   FORMAT STATEMENTS   ***************
   90 FORMAT(//'DO YOU WISH TO ENTER THE TEST DATA'/
     *'    1. FROM A KEYBOARD'/
     *'    OR'/
     *'    2. FROM A FILE?'/
     *'PLEASE ENTER THE INTEGER AT THE LEFT '/
     *'OF THE DESIRED ENTRY MODE.')
   91 FORMAT(//'PLEASE TYPE THE NUMBER OF TRIALS OBSERVED '/
     *'DURING TEST',I2,'.')
   92 FORMAT(//'PLEASE TYPE (IN CHRONOLOGICAL ORDER) THE INPUT/OUTPUT '/
     *'TIMES OBSERVED DURING TEST',I2,'. PRESS THE RETURN KEY  '/
     *'AFTER EACH ENTRY.')
   98 FORMAT(//'THE NUMBER OF TESTS MUST BE 2,3,4,5, OR 6.')
   99 FORMAT(//'YOU FAILED TO TYPE EITHER YES OR NO.')
  100 FORMAT(//'PLEASE TYPE THE NUMBER OF TESTS.'/
     *' THIS NUMBER MUST BE GREATER THAN 1 AND '/
     *'LESS THAN 7.')
  101 FORMAT(//'PLEASE TYPE THE NUMBER OF DELAYS OBSERVED '/
     *'DURING TEST',I2,'.')
  102 FORMAT(//'PLEASE TYPE (IN CHRONOLOGICAL ORDER) THE DELAYS'/
     *'OBSERVED DURING TEST',I2,'. PRESS THE RETURN KEY '/
     *'AFTER EACH ENTRY.')
  103 FORMAT(//'  WITH 95% CONFIDENCE THE TRIALS CAN BE CONSIDERED TO'/
     *'COME FROM THE SAME POPULATION. HENCE, THEY CAN ALL BE GROUPED'/
     *'TO DETERMINE A SMALLER CONFIDENCE INTERVAL.')
  104 FORMAT(//'WITH 95% CONFIDENCE THE TRIALS CANNOT BE CONSIDERED'/
```

Figure C-7. Continued.

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```
      *'TO COME FROM THE SAME POPULATION. HENCE, THEY CANNOT BE '/
      *'GROUPED TO DETERMINE A SMALLER CONFIDENCE INTERVAL. '/
      *'   IF YOU WISH, OMIT DATA FROM ONE OR MORE TESTS THAT ARE'/
      *'THOUGHT TO CAUSE REJECTION, AND RE-ACCESS THIS PROGRAM.'/
      *'YOUR CODE WILL REMAIN 40.')
  105 FORMAT(//'IF YOU WISH TO CONTINUE (USING GROUPED DELAYS TO'/
      *'OBTAIN A BETTER ESTIMATE), YOU MUST ENTER THE USER-'/
      *'FRACTION OF EACH DELAY'/
      *'(IN CHRONOLOGICAL ORDER).'/
      *'IF YOU WISH TO CONTINUE, TYPE YES. OTHERWISE, TYPE NO.')
  106 FORMAT(//'IF YOU WISH TO CONTINUE (USING GROUPED INPUT/OUTPUT '/
      *'TIMES TO OBTAIN A BETTER ESTIMATE), YOU MUST ENTER'/
      *'       *THE USER-FRACTION OF EACH INPUT/OUTPUT TIME'/
      *'       *THE NUMBER OF BITS TRANSFERED IN EACH TRIAL'/
      *' IF YOU WISH TO CONTINUE, TYPE YES. OTHERWISE, TYPE NO.')
  107 FORMAT(//'PLEASE TYPE THE USER-FRACTION OF EACH OF'/
      *'THE',I2,' TRIALS (IN CHRONOLOGICAL ORDER). PRESS THE RETURN'/
      *'KEY AFTER EACH ENTRY.')
  108 FORMAT(//'PLEASE TYPE THE NUMBER OF BITS TRANSFERED'/
      *'IN EACH OF THE',I2,' TRIALS.')
  109 FORMAT(//' YOUR TESTS RESULTED IN AN ESTIMATED MEAN DELAY OF'/
      *,E10.5,' . YOU CAN BE ',I2,' PERCENT CONFIDENT THAT THE'/
      *'TRUE MEAN DELAY IS BETWEEN ',E10.5,' AND ',E12.5,' .'/
      *'   YOUR TESTS RESULTED IN AN ESTIMATED MEAN USER-FRACTION OF'/
      *'THE DELAY OF',E10.5,' YOU CAN BE',I3,' PERCENT CONFIDENT'/
      *'THAT THE TRUE MEAN IS BETWEEN ',E10.5,' AND ',E10.5,'.')
  110 FORMAT(//'YOUR TESTS RESULTED IN AN ESTIMATED MEAN INPUT/OUTPUT '/
      *'TIME OF ',E10.5,' . YOU CAN BE ',I2,' PERCENT CONFIDENT '/
      *'THAT THE TRUE MEAN INPUT/OUTPUT TIME IS BETWEEN ',E10.5,' AND '/
      *,E10.5,' .'/
      *'   YOUR TESTS RESULTED IN AN ESTIMATED MEAN USER-FRACTION'/
      *'INPUT/OUTPUT TIME OF ',E10.5,' YOU CAN BE ',I2,' PERCENT  '/
      *'CONFIDENT THAT THE TRUE MEAN IS BETWEEN ',E10.5,' AND ',E10.5,/
      *'   YOUR TESTS RESULTED IN AN ESTIMATED MEAN TRANSFER'/
      *'RATE OF ',E10.5,'. YOU CAN BE ',I2,' PERCENT CONFIDENT THAT THE'/
      *'TRUE MEAN TRANSFER RATE IS BETWEEN ',E10.5,' AND '/
      *,E10.5,'.')
       END
```

Figure C-7.   Continued.

```
      SUBROUTINE LIMIT(N,R,A)
C     TO AVOID AN ARTIHMETIC UNDERFLOW BY A=R**N, THIS SUBROUTINE ASSIGNS
C     TO A THE MAXIMUM OF R**K OR 1.E-20.
      K=0
      A=1.
    2 A=A*R
      K=K+1
      IF(A.LT.1.E-20)RETURN
      IF(K.LT.N)GO TO 2
      RETURN
      END
```

Figure C-8. Listing of subroutine LIMIT.

114

```fortran
      SUBROUTINE MULTIP
      COMMON/SO/CODE,PARAM,W(200),TUR(200),BIT(200)
      INTEGER  CONLEV, PARAM
C     THIS SUBROUTINE COORDINATES THE TWO STATISTICAL TESTS
C     (CHISQR FOR FAILURE PARAMETERS AND FTEST FOR TIME
C     PARAMETERS) TO DETERMINE IF DATA FROM MULTIPLE TESTS
C     COME FROM THE SAME POPULATION.
    1 WRITE(*,101)
      CALL ENTERI(PARAM)
      IF((PARAM.EQ.1).OR.(PARAM.EQ.2).OR.(PARAM.EQ.3))GO TO 3
      WRITE(*,102)
      GO TO 1
    3 WRITE(*,103)
      CALL ENTERI(CONLEV)
      IF(PARAM.LE.2)CALL FTEST(CONLEV)
      IF(PARAM.EQ.3)CALL CHISQR(CONLEV)
      RETURN

C     **************  FORMAT STATEMENTS   ****************

  101 FORMAT(//+DO YOUR TESTS MEASURE+/
     *+   1. DELAYS,+/
     *+   2. RATES,+/
     *+OR+/
     *+   3. FAILURES++/
     *+PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE APPROPRIATE+/
     *+PARAMETER.+)
  102 FORMAT(//+YOU FAILED TO TYPE 1, 2, OR 3.+)
  103 FORMAT(//+THE MEAN OF THE  PERFORMANCE PARAMETER THAT YOU  +/
     *+SELECTED CAN BE MEASURED WITH ONE OF THE FOLLOWING LEVELS+/
     *+OF CONFIDENCE:+/
     *+    1. 90%+/
     *+    2. 95%+/
     *+   PLEASE TYPE THE INTEGER LISTED AT THE LEFT OF THE CONFID-+/
     *+ENCE LEVEL THAT YOU HAVE SELECTED.+)
      END
```

Figure C-9. Listing of subroutine MULTIP.

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```
      SUBROUTINE POISS(CONLEV,U,S,YL,YU,D)
      INTEGER CONLEV, S
C     THIS SUBROUTINE DETERMINES THE VALUES OF D FOR THE POISSON
C     APPROXIMATION UPPER CONFIDENCE LIMITS AND THE CONFIDENCE
C     LIMITS FOR THE MEAN OF A POISSON DISTRIBUTION (CALLED YL AND YU).
      DIMENSION CL1(45),CU1(45),CL2(45),CU2(45)
      DATA CL1/.000,.051,.36,.82,1.37,1.97,2.61,3.3,4.0,4.7,5.4,
     *6.2,6.9,7.7,8.5,9.2,10.0,10.8,11.6,12.4,13.3,
     *14.1,14.9,15.7,16.6,17.4,18.2,19.1,19.9,20.7,21.6,
     *25.9,30.2,34.6,39.0,43.,48.,52.,57.,61.,66.,70.,75.,80.,84./
      DATA CU1/3.00,4.7,6.3,7.8,9.2,10.5,11.8,13.1,14.4,15.7,17.0,
     *18.2,19.4,20.7,21.9,23.1,24.3,25.5,26.7,27.9,29.1,
     *30.2,31.4,32.6,33.8,35.,36.,37.,38.,40.,41.,
     *46.,52.,58.,63.,69.,74.,80.,85.,91.,96.,102.,107.,113.,118./
      DATA CL2/.000,.025,.242,.62,1.09,1.62,2.20,2.81,3.5,4.1,4.8,
     *5.5,6.2,6.9,7.7,8.4,9.1,9.9,10.7,11.4,12.2,
     *13.0,13.8,14.6,15.4,16.2,17.0,17.8,18.6,19.4,20.2,
     *24.4,28.6,32.8,37.1,41.,46.,50.,55.,59.,63.,68.,72.,77.,81./
      DATA CU2/3.7,5.6,7.2,8.8,10.2,11.7,13.1,14.4,15.8,17.1,18.4,
     *19.7,21.0,22.2,23.5,24.7,26.0,27.2,28.4,29.7,31.,
     *32.,33.,35.,36.,37.,38.,39.,40.,42.,43.,
     *49.,54.,60.,66.,72.,77.,83.,88.,94.,100.,105.,111.,116.,122./

C     **********  DETERMINE THE CONFIDENCE LIMITS, YL AND YU,  ***********
C     **********  FOR THE MEAN OF A POISSON DISTRIBUTION.  ***************


      IF(S.GT.100)GO TO 33
      IF(S.GT.30)GO TO 40
      IT=S+1
      IF(CONLEV.EQ.1)YL=CL1(IT)
      IF(CONLEV.EQ.1)YU=CU1(IT)
      IF(CONLEV.EQ.2)YL=CL2(IT)
      IF(CONLEV.EQ.2)YU=CU2(IT)
      GO TO 35
   40 IT=(S-30)/5+31
      IF(CONLEV.EQ.1)THEN
      YL1=CL1(IT)
      YU1=CU1(IT)
      YL2=CL1(IT+1)
      YU2=CU1(IT+1)
      END IF
      IF(CONLEV.EQ.2)THEN
      YL1=CL2(IT)
      YU1=CU2(IT)
      YL2=CL2(IT+1)
      YU2=CU2(IT+1)
      END IF
      S1=(S/5)*5.
      S2=S1+5.
C     INTERPOLATE
      YL=YL1*(S2-S)/5.+YL2*(S-S1)/5.
      YU=YU1*(S2-S)/5.+YU2*(S-S1)/5.
      GO TO 35
   33 QL=S-0.5
      QU=S+0.5
      YL=QL+0.375*U*U-U*SQRT(QL+0.125*U*U)
      YU=QU+0.375*U*U+U*SQRT(QU+0.125*U*U)

C     *******  DETERMINE, D, A VALUE USED FOR THE POISSON APPROXIMATE  *******
C     *******  UPPER CONFIDENCE LIMIT.  **************************************

   35 IF((S.LE.3).AND.(CONLEV.EQ.1))D=0.012*(S+1.)
      IF((S.LE.3).AND.(CONLEV.EQ.2))D=0.019*(S+1.)
```

Figure C-10. Listing of subroutine POISS.

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```
IF((S.GE.4).AND.(CONLEV.EQ.1))D=0.062
IF((S.GE.4).AND.(CONLEV.EQ.2))D=0.093
IF(S.GT.50)D=0.
RETURN
END
```

Figure C-10.   Continued.

```
      SUBROUTINE SSDFLR(CODE,NODE,CONLEV,N,S,R,A,PH,PL,PU)
      INTEGER CODE, CONLEV, S, R, SO
C     THIS SUBROUTINE DETERMINES THE SAMPLE SIZE REQUIRED TO ACHIEVE
C     A GIVEN RELATIVE PRECISION AND CONFIDENCE LEVEL FOR THE MEAN
C     PROBABILITY OF A FAILURE.
      CHARACTER MY*3, MN*2, MAX*3
      DATA MY/↑YES↑/,MN/↑ NO↑/
      AQ(A,U)=0.0001*(A/U)**2.
      BQ(X)=-(1.+2.*X/3.)
      CQ(X,U)=0.5-0.125*(U**2.)+X/9.
      SINDF(A,B,C)=(-B+SQRT(B*B-4.*A*C))/(2.*A)
C     DETERMINE SIND
      IF(CONLEV.EQ.1)U=1.645
      IF(CONLEV.EQ.2)U=1.960
      A1=AQ(A,U)
      B1=BQ(A1)
      C1=CQ(A1,U)
      SIND=SINDF(A1,B1,C1)
      IF(CODE.GT.0)GO TO 200

C     *********   DETERMINE THE SAMPLE SIZE FOR THE TEST   **********

    1 WRITE(*,100)
      CALL ENTERA(MAX)
      IF(MAX.EQ.↑NO↑) GO TO 2
      IF(MAX.EQ.↑YES↑)GO TO 4
      WRITE(*,99)
      GO TO 1
    2 WRITE(*,101)
      CALL ENTERX(B)
      S=INT((1.645/(2.*B))**2.)+1                                    (A-10)
      CODE=NODE+2
      RETURN

    4 WRITE(*,103)
      CALL ENTERX(P11MAX)
      S=INT(SIND*(1.+P11MAX)/(1.-P11MAX))+1                          (A-9)
      CODE=NODE
      RETURN

C     *************   DETERMINE IF ANOTHER SAMPLE IS NECESSARY   **************

  200 CALL ANZFLR(CONLEV,N,S,R,PH,PL,PU,P11U)
      SO=INT(SIND*(1.+P11U)/(1.-P11U))+1                             (A-13)
      IF(SO.LT.S)THEN
      CODE=CODE-2
      RETURN

      END IF
      S=SO-S                                                         (A-14)
      RETURN

C     ****************   FORMAT STATEMENTS   ****************

   99 FORMAT(//↑YOU FAILED TO TYPE EITHER YES OR NO.↑)
  100 FORMAT(//↑   CAN YOU ESTIMATE  THE MAXIMUM VALUE OF THE↑/
     *↑CONDITIONAL PROBABILITY OF A FAILURE GIVEN THAT↑/
     *↑A FAILURE OCCURRED IN THE PREVIOUS TRIAL↓ PLEASE↑/
     *↑TYPE YES OR NO.↑)
```

Figure C-11. Listing of subroutine SSDFLR.

```
101 FORMAT(//4X,↑SINCE YOU CANNOT ESTIMATE THE MAXIMUM VALUE OF THE↑/
   *↑CONDITIONAL PROBABILITY,  SPECIFY THE INTERVAL ABOVE↑/
   *↑THE CONDITIONAL PROBABILITY THAT YOU EXPECT IT TO EXCEED ↑/
   *↑ONLY 5% OF THE TIME. PLEASE TYPE THIS VALUE IN THE FORM 0.XXX.↑)
103 FORMAT(//↑PLEASE TYPE THIS DECIMAL VALUE IN THE FORM 0.XXX.↑)
   END
```

Figure C-11. Continued.

```
      SUBROUTINE SSDTIM(NODE,CONLEV,N,ABS)
      INTEGER CODE, CONLEV
C     THIS SUBROUTINE DETERMINES THE SAMPLE SIZE REQUIRED TO ACHIEVE
C     A GIVEN ABSOLUTE PRECISION AND CONFIDENCE LEVEL FOR THE MEAN
C     DELAY.
      COMMON/SQ/CODE,PARAM,W(200),TUR(200),BIT(200)
      CHARACTER MY*3, MN*2, ANS*3, IND*3, AUT*3
      DATA MY/'YES'/, MN/'NO'/
      IF(CODE.GT.0)GO TO 41

C     **********   DETERMINE THE SAMPLE SIZE FOR THE TEST  ************

      CODE=NODE
    1 IF(CODE.LE.14)WRITE(*,99)
      IF(CODE.GE.21)WRITE(*,100)
      CALL ENTERA(ANS)
      IF(ANS.EQ.'NO')GO TO 3
      IF(ANS.EQ.'YES')GO TO 4
      WRITE(*,101)
      GO TO 1
    3 CODE=NODE+2
      N=10
      RETURN

    4 WRITE(*,109)
      CALL ENTERX(SIGMAX)
      IF(CONLEV.EQ.1)U=1.645
      IF(CONLEV.EQ.2)U=1.960
      NIND=INT((U*SIGMAX/ABS)**2.)+1                                      (A-1)
  190 WRITE(*,105)
      CALL ENTERA(IND)
      IF(IND.EQ.'YES')GO TO 207
      IF(IND.EQ.'NO')GO TO 211
      WRITE(*,101)
      GO TO 190
  207 CODE=NODE
      N=NIND
      RETURN

  211 WRITE(*,201)
      CALL ENTERA(AUT)
      IF(AUT.EQ.'YES')GO TO 212
      IF(AUT.EQ.'NO')GO TO 222
      WRITE(*,101)
      GO TO 211
  222 CODE=NODE+2
      N=10
      RETURN

  212 WRITE(*,202)
      CALL ENTERX(X)
      CODE=NODE
      N=INT(NIND*(1.+X)/(1.-X))+1                                         (A-2)
      RETURN

C     *************   DETERMINE IF ANOTHER SAMPLE IS NECESSARY  ************

   41 WAVE=0.
      DO 43 I=1,N
      WAVE=WAVE+W(I)
   43 CONTINUE
      WAVE=WAVE/N                                                         (A-4)
      SD=0.
```

Figure C-12.   Listing of subroutine SSDTIM.

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```
      DO 44 I=1,N
      SD=SD+(W(I)-WAVE)*(W(I)-WAVE)
   44 CONTINUE
      NA=N-1
      SD=SQRT(SD/NA)                                                    (A-5)
      WA=0.
      DO 92 I=1,NA
      WA=WA+(W(I)-WAVE)*(W(I+1)-WAVE)
   92 CONTINUE
      X=WA/(SD*SD*NA)                                                   (A-3)
      CALL STUDNT(CONLEV,N,T)
      AR1=T*SD*SQRT((1.+X)/((1.-X)*N))                                  (A-6)
      IF(AR1.LE.ABS)THEN
      CODE=CODE-2
      RETURN

      END IF
      N=INT((T*SD/ABS)**2.*(1.+X)/(1.-X))+1-N                           (A-8)
      RETURN

C     ***************   FORMAT STATEMENTS   *****************

   99 FORMAT(//+DO YOU KNOW THE MAXIMUM VALUE OF THE STANDARD +/
     *+DEVIATION OF THE DELAYS+ PLEASE TYPE YES OR NO.+)
  100 FORMAT(//+DO YOU KNOW THE MAXIMUM VALUE OF THE STANDARD +/
     *+DEVIATION OF THE RATES+ PLEASE TYPE YES OR NO.+)
  101 FORMAT(//+YOU FAILED TO TYPE EITHER YES OR NO.+)
  105 FORMAT(//+ARE THE TRIALS STATISTICALLY INDEPENDENT+ (IF THEY+/
     *+ARE, THEY ARE UNCORRELATED). PLEASE TYPE YES OR NO.+)
  109 FORMAT(//+PLEASE TYPE THE MAXIMUM VALUE OF THE+/
     *+STANDARD DEVIATION IN THE FORM XXX.XXX.+)
  201 FORMAT(//+SINCE THE TRIALS ARE NOT STATISTICALLY INDEPENDENT,+/
     *+THE AUTOCORRELATION IS NOT ZERO. DO YOU KNOW THE VALUE OF+/
     *+THE AUTOCORRELATION OF LAG 1+ PLEASE TYPE YES OR NO+)
  202 FORMAT(//+PLEASE TYPE THIS VALUE IN THE FORM 0.XX OR -0.XX.+)
      END
```

Figure C-12. Continued.

```
      SUBROUTINE STUDNT(CONLEV,N,T)
C     THIS SUBROUTINE DETERMINES THE STUDENT T STATISTIC.
      INTEGER CONLEV
      DIMENSION T90(9),T95(9),NP(9)
      DATA NP/1,2,3,4,5,10,20,40,400/
      DATA T90/6.314,2.920,2.353,2.132,2.015,1.812,1.725,1.684,1.645/
      DATA T95/12.706,4.303,3.182,2.776,2.571,2.228,2.086,2.021,1.960/

C     ******   FIND TWO CONSECUTIVE POINTS OF TABLE THAT BRACKET N-1.*****

      DO 60 I=1,8
      IF((N-1.GE.NP(I)).AND.(N-1.LE.NP(I+1)))GO TO 61
   60 CONTINUE

C     *******   SELECT TABLE ACCORDING TO CONFIDENCE LEVEL   *******

   61 IF(CONLEV.EQ.1)T1=T90(I)
      IF(CONLEV.EQ.1)T2=T90(I+1)
      IF(CONLEV.EQ.2)T1=T95(I)
      IF(CONLEV.EQ.2)T2=T95(I+1)

C     *****   T IS DETERMINED BY HYPERBOLIC INTERPOLATION FOR   ************
C     *****   DEGREES OF FREEDOM BETWEEN THOSE LISTED IN THE TABLE.   ******

      A=((T1-T2)*NP(I)*NP(I+1))/(NP(I+1)-NP(I))
      B=(T2*NP(I+1)-T1*NP(I))/(NP(I+1)-NP(I))
      T=A/(N-1)+B
      RETURN
      END
```

Figure C-13. Listing of subroutine STUDNT.

```
      SUBROUTINE XFILE(W,TUR,BIT,I)
      DIMENSION W(200),TUR(200),BIT(200)
      CHARACTER  FILENM*6
      WRITE(*,101)
      CALL ENTERA(FILENM)
      CLOSE(UNIT=1)
      OPEN(UNIT=1,FILE=FILENM)
      I=0
   10 READ(1,102)WF,TURF,BITF
      IF(WF.NE.-10.)THEN
      I=I+1
      W(I)=WF
      TUR(I)=TURF
      BIT(I)=BITF
      GO TO 10
      END IF
      RETURN

C     ********     FORMAT STATEMENTS     *********

  101 FORMAT(//+PLEASE ENTER THE NAME OF THE FILE CONTAINING+/
     *+DATA FROM THE TEST . THIS NAME SHOULD BE A +/
     *+ CHARACTER NAME OF THE FORM AAAAAA.+)
  102 FORMAT(2F16.3,F16.0)
      END




      SUBROUTINE YFILE(W,TUR,BIT,J,I)
      DIMENSION W(200),TUR(200),BIT(200)
      CHARACTER  FILENM*6
      WRITE(*,101)J
      CALL ENTERA(FILENM)
      CLOSE(UNIT=1)
      OPEN(UNIT=1,FILE=FILENM)
      I=0
   10 READ(1,102)WF,TURF,BITF
      IF(WF.NE.-10.)THEN

      I=I+1
      W(I)=WF
      TUR(I)=TURF
      BIT(I)=BITF
      GO TO 10
      END IF
      RETURN

C     ********     FORMAT STATEMENTS     *********

  101 FORMAT(//+PLEASE ENTER THE NAME OF THE FILE CONTAINING+/
     *+DATA FROM THE TEST ON DAY+,I2,+. THIS NAME+/
     *+SHOULD BE A CHARACTER NAME OF THE FORM AAAAAA.+)
  102 FORMAT(2F16.3,F16.0)
      END
```
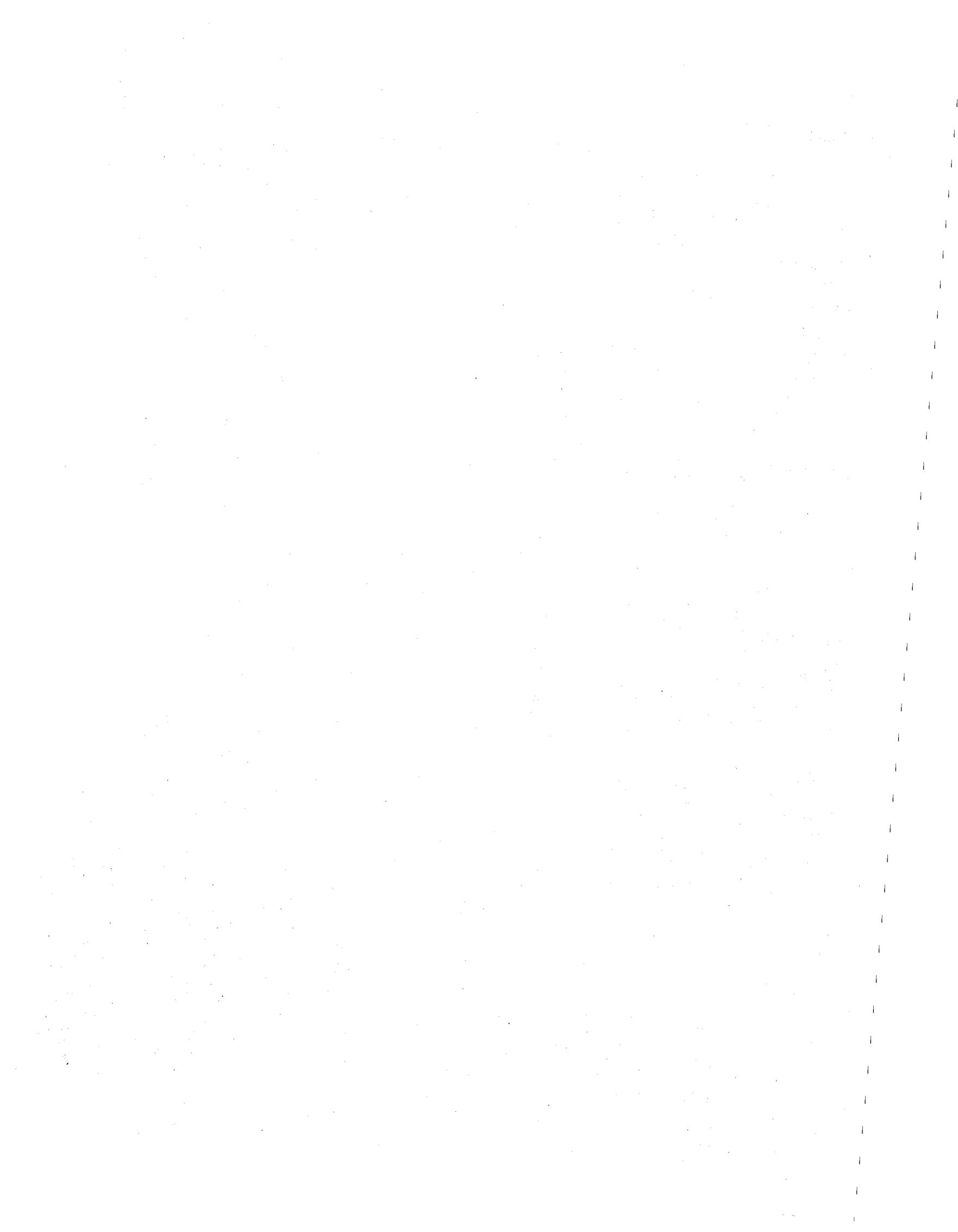
Figure C-14.   Listing of subroutines XFILE and YFILE.

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15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

This report describes an interactive computer program that facilitates efficient measurement of communication system performance parameters. The program performs three primary functions: (1) determines the minimum sample size required to achieve a desired precision in estimating delay, rate, or failure probability parameters; (2) analyzes measurement results to determine the precision achieved; and (3) tests independent sets of measured data for statistical homogeneity. The report discusses statistical concepts underlying the program, shows how each function is performed, and provides comprehensive program documentation in the in the form of mathematical formulas, structured design diagrams, and the program listing. The program was written specifically to facilitate measurement of the performance parameters defined in a newly approved Data Communication Standard, American National Standard X3.102. The statistical techniques implemented in it may also be applied to any other delay, rate, or failure probability measurement. "continued on reverse"

16. Key Words *(Alphabetical order, separated by semicolons)*

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The program is written in ANSI (1977) standard FORTRAN to enhance its portability. It is available from the author at duplication cost.