NTIA Report 91-273

Mean Synchronization Times for ATM Cells: Derivations and Computational Background

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March 1991

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MEAN SYNCHRONIZATION TIMES FOR ATM CELLS: DERIVATIONS AND COMPUTATIONAL BACKGROUND

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ABSTRACT

The mean synchronization (sync) duration time and the mean sync acquisition times are derived and computed for the Asynchronous Transfer Mode of the proposed Broadband ISDN. These waiting times are expressed as functions of channel quality, i.e., the bit error probability and related event probabilities, and as a function of two system sync thresholds.

Key words: acquisition time; asynchronous transfer mode (ATM); broadband ISDN; cell; header error control (HEC); in-sync duration time; synchronization

1. INTRODUCTION

The CCITT adopted recommendation, I.121, and proposed recommendation, I.432, specify a new method for broadband ISDN (B-ISDN) cell synchronization (sync) or delineation in the Asynchronous Transfer Mode (ATM) (CCITT, 1988 and 1990; T1S1, 1989). Cell identification is accomplished by means of an 8-bit header error control (HEC) field in a 40-bit header of the cell. The cell itself consists of 424 bits, or 53 bytes, including the header. The sync process, by design, occupies one of three mutually exclusive system states: HUNT, PRESYNC, and SYNC. Given specified HEC events, the process either stays in its existing state or it transits to another state. Figure 1 presents the state-transition diagram for the process.

In the HUNT state, the sync process performs a bit-by-bit search for the event HEC = 0 (i.e., for the condition where the parity check syndrome is zero). Once the HEC = 0 condition is observed, the system enters the PRESYNC state. In the PRESYNC state, header parity checks are carried out only once per cell, or after every 424 bits. This is continued until either a violation (HEC $\neq$ 0) occurs, or δ consecutive correct HEC's are realized. In the first case, the search abandons the PRESYNC state and returns to the HUNT state. In the second case, the system enters the SYNC state. Parameter δ is thus an important sync acquisition threshold. It affects the average waiting time for sync acquisition.

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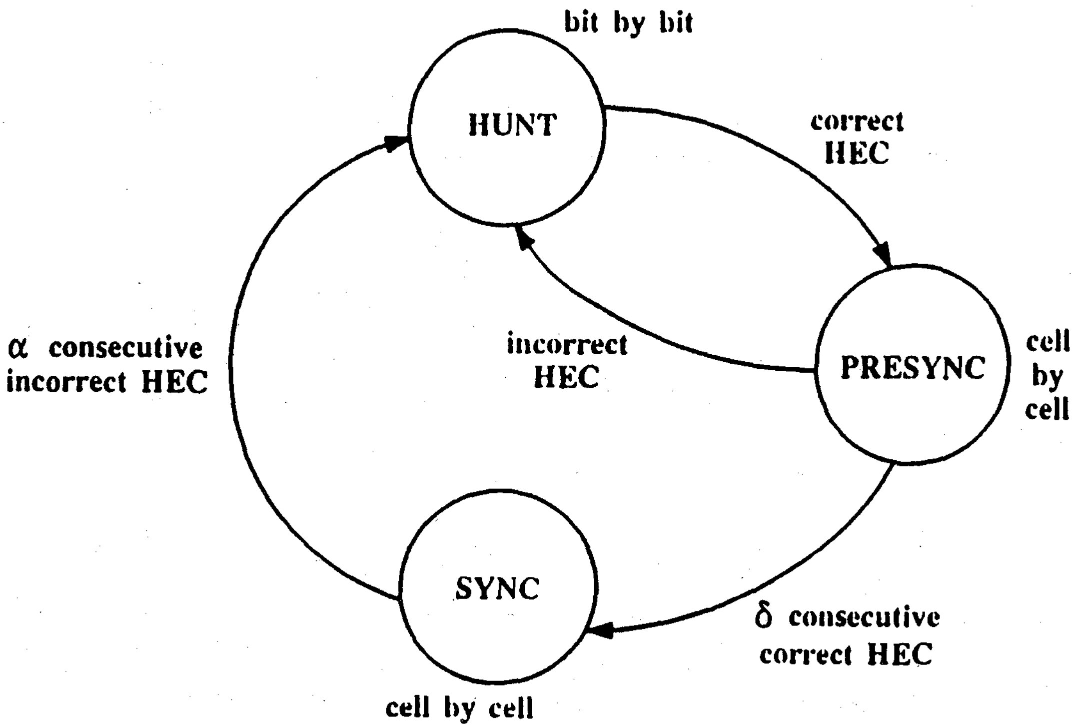

Figure 1. State diagram for ATM cell synchronization.

Once the system is in the SYNC state, cell synchronization is assumed for operational purposes. However, the HEC = 0 versus HEC $\neq$ 0 condition is monitored to identify sync violations. As in the PRESYNC state, the system still performs cell-by-cell HEC parity checks. The system stays in the SYNC state, unless α consecutive incorrect (i.e., HEC $\neq$ 0) cells occur. If so, then sync is assumed to be lost and the system returns to the HUNT state. In the HUNT state, the previously described bit-by-bit search is started all over again. Parameter α is another important synchronization threshold. It affects the statistics of the in-sync duration time.

This study presents equations and curves for the mean in-sync duration time $T_d(\alpha)$ and the mean sync acquisition (or waiting) time $T_a(\delta)$. These two quantities, as already indicated, are functions of threshold parameters α and δ, respectively. They also depend on channel bit error probability, p_e. Since values $\alpha = 7$ and $\delta = 6$ have been proposed, the numerical work here considers ranges $\alpha = 5, 7, 9$ and $\delta = 4, 6, 8$. For p_e, a realistic computation range from 10^{-6} to 10^{-2} is assumed. Likewise, it is assumed that the occurrences of HEC errors are independent random events.

The analytical methods employed here will be largely associated with the, so called, probability generating functions (gf). Section 2 introduces the gf's and their key properties.

Section 3 derives expressions for the mean in-sync duration time $T_d(\alpha)$. Numerical evaluation leads to different duration for different (e.g., HEC = 0 or HEC $\neq$ 0) starting conditions in the SYNC state.

Section 4 gives approximations and plots of the mean sync acquisition waiting time, $T_a(\delta)$, as a function of p_e. In recent submissions to CCITT these characteristics have been indicated to represent close upper bounds to the true $T_a(\delta)$.

Section 5 solves the exact problem for $T_a(\delta)$. While retaining the same general features, these curves are slightly more favorable than the approximate curves of section 4.

Finally, section 6 gives a brief introduction to variances for both in-sync duration and sync acquisition times.

2. GENERATING FUNCTIONS

If p(n) is the probability that a random event occurs at time n, where n = 0, 1, 2, . . . , then the probability generating function, or gf, for this event is defined (Feller, 1968) as

$$G(s) = \sum_{n=0}^{\infty} p(n)\, s^n. \tag{1}$$

If there are a number of mutually exclusive classifications of these random events, then separate gf's may be assigned to the individual classes of events. For example, if i = 0, 1, 2, . . . identifies different classes of events, then $G_i(s)$ can represent the gf for the i-th class.

When dealing with event-generating systems, it seems natural to use the term "state." Thus, when an event of class i occurs, the system is said to be in state i. In this report, state-transition diagrams will be used to represent the states and to define transitions between the states. For systems of interest, the number of states will be finite, but not necessarily a known fixed number.

Recent texts, such as Gupta (1966), Cadzow (1973), and Kleinrock (1975), refer to the gf as the z-transform and apply it to analysis of finite-state machines or systems.

There are at least three types of system states that occur in analysis of synchronization systems: the recurrent states, the nonrecurrent states, and the irrelevant states. The recurrent states are those that occur any number of times. Thus, state i is recurrent if events of class i can be generated either 0, or 1, or 2, or any number of times by the system. Nonrecurrent states can happen no more than once. Finally, irrelevant states are those with no direct bearing on a given problem. Among them one finds dead-end states, escape (or exit) states, and states that can never be reached.

Recurrent states occur frequently in the work to follow. However, the sought synchronization times are more closely associated with nonrecurrent states. For nonrecurrent states, any arrival is the first and only arrival. Therefore, given a nonrecurrent state with gf G(s), the expected or mean waiting time, T, for the first arrival event is given by the derivative of G(s) evaluated at s = 1:

$$T = \sum_{n=1}^{\infty} n\, p(n) = G'(1). \tag{2}$$

As an illustration of gf construction and application to a waiting time problem, Figure 2 presents an example of a state-transition diagram. This diagram has four states, i = 0, 1, 2, and 3. Of these, states 0 and 1 are recurrent, state 2 is nonrecurrent, and state 3 provides escape (it is irrelevant). The one-way arrows between states indicate state-to-state transitions that are allowed to occur at discrete times. The descriptor associated with every transition, such as $p_i s^d$, is a linear attribute. It is additive for parallel transition paths and multiplicative for serial paths through the system (Zadeh and Desoer, 1963). The typical attribute, $p_i s^d$, denotes that the transition occurs with probability p_i and that its duration (delay) is d. Note that descriptor 1, indicated on the state 2 - to - state 3 transition in the diagram, represents $p_i = 1$ and $d = 0$. This transition is certain and immediate.

Finally, Figure 2 shows two initial state probabilities a_0 and a_1. That means that at time n = 0, the system starts in state 0 or 1, with probability a_0 and a_1, respectively. No other initial states are permitted in this example, so

$$a_0 + a_1 = 1. \tag{3}$$

Let us continue the illustration by deriving the mean waiting time expression for the system to arrive in state 2. According to equation (2), this waiting time is given by $G'_2(1)$, where $G_2(s)$ is the gf of state 2.

Figure 2 shows that $G_2(s)$ depends on $G_0(s)$ and $G_1(s)$, but not on $G_3(s)$. Specifically, at state 0, the event probability $p_0(n)$ at time n must satisfy

$$\begin{aligned} p_0(n) &= a_0 && \text{if } n = 0, \\ &= p_0\, p_0(0) && \quad\;\;\, = 1, \\ &= p_0\, p_0(n-1) + p_2\, p_1(n-2) && \quad\;\;\, \geq 2, \end{aligned} \tag{4}$$

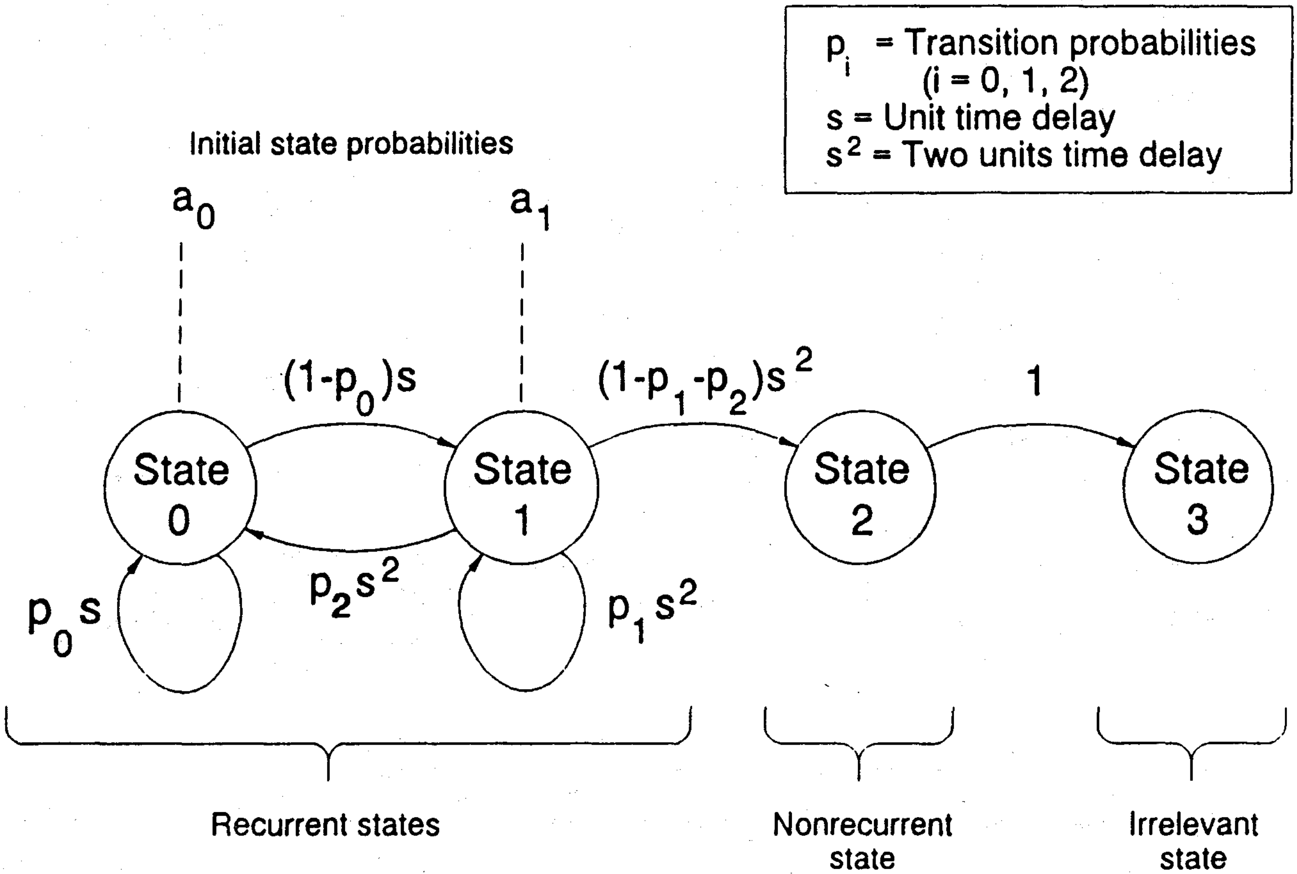

Figure 2. Illustrative state-transition diagram.

where $p_1(n)$ is the corresponding occupancy probability for state 1 at time n. One concludes that the gf for state 0 must equal

$$
\begin{aligned}
G_0(s) &= \sum_{n=0}^{\infty} p_0(n)\ s^n \\
&= a_0 + p_0 p_0(0) s + \sum_{n=2}^{\infty} [p_0 p_0(n-1) + p_2 p_1(n-2)]\ s^n \\
&= a_0 + p_0 s\, p_0(0) + p_0 s \sum_{n=1}^{\infty} p_0(n)\, s^n + p_2 s^2 \sum_{n=0}^{\infty} p_1(n)\, s^n \\
&= a_0 + p_0 s\, G_0(s) + p_2 s^2 G_1(s)\ .
\end{aligned}
\tag{5}
$$

Inspection of Figure 2, state 0, confirms that the gf for state 0 is equal to the sum of the initial condition plus all "transition inputs." The transition inputs come from all indicated states (including 0 itself), and they are multiplied by their linear transition attributes, such as $p_0 s$ and $p_2 s^2$.

Similar derivations apply to states 1 and 2. One obtains

$$
\begin{aligned}
G_1(s) &= a_1 + (1-p_0)s\ G_0(s) + p_1 s^2 G_1(s), \\
G_2(s) &= (1 - p_1 - p_2)s^2 G_1(s).
\end{aligned}
\tag{6}
$$

Note that in equations (5) and (6) one has three linear equations for three unknowns: $G_0(s)$, $G_1(s)$, and $G_2(s)$. The gf for state 3 is not involved here and therefore can be ignored. Yet, the presence of state 3 is significant in that it assures us that state 2 cannot occur more than once.

A standard way to solve for $G_2(s)$ is to express (5) and (6) in a matrix form and then to invert the matrix. In the present example, let

$$
G = \begin{pmatrix} G_0(s) \\ G_1(s) \\ G_2(s) \end{pmatrix}, \tag{7}
$$

$$
A = \begin{pmatrix} a_0 \\ a_1 \\ 0 \end{pmatrix}, \tag{8}
$$

and

$$M = \begin{pmatrix} 1-p_0 s & -p_2 s^2 & 0 \\ -(1-p_0)s & 1-p_1 s^2 & 0 \\ 0 & -(1-p_1-p_2)s^2 & 1 \end{pmatrix}. \quad (9)$$

Then (5) and (6) are equivalent to

$$M \cdot G = A, \quad (10)$$

and

$$G = M^{-1} \cdot A. \quad (11)$$

It is illustrative to work out this example in detail. First, by any of several standard techniques, the determinant $|M|$ is seen to be

$$|M| = 1-p_0 s - p_1 s^2 - (p_2 - p_0 p_1 - p_0 p_2)\, s^3. \quad (12)$$

Note that for s=1, this determinant equals $(1-p_0)(1-p_1-p_2)$. This means that there is no finite mean waiting time solution for state 2 whenever the determinant vanishes, i.e., when either $p_0 = 1$ or $p_1 + p_2 = 1$. These are the necessary and sufficient conditions for the system to be stuck in the recurrent states, and never being able to reach the nonrecurrent state 2.

More generally, solutions will always exist, as long as the determinant $|M|$ does not vanish at $s = 1$. Violations of this rule and subsequent infinite waiting times typically occur in transition diagrams with periodic, separated, or dead-end states. The cases to be considered here will avoid such pitfalls by verifying that $|M| \neq 0$ holds at $s = 1$, and that the adverse $|M| = 0$ would occur only under justifiable circumstances.

The matrix inverse is given by

$$M^{-1} = \frac{1}{|M|}\begin{pmatrix} 1-p_1s^2 & p_2s^2 & 0 \\ (1-p_0)s & 1-p_0s & 0 \\ (1-p_0)(1-p_1-p_2)s^3 & (1-p_1-p_2)(1-p_0s)s^2 & |M| \end{pmatrix}, \tag{13}$$

as can be verified by multiplication with M in (9).

The gf for state 2 follows from (8), (11), (12), and (13),

$$G_2(s) = \frac{(1-p_1-p_2)\,[a_0(1-p_0)s^3 + a_1(1-p_0s)s^2]}{1-p_0s-p_1s^2-(p_2-p_0p_1-p_0p_2)s^3}, \tag{14}$$

and the mean waiting time for first arrival at state 2 is therefore

$$G'_2(1) = 3a_0 + 2a_1 + \frac{p_0+2p_1+3p_2-3p_0(p_1+p_2)}{(1-p_0)(1-p_1-p_2)}. \tag{15}$$

The following sections derive expressions for mean in-sync duration times and acquisition times for ATM cells. The method is identical to that illustrated here. One begins with the state-transition diagram. The diagram generates linear equations for the unknown gf's. Matrix inversion produces the desired first-arrival gf. Finally, the derivative of the generating function evaluated at $s = 1$ is the sought mean waiting time. The approach for waiting time variances differs slightly from the above. It requires second derivatives evaluated at $s = 1$.

3. MEAN IN-SYNC DURATION TIME

Also known as the mean time for sync loss, duration time $T_d(\alpha)$ is deduced from the state-transition diagram shown in Figure 3. In this diagram:

s = unit delay operator,

α = threshold for sync loss,

p = probability of incorrect header (HEC $\neq$ 0),

q = probability of correct header (HEC = 0), which satisfies $p + q = 1$,

T = delay counter that advances one count for each ATM cell delay,

a_j = initial probability for state j, where $j = 0, 1, 2, \ldots, \alpha - 1$,

and the initial condition

$$a_0 + a_1 + a_2 + \ldots + a_{\alpha-1} = 1. \tag{16}$$

The matrix equations require $(\alpha + 1)$-dimensional vectors

$$G = \begin{pmatrix} G_0(s) \\ G_1(s) \\ G_2(s) \\ \vdots \\ G_\alpha(s) \end{pmatrix} \tag{17}$$

and

$$A = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{\alpha-1} \\ 0 \end{pmatrix}, \tag{18}$$

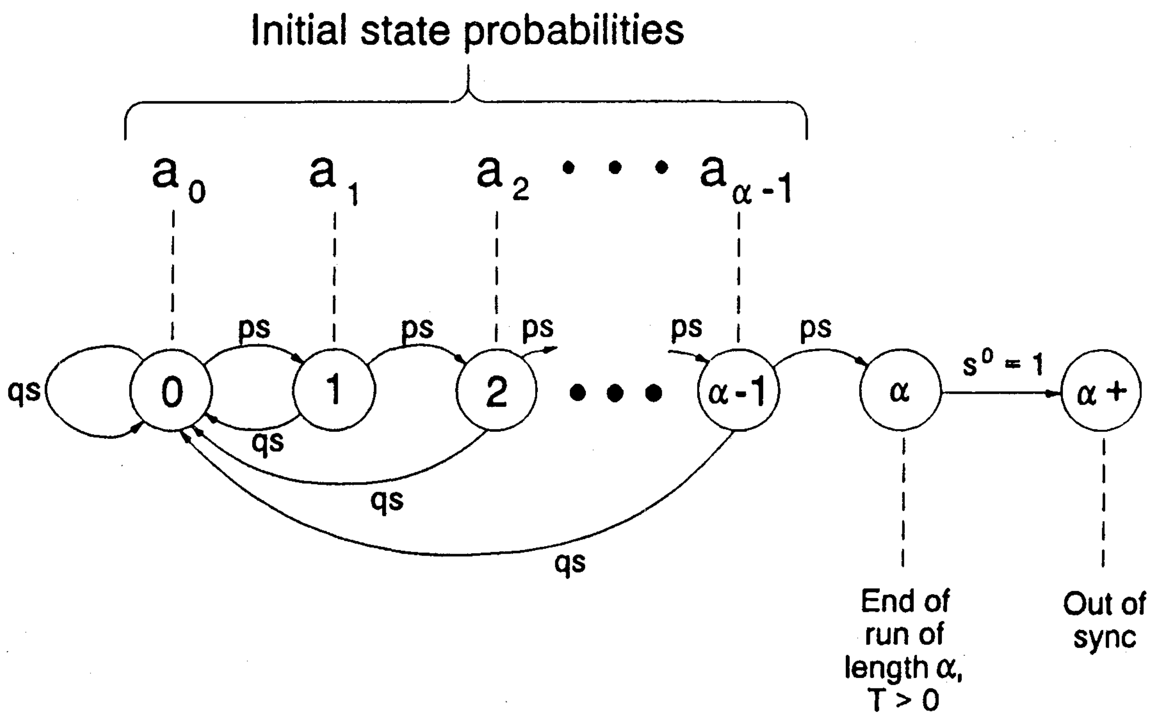

Figure 3. State-transition diagram for sync loss.

and an $(\alpha + 1) \times (\alpha + 1)$ matrix

$$M = \begin{pmatrix} 1-qs & -qs & -qs & & -qs & 0 \\ -ps & 1 & 0 & \cdots & 0 & 0 \\ 0 & -ps & 1 & & 0 & 0 \\ & \vdots & & & \vdots & \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & & -ps & 1 \end{pmatrix}. \tag{19}$$

It follows from addition of row (column) multiples to other rows (columns) that the determinant $|M|$ is given by

$$|M| = \frac{1-s+qp^{\alpha}s^{\alpha+1}}{1-ps} \tag{20}$$

For $s = 1$, this determinant equals p^{α}. Thus, the inverse matrix always exists for finite α and $p \neq 0$.

$$
M^{-1} = \frac{1}{(1-ps)\,|M|}
\begin{bmatrix}
1-s+qs & (1-p^{\alpha-1}s^{\alpha-1})qs & (1-p^{\alpha-2}s^{\alpha-2})qs & \cdots & (1-ps)qs & 0 \\
(1-ps)ps & 1-s+qps^2 & (1-p^{\alpha-2}s^{\alpha-2})qps^2 & \cdots & (1-ps)qps^2 & 0 \\
(1-ps)p^2s^2 & (1-s+qps^2)ps & 1-s+qp^2s^3 & \cdots & (1-ps)qp^2s^3 & 0 \\
 & \vdots & & & \vdots & \\
(1-ps)p^{\alpha-1}s^{\alpha-1} & (1-s+qps^2)p^{\alpha-2}s^{\alpha-2} & (1-s+qp^2s^3)p^{\alpha-3}s^{\alpha-3} & \cdots & 1-s+qp^{\alpha-1}s^{\alpha} & 0 \\
(1-ps)p^{\alpha}s^{\alpha} & (1-s+qps^2)p^{\alpha-1}s^{\alpha-1} & (1-s+qp^2s^3)p^{\alpha-2}s^{\alpha-2} & \cdots & (1-s+qp^{\alpha-1}s^{\alpha})ps & 1-s+qp^{\alpha}s^{\alpha+1}
\end{bmatrix}
\quad (21)
$$

Entity $G_\alpha(s)$ depends on the last row of M^{-1}. From equations (11), (18), (20), and (21) follows

$$G_\alpha(s) = \frac{\sum_{j=0}^{\alpha-1} a_j(1-s+qp^j s^{j+1})(ps)^{\alpha-j}}{1-s+qp^\alpha s^{\alpha+1}} . \tag{22}$$

One identifies the expected value of the random count T for state α with the mean in-sync duration time. It follows from (2) and (22) that the in-sync duration time, denoted by $T_d(\alpha)$, is given in cell units as

$$T_d(\alpha) = \frac{1}{qp^\alpha}\left(1-\sum_{j=0}^{\alpha-1} a_j\, p^{\alpha-j}\right). \tag{23}$$

Two particular cases of this duration time seem of immediate interest. The first case arises when the system starts in state 0 (i.e., when the most recent cell header satisfies HEC = 0). The second case is conditioned only on the premise that the system is in one of the states a_0, a_1, . . . , $a_{\alpha-1}$.

In the first case, $a_0 = 1$ and $a_j = 0$ for all $j \neq 0$. Then

$$T_{d1}(\alpha) = \frac{1-p^\alpha}{qp^\alpha}. \tag{24}$$

In the second state, while the system is in SYNC the previous headers could have generated none, one, or more HEC ≠ 0. The initial probability for state j is then estimated from the approximation $a_j/a_{j-1} \approx p$, which must be valid in Figure 3 for all $j = 1, 2, . . . , \alpha - 1$. Then

$$a_j = \frac{qp^j}{1-p^\alpha}, \qquad j = 0, 1, . . . , \alpha-1. \tag{25}$$

Substitution into (23) yields the in-sync duration time $T_{d2}(\alpha)$ for the second case,

$$T_{d2}(\alpha) = \frac{1}{qp^\alpha}\left(1 - \frac{\alpha qp^\alpha}{1-p^\alpha}\right). \tag{26}$$

Both duration times, $T_{d1}(\alpha)$ and $T_{d2}(\alpha)$, depend on channel bit error-probability, p_e. For a 40-bit header, transmitted over a binary symmetric channel,

$$q = (1 - p_e)^{40},$$
$$p = 1 - (1 - p_e)^{40}. \tag{27}$$

The work of CCITT Study Group XVIII has suggested a value $\alpha = 7$ for the threshold parameter. To illustrate the effect of different α values, we have computed (24) and (26) for α = 5, 7, and 9. Figure 4 shows $T_{d1}(\alpha)$ as a function of p_e for these three α values. Note that $T_{d1}(\alpha)$ increases monotonically as p_e decreases. For very small p_e, the asymptote $T_{d1}(\alpha) \sim (40\, p_e)^{-\alpha}$ is valid. Thus, for all $\alpha > 0$ $T_{d1}(\alpha)$ tends to infinity as p_e tends to zero. On the other hand, $T_{d1}(\alpha) \geq \alpha$ applies over the entire range of $0 \leq p_e \leq 1$.

Figure 5 shows the very similar behavior of $T_{d2}(\alpha)$. Note that $T_{d2}(\alpha)$ is always less than $T_{d1}(\alpha)$, while $T_{d2}(\alpha) \geq \frac{1}{2}(\alpha+1)$ is of particular interest for large p_e.

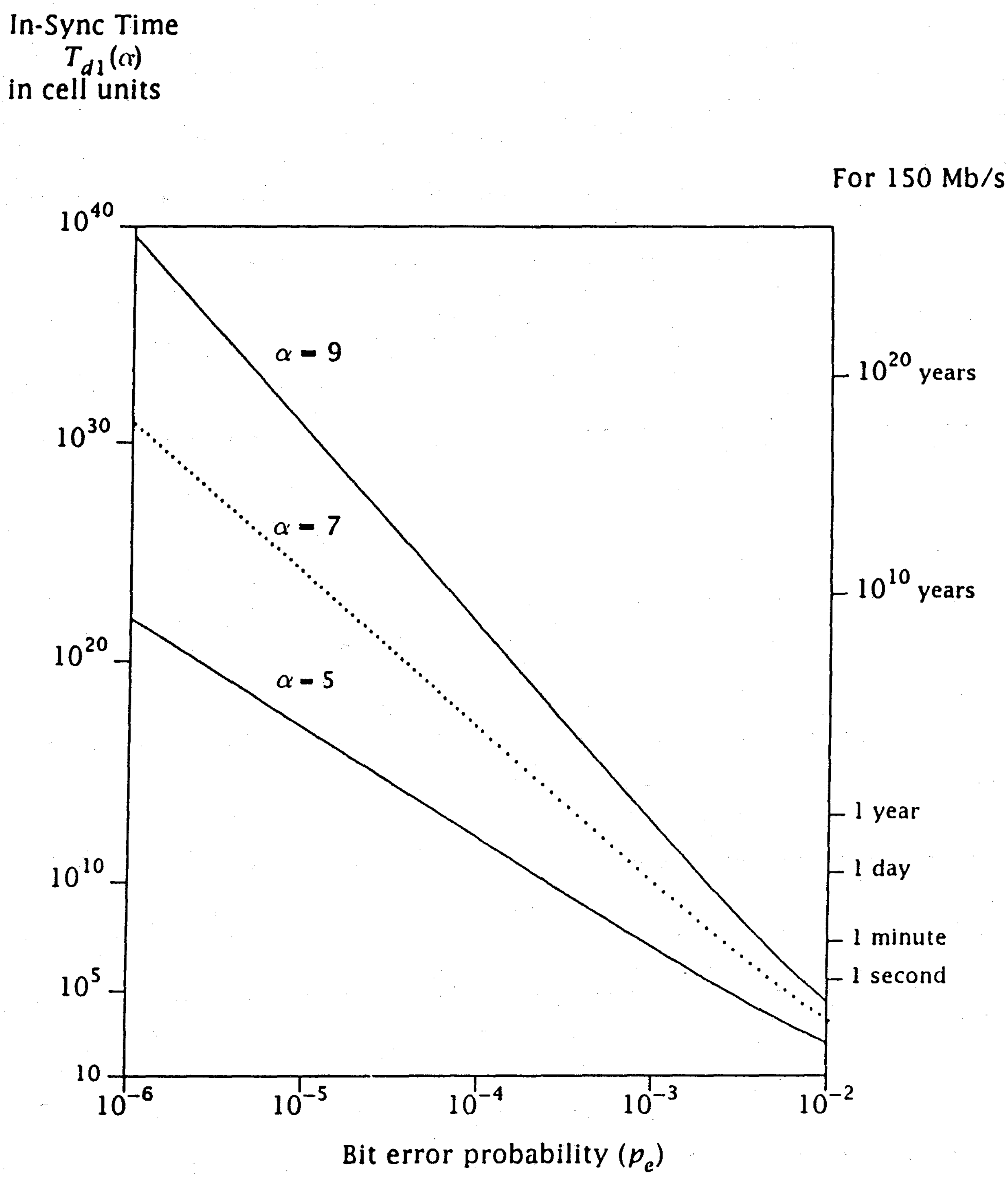

Figure 4. In-sync time vs bit error rate, given that the system starts with an error free cell.

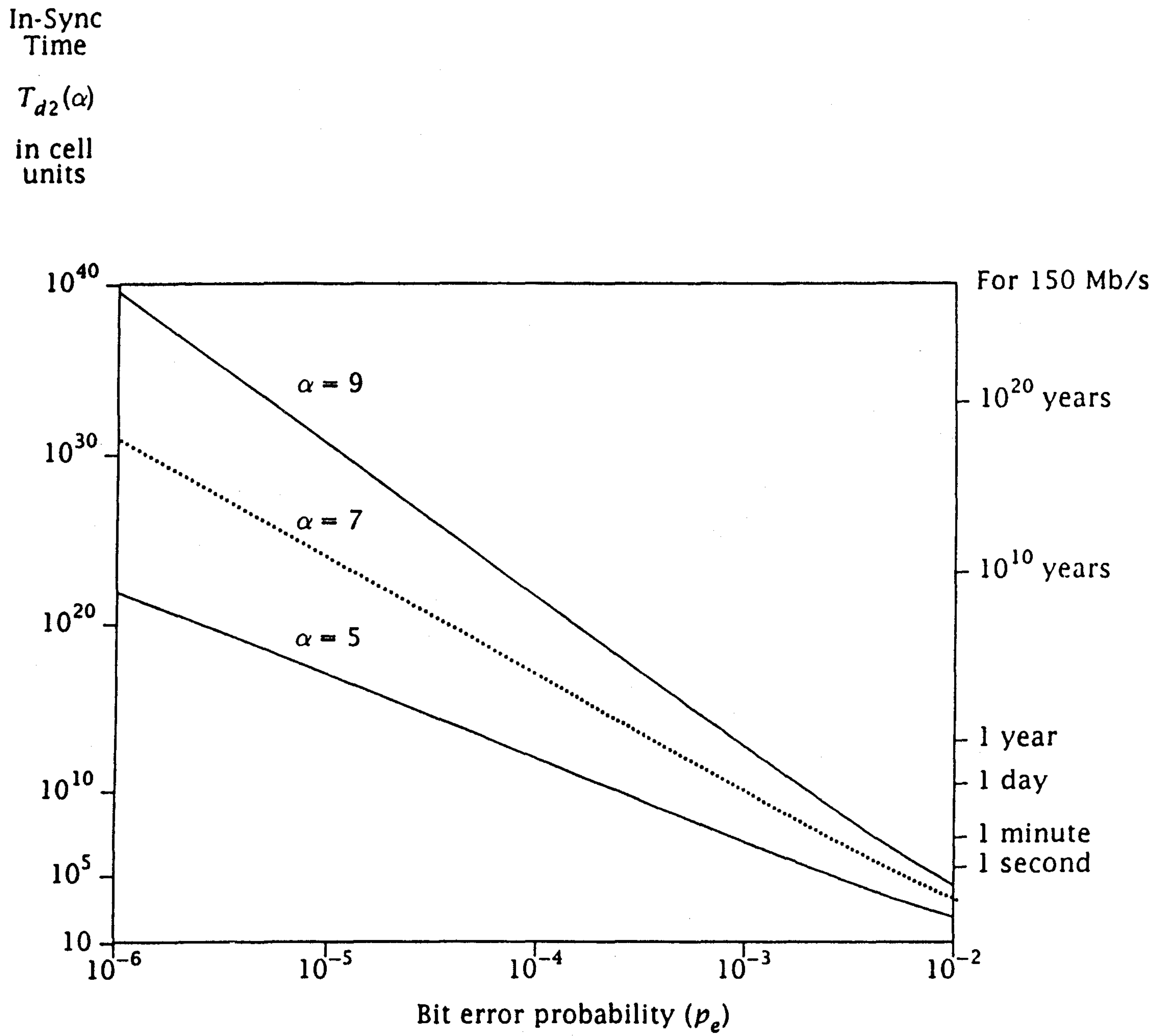

Figure 5. In-sync time vs bit error rate, given that the system starts anywhere in the SYNC state.

4. MEAN SYNC ACQUISITION TIME: AN APPROXIMATION

The sync acquisition process starts in the HUNT state, where a bit-by-bit search for HEC=0 takes place. Since there are 424 bits in the ATM cell, there are 423 misaligned or incorrect bit positions that may require attention. Once a HEC = 0 position has been located, the search enters the PRESYNC state. Now the header inspection is reduced in frequency. As described in the Introduction, 423 bits are skipped over and only the 424-th bit is inspected on a cell-by-cell basis. If HEC = 0 is observed in $\delta \geq 1$ consecutive cells, the system exits PRESYNC and enters the SYNC state. If $H \neq 0$ occurs before SYNC is attained, the system returns to the HUNT state. Here, one seeks the mean waiting or synchronization time, $T_a(\delta)$, required for the first time to reach the SYNC from the HUNT state. Suggested values for threshold δ are centered around $\delta = 6$.

Consider the approximate model defined in Figure 6. This is a $(\delta + 4)$-state model. It assumes that the HUNT starting time, being random over the 424 bits in a cell, can effectively be approximated as being the cell midpoint. The model also assumes that the events associated with the less frequent false alarms (fa) take place, more or less, in the center of a cell.

In the approximate model of Figure 6:

s = unit delay operator,
z = half-unit delay operator ($z^2=s$),
δ = sync acquisition threshold,
p = probability of incorrect header (HEC $\neq$ 0),
q = probability of correct header (HEC = 0), which satisfies $p + q = 1$,
u = probability of no fa in a half-cell (212 bits),
v = probability of HEC $\neq$ 0 in a given single bit of a random word,
y = probability of no fa in 423 bits,
$G_s(s)$ = gf of the start state (s),
$F(s)$ = gf of the false alarm state (fa),
$G_0(s)$ = gf of the pre-lineup state (0),
$G_x(s)$ = gf of the post-lineup state (x),
$G_n(s)$ = gf of the n-th presync state ($n = 1, 2, \ldots \delta - 1$),
$G_\delta(s)$ = gf of the sync state,
a_s = 1, the initial probability for the start state (s).

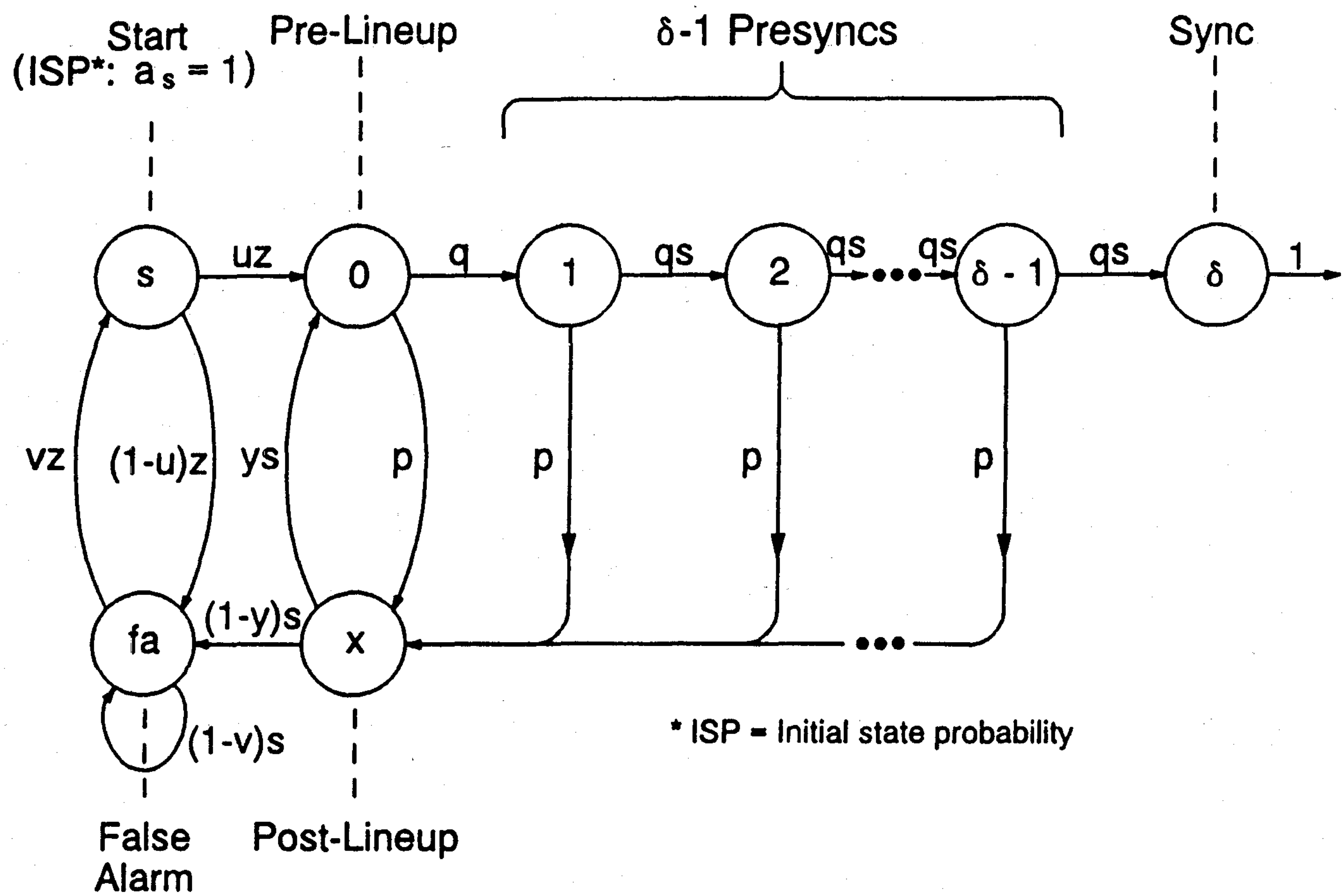

Figure 6. Approximate state-transition diagram for sync acquisition.

Starting from state s, the process reaches the pre-lineup state 0 with probability u and a half-cell delay. At 0, a nearly instantaneous header test takes place. If HEC = 0, which is an event with probability q, then the system enters PRESYNC state 1. From there it may sequentially proceed to states 2, 3, . . . , and eventually to the SYNC state δ.

If instead, at pre-lineup HEC $\neq$ 0 materializes, the system goes to the post-lineup state x. At this time the cell synchronization counters are still in alignment, but the system does not know it. From post-lineup x the system may go back to pre-lineup 0 if there are no fa's in the next 423 bits of the cell (this happens with probability y and one cell delay). Or, if false alarms do occur, the system must pass through the fa state. There are several ways in which state 0 can be reached from x and via fa, for example:

x-fa-s-0 with probability (1-y)vu and delay 2,

x-fa-fa-s-0 with probability (1-y)v(1-v)u and delay 3,

$\vdots$

x-fa-s-fa-s-0 with probability $(1-y)v^2(1-u)u$ and delay 3.

$\vdots$

In all of these paths, the system traverses cell centers, effectively represented by state s. And, of course, the system, while starting at s, need not go to 0 directly. It may experience one or more false alarms first. An example may be:

s-fa-s-0 with probability (1-u)vu and delay 3/2.

With the aid of the previously introduced matrix method one next derives $G_\delta(s)$. Let

$$G = \begin{pmatrix} G_s(s) \\ F(s) \\ G_0(s) \\ G_x(s) \\ G_1(s) \\ \vdots \\ G_\delta(s) \end{pmatrix} \qquad (28)$$

be a (δ + 4) - dimensional vector of unknown gf's. Let A be another (δ + 4) - dimensional vector,

$$A = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{29}$$

The $(\delta + 4) \times (\delta + 4)$ - dimensional transition matrix M is now more elaborate than it was in earlier (9) or (19). From Figure 6 one gets,

$$M = \begin{bmatrix} 1 & -vz & 0 & 0 & 0 & & 0 & 0 \\ -(1-u)z & 1-(1-v)s & 0 & -(1-y)s & 0 & & 0 & 0 \\ -uz & 0 & 1 & -ys & 0 & & 0 & 0 \\ 0 & 0 & -p & 1 & -p & \cdots & -p & 0 \\ 0 & 0 & -q & 0 & 1 & & 0 & 0 \\ 0 & 0 & 0 & 0 & -qs & & 0 & 0 \\ & & \vdots & & & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & & -qs & 1 \end{bmatrix}. \tag{30}$$

The complete inverse M^{-1} can be derived, but it does turn out quite complicated. Fortunately, most of the matrix inverse is really not needed. Since A vanishes in all but the first position, only the first column of M^{-1} is needed to determine vector G. The derivation of this column is still relatively long, but its validity can be checked (as we have done) by substitution into defining equations (10), (28), (29), (30), and $M^{-1} \cdot M = I$.

To simplify, introduce

$$D(s) = 1 + q\,\frac{1-(qs)^{\delta-1}}{1-qs} \tag{31}$$

and claim that the determinant of M is

$$|M| = 1 - (1-uv)s - ps[y-(y-uv)s]D(s). \tag{32}$$

It follows that at $s = 1$ the determinant is uvq^{δ}, which is non-zero for non-vanishing u, v, q, and finite δ.

With the aid of D(s) and $|M|$, the left-hand column of M^{-1} can be expressed as

$$M^{-1} = \frac{1}{|M|}\begin{bmatrix} [1-(1-v)s]\,[1-pys\,D(s)] & \cdots \\ (1-u)z + p(u-y)\,zs\,D(s) & \cdots \\ uz[1-(1-v)s] & \cdots \\ puz[1-(1-v)s]D(s) & \cdots \\ quz[1-(1-v)s] & \cdots \\ q^2uzs[1-(1-v)s] & \cdots \\ \vdots & \\ q^{\delta-1}uzs^{\delta-2}[1-(1-v)s] & \cdots \\ q^{\delta}uzs^{\delta-1}[1-(1-v)s] & \cdots \end{bmatrix}. \tag{33}$$

It follows immediately that the gf for the mean sync acquisition time is

$$G_{\delta}(s) = \frac{1}{|M|}\, uq^{\delta} s^{\delta-\frac{1}{2}}\, [1-(1-v)s]. \tag{34}$$

The derivative with respect to s is a straightforward matter. With the aid of (2) one gets the approximate mean sync acquisition time,

$$T_a(\delta) \approx \delta - \tfrac{1}{2} + \frac{1-y+uv\left(1+q^2\dfrac{1-q^{\delta-1}}{1-q}\right) - (u+\delta uv-y)q^{\delta}}{uvq^{\delta}}. \tag{35}$$

Since the probability of a false alarm in a single bit is relatively small, the quantity v is close to unity. For practical purposes it appears justified to set $v = 1$ and $(u - y)(1 - q^{\delta}) = 0$ in (35). If so, then a simpler form,

$$T_a(\delta) \approx \delta - \tfrac{1}{2} + \frac{1 + u\left(q^2\dfrac{1-q^{\delta-1}}{1-q} - \delta q^{\delta}\right)}{uq^{\delta}} \tag{36}$$

materializes.

We have used both forms (35) and (36) to compute and to plot $T_a(\delta)$ as a function of p_e. The difference between the two approximations appears insignificant, especially in a graphical presentation.

In the ATM cell case, the dependence on p_e and on the frame format is given through:

$$
\begin{aligned}
q &= (1 - p_e)^{40}, \\
p &= 1 - (1 - p_e)^{40}, \\
u &\approx (1 - 2^{-8})^{212}, \\
v &\approx 1 - 2^{-8}, \\
y &\approx (1 - 2^{-8})^{423}.
\end{aligned}
\tag{37}
$$

Curves based on these numbers are shown in Figure 7. Each curve represents the indicated value for threshold parameter δ. Of the three values shown, the middle value $\delta = 6$ has been proposed and conditionally accepted as a starting point for future B-ISDN planning by Study Group XVIII of CCITT (CCITT, 1990).

The left ordinate in Figure 7 is in cell units, each cell assumed to be 424 bits. On the right side, the ordinate shows the waiting times in μs for a transmission rate of 150 MB/s. If, on different links, the actual rate turns out different, then these times must be appropriately scaled. For instance, a B-ISDN rate of 600 Mb/s warrants all times to be divided by 4, while a 50 Mb/s channel requires 3 times larger waiting times.

The ideal error-free channel has $p_e = 0$. But, note that in the limit $p_e = 0$ the sync acquisition time does not vanish. Instead, as seen from equations (35), (36), and Figure 7, the asymptotic limit is

$$
T_a(\delta) \xrightarrow[p_e \to 0]{} \delta - \tfrac{1}{2} + \frac{1 - u}{uv}
\tag{38}
$$

For the numbers given in (37), the error-free channel implies a waiting time of $\delta + .80$ cell units.

Finally, observe that $p_e > 10^{-3}$ places the acquisition process in a non-linear region, where the waiting time starts to grow at an accelerated rate.

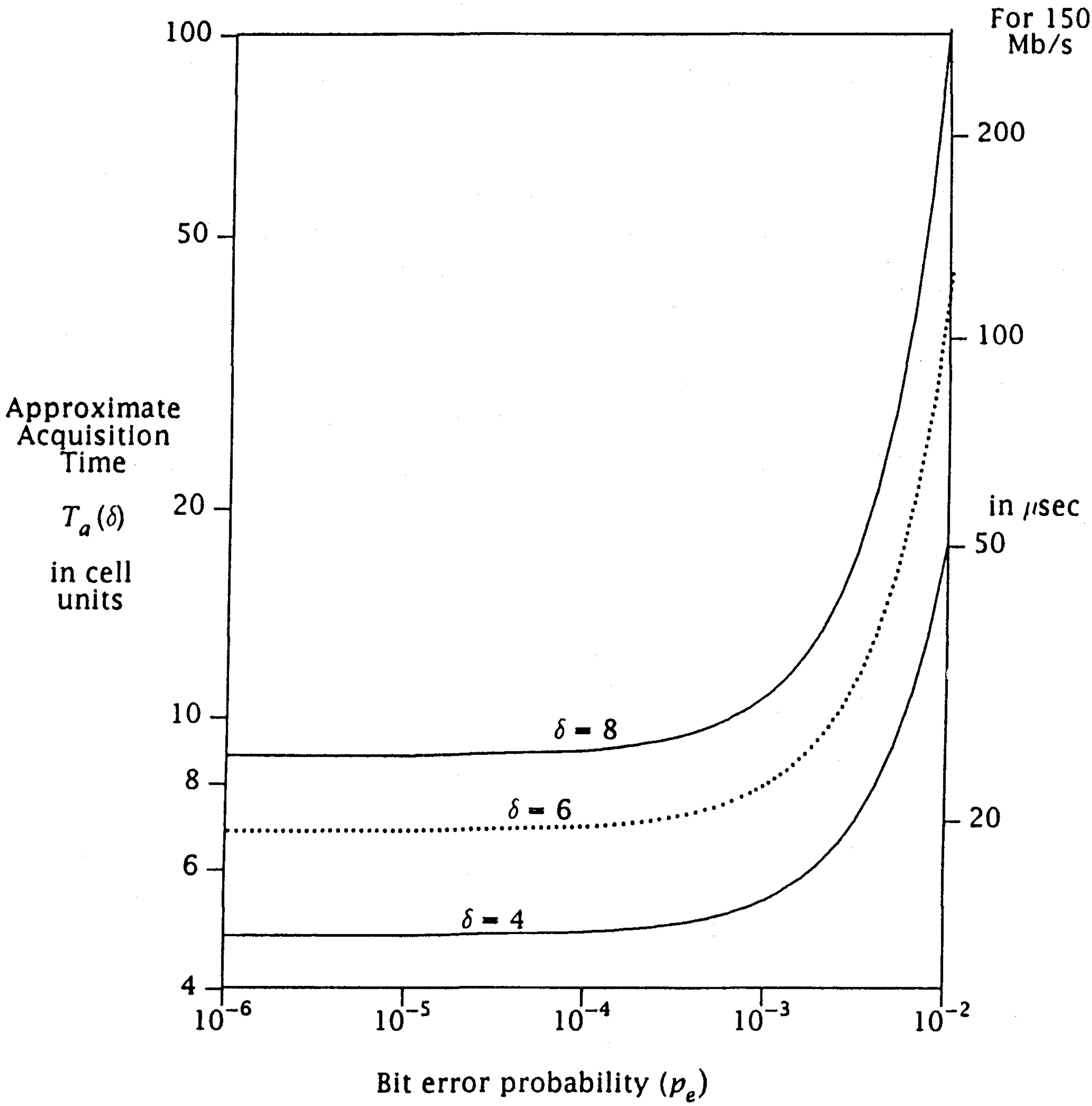

Figure 7. Acquisition time vs bit error rate for the approximate model.

5. MEAN SYNC ACQUISITION TIME: THE BASIC APPROACH

In this section, a detailed and therefore more complex approach is taken to the derivation of the mean acquisition time. Basically, every bit position in an ATM cell is now allowed to be the starting point for the search process. That beginning point is somewhere in the HUNT state, where bit-by-bit scrutiny takes place. Once out of the HUNT state (see Figure 1), only cell-to-cell search is performed in the PRESYNC state.

Consider the bit/cell format of Figure 8. If the acquisition process starts with probability a_k at some bit k (k = 1, 2, . . . , m) within a cell, then subject to HEC $\neq$ 0, the process suffers a bit-to-bit delay z. Since there are m bits in a cell (we shall set m = 424 subsequently), the cell-to-cell delay s must satisfy

$$s = z^m. \tag{39}$$

The initial probability distribution $\{a_m\}$ is quite general. It is only constrained by

$$a_1 + a_2 + \ldots + a_m = 1. \tag{40}$$

For numerical examples, one may start with the uniform distribution, where $a_k = 1/m$ for all k. Or, perhaps, one should not overlook the single-bit sync slip (forward and backward from m) characterized by $a_1 = a_{m-1} = 1/2$, and zero elsewhere.

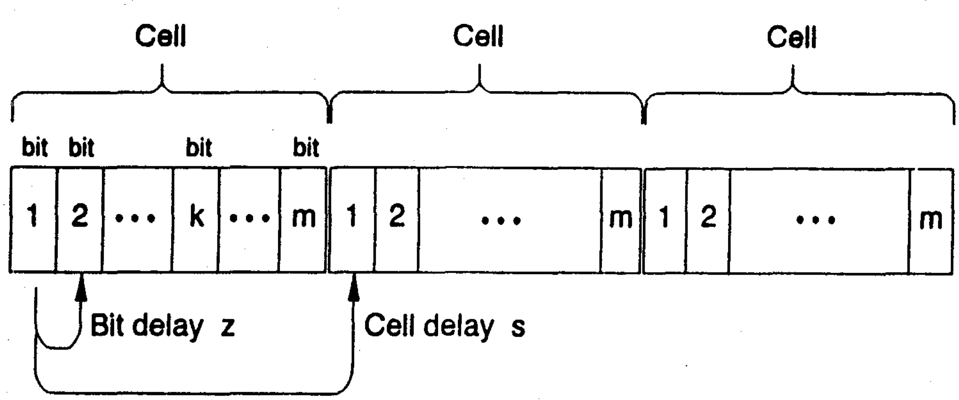

Figure 8. Bit/cell format.

Figure 9 presents the state-transition diagram assumed in this section. The notation is:

s = unit delay operator,

z = 1/m of the unit delay,

p = probability of the incorrect header (HEC $\neq$ 0),

q = probability of the correct header (HEC = 0), which satisfies $p + q = 1$,

v = probability of HEC $\neq$ 0 in a given single bit of a random word,

w = probability of HEC = 0 in a given single bit of random word ($v + w = 1$),

m = number of bits per cell ($m = 424$),

Bk = bit state k ($k = 1, 2, \ldots, m$),

Cn = cell state n ($n = 1, 2, \ldots, \delta$),

δ = sync threshold,

S = sync state,

BC = same as state Bm /C1 (a state that is simultaneously the Bm and the C1 state),

a_k = initial probability of state Bk ($k = 1, 2, \ldots, m$).

As done in earlier sections, one next follows the gf and matrix approach. Let

$$G = \begin{pmatrix} G_{B1}(s) \\ G_{B2}(s) \\ \vdots \\ G_{BC}(s) \\ G_{C2}(s) \\ \vdots \\ G_{C\delta}(s) \\ G_{S}(s) \end{pmatrix} \tag{41}$$

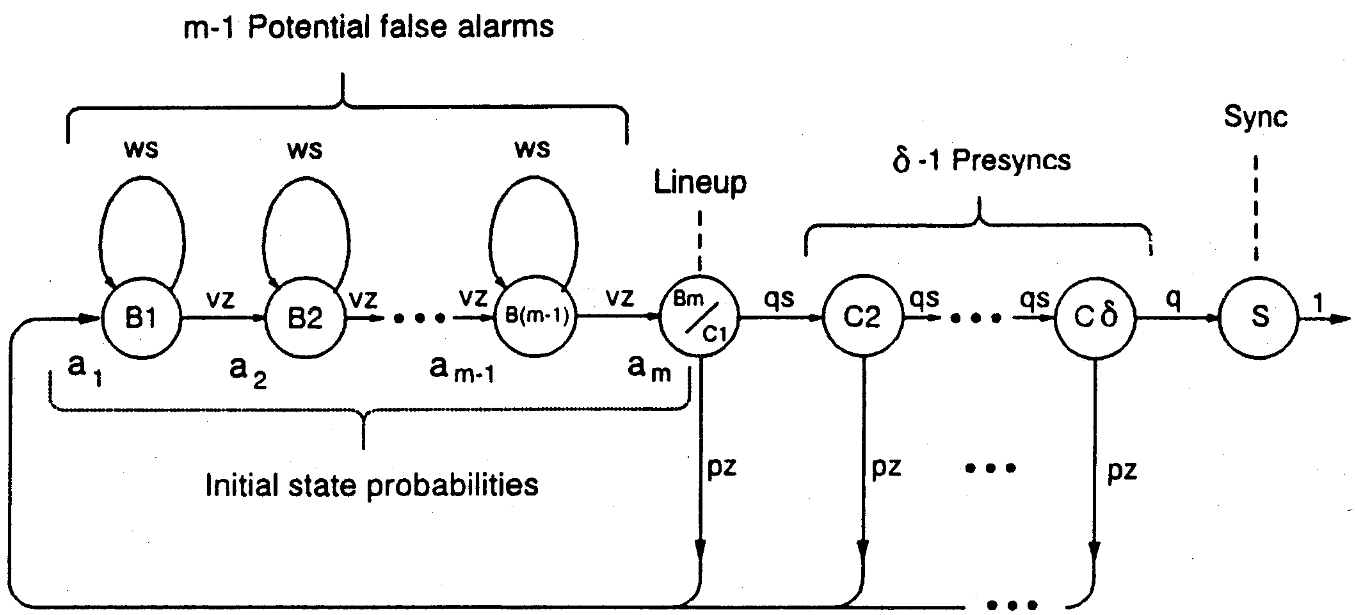

Figure 9. The basic state-transition diagram for cell sync acquisition.

and

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad (42)$$

be two (m + δ) - dimensional vectors. The corresponding matrix M is (m + δ) x (m + δ). Matrix M follows from Figure 9. It is a rather complex matrix that can be expediently partitioned into two submatrices M_1 and M_2, as in

$$M = [M_1 \mid M_2], \qquad (43)$$

where M_1 is the (m + δ) x (m - 1) dimensional part

$$M_1 = \left[\begin{array}{ccccccc} 1-ws & 0 & 0 & & 0 & 0 & 0 \\ -vz & 1-ws & 0 & \cdots & 0 & 0 & 0 \\ 0 & -vz & 1-ws & & 0 & 0 & 0 \\ & \vdots & & & & \vdots & \\ 0 & 0 & 0 & & 0 & -vz & 1-ws \\ 0 & 0 & 0 & \cdots & 0 & 0 & -vz \\ \hline 0 & 0 & 0 & & 0 & 0 & 0 \\ & \vdots & & & & \vdots & \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \end{array}\right] \begin{array}{c} \\ \\ \\ m \\ \\ \\ \hline \\ \delta \\ \\ \end{array} \qquad (44)$$

and M_2 is the (m + δ) x (δ + 1) dimensional remainder

$$M_2 = \left[\begin{array}{cccccccc} -pz & -pz & -pz & & -pz & -pz & 0 \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ & \vdots & & \cdots & & \vdots & \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ 1 & 0 & 0 & & 0 & 0 & 0 \\ \hline -qs & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -qs & 1 & & 0 & 0 & 0 \\ & \vdots & & & & \vdots & \\ 0 & 0 & 0 & \cdots & -qs & 1 & 0 \\ 0 & 0 & 0 & & 0 & -q & 1 \end{array}\right] \begin{array}{c} \\ \\ m \\ \\ \\ -- \\ \\ \delta \\ \\ \\ \end{array} . \tag{45}$$

Unfortunately, derivation of M^{-1} is too long to be repeated here. The validity of the inverse, however, can be rather easily proved by verifying that $M \cdot M^{-1} = I$ holds. Introduce

$$\varphi = \frac{vz}{1 - ws} \tag{46}$$

and

$$\theta = pz \, \frac{1-(qs)^{\delta}}{1-qs} \, \varphi^{m-1} , \tag{47}$$

where φ and θ are abbreviations for often-to-be-used functions of s and z.

The determinant of M is

$$|M| = (1-ws)^{m-1} \, (1-\theta). \tag{48}$$

For s = 1, this determinant equals $v^{m-1}q^{\delta}$. It is clearly larger than zero for non-zero v and q, while m and δ are required to be finite. Thus the inverse M^{-1} again exists. We claim that it can be written as

$$M^{-1} = \frac{1}{1-\theta}\begin{bmatrix} M_1^{-1} \\ \hline M_2^{-1} \end{bmatrix}, \tag{49}$$

where the sub-matrices M^{-1}_1 and M^{-1}_2 are not inverses of M_1 and M_2, respectively, but instead are convenient partitions of M^{-1}. Actually, we shall not present the entire M^{-1}_1 and M^{-1}_2. They are too big and awkward for that. Instead, we present only the first m columns of matrix M^{-1}. The remaining δ columns are superfluous for determination of G.

The first m columns of matrix M^{-1}_1 are given by an (m-1)x m array

$$M_1^{-1} = \frac{1}{vz}\begin{bmatrix} \varphi & \theta & \frac{\theta}{\varphi} & & \frac{\theta}{\varphi^{m-3}} & \frac{\theta}{\varphi^{m-2}} & \cdots \\ \varphi^2 & \varphi & \theta & \cdots & \frac{\theta}{\varphi^{m-4}} & \frac{\theta}{\varphi^{m-3}} & \cdots \\ \varphi^3 & \varphi^2 & \varphi & & \frac{\theta}{\varphi^{m-5}} & \frac{\theta}{\varphi^{m-4}} & \cdots \\ & \vdots & & & & \vdots & \\ \varphi^{m-2} & \varphi^{m-3} & \varphi^{m-4} & \cdots & \theta & \frac{\theta}{\varphi} & \cdots \\ \varphi^{m-1} & \varphi^{m-2} & \varphi^{m-3} & & \varphi & \theta & \cdots \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \\ \leftarrow \textit{row } m-1 \end{matrix} \tag{50}$$

column m ↑

Likewise, the m columns of matrix $M^{-1}{}_2$ constitute a $(\delta + 1) \times m$ dimensional array

$$M_2^{-1} = \begin{bmatrix} \varphi^{m-1} & \varphi^{m-2} & \varphi^{m-3} & & \varphi & 1 & \cdots \\ qs\varphi^{m-1} & qs\varphi^{m-2} & qs\varphi^{m-3} & \cdots & qs\varphi & qs & \cdots \\ (qs)^2\varphi^{m-1} & (qs)^2\varphi^{m-2} & (qs)^2\varphi^{m-3} & & (qs)^2\varphi & (qs)^2 & \cdots \\ & \vdots & & & & \vdots & \\ (qs)^{\delta-1}\varphi^{m-1} & (qs)^{\delta-1}\varphi^{m-2} & (qs)^{\delta-1}\varphi^{m-3} & \cdots & (qs)^{\delta-1}\varphi & (qs)^{\delta-1} & \cdots \\ q(qs)^{\delta-1}\varphi^{m-1} & q(qs)^{\delta-1}\varphi^{m-2} & q(qs)^{\delta-1}\varphi^{m-3} & & q(qs)^{\delta-1}\varphi & q(qs)^{\delta-1} & \cdots \end{bmatrix} \text{row } \delta + 1 \quad . \tag{51}$$

column m ↑

Substitution of (46) through (51), as well as (42), into (11) yields the entire G vector (defined in equation (41)). For our purposes, the only entity that really matters is the last component $G_s(s)$:

$$G_s(s) = \frac{q^\delta s^{\delta-1}}{1-\theta} \sum_{k=1}^{m} a_k \varphi^{m-k} \quad . \tag{52}$$

The derivation of the mean sync acquisition time is next. Set $s = 1$ and note that

$$\begin{aligned} \varphi(1) &= 1, \\ \theta(1) &= 1 - q^\delta, \\ G_s(1) &= 1. \end{aligned} \tag{53}$$

Further,

$$\begin{aligned} \varphi'(1) &= \frac{1}{m} + \frac{w}{v}, \\ \theta'(1) &= -\delta q^\delta + (1 - q^\delta)\left(\frac{1}{1-q} + (m-1)\frac{w}{v}\right), \end{aligned} \tag{54}$$

which, in the derivative

$$G_s'(s) = \left(\frac{\theta'}{1-\theta} + \frac{\delta-1}{s}\right) G_s(s) + \frac{\varphi'}{\varphi} \frac{q^\delta s^{\delta-1}}{1-\theta} \sum_{k=1}^{m-1} (m-k)\, a_k\, \varphi^{m-k} \tag{55}$$

produces the desired mean acquisition time:

$$T_a(\delta) = \left(\frac{1}{m} + \frac{w}{v}\right) \sum_{k=1}^{m-1} k\, a_{m-k} + \frac{(1-q^\delta)\left(1 + (m-1)\frac{pw}{v}\right)}{(1-q)\, q^\delta} - 1. \tag{56}$$

One can view this expression (56) for the mean sync acquisition time as the primary objective of the last two sections. It has a number of limit properties that are, at least intuitively, quite acceptable. For instance, as p_e tends to 1 and q tends to 0, $T_a(\delta)$ tends to infinity. At the other extreme, as p_e tends to 0 and q tends to 1, the acquisition wait becomes

$$T_a(\delta) = \delta - 1 + \left(\frac{1}{m} + \frac{w}{v}\right) \sum_{k=1}^{m-1} k\, a_{m-k}. \tag{57}$$

Let us consider three different initial probability distributions $\{a_k\}$. Let

$$\begin{aligned}
&\text{(i)}\ a_k = 1/m && \text{for all } k = 1, 2, \ldots, m, \\
&\text{(ii)}\ a_k = 1 && \text{for } k = m/2, \\
&\phantom{\text{(ii)}\ a_k} = 0 && \text{otherwise}, \\
&\text{(iii)}\ a_k = 1/2 && \text{for } k = 1 \text{ and } m-1, \\
&\phantom{\text{(iii)}\ a_k} = 0 && \text{otherwise}.
\end{aligned} \tag{58}$$

Case (i) represents the uniformly distributed initial condition (IC), where the HUNT mechanism must face every bit in the m-bit span as equally likely to be the true cell beginning. Case (ii) assumes that the search process starts exactly in the middle of a cell, while case (iii) postulates an equally likely single-bit synchronization slip in either direction, +1 or -1 bit.

The mean acquisition times for these three cases follow immediately from (56),

$$T_a(\delta)|_{(i)} = \left(\frac{1}{m} + \frac{w}{v}\right)\frac{m-1}{2} - 1 + \frac{(1-q^\delta)\left(1 + (m-1)\dfrac{pw}{v}\right)}{(1-q)q^\delta} \tag{59}$$

and

$$T_a(\delta)|_{(ii)} = T_a(\delta)|_{(iii)} = T_a(\delta)|_{(i)} + \tfrac{1}{2}\left(\frac{1}{m} + \frac{w}{v}\right). \tag{60}$$

Hence, the uniform IC (i) yields a waiting time that is $\frac{1}{2}\left(\frac{1}{m} + \frac{w}{v}\right) = .003$ cell units shorter than the identical waits (ii) and (iii). For the assumed ATM cell scenario, the quantity $T_a(\delta)|_{(i)}$ is plotted in Figure 10 versus bit error probability. For comparison with the approximate model of Figure 7, the same coordinate axes and the same threshold parameter values for δ are used in Figure 10.

One observes that in this exact model, as p_e tends to zero and q to unity, the asymptotic mean waiting time tends to

$$T_a(\delta)|_{(i)} = \delta - 1 + \left(\frac{1}{m} + \frac{w}{v}\right)\frac{m-1}{2} \approx \delta + .328. \tag{61}$$

The fact that this mean delay is nearly 0.5 cell units lower than the estimate made in the section 4, where an approximate model was postulated, leads to two comments. First, the difference could be due to the imprecise handling of the false alarm states in Figure 6. And second, the approximate model produces an upper-bound for the mean acquisition delay.

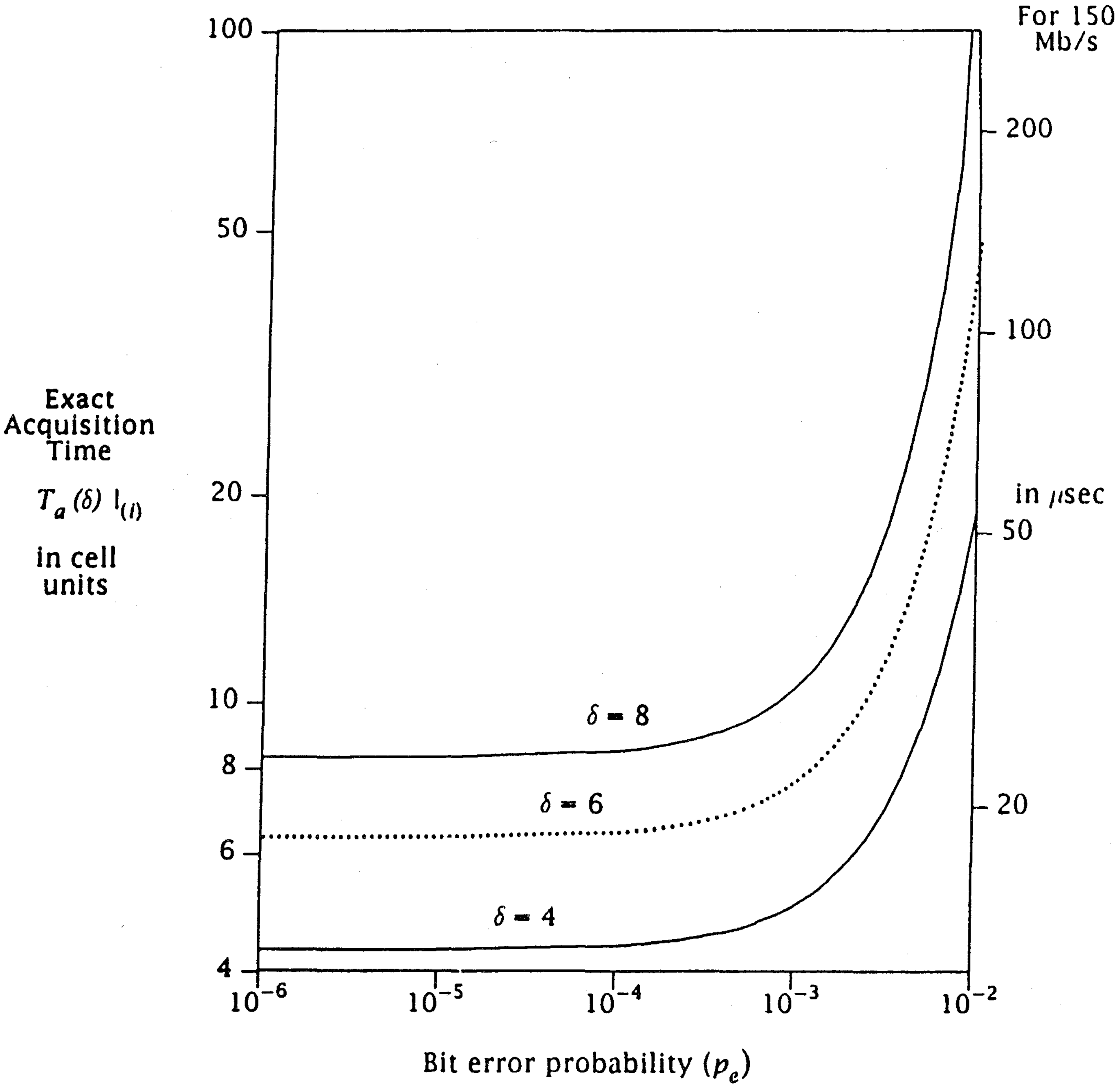

Figure 10. Acquisition time vs bit error rate in the basic model (case (i) of the initial probability distribution).

6. VARIANCES

Mean or average values are of primary interest for random synchronization times, as they are for most random variables. However, occasionally need may arise for values of their variances. Given a probability gf G(s), the variance σ^2 follows from the relationship (Feller, 1968).

$$\sigma^2 = G''(1) - [G'(1)]^2 + G'(1). \qquad (62)$$

For in-sync duration time, the gf was given earlier in (22). The corresponding variance in units of $(\text{cell})^2$ is then

$$\sigma_d^2 = \frac{2}{qp^{\alpha}}\left[T_d(\alpha) - \alpha + \sum_{j=0}^{\sigma-1} j\, a_j\, p^{\alpha-j}\right] - T_d^2(\alpha) - T_d(\alpha), \qquad (63)$$

where the mean duration time $T_d(\alpha)$ is as defined in equation (23). Entity σ_d^2 becomes unbounded as q tends to 1, and p to 0. At the other extreme, namely at $q = 0$ and $p = 1$, σ_d^2 assumes its lowest value, which is

$$\sigma_d^2\,\Big|_{q=0} = \sum_{j=0}^{\alpha-1} (\alpha-j)^2\, a_j - \left[\sum_{j=0}^{\alpha-1} (\alpha-j)\, a_j\right]^2. \qquad (64)$$

Thus, $\sigma_d^2 = 0$ occurs when both $q = 0$ and $a_j = 1$ for some value of j, $0 \le j \le \alpha - 1$.

Variances can also be computed for both the approximate and the exact sync acquisition times. The relevant generating functions are given in equations (34) and (52) respectively. Both cases lead to quite lengthy and complex expressions. Because of its imprecise character, the variance around an approximate mean seems to be of least interest. It is not included here.

We next present the variance σ_a^2, which is valid for the exact, or basic, sync acquisition model. We assume the additional condition that $p \ll 1$. As defined earlier, p represents the probability of a HEC $\neq$ 0 event. Equation (37) shows that $p \ll 1$ is equivalent to the channel error probability p_e being much less than 1/40. For practical purposes this implies that the σ_a^2 expression, to be given next, must be valid at least for all sufficiently small p_e.

Application of equation (62) to the acquisition gf of (52) yields for $p \ll 1$

$$\sigma_a^2 = T_a(\delta)\left[2\delta - 1 + \delta\left(\delta + 1 + (m-1)\frac{w}{v}\right)p\right] - T_a^2(\delta)$$
$$+\left(-\frac{1}{m} + \frac{w^2}{v^2}\right)\sum_{k=1}^{m}(m-k)\,a_k + \left(\frac{1}{m} + \frac{w}{v}\right)^2\sum_{k=1}^{m}(m-k)^2\,a_k \qquad (65)$$
$$-\delta(\delta-1)\,[1 + (3\delta+4)\,p] - \delta(m-1)\left[7\delta - 4 - (m-2)\frac{w}{v}\right]\frac{w}{v}\,p,$$

where $T_a(\delta)$ is the mean acquisition time of equation (56) and of Figure 10.

When the channel is error-free, $p = p_e = 0$, then this variance becomes

$$\sigma_a^2 = \left(\frac{1}{m} + \frac{w}{v}\right)^2\left[\sum_{k=1}^{m}(m-k)^2 a_k - \left(\sum_{k=1}^{m}(m-k)\,a_k\right)^2\right] + \frac{w}{v}\left(1 + \frac{w}{v}\right)\sum_{k=1}^{m}(m-k)\,a_k\,. \qquad (66)$$

Even if the search process is known to start from a fixed bit position, the variance of (66) does not vanish. Instead, given $a_k = \delta_{k,j}$, one obtains for $p = 0$

$$\sigma_a^2 = \frac{w}{v}\left(1 + \frac{w}{v}\right)(m-j). \qquad (67)$$

Here, entity $\frac{w}{v}\left(1 + \frac{w}{v}\right)$ represents the "potential false alarm" contribution from a single bit in the bit-by-bit search of the HUNT state (compare earlier Figures 1, 8, and 9). For the system numbers assumed, see (37), $\frac{w}{v}\left(1 + \frac{w}{v}\right) \approx .0039$.

Although this may look negligible at first, its contribution should not be ignored. The number of bits in an ATM cell, m=424, reduces (67) to $\sigma_a^2 \approx 1.669 - .004\,j$, where $1 \le j \le m$. If one chooses to ignore this effect of a false alarm due to a single random bit, the expected underestimate of the standard deviation σ_a is apt to be on the order of one ATM cell.

7. CONCLUSION

The methods of probability generating functions, also known as z-transforms, and matrix inversions have been used to compute the mean synchronization times for the Asynchronous Transfer Mode (ATM). First, the mean in-sync duration characterizes the time that the system is expected to spend in cell synchronism, given that a valid ATM cell exchange takes place throughout the period. Second, the mean sync acquisition time is the average wait necessary to achieve ATM cell synchronization, given that the search starts in a state of ignorance about the true epochs of cell demarcation.

There are two results presented for the mean acquisition time. To start, a simpler approximation is derived first in section 4. That is followed in section 5 by a more complex, but exact, acquisition characterization. Comparison of the two results shows that the simpler approximation represents an upper bound on the average waiting time for cell acquisition.

Future studies should answer questions about probabilities of both desired and undesired events in ATM cell synchronization. For specified values of time, T, these could be probabilities that synchronization is abandoned at T, given that the signal is either present or absent. Or one may be interested in probabilities of the system declaring sync acquisition at time T, conditioned again either on signal presence or its absence. The latter, known as false lock or false alarm probability, is of interest in several applications. The probability generating functions are also expected to be useful in the analysis of such probabilities.

The synchronization times derived here should become of increased concern and significance, if and when the Asynchronous (instead of the Synchronous) Transfer Mode ever becomes accepted internationally or nationally as the main transport mechanism for broadband ISDN.

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FORM NTIA-29
(4-80)

U.S. DEPARTMENT OF COMMERCE
NAT'L. TELECOMMUNICATIONS AND INFORMATION ADMINISTRATION

BIBLIOGRAPHIC DATA SHEET

	1. PUBLICATION NO.	2. Gov't Accession No.	3. Recipient's Accession No.

4. TITLE AND SUBTITLE

Mean Synchronization Times for ATM Cells: Derivations and Computational Background

5. Publication Date

6. Performing Organization Code

NTIA/ITS.N1

7. AUTHOR(S)

Martin Nesenbergs and Darren L. Smith

8. PERFORMING ORGANIZATION NAME AND ADDRESS

National Telecommunications and Information Admin.
Institute for Telecommunication Sciences
325 Broadway
Boulder, CO 80303

9. Project/Task/Work Unit No.

10. Contract/Grant No.

11. Sponsoring Organization Name and Address

National Telecommunications and Information Admin.
Herbert C. Hoover Bldg.
14th & Constitution Ave., NW
Washington, DC 20230

12. Type of Report and Period Covered

13.

14. SUPPLEMENTARY NOTES

15. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

The mean synchronization (sync) duration time and the mean sync acquisition times are derived and computed for the Asynchronous Transfer Mode of the proposed Broadband ISDN. These waiting times are expressed as functions of channel quality, i.e., the bit error probability and related event probabilities, as a function of two system sync thresholds.

16. Key Words *(Alphabetical order, separated by semicolons)*

acquisition time; asynchronous transfer mode (ATM); broadband ISDN; cell; header error control (HEC); in-sync duration time; synchronization

17. AVAILABILITY STATEMENT	18. Security Class. *(This report)*	20. Number of pages
☒ UNLIMITED.	Unclassified	38
☐ FOR OFFICIAL DISTRIBUTION.	19. Security Class. *(This page)* Unclassified	21. Price: